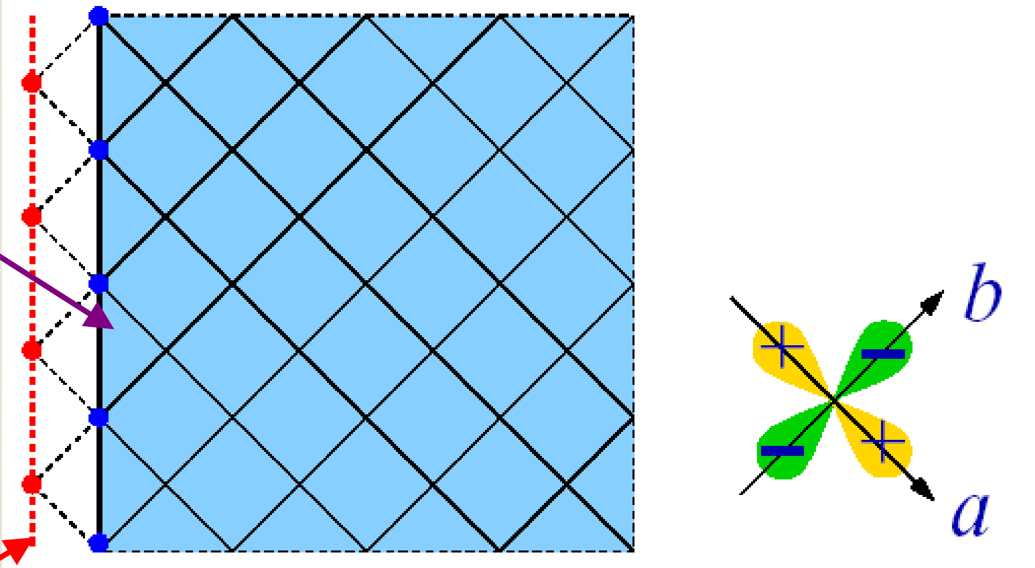
Physics at the edge

清華大學物理系牟中瑜

1. What does a d-wave superconductor look like in the mirror?

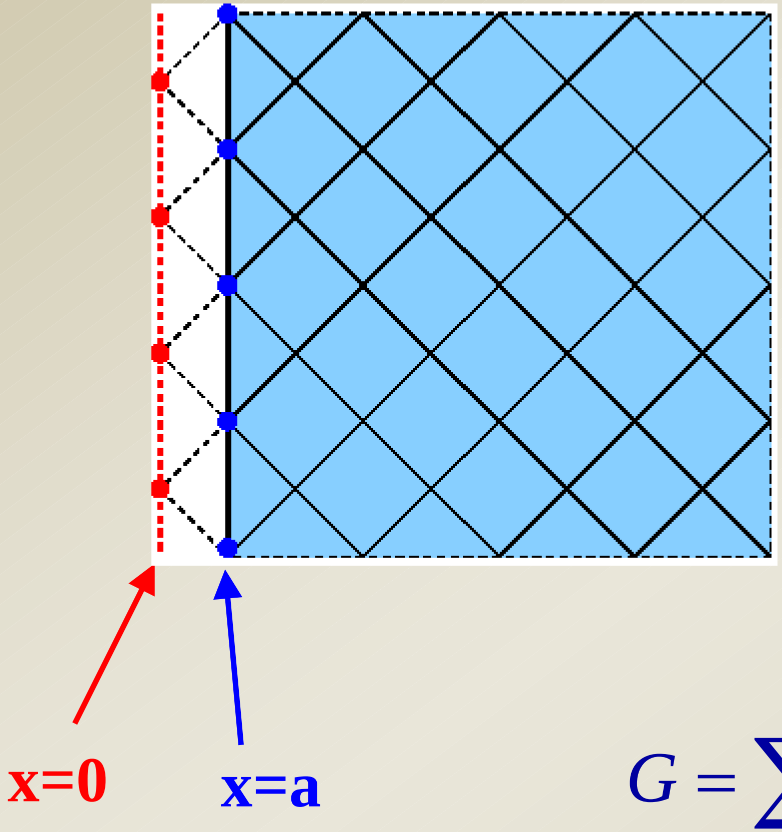
Local density of state = ?



(110) orientation

mirror

Naïve Expectation

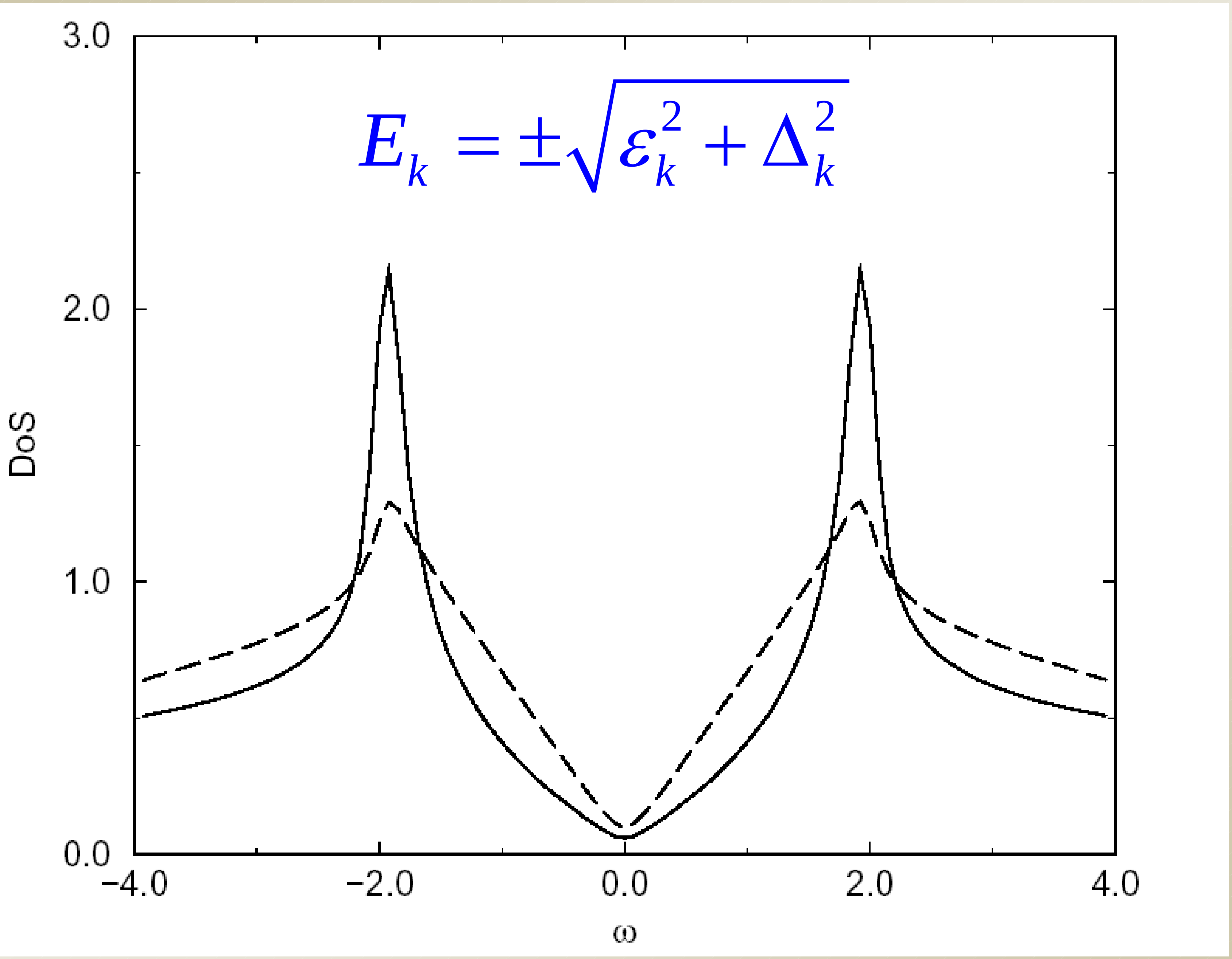


$$\because \psi(x=0)=0,\ \ \psi(\mathrm{x}) \propto \sin(k_x x)$$

$$\therefore LDOS = \sum_{k_x} \delta(\omega - E_{k_x}) \,|\, \psi(\mathrm{x})|^2$$

