Bose-Fermi Mixtures in Ultracold Atoms

Daw-Wei Wang
National Tsing-Hua University

Cooperators:

Prof. E. Demler
Prof. M. Lukin (Harvard University)
Mr. L. Mathey

Outline

(I) Introduction to cold atoms

(II) Some interesting physics in BFM

(1) Phase instability

(2) Superfluidity of fermion pair

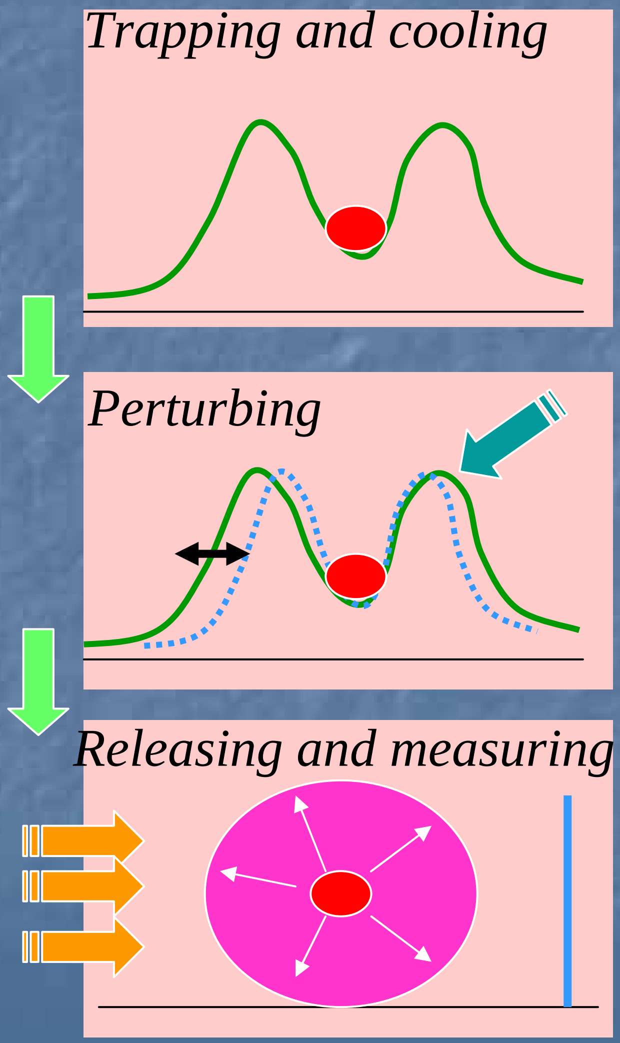
(3) Polaronic effects

(4) Charge/spin density waves

(III) Experimentally related issues

(IV) Summary and outlook

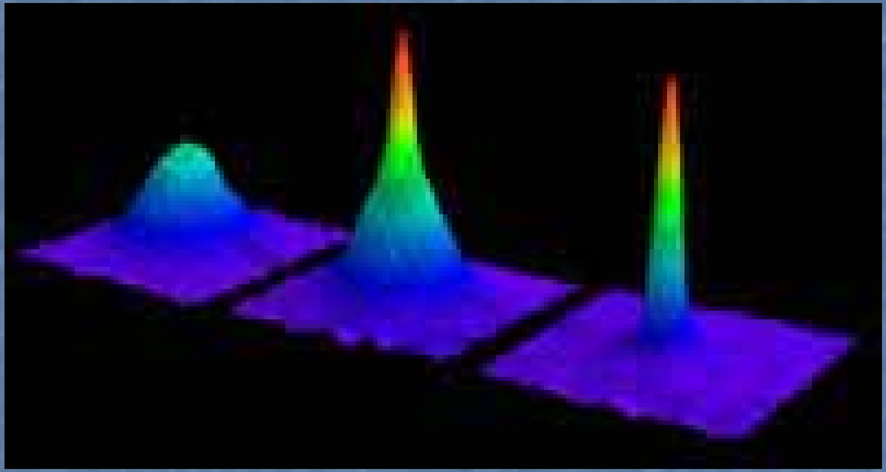
How does it work ?



(i) Magnetic-Optical trap
(ii) Optical lattice
(iii) Laser cooling
(iv) Evaporative cooling
(v) Optical transition
(vi) Feshbach resonance
(vii) Optical lattice
(viii) Time-of-flight measurement
(ix) Bragg scattering
(x) Effective magnetic field

…………..

Introduction and references



(1) Dilute atoms

(2) Magnetic/optical
trap potential

(3) Ultralow T
(<1 micro-K)

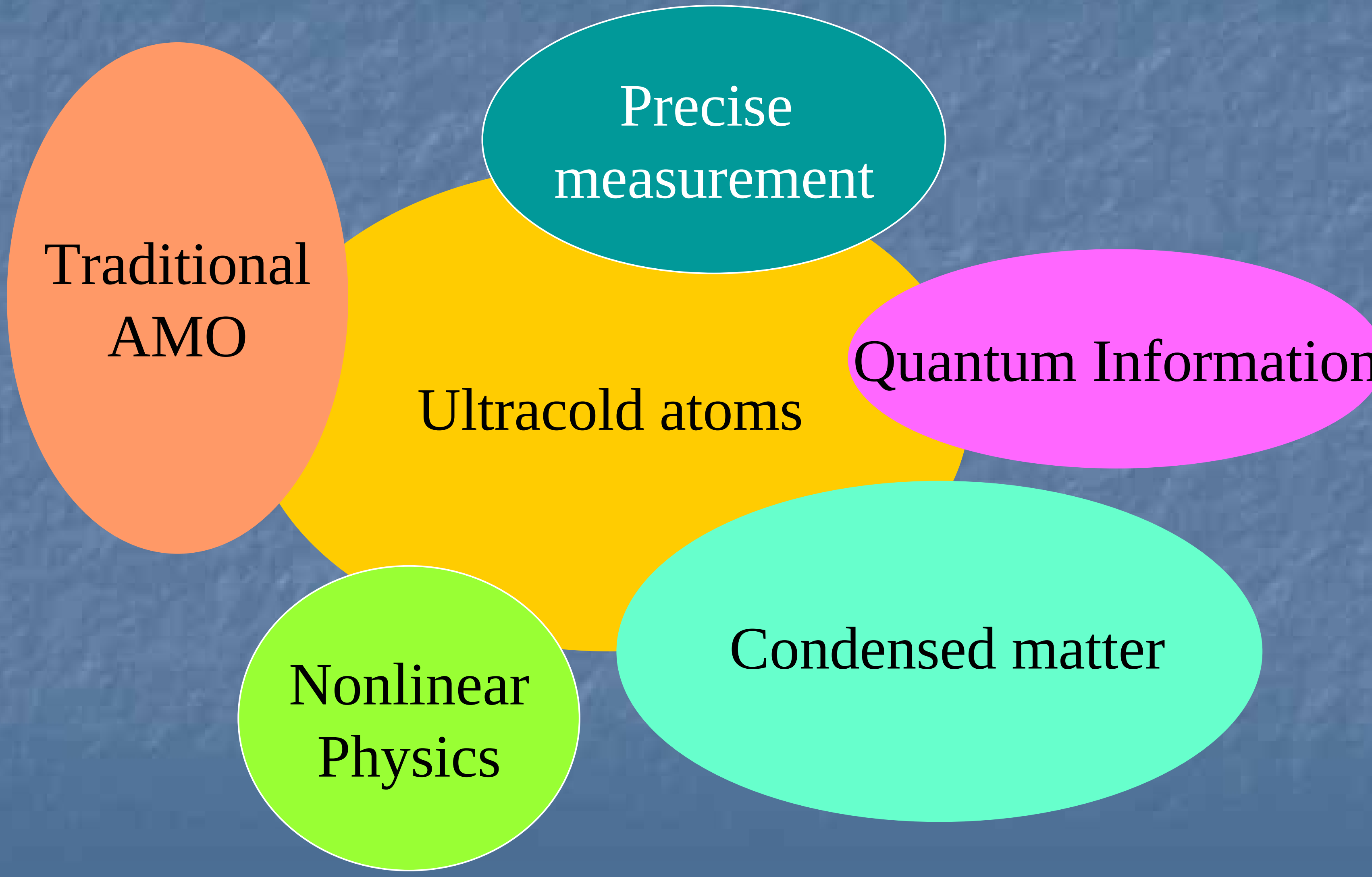
• F. Dalfovo, S. Giorgini, L.P. Pitaevskii, and A. Stringari, Rev. Mod. Phys. **71**, 463 (1999)

• A.J. Leggett, Rev. Mod. Phys. **73**, 307 (2001)

•MIT/Harvard University: Center for ultracold atoms
http://www.rle.mit.edu/cua/

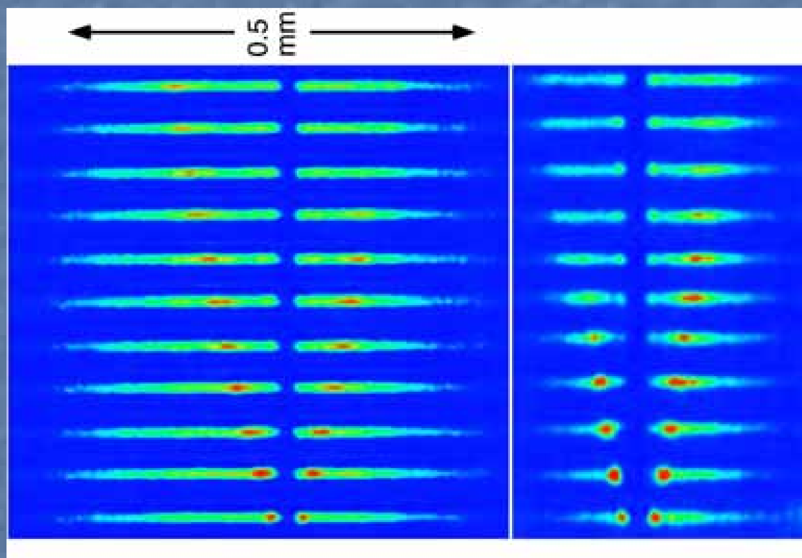
• Nature, **416**, 205 (2002)

