High-brightness Electron Radiation

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Outline

1. Relativistic Electron Radiation
   - Laser Synchrotron
   - Undulator Radiation
   - Smith-Purcell Radiation

2. Superradiance: Radiation from Bunched Electron Beam

3. High-brightness Radiation from x-ray to THz

4. Summary
Compton Effect: \[ \lambda' - \lambda = \frac{h}{m_0c} (1 - \cos \theta) \]

- $\lambda$: wavelength
- $h$: Planck's constant
- $m_0$: electron rest mass
- $c$: vacuum wave speed
- $p$: momentum

Compton Scattering

Electron

Scattered photon

Scattered electron
Thomson Scattering $\Rightarrow$ laser synchrotron

Backward scattered photon

$\lambda: \text{laser wavelength}$

Colliding Laser photon

$\lambda_r: \text{laser synchrotron wavelength}$

$\nu_z$

Colliding electron

$\frac{f}{f_0} = \sqrt{\frac{1+\beta_z}{1-\beta_z}}$

Double Doppler shifted wavelength $\lambda_r = \frac{\lambda}{4\gamma_z^2}$

Given $\lambda = 800 \text{ nm (Ti:sapphire laser)}, \gamma \sim \gamma_z = 45 \ (23 \text{ MeV beam}), \lambda = 1 \text{ Å (hard x-ray!)}$

Lorentz factor $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$

Longitudinal Lorentz factor $\gamma_z \equiv \frac{1}{\sqrt{1-\beta_z^2}}$

where $\beta \equiv \frac{\nu}{c}$

where $\beta_z \equiv \frac{\nu_z}{c}$
Undulator/Wiggler Radiation

In laboratory frame

In electron rest frame

\[ \vec{E}' = \gamma \beta \times \vec{B}, \quad \vec{B}' = \gamma \vec{B} \]

(Sec. 11.10 in J. D. Jackson’s text)
In the Electron Rest Frame

All harmonics

Dipole radiation pattern

Odd harmonics

Figure-8 motion

Dipole radiation pattern

In the laboratory frame

(γ >> 1)
Virtual photon

Electron Rest Frame

\[ \lambda_w / \gamma_z \]

Virtual photon

Electron oscillation frequency

\[ f' = \frac{1}{T'} = \left( \frac{\lambda_w / \gamma_z}{v_z} \right)^{-1} \]

Laboratory Frame

Doppler Shift

\[ f = f' \sqrt{\frac{1 + \beta_z}{1 - \beta_z}} \]

\[ \lambda = \lambda_w \left( \frac{1}{\beta_z} - 1 \right) \approx \frac{\lambda_w}{2 \gamma_z^2} \]

For \( \lambda_w \sim 1 \) cm, \( \gamma_z \sim 200 \) (100 MeV), \( \Rightarrow \lambda = 125 \) nm (deep UV)

• Relativistic undulator radiation is an effective means for producing expensive short-wavelength photons from economic long-wavelength virtual photons
Wavelength Tuning

In a magnetic field, $\gamma$ is a constant and $\gamma_z$ is a function of $B$ field

$$\gamma_z = \frac{1}{\sqrt{1 - \beta_z^2}} = \frac{1}{\sqrt{1 - v_z^2 / c^2}}, \text{ and } \lambda \approx \frac{\lambda_w}{2\gamma_z^2}$$

$$\frac{1}{\gamma_z^2} = \frac{1 + a_w^2}{\gamma^2} \text{ where } a_w = 0.093 B_{rms} \text{ (kgauss)} \times \lambda_w \text{ (cm)}$$

is called the wiggler parameter

$$\lambda = \frac{1 + a_w^2}{2\gamma^2} \lambda_w : \text{FEL synchronism condition}$$

Radiation wavelength can be tuned by magnetic field $B$, wiggler period $\lambda_w$, and electron energy $\gamma$
Free-electron Laser (Stimulated Compton Scattering)
Free-electron Laser

To have gain $\Delta W = e \int_{t=L_w/v_z}^{\infty} \vec{E} \cdot \vec{v} \, dt < 0$

$\vec{v} \cdot \vec{E} > 0$

By

$\tau = \frac{\lambda_w / 2 + \lambda_z / 2}{c} = \frac{\lambda_w / 2}{v_z}$

$\lambda = \lambda_w \left( \frac{1}{\beta_z} - 1 \right) \approx \frac{\lambda_w}{2\gamma_z^2}$
Electron-Wave Energy Exchange

