Electroweak Interaction

**Weak Interaction before gauge theory** (Fermi 1934)

Neutron $\beta$-decay: $n \rightarrow p + e^- + \bar{\nu}_e$

Fermi proposed the following Lagrangian:

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \left[ \bar{\nu}_e \gamma^\mu (1 - \gamma^5) e + \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu \right] + h.c$$

$G_F \approx \frac{10^{-5}}{M_W^2}$

1956 Parity violation (Lee and Yang)

**V-A theory**

$$\mathcal{L}^{\text{eff}} = \frac{G_F}{\sqrt{2}} J^\mu_i \tilde{J}^\mu_i + h.c.$$  

$$J^\mu_i (x) = J_{\lambda i} (x) + J_{\lambda i} (x)$$

leptonic current:  

$$J^{\lambda e}_i (x) = \bar{e}_i \gamma^\mu (1 - \gamma^5) e_i + \bar{\nu}_\mu \gamma^\mu (1 - \gamma^5) \mu_i$$

hadronic current:  

$$J^{A i}_i (x) = \bar{u} \gamma^\mu (1 - \gamma^5) (c \delta_{i2} + s \delta_{i3})$$

$\theta_c$: Cabibbo angle.

Note that in V-A form the fermion fields are all left-handed.

$$\nu_e = \frac{1}{2} (1 - \gamma^5) \psi$$

$$J^\lambda_e = 2 \bar{\nu} \gamma^\mu e + 2 \bar{\nu}_\mu \gamma^\mu \mu$$

Phenomenologically, the V-A theory has been quite successful in most of the weak interactions phenomena.

**Difficulties:**

1) Not renormalizable

4-fermion interaction operator has dimension 6

E.g. $\mu$-decay:

$$\mathcal{L} = G_F \bar{\nu}_e \gamma^\mu (1 - \gamma^5) e + \frac{G_F^2}{2} \bar{\nu}_e \gamma^\mu \gamma^\nu (1 - \gamma^5) \nu_e + \frac{G_F^2}{2} \bar{\nu}_\mu \gamma^\mu \gamma^\nu (1 - \gamma^5) \mu$$

2) Violate unitarity

Tree amplitude for $\nu_e e \rightarrow \mu e$ has only $J = 1$ partial wave at high energies and cross section

$$\sigma (\nu_e e) \sim G_F^2 S$$

$$S = 2m_e E$$

Unitarity for $J = 1$ cross section: $\sigma (J = 1) < \frac{1}{S}$

Thus $\sigma (\nu_e e)$ violates unitarity for $E \sim 300$ GeV

**Intermediate Boson Theory (IVB)**

In analogy with QED, we can introduce vector boson $W$ to couple to the $V-A$ current,
\[ L = g (J_\mu W^\mu + h.c) \]

For example, the \( \mu \) decay is now mediated by \( W \)-exchange:

\[
\begin{array}{c}
\mu^+ \\
\downarrow \\
W^+ \\
\uparrow \\
\nu_e \\
\end{array}
\quad \begin{array}{c}
\rightarrow \\
\wedge \\
\rightarrow \\
\wedge \\
\nu \bar{\nu} \\
\end{array}
\begin{array}{c}
e^+ \\
\uparrow \\
W^-
\end{array}
\]

Since weak interaction is short-range, we need \( W \)-boson to be massive \( M_w \) to \( W \)-boson propagator:

\[
\frac{-g^{\mu\nu} k^\mu k^\nu}{M_w^2} \rightarrow \frac{\pm g^{\mu\nu}}{M_w^2} \quad \text{when} \quad |k_\mu| \ll M_w
\]

This reproduces 4-fermion interaction with \( \frac{g^2}{M_w} = \frac{g_F}{\sqrt{2}} \).

