NTHU Physics Colloquim 11/4/2009

Precision Measurement in Atomic Physics

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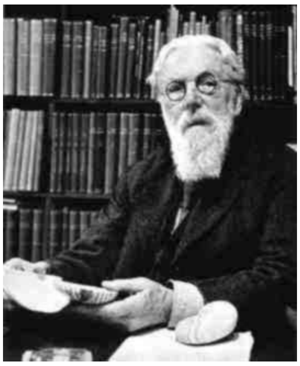
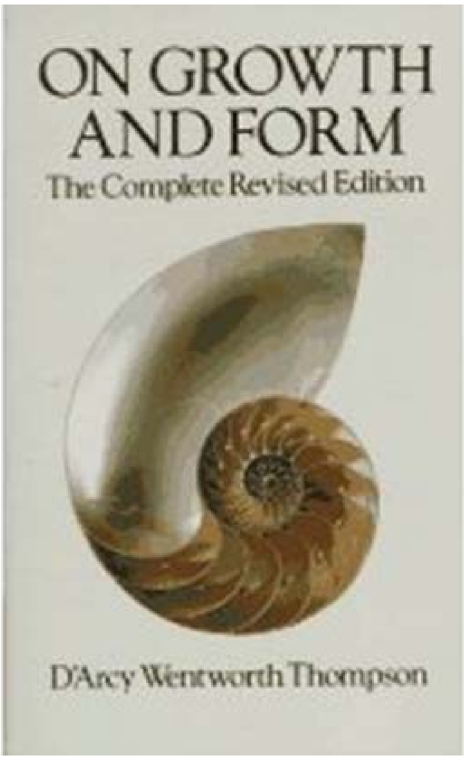


Thompson, D'Arcy Wentworth

On Growth and Form, 1917

…numerical precision is the very soul of science…



ON GROWTH AND FORM
The Complete Revised Edition

D'Arcy Wentworth Thompson

How precise?

- Magnetic moment of electron,
  g_e (exp) = 2.0023193043617(15)
- Rydberg constant = $\frac{2\pi^2 m_e e^4}{h^3 c}$
  = 109,737.31568639(91)
- Electric dipole moment of electron
  $|d_e| < 1.6\times 10^{-27}\ e\cdot\text{cm}$
- Time variation of fine structure constant
  $\delta\alpha/\alpha = (-6.2 \pm 6.5)\times 10^{-17}/\text{year}$ $\alpha = e^2/\hbar c \approx 1/137$

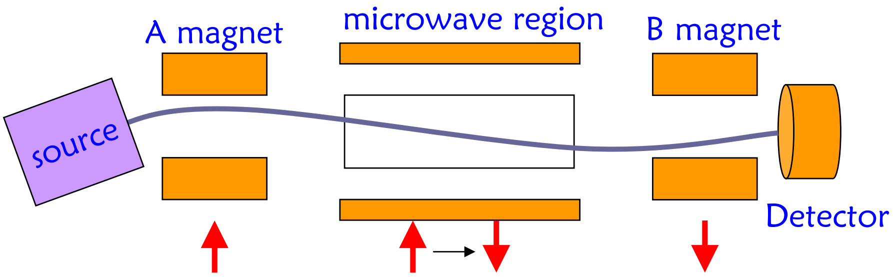
[1] G. Gabrielse et al., *Phys. Rev Lett.* **97**, 30802 (2006)
[2] Th. Udem et al., *Phys. Rev. Lett*. **79**, 2646 (1997)
[3] B. C. Regan et al., *Phys. Rev. Lett*. **88**: 071805 (2002)
[4] T. M. Fortier *et al.*, *Phys. Rev. Lett.* **98**, 070801 (2007)

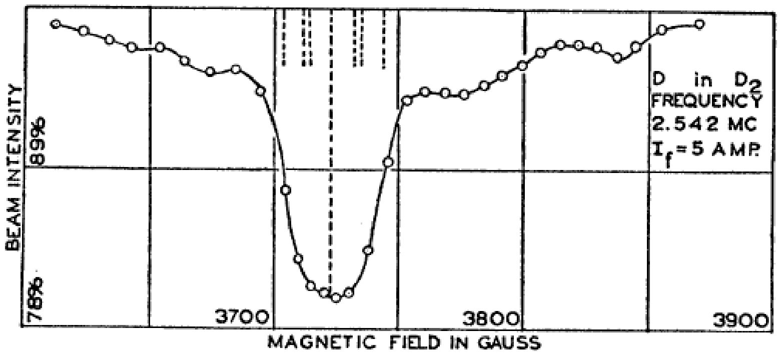
Review

- Historical Review:
  Precision measurement lead to new physics
- Atomic physics: (simple system)
  Techniques utilizing resonance, spectroscopy, time and frequency measurement could be very precise
- Compare: HEP, NP 1/1000 measurement

New Physics Discovered by Atomic Spectroscopy

- First Motivated by Stern-Gerlach experiment, 1922
- First atomic spectroscopy by resonance method NMR, Rabi and Ramsey, 1934-1939





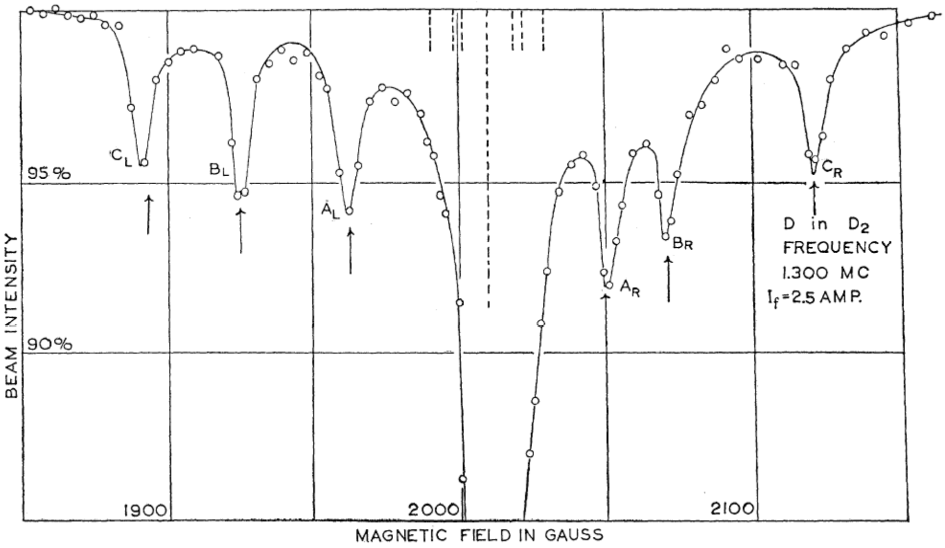
NMR for Deuteron

Isidor Isaac Rabi

Norman F. Ramsey

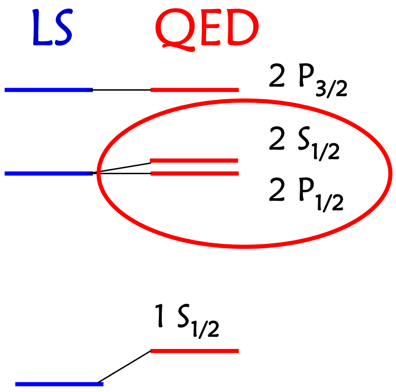
Electric Quadrupole Moment of Deuteron

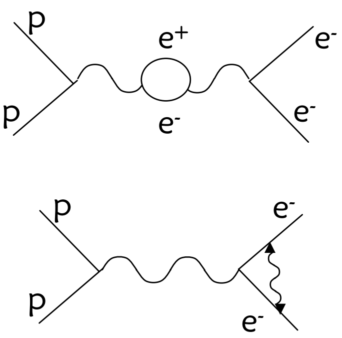
- Electric Quadrupole moment of Deuteron discovered
- Nuclear force is Non-central! Tensor force