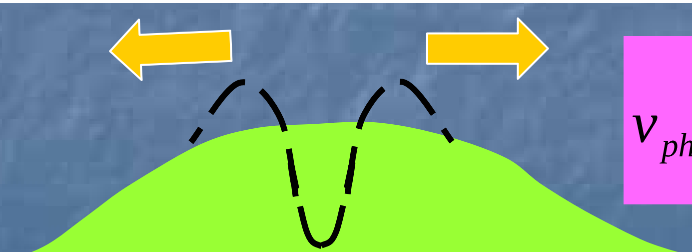
Interdisciplinary field



Phonons and vortices in BEC

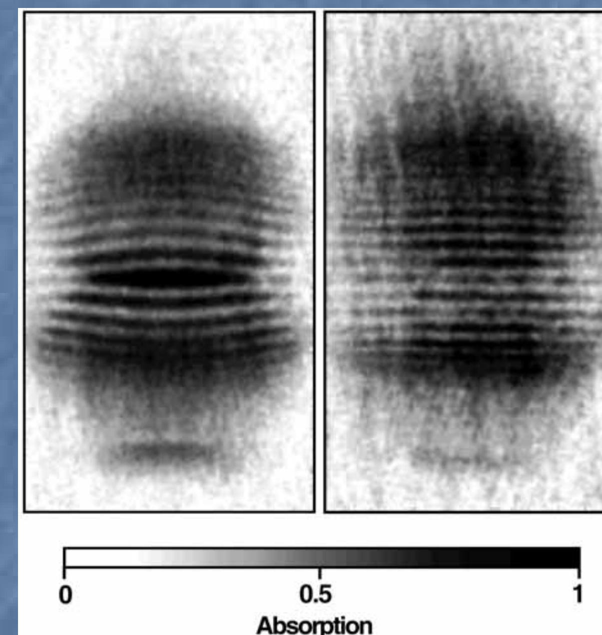
Phonon=density fluctuation





$$v_{ph} = \sqrt{\frac{n_0 U}{m}}$$

Interference

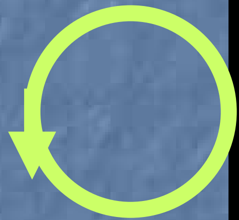


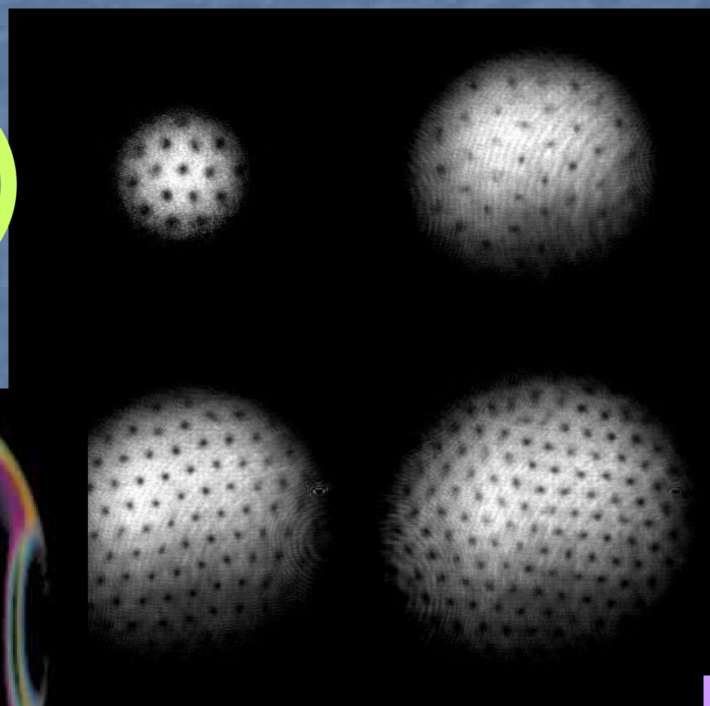
(after Science **275**, 637 ('97))

Matter waves !

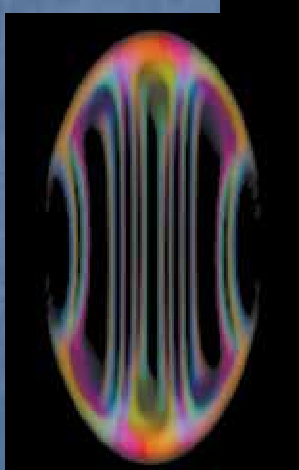
Vortices in condensate

Vortex = topological disorder

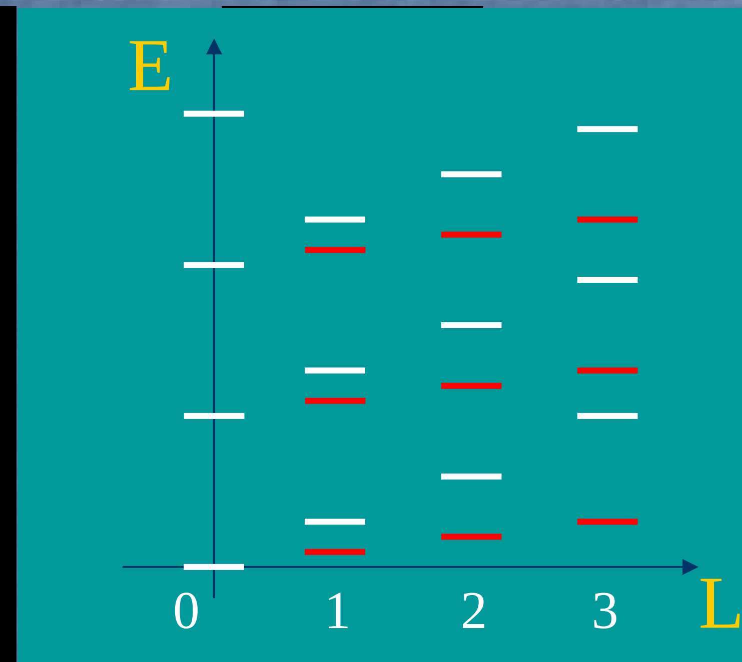




(after Science **292**, 476 ('01))



(after PRL **87**, 190401 ('01))

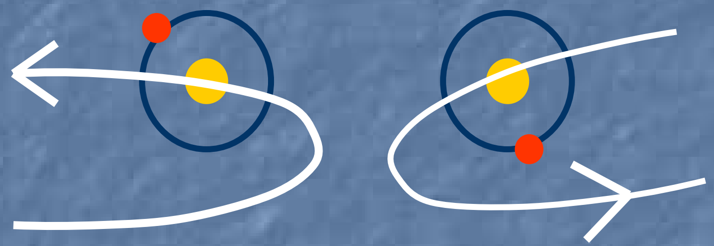


$$E_{n,l} = \omega_0 \sqrt{l + 3n + 2nl + 2n^2} - l\omega_{ext}$$

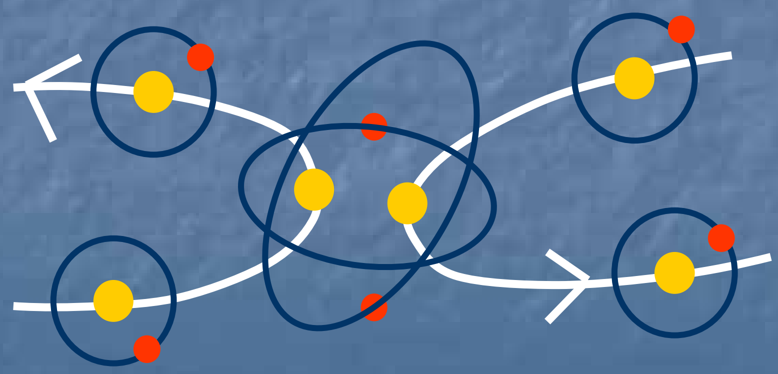
Vortices melting, quantum Hall regime ?

Feshbach Resonance

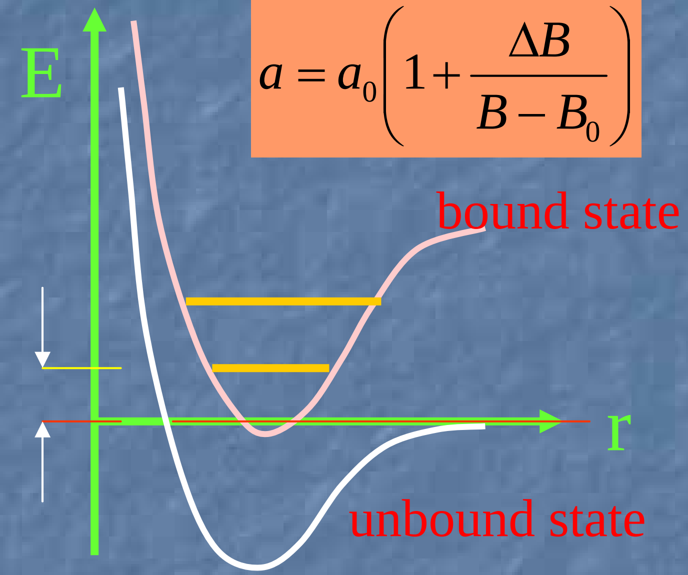
(i) Typical scattering:

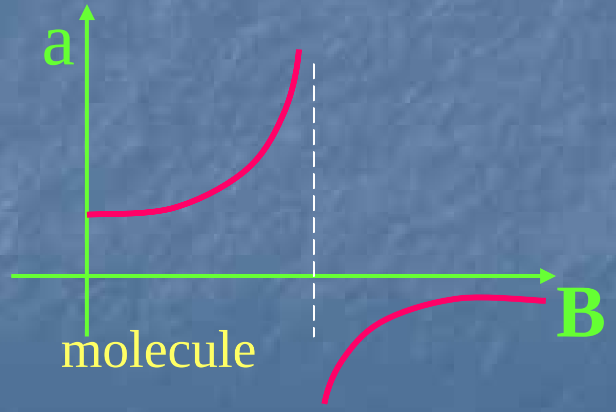


(ii) Resonant scattering:

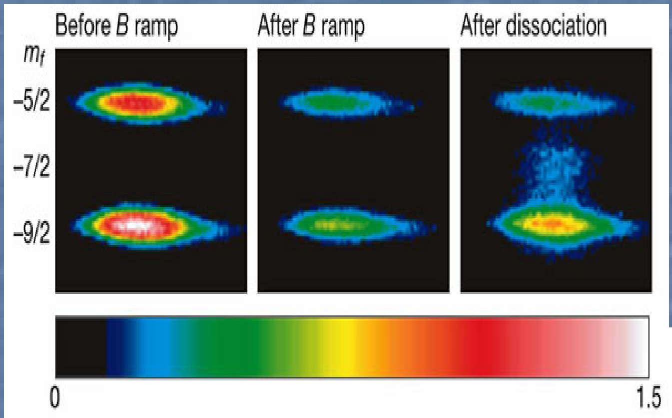


$$a = a_0\left(1 + \frac{\Delta B}{B - B_0}\right)$$

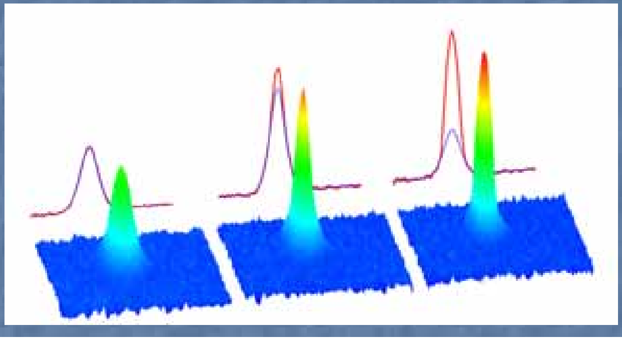




Molecule and pair condensate



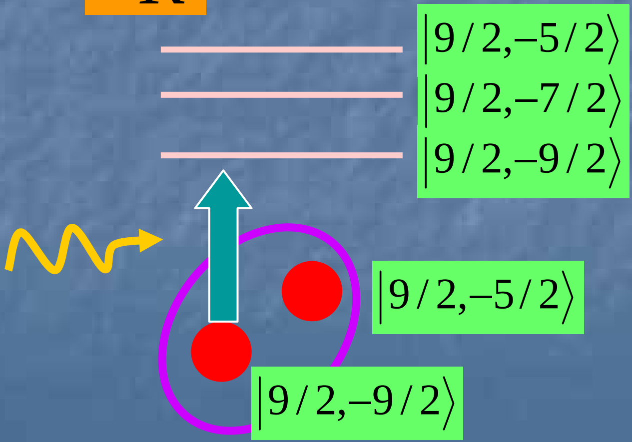
(JILA, after Nature **424**, 47 ('03))