In this theory, the scattering \( \nu_e \rightarrow \mu^+ \) no longer violates unitarity. But the violation of unitarity shows up in other processes like:

\[
\nu + \bar{\nu} \rightarrow W^+ W^-
\]

and the theory is still non-renormalizable.
3) Choice of group

In \( \text{IVB theory} \):
\[
S_{\text{IVB}} = \frac{g}{2} \mathcal{J}_{\mu} W^{\mu} + \text{h.c.}
\]

For simplicity we neglect all other fermions except \( \nu, e \Rightarrow \)
\[
\mathcal{J}_{\mu} = \bar{\nu} Y_{\mu} (\nu - \bar{\nu}) e
\]

Recall that in the electromagnetic interaction, we have
\[
\mathcal{L}_{\text{em}} = e \bar{\nu} \gamma_{\mu} \nu, \quad \mathcal{J}_{\mu}^{\text{em}} = e \gamma_{\mu} e
\]

Define the em and weak charges as
\[
T_{+} = \frac{1}{2} \int d^3 \mathbf{x} J_{\mu}^{\text{em}}(\nu - \bar{\nu}) e, \quad T_{-} = (T_{+})^{\dagger}
\]
\[
Q = \frac{1}{4} \int d^3 \mathbf{x} \left( \mathcal{J}_{\mu}^{\text{em}}(\nu - \bar{\nu}) e \right) = \int d^3 \mathbf{x} e e
\]

We can compute the commutator
\[
[T_{+}, T_{-}] = 2 T_{3}
\]
\[
T_{3} = \frac{1}{2} \int d^3 \mathbf{x} \left[ \mathcal{J}_{\mu}^{\text{em}}(\nu - \bar{\nu}) e \right] = Q
\]

This means that \( T_{+}, T_{-}, \) and \( Q \) do not form an \( SU(2) \) algebra.

We need to include \( T_{3} \) as part of generators, i.e. \( T_{+}, T_{-}, T_{3}, Q \). This leads to the gauge group

\( SU(2) \times U(1) \)

The Lagrangian for the gauge fields is then
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} G_{\mu\nu} G^{\mu\nu}
\]

where
\[
F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + g e^{ij} A_{\mu}^{i} A_{\nu}^{j}
\]
\[
G_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}
\]

Fermions:

Clearly, \( \nu, e \) form a doublet under \( SU(2) \)
\[
\psi = \begin{pmatrix} \nu_e \\ \bar{e}_\nu \end{pmatrix}
\]

For convenience, we introduce left-handed and right-handed fields
\[
\psi_L = \frac{1}{2} (\nu - \bar{\nu}) \psi, \quad \psi_R = \frac{1}{2} (\nu + \bar{\nu}) \psi \Rightarrow \psi = \psi_L + \psi_R
\]

We can write
\[
T_{+} = \int (\psi_L^\dagger e_e) d^3 \mathbf{x}, \quad T_{-} = \int (e_L^\dagger \psi_L) d^3 \mathbf{x}, \quad Q = \int (e_L^\dagger e_L + e_R^\dagger e_R) d^3 \mathbf{x}
\]

Note that
\[
Q - T_3 = \int \left[ -\frac{1}{2} (\nu_L^\dagger \nu_L + \bar{e}_e^\dagger \bar{e}_e) - e_R^\dagger e_R \right] d^3 \mathbf{x}
\]

It is straightforward to show that
\[ [Q - T_3, T_3] = 0, \quad i = 1, 2, 3 \]

Thus we can take \((Q - T_3)\) to be the \(U(1)\) charge

\[ Y = 2(Q - T_3) \]

The \(Y\) charges for fermions are

\[
\begin{pmatrix}
Y_L \\
Y_R
\end{pmatrix}, \quad e_R, \quad \gamma = -2
\]

The Lagrangian for gauge coupling

\[ \mathcal{L}_2 = \bar{\Psi} \gamma^\mu D_\mu \Psi = \bar{\Psi} \gamma^\mu \partial_\mu \Psi + \bar{\Psi} \gamma^\mu \partial_\mu \psi \]

where

\[ D_\mu \psi = (\partial_\mu - i g' T^A \frac{A_\mu^A}{2} + i g B_\mu) \psi \]

\[ e_R \quad D_\mu \psi = (\partial_\mu - i g T^A \frac{A_\mu^A}{2} + i g' B_\mu) \psi \]