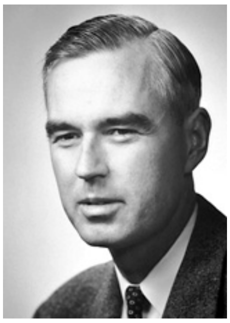
J. M. Kellogg, I. I. Rabi, N. F. Ramsey, and J. R. Zacharias Phys. Rev. 57, 677 (1940)

The Lamb Shift

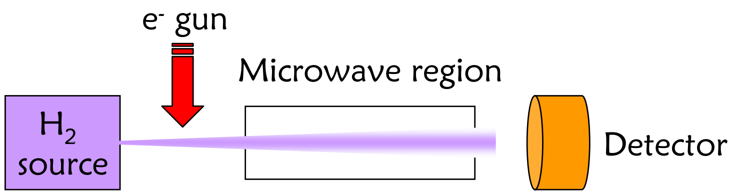




1947 by Lamb ~1060 MHz
now: 1057.846(4) MHz



Willis Eugene Lamb
Nobel Prize in 1955
"for his discoveries concerning the fine structure of the hydrogen spectrum"



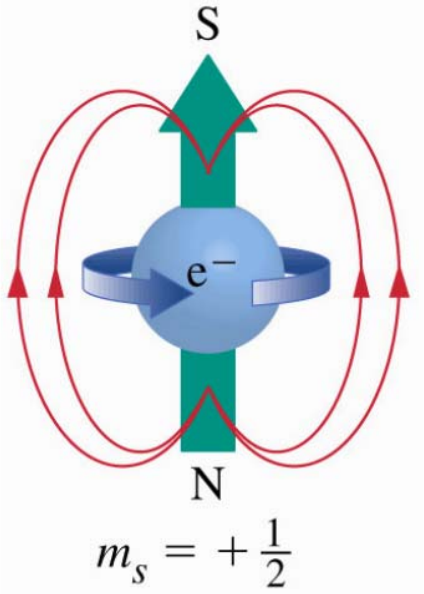
Selected topics in precision measurement

- Test QED: the g-2 of electron and muon
- New source of CP violation: electric dipole moment of electron and nucleon
- Strong interaction: size of proton
- Time and frequency standard: making a better clock
- Constancy of fundamental constant: can speed of light change?
- Weak interaction: parity violation in atom (time permitted)

g-2 of electron

What is g-factor?

The magnetic moment of the electron is proportional to the electron spin



$$\frac{|\bar{\mu}|}{\mu_B} = g \cdot \frac{|\bar{s}|}{\hbar}$$

- Classical non-relativistic g=1
- In QM, Dirac Theory predict g=2

In reality, g= 2.002319304….. Why??

QED

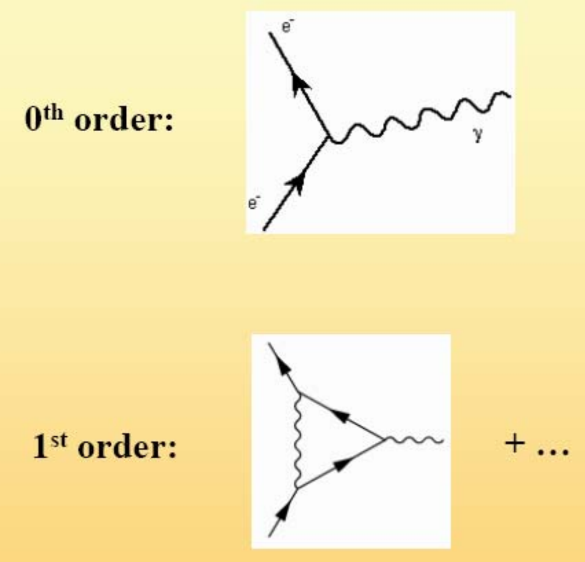
- Quantum Electrodynamics
- The most precisely tested theory

Sin-Ichiro Tomonaga (朝永振一郎), Julian Schwinger, Richard P. Feynman
Nobel Prize in Physics 1965

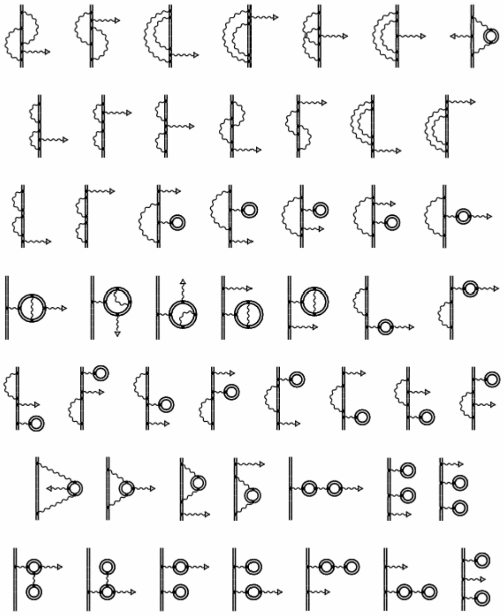
"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"

QED and g factor

- Vacuum is not real vacuum



and a lot more….



Why measure g factor

- Test QED
- Search for structure of electrons
- g can be expanded as a function of α

$$\frac{g}{2}=1+C_2\left(\frac{\alpha}{\pi}\right)+C_4\left(\frac{\alpha}{\pi}\right)^2+C_6\left(\frac{\alpha}{\pi}\right)^3+\ldots\ldots$$

