Quantum turbulence in a Two-dimensional Trapped Bose-Einstein Condensates

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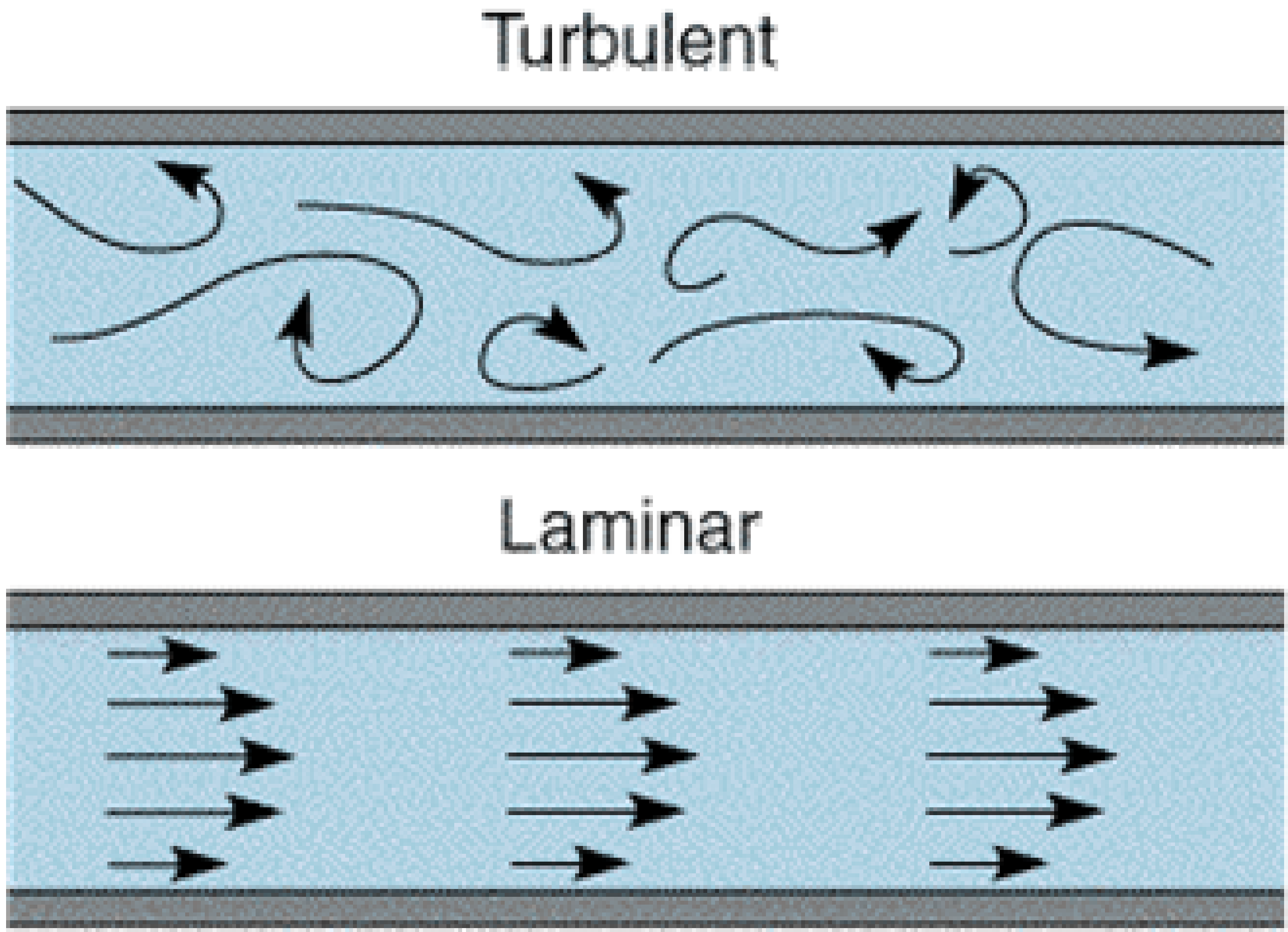
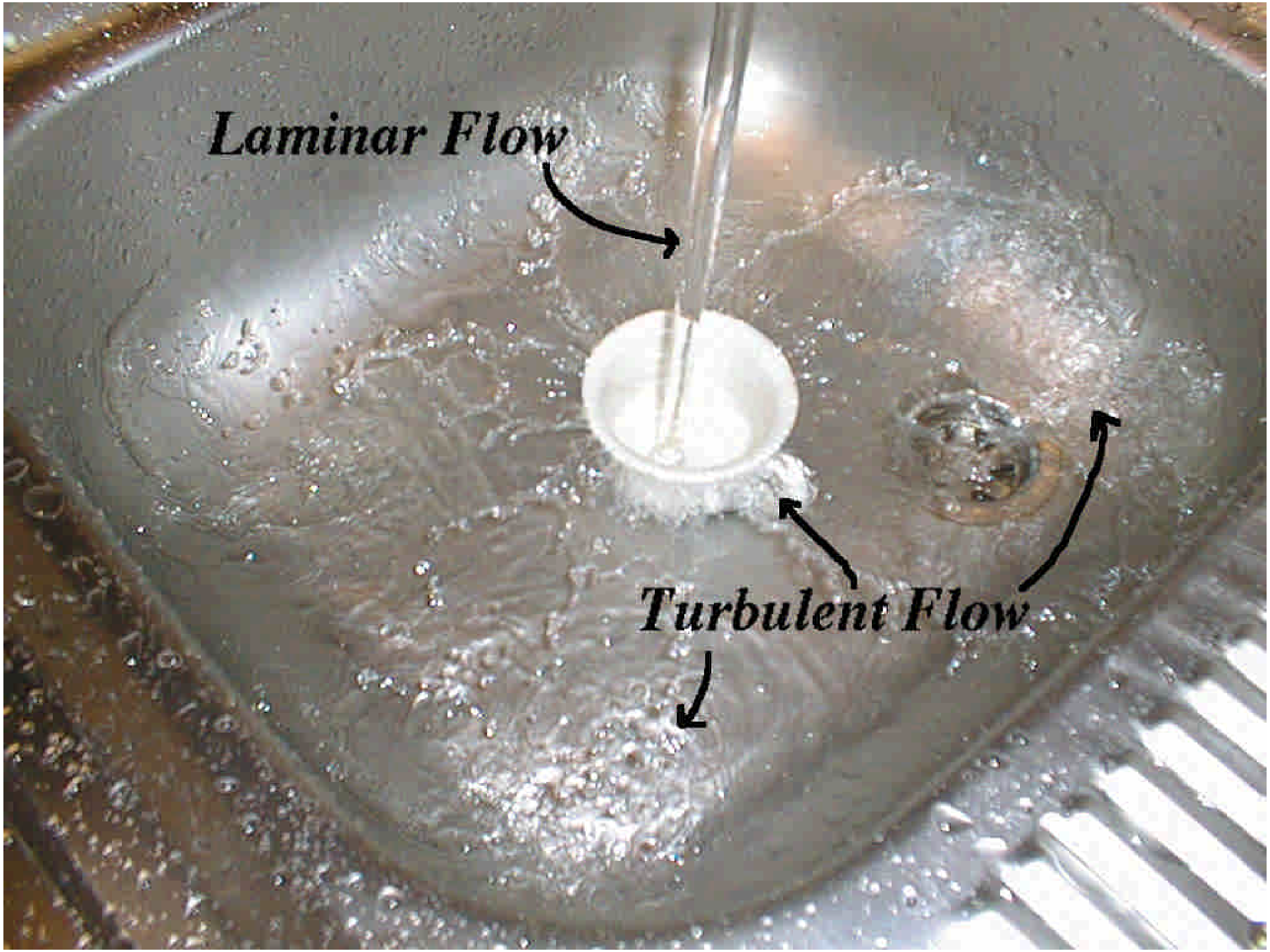
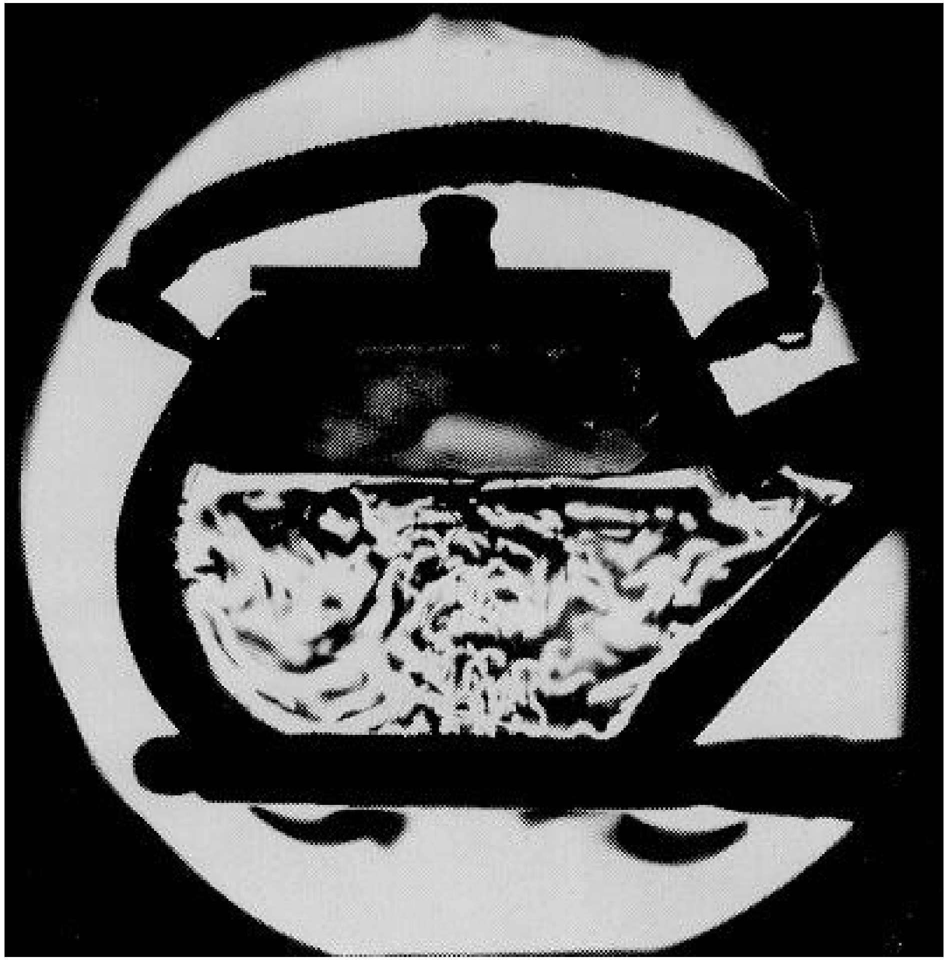
NTHU, Apr 15, 2009