2) Spontaneous Symmetry Breaking

The symmetry breaking pattern we want is

\[ SU(2) \times U(1) \rightarrow U(1)_{\text{em}} \]

Choose scalar fields in \(SU(2)\) doublet

\[ \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad Y = 1 \]

The Lagrangian containing \(\phi\)

\[ \mathcal{L}_3 = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - V(\phi) \]

\[ D_\mu \phi = (\partial_\mu - i g T^A \frac{A_\mu^A}{2} - i g' B_\mu) \phi \]

\[ V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \]

\[ \mathcal{L}_4 = \int \bar{\phi} \phi \, e_\pm \, h.c. \]

As we have seen before, the spontaneous symmetry breaking is generated by the vacuum expectation value

\[ \langle \phi \rangle_0 \equiv \langle \phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix}, \quad V = \sqrt{\frac{\lambda}{\mu^2}} \]

Write the scalar field in the form,

\[ \phi(\omega) = U(\frac{\omega}{\nu}) \begin{pmatrix} 0 \\ \nu \end{pmatrix} \]

where

\[ U(\frac{\omega}{\nu}) = \exp \left[ \frac{i \bar{\phi} \phi}{\nu} \right] \]

Gauge transformation

\[ \phi' = U(\frac{\omega}{\nu}) \phi \]

\[ \bar{\phi}' = U(\frac{\omega}{\nu})(\bar{\phi} + \nu \phi) \]

\[ \bar{\phi}' \bar{\phi}' = U(\frac{\omega}{\nu})(\bar{\phi} + \nu \phi) U(\frac{\omega}{\nu}) U^* \]

\[ \bar{\phi}' \bar{\phi}' = \frac{\nu}{\nu + \nu \phi} \]

\[ \bar{\phi}' \bar{\phi}' : \text{disappear from the Lagrangian}, \]

From \(\mathcal{L}_4\) (Yukawa coupling), YEV gives
From $L_4$ (Yukawa coupling), $\nu$ gives

$$ L_4 = \frac{g}{\sqrt{2}} \langle \phi \rangle (\bar{e} \gamma^\mu (e_c + h.c.) + \bar{\nu} \gamma^\mu (\bar{e} \gamma^\mu + h.c.) ) $$

electron mass $m_e = f <\phi>$

**Mass spectrum**

1) Fermion mass $m_e = \frac{\rho \nu}{\sqrt{2}}$

2) Scalar mass ($Higgs$)

$$ V(\phi') = \frac{\lambda^2}{2} \phi^2 + \frac{3}{2} \lambda' \phi^4 \quad \Rightarrow \quad m_\phi = \sqrt{\frac{2}{3}} \mu $$

3) Gauge boson masses

From the covariant derivative in $\xi_3$

$$ \xi_3 = \frac{1}{\sqrt{2}} \chi^\mu \left( \frac{g}{\sqrt{2}} A_\mu + \frac{g'}{2} B_\mu \right) \left( \frac{g}{\sqrt{2}} A^\mu + \frac{g'}{2} B^\mu \right) \chi $$

$$ = \frac{\sqrt{2}}{\sqrt{2}} \left( \frac{g}{\sqrt{2}} A_\mu + \frac{g'}{2} B_\mu \right)^2 + \left( \frac{g}{\sqrt{2}} A^\mu + \frac{g'}{2} B^\mu \right)^2 

= \frac{M_\xi^2}{\sqrt{2}} W^\pm W^\mp + \frac{1}{2} M_2^2 Z^\mu Z^\mu $$

where $W^\pm \mu = \frac{1}{\sqrt{2}} \left( A_\mu - i A_\mu^0 \right)$, $M_2^2 = \frac{g^2 v^2}{4}$, $Z_\mu = \frac{1}{\sqrt{2}} \left( \frac{g}{\sqrt{2}} A_\mu + \frac{g'}{2} B_\mu \right)$, $M_2^2 = \left( \frac{g^2 + g'^2}{4} \right) v^2$

$$ A_\mu = \frac{1}{\sqrt{2} g'} \left( g' A^3 + g B_\mu \right) \quad \text{photons massless} $$