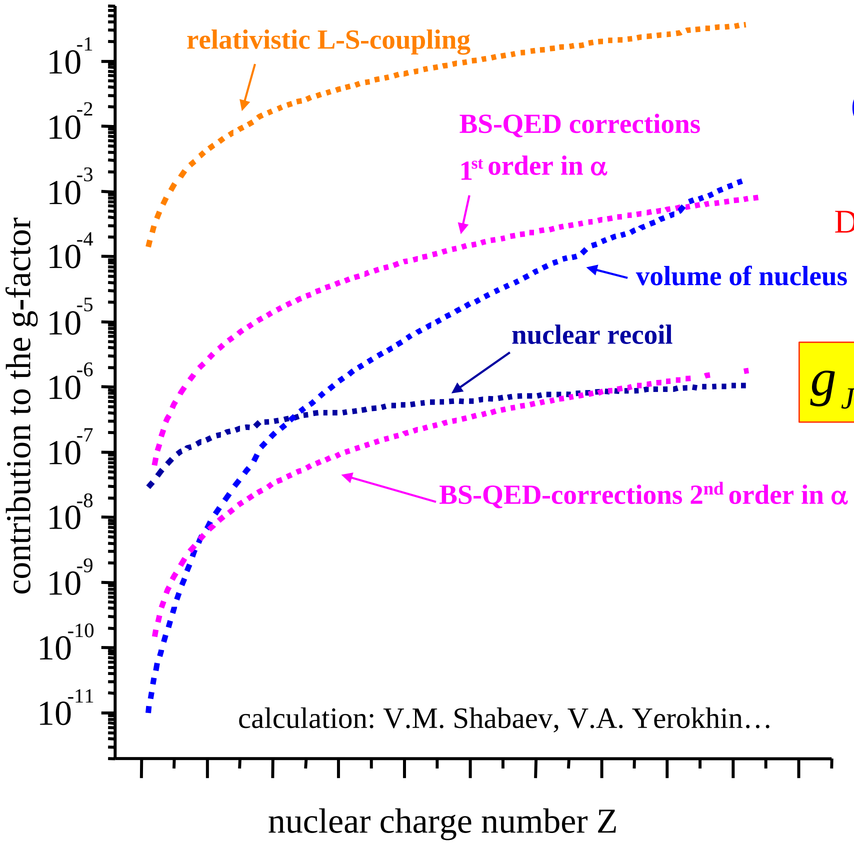
$$+a_{\mu\tau}+a_{hadronic}+a_{weak}$$

$$\alpha={e^2}/{\hbar c}\approx {1}/{137}$$

α^{-1} = 137.035 999 070 (98)

Gabrielse, Hanneke, Kinoshita, Nio, and Odom, *Phys Rev Lett* 97, 30802 (2006)

Measure electron in free space



calculation: V.M. Shabaev, V.A. Yerokhin…

QED of free electron (exp. Dehmelt et al., Gabrielse et al. theo. Kinoshita)

Dirac value

Nuclear correction

$$\boxed{g_J = 2 + a_e + a_b + b + c}$$

Relativistic correction

QED of bound electron

How to measure g

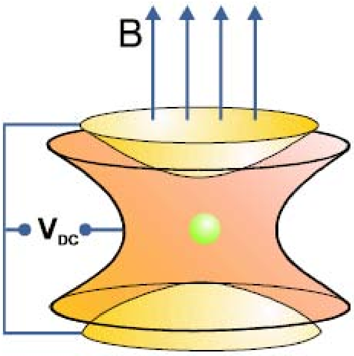
Spin precession (Larmor) frequency

$$\hbar\omega_L = m_s g \cdot \mu_B \cdot B \qquad \mu_B = \frac{e\hbar}{2m}$$

Calibration of magnetic field by cyclotron frequency:

$$\hbar\omega_C = \frac{q}{M} B$$

$$\boxed{g = 2\frac{\omega_L}{\omega_C}\frac{q}{e}\frac{m}{M} = 2\frac{\omega_L}{\omega_C}}$$



Measurement performed on a single electron in Penning trap

Current status

g-2 experiment of electron and muon:

g_e (exp) = 2.0023193043617(15),
g_e (th) to determine fundamental constant

$$\left.\begin{aligned} g_\mu\ (\text{exp}) &= 2.0023318416(12)^* \\ g_\mu\ (\text{th}) &= 2.0023318367(13)^* \end{aligned}\right\}\ 5\,\sigma\ \text{deviation}$$

G.W. Bennett et al., *Phys Rev Lett* 92, 1618102 (2004)

Permanent electric dipole moment

- 1957 Landau first pointed out the electric dipole moment (EDM) of a fundamental particle would suggest P and T violation

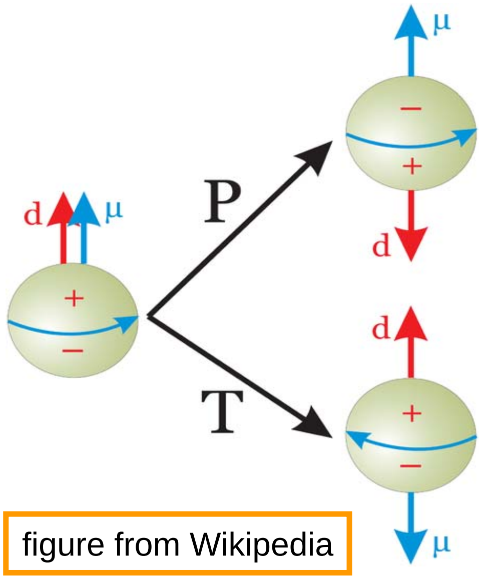


figure from Wikipedia

d: electric dipole moment
= vector
u: spin or magnetic moment
= pseudo-vector, (like **r** x **p**)

