Looking for Higgs Particle

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2. Symmetry and Conservation Law
3. Explicit vs Spontaneous Symmetry Breaking
4. Local vs global symmetries
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Introduction

Fundamental Interactions in nature

1. **Strong interactions**–Quantum Chromodynamics(QCD) : gauge theory based on $SU(3)$ symmetry
2. **Electromagnetic interaction** / **Weak interaction** $\}$ **Electroweak interaction**–gauge theory based on $SU(2) \times U(1)$ symmetry with spontaneous symmetry breaking
3. **Gravitational interacton**: gauge theory of coordinate transformation

Symmetry and Conservation Law

Noether's Theorem: Any continuous transformation which leaves the action

$$S = \int d^4x \mathcal{L}$$

invariant, will give a conserved charge.

Symmetry Transformation	Conserved Charge
time translation $t \longrightarrow t + a$	Energy
space translation $\overrightarrow{x} \longrightarrow \overrightarrow{x} + \overrightarrow{b}$	Momentum
rotation	Angular momentum
$\cdots$	$\cdots$

Other conserved quantities: Electric charge, Baryon number,. . .

2)Explicit vs Spontaneous Symmetry Breaking

Most of the symmetries in nature are approximate symmetries.

(a) **Explicit breaking**–

Add small non-symmetric terms to the Hamiltonian

e.g. Isospin symmetry is broken by electromagnetic interaction

(b) **Spontaneous breaking**:

Ground state does not have the same symmetry as the Hamiltonian (Nambu 1960, Goldstone 1961)

Goldstone theorem: Spontaneous breaking of continuous symmetry implies the existence of **massless** particle (Goldstone boson).

For example, the effective potential given by

$$V\left(\vec{\phi}\right) = -\mu^2\left(\vec{\phi}\cdot\vec{\phi}\right) + \lambda\left(\vec{\phi}\cdot\vec{\phi}\right)^2$$

has $O\left(3\right)$ symmetry. The classical mininmum is located at

$$\vec{\phi}\cdot\vec{\phi} = v^2 = \frac{\mu^2}{2\lambda}$$

.

Choose $\left\langle \vec{\phi} \right\rangle = (0, 0, v)$ the symmetry is broken from $O(3)$ to $O(2)$.

Define the quantum field $\vec{\phi}'$ by

$$\vec{\phi}' = \vec{\phi} - \left\langle \vec{\phi} \right\rangle$$

Then ϕ_1' and ϕ_2' are Goldstone bosons.

- Note that massless particles imply long range forces which do not seem to show up in nature very often.
- The pattern of symmetry breaking depends on the chosen scalar fields. Generally, we have

  $$\# \text{ of Goldstone bosons} = \# \text{ of broken generators}$$

  and there always be scalar fields which are not Goldstone bosons.

3) Local vs global symmetries

a) **Global symmetry**: symmetry transformation independent of space-time

Example: phase transformation $\phi \longrightarrow \phi' = e^{i\alpha}\phi$, α : some constant

Then Lagrangian given by

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \mu^2 \phi^\dagger \phi - \lambda \left(\phi^\dagger \phi\right)^2$$

is invariant under the phase transformation and the charge

$$Q = \int d^3 x i \left[\phi^\dagger \partial_0 \phi - \partial_0 \phi^\dagger \phi\right]$$

is conserved. (Noether's theorem)

This is an example of theory with $U\left(1\right)$ symmetry.

In nature, many approximate symmetries, e.g. lepton number, isospin, Baryon number,$\cdots$ are probably realized in the form of global symmtries.

b) **Local symmetry**: **guage smmetry**

The phase transformation is now space-time dependent,

$$\phi \rightarrow \phi' = e^{ig\alpha(x)}\phi$$

The derivative transforms as

$$\partial^{\mu}\phi \rightarrow \partial^{\mu}\phi^{'} = e^{i\alpha(x)}\left[\partial^{\mu}\phi + ig\left(\partial^{\mu}\alpha\right)\phi\right],$$

which is not just a phase transformation on the derivative.
Introduce a vector field A^{μ}, gauge field, with the transformation

$$A^{\mu} \rightarrow A'^{\mu} = A^{\mu} - \partial^{\mu}\alpha$$

The combination

$$D^{\mu}\phi \equiv \left(\partial^{\mu} - igA^{\mu}\right)\phi, \qquad \text{covariant derivative}$$

will be transformed by a phase,

$$D^{\mu}\phi' = e^{ig\alpha(x)}\left(D^{\mu}\phi\right)$$

and the combination

$$D_{\mu}\phi^{\dagger}D^{\mu}\phi$$

is invarianat under the phase transformation.

If we define anti-symmetric tensor for the gauge field by

$$\left(D_\mu D_\nu - D_\nu D_\mu\right)\phi = gF_{\mu\nu}\phi, \qquad \text{with} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

We can use the property of the covariant derivative under the gauge transformation to show that

$$F'_{\mu\nu} = F_{\mu\nu}$$

The complete Lagragian for this theory is

$$\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

where $V(\phi)$ does not depend on derivative of ϕ.

