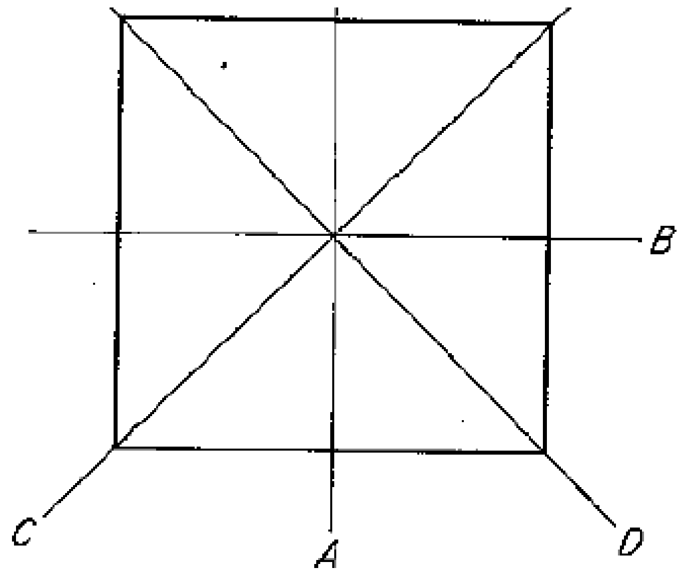

Homework set 1, Due Fri Oct 12

1. Consider the symmetry group of the proper covering operations of a square (D_4). There are 8 elements in the group, E : identity,
A, B, C, D: rotations by π about the axes as shown in the figure
F, G, H : rotations in the plane of the square by $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$ respectively.

   (a) How many inequivalent irreps are there? What are their dimensionalities?

   (b) Work out the representation matrice by considering their effects on the coodiantes, x, y, z.

   (c) Consider the function $f(x, y) = x^2 - y^2$, Work out $P_{A_i} f\left(\vec{x}\right) = f\left(A_i^{-1}\vec{x}\right)$ for all elements of the group.

   (d) Construct all the inequivalent classes.

2. Consider the group S_3, the permutation group of 3 objects.

   (a) Show that the collection of even permutations form a subgroup, called A_3.

   (b) Show that A_3 is an invariant subgroup.

   (c) Find the quotient group S_3/A_3.

3. Consider a group with 4 elements $\{e, a, b, c\}$ with the properties;

$$a^2 = b^2 = c^2 = e, \qquad e: \text{ identity}$$

$$ab = c, \qquad bc = a, \qquad ca = b$$

   Show that this group is Abelian.

4. In the symmetry group D_3 of the regular triangle, we have worked out the transfomation properties of the coordinates, (x, y, z) under this group.

   (a) Compute the transformation properties of $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$,

   (b) Work out the transformation properties of the combinations, $x_i \frac{\partial}{\partial x_j}$, where $x_i = (x, y, z)$, $\frac{\partial}{\partial x_j} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$.

5. If $\phi_i^{(\lambda)}(x)$, for $i = 1, 2, \cdots l_\lambda$ are the basis functions for unitary irrep $D^{(\lambda)}$ with dimension l_λ. Show that

$$\psi(x) = \sum_{i=1}^{l_\lambda} \left|\phi_i^{(\lambda)}(x)\right|^2$$

is invariant under the group transformation, i.e.

$$P_R \psi(x) = \psi(x), \qquad \text{for all } R \in G.$$