Neutron EDM: history

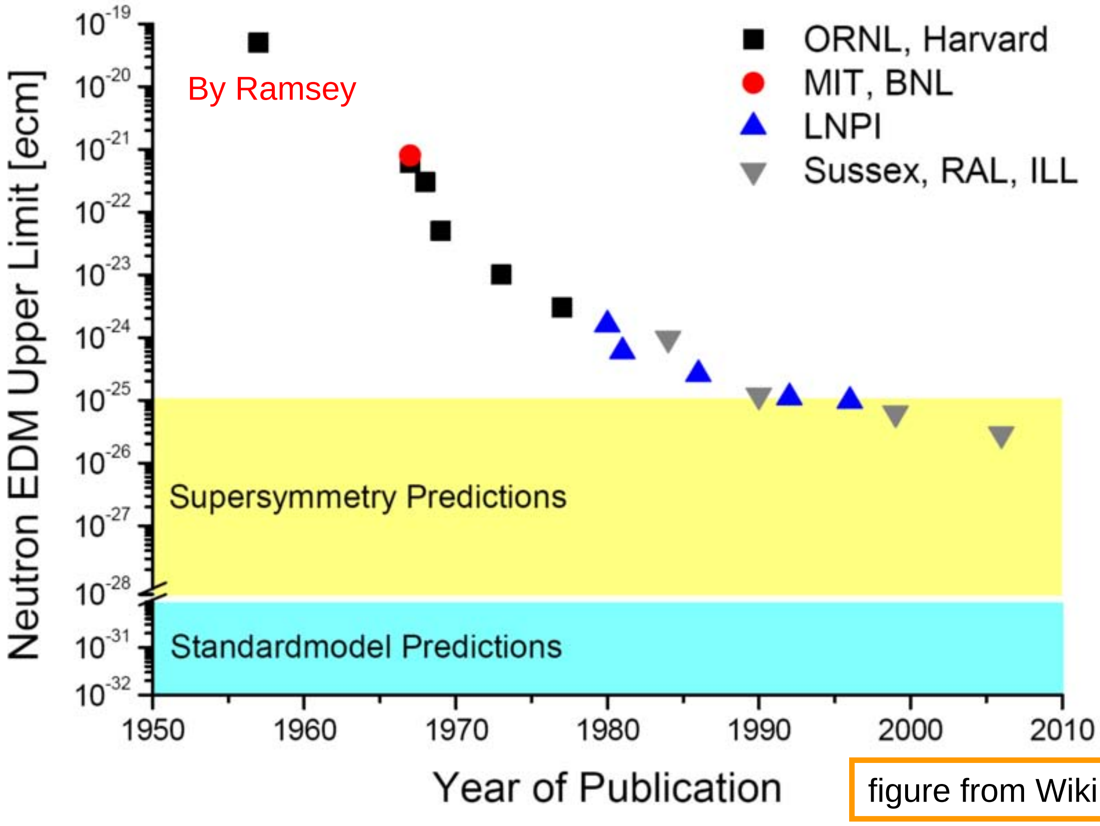


figure from Wikipedia

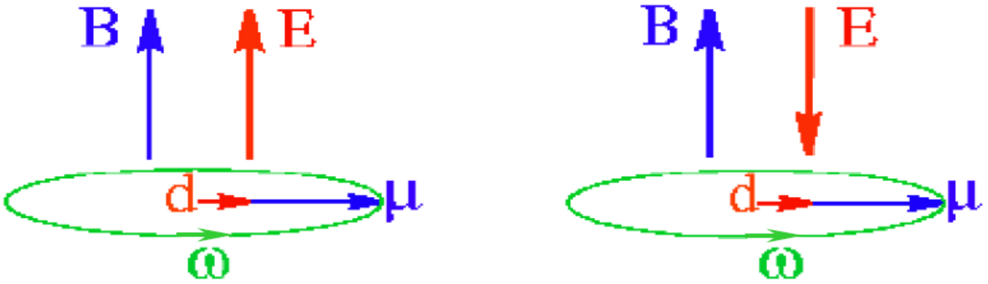
CP violation without strangeness

- Matter–antimatter asymmetry suggest another source of CP violation
  - Electron: intrinsic ?
  - Quark: intrinsic ?
  - Neutron/proton:
    from quark EDM ? New interaction?
  - Atoms, molecules:
    large enhancement factors

Atoms: Tl, Cs, Hg, Xe, Rn, Ra, Fr
Molecule: PbO, YbF, TlF
Ions, solid state systems, etc…

How to measure EDM

$$H = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$$



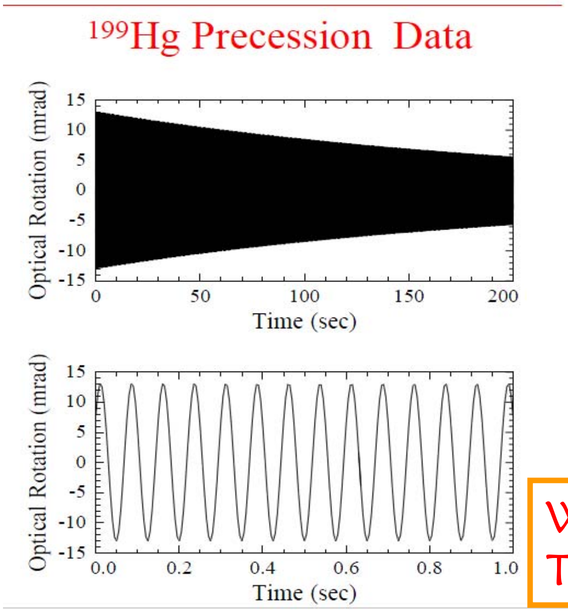
$$\omega_1 = \frac{2\mu B + 2dE}{\hbar} \qquad \omega_2 = \frac{2\mu B - 2dE}{\hbar}$$

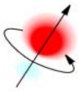
$$\omega_1 - \omega_2 = \frac{4dE}{\hbar}$$

figure from M. Romalis, Princeton

Problem:
reverse E field cause leakage current,
huge noise from B field

Current status





Typical value
E ~ 10 kV/cm
ω~ 10Hz, δω ~ nHz

Sensitivity approaching
some Standard Model extension

What if someone observes EDM?
Theories ready with many parameters

figure from M. Romalis, Princeton

Size of proton

- Fundamental quantity bothering for decades
- Very important for nuclear calculation
- Lattice QCD not able to calculate precisely

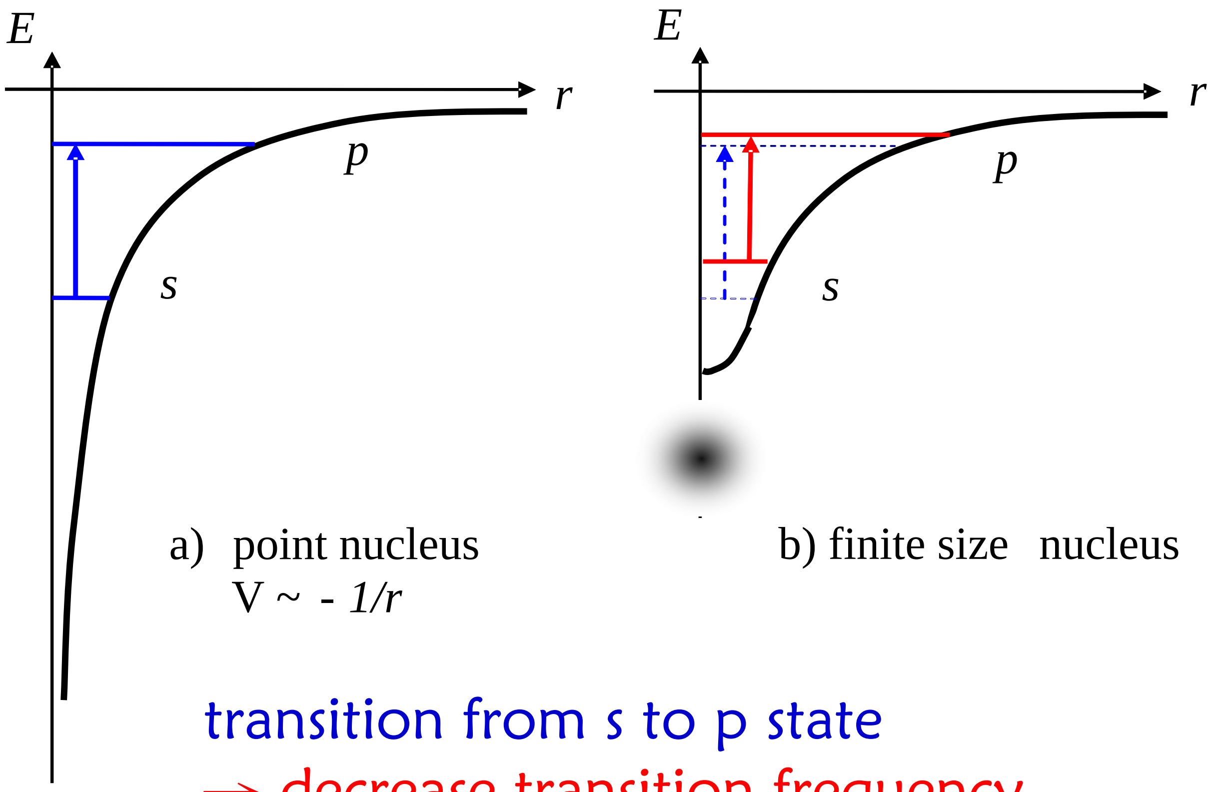
Charge radius of proton by electron scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{ \left(\frac{G_E^2(Q^2) + \frac{Q^2}{4M^2} G_M^2(Q^2)}{1 + \frac{Q^2}{4M^2}} \right) + \frac{Q^2}{2M^2} \cdot G_M^2(Q^2) \cdot \tan^2(\theta/2) \right\}$$

$$\Rightarrow \quad < r_c^{\,2} > = -6 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