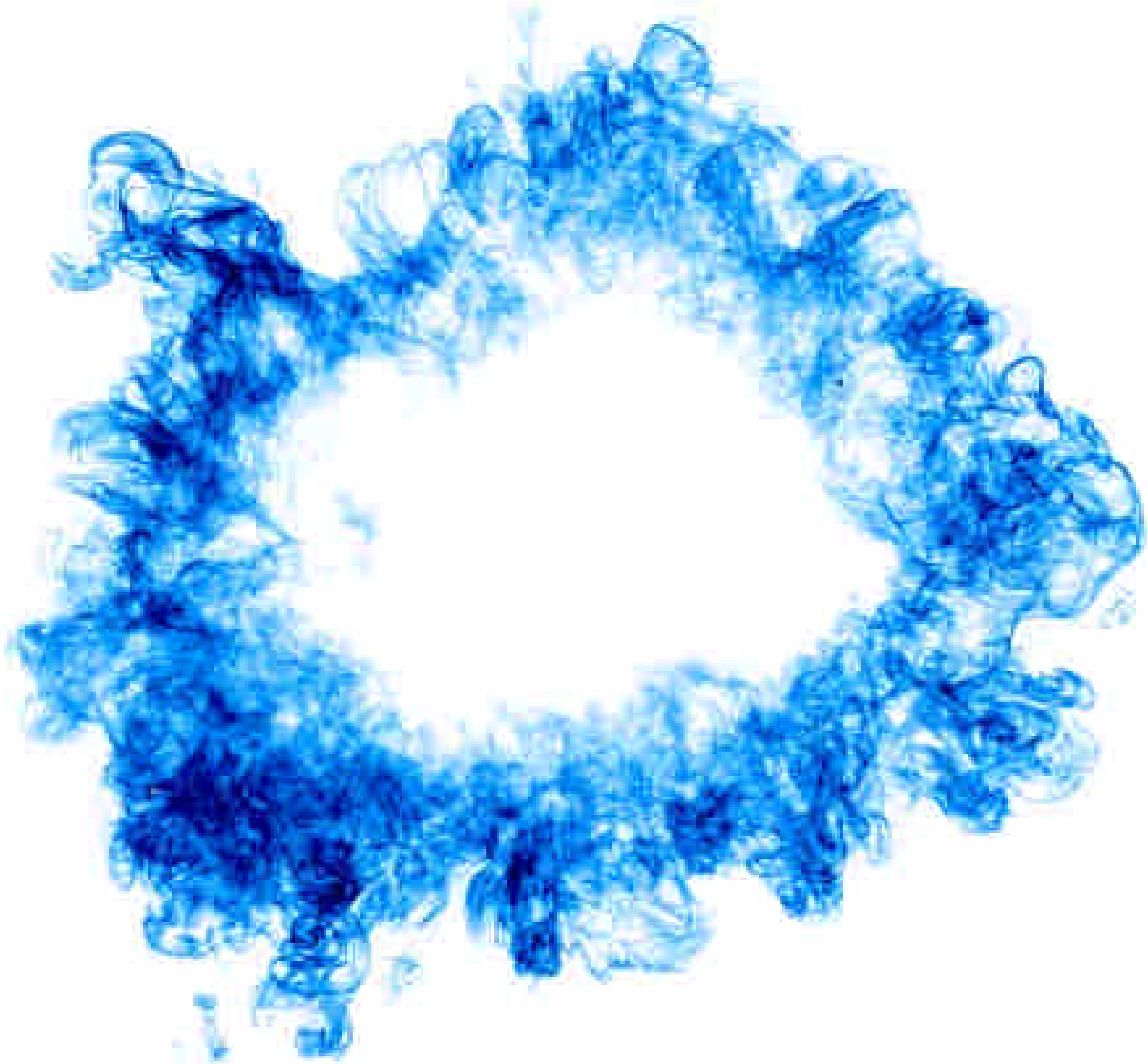

Turbulent
Laminar


Laminar Flow
Turbulent Flow

Turbulence in everyday life



Turbulence in Nature



Turbulence in Nature

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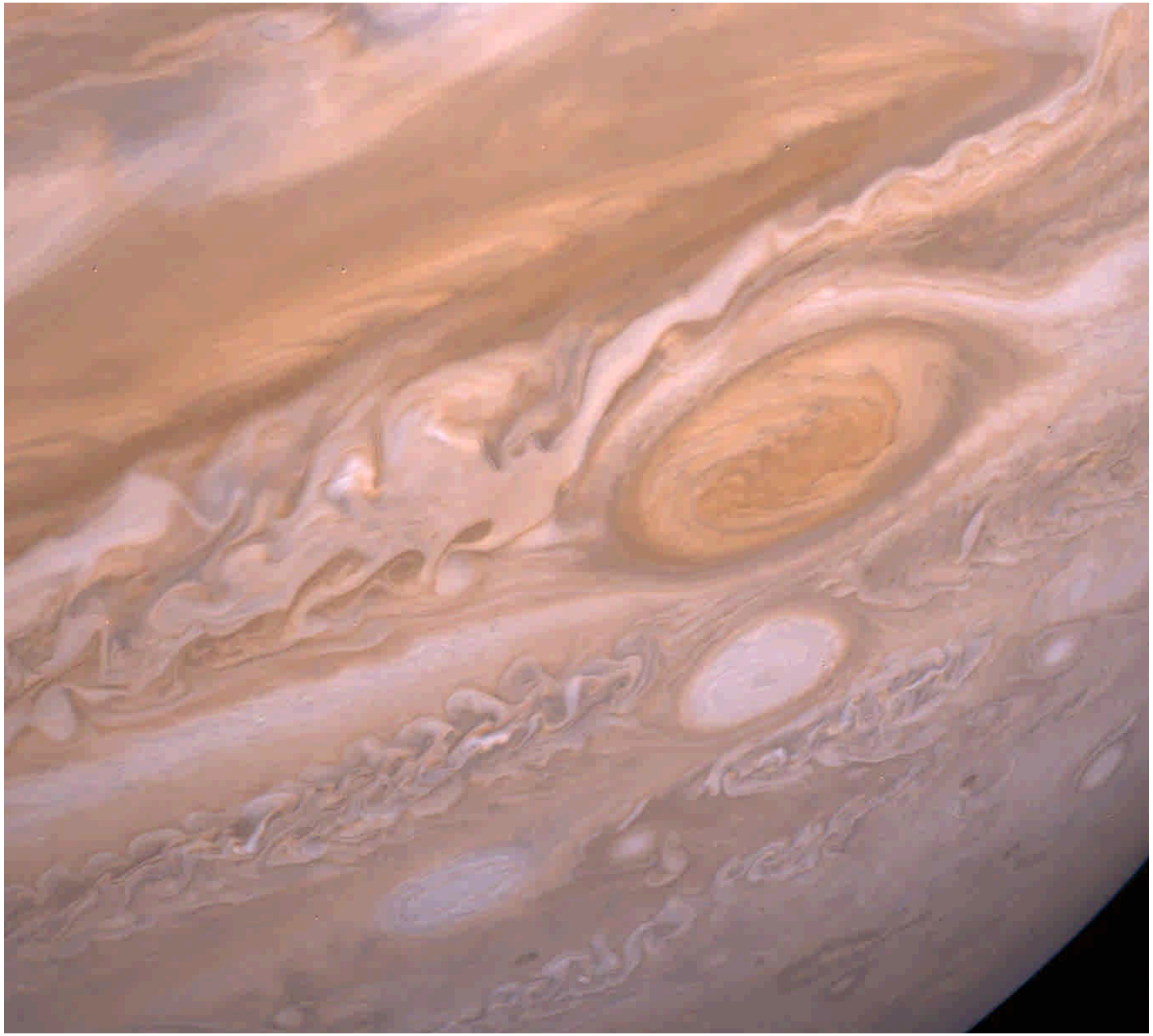
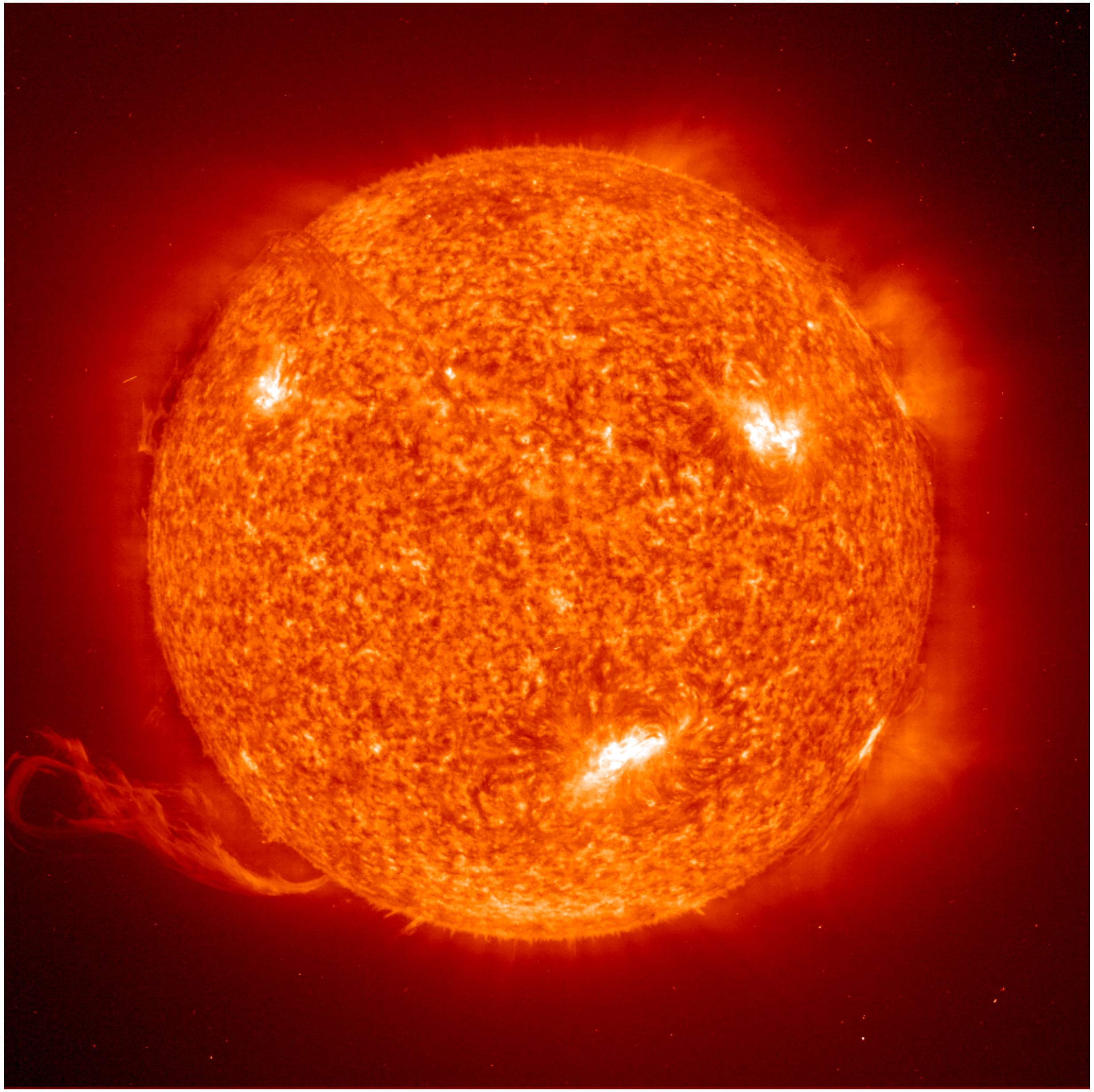
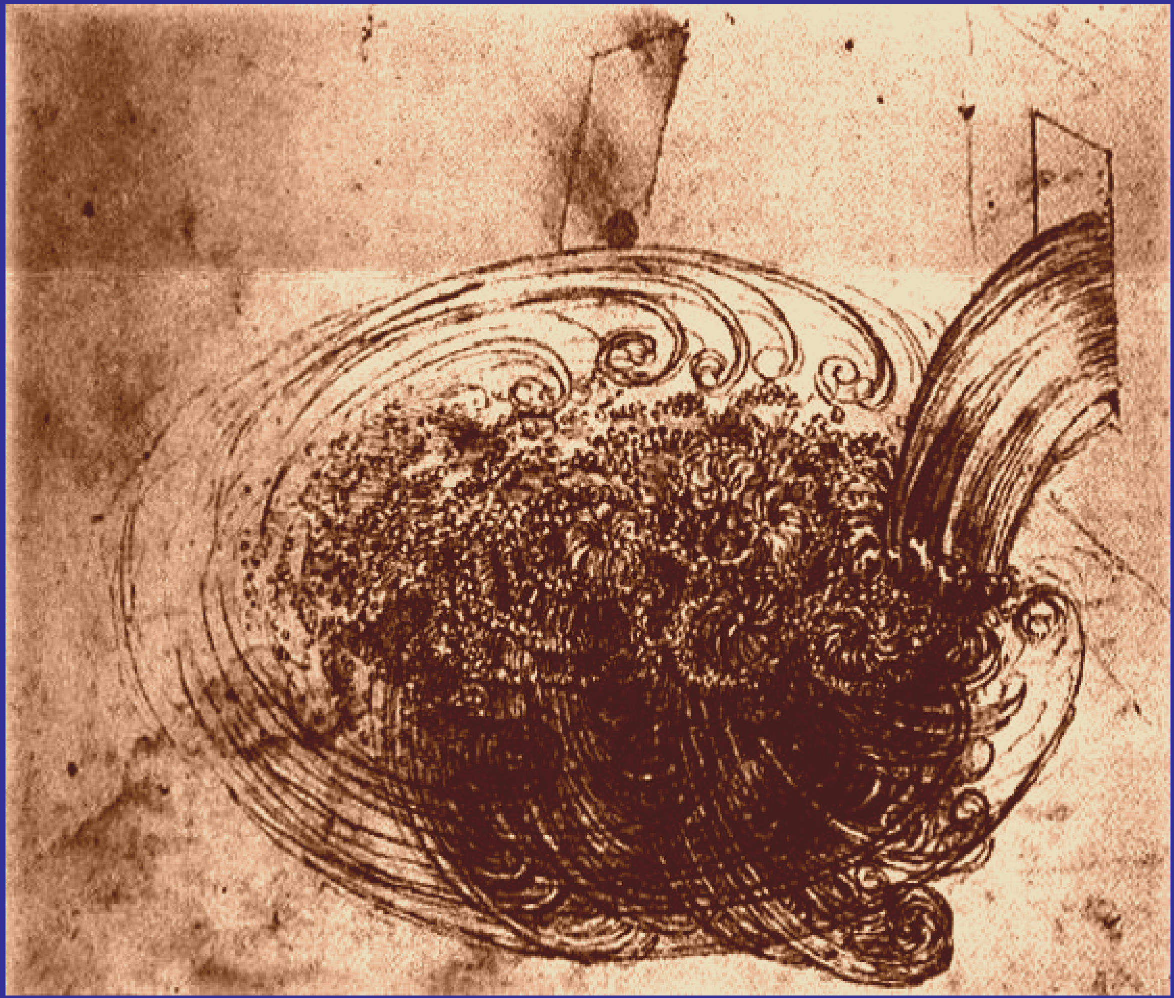


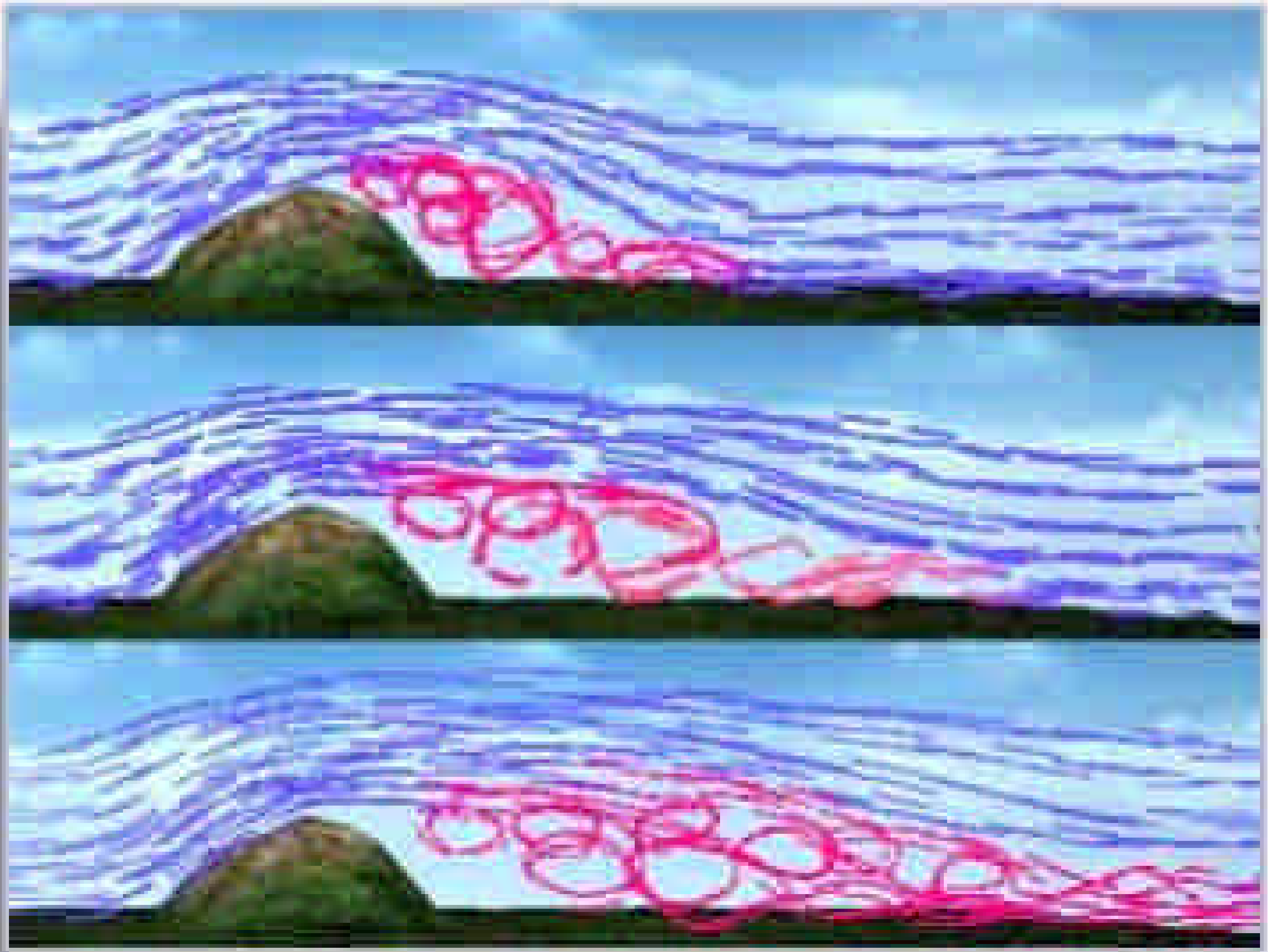
Turbulence in Nature



Da Vinci's code on classical turbulence



Turbulence is not simply a disordered state, but rather some kind of self-organized motion with vortices.



Reynolds number as a measure for the formation of turbulence

density of fluid

characteristic length of the problem

$$\mathrm{Re} = \frac{nLU}{\nu}$$

characteristic velocity of the problem

viscosity

Taylor Vortex Flow (Re = 177)

Wavy Vortex Flow (Re = 505)

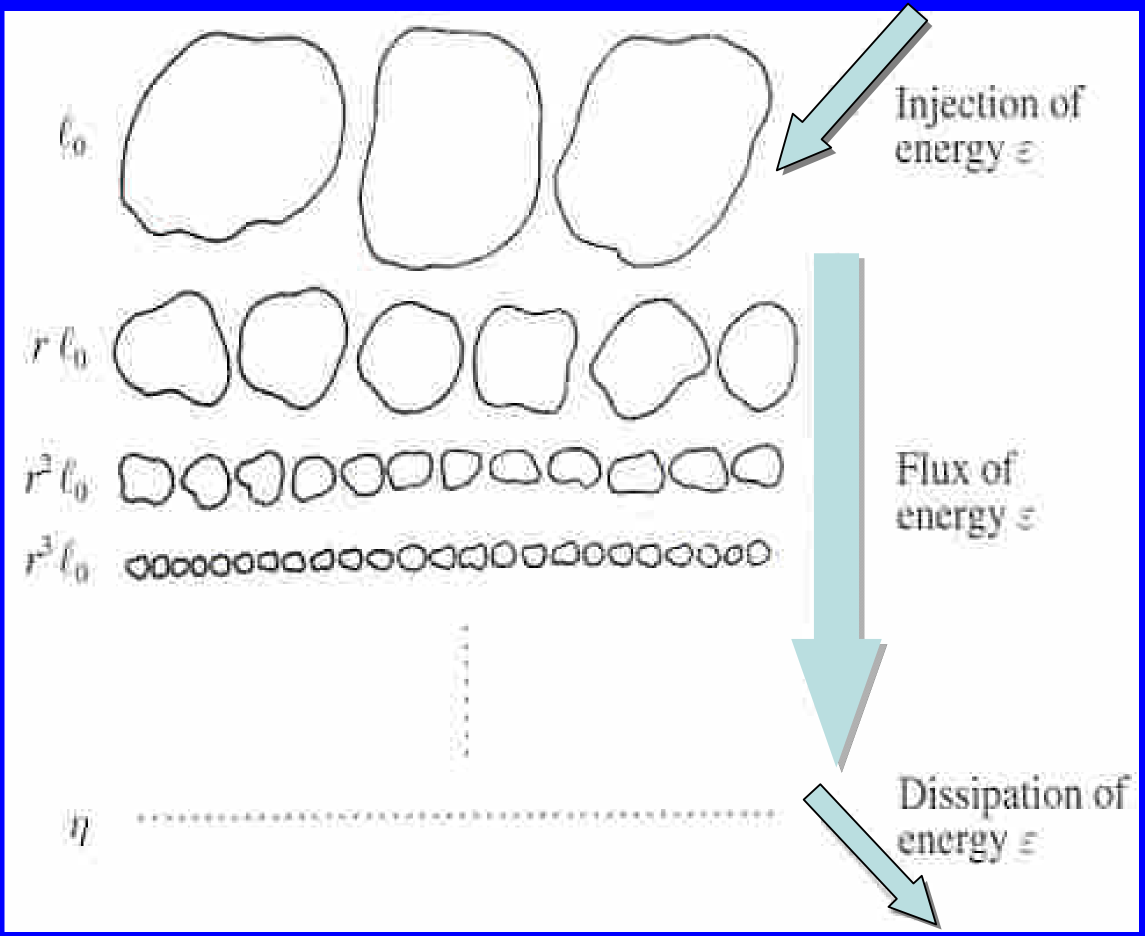
Weakly Turbulent Vortex Flow (Re = 3027)

Turbulent Vortex Flow (Re = 8072)

Reynolds numbers in everyday life

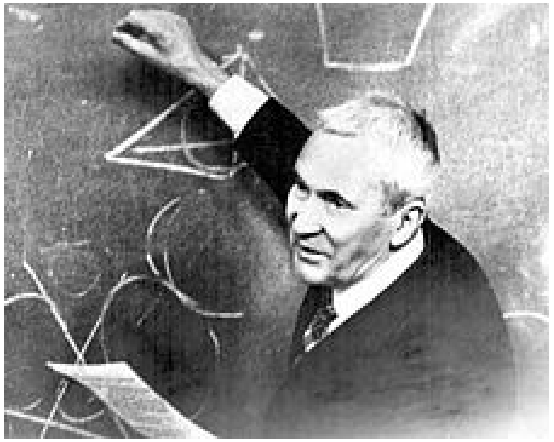
	Re
Water flow in a 2.5cm diameter bathroom supply pipe	2×10^4
A baseball thrown by a major league picture	2×10^5
A mild 10m/s breeze	3×10^5
An automobile driving at highway speeds	10^7
Wing of a commercial jet airplane	2×10^8
Typical atmospheric low-pressure system	10^{12}

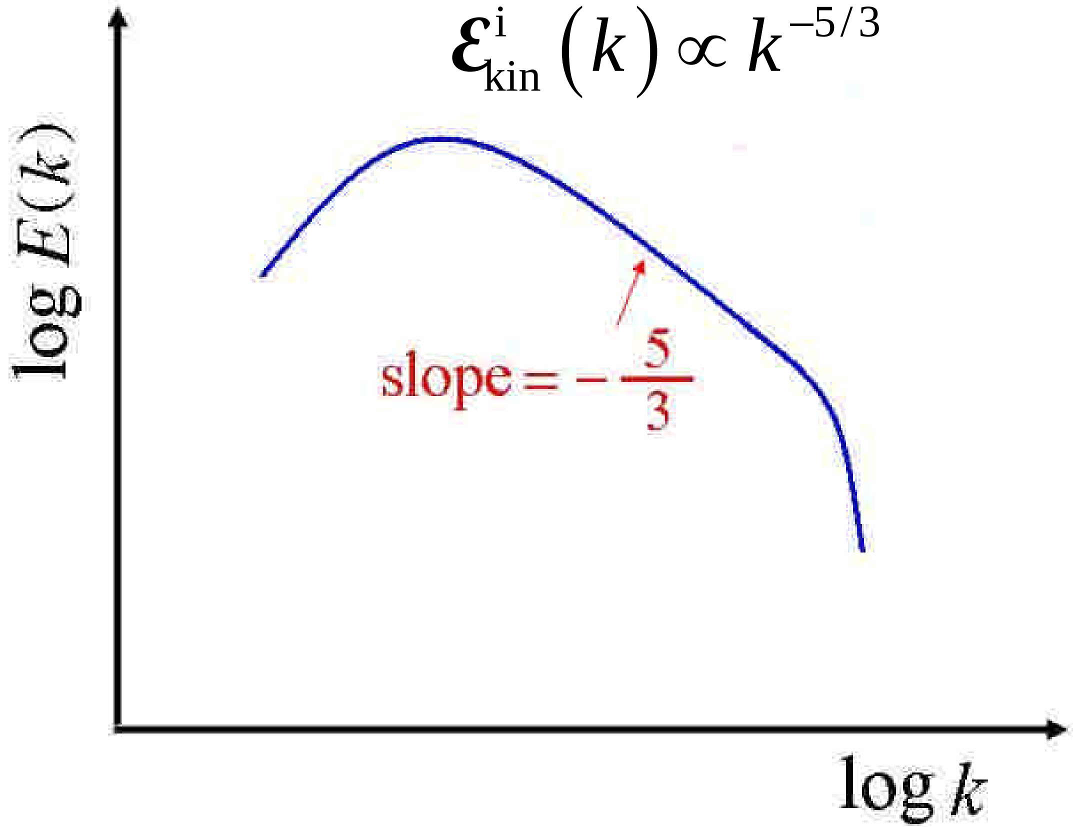
Energy cascade in the turbulent flow



NOTE:It is believed that turbulence is sustained by this Richardson cascade. However, this is only a cartoon; nobody has ever confirmed it clearly. One reason is that it is too difficult to identify each individual eddy in the fluid.