Define $\tan \theta_W = \frac{g'}{g}$

then we can write

$$ Z_\mu = \cos \theta_W A_\mu^0 - \sin \theta_W B_\mu$$

$$ A_\mu = \sin \theta_W A_\mu^0 + \cos \theta_W B_\mu$$

Note that $\mu = \frac{M_\xi^2}{\sqrt{2}} \cos^2 \theta_W = 1$

**Charged current**

$$ L_{cc} = \frac{g}{\sqrt{2}} \left( J^\mu_+ W^\pm_\mu + h.c. \right)$$

$$ J^\pm_\mu = j^\mu_+ + j^\mu_- = \frac{i}{2} \bar{\nu} \gamma^\mu (\bar{e} - e) e $$

Again to get 4-fermion interaction as low energy limit, we require

$$ \frac{g^2}{8 M_\nu^2} = \frac{g^2}{\sqrt{2}} \Rightarrow \frac{g^2}{8 M_\nu^2} = \sqrt{\frac{2}{\sqrt{2}}} = 2 \sqrt{2} \text{ GeV} $$

**Neutral Current**

$$ L_{nc} = g J^\mu_+ A^\mu_+ + g J^\mu_+ B^\mu_+ = e J^\mu_+ A^\mu_+ + \frac{g}{\sin \theta_W} J^\mu_+ Z^\mu_+ $$

Where $e = e_\sin \theta_W$

$$ J^\mu_0 = J^\mu_+ - \sin \theta_W J^\mu_- $$

WK Int Page 5
\[ L_{\text{NC}} = g J^3 \mathcal{A}^3 + 2' J^Y B^Y = e J^e A^e + \frac{g}{\cos^2 \theta_w} J^Z Z^Z \]

where \( e = g \sin \theta_w \)

\[ J^Z = J^3 - \sin \theta_w J^e \]

\( J^e \) weak neutral current
Neutral current

Neutral current interaction
\[ \sigma_W = \frac{1}{2} J^\mu \, Z^\mu \]
where
\[ J^\mu = \bar{f} \gamma^\mu f \]
weak neutral current
\[ Q^2 = \int \frac{4}{3} T_{\mu} = (T_{1} - \sin^2 \theta_W) \]

This means that the coupling strength of fermions to Z-boson is proportional to the
quantum number \( (T_{1} - \sin^2 \theta_W) \).

In particular, Z-boson can contribute to the scattering
\[ e^+ e^- \rightarrow \mu^+ \mu^- \]

The measurement of the cross section in the 1970's gives
\[ \sin^2 \theta_W = 0.22 \]

This yields
\[ M_W \approx 80 \text{ GeV} \quad M_Z \approx 90 \text{ GeV} \]

Generalization to more than one family

From 4-fermion and ZV theory, the weak current of leptons and hadrons give
the following multiplets,
\[ (\nu_e), (\mu_e), (\tau_e), \]
\[ (\mu_d), (\mu_u), (\tau_d), (\tau_u), \]

where
\[ d = d^0_d, d + s \]

the neutral current in the down quark sector is of the form
\[ \bar{e}_e \left[ -\frac{1}{2} + \sin^2 \theta_W \right] \bar{d}^0_d - \sin^2 \theta_W \bar{d}_s \left( \bar{d}_s \gamma_5 d_s + \frac{1}{3} \gamma_5 \gamma_4 d_s \right) \]

\[ (\frac{1}{3} + \sin^2 \theta_W) \left( \bar{d}_s \gamma_5 d_s \right) + \sin^2 \theta_W \bar{d}_s \left( \bar{d}_s \gamma_5 d_s + \frac{1}{3} \gamma_5 \gamma_4 d_s \right) \]
gives rise to \( AS = 1 \) neutral current processes e.g. \( \tau_+ \rightarrow \mu^+ \mu^- \)
with some order of magnitude as charged

current

Exp.
\[ R = \frac{\Gamma(K_e \rightarrow \mu^+ \mu^-)}{\Gamma(K^+ \rightarrow \mu^+ \nu)} \leq 10^{-3} \]

Thus we can not have \( AS = 1 \) neutral current process at the same order of magnitude
as the charged current process.