- Simon et al, (1980) R_{rms} = 0.862(12) fm
- I. Sick, (2003). R_{rms} = 0.895(18) fm

Nuclear size effect



a) point nucleus
V ~ - 1/r

b) finite size nucleus

transition from s to p state
→ decrease transition frequency

Laser spectroscopy method

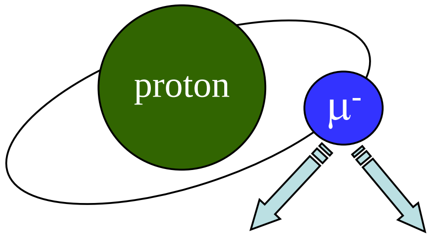
- Measure H transition frequency 1s →2s at 121nm, uncertainty: (th) 16kHz, (exp) 22kHz

→ charge radius of proton R_{rms} = 0.883(14) fm

Lack of precision (only1~2%) very annoying!

Kirill Melnikov and Timo van Ritbergen, Phys. Rev. Lett. **84**, 1673(2000)

Muonic hydrogen



muon decay $\mu^- \to e^-\nu\bar{\nu}$

- $m_\mu/m_e \sim 200$
- Bohr radius $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$
- Energy level $E_n = -\frac{m_e e^4}{2(4\pi\varepsilon_0\hbar)^2}\frac{m_e}{n^2}$
- Wave function $\Psi(r) \sim a_0^{-3/2} e^{-r/a_0}$
- Energy shift due to nuclear size~ $|\Psi(0)|^2 \langle r^2 \rangle$
- Sensitivity ~ $(m_\mu/m_e)^2$

Ongoing project at PSI, aiming 0.1% measurement

Frequency standard

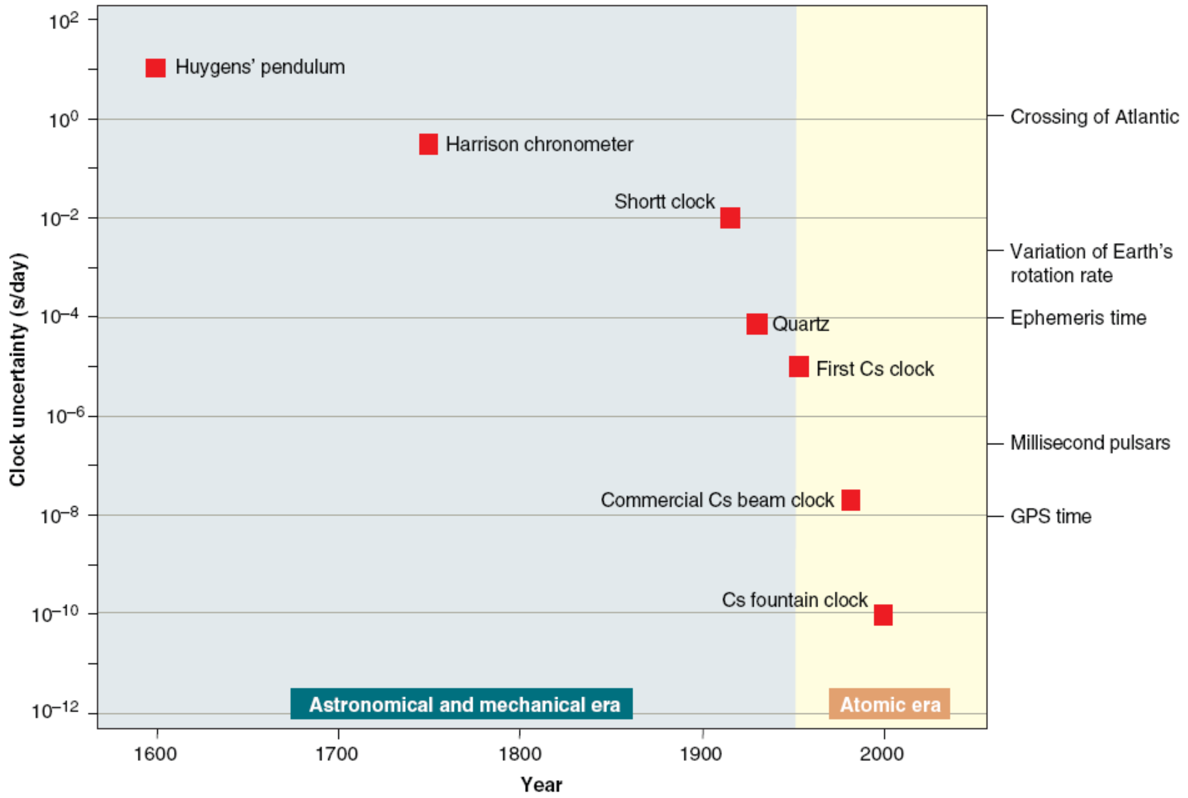
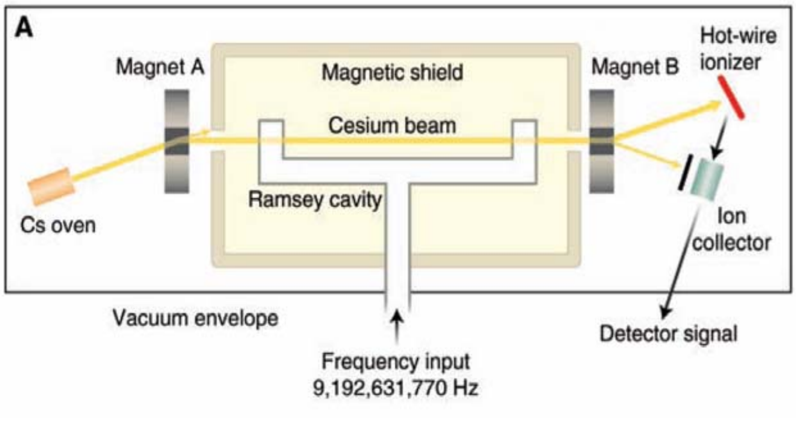


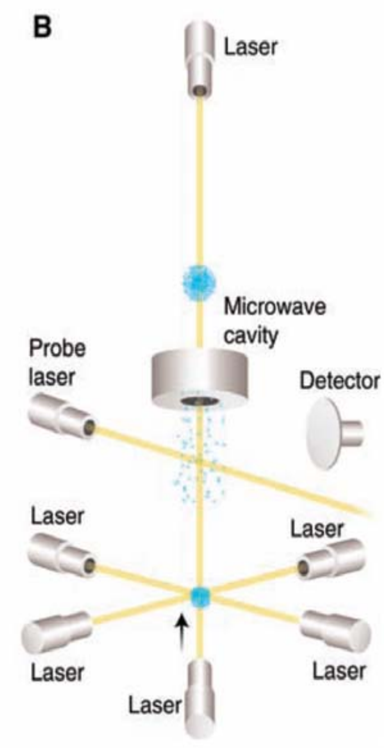
Figure: S.A. Diddams et al.,Science 306, 1318, 2004

Why better clock?