Kolmogorov's scaling law of incompressible kinetic energy





$$\text{Kinetic Energy} = \frac{1}{2}\int \rho \mathbf{v}^2 d\mathbf{r} = \int \mathcal{E}^{\mathrm{i}}_{\mathrm{kin}}(k)\, dk$$

Kolmogorov's first universality assumption:

At very high, but not infinite Reynlds numbers, all the small-scale statistical properties are uniquely and universally determined by the scale $1/k$, the mean energy dissipation rate ε and the viscosity ν.

In the inertial range, the energy spectrum takes the form

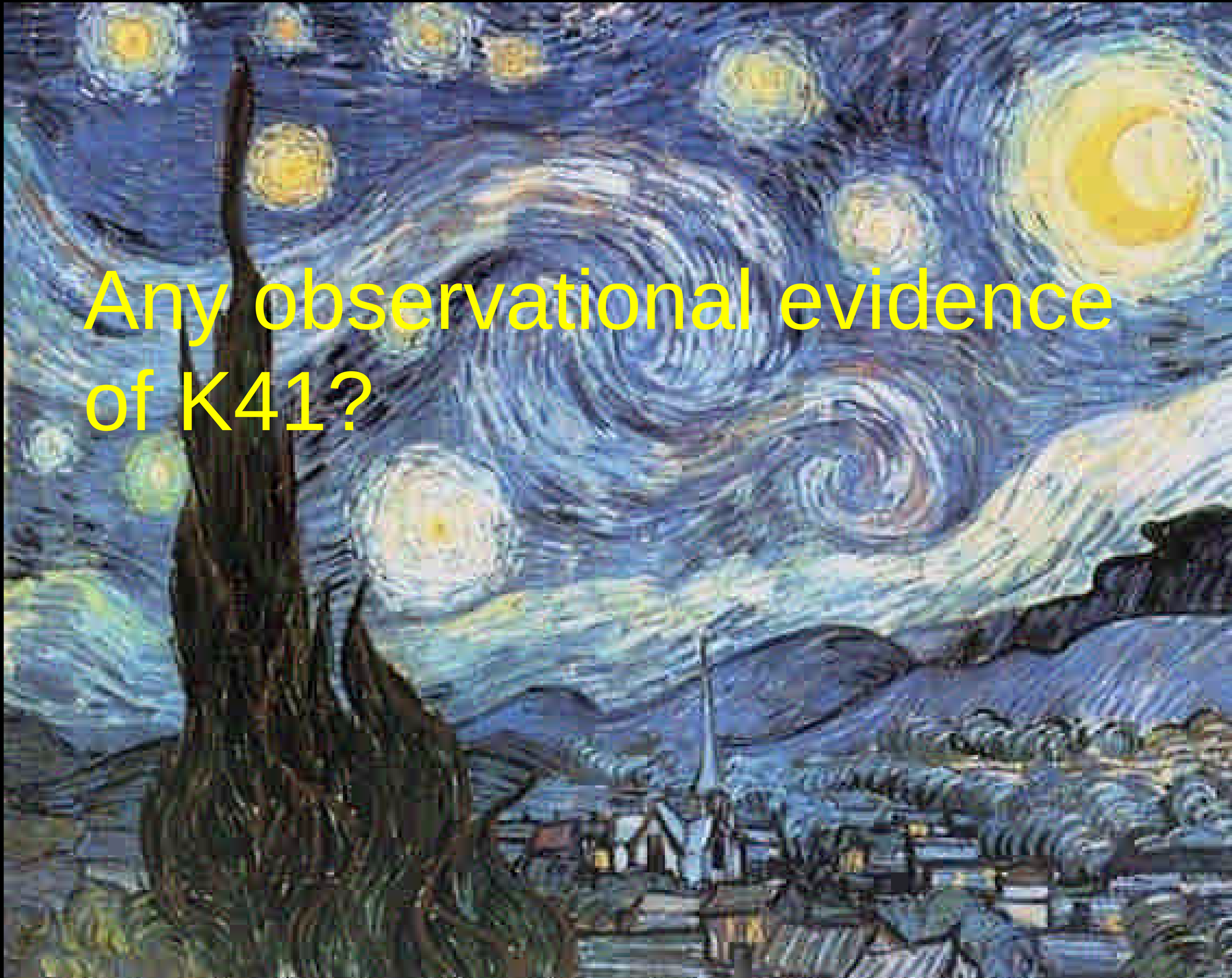
$$\mathcal{E}^{\mathrm{i}}_{\mathrm{kin}}(k) = F(k\eta)\,\varepsilon^{\alpha} k^{\beta}$$

$$\begin{cases} F(k\eta)\text{: dimensionless function} \\ \eta = \left(\nu^3/\varepsilon\right)^{1/4}\text{: Kolmogorov length} \end{cases}$$

Simple dimensional argument leads to

$$\mathcal{E}^{\mathrm{i}}_{\mathrm{kin}}(k) \sim \varepsilon^{2/3} k^{-5/3} \qquad \text{(K41)}$$

Any observational evidence of K41?

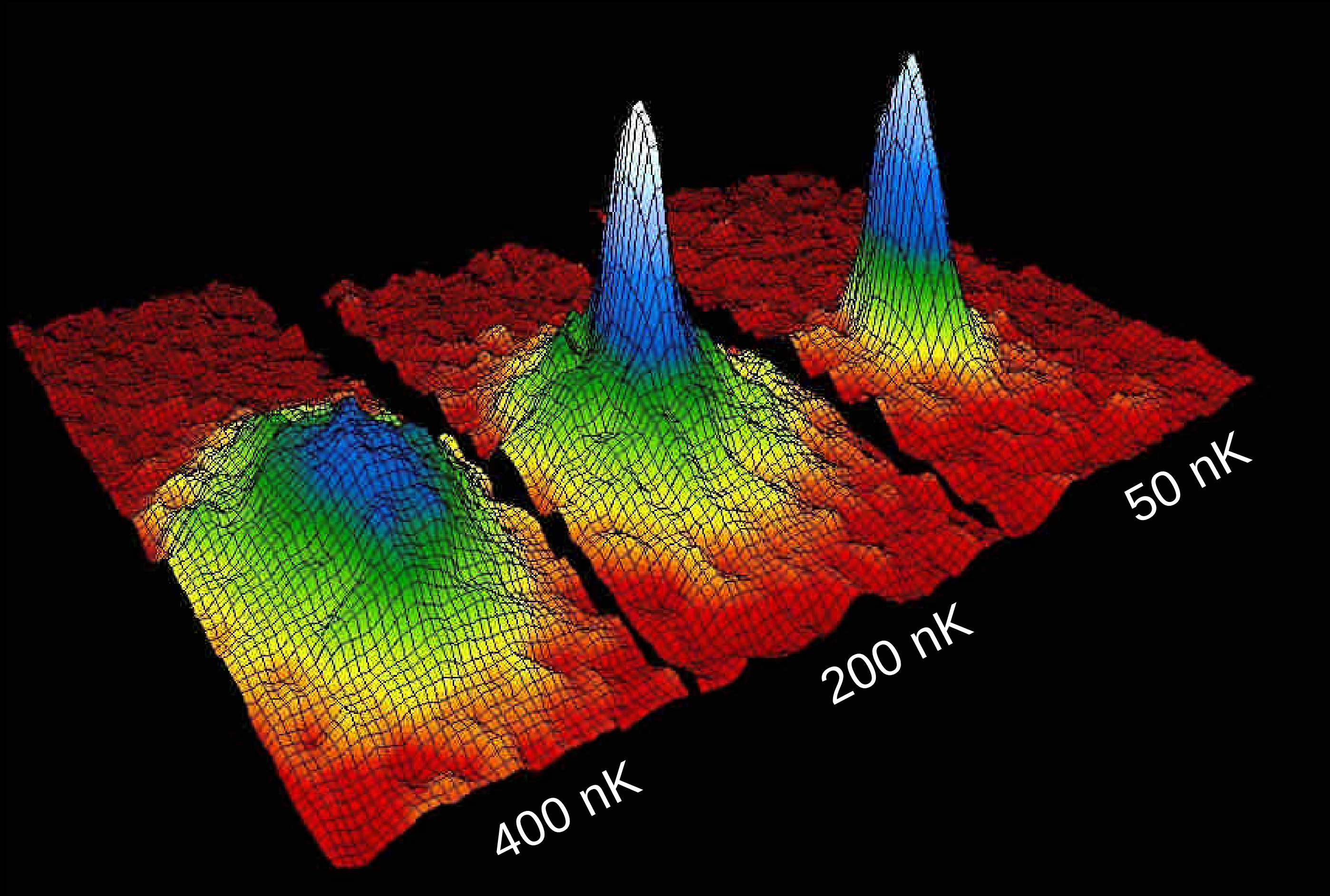


See ***Plus e-zine*** News, 21/07/2006

Troubled minds and perfect turbulence

Two Mexican physicists, J. L. Aragón and G. Naumis, have examined the patterns in van Gogh's *Starry Night* and concluded that the pair distribution function of luminosity follows a Kolmogorov -5/3 scaling law.

BEC — the Holy Grail of Atomic Physics!



M. H. Anderson, et. al., Science **269**, 198 (1995)

Nobel Prize in Physics, 2001

“for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

Eric Cornell
JILA, University of Colorado

Wolfgang Ketterle
MIT

Carl Wieman
JILA, University of Colorado

Characterization of superfluidity

1. Existence of a critical velocity v_c such that the viscosity disappears if the relative velocity between the fluid and the wall of container is smaller than v_c (Landau criterion)

2. Existence of vortices with quantized circulation (Feynman-Onsager criterion)

Feynman's theorem of circulation quantization

$$\Psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\theta(\mathbf{r})}$$

order parameter

$$\mathbf{v}_s = \frac{2\pi\hbar}{m}\nabla\theta(\mathbf{r})$$

superfluid velocity

$$\oint_C \mathbf{v}_s \cdot d\mathbf{r} = n\frac{2\pi\hbar}{m}$$

circulation quantization
around the singularity

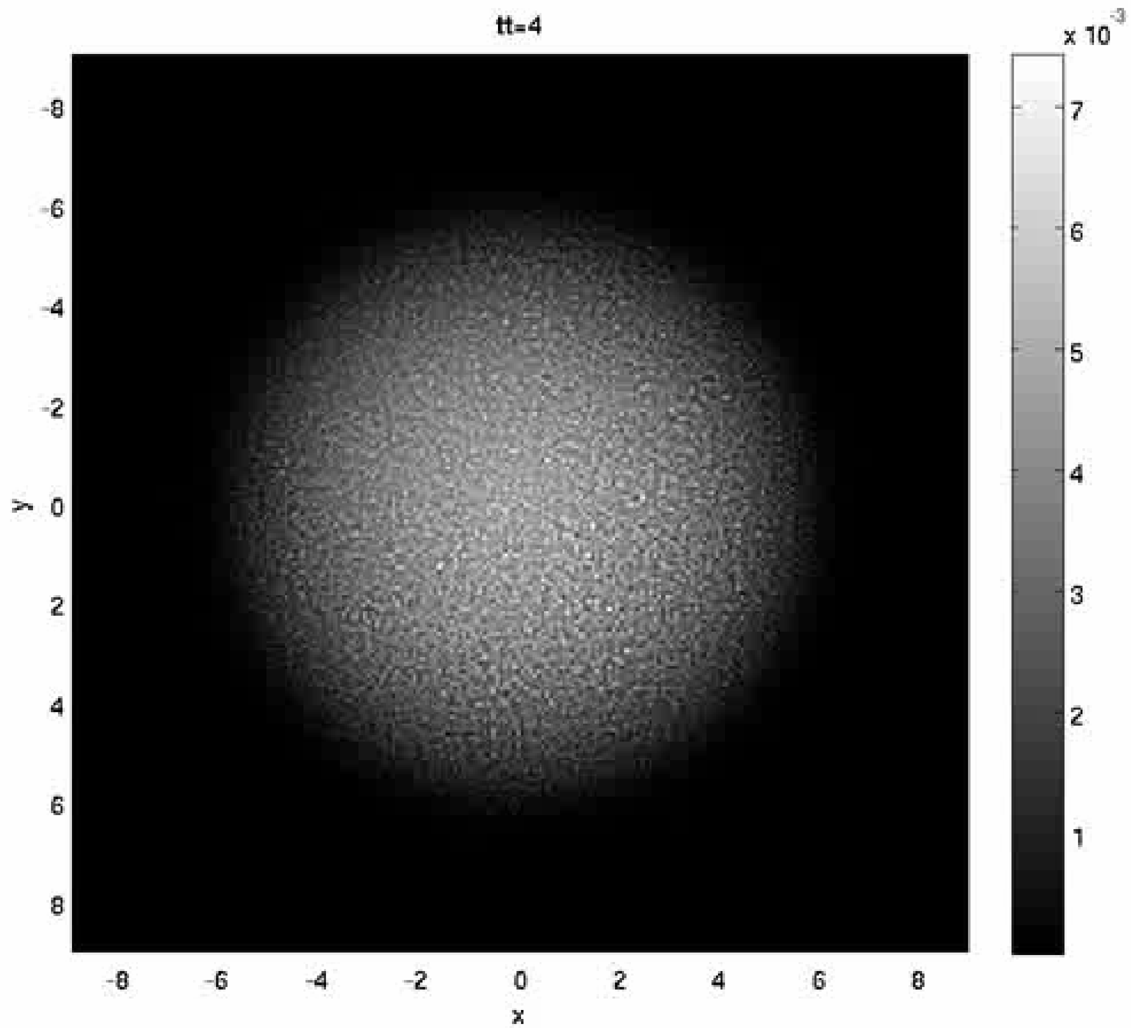
Feynman's theorem of circulation quantization

$$\Psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})} e^{i\theta(\mathbf{r})}$$ order parameter

$$\mathbf{v}_s = \frac{2\pi\hbar}{m} \nabla\theta(\mathbf{r})$$ superfluid velocity

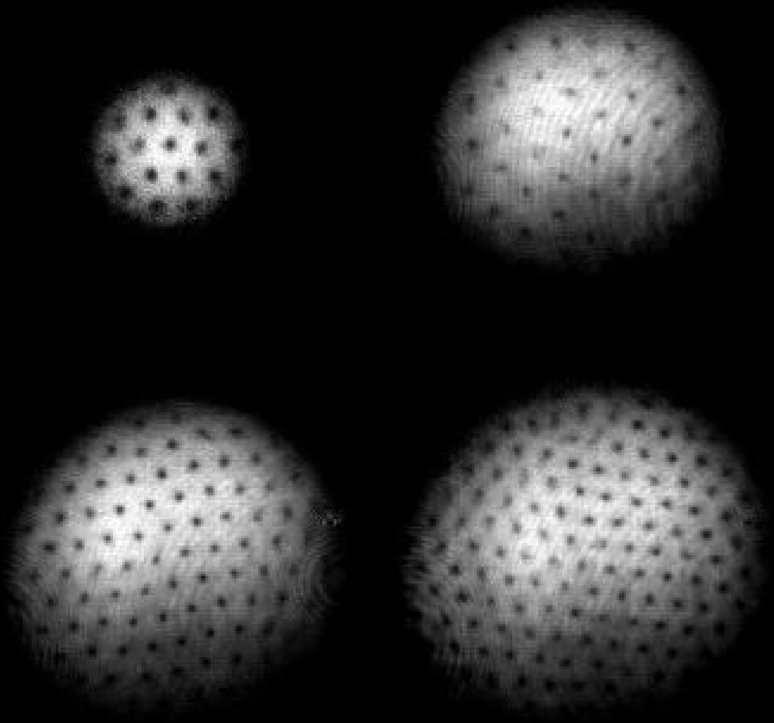
$$\oint_C \mathbf{v}_s \cdot d\mathbf{r} = n\frac{2\pi\hbar}{m}$$ circulation quantization around the singularity

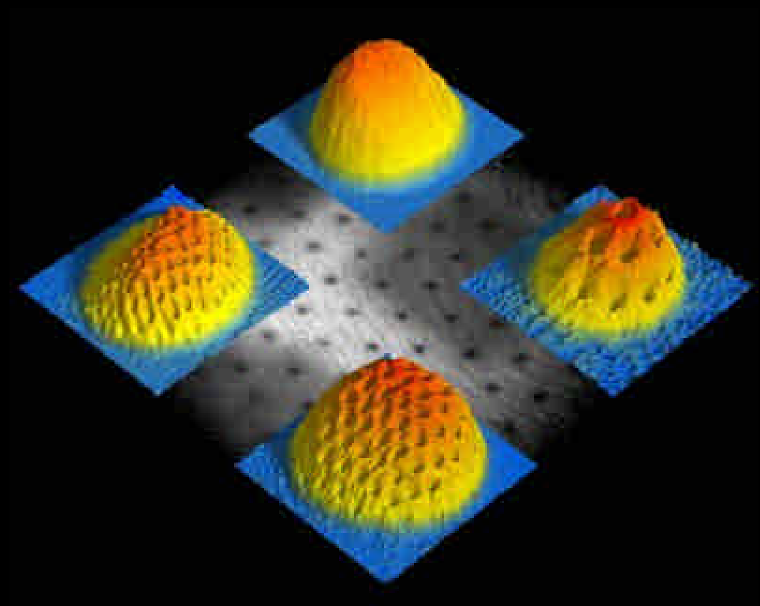
Formation of vortex lattice in a quenched rotating BEC



(Simulation based on the stochastic GPE formalism by NCUE group) $T_f = 11nk,\ \Omega = 0.8\omega_r,\ \mu = 8$

Large Abrikosov Vortex Lattices

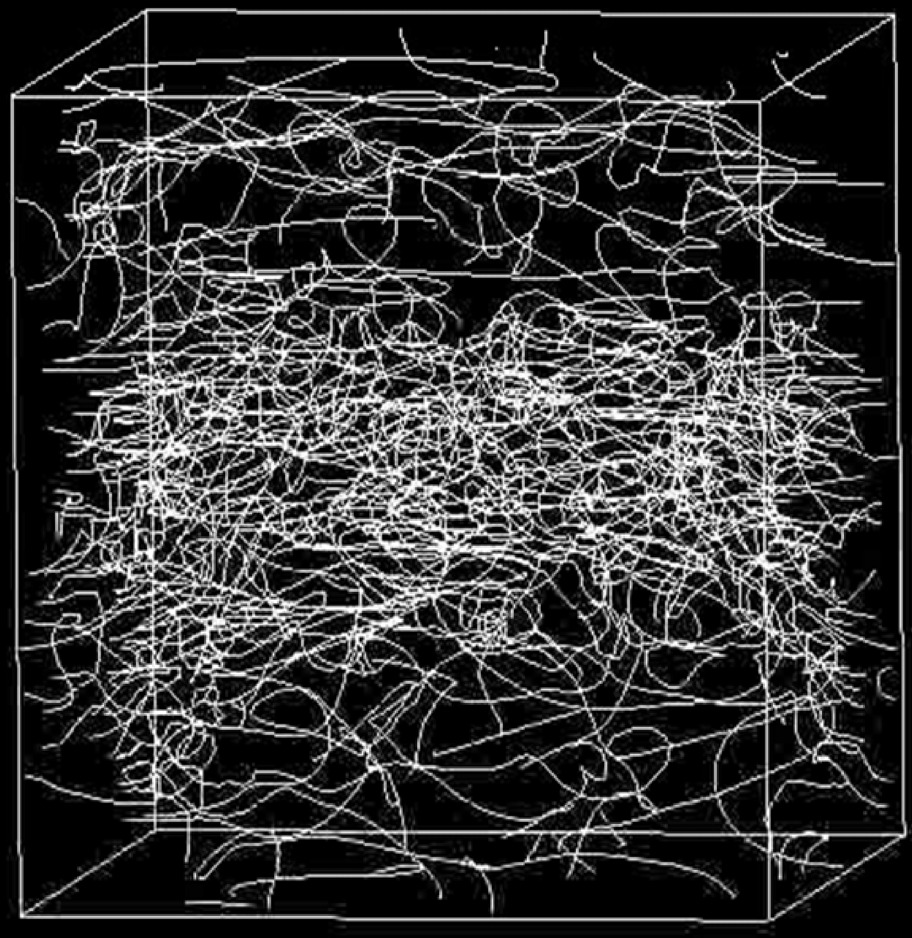




Up to 150 vortices!