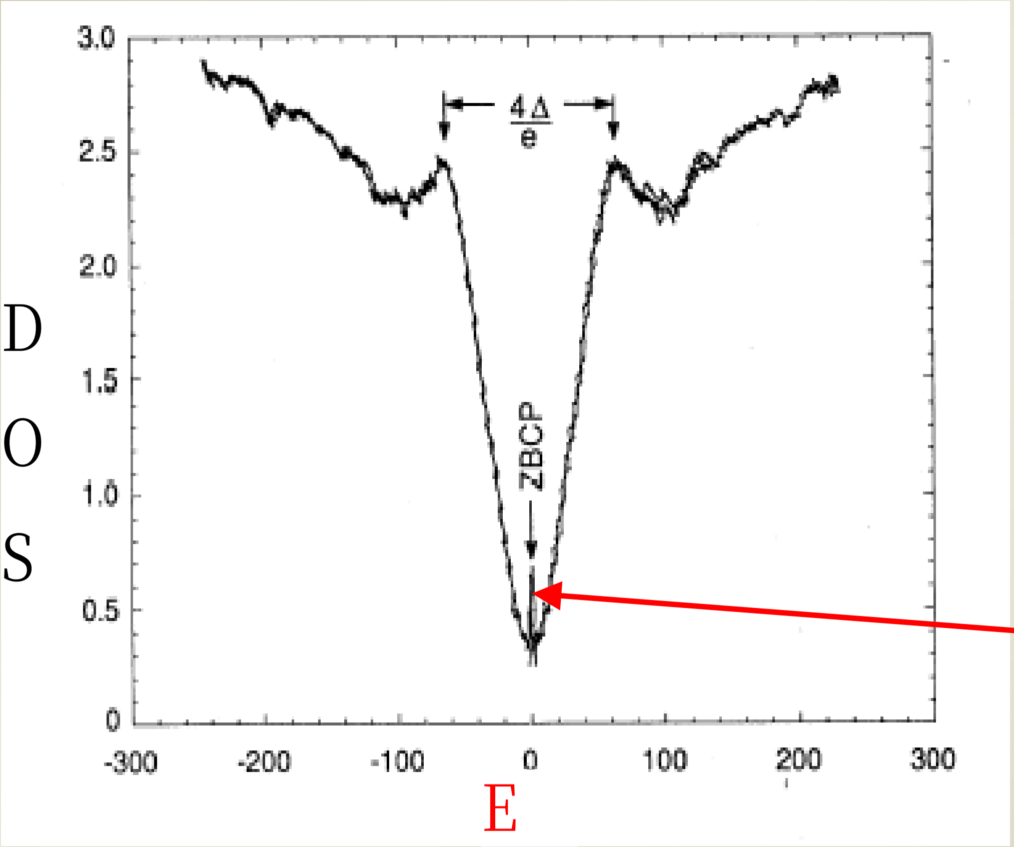
$$\propto \sum_{k_x} \delta(\omega - E_{k_x}) \sin^2(k_x x)$$

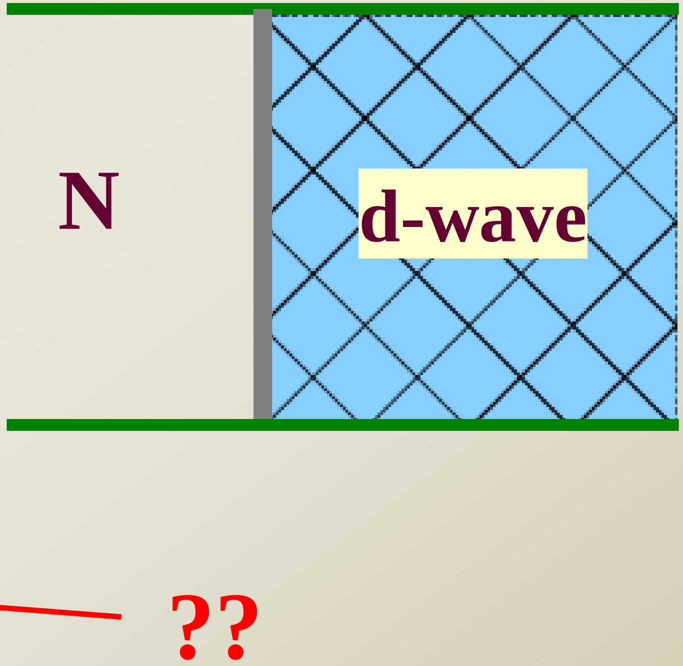
$$G = \sum_n \frac{\psi_n^*(x)\psi_n(x')}{\omega - E_n + i\varepsilon},\ \ LDOS = -\frac{\mathrm{Im}\, G(x,x)}{\pi}$$



$E_k = \pm\sqrt{\varepsilon_k^2 + \Delta_k^2}$
DoS
ω
3.0
2.0
1.0
0.0
−4.0
−2.0
0.0
2.0
4.0

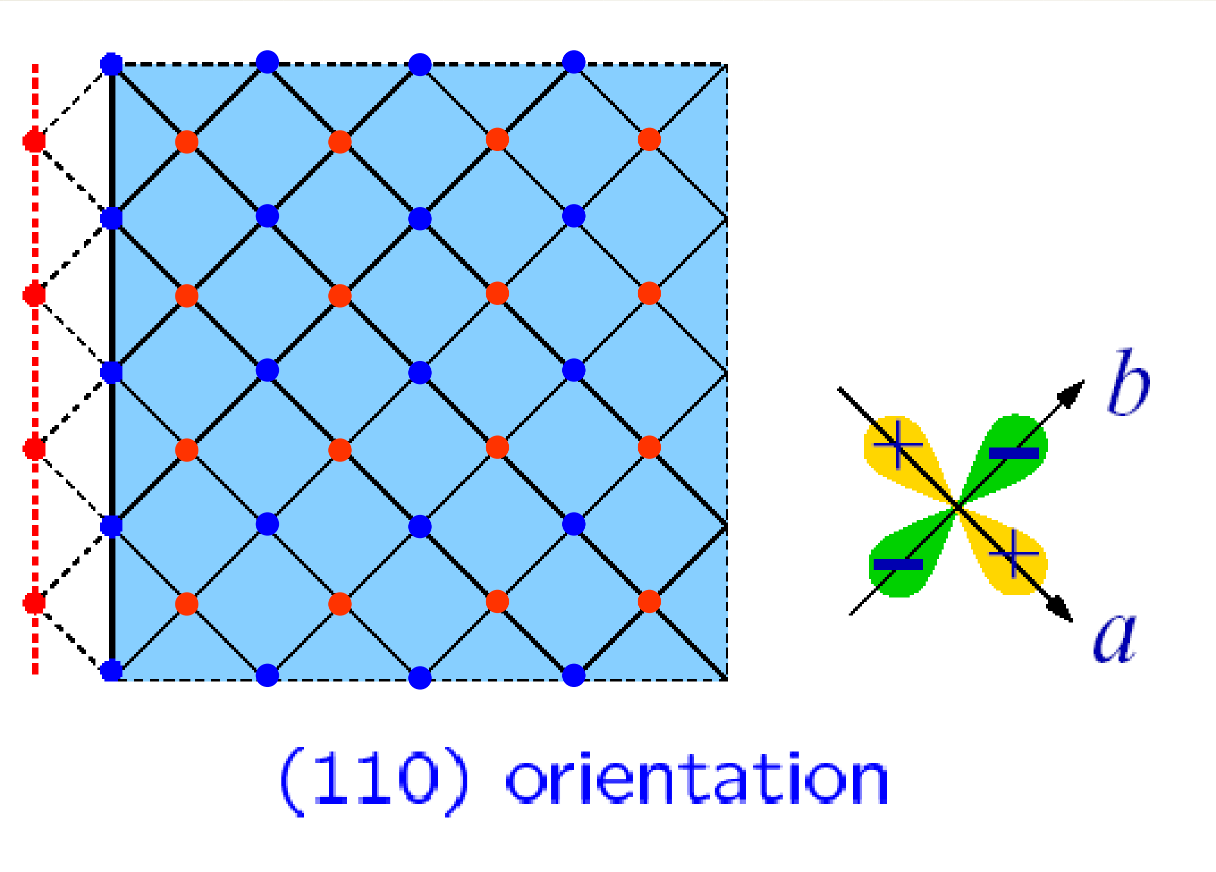
This does not work!