GIM

Glashow, Iliopoulos and Maiani (1970) suggested the existence of 4-th quark,
the charm quark \( c \), which couples to the orthogonal combination \( \overline{S_0} = -\frac{1}{2} d + \frac{1}{2} u \)

\[ (\frac{1}{2} - \frac{1}{2} \sin^2 \theta_W) \overline{d}_s \left( \bar{d}_s \gamma_5 d_s \right) \]

As a result, the \( AS = 3 \), neutral current is canceled out,
\[ \overline{d}_s \left( \frac{1}{2} + \frac{1}{2} \sin^2 \theta_W \right) \overline{d}_s + \overline{d}_s \left( \frac{1}{2} - \frac{1}{2} \sin^2 \theta_W \right) \overline{d}_s \]

\[ = \left( \frac{1}{2} + \frac{1}{2} \sin^2 \theta_W \right) (\bar{d}_s \gamma_5 d_s) \]
Quark mixing

Before the spontaneous symmetry breaking, fermions are all massless because $\frac{V}{h}$ and $\frac{V}{Q}$ have different quantum numbers. So the mass term ($\frac{V}{h}$, $\frac{V}{Q}$) is not invariant under SU(2) x U(1) groups. When we have more than one doublets $\frac{V}{h}$, $\frac{V}{Q}$ all have definite quantum numbers with respect to SU(2) x U(1) group, then they are called "weak eigenstates." When spontaneous symmetry breaking takes place, fermions obtain their masses through Yukawa coupling.

$$y \phi \left( \frac{V}{h} \phi, \frac{V}{Q} \phi + f \frac{V}{Q} \phi \delta \phi \right) + h.c.$$ Since the Yukawa coupling constants are arbitrary, the fermion mass matrices are in general not diagonal. When mass matrices are diagonalized, we obtain the mass eigenstates which are not the same as the weak eigenstates. The mass matrices in the up and down sectors are given by

$$m_{ij}^{(u)} = f_{ij}^u = \left( \frac{1}{\sqrt{2}} \right), \quad m_{ij}^{(d)} = f_{ij}^d = \frac{1}{\sqrt{2}}$$

These matrices which are sandwiched between left and right handed fields can be diagonalized by bi-unitary transformations, i.e. given $m_{ij}$, a unitary matrices $S$ and $T$ such that

$$S^* M T = M_D$$

is diagonal. Basically, $S$ is the unitary matrix which diagonalizes the hermitian matrix $m_{ij}$, i.e.

$$S^* (m_{ij}^*) S = m_D^{ij}$$

If we write the left-handed doublets, (weak eigenstates) as

$$\phi_{u} = (d^u)_{L}, \quad \phi_{d} = (s^u)_{L}$$

These weak eigenstates are related to mass eigenstates by unitary transformations,

$$\begin{align*}
(d^u) &= S (d^u) \\
(s^u) &= S (s^d)
\end{align*}$$

Note that in the coupling to charged gauge boson $W^\pm$, we have

$$W^\pm = \sqrt{\frac{2}{3}} \nu \left[ \bar{d} \gamma^\mu \gamma^5 \begin{pmatrix} \nu \end{pmatrix} + \bar{d} \gamma^\mu \gamma^5 \begin{pmatrix} \nu \end{pmatrix} \right] + h.c.$$ and is invariant under unitary transformation in $\bar{d}_L, d_L$ space, i.e.

$$W^\pm V = V^* W^\pm = T V = V' = 1$$

We can use this feature to put all the mixing in the down quark sector.

$$\begin{align*}
\phi_{d} &= (d^u)_{L}, \quad (s^u)_{L} \\
\phi_{d} &= U (d^u)_{L}, \quad (s^u)_{L}
\end{align*}$$

Clearly, we can extend this to 3 generations with result

$$\begin{align*}
\phi_{d} &= (d^u)_{L}, \quad (s^u)_{L}, \quad (u^u)_{L} \\
\phi_{d} &= U (d^u)_{L}, \quad (s^u)_{L}, \quad (u^u)_{L}
\end{align*}$$