J. R. Abo-Shaeer, *et al.* Science **292**, 476 (2001)

There are two main cooperative phenomena of quantized vortices; vortex array under rotation and vortex tangle. Feynman considered the latter as the source of quantum turbulence.



Recently, it has been found theoretically that QT follows Komogorov's -5/3 law even at T=0.

●***Decaying Kolmogorov turbulence in a model of superflow***

C. Nore, et,al. Phys. Fluids 9, 2644 (1997)

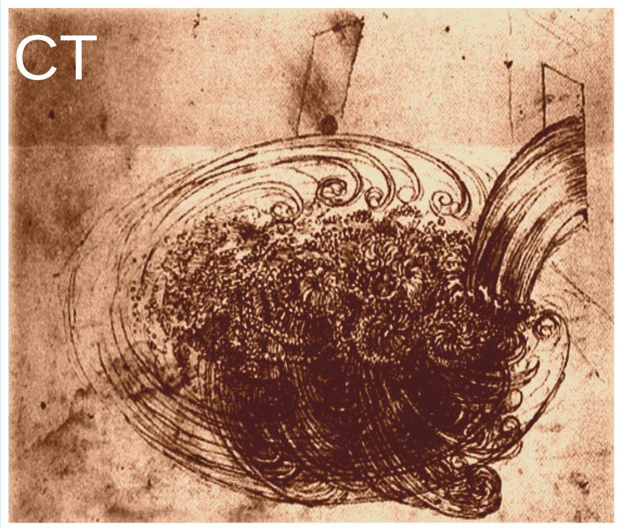
●***Energy Spectrum of Superfluid Turbulence with No Normal-fluid component***

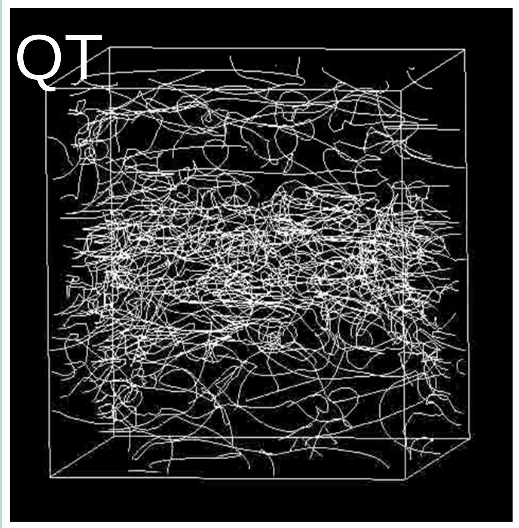
T. Araki, M. Tsubota, and S. K. Nemirovskii, Phys. Rev. Lett. 89, 145301 (2002)

●***Kolmogorov Spectrum of Superfluid Turbulence: Numerical Analysis of the Gross-Pitaevskii Equation with a Small Dissipation***

M. Kobayashi and M. Tsubota, Phys. Rev. Lett. 94, 065302 (2005).

Classical Turbulence vs Quantum Turbulence





- The quantized vortices are stable topological defects
- Every vortex has the same circulation
- Circulation is conserved

QT is much simpler than CT, because each element of turbulence is definite.

Gross-Pitaevskii Equation for a BEC

$$i\hbar \frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) + g\left|\Psi(\mathbf{r},t)\right|^2 \right]\Psi(\mathbf{r},t)$$

L.P. Pitaevskii

1. Gross, E. P., 1961, Nuovo Cimento 20, 454.
2. Pitaevskii, L. P., 1961, Zh. Eksp. Teor. Fiz. 40, 646 [Sov. Phys. JETP 13, 451 (1961)].

Thomas-Fermi approximation for GPE

For stationary solutions

$$\Psi(\mathbf{r},t) = \Psi(\mathbf{r})e^{-i\mu t/\hbar}$$

Time-independent GPE

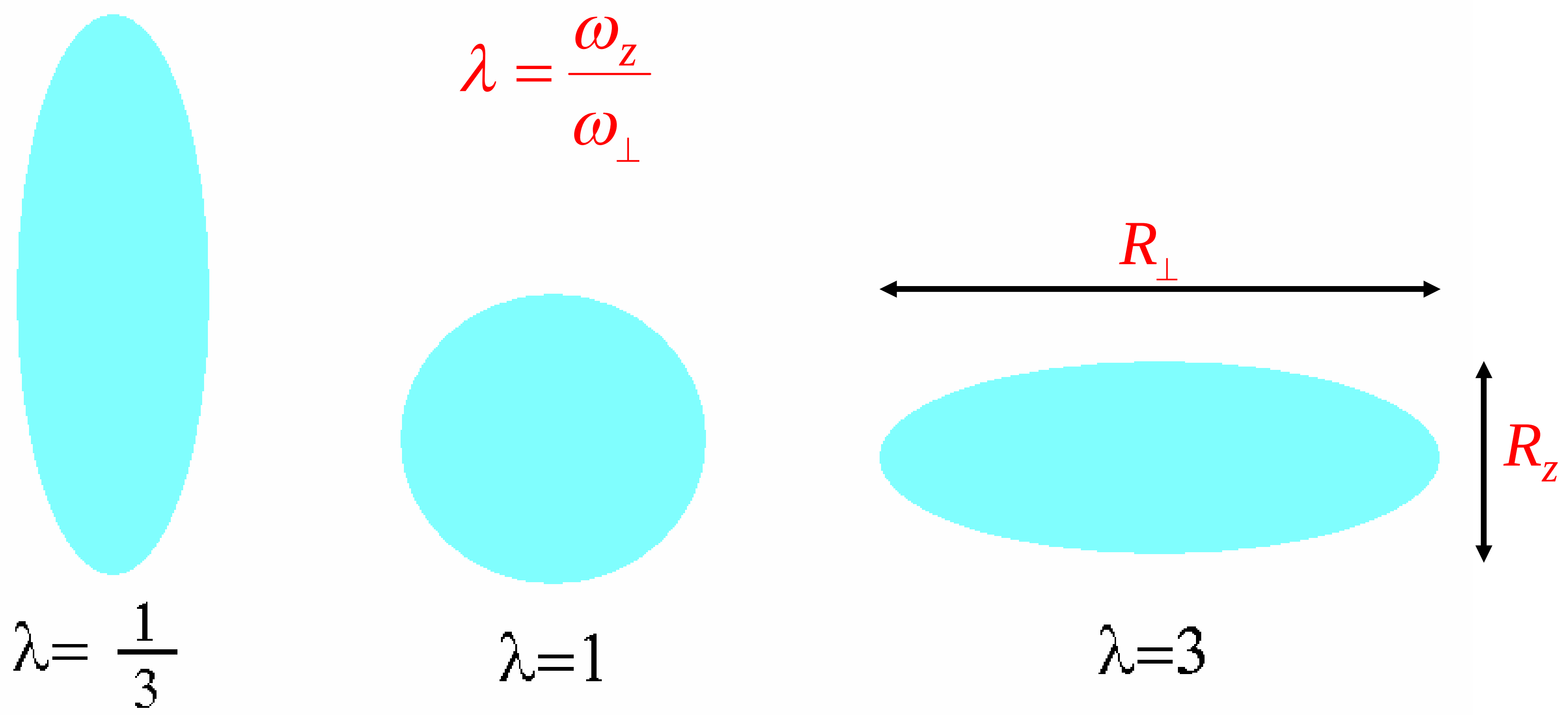
$$\Rightarrow \left[-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) + g\left|\Psi(\mathbf{r})\right|^2\right]\Psi(\mathbf{r}) = \mu\Psi(\mathbf{r})$$

Neglecting the kinetic term

$$\Rightarrow \left|\Psi_{TF}(\mathbf{r})\right|^2 = \frac{\mu - V_{ext}(\mathbf{r})}{g}$$

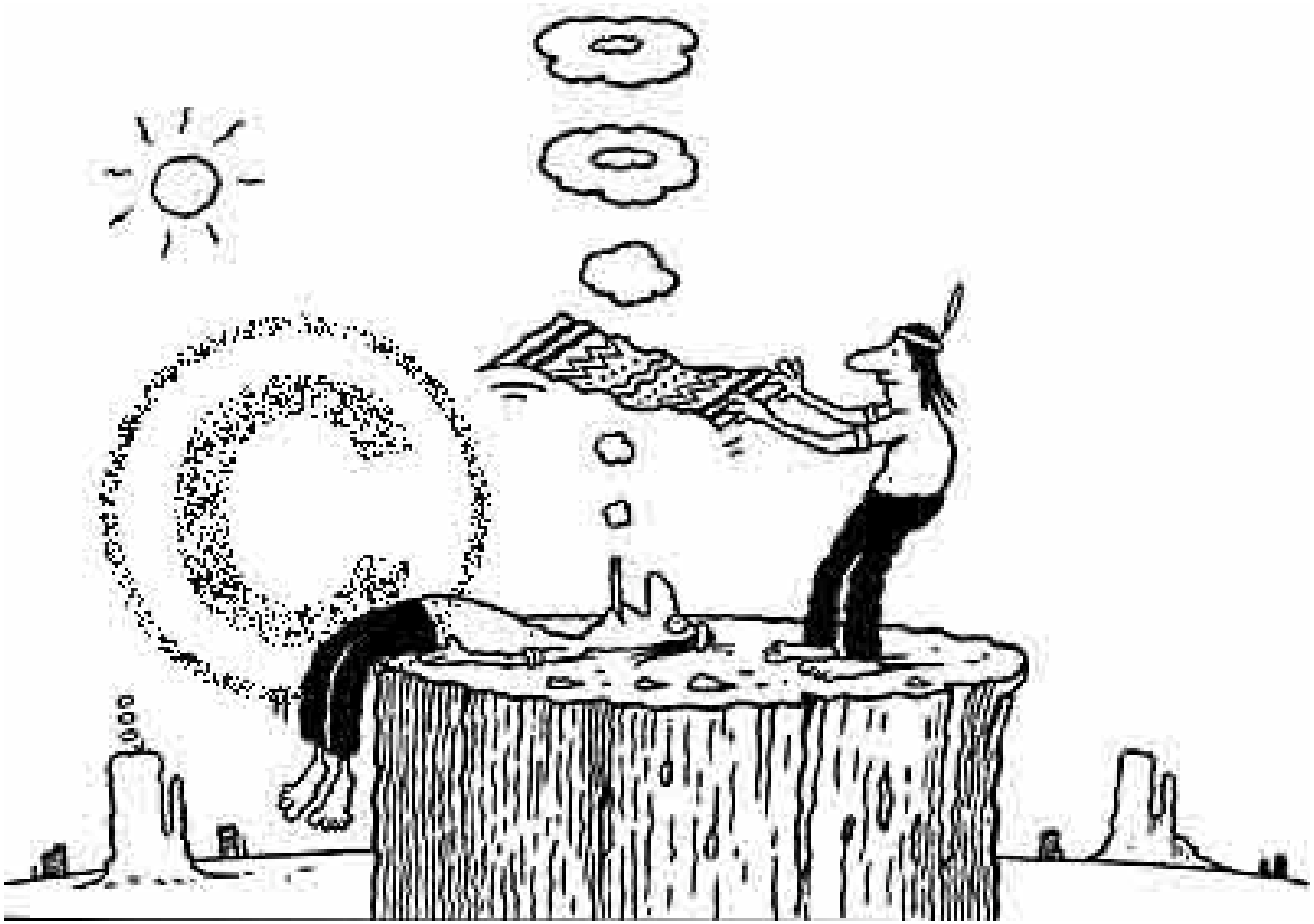
Thomas-Fermi approximation

Aspect ratio and the shape of the trapped condensate



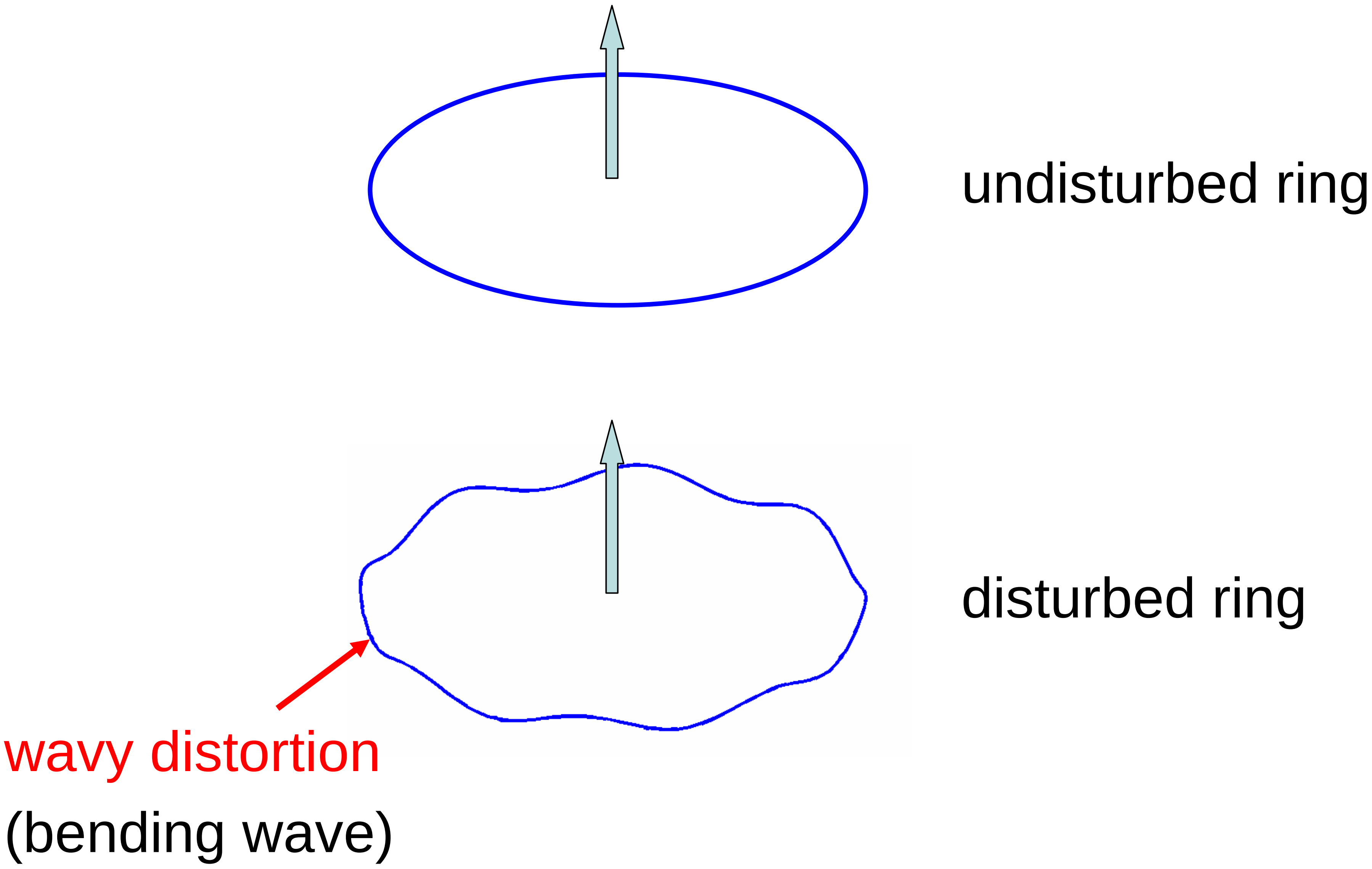
(cigar) (spherical) (pancake)