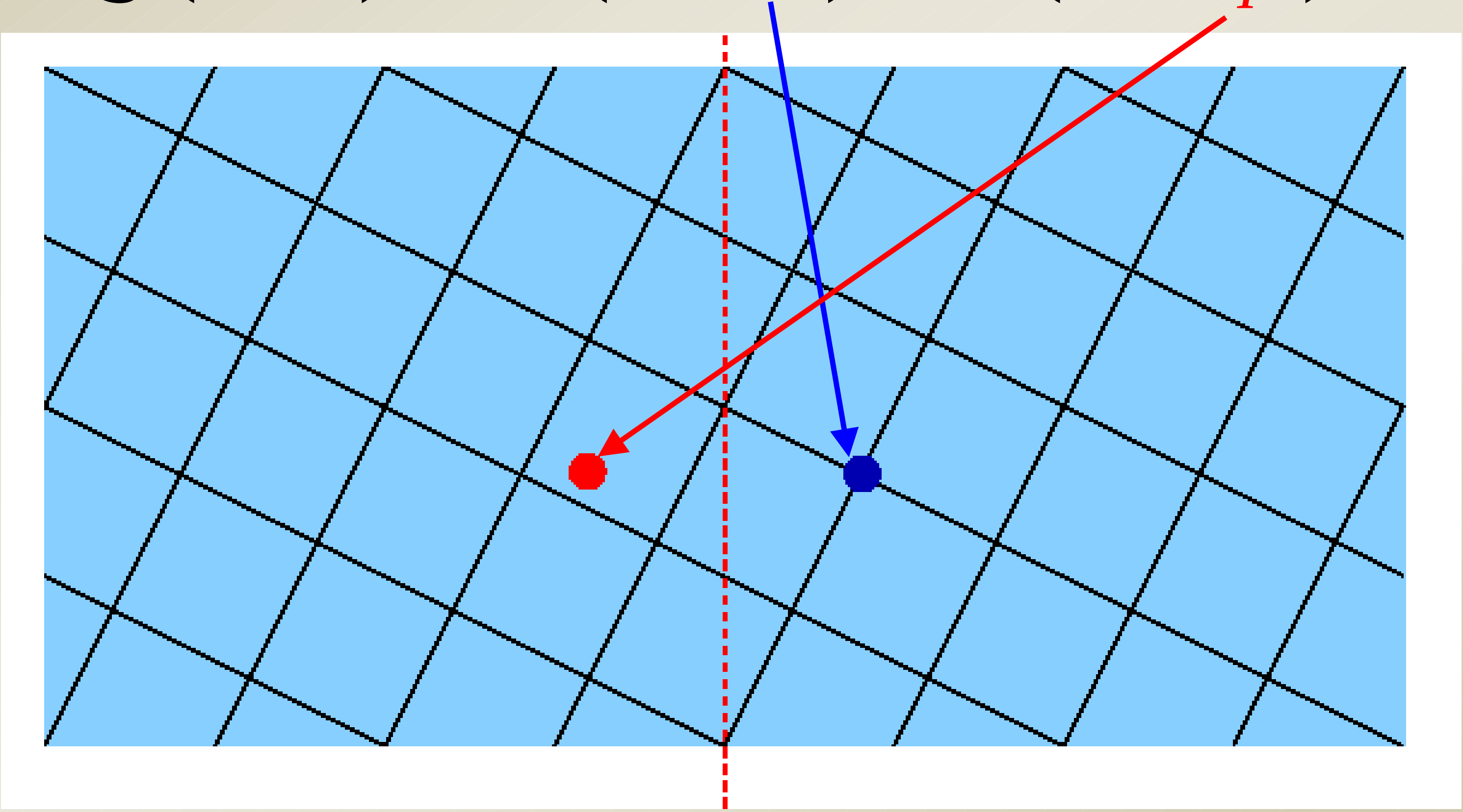
Walsh et. al. Phys. Rev. Lett. 66, 516 (1991)

Simple reason of why it does not work



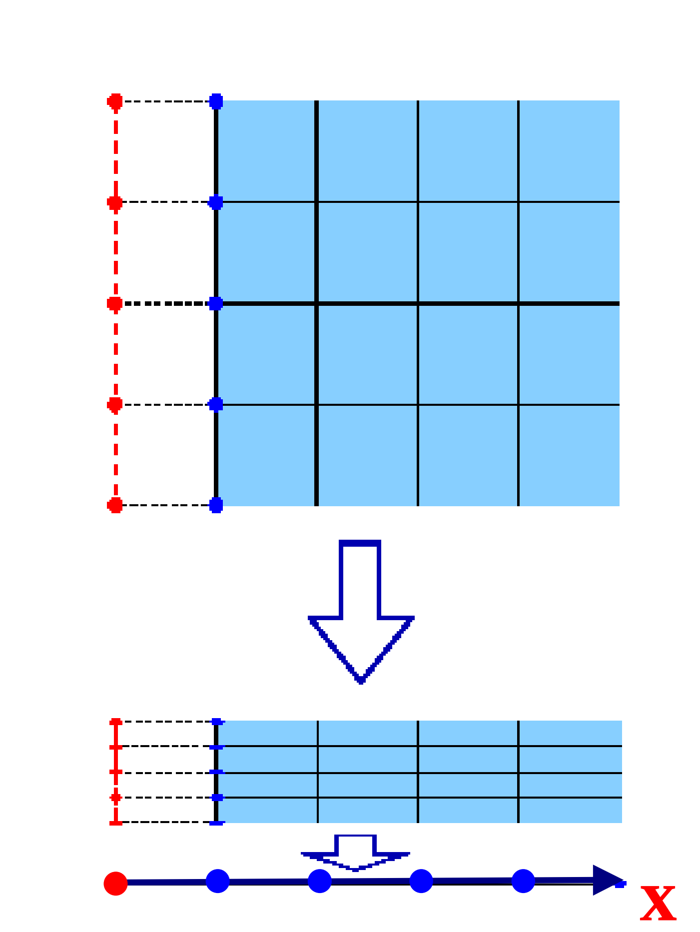
Difficulty with conventional method of image on lattice

$$g(r,r') = G(r - r') - G(r - r_I')$$



The image is not at a lattice point!

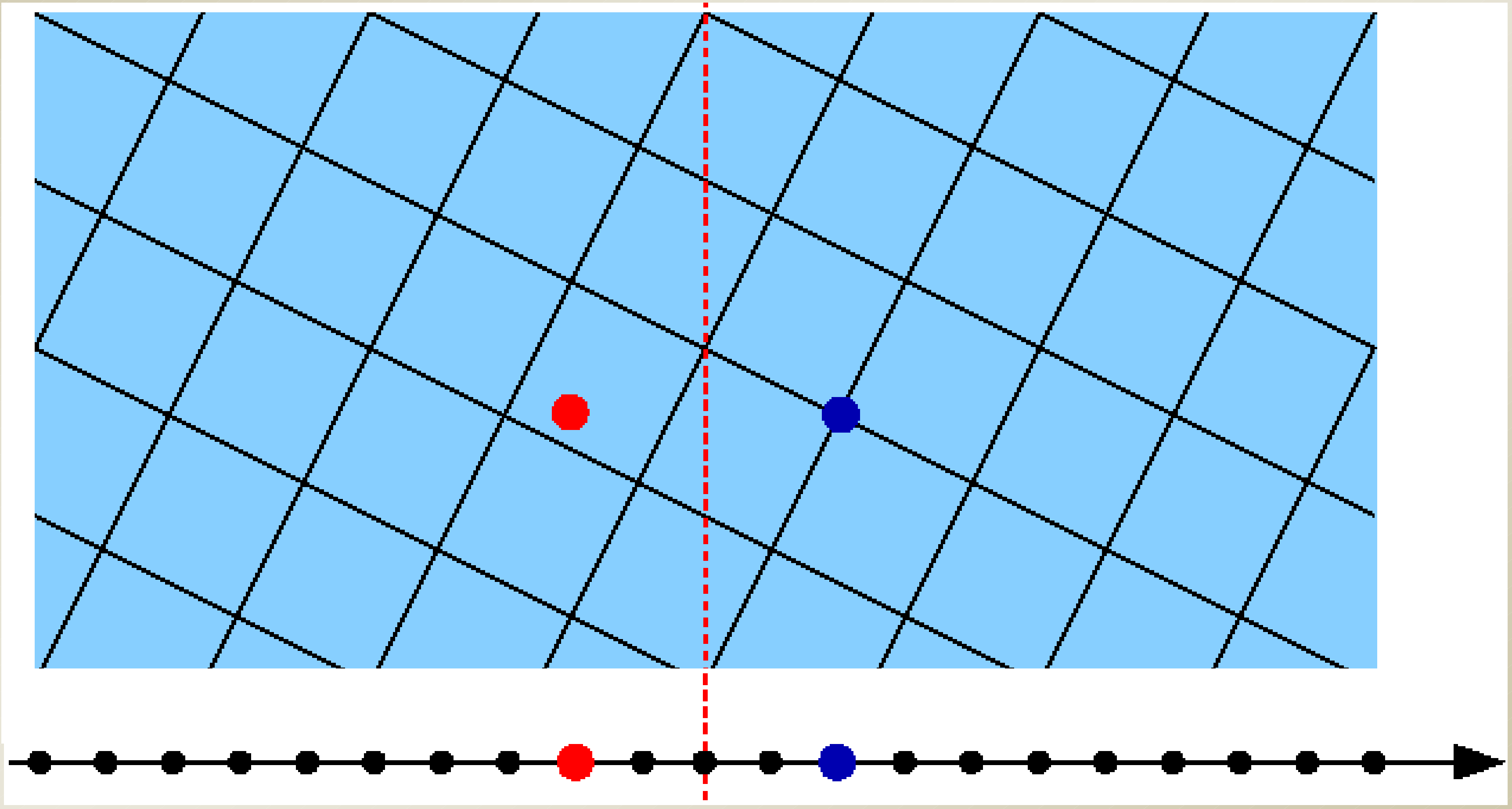
The Generaized Method of Image
: (1) Fourier transform along the interface



