Symmetry and Conservation Law

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Conservation Laws–all come from experiments directly or indirectly

1. Exact
    1. Energy Conservation
    2. Momentum Conservation
    3. Electric Charge
    4. Baryon Number
2. Approximate–Valid only in some approximations
    1. Parity
    2. Charge Conjugation
    3. Lepton Number
    4. Isospin
    5. ......

Symmetry

In daily life, symmetry⇔beauty

In physics,

$$\text{symmetry} \Longrightarrow \begin{cases} \text{conservation law} \\ \text{degeneracy} \end{cases}$$

$$\text{symmetry transformations} \begin{cases} \text{physical space} \\ \text{abstract space—internal Symmetry} \end{cases}$$

Example: Energy Conservation

For simple case, Newton's law gives

$$m\frac{d^2\vec{x}}{dt^2} = \vec{f}\left(\vec{x}, t\right)$$

If $\vec{f}\left(\vec{x}, t\right)$ is independent of t and $\vec{f}\left(\vec{x}, t\right) = -\vec{\nabla}V\left(\vec{x}\right)$, then

$$m\frac{d^2\vec{x}}{dt^2} \cdot \frac{d\vec{x}}{dt} = -\vec{\nabla}V\left(\vec{x}, t\right) \cdot \frac{d\vec{x}}{dt} \quad \Longrightarrow \quad \frac{d}{dt}\left[\frac{1}{2}m\left(\frac{d\vec{x}}{dt}\right)^2 + V\right] = 0$$

Noether's theorem: If action

$$S = \int dtL, \qquad L : \text{Lagrangian}$$

is invariance under continuous symmetry tranformation $\Rightarrow$ conservation law

time translation	$t \longrightarrow t + a$	energy conservation
spatial tranlation	$\overrightarrow{x} \longrightarrow \overrightarrow{x} + \overrightarrow{b}$	momentum
spatial rotation	$x_i \longrightarrow R_{ij}x_j, \quad RR^T = 1$	angular mometum

Internal Symmetry-symmetry transformation in abstract space

Example: isospin symmetry

Motivation: nuclear force seems to be the same for neutron and proton

symmetry transfomation:

$$\begin{pmatrix} n(x) \\ p(x) \end{pmatrix} \to U \begin{pmatrix} n(x) \\ p(x) \end{pmatrix}, \qquad 2\times 2 \quad \text{unitay matrix indep of } x^{\mu}$$

Consequence: $m_p = m_n$

Similarly, $\{\pi^-, \pi^0, \pi^+\}$, $I = 1$ triplet,

$\{K^0, K^+\}$, $I = 1/2$ doublet

....

Quark model: search for more fundamental constituents

All hadrons are made out of quarks, u, d, s

SU(3) symmetry:

$$\begin{pmatrix} u \\ d \\ s \end{pmatrix} \to U \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad U: \; 3\times 3 \quad \text{unitary matrix}$$

Baryon number

Why proton is stable? $p \longrightarrow e^{+} + \gamma$ does not violate any physical law

Baryon number conservation: $B(p) = 1$, $B(e^{+}) = 0$, $B(\gamma) = 0$,

In the universe at large, only baryons and no anti-baryons

At beginning, maybe $B = 0$ for the universe as whole, because

$$\gamma + \gamma \rightleftarrows p + \overline{p}$$

To get $B \neq 0$ now, we need baryon number non-sonservation (Sakharov)

In Grand Unified Theory, it is possible to have

$$p \longrightarrow \pi^{0} + e^{+}$$

Many experiments(IMB, Sudane, Kamiokonde...) search for this decay with null result,

$$\tau\left(p \longrightarrow \pi^{0} + e^{+}\right) > 10^{31} \quad \text{years}$$

Lepton number

Neutrino was first introduced to save energy momentum conservation in β decay,

$$n \longrightarrow p + e^- + \nu$$

Later neutrino was also "seen" in other decays, e.g

$$\pi^+ \longrightarrow \mu^+ + \nu$$

Are these two neuttinos the same? (Hint from absence of $\mu^+ \longrightarrow e^+ + \gamma$)
Two neutrino experiment: use ν from π decay to see whether it can induce inverse β decay

$$\pi^+ \longrightarrow \mu^+ + \nu$$

$$\nu + n \longrightarrow p + e^-$$

Result: $\nu_e \neq \nu_\mu, \quad \Longrightarrow$ muon number conservation
Recent years, the discovery of neutrino oscillation $\Longrightarrow$ muon number non-conservation

Local Symmetry

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \qquad \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \vec{j}$$

To solve the source free equations, introduce $\vec{A}$, and ϕ

$$\vec{B} = \vec{\nabla} \times \vec{A}, \qquad \vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

But we get the same $\vec{E}$,and $\vec{B}$, if

$$\vec{A} \to \vec{A}' = \vec{A} - \vec{\nabla}\alpha, \qquad \phi \to \phi' = \phi + \frac{\partial \alpha}{\partial t}, \qquad \text{gauge transformation}$$

Connection with symmetry: quantum mechanics for charged particle,

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\vec{\nabla}-e\vec{A}\right)^2+e\phi\right]\psi\left(\vec{x},t\right)=i\hbar\frac{\partial\psi}{\partial t}$$