Vortex rings

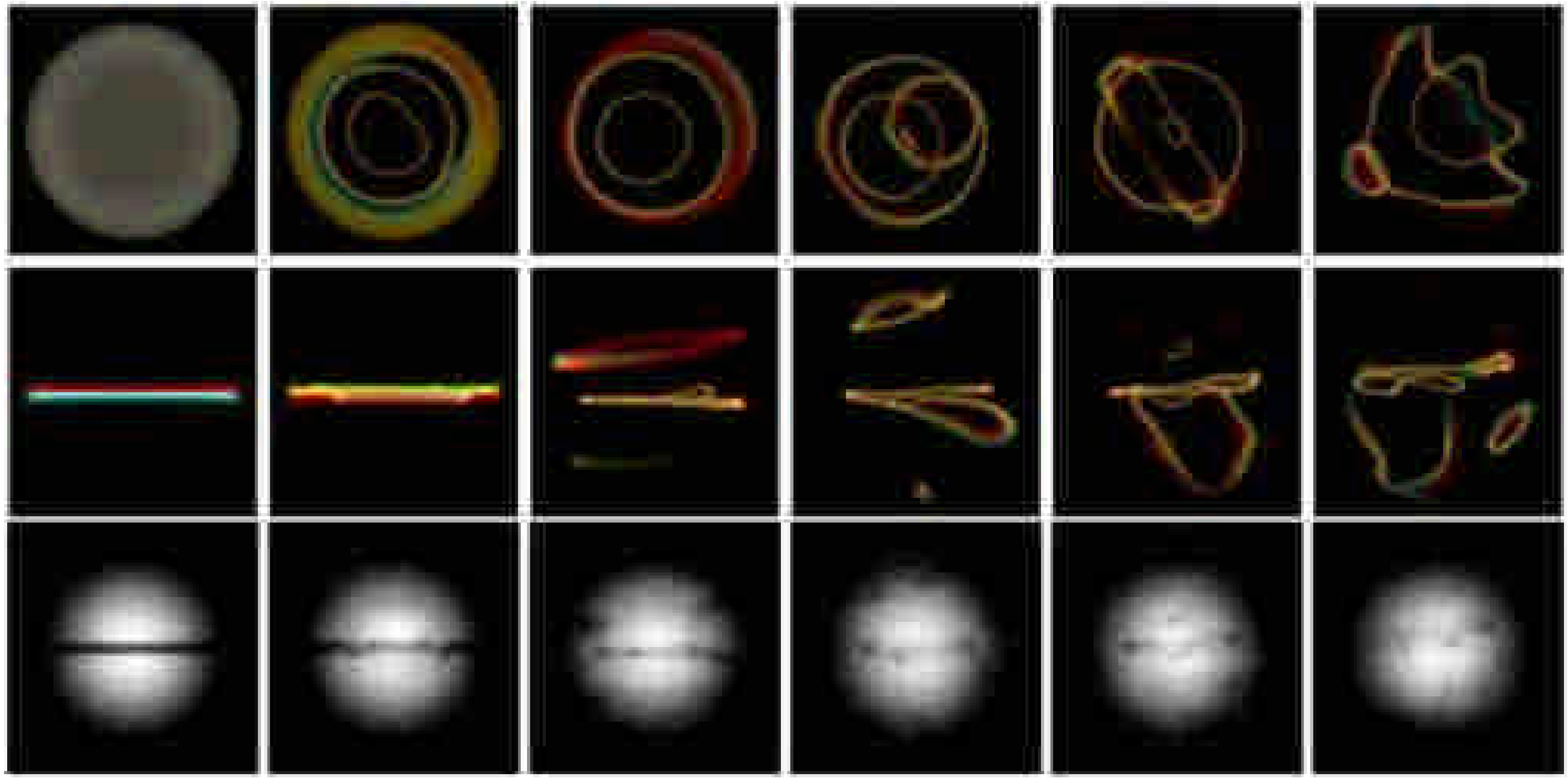


Vortex rings underwater

Bending waves on a vortex ring



Using the snake instability of a dark soliton to generate vortex rings in BEC



B.P. Anderson et al., PRL **86**, 2926 (2001)

D.L. Feder et al., PRA **62**, 053606 (2000)

RAPID COMMUNICATIONS

PHYSICAL REVIEW A **74**, 041603(R) (2006)

Bending-wave instability of a vortex ring in a trapped Bose-Einstein condensate

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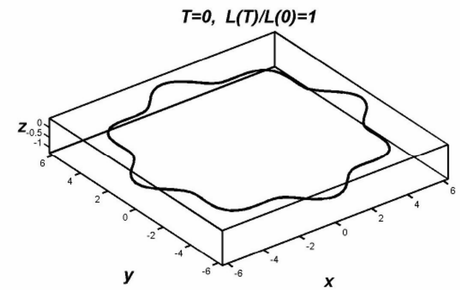
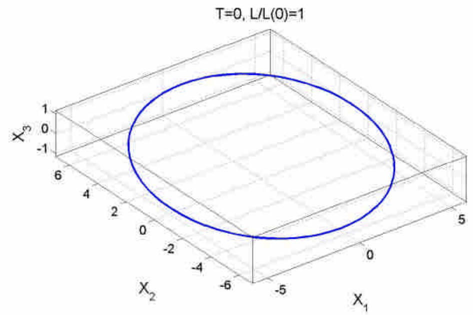
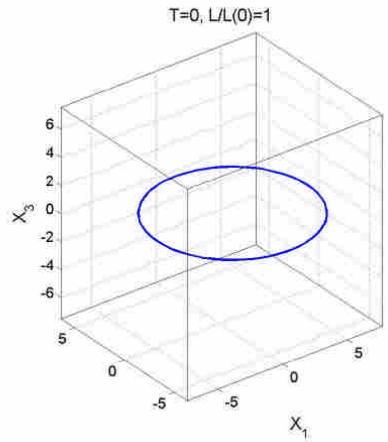
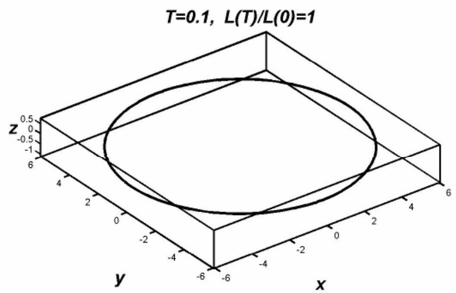
Using a velocity formula derived by matched asymptotic expansion, we study the dynamics of a vortex ring in an axisymmetric Bose-Einstein condensate in the Thomas-Fermi limit. The trajectory for an axisymmetrically placed and oriented vortex ring shows that it generally precesses in a condensate. The linear instability due to bending waves is investigated both numerically and analytically. General stability boundaries for various perturbed wave numbers are computed. Our analysis suggests that a slightly oblate trap is needed to prevent the vortex ring from becoming unstable.

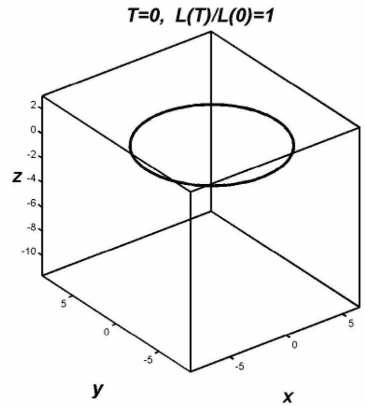
DOI: 10.1103/PhysRevA.74.041603 PACS number(s): 03.75.Lm, 03.75.Kk, 67.40.Vs





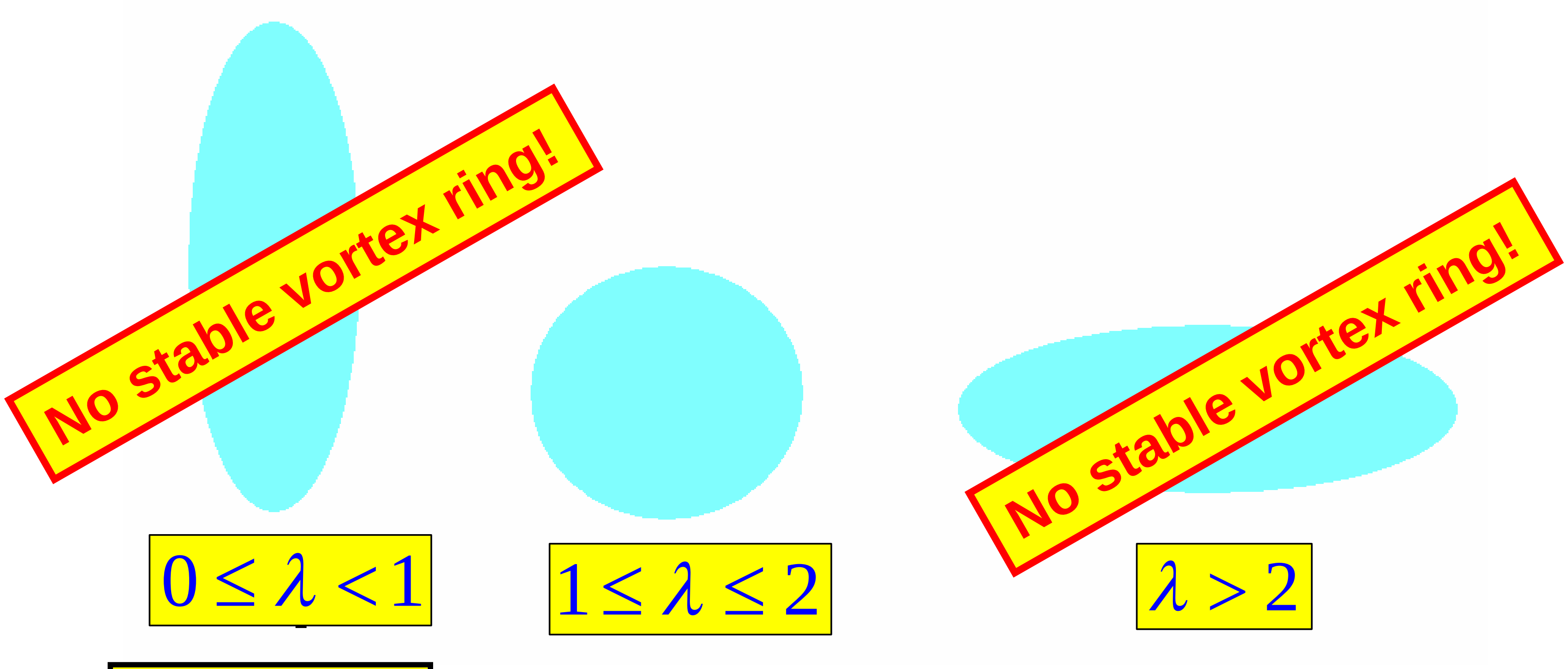






Conclusion:

A trap of slightly oblate geometry ($1 \le \lambda \le 2$) is necessary to ensure the stability of a vortex ring in a trapped BEC



Question:

What will BEC become after the disruption of vortex ring?

IOP PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **40** (2007) 4561–4571 doi:10.1088/0953-4075/40/24/002

Vortex-ring solutions of the Gross–Pitaevskii equation for an axisymmtrically trapped Bose–Einstein condensate

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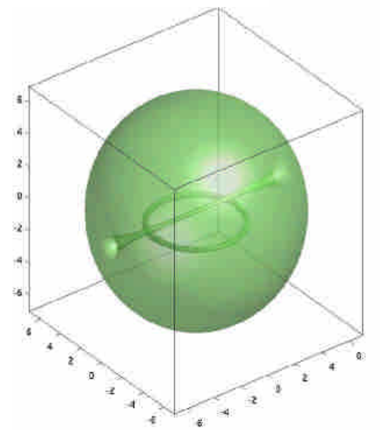
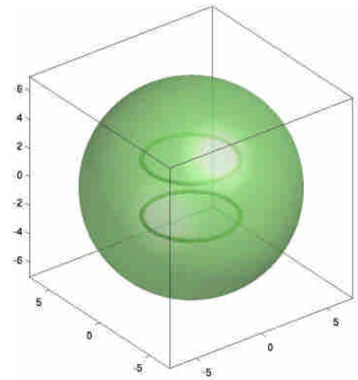
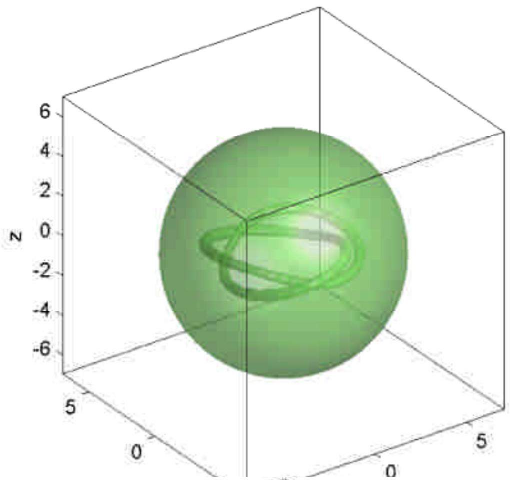
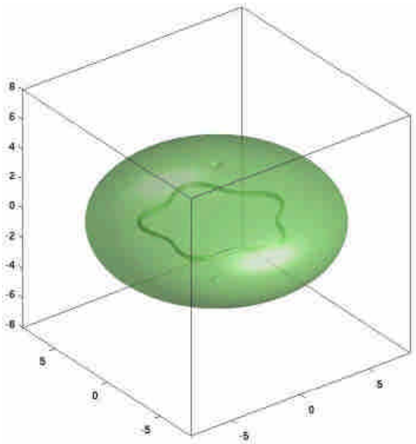
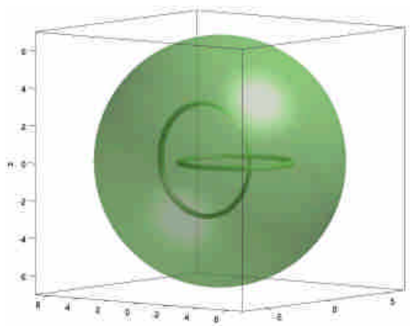
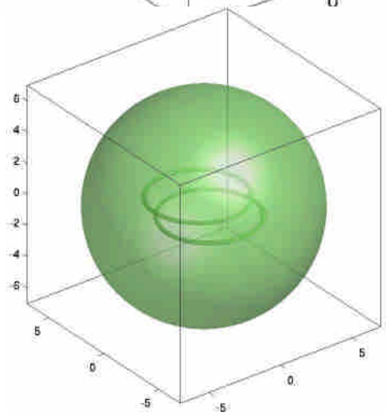




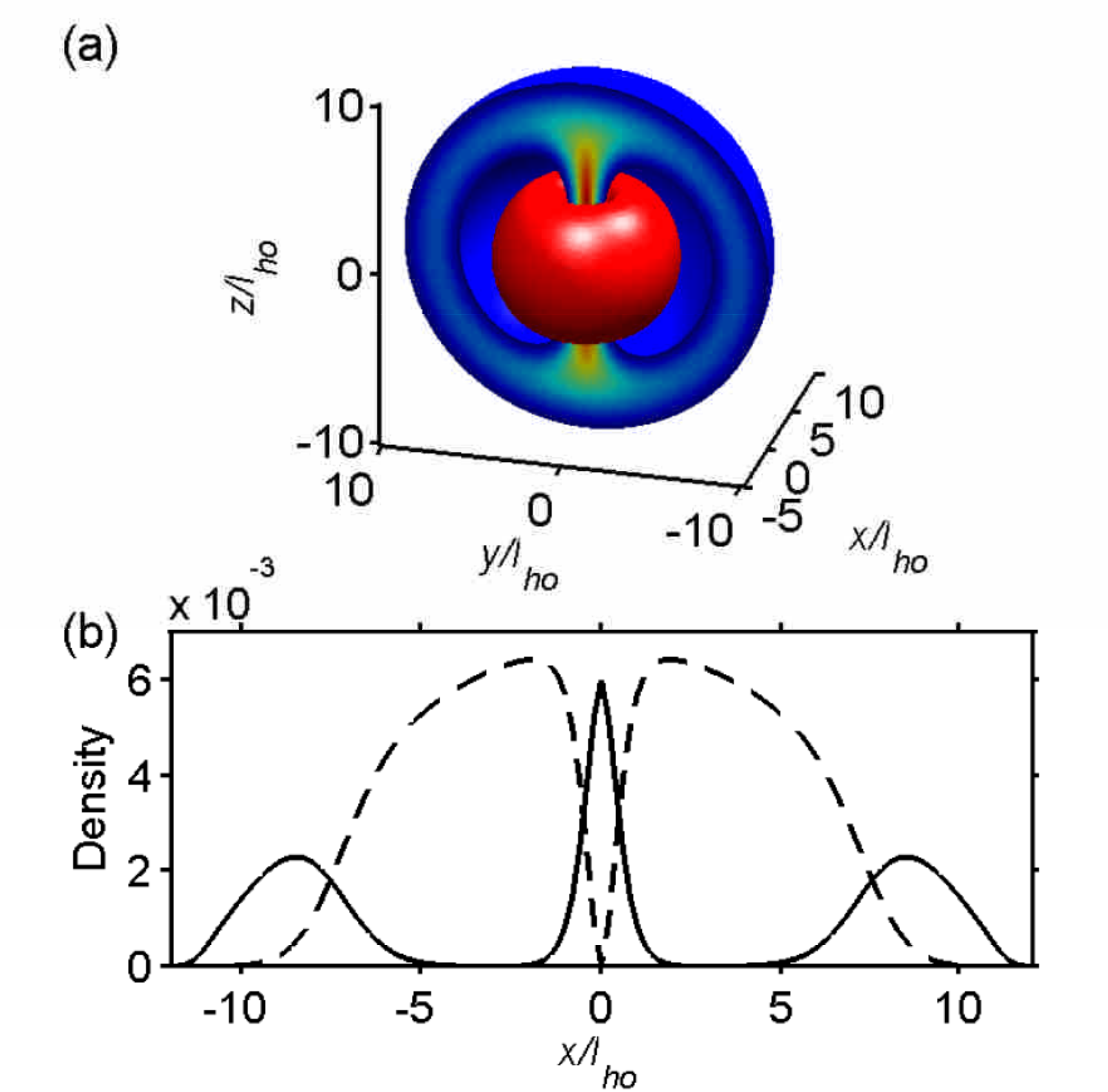








Skyrmion in a trapped two-component BEC



J. Ruostekoski and J.R. Anglin, PRL **86**, 3934 (2000)

C.-H. Hsueh et al., J. Phys. B :At. Mol. Opt. Phys. **40**, 4561(2007)

PHYSICAL REVIEW A **77**, 063625 (2008)

Transition to quantum turbulence in a Bose-Einstein condensate through the bending-wave instability of a single-vortex ring

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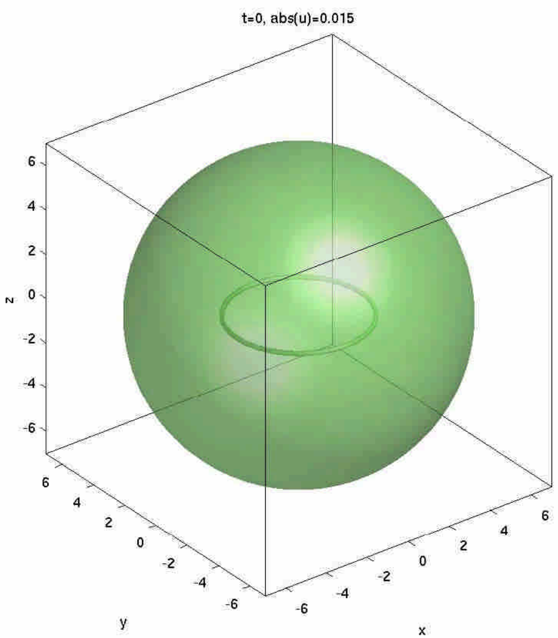
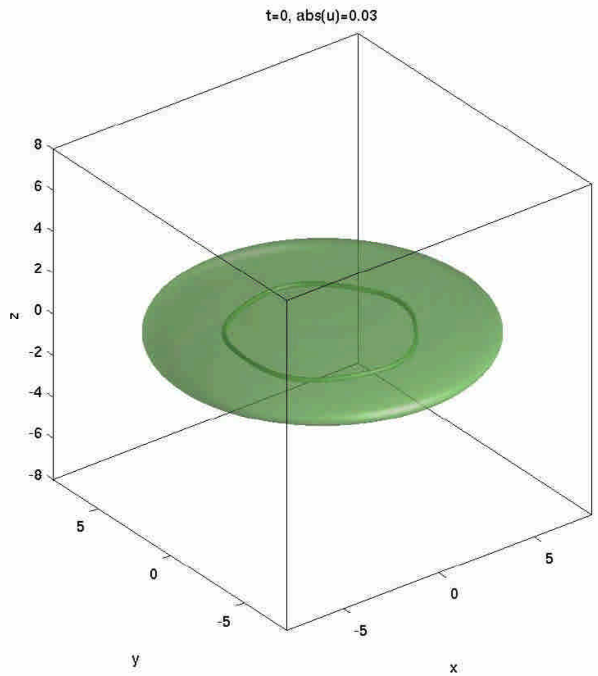
[2]*Department of Physics, National Changhua University of Education, Changhua 50058, Taiwan*
(Received 25 April 2008; published 30 June 2008)