CP violation Phase

CP violation can come from complex coupling to gauge bosons. The gauge coupling of $W^\pm$ to quarks is governed by the 3x3 unitary matrix $U$.

discussed above. This unitary matrix $U$ can have many complex entries. However, in diagonalizing the mass mass,
There is ambiguity in the matrix $S$ in the form of diagonal phases. In other words, if $S$ diagonalizes the mass matrix, so does $S'$

$$S' = S \begin{pmatrix} e^{i \phi_1} & & \\ & \ddots & & \\ & & e^{i \phi_n} \end{pmatrix}$$

We can use this property to redefine the quark fields to reduce the phases in $U$. It turns out that for an $n \times n$ unitary matrix, number of independent physical phases left over is

$$\frac{(n-1)(n-2)}{2}$$

Thus to get CP violation we need to go to 3 generations or more (Kobayashi-Maskawa).
Standard Model phenomenology

\[ W, Z \text{ gauge bosons} \]

\[
W^\pm = \frac{2}{\sqrt{3}} \left( \frac{e^2}{\sqrt{G_F}} \right) \frac{W}{\sin^2 \theta_W} = 37.3 \text{ GeV} \quad \text{for } \sin^2 \theta_W = 0.23
\]

\[
M_Z = \left( \frac{e^2}{\sqrt{G_F}} \right) \frac{M^2}{\sin^2 \theta_W} = 91.2 \text{ GeV} \quad \text{(Exp)}
\]

**W decays**

\[
\omega_W = \frac{g}{2\sqrt{2}} W^\pm \left[ (\ell^c, \ell^c, \ell^c) Y^{(\ell)}(\ell^c) \left( \frac{m^2}{2} \right) + (\ell^{c*} \ell^c) Y^{(\ell^* \ell)}(\ell^c) U \left( \frac{m}{2} \right) \right] + \text{h.c.}
\]

\[
\Gamma(W \rightarrow e^+\nu_e) = \frac{G_F M_W^2}{4 \pi} (\frac{m^2}{2}) \approx 0.25 \text{ GeV}
\]

\[ W^+ \rightarrow e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau \]

\[ \rightarrow u_d, u_s, u_b \]

\[ \rightarrow c\bar{d}, s\bar{c}, b\bar{c} \]

\[ uW = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

\[ \Gamma(W \rightarrow u_d, u_s, u_b) = 3.6e \]

Similar \[ \Gamma(W \rightarrow c\bar{d}, s\bar{c}, b\bar{c}) = 3.7e \]

**\[ \Gamma(\text{total}) = 9.1e \]**

\[ B(W \rightarrow e^+\nu_e) = \frac{1}{3} \approx 11.1\% \quad \text{exp} (10.8 \pm 0.9)\% \]

\[ B(W \rightarrow \text{hadrons}) = \frac{6}{\sqrt{2}} \approx 66.7\% \quad \text{exp} (67.6\%) \]

**Z decays**

\[ Z \rightarrow \ell^+\ell^- \]

Theor. \[ B(Z \rightarrow \ell^+\ell^-) = 20.5\% \]

Exp. \[ B(Z \rightarrow \ell^+\ell^-) = (20.00 \pm 0.06)\% \]

The significance of this measurement is that it limits the number of light neutrinos to 3. Any possible 4th generation neutrinos have to be above 1/2 of the Z mass.

3. Higgs particle

- Higgs couplings to fermions

\[ \mathcal{L}_f = \sum_f \bar{f} \phi \psi_f + \text{h.c.} \]

- Spontaneous symmetry breaking

\[ \phi = \frac{v}{\sqrt{2}} (\xi + \eta) \]

\[ \mathcal{L}_f = \sum_f \bar{f} \left( \frac{m_f^2}{2} \right) \phi \psi_f + \frac{1}{2} (\xi + \eta) \phi \psi_f + \text{h.c.} \]

- Diagonalization of the mass matrix

\[ m_f = \frac{\sqrt{2} v}{2} f_i \]

- Mass term

\[ \sum_i \bar{f}_i m_i \phi \psi_i = \sum_i \bar{f}_i m_i \phi \psi_i = \sum_i \bar{f}_i m_i \phi \psi_i + \text{h.c.} \]