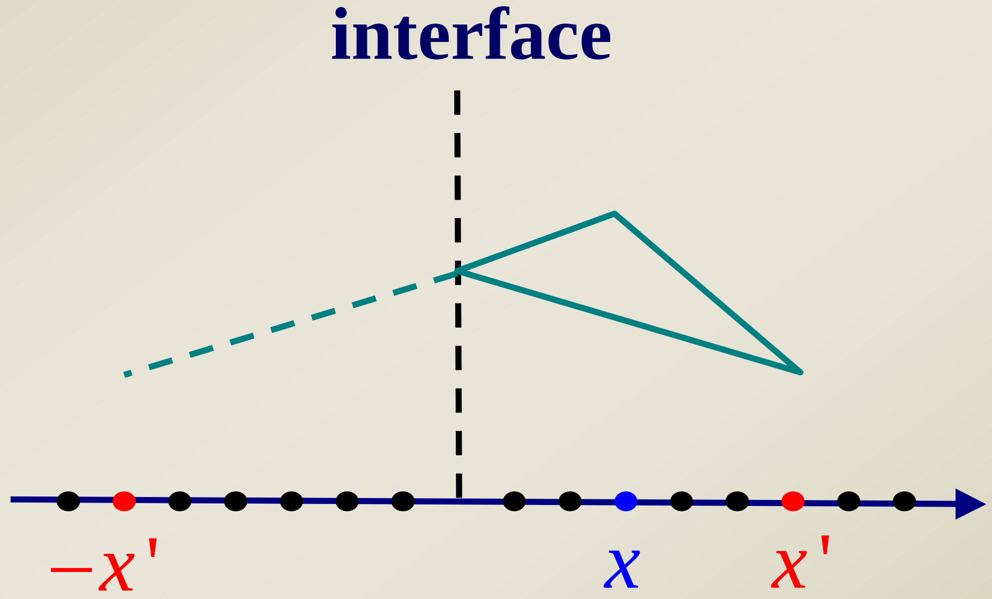
$\Rightarrow$become 1D problem

need to find $g(x_i, x_j, k_y, \omega)$

The difficulty on lattice is resolved!