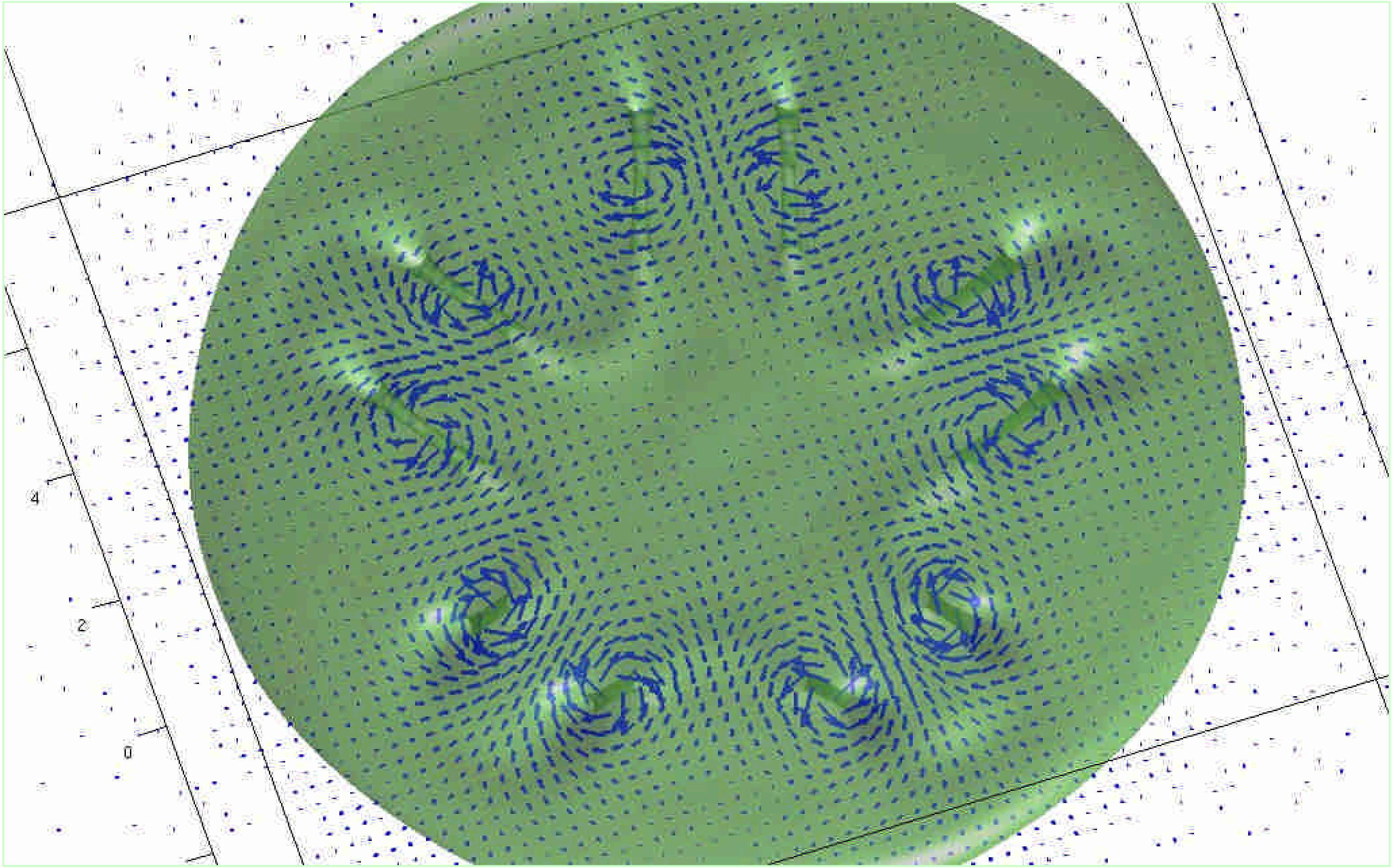
We investigate the dynamics of an unstable vortex ring in a pancake-shaped Bose-Einsten condensate by solving the Gross-Pitaevskii equation numerically. It is found that a quasisteady turbulent state with long relaxation time can be achieved through the disruption of a perturbed vortex ring in the condensate owing to the bending-wave instability. We verify that this quantum turbulent state is characterized by Kolmogorov energy spectrum.

DOI: 10.1103/PhysRevA.77.063625 PACS number(s): 03.75.Kk, 47.32.C−, 47.37.+q, 67.25.dk

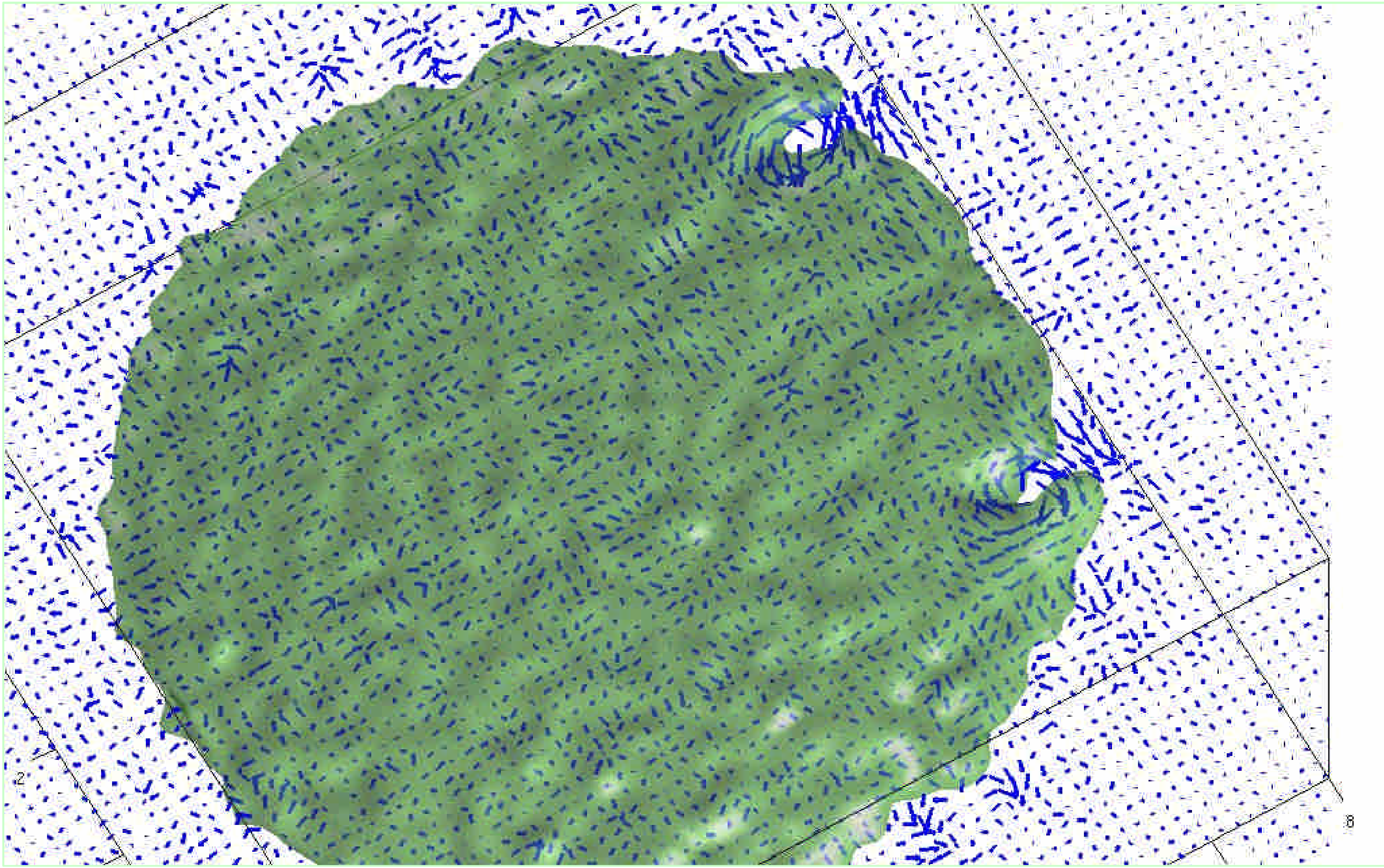


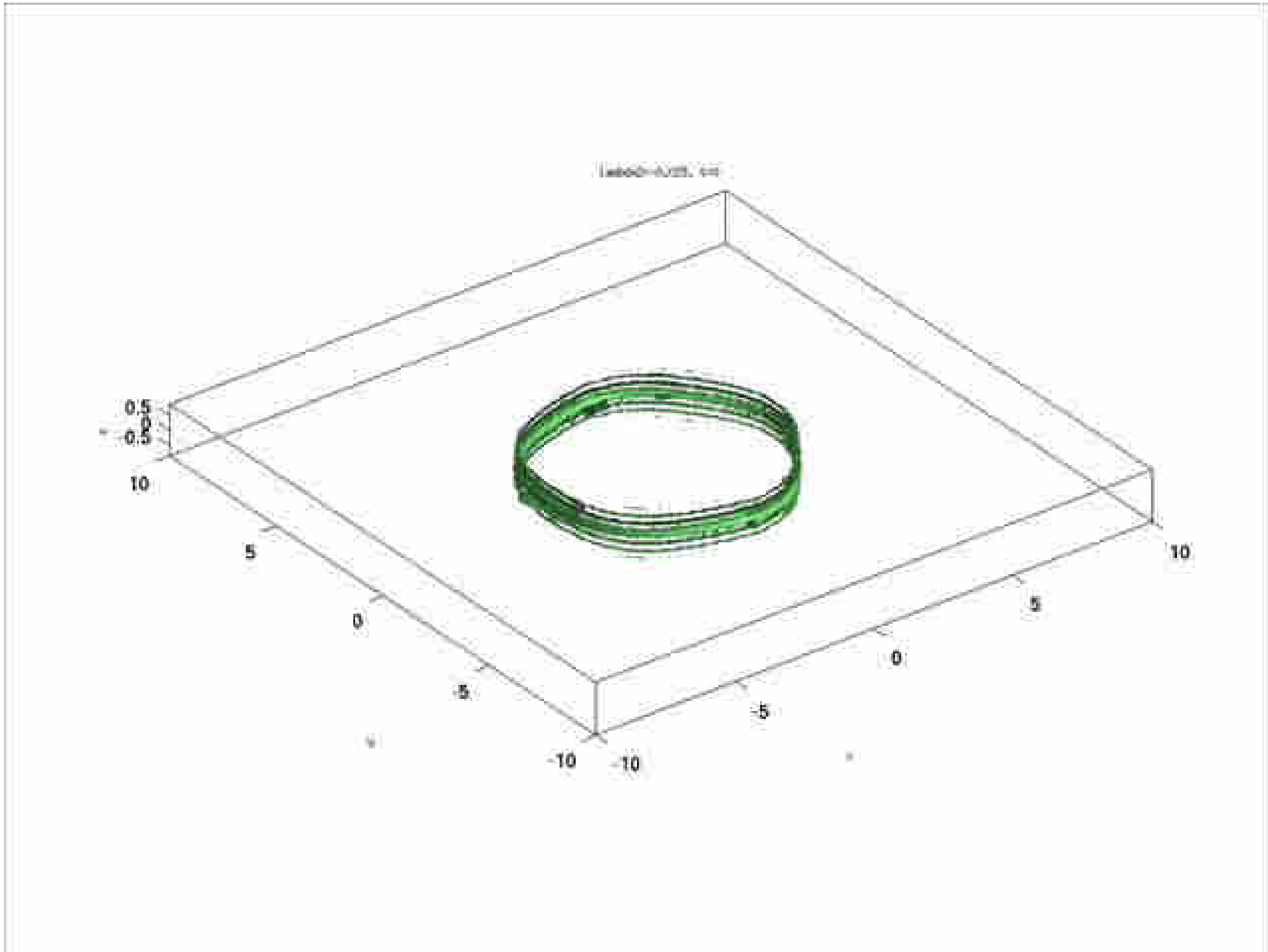


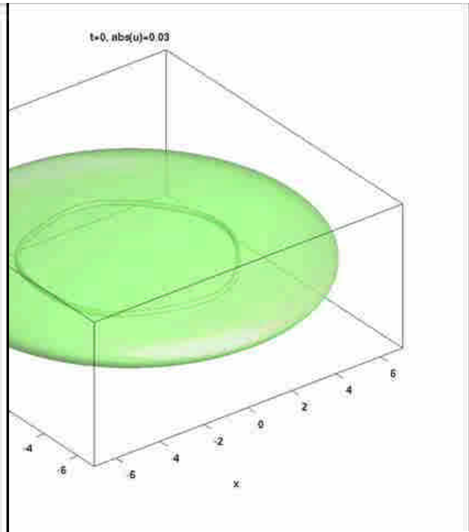
Velocity field in the non-turbulent regime



Velocity field in the turbulent regime







Where have all the vortices gone?

They all went to the outer region of BEC!