- When you measure something very precisely, new physics will come out by itself!
- Applications:
  - GPS
  - Test special relativity, gravitational red-shift
  - Gravitational wave radiation in binary pulsar
  - Test linearality of quantum mechanics
  - Variation of fundamental constant

Microwave Cs clock





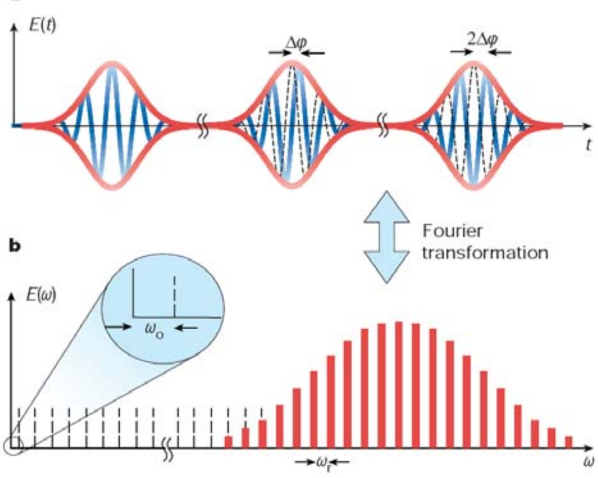
NIST Cs fountain

A: Microwave resonance of Cs beam
$\delta\nu/\nu_0 = 3\times10^{-15}$
state selection, interaction time
B: Cs fountain clock
$\delta\nu/\nu_0 = 4\times10^{-16}$
> To improve $\delta\nu/\nu_0$?

Optical Atomic Clock

- Frequency comb: pioneering work by T.W. Hänsch made direct frequency measurement possible

Frequency Comb basics



Picture from Th. Udem, R. Holzwarth, T.W. Hänsch, Nature 416, 233, 2002

$\omega_n = n\,\omega_r + \omega_{offset}$
ω_r : 50MHz ~ 1GHz

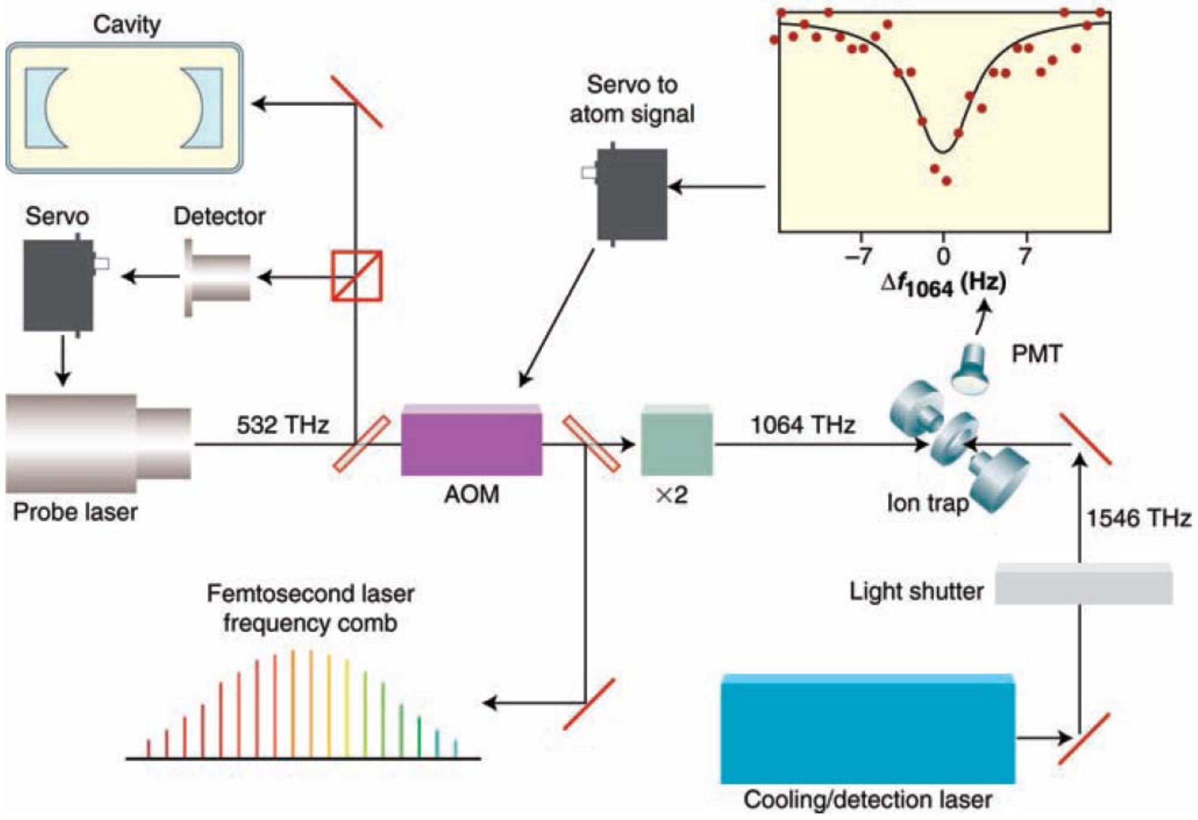
ω_r:
Determined by cavity length

ω_{offset}:
measured by f-2f technique

Optical Atomic Clock

- Trapped ions: Hg$^+$, Ca$^+$, Sr$^+$, Yb$^+$, Ba$^+$, In$^+$…..
  - Long interaction time, can be laser cooled
  - Single ion detection, poor S/N
- Trapped neutral atoms: Ca, Sr, Yb, Mg, H…..
  - Large number of atoms, very high S/N
  - Complicated systematics
- Precision ~ 10^{-16} – 10^{-17} (Hz$^{1/2}$)