Wave Amplification
\[ \Delta W = e \int_{\tau=L/v_{\parallel}}^{\tau} \vec{E} \cdot \vec{v} \, dt < 0 \]

Particle Acceleration
\[ \Delta W = e \int_{\tau=L/v_{\parallel}}^{\tau} \vec{E} \cdot \vec{v} \, dt > 0 \]

Transverse Coupling
(Eg. Compton/Thompson/undulator radiation etc.)
\[ \Delta W = e \int_{\tau=L/v_{\parallel}}^{\tau} \vec{E}_{\perp} \cdot \vec{v}_{\perp} \, dt \]

Longitudinal Coupling
(Eg. Smith-Purcell radiator, backward-wave oscillator etc.)
\[ \Delta W = e \int_{\tau=L/v_{\parallel}}^{\tau} \vec{E}_{\parallel} \cdot \vec{v}_{\parallel} \, dt \]
\[ \Delta W = e \int_{\tau = L/v_z} \vec{E}_z \cdot \vec{v}_z \, dt < 0 \]

**Smith-Purcell Radiator**

\[ \lambda = \Lambda g \left( \frac{1}{\beta} - \sin \theta \right) \]

**Backward-wave Oscillator**
Incoherence Radiation $\sigma_z \gg \lambda_r$

$$
\left( \frac{dW}{d\omega} \right)_{inc,N} = \left( \frac{dW}{d\omega} \right)_1 \sum_{j=1}^{N} e^{i\omega_j t} \right)^2
$$

Spectral Energy

$$
\left( \frac{dW}{d\omega} \right)_{inc,N} = N \left( \frac{dW}{d\omega} \right)_1
$$

$N$: number of electrons

(Undulator with $N_u$ periods)

$$
\left( \frac{dW}{d\omega} \right)_1
$$

$\omega_r / N_u$
Superradiance: coherent radiation

\[ \sigma_z \ll \lambda_r \]

\[ \sigma_z, \tau_b \quad \text{and} \quad \lambda_r \]

Spectral Energy

\[ \left( \frac{dW}{d\omega} \right)_{SR, N_b} = N_b^2 \left( \frac{dW}{d\omega} \right)_{1} M_b^2(\omega) \]

\( M_b(\omega) \): Fourier transform of bunch shape

\( N_b \): number of bunched electrons

* For 1 nC in 10 ps and \( \lambda_r = 1 \mu m \), \( N_b = 2 \times 10^6 \)!

\[ M_b(\omega) = \exp \left( -\frac{\omega^2 \tau_b^2}{4} \right) \] for Gaussian bunch shape function

\[ f(t) = \frac{\exp \left( -t^2 / \tau_b^2 \right)}{\sqrt{\pi \tau_b}} \]
Superradiance from a Periodically Bunched Beam

\[ \left( \frac{dW}{d\omega} \right)_{SR,N_p \times N_b} = \left( N_p N_b \right)^2 \left( \frac{dW}{d\omega} \right) \frac{M_b^2(\omega) M_p^2(\omega)}{M_p^2(\omega)} \]

\[ M_p(\omega) = \frac{\sin(N_p \pi \omega / \omega_b)}{N_p \sin(\pi \omega / \omega_b)} \]

Coherent sum of \( N_p \) bunches with bunching freq. \( \omega_b \)

* For 10-ps macro-bunch length and \( \lambda_r = 1 \) \( \mu \)m, \( N_p = 3 \times 10^3 \)

Spectral narrowing from periodic bunched beam

\[
\frac{dW}{d\omega}_{SR,Np\times Nb} = (N_p N_b)^2 \left( \frac{dW}{d\omega} \right)_1 M_b^2(\omega) M_p^2(\omega)
\]

where \( M_p(\omega) = \sin(N_p \pi \omega / \omega_b) / \sin(\pi \omega / \omega_b) / N_p \)

Assume undulator radiation \( \Rightarrow \left( \frac{dW}{d\omega} \right)_1 \propto \text{sinc}^2[2N_u (\omega / \omega_r - 1)] \)

for undulator with period \( N_u \)

Assume \( \omega_b = 1 / \tau_b \Rightarrow M_b(\omega) = \exp\left(-\omega^2/(4\omega_b^2)\right) \) \( \text{(3)} \quad \omega_r = \omega_b \)
Example: High Gain Harmonic Generation (HGHG, L. H. Yu et. @ BNL)