Vortex-sound separation

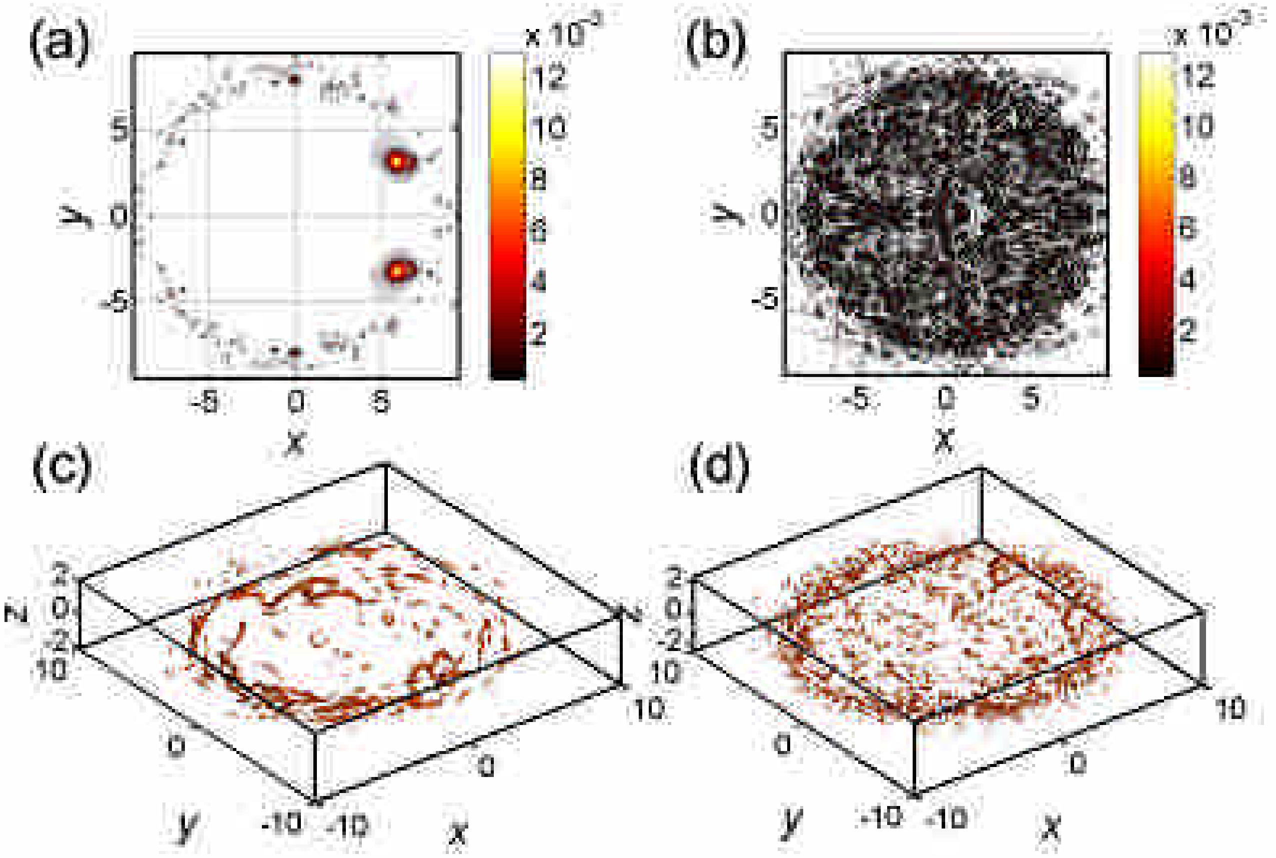
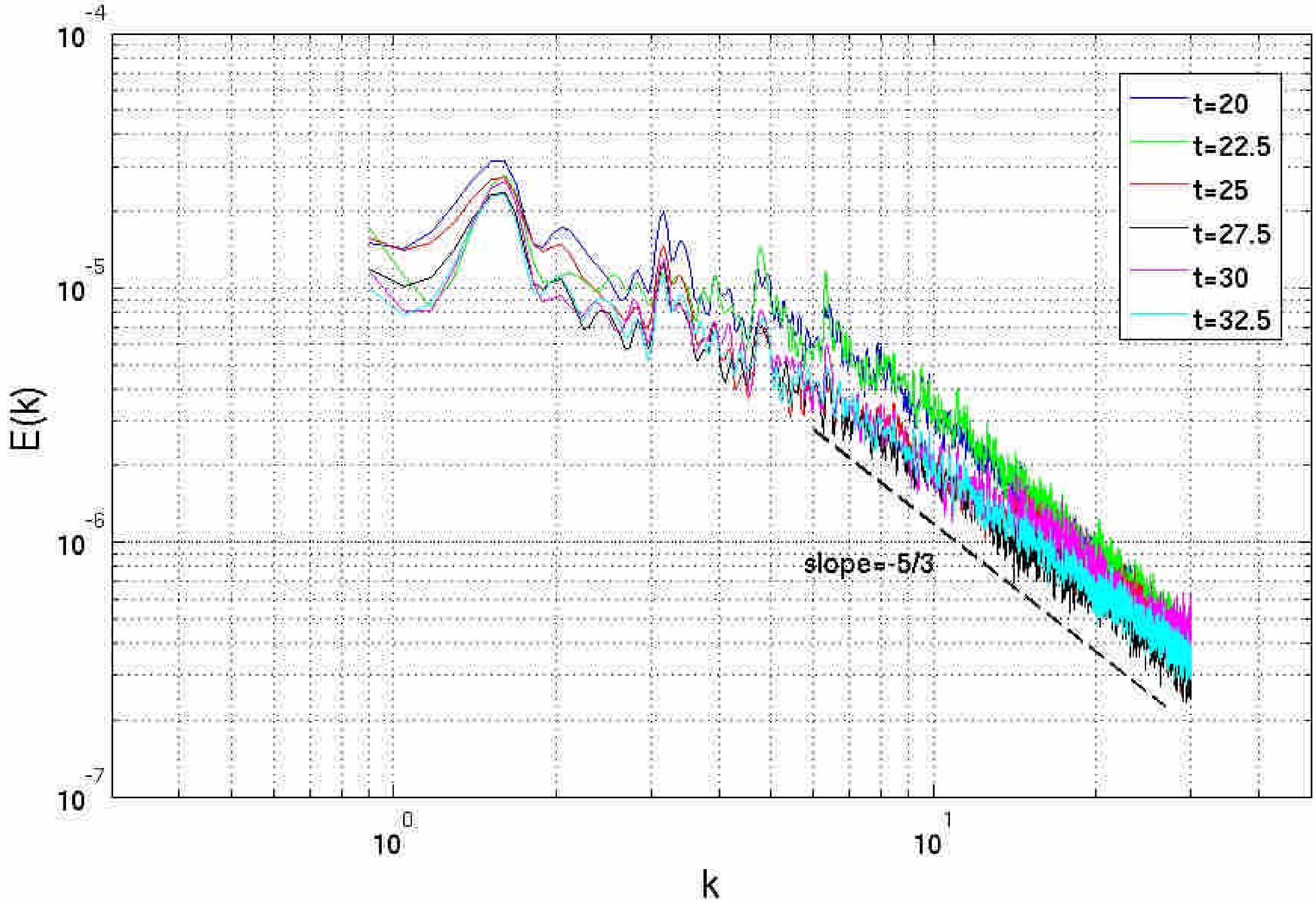
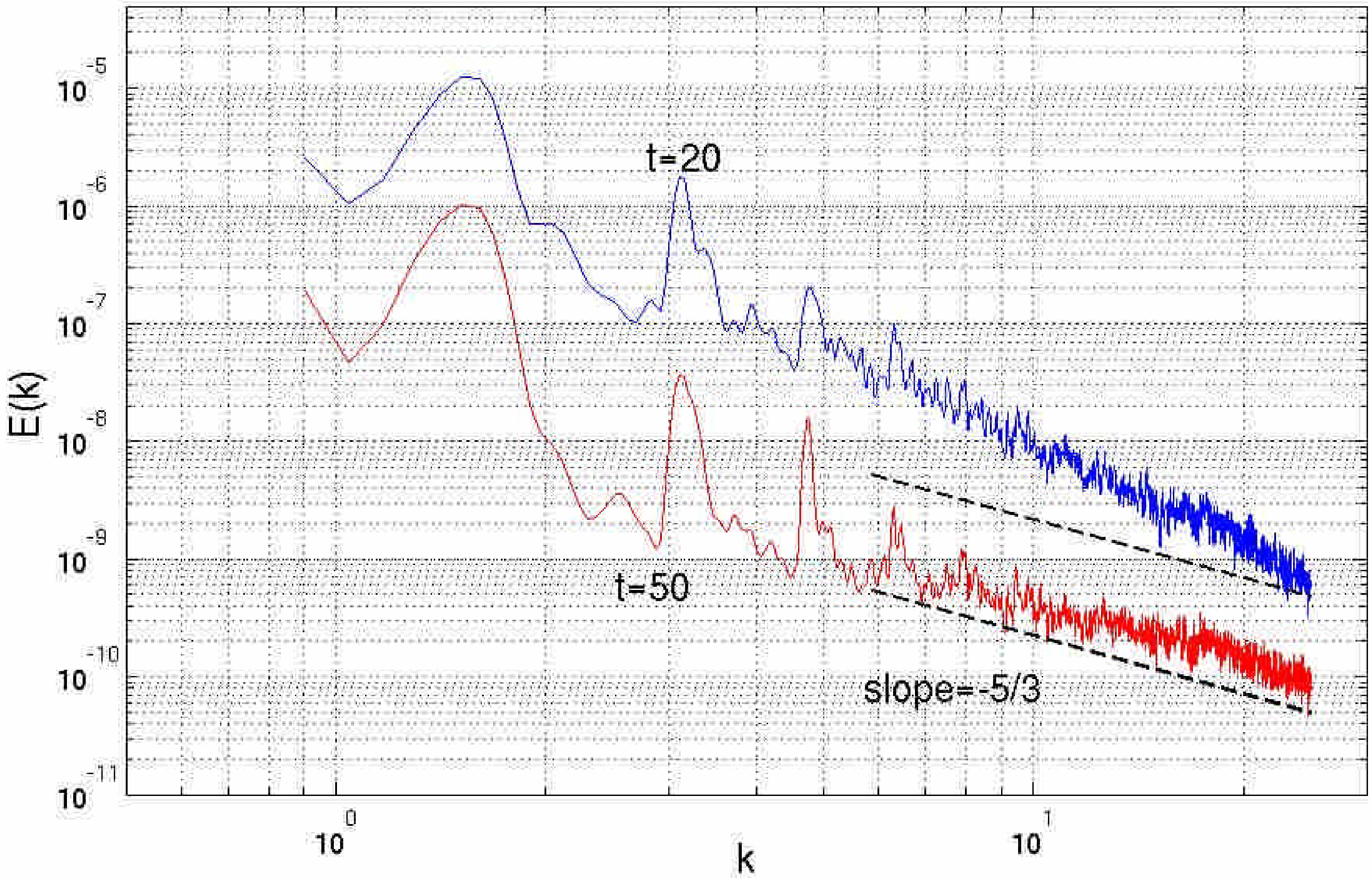


FIG. 4. (Color online) Contour plot for the kinetic energy densities (a) incompressible field and (b) compressible field, on the xy plane at $t=50$, and the perspective view of three-dimensional spatial distribution of vortices at (c) $t=20$ and (d) $t=50$.

Kolmogrov -5/3 law for the kinetic energy spectrum of the vortex





E(k)
k
t=20
t=50
slope=-5/3
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{0}
10^{1}

Conclusions:

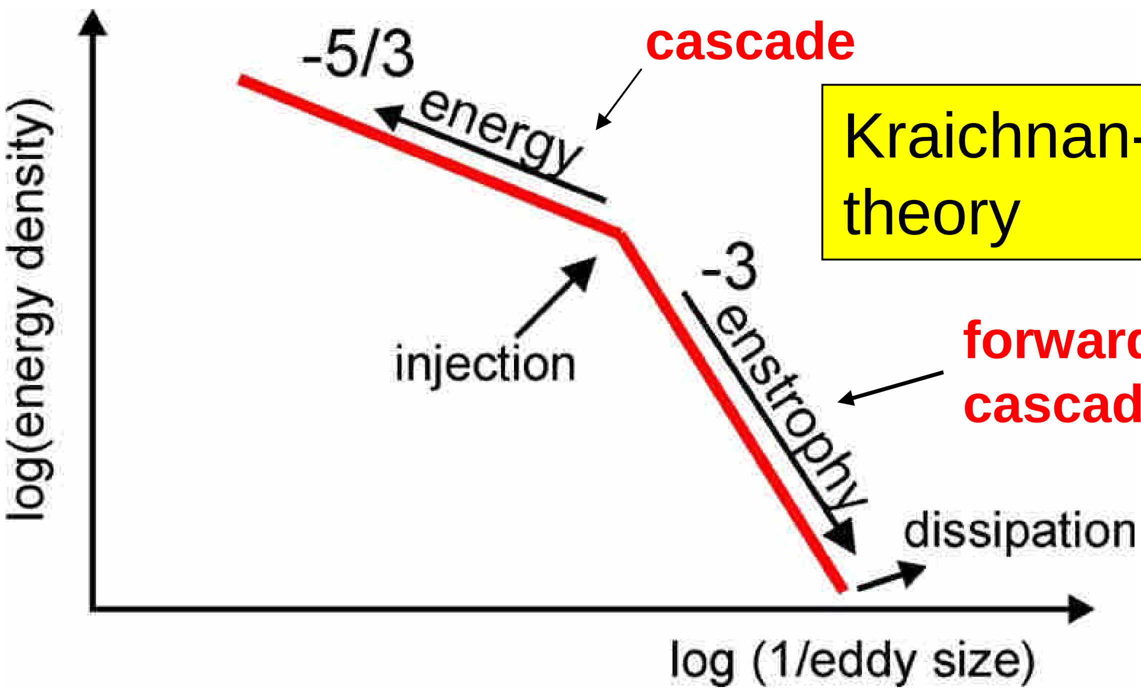
1. New prototype of QT characterized by vortex-sound separtion.
2. Kolmogorov's -5/3 law holds in this new type of QT.

Questions:

1. When $\lambda \to \infty$, we get a 2D superfluid, what are the corresponding scaling laws for QT?
2. Is this new 2D QT similar with 2D CT?

2D CT vs. 3D CT

- **2D Turbulence**
  - **Energy & Enstrophy conserved**
  - ***No* vortex stretching**
- **3D Turbulence**
  - **Enstrophy *not* conserved**
  - **Vortex stretching present**



inverse cascade

forward cascade

Kraichnan-Bachelor theory

vorticity vector $\boldsymbol{\omega} = \nabla \times \mathbf{v}$

$\boldsymbol{\omega}$ is a conserved quantity in 2D

$$\text{Enstrophy} = \frac{1}{2}\int \left|\boldsymbol{\omega}\right|^2 d\mathbf{r} = \frac{1}{2}\int \left|\nabla \times \mathbf{v}\right|^2 d\mathbf{r}$$

2D Turbulence

- **Standard 2D turbulence theory predicts:**
  - **Upscale energy cascade from the point of energy injection (spectral slope –5/3)**
  - **Downscale enstrophy cascade to smaller scales (spectral slope –3)**

However, 2D CT still remains open to question!

Two-dimensional quantum turbulence in a nonuniform Bose-Einstein condensate

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We investigate the dynamics of turbulent flow in a two-dimensional trapped Bose-Einsten condensate by solving the Gross-Pitaevskii equation numerically. The development of the quantum turbulence is activated by the disruption of an initially embedded vortex quadrupole. By calculating the incompressible kinetic energy spectrum of the superflow, we conclude that this quantum turbulent state is unviserally characterized by the Kolmogorov-Saffman scaling law in the wavenumber space. Our study predicts the coexistence of two sub-inertial ranges responsible for the energy cascade and enstrophy cascade in this new protype of two-dimensional quantum turbulence.

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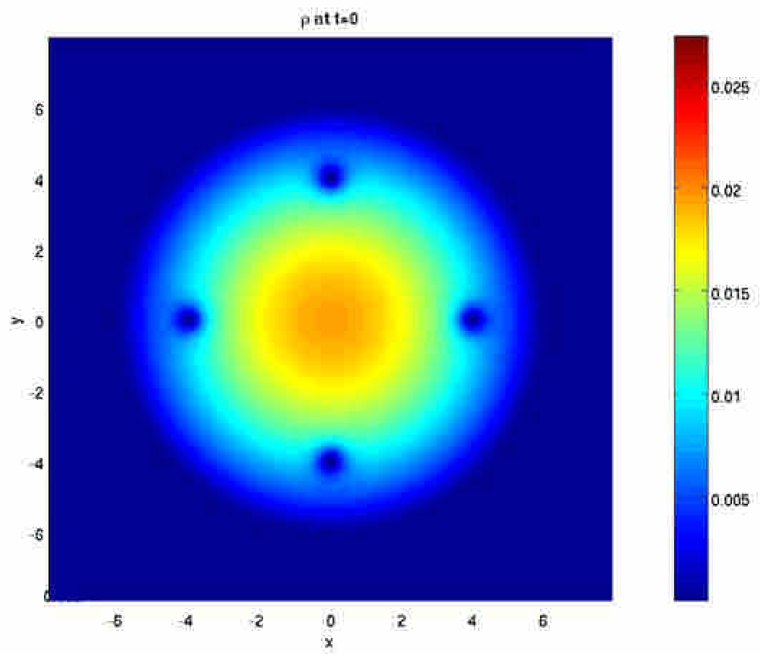
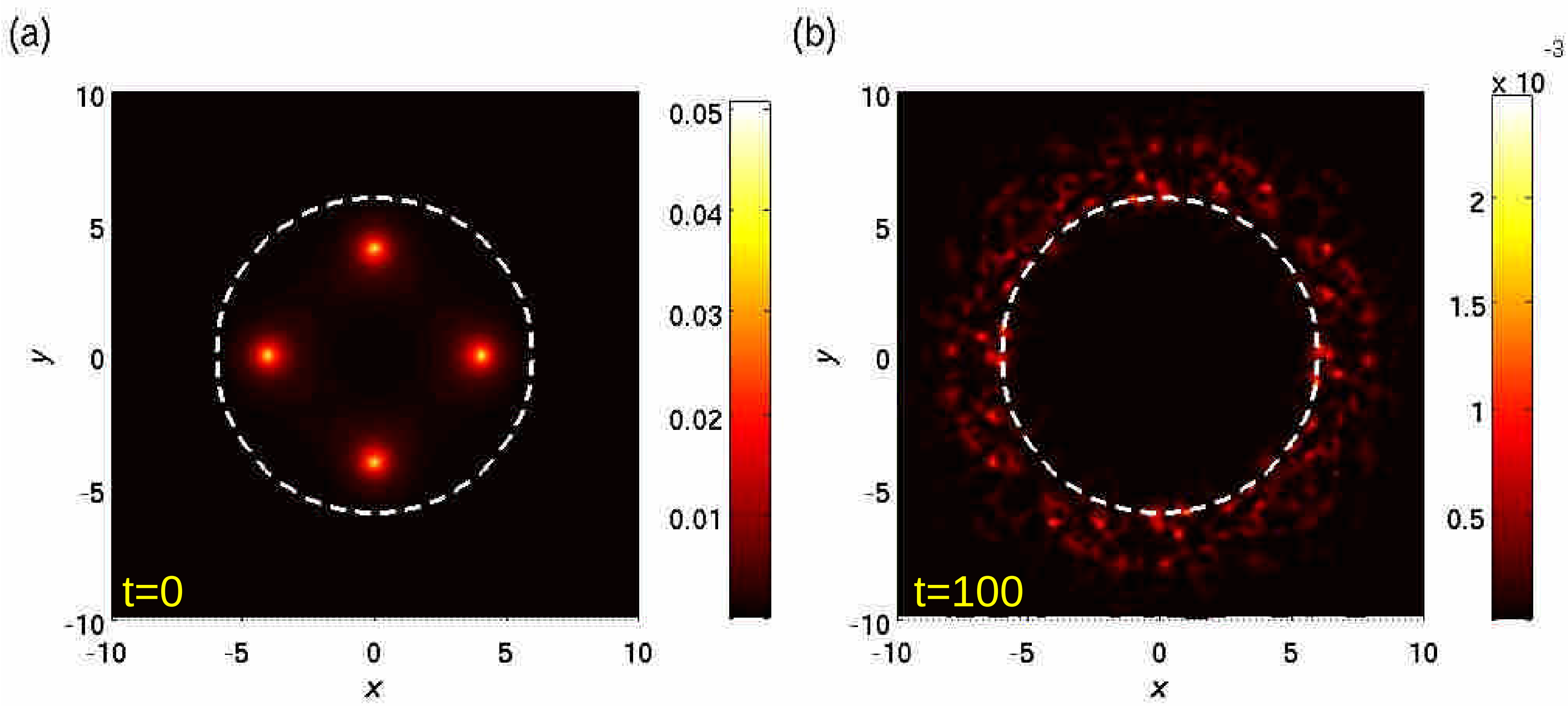
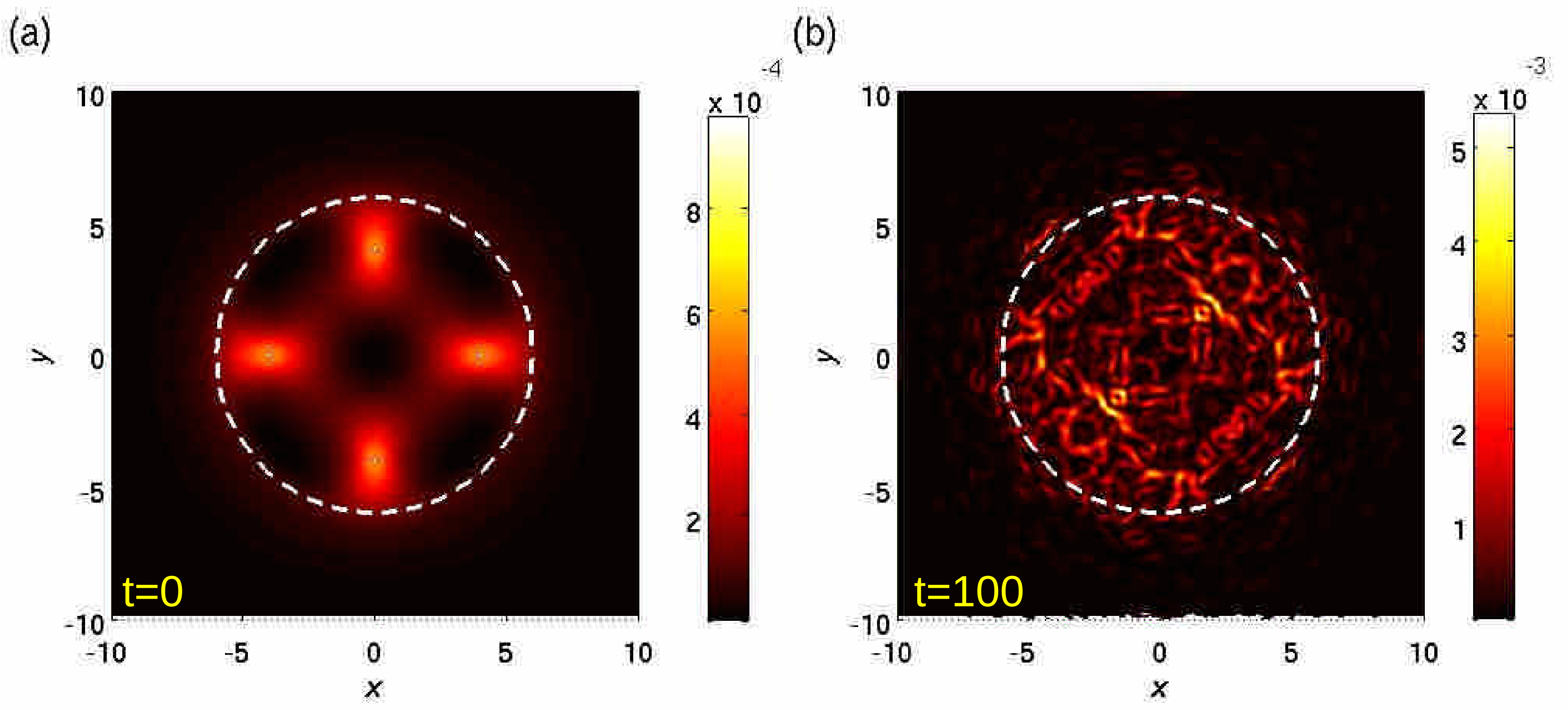