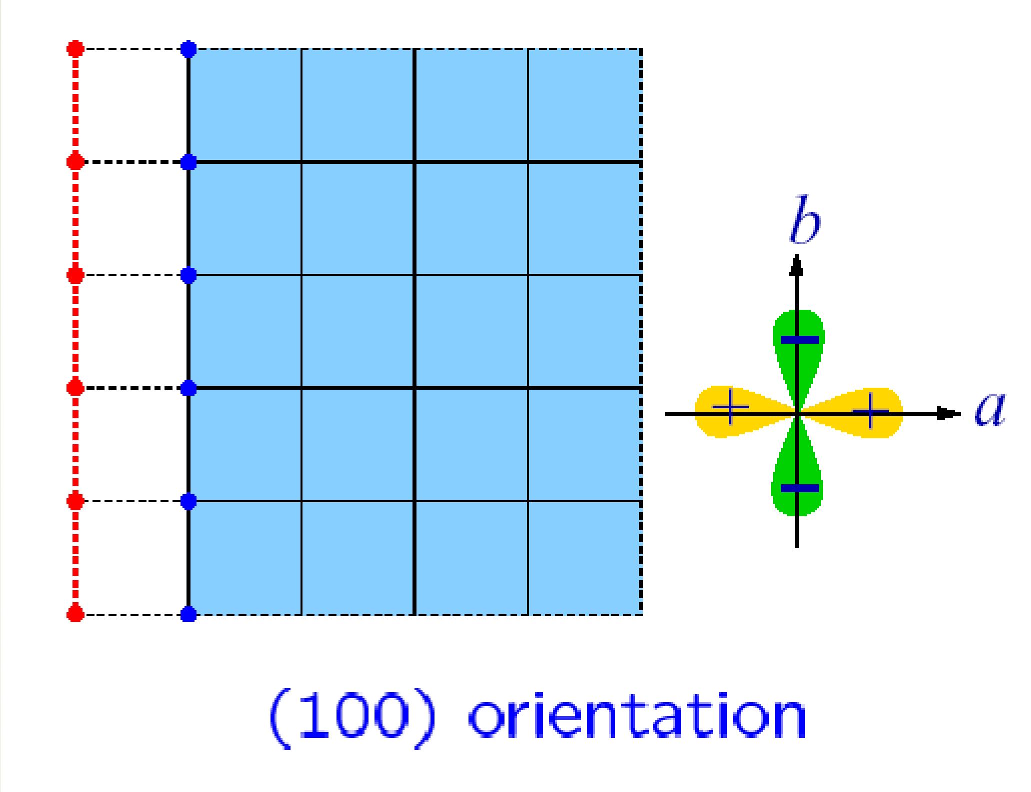
(2) The Generaized Method of Image



$$g(x,x') = G(x,x') - G(x,-x')\alpha(x')$$

with $\alpha(x') = G^{-1}(0,-x')G(0,x')$

Cases with reflection symmetry



$$G(x') = G(-x')$$

$$\alpha(x') = G^{-1}(x')G(-x') = 1$$

Recovering the naïve expectation

$$g(x,x') = G(x,x') - G(x,-x')$$

$$\therefore g(x=a,x'=a)$$

$$= \sum_{k_x} (1 - e^{i2k_x a}) G_{bulk}(k_x)$$

$$= \sum_{k_x} 2\sin^2(k_x a) G_{bulk}(k_x)$$

Cases without reflection symmetry

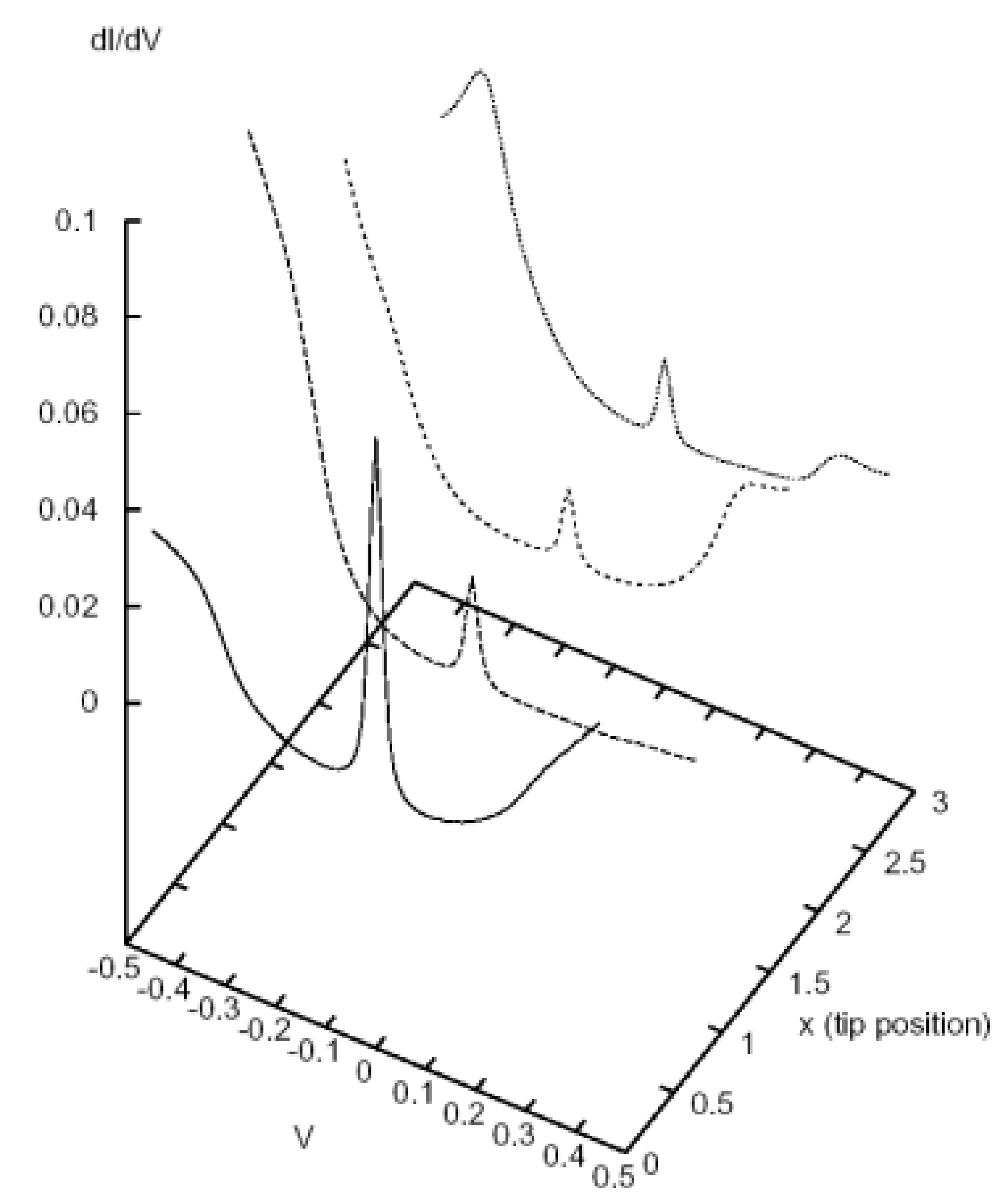
$$G(x') \neq G(-x')$$

$$\alpha(x') = G^{-1}(x')G(-x') \neq 1$$

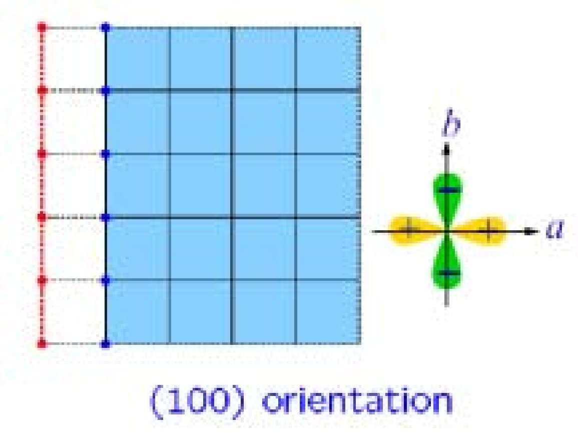
Must keep $\alpha(x')$!

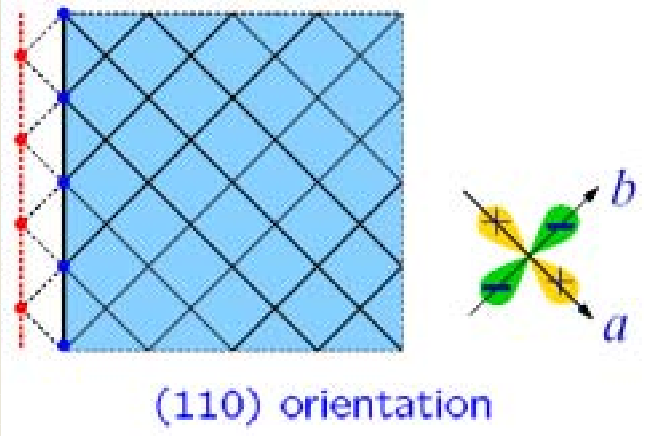
$$g(x,x') = G(x-x') - G(x+x')\alpha(x')$$

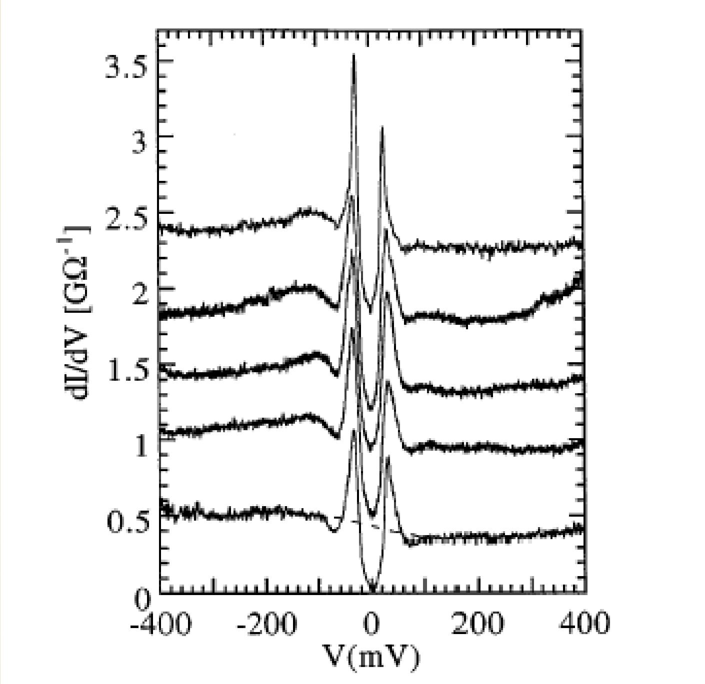
But α may contains poles: $\alpha(x') = G^{-1}(x')G(-x')$

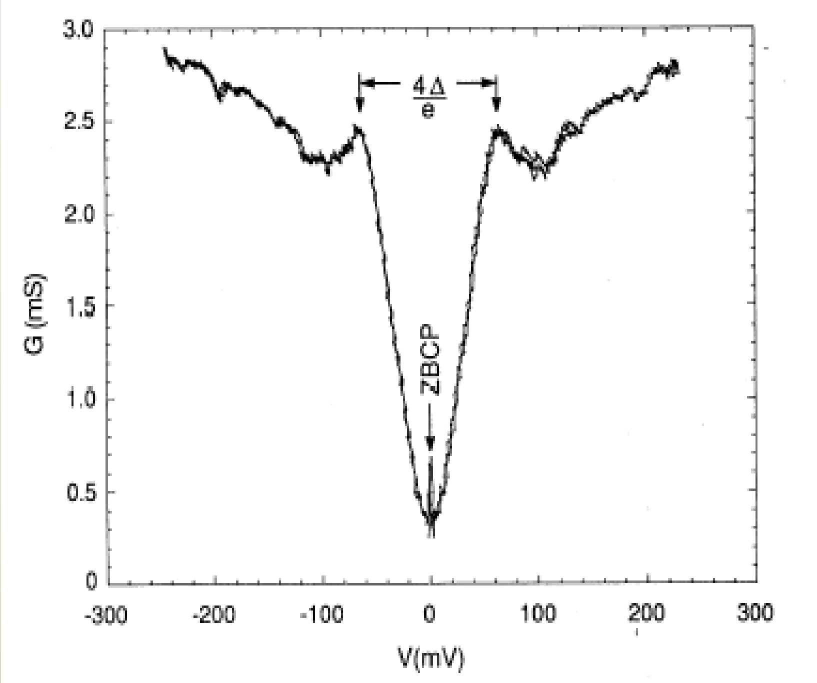


(100) versus (110)



