# Homework set 4, Due Fri Dec 14

1. The quarks have spin $\frac{1}{2}$. Construct 3 quarks states with spin $\frac{3}{2}$ and $\frac{1}{2}$ respectively.

2. The Hamiltonian for a 3-dimensional isotropic harmonic oscillator is of the form,

$$H = \hbar\omega\left(a_1^\dagger a_1 + a_2^\dagger a_2 + a_3^\dagger a_3 + \frac{3}{2}\right)$$

where

$$\left[a_i^\dagger, a_j\right] = -\delta_{ij}, \qquad [a_i, a_j] = 0, \qquad \left[a_i^\dagger, a_j^\dagger\right] = 0$$

   (a) Show that the Hamiltonian is invariant undet the $U(3)$ transformation, i.e.

$$[H, U_{ij}] = 0, \qquad \text{with} \quad U_{ij} = a_i^\dagger a_j, \qquad i, j = 1, 2, 3$$

   (b) Show that

$$[U_{ij}, U_{rs}] = U_{is}\delta_{jr} - U_{rj}\delta_{is}$$

   (c) Express $U_{ij}$ in terms of the usual generators of $SU(3)$, $(F_1, \cdots F_8$ or $T_\pm, U_\pm, V_\pm$ ).

3. Write the general Lie algebra as

$$[X_a, X_b] = if_{abc}X_c$$

where $f'_{abc}s$ are structure constants.

   (a) Show that the generators satisfy the Jacobi identity

$$[X_a, [X_b, X_c]] + [X_b, [X_c, X_a]] + [X_c, [X_a, X_b]] = 0$$

   (b) From Jacobi identity show that the matrices of the form

$$(T_a)_{bc} = -if_{abc}$$

   satisfy the Lie algebra, i. e.

$$[T_a, T_b] = if_{abc}T_c$$

4. For any $2 \times 2$ unitary matrix $U$ with unit determinant, show that there exists a matrix $S$ which takes $U$ to its complex conjugate $U^*$ through similarity transformation,

$$S^{-1}US = U^*$$

Find such matrix.

5. Let the hermitian traceless $n \times n$ matices be $\lambda_a, a = 1, 2, \cdots, n^2 - 1$, with normalization

$$Tr(\lambda_a\lambda_b) = 2\delta_{ab}$$

Show that

$$\sum_a (\lambda_a)_{ij}(\lambda_a)_{kl} = 2\delta_{il}\delta_{kj} - \frac{2}{n}\delta_{ij}\delta_{kl}$$