![Diagram of laser system and Superdiabiace/SASE FEL](image)

- Seed laser
- Modulator undulator
- Dispersion section
- Superdiance/SASE FEL
- Laser system
- RF Photocathode gun
- Linac

Mathematical expression:

\[
\omega / \omega_b
\]

Graph showing:
- \(M_b^2(\omega)\) green
- \((dW / d\omega)\) red
- \(M_p^2(\omega)\) blue

Graph axes:
- \(\omega / \omega_b\)
- \(\omega_b\)
Idea: Laser-beat-wave Bunched-beam Accelerator
(B³ technique)

Laser beat-wave with a variable beat frequency

Dispersion section
(further bunch compression)

For an alpha magnet, compression ratio
50 ~ 100

Bunched beam to radiation insertion device

Photocathode gun

An Open Question: How fast could a copper cathode respond?
• Electron relaxation time in a conductor: ~ 10⁻¹⁸ sec
• Plasma frequency in copper ~ 2.6×10¹⁵ Hz

Accelerator
(further particle acceleration)
**Laser Beat Wave:** beating of two waves

Superposition of two fields with \( \Delta \omega \)
\[
E = E_0 \cos \omega t + E_0 \cos(\omega - \Delta \omega) t
\]

Instantaneous intensity
\[
I = I_0 \cos^2[(\omega - \frac{\Delta \omega}{2})t] \times \cos^2(\frac{\Delta \omega}{2} t)
\]
beating at frequency \( \Delta \omega \)

*Drawn with \( \Delta \omega = \omega / 2 \)
Laser Beat Wave: multi-wave beating

\[ I = I_0 \left| \exp(j \omega_0 t) \{1 + \exp(-j \Delta \omega t) + \ldots + \exp[-j(N-1)\Delta \omega t]\} \right|^2 = I_0 \frac{\sin^2 \left( \frac{N_p \Delta \omega t}{2} \right)}{\sin^2 \left( \frac{\Delta \omega t}{2} \right)} \]

Assume a Nd:YLF laser

\( \lambda = 1053 \text{ nm} \) (Fundamental)

Second harmonic generation (SHG)

\( \lambda = 526.5 \text{ nm} \) (2\( \omega \))

\( \lambda = 263.25 \text{ nm} \) (4\( \omega \))

\( \lambda = 351 \text{ nm} \) (3\( \omega \))

\( \omega_{\text{p}} = \frac{2 \pi}{N_p \Delta \omega} \)

\( \omega_{\text{p}} = \frac{2 \pi}{\Delta \omega} \)

Eg. Raman material with Stokes shift \( \Delta \omega \)

Coherent Stokes and anti-Stokes

Pump laser

\( 1 \text{ fs} \Leftrightarrow 300 \text{ nm} \)
Frequency Tunable Beat-wave Laser System

- Narrow line fixed-wavelength laser
  - DFB laser
  - ECDL
- Narrow-line wavelength-tunable laser (ECDL)
- CW laser amplifier (Erbium-doped fiber amplifier, ~20 dB gain)
  - EDFA
- Broadband Optical parametric amplifier
  - QPM OPA
- Broadband second harmonic generator
  - QPM SHG
- Harmonic generators
- Laser Beat waves
- Telecom diode lasers
- Quasi-phase-matched nonlinear optics
- Broadband second harmonic generator
- Ti:sapphire amplifier
- Laser amplifier
Seed Laser System

- **Distributed-feedback diode laser with kHz linewidth**

- **External-cavity tunable diode laser 1.4-1.6 μm with MHz linewidth**

- **Erbium doped fiber amplifier 1.48 ~1.62 μm (20 THz bandwidth)**

Seed laser spectrum
Quasi-phase-matched (QPM) Nonlinear Optical Crystals

Phase-matching wavelengths can be engineered by using lithographic technology.
Broadband Optical Parametric Amplification: Amplified Beat-wave Spectrum

Input: 60 μJ/pulse at 1064-nm
Output: 11 μJ/pulse at 1.55-6 μm
Question: Would particle acceleration smear out particle distribution?

Simulation (ASTRA) 1: 1-1/2 Cell S-band SSRL RF gun (input)

Input
- 25 periodic bunches over 11 ps or 0.44 ps/bunch
- Gaussian bunch with rms bunch length = 72 fsec
- rms radius = 0.75 mm at cathode with radial distribution
- Total charge 0.1 nC or 4 pC/bunch
- Space