- the usual mass term of the form $A^\mu A_\mu$ is not gauge invariant $\Rightarrow$ gauge field gives massless particle$\Rightarrow$long range force
- the coupling of gauge field to other field is universal
- The extension to non-abelian local symmetry, Yang-Mills fields, **(**Yang & Mills 1954) has led to many new interesting properties.

4) Local symmetry & spontaneous symmetry breaking

If we combine the local symmetry with spontaneous symmetry breaking, an interesting new phenonmena happens, namely **Higgs phenomenon**.(Higgs (1964), Englert & Brout (1964), Guralnik, Hagen, & Kibble (1964)) Take the Lagrangain to be of the form,

$$\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)$$

with

$$D^\mu\phi \equiv \left(\partial^\mu - igA^\mu\right)\phi, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$V(\phi) = -\mu^2\left(\phi^\dagger\phi\right) + \lambda\left(\phi^\dagger\phi\right)^2$$

This Lagrangian is invariant under the local symmetry transfomation,

$$\phi \to \phi' = e^{ig\alpha(x)}\phi, \qquad A'^\mu = A^\mu - \partial^\mu\alpha$$

The classical minimum is at

$$\left(\phi^\dagger\phi\right) = v^2 = \frac{\mu^2}{2\lambda}$$

If we define the quantum field by

$$\phi' = \phi - v, \qquad \text{so that} \quad \langle \phi' \rangle = 0$$

then covariant derivative term will give

$$D_\mu \phi^\dagger D^\mu \phi \longrightarrow \left[\left(\partial_\mu + igA_\mu \right) \phi \right] \left[\left(\partial^\mu - igA^\mu \right) \phi \right] \longrightarrow g^2 v^2 \left(A^\mu A_\mu \right) + \cdots$$

which is a mass term for the gauge boson. So there is no more long range force associated with gauge boson. In fact, one
can make a gauge transformation to get rid of the Goldstone boson. First write the ϕ as

$$\phi = e^{i\xi/v} \left(v + \eta \right)$$

Make a gauge transformation,

$$\phi' \left(x \right) = e^{-i\xi/v} \phi, \qquad B_\mu = A_\mu - \frac{1}{gv} \partial_\mu \xi$$

Then $\xi \left(x \right)$ field, the Goldstone bosons, disappears from the Lagrangian because of the gauge invariant. What happens is that the massless A_μ field combines with ξ field to become massive.

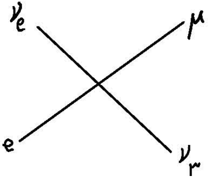
5) Standard Model of Electroweak Interactions

Weak Interactions before gauge theory :

1) Four-fermion interaction

$$\mathcal{L}_{wk} = \frac{G_F}{\sqrt{2}} \left(J^{\mu} J_{\mu}^{\dagger} + h.c. \right) \qquad \text{where } J_{\mu} = \left[\bar{\nu} \gamma_{\mu} \left(1 - \gamma_5 \right) e \right] + \cdots$$

where $G_F \simeq \frac{10^{-5}}{M_p^2}$ is the Fermi constant. Note that only the left-handed currents come in here.

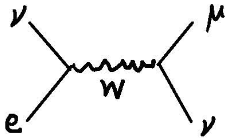


This theory has many sucesses phenomenolgically but behaves badly at high energies. In particular, it non-renormalizable and violates unitarity around 300 *Gev*.

2) Intermediate Vector Boson theory

Here the weak interaction is mediated by a massive vector bosons W

$$\mathcal{L}_{wk} = g\left(J_{\mu}W^{\mu} + h.c.\right), \qquad \frac{g^2}{M_w^2} = \frac{G_F}{\sqrt{2}}$$



But the bad high energy behavior persists.

Construction of Standard Model (Weinberg 1967, 't Hooft 1971)
This is a gauge theory with spontaneous symmetry breaking. The bad high energy behavior is avoided as the intermediate vector meson W gets its mass through spontaneous symmetry breaking.