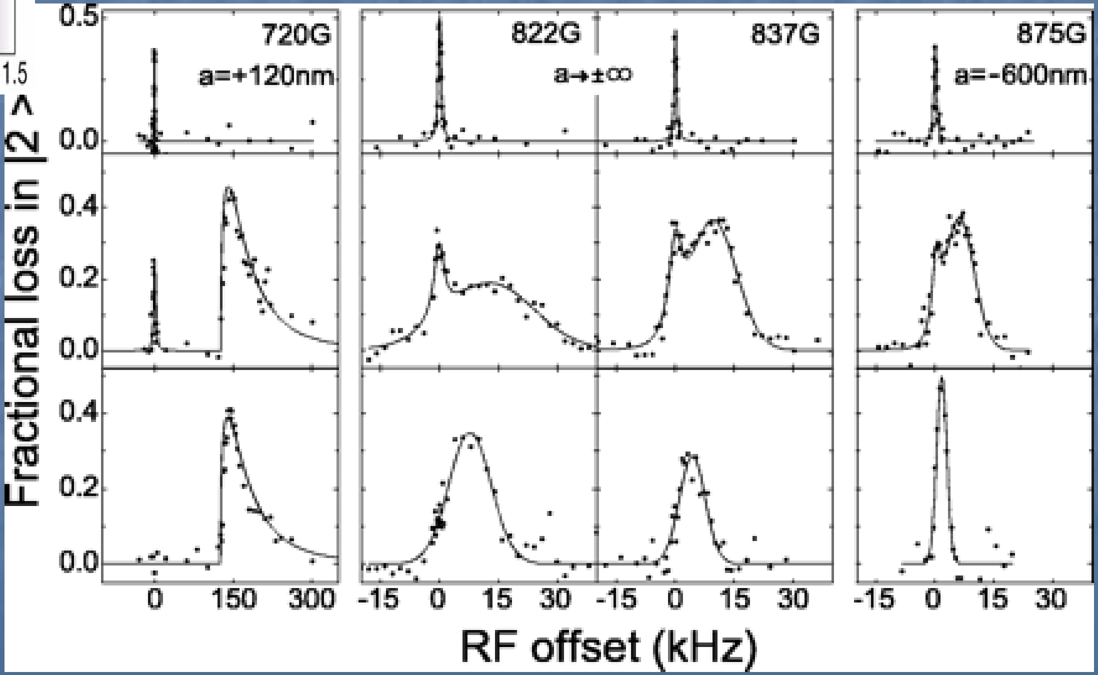


(MIT group, PRL **92**, 120403 ('04))

^{6}Li

^{40}K

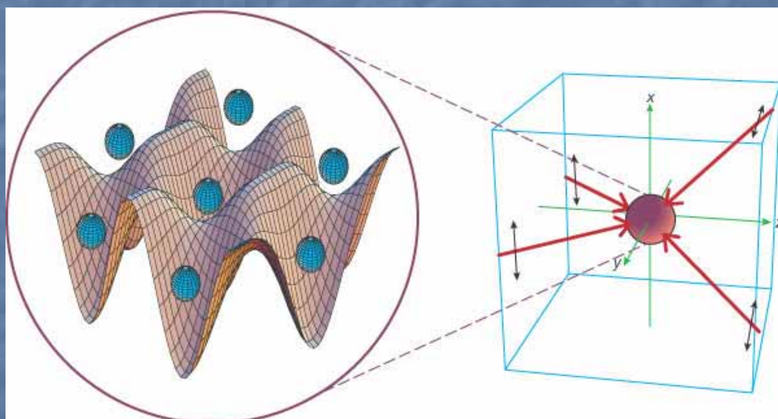




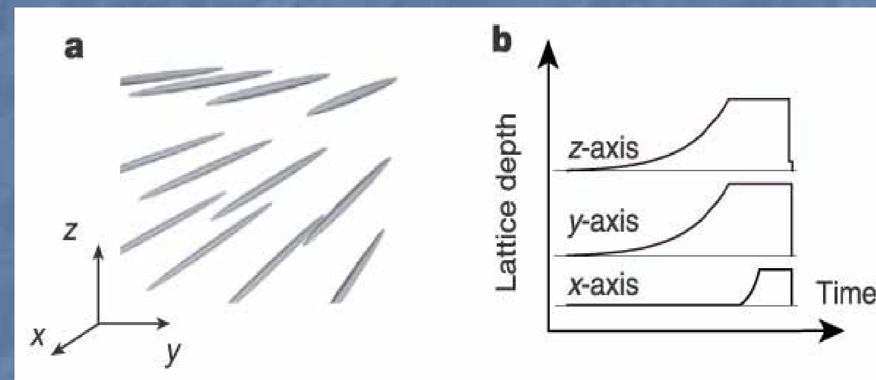
(Innsbruck, after Science **305**, 1128 ('04))

Optical lattice

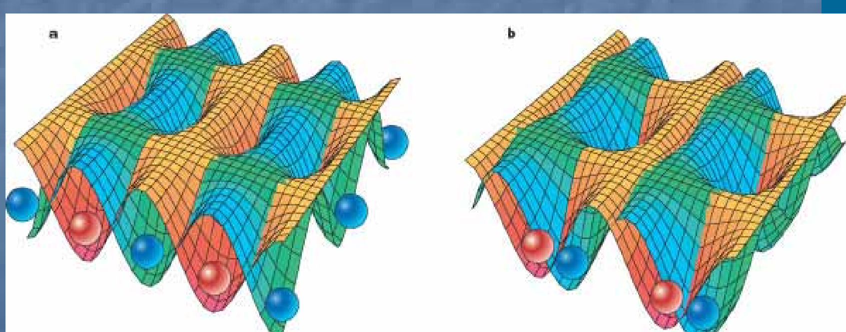
3D lattice

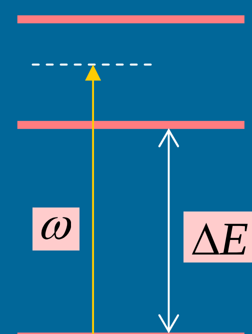


1D lattice

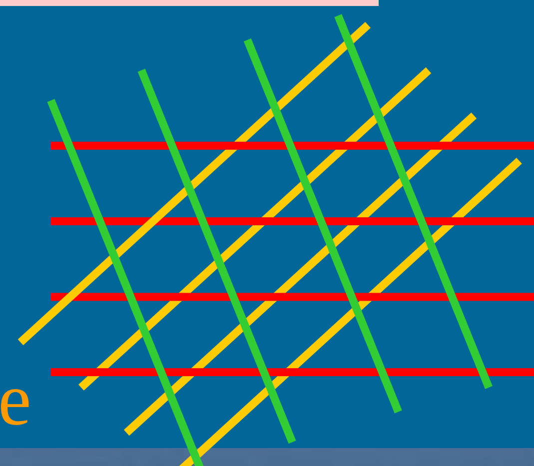


Entanglement control





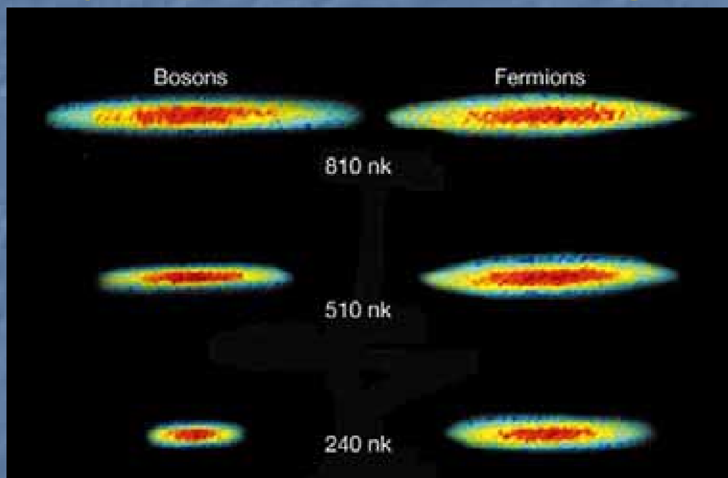
$$V_0 \propto -\frac{\left|\Omega_R\right|^2(\omega-\Delta E)}{(\omega-\Delta E)^2+\Gamma^2}$$



other lattice

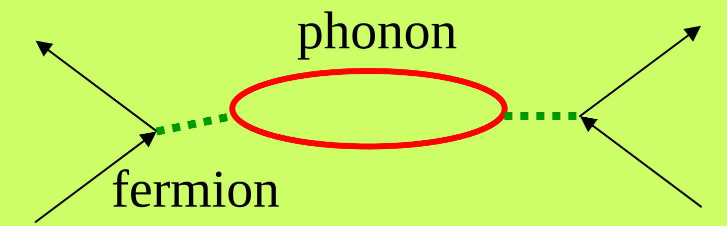
Boson-fermion mixtures

Sympathetic cooling



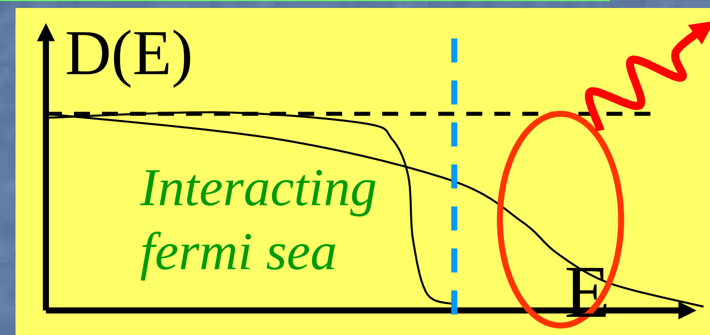
(after Science **291**, 2570 ('01))