This clearly not gauge invariant unless the wavefunction also transforms,

$$\psi\rightarrow\psi'=e^{ie\alpha/\hbar}\psi,\qquad U\left(1\right)\ \text{symmetry}$$

$\alpha=\alpha\left(x\right)$ space time dependent $\Longrightarrow$ symmetry is local
The combination

$$\vec{D}=\vec{\nabla}-ie\vec{A}$$

is usually referred to as <u>covariant derivative</u>.
Important features of theory with local symmetry

1. Gauge particle (e.g. photon) is massless–long range force
2. Coupling e is universal

Spontaneous Symmetry Breaking

Hamiltoian has the symmetry, $[Q, H] = 0$ but $Q|0\rangle \neq 0$

Example: ferromagnetism

For $T > T_C$, Curie tempeture, magnetic dipoles are randomly oriented in the ground state .

For $T < T_C$, dipoles are aligned (spontaneous magnetization) in the ground state and is not rotationally invariant.

Landau-Ginsberg's mean field theory. free energy density in terms of magnetization $\vec{M} = (M_x, M_y, M_z)$

$$u\left(\vec{M}\right) = \left(\partial_t \vec{M}\right)^2 + V\left(\vec{M}\right),$$

where

$$V\left(\vec{M}\right) = \alpha_1(T)\left(\vec{M}\cdot\vec{M}\right) + \alpha_2\left(\vec{M}\cdot\vec{M}\right)^2, \qquad \alpha_2 > 0,$$

This is invariant under rotation

Assume

$$\alpha_1(T) = \alpha(T - T_C) \qquad \text{with } \alpha > 0$$

then for $T < T_C$ (i.e. $\alpha_1 < 0$), the minimum is at

$$\left|\vec{M}\right| = \sqrt{-\frac{\alpha_1}{2\alpha_2}} \qquad \text{spontaneous magnetization}$$

If we choose

$$\vec{M} = \left(0, 0, \sqrt{-\frac{\alpha_1}{2\alpha_2}}\right), \qquad \text{not invariant under rotation}$$

Goldstone theorem: continuous symmetry broken spontaneously $\Rightarrow$ massless particle(Goldsone boson)–long range force
In relativistic theory, if the potential energy for a scalar fielf ϕ has the form,

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda\left(\phi^\dagger\phi\right)^2$$

This has the symmetry

$$\phi \rightarrow e^{i\alpha}\phi \qquad U(1) \quad \text{symmetry}$$

The minimum is located at

$$|\phi| = \sqrt{\frac{\mu^2}{\lambda}} \equiv v$$

If we choose

$$\operatorname{Re}\phi = v, \qquad \operatorname{Im}\phi = 0$$

then $U(1)$ symmetry is broken spontaneously. It turns out that $\operatorname{Im}\phi$ corresponds to Goldstone boson.

Higgs Phenomena : local symmetry+spontaneous symmetry breaking

massless gauge boaon + massless Goldstone boson=massive gauge boson

Coupling of gauge particle to scalar field is in the form of the covariant derivative,

$$D_\mu \phi = \left(\partial_\mu - eA_\mu\right)\phi$$

In the Lagrangian for scalar field

$$L = \left(D_\mu \phi\right)^\dagger D_\mu \phi + V\left(\phi\right)$$

the choice $\mathrm{Re}\,\phi = v$ will give rise to $e^2 v^2 \left(A_\mu A^\mu\right)$ —a mass term for gauge field.
No more long-range force

Standard Model of Electroweak Interaction

Important observed properties of weak interaction:

1. Weak coupling is universal—gauge coupling?
2. Weak interaction is short ranged—massive mediator

These propperties $\Longrightarrow$ massive gauge boson $\Longleftarrow$ Higgs phenomena

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Gauge group: $SU(2) \times U(1)$ (1967)
Spontaneous symmetry breaking:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

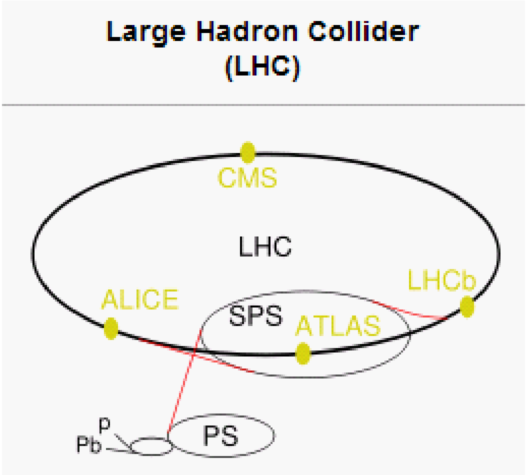
with

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}$$

For the ground state,

$$\langle \phi \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

break the symmetry: $SU(2) \times U(1) \longrightarrow U(1)$
Gauge bosons: massive :$W^{\pm}$, Z, (found 1983), massless :γ
Higgs particle: scalar field left over after symmetry breaking
LHC search:
LHC(Large Hadron Collider) : 7 *Tev* on 7 *Tev* proton machine



The accelerator chain of the Large Hadron Collider

$$p + p \longrightarrow H + ...$$