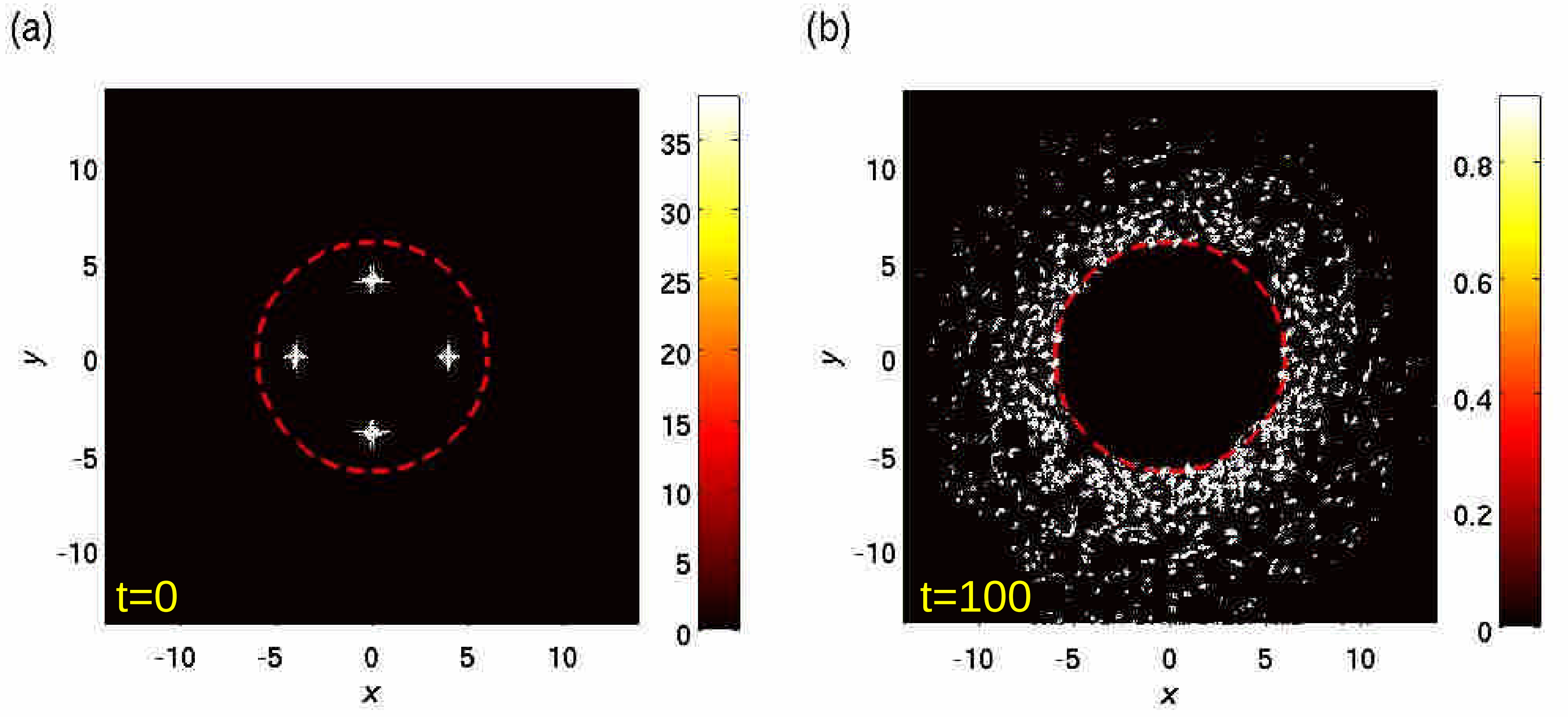
Distribution of incompressible kinetic energy



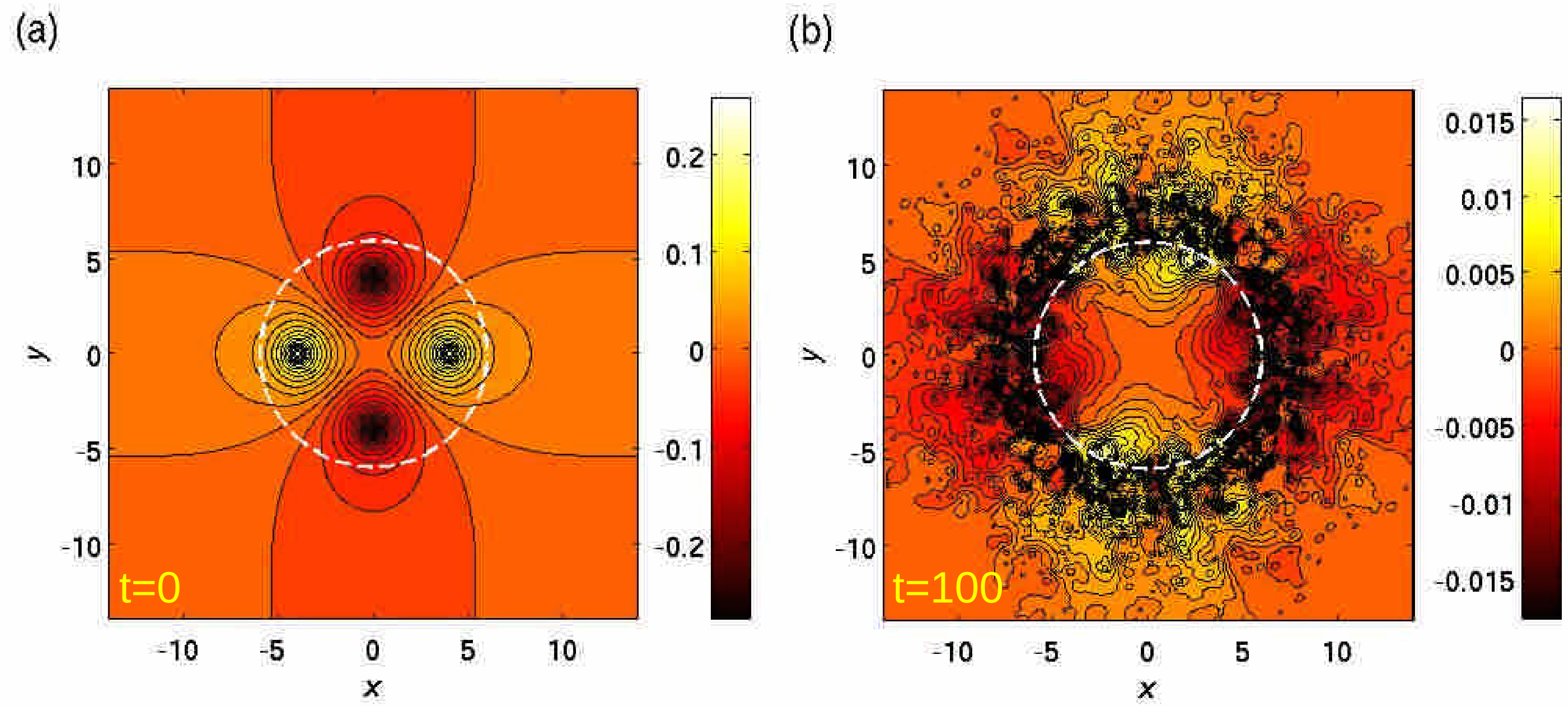
Distribution of compressible kinetic energy



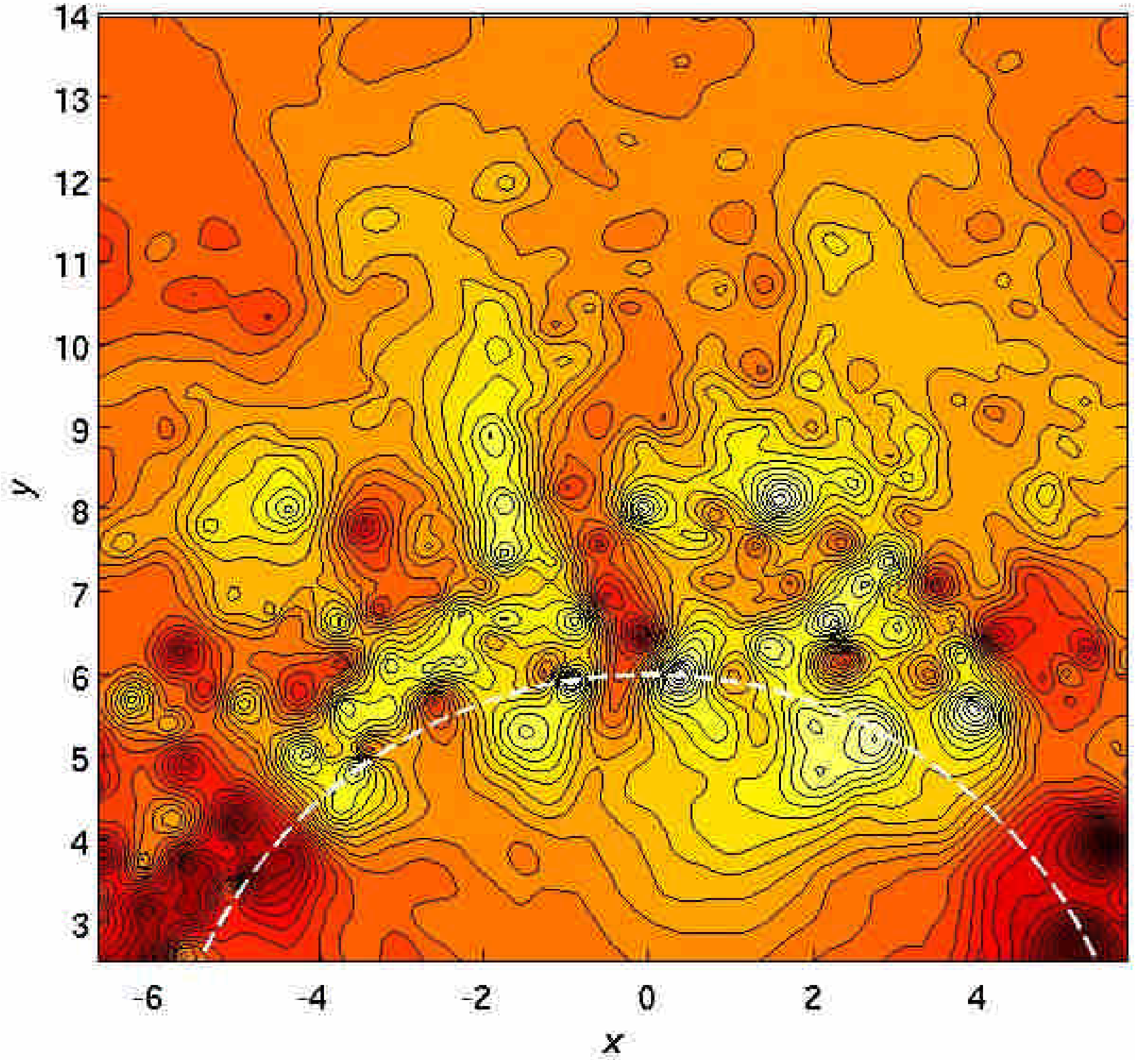
Distribution of enstrophy



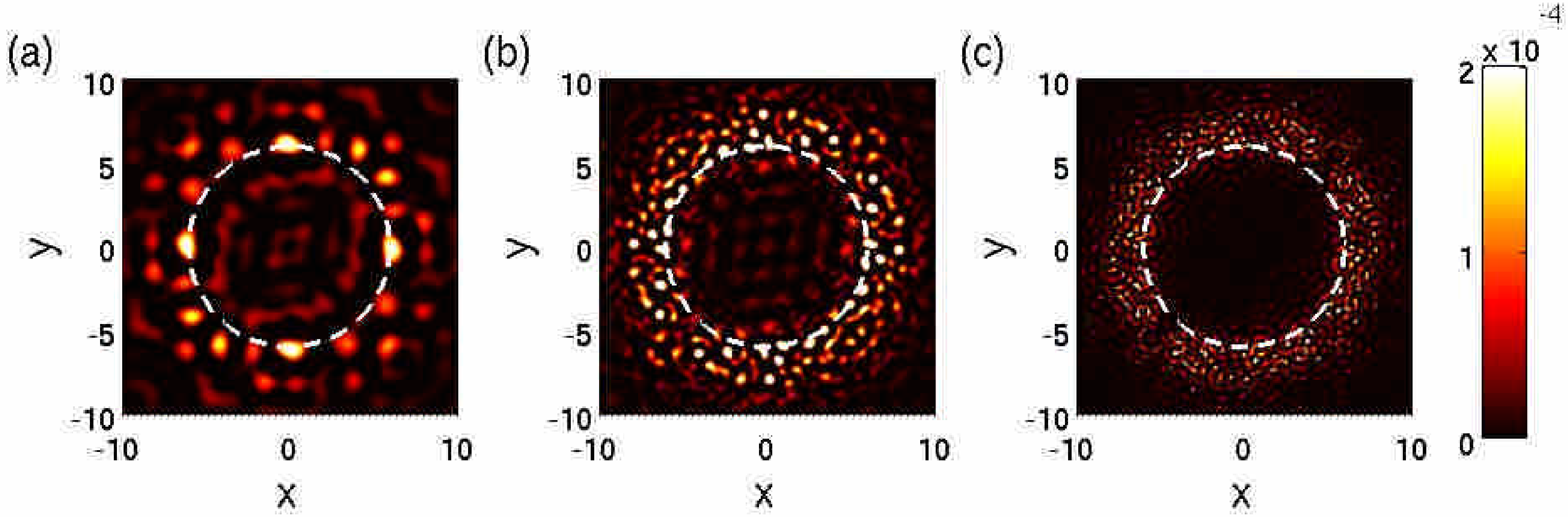
Contours of stream lines



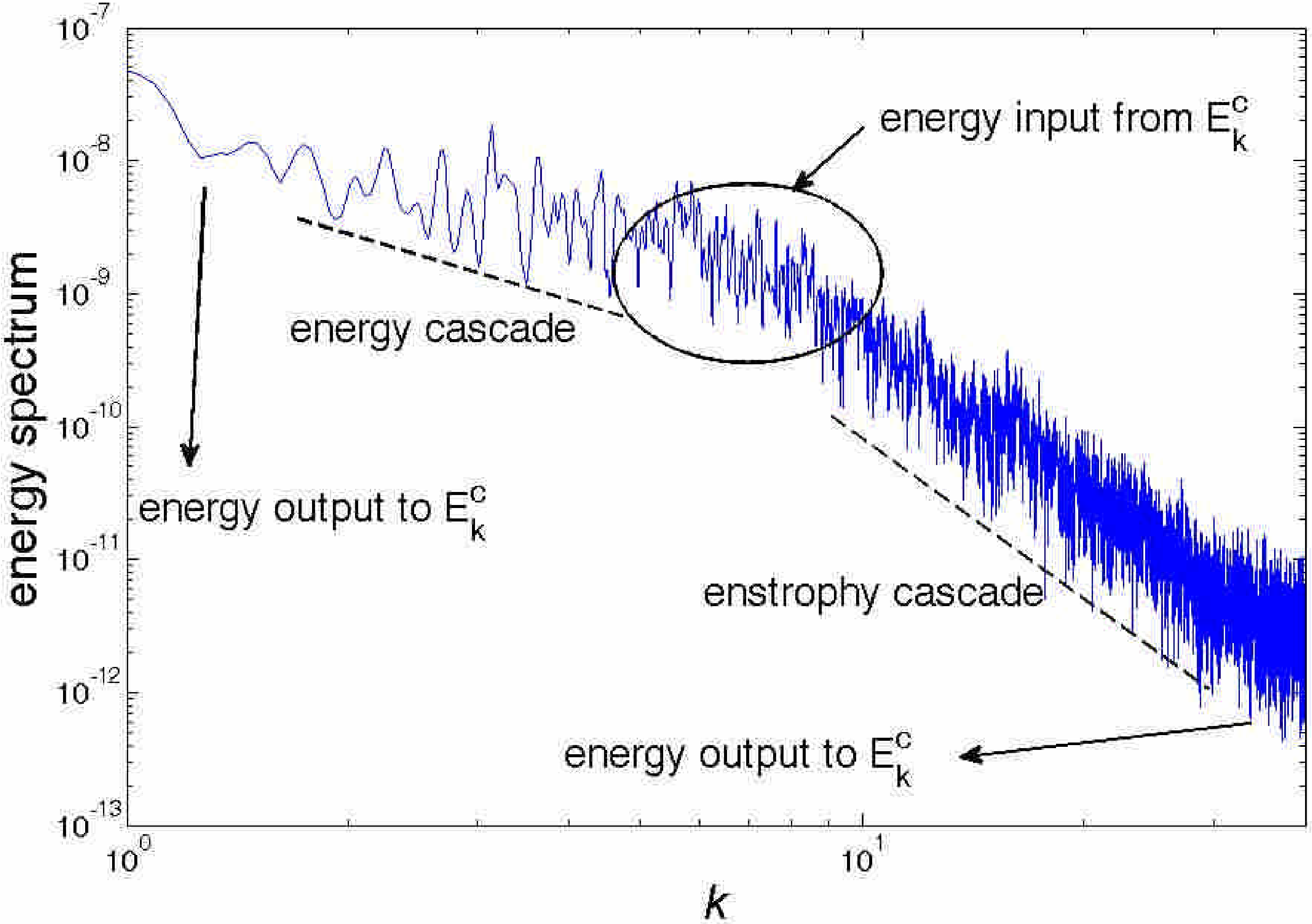
Distribution of stream lines



Cumulative spatial distribution of incompressible kinetic energy at different scales



$2 < k < 4$ $4 < k < 10$ $k > 10$



energy spectrum
k
energy input from E_k^c
energy cascade
energy output to E_k^c
enstrophy cascade
energy output to E_k^c
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{0}
10^{1}

Conclusions:

1. New prototype of compressible QT characterized by vortex-sound separtion.
2. There are two inertial ranges, corresponding to inverse cascade and forward cascade.
3. The inverse cascade follows Kolmogorov's -5/3 law and the forward cascade follows a k^{-4} law.
4. The scaling behavior are universal.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