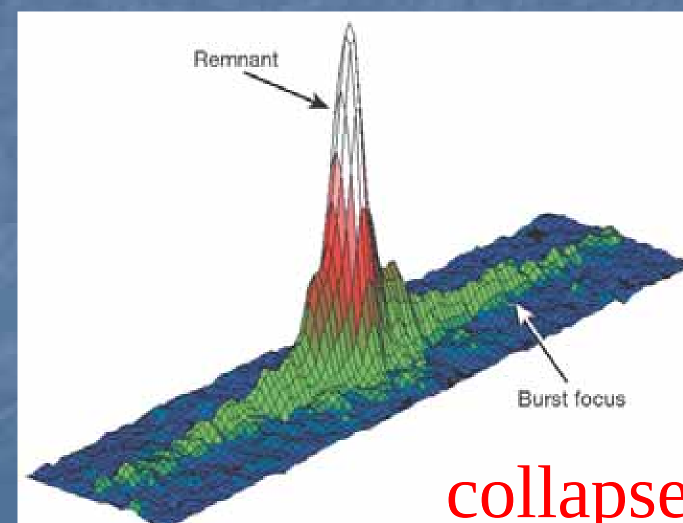
Fermions are noninteracting !



phonon-mediated interaction

$^{40}K-^{87}Rb$, $^{6}Li-^{7}Li$, or $^{6}Li-^{23}Na$





(after Nature **412**, 295 ('01))

Phase instability

$$H=\sum_{k,s}\varepsilon^{f}_{k,s}f^{+}_{k,s}f_{k,s}+\sum_{k}\varepsilon^{b}_{k}b^{+}_{k}b_{k}+\frac{U_{bb}}{2\Omega}\sum_{k}\rho^{b}_{k}\rho^{b}_{-k}+\frac{U_{bf}}{\Omega}\sum_{k}\rho^{b}_{k}(\rho^{f}_{-k\uparrow}+\rho^{f}_{-k\downarrow})+\frac{U_{ff}}{\Omega}\sum_{k}\rho^{f}_{-k\uparrow}\rho^{f}_{k\downarrow}$$

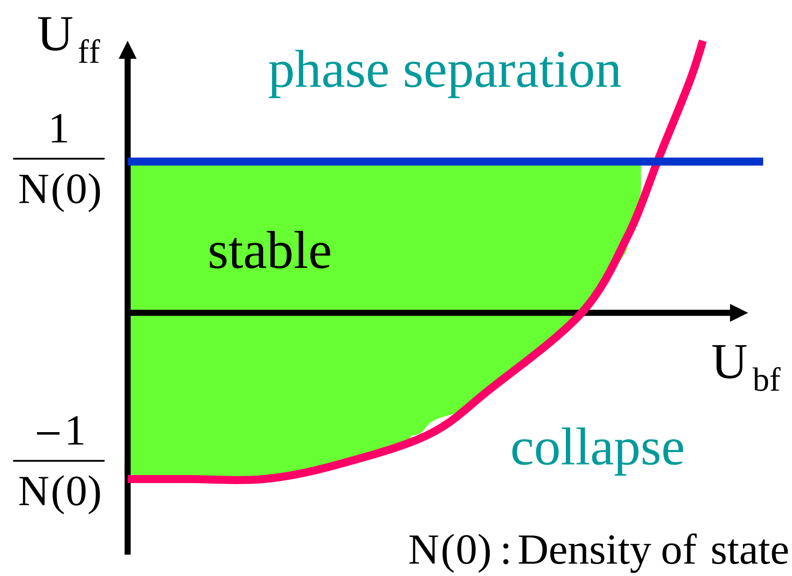
$$\Delta E=\int d\vec{r}\,\varepsilon(n_1+\delta n_1,n_2+\delta n_2)-\int d\vec{r}\,\varepsilon(n_1,n_2)$$

$$=\int d\vec{r}\left[\cancel{\frac{\delta\varepsilon}{\delta n_1}\delta n_1}+\cancel{\frac{\delta\varepsilon}{\delta n_2}\delta n_2}\right]+\frac{1}{2}\int d\vec{r}\;\left[\frac{\delta^2\varepsilon}{\delta n_1^2}\delta n_1^2+\frac{\delta^2\varepsilon}{\delta n_2^2}\delta n_2^2+2\frac{\delta^2\varepsilon}{\delta n_1\delta n_2}\delta n_1\delta n_2\right]+\cdots>0$$

condition for stability

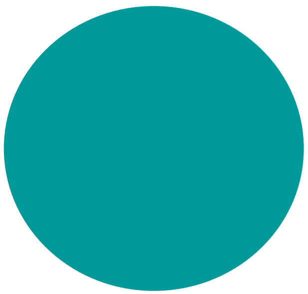
eigenvalues of $\begin{bmatrix}\dfrac{\delta^2\varepsilon}{\delta n_1^2} & \dfrac{\delta^2\varepsilon}{\delta n_1\delta n_2}\\ \dfrac{\delta^2\varepsilon}{\delta n_1\delta n_2} & \dfrac{\delta^2\varepsilon}{\delta n_2^2}\end{bmatrix}$ >0

$$\left|U_{ff}\right|<\frac{1}{N(0)}\qquad U_{ff}-\frac{2U_{bf}^2}{U_{bb}}>\frac{-1}{N(0)}$$

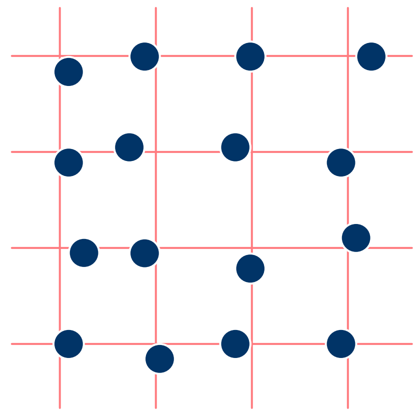


N(0) : Density of state

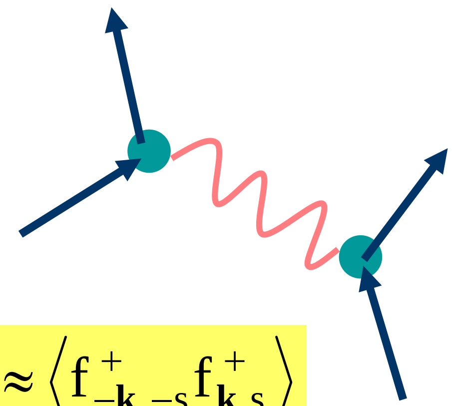
Introduction to superconductors



+



=



$$\Delta(\mathbf{k}) \approx \left\langle f^{+}_{-\mathbf{k},-s} f^{+}_{\mathbf{k},s} \right\rangle$$

Fermi sea | lattice phonon | superconductor

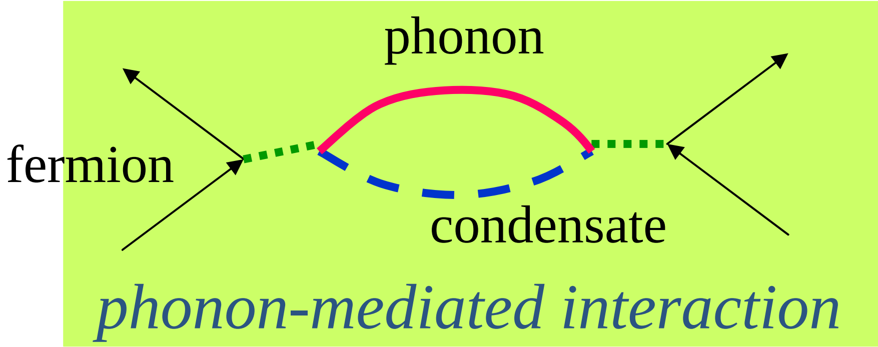
BCS theory:

$$T_c \approx 1.13\,\omega_D e^{-1/N(0)V_0}$$

$$T_c \approx 0.61\,E_F e^{-1/N(0)U_0}$$

(contact attractive interaction)

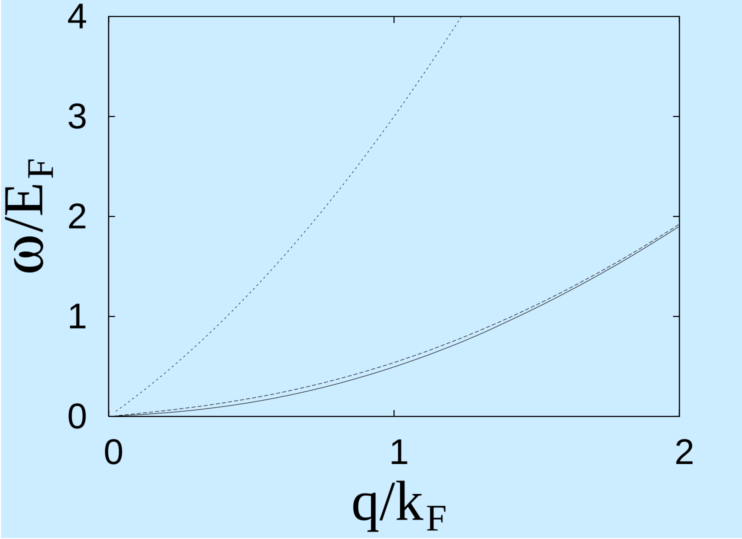
Phonons in BFM



$$\omega_{\mathbf{k}} = \sqrt{\frac{\mathbf{k}^2}{2m_b}\left(\frac{\mathbf{k}^2}{2m_b} + 2n_b U_{bb}\right)}$$

$$= c\,|\mathbf{k}| \quad \text{for } |k| << 1$$

But....