How optical clock work



S.A. Diddams, J.C. Bergquist, S.R. Jefferts, C.W. Oats, Science 306, 1318, 2004

Constancy of Fine Structure Constant

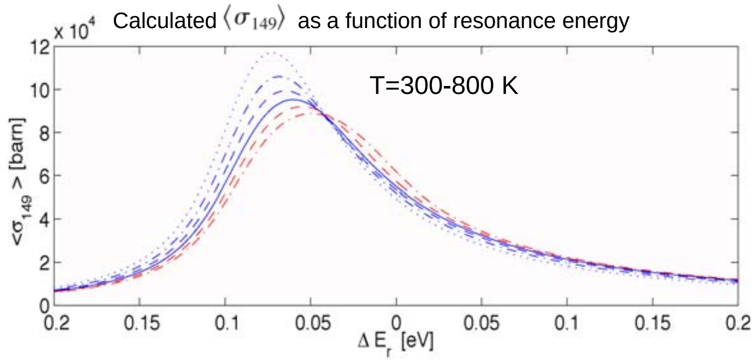
$$\alpha = e^2/\hbar c \approx 1/137$$

$$\alpha^{-1} = 137.035\,999\,070\,(98)$$

- How constant is the constant?
- Two reports show the constant may change in time
- Oklo phenomenon
- Quasar spectra

The Oklo Natural Nuclear Reactor

- Natural nuclear reactor 1.9 billion years ago
- ^{235}U abundance very low
- Fission of ^{235}U, neutron slowed by water
- $n + {}^{149}Sm \to {}^{150}Sm + \gamma$, $E_r = 97.3$ meV

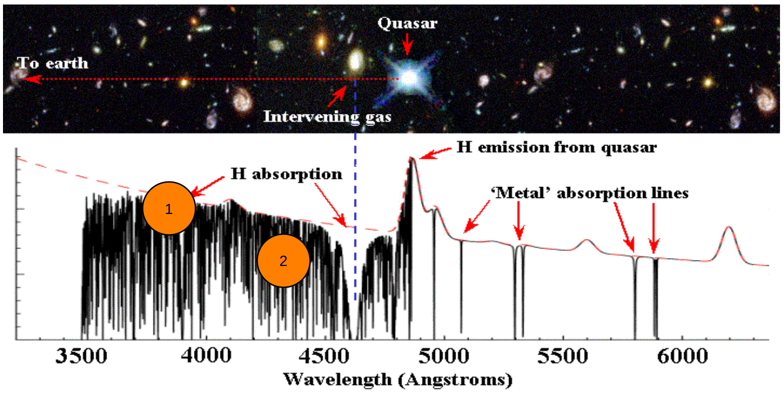

Calculated $\langle\sigma_{149}\rangle$ as a function of resonance energy

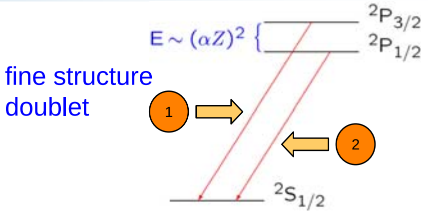
The uranium isotopes found at Oklo strongly resemble those in the spent nuclear fuel generated by today's nuclear power plants.

- ^{149}Sm isotope abundance, → neutron absorption cross section σ_{149}
- From $\sigma_{149} \to \Delta E_r$, assume reasonable T

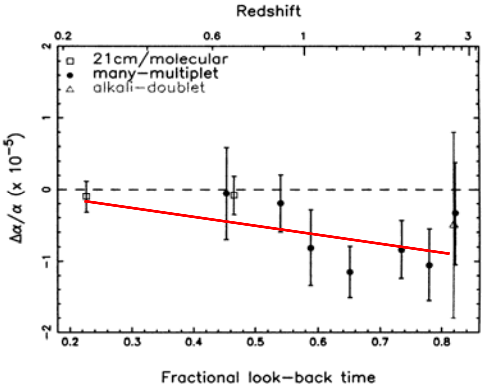
$$\frac{\Delta E_r}{1.1 MeV} = \frac{\Delta\alpha}{\alpha} \qquad \Delta E_r < 0.1\ \text{eV},\ \Delta\alpha/\alpha < 10^{-7}$$

Quasar Absorption Spectra





$\Delta\alpha/\alpha$	z	t
$(7.2 \pm 1.8) \cdot 10^{-6}$	0.5–2.6	0.38–0.82
$\longrightarrow (6.4 \pm 1.4) \cdot 10^{-16}$/yr		



(1) Atoms located in different regions with different velocities
→ different Doppler red shift

(2) Isotopic abundance in the past may change
→ shift the position of absorption line

(3) Magnetic field shift the energy levels by Zeeman effect

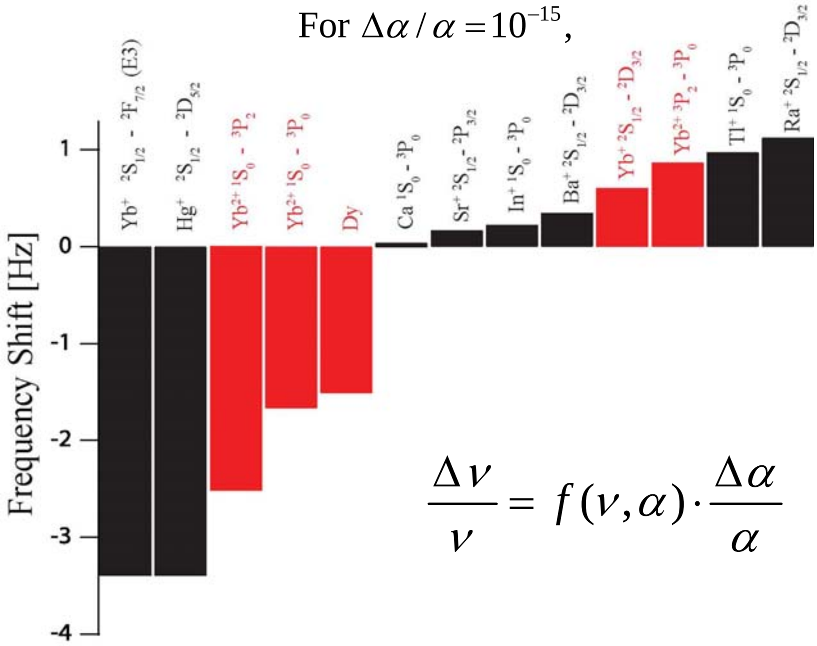
Results From Searches for dα/dt

Δα/α per year	method	quantity	ref
$-(2.3\ +0.7/-0.3)\ 10^{-17}$	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	1
$< 0.8\ 10^{-17}$	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	2
$(7.2 \pm 1.8)\ 10^{-16}$	Quasar spectra	α	3
$(6.4 \pm 1.4)\ 10^{-16}$	Quasar spectra	α	4
$(0.6 \pm 0.6)\ 10^{-16}$	Quasar spectra	α	5
$(4.4 \pm 1.6)\ 10^{-16}$	Quasar spectra	α	6
$(-2.7 \pm 2.6)\ 10^{-15}$	Dy atomic beam	α	7
$(-0.9 \pm 2.9)\ 10^{-15}$	Opt-H ↔ Cs	$g_{Cs} m_e/m_p \alpha^{2.8}$	8
$(0.3 \pm 2.0)\ 10^{-15}$	Opt-Yb$^+$ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{1.9}$	9
$(0.02 \pm 0.7)\ 10^{-15}$	Rb ↔ Cs	$g_{Cs}/g_{Rb} \alpha^{0.44}$	10
$(-6.2 \pm 6.5)\ 10^{-17}$	Opt-Hg$^+$ ↔ Cs	$g_{Cs} m_e/m_p \alpha^{6.0}$	11