Gauge group: $SU(2) \times U(1)$ gauge bosons: $\overrightarrow{A}_\mu, B_\mu$

Scalar field: $\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$,

Spontaneous symmetry breaking: $SU(2) \times U(1) \longrightarrow U(1)_{em}$

$$\phi = \exp\left(i\overrightarrow{\tau} \cdot \overrightarrow{\xi}(x)/v\right) \begin{pmatrix} 0 \\ v + \eta(x) \end{pmatrix}$$

Here $\overrightarrow{\xi}(x)$ are Goldstone bosons and will be eaten up by gauge bosons to become massive. The left over field $\eta(x)$ is usually called **Higgs Particle.**
Massive gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right) \qquad W - \text{boson}$$
$$Z_\mu = \cos\theta_W A_\mu^3 - \sin\theta_W B_\mu \qquad Z - \text{boson}$$
$$A_\mu = \sin\theta_W A_\mu^3 + \cos\theta_W B_\mu \qquad \text{Photon}$$

Fermions:

a) Leptons

$$L_i = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \qquad R_i = e_R, \mu_R, \tau_R,$$

b) Quarks (Glashow, Iliopoupos, and Maiani, Kobayashi and Maskawa)

$$q_{iL} = \begin{pmatrix} u' \\ d \end{pmatrix}_L, \begin{pmatrix} c' \\ s \end{pmatrix}_L, \begin{pmatrix} t' \\ b \end{pmatrix}_L, \qquad U_{iR} = u_R, c_R, t_R, \quad D_{iR} = d_R, s_R, b_R$$

All left-handed fermions are in $SU(2)$ doublets and right-handed fermions are all singlets.

Yukawa coupling:

$$\mathcal{L}_Y = f_{ij} \bar{L}_i R_j \phi + h.c. + \cdots$$

Fermions get their masses from spontaneous symmetry breaking through Yukawa couplings,

$$m_{ij} = f_{ij} v$$

This implies that the Yukawa couplings $\propto$ masses.

Highlights of the Sucess of Standard Model

- W and Z were discovered in1983 at SPS in CERN, their masses agree well with theoretical prediction
- Z boson mediates weak neutral current interactions,e.g.

  $$\nu_\mu + e \longrightarrow \nu_\mu + e, \qquad \nu + N \longrightarrow \nu + X, \cdots$$

  These processes were discovered and being studies extensively in 1970's.
- t and b quarks were predicted and subsequently found
- Z bosons are studied extensively in e^+e^- machine and the results agree with theory
- ...

Higgs Particle H

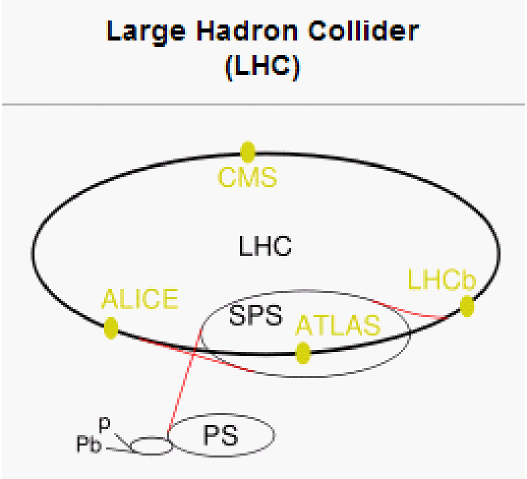
- Mass is not predicted by theorectical consideration
- Coupling to other particle is proportional to the other particle's mass $\Rightarrow$ Higgs will decay into heavies particles allowed by kinematics.
- Production at hadron machine:

$$(i) \qquad \text{Gluon fusion :} \qquad pp \longrightarrow gg \longrightarrow H,$$

$$(ii) \qquad \text{V V fusion :} \qquad pp \longrightarrow VV \longrightarrow H,$$

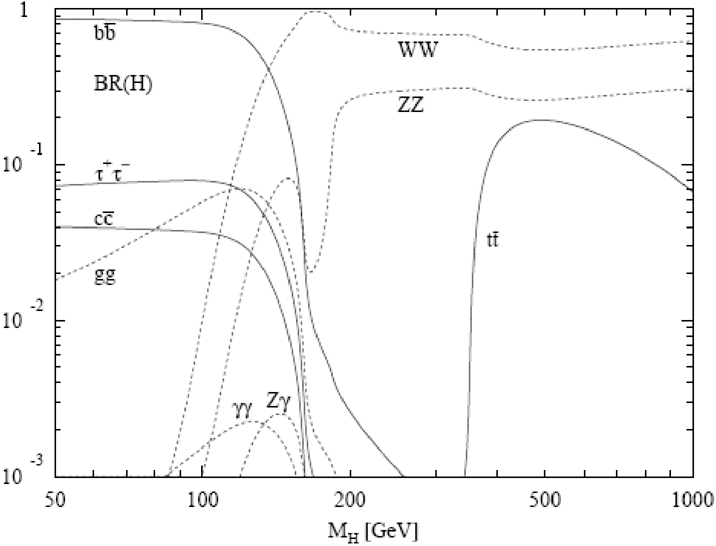
$$(iii) \qquad \text{Association with V} \qquad pp \longrightarrow qq\prime \longrightarrow VH.$$

LHC(Large Hadron Collider) : 7 *Tev* on 7 *Tev* proton machine



The accelerator chain of the Large Hadron Collider

- Decay of Higgs



Anything beyond Standard Model?

1. Neutrino masses(confirmed)
2. More complicate Higgs structure
3. Supersymmetry
4. Grand unification
5. String