Anti-adiabatic regime: $c >> v_F$

$$V_{eff}(\vec{r}) = \frac{-U_{bf}^2}{U_b} \times \frac{1}{r} e^{-r/\zeta}$$

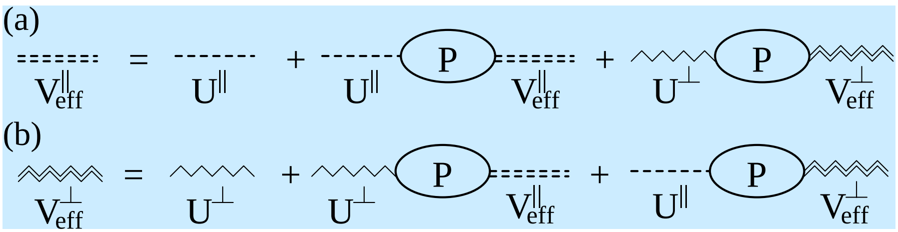
$$\zeta = \sqrt{1/4m_b\ n_b U_{bb}}$$

$$T_c \approx 0.61\, E_F e^{1/N(0)(U_{ff} - V_{eff})}$$

Phonon induced interaction

(DWW, *et. al.* in preparation)

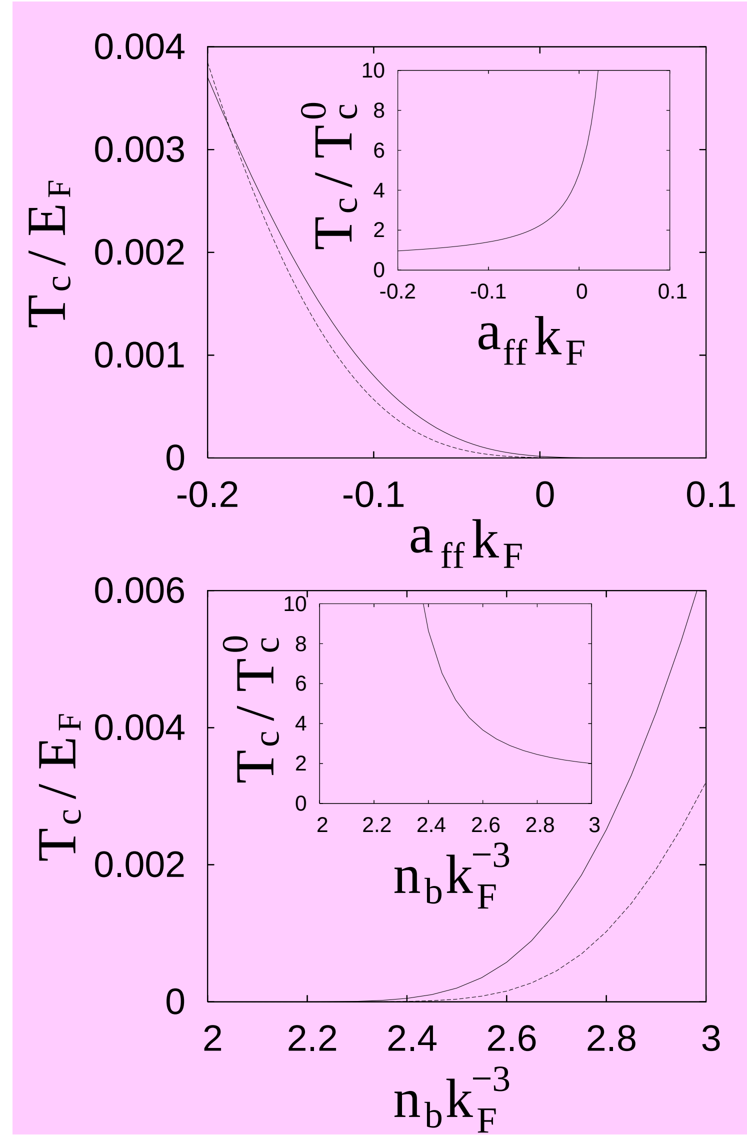
Strong coupling regime + retardation effect:



Solving Migdal-Elashberg equation with short-range onsite repulsion:

$$T_c \approx \left(\frac{8\gamma E_F}{\pi e^2 (1+\lambda_{\parallel})}\right)^{\frac{N(0)U_{ff}}{N(0)U_{ff}-\lambda_{\perp}}} \left(\frac{2\gamma D}{\pi}\right)^{\frac{-\lambda_{\perp}}{N(0)U_{ff}-\lambda_{\perp}}}$$

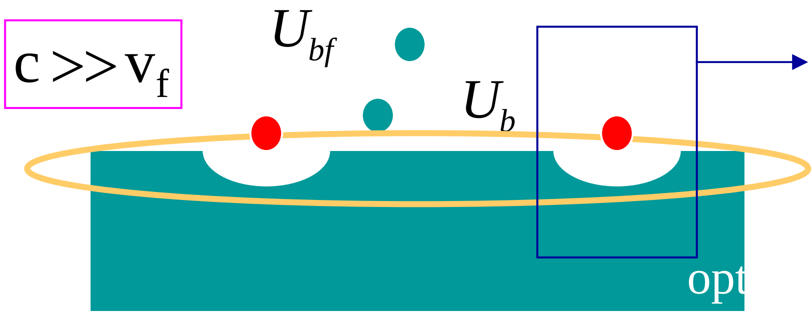
$$\times \exp\left[\frac{1+\lambda_{\parallel}+\lambda_{log}}{N(0)U_{ff}-\lambda_{\perp}}\right]$$



Polaronic effects (1D case)

(L. Mathey, DWW, et. al. Phys. Rev. Lett. 93, 120404 (2004).)

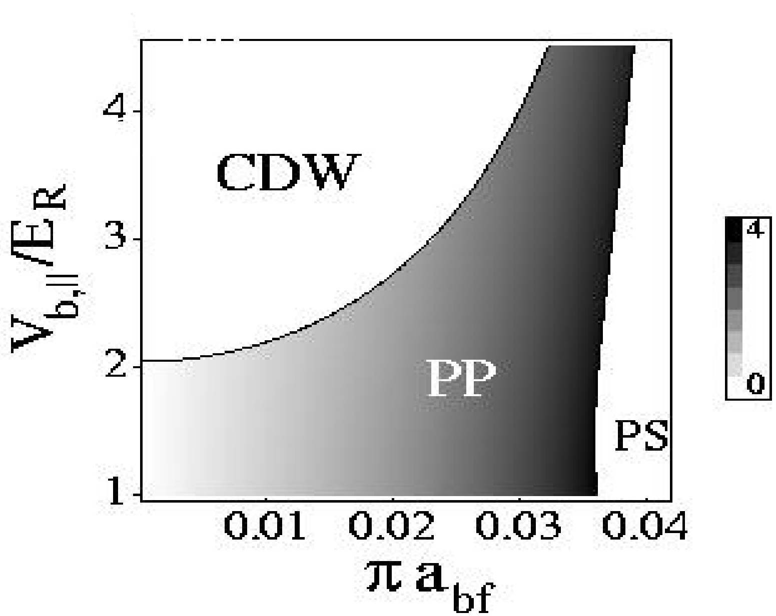
$c >> v_f$

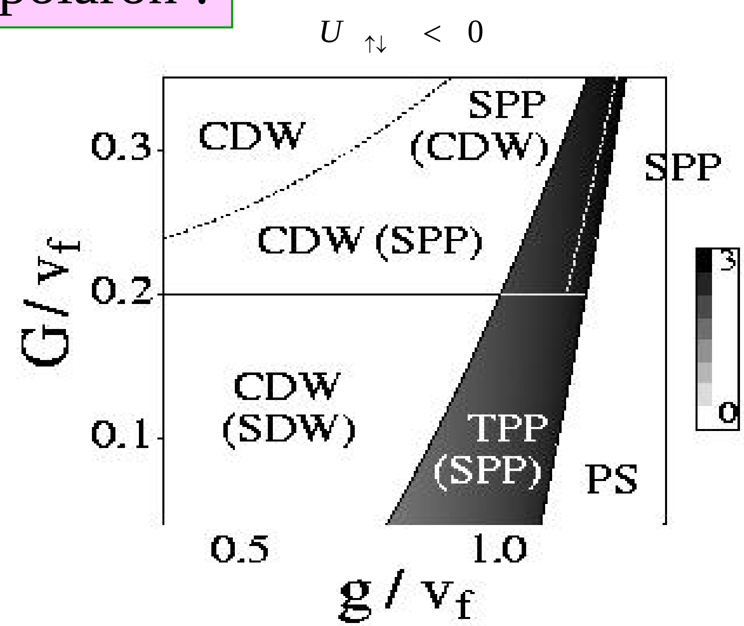


$$\tilde{f}_\lambda = f \, e^{-i\lambda \Phi_b}$$

$$\lambda \sim U_{bf} / U_b$$

Luttinger liquid of polaron !





Polaronic effects (more general case)

(DWW, *et. al.* in preparation)

$$H = \sum_{\mathbf{k}\neq 0} \omega_{\mathbf{k}} \beta_{\mathbf{k}}^{+} \beta_{\mathbf{k}} + \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} f_{\mathbf{k}}^{+} f_{\mathbf{k}} + \sum_{\mathbf{k}\neq 0} g_{\mathbf{k}} \left(\beta_{\mathbf{k}}^{+} \rho_{\mathbf{k}} + \beta_{\mathbf{k}} \rho_{\mathbf{k}}^{+} \right)$$

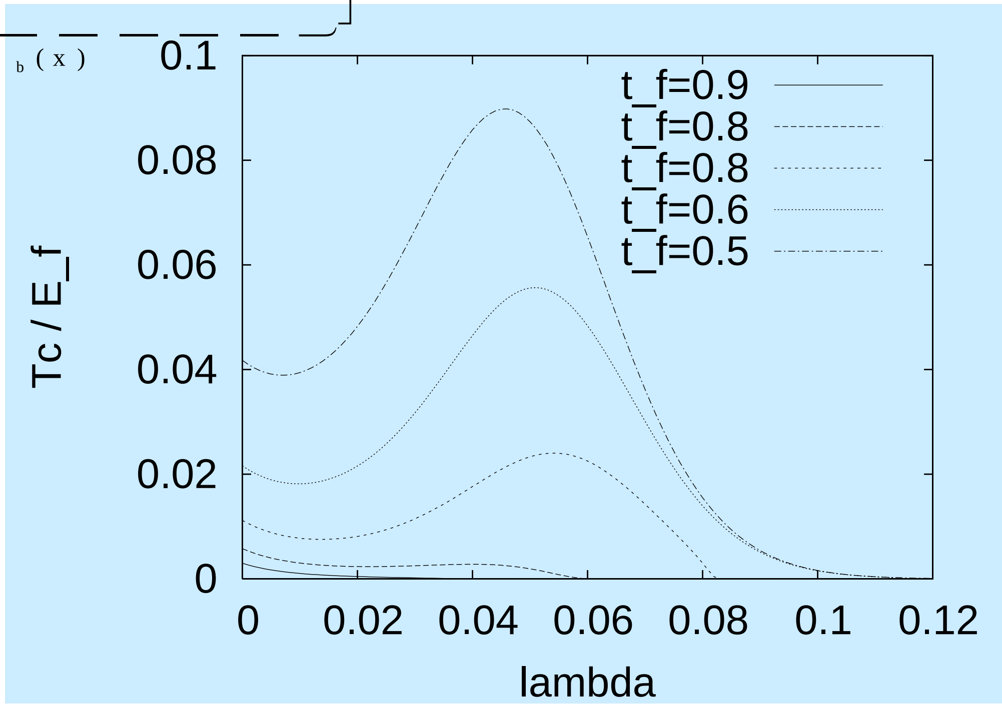
phonon fermion phonon fermon

$$S(\lambda) = \exp\left[-\lambda \sum_{\mathbf{k}\neq 0} \frac{g_{\mathbf{k}}}{\omega_{\mathbf{k}}} \left(\beta_{\mathbf{k}}^{+} \rho_{\mathbf{k}} - \beta_{\mathbf{k}} \rho_{\mathbf{k}}^{+} \right) \right]$$

$$\beta_{\mathbf{k}} \to \beta_{\mathbf{k}} - \frac{\lambda g_{\mathbf{k}}}{\omega_{\mathbf{k}}} \rho_{\mathbf{k}}$$