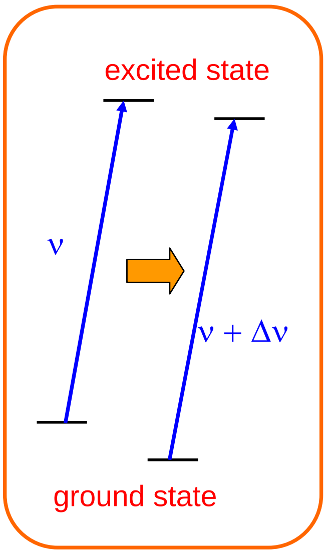
[1] S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D **69**, 121701R (2004).
[2] Y. Fujii *et al.*, Nuclear Physics B **573, 377** (2000).
[3] J. K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).
[4] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**,609 (2003).
[5] R. Srianand *et al.*, Phys. Rev. Lett. **92**, 121302 (2004).
[6] M. T. Murphy, J. K. Webb, and V. V. Flambaum, astro-ph/0612407 (2006).
[7] A. Cingöz *et al.*, Phys. Rev. Lett. **98**, 040801 (2007).
[8] M. Fischer *et al.*, Phys. Rev. Lett. **92**, 230802 (2004).
[9] E. Peik *et al.*, Phys. Rev. Lett. **93**, 170801 (2004).
[10] H. Marion *et al.*, Phys. Rev. Lett. **90**, 150801 (2003).
[11] T. M. Fortier *et al.*, Phys. Rev. Lett. **98**, 070801 (2007).

Laboratory Test

- Laboratory and controllable test strongly desired
- Ultra-cold trapped ions and atoms best testing ground



$$\frac{\Delta\nu}{\nu} = f(\nu,\alpha)\cdot\frac{\Delta\alpha}{\alpha}$$



Results From Searches for dα/dt

$\Delta\alpha/\alpha$ per year	method	quantity	ref
-(2.3 +0.7/-0.3) 10^{-17}	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	1
< 0.8 10^{-17}	Oklo nuclear reactor	$\alpha(m_\pi/\Lambda_{qcd})^{0.5}$	2
(7.2 ± 1.8) 10^{-16}	Quasar spectra	α	3
(6.4 ± 1.4) 10^{-16}	Quasar spectra	α	4
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(4.4 ± 1.6) 10^{-16}	Quasar spectra	α	6
(-2.7 ± 2.6) 10^{-15}	Dy atomic beam	α	7
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(0.3 ± 2.0) 10^{-15}	Opt-Yb^+ ↔ Cs	$g_{Cs}m_e/m_p\alpha^{1.9}$	9
(0.02 ± 0.7) 10^{-15}	Rb ↔ Cs	$g_{Cs}/g_{Rb}\alpha^{0.44}$	10
(-6.2 ± 6.5) 10^{-17}	Opt-Hg^+ ↔ Cs	$g_{Cs}m_e/m_p\alpha^{6.0}$	11

[1] S. K. Lamoreaux and J. R. Torgerson, Phys. Rev. D **69**, 121701R (2004).
[2] Y. Fujii *et al.*, Nuclear Physics B **573, 377** (2000).
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[4] M. T. Murphy, J. K. Webb, and V. V. Flambaum, Mon. Not. R. Astron. Soc. **345**,609 (2003).
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Conclusion and Outlook

- Precision atomic measurement keeps producing interesting physics
- Our direction in NTHU: Precision spectroscopy with aid of optical frequency comb

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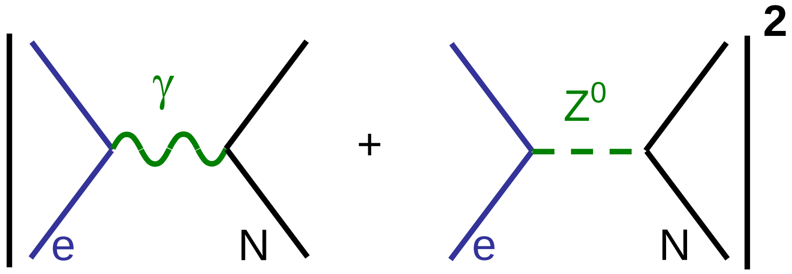
Parity violation in atom

- interaction between nucleus and electron involves coherent sum of neutral weak and electromagnetic amplitudes

$|\mathfrak{M}|^2 \propto$



- opposite parity states (e.g. 2s and 2p of H) are mixed by weak interaction

In Hydrogen

- conventional characterization in atomic physics: weak charge

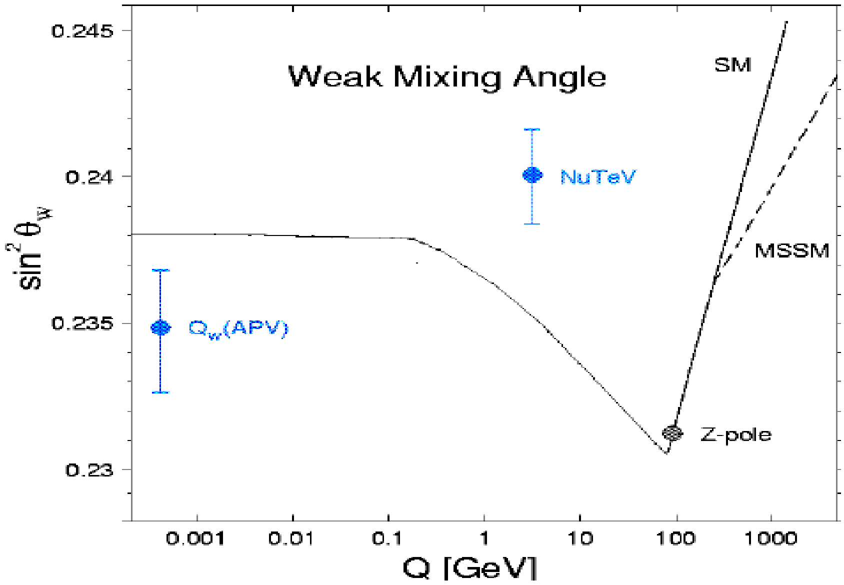
$$Q_W(Z,N) = (1 - 4\sin^2\theta_W)Z - N$$

- Mixing between 2s and 2p state

$$\varepsilon \sim \frac{\langle 2\hat{s}|H_{weak}|2\hat{p}\rangle}{\Delta E} \propto \frac{Q_W}{L} \cong \frac{1/80\ Hz}{L} \cong 10^{-11}$$

- Measure asymmetry in transition rate under reversed E, B field
- Weak mixing angle $\mathrm{Sin}\theta_W$

Weinberg angle



- Only success: Cs APV by C. Wieman
- Recently, D. Budker, large APV effect in Yb

Bismuth: M. Macpherson et al, PRL 67, 2684 (1991)
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