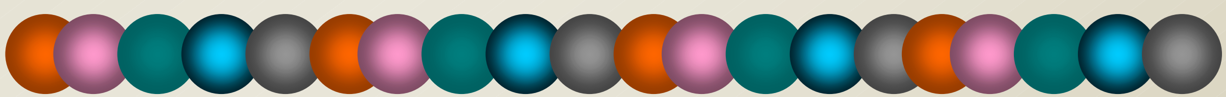


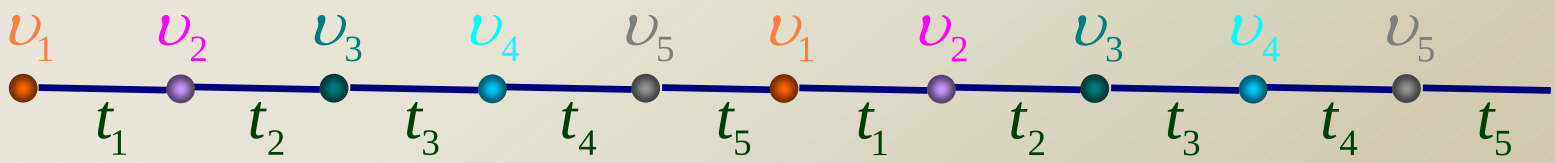




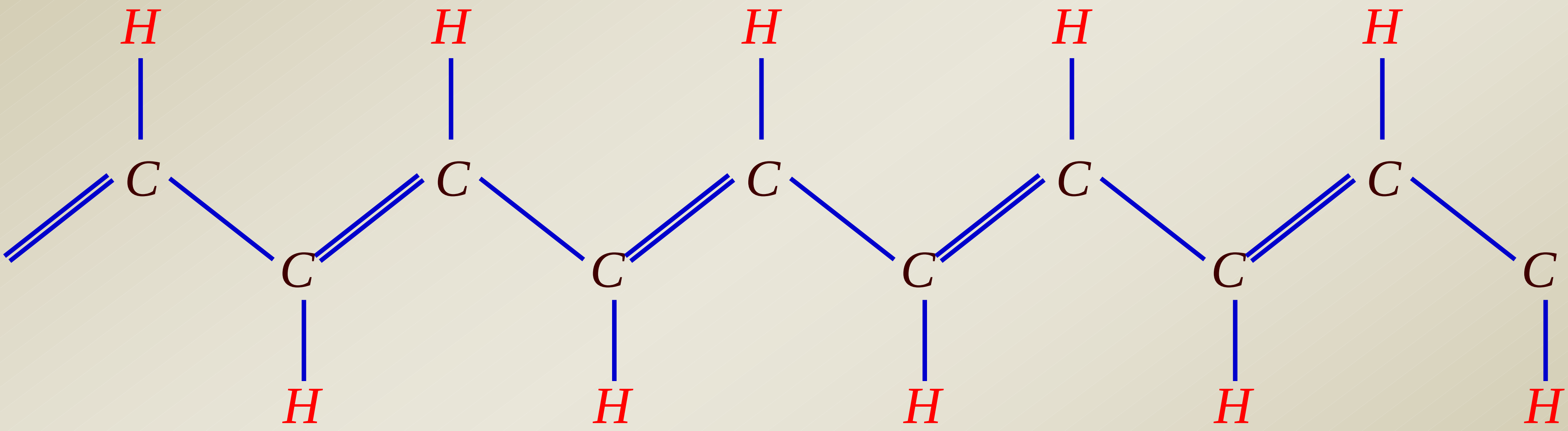
2. Supersymmetric Nanowires

Generalized 1D Chain with broken reflection symmetry:



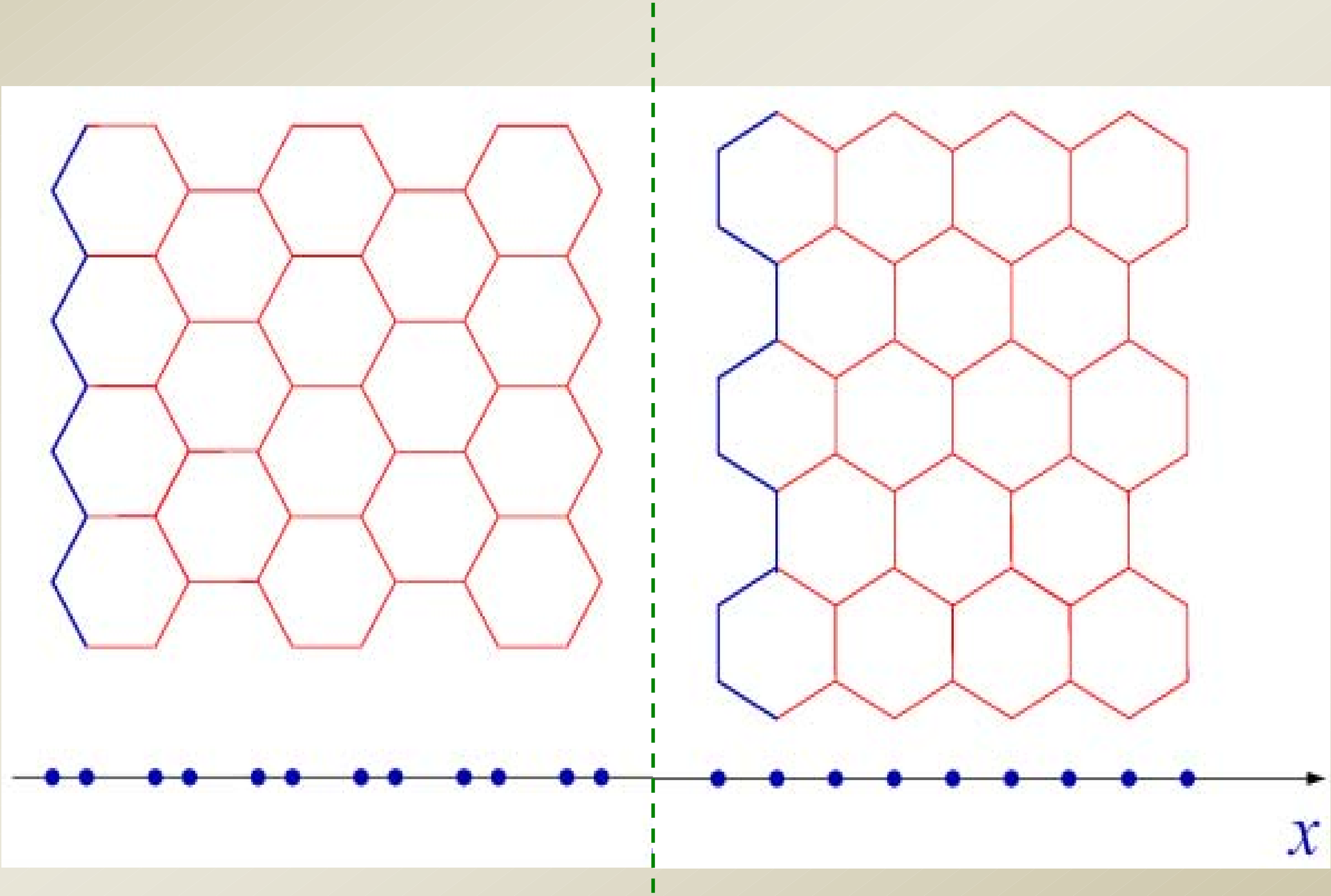


Examples: The Polyacetylene

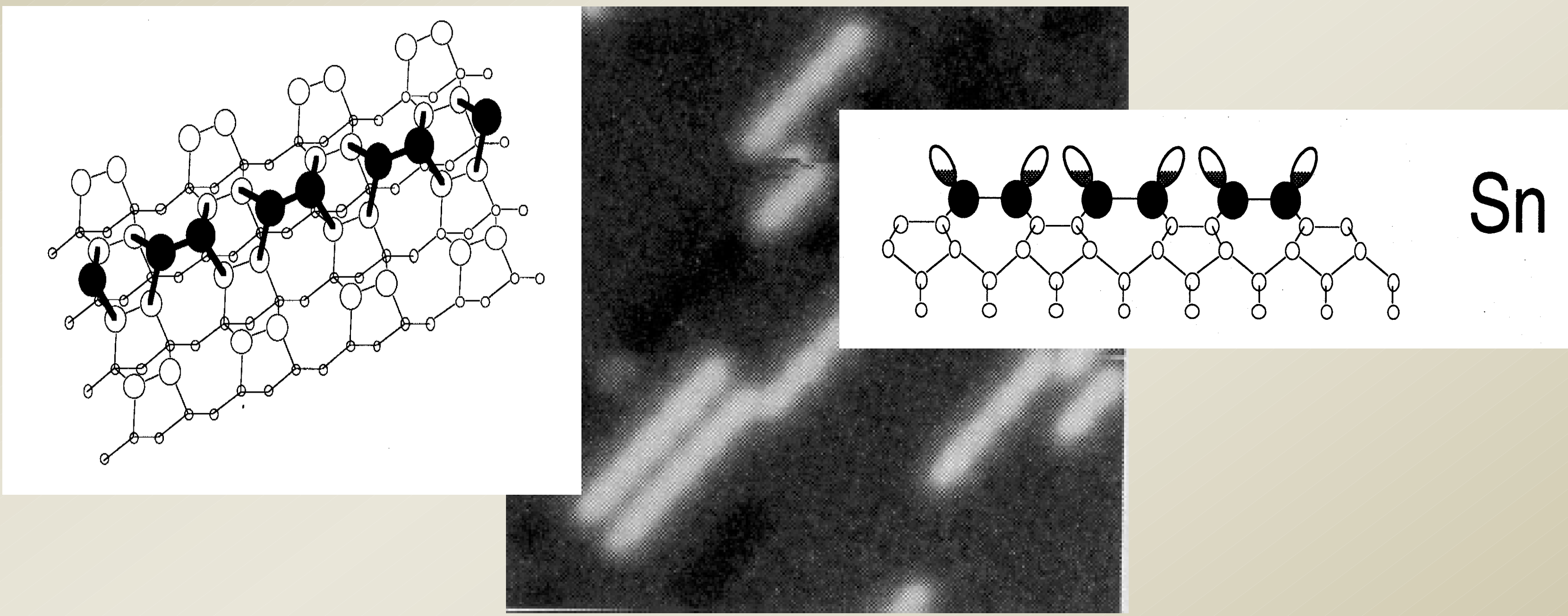


Su, Schrieffer, Heeger, PRL42, 1698 (1979)

The Graphite Sheets



Metal Wire on Si (001) surface

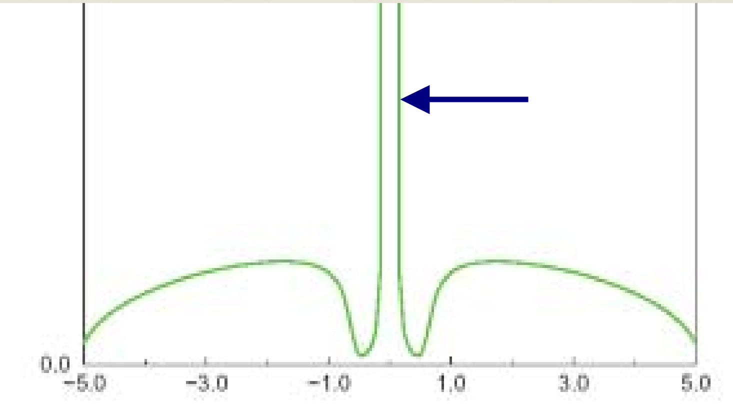


Ref. J. Nogami, Atomic and Molecular Wires edited by Joachim and Roth (1997)

What are these generalized mid-gap states?

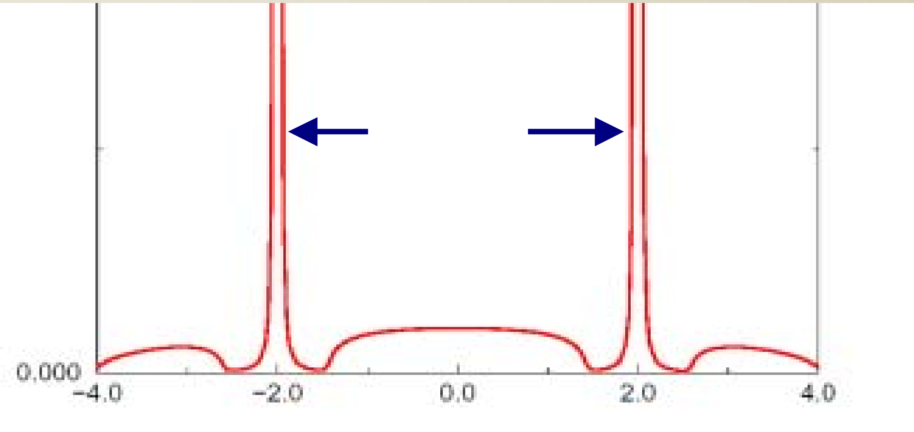
- $t_1 t_2\ t_1 t_2\ t_1 t_2\ t_1 t_2 t_1 t_2 \ldots$