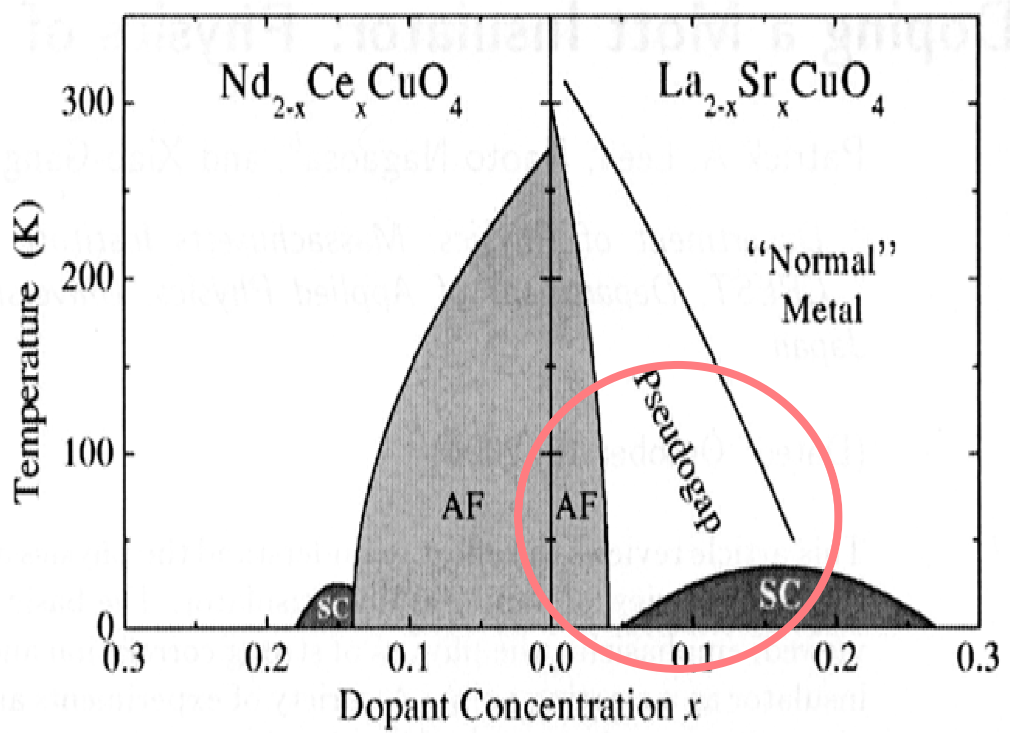
$$f(\mathbf{r}) \to f(\mathbf{r}) \underbrace{\exp\left[\lambda \sum_{\mathbf{k}} \frac{g_{\mathbf{k}}}{\omega_{\mathbf{k}}} \left(e^{-i\mathbf{k}\cdot\mathbf{r}} \beta_{\mathbf{k}} - e^{i\mathbf{k}\cdot\mathbf{r}} \beta_{\mathbf{k}}^{+} \right) \right]}_{= e^{-i\lambda \Phi_b(x)}}$$

If we treat λ as a variational parameter, T_c could be enhaced due to mass enhancement of polaronic effects.

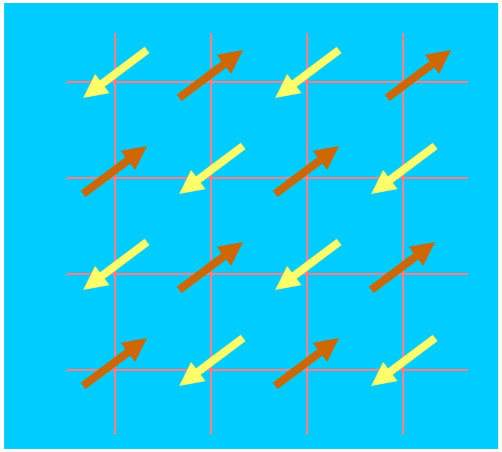


Motivation: High T_c superconductor

electron doped　　hole doped

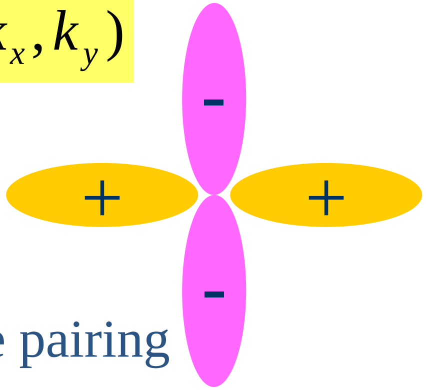


P.A. Lee, N. Nagaosa, and X.-G. Wen, cond-mat/0410445



Antiferromagnetic

$\Delta(k_x, k_y)$



D-wave pairing

Unknown questions in high *Tc* ...

(1) The origin of superconductivity:

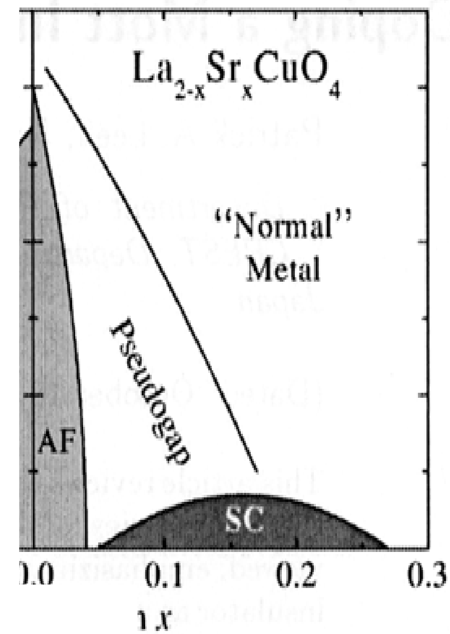
(i) Is the proximity to AF crucial ?

(ii) Can one get superconductivity from purely repulsive interaction ?

$$H = -t \sum_{<ij>,s} c_{i,s}^{+} c_{j,s} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} \quad (U > 0)$$

(iii) The effects of phonons

(iv) Other exotic order (RVB, stripe...)?



(2) Non-BCS behavior:

(i) d-wave pairing

(ii) Small coherent length

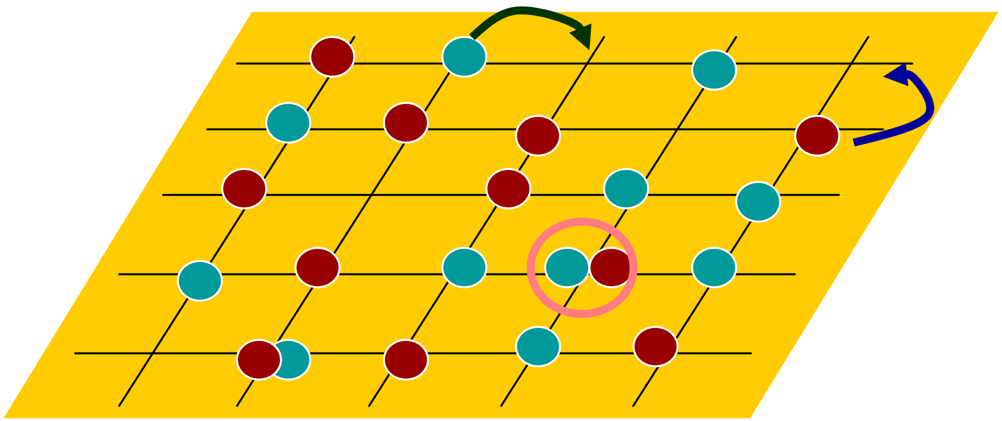
(iii) Pseudo-gap regime

(3) Non-Fermi liquid in normal state

* A unified mechanism is still poorly understood.

B-F mixtures in 2D optical lattice

(DWW, M. Lukin, E. Demler, cond-mat/0410494)

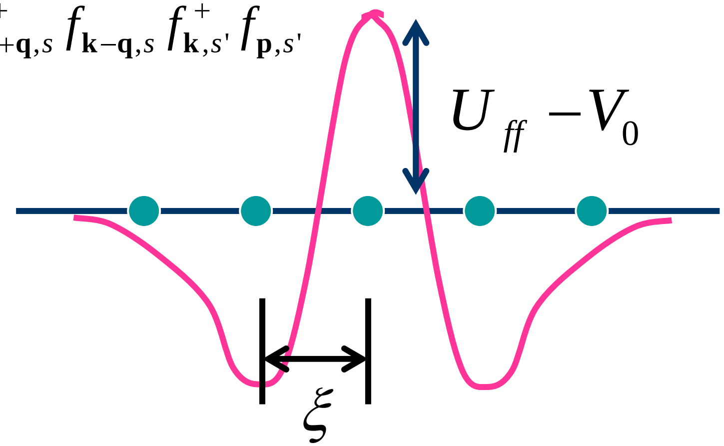


- $^{40}K \; |9/2, -9/2\rangle$
- $^{40}K \; |9/2, -7/2\rangle$
- $^{87}Rb \; |1,1\rangle$ or $^{23}Na \; |1,1\rangle$

Effective H: $$H = -t \sum_{\langle ij \rangle, s} f_{i,s}^{+} f_{j,s} + \frac{1}{2} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} V_{\text{eff}}(\mathbf{q}) \, f_{\mathbf{p}+\mathbf{q},s}^{+} f_{\mathbf{k}-\mathbf{q},s} f_{\mathbf{k},s'}^{+} f_{\mathbf{p},s'}$$

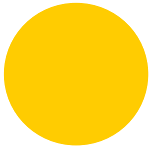
Effective interaction: $$V_{\text{tot}}(k) = U_{ff} + \frac{-V_0}{1 + k^2 \xi^2}$$

$$V_0 = U_{bf}^2 / U_b, \quad \xi = \sqrt{t_b / 2 n_b U_{bb}}$$

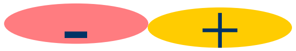


Possible many-body phases

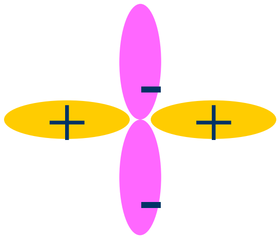
(1) s-wave pairing $\Delta(\mathbf{k}) = \Delta_0 + \Delta_1(\cos k_x + \cos k_y)$