- Similarly

\[ U \phi V^\dagger \text{ is also diagonal} \]

\[ \mathcal{L}_f = \sum_f \bar{f}_i \phi \psi_i + \text{h.c.} \]

Thus Higgs coupling to fermions has the properties

1. Proportional to fermion mass
(1) Coupling to $Y_i$ is proportional to fermion mass

(2) Couplings conserve flavors and parity

(3) Coupling to gauge bosons

\[ L_{\text{Int}} = g \left[ W^+ W^- + \frac{1}{2 \sin^2 \theta W} \right] Z \bar{Z}_F \]

again the couplings are proportional to masses

(c) mass of Higgs particle

\[ m_H = \sqrt{2} \mu = 15 \times \nu \quad \nu = 250 \text{ GeV} \]

Experimentally, there is no information on the parameter $\lambda \rightarrow m_H$ is not constrained.

3) Neutrino oscillations,

If neutrino masses are all massless, there is no mixing in the lepton sector

\[ (\nu_e), (\nu_\mu), (\nu_\tau) \]

Suppose neutrino masses are all massive, then in analogy to the quark sector, they are linear combinations of mass eigenstates $\nu_1, \nu_2, \nu_3$

\[ (\nu_e^L, \nu_\mu^L, \nu_\tau^L) = U (\nu_e^H, \nu_\mu^H, \nu_\tau^H) \]

If at time $t = 0$, a beam of pure $\nu_e$ is produced, we can write

\[ |\nu_e(0)\rangle = U_{e1} |\nu_1\rangle + U_{e2} |\nu_2\rangle + U_{e3} |\nu_3\rangle \]

The time evolution is controlled by energy eigenvalues,

\[ |\nu_e(t)\rangle = U_{e1} e^{-iE_1 t} |\nu_1\rangle + U_{e2} e^{-iE_2 t} |\nu_2\rangle + U_{e3} e^{-iE_3 t} |\nu_3\rangle \]

\[ E_1 = E_2 = m_\nu_1 \]

Assume $p \gg m_\nu$, we get

\[ E_1 = p + m_\nu^2 / p \]

Define the oscillation length $\lambda_{ij}$ as

\[ \lambda_{ij} = \frac{2 \pi}{E_i - E_j} = \frac{4 \pi \Delta m_{ij}^2}{E_i - E_j} \]

Consider the simple case of 2 neutrino species.

\[ |\nu_e(0)\rangle = \cos \theta_1 |\nu_1\rangle + \sin \theta_1 |\nu_2\rangle \]

\[ |\nu_\mu(0)\rangle = -\sin \theta_1 |\nu_1\rangle + \cos \theta_1 |\nu_2\rangle \]

\[ |\nu_e(t)\rangle = \cos \theta_1 e^{-iE_1 t} |\nu_1\rangle + \sin \theta_1 e^{-iE_2 t} |\nu_2\rangle = e^{-iE_2 t} (\cos \theta_1 |\nu_1\rangle + \sin \theta_1 e^{-i\frac{\Delta m_{21}^2}{2E_2} t} |\nu_2\rangle) \]

\[ \langle \nu_e | \nu_\mu \rangle = e^{-i\Delta m_{21}^2 / 2E_2} \]

\[ \Delta m_{21}^2 = \frac{(\Delta m_{\mu e}^2)^2}{2E_2} \]

\[ \lambda_{21} = \frac{2 \pi}{\Delta m_{21}^2 / 2E_2} \]
$$P_{\nu e} \rightarrow \nu_e = \langle \nu_e | \nu_e(e) \rangle^2 = 1 - 2 \sin^2 \theta \cos^2 \left( \phi - \cos \frac{2 \pi}{e_{\nu_e}} \times \right)$$

$$P_{\nu e} \rightarrow \nu_{\mu} = 1 - \langle \nu_e | \nu_{\mu} \rangle = 2 \sin \theta \cos \left( \phi - \cos \frac{2 \pi}{e_{\nu_e}} \times \right)$$