Period = 2
polyacetylene,
graphite



- $t_1 t_2\ t_3\ t_1 t_2\ t_3\ t_1 t_2\ t_3 \ldots$

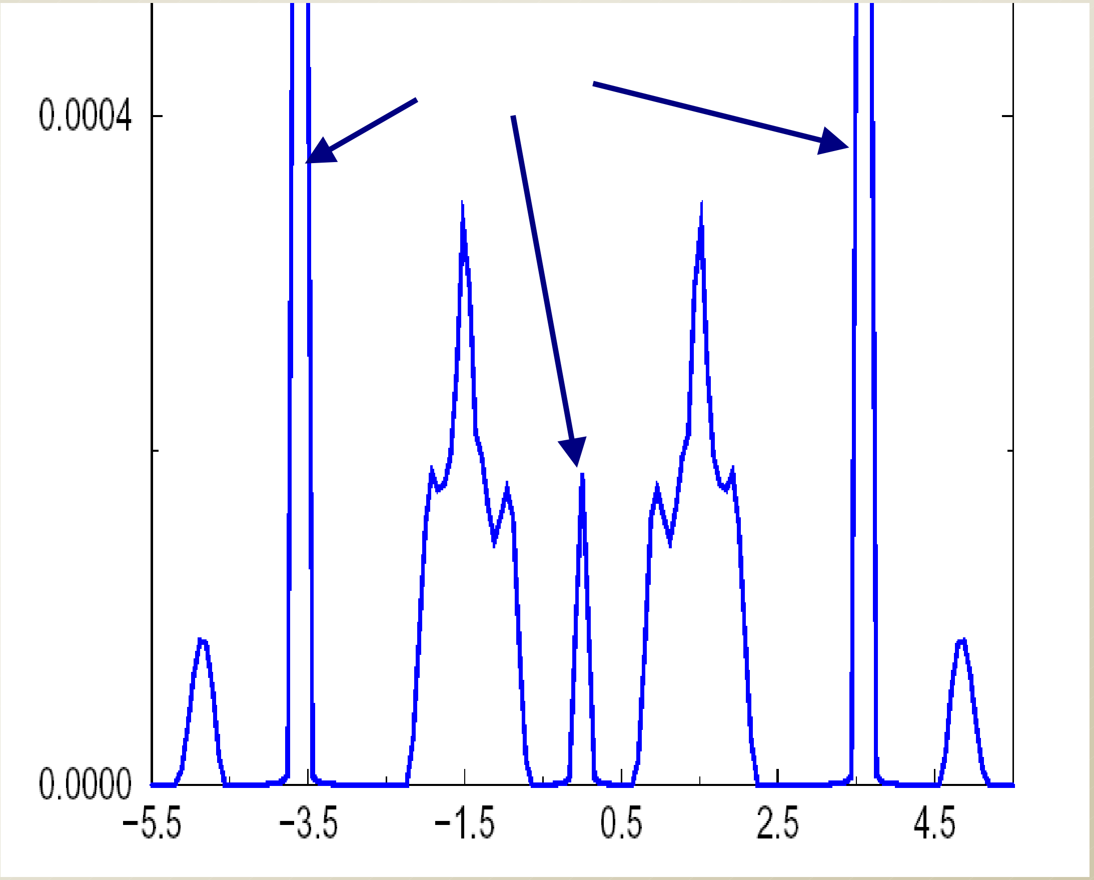
Period = 3



- $t_1 t_2\ t_3\ t_4\ t_1 t_2\ t_3\ t_4\ t_1 t_2\ t_3\ t_4 \ldots$

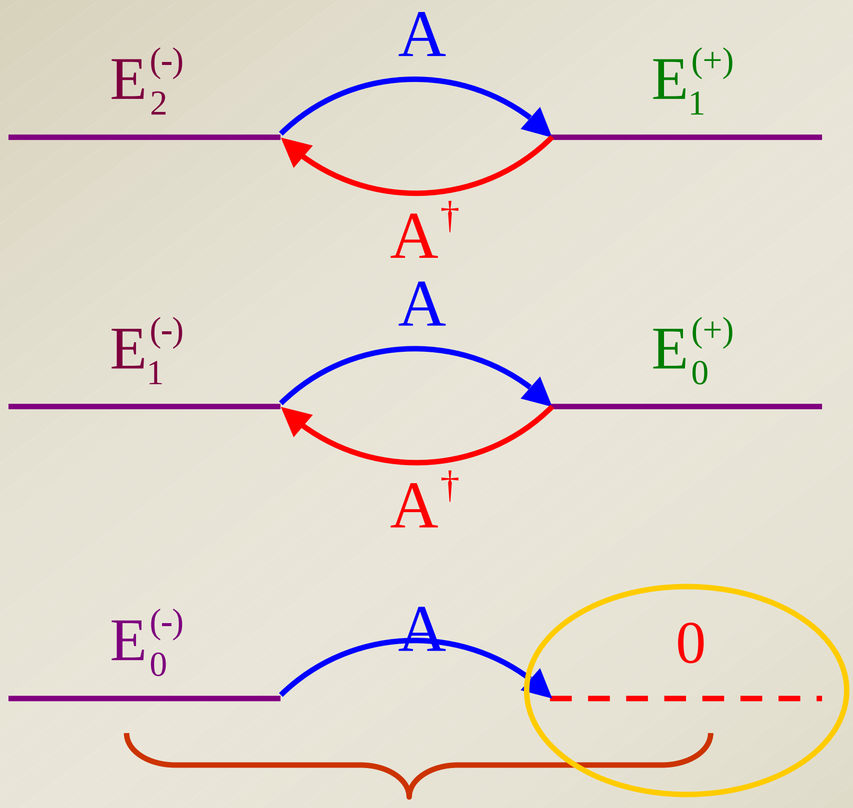
Period = 4

DNA,..



Witten's SUSY Quantum Mechanics

$H_- = A^\dagger \hat{A}$ $\qquad$ $H_+ = \hat{A} A^\dagger$



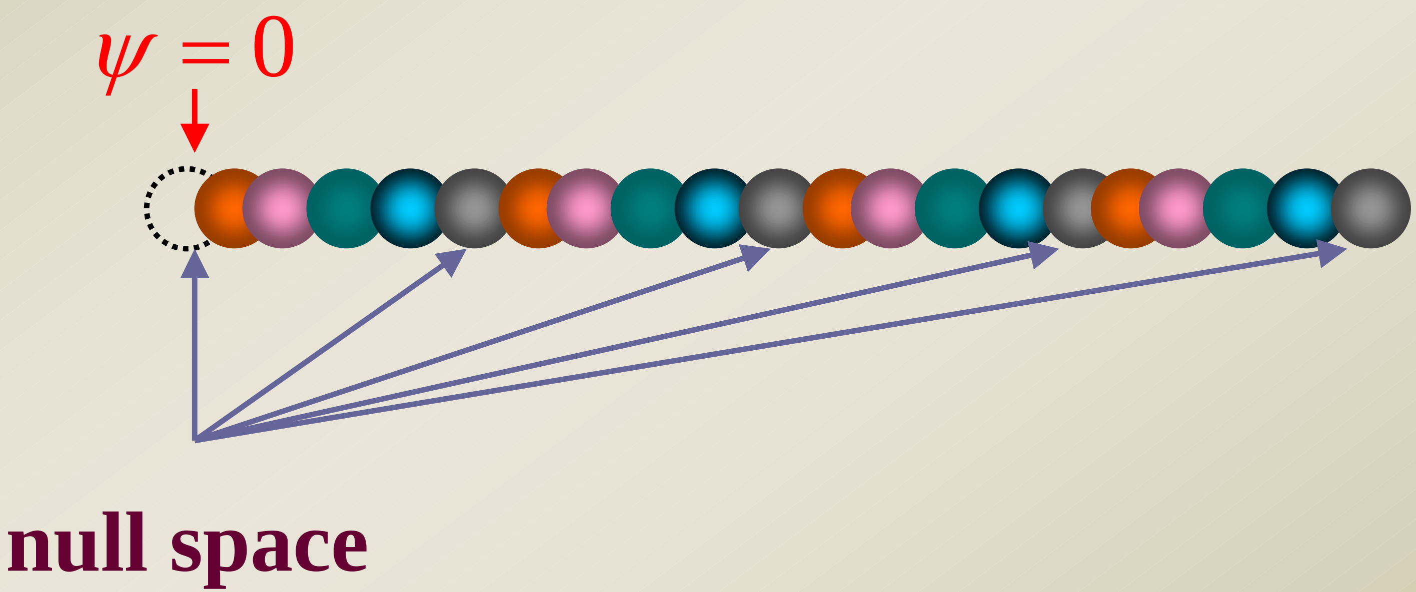
SUSY partners

$$Q = \begin{pmatrix} 0 & \hat{A} \\ 0 & 0 \end{pmatrix} = \hat{A} \oplus \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

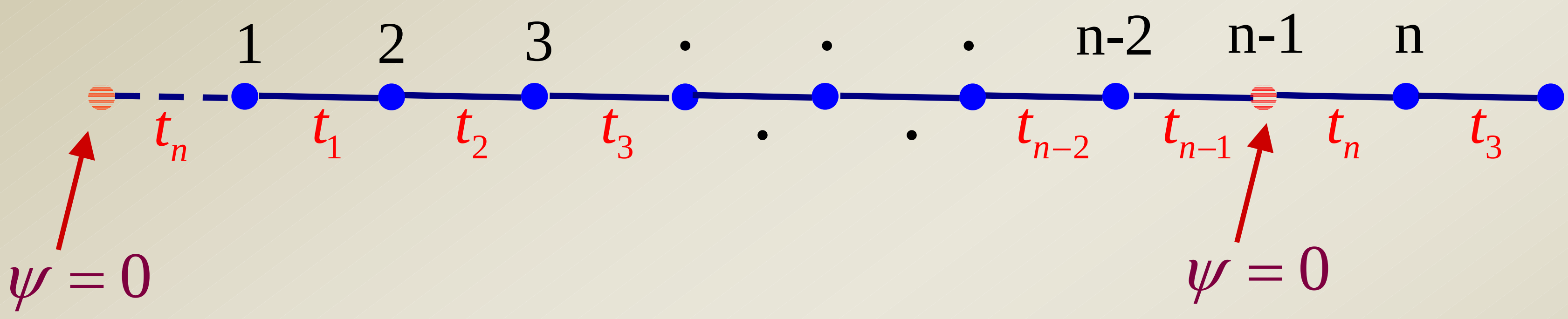
boson

σ_+ (fermion)

SUSY partners are all in one nanowire



Positions of mid-gap states

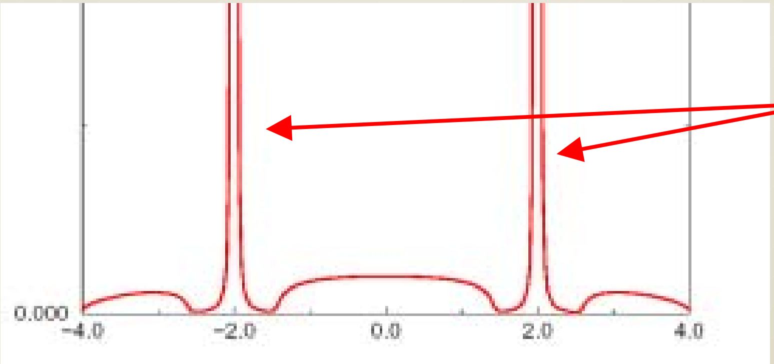


$$\text{positions} = \text{eigenvalues of} \begin{pmatrix} 0 & t_1 & 0 & 0 & 0 & . & . \\ t_1 & 0 & t_2 & 0 & 0 & . & . \\ 0 & t_2 & 0 & t_3 & 0 & . & . \\ 0 & 0 & t_3 & 0 & t_4 & . & . \\ 0 & 0 & 0 & t_4 & 0 & . & . \\ . & . & . & . & . & . & t_{n-2} \\ . & . & . & . & . & t_{n-2} & 0 \end{pmatrix}$$

Examples

The t1-t2-t3 model

$$\text{eigenvalues of} \begin{pmatrix} 0 & t_1 \\ t_1 & 0 \end{pmatrix} = \pm t_1$$



(x,x,0,x,x,0,x,x0,x,x0,…)

$-t_1$ t_1

The t1-t2-t3-t4 model

$$\text{eigenvalues of} \begin{pmatrix} 0 & t_1 & 0 \\ t_1 & 0 & t_2 \\ 0 & t_2 & 0 \end{pmatrix} = 0, \pm\sqrt{t_1^2 + t_2^2}$$