(2) p-wave pairing $\Delta(\mathbf{k}) = \Delta_p \sin k_x$



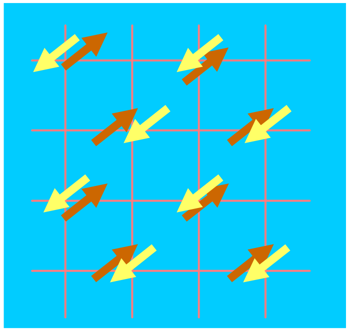
(3) d-wave pairing $\Delta(\mathbf{k}) = \Delta_d (\cos k_x - \cos k_y)$



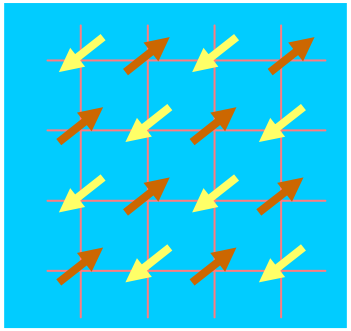
(4) charge-density-wave (5) spin-density wave

(most effective near half-filling)

$\langle f^+_{\mathbf{k}+\mathbf{Q},s} f_{\mathbf{k},s} \rangle \neq 0$



$\langle f^+_{\mathbf{k}+\mathbf{Q},s} f_{\mathbf{k},-s} \rangle \neq 0$



(6) phase separation/collapse $\dfrac{\partial^2 E_{tot}}{\partial n^2} < 0$

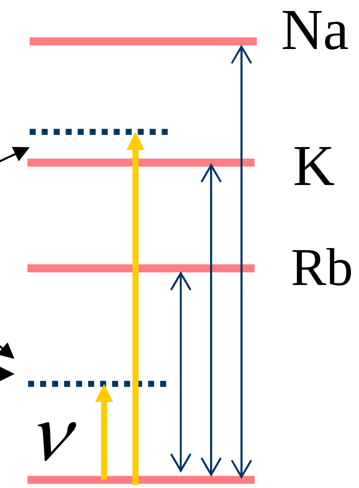
Systems and parameters

(1) Systems we consider:

(a) ^{40}K-^{87}Rb $\lambda = 1.06\ \mu m$ (Nd : YAG)

(b) ^{40}K-^{87}Rb $\lambda = 0.7655\ \mu m$

(c) ^{40}K-^{23}Na $\lambda = 1.06\ \mu m$ (Nd : YAG)



(2) Parameters used in calculation:

(a) $^{40}\text{K}\ \ |9/2,-7/2\rangle + |9/2,-9/2\rangle$ + $^{87}\text{Rb}\ \ |1,1\rangle$ or $^{23}\text{Na}\ \ |1,1\rangle$

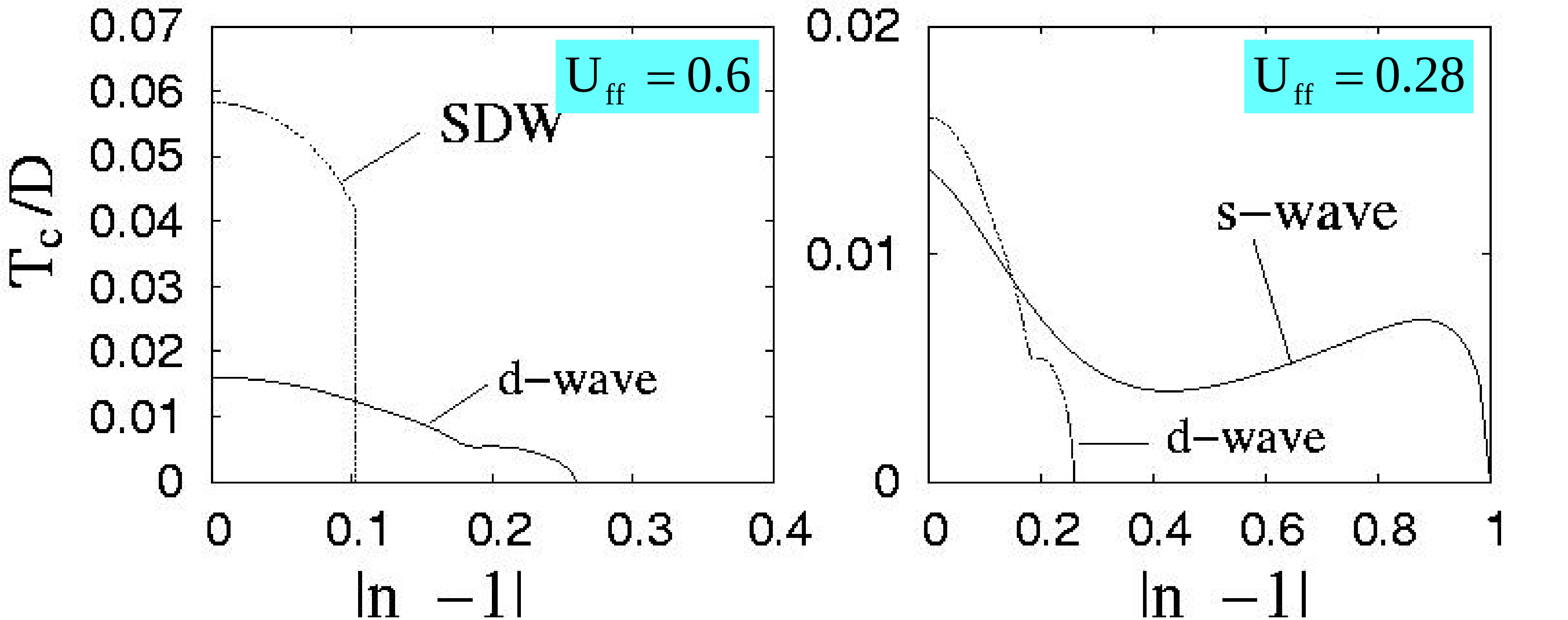
(b) $V^K_{xy}/E_R = 5,\ V^K_z / E_R = 30$ $U_{ff} > 0$

(c) Boson density is adjusted to optimize Tc

(d) Phase diagrams plotted in filling fraction n and $|U_{bf}| / E_R$

T_c for different phases

$^{40}K-^{23}Na$

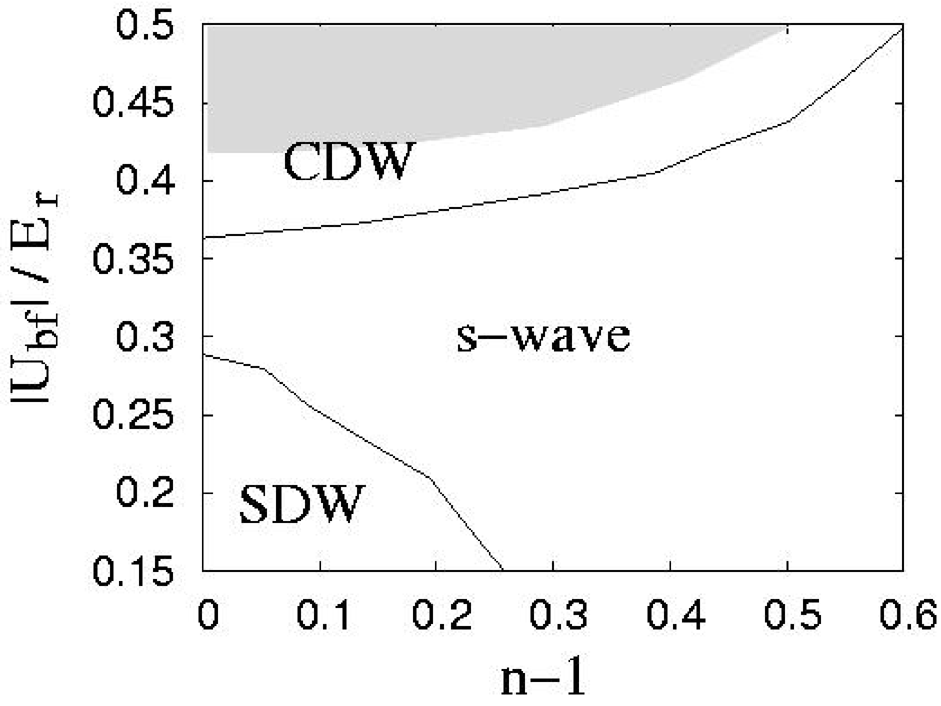


(a) Competition between *d*-wave and SDW (AFM) phases is similar to the one in high *Tc* cuprates.

(b) Competition between *s*- and *d*-wave phases can be observed.

Phase diagrams for K-Rb system (I)

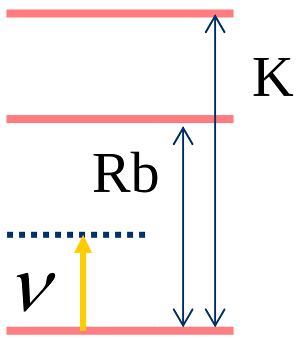
$^{40}\text{K}\quad |9/2,-7/2\rangle + |9/2,-9/2\rangle \qquad ^{87}\text{Rb}\quad |1,1\rangle$



$\lambda = 1.06\ \mu\text{m}$

$\text{V}_{\text{xy}}^{\text{K}}/\text{E}_{\text{R}} = 5,\ \ \text{V}_{\text{z}}^{\text{K}} / E_R = 30$

$n_b = 1, \qquad \xi / a = 0.12$

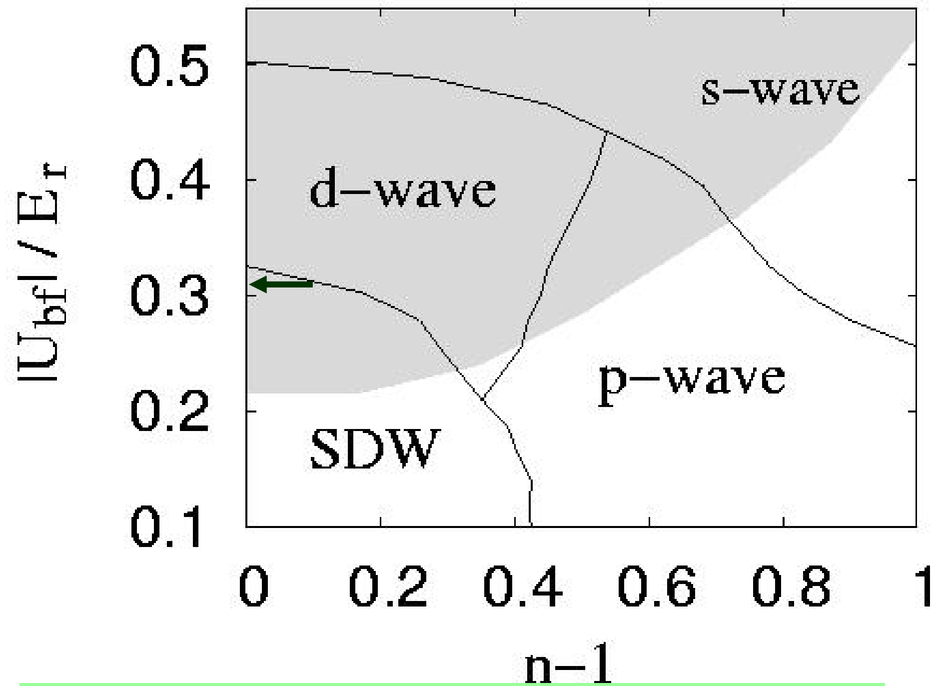


(a) *s*-wave can dominate near *n=1*:

(b) No unconventional pairing phase $(\xi / a = 0.12 << 1)$

Phase diagrams for K-Rb system (II)

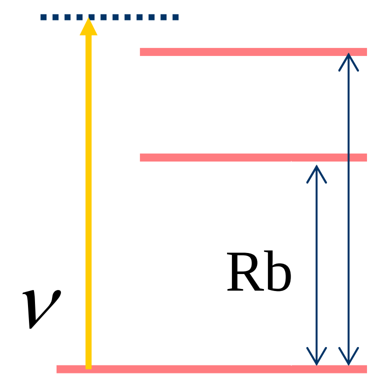
Idea: to make the boson "effective mass" lighter



$\lambda = 765.5\,\mathrm{nm}$

$V^{K}_{xy}/E_R = 5,\;\; V^{K}_{z}/E_R = 30$

$n_b = 1, \qquad \xi / a = 1.5$



(a) *d*-wave can dominate near *n=1*:

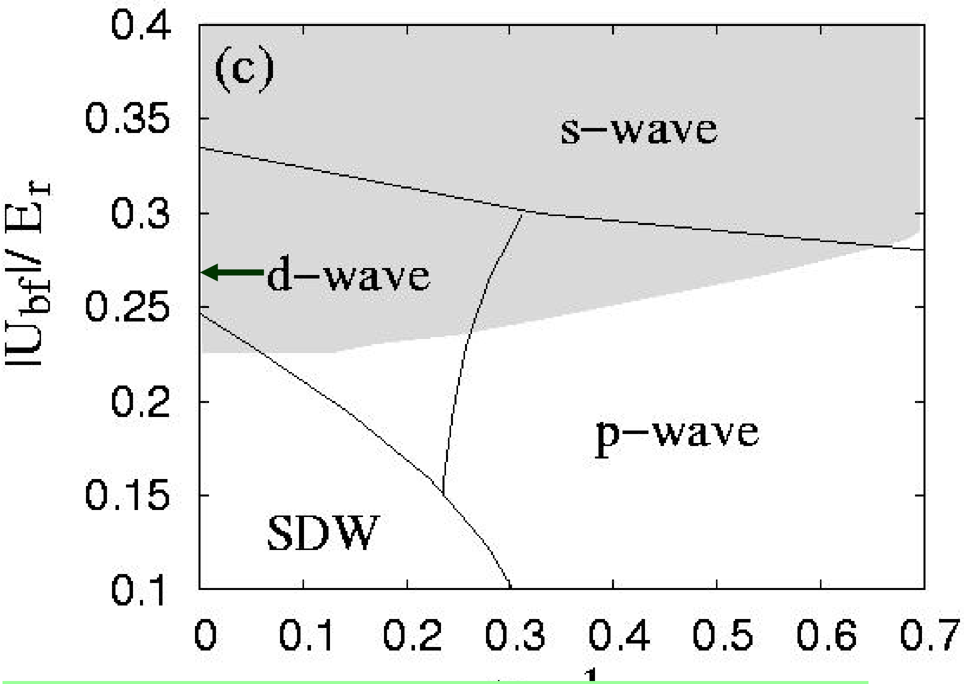
(b) *p*-wave can dominate away from *n=1*:

(c) Collapse regime is overestimated in meanfield theory

(Phys. Rev. A **67**, 063606 (2003).

Phase diagrams for K-Na system

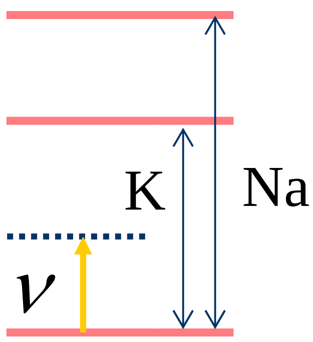
$^{40}\mathrm{K}\ \ |9/2,-7/2\rangle + |9/2,-9/2\rangle \qquad ^{23}\mathrm{Na}\ \ |1,1\rangle$



$\lambda = 1.06\ \mu\mathrm{m}$

$V_{xy}^{K}/E_R = 5,\ \ V_z^{K}/E_R = 30$

$n_b = 9, \qquad \xi / a = 0.6$



(a) *d*-wave can dominate near *n=1*:

(b) *p*-wave can dominate away from *n=1*:

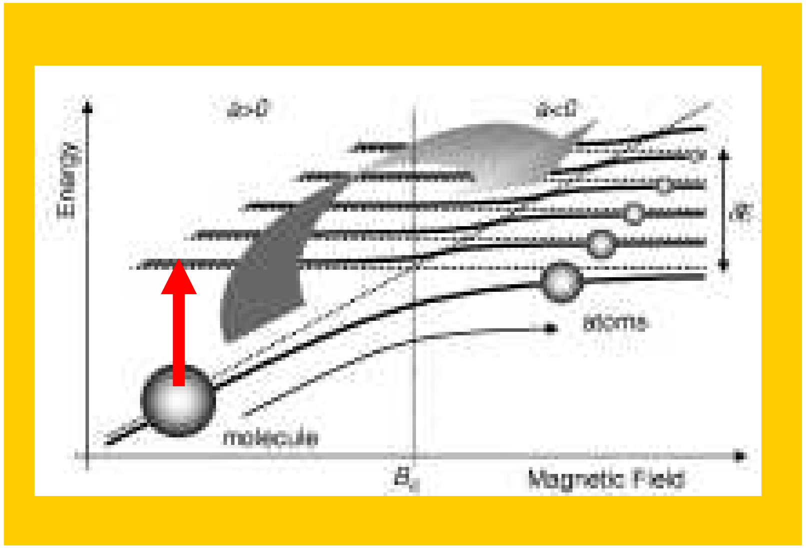
(c) d-wave phase can be observed in a wider range

System preparation (I)

(DWW, M. Lukin, E. Demler, cond-mat/0410494)

(I) Traditional method:

(1) Prepare B-F mixtures
(2) Sympathetic cooling
(3) Adiabatic cooling by turning the lattice on
(4) Tuning the magnetic field



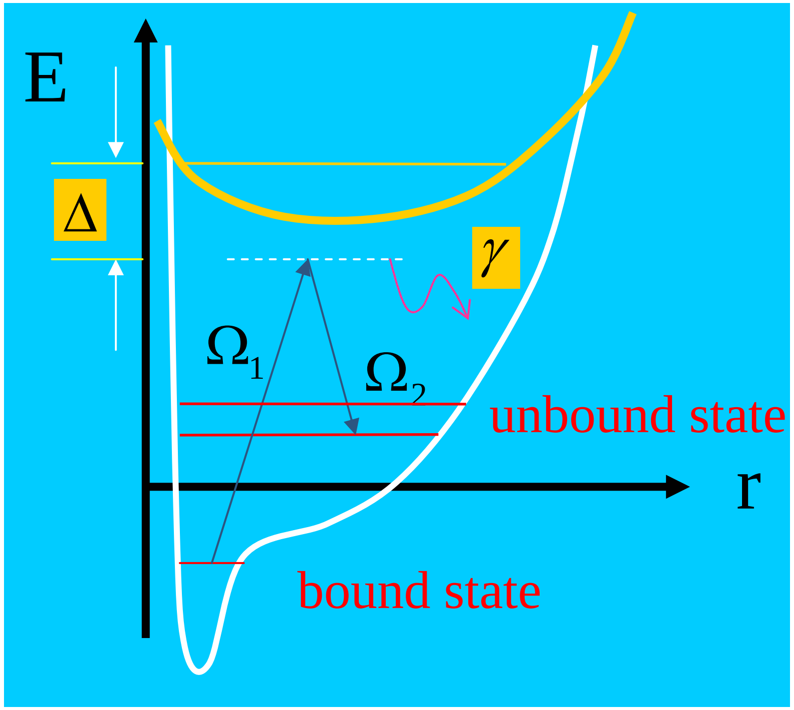
(II) Proposed method:

(1) Prepare (bi-fermion) molecule and boson mixtures
(2) Sympathetic cooling to condensation of bosons and molecules
(3) Adiabatically turn on the optical lattice to strong laser amplitude
(4) Apply two-photon Raman scattering to dissociate molecules
(5) Reduce lattice potential to lower value
(6) Tuning the magnetic field

System preparation (II)

--- Two-color Raman scattering, two-photon disassociation, or STIRAP (stimulated Raman adiabatic passage)

P.D. Drummond *et al*., PRA **65**, 63618 ('02); M. Mackie *et al*., PRL **84**, 3803 ('00); D. Jaksch *et al*., PRL **89**, 40402 ('02); R. Wynar, *et al*., Science **287**, 1016 ('00)



Effective Rabi frequency and linewidth

$$\Omega_{eff} = \Omega_1 \Omega_2 / 2\Delta$$

$$\gamma_{\mathrm{eff}} = \gamma\, \Omega_1^2 / 4\Delta^2$$

Effective linewidth can be reduced by Tuning the laser amplitude and detuning

Detection of exotic phases

(1) charge-density-wave $\left\langle f^{+}_{\mathbf{k}+\mathbf{Q},s} f_{\mathbf{k},s} \right\rangle \neq 0$

→ Time of flight measurement of bosons →peak at **k=Q**

(2) spin-density wave $\left\langle f^{+}_{\mathbf{k}+\mathbf{Q},s} f_{\mathbf{k},-s} \right\rangle \neq 0$

→ Noise correlation measurement of fermions →peak at **k=Q**

E. Altman, *et. al*., Phys. Rev. A **70**, 013603 (2004).

(3) Pairing phases: $\left\langle f_{\mathbf{k},s} f_{-\mathbf{k},s'} \right\rangle \neq 0$

→ Noise correlation measurement of fermions →peak at **k=0**

(4) Gap symmetry of pairing phases:

→ Bragg scattering →zero excitation between two nodal points

(5) Single particle gap:

→ rf spectroscopy →nonzero absorption above a critical frequency

Summary

(1) In 1D BFM system, polaronic effect can be solved exactly and the system can be understood as *Luttinger liquid of polarons*.

(2) Fermion-pairing phase could have higher T_c when including the polaronic effects and *strong coupling theory*.

(3) Many exotic quantum phases (like *s-,p-,d-wave pairing superfluidity, CDW and SDW*) can be observed in Bose-Fermi mixtures in 2D optical lattice by tuning various atomic or/and lattice parameters.

(4) *Two-color Raman scattering* can be used to created unconventional pairing phase.

(5) 2D B-F system can provide some insights to the understanding of *high T_c cuprates*.