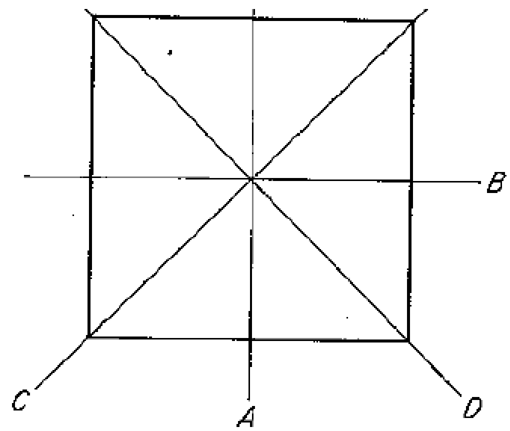
Homework set 2, Due Fri Oct 26

1. Consider the symmetry group of the proper covering operations of a square (D_4) again. There are 8 elements in the group,



E : identity,
A, B, C, D: rotations by π about the axes as shown in the figure
F, G, H : rotations in the plane of the square by $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$ respectively.

   (a) Work out the character table.
   (b) Write these rotations in terms of permutations of 4 objects in the cycle notation.
   (c) Work out the classes in terms of permutations.

2. Show in general that

$$\sum_R \chi^{(j)}(R) = 0$$

for all representations except the identity representation $\Gamma^{(1)}$.

3. Z_n is the cyclic group of order n.

   (a) Work out the character table for the group Z_3.
   (b) Work out the character table for the group Z_4.

4. Consider the group D_3. Work out all the direct product

$$\Gamma^{(i)\times}\Gamma^{(j)} = \sum_k c_{ijk}\Gamma^{(k)}$$

of the irreducible representations.

5. Consider again the group with 4 elements $\{e, a, b, c\}$ with the properties;

$$a^2 = b^2 = c^2 = e, \qquad e: \text{ identity}$$

$$ab = c, \qquad bc = a, \qquad ca = b$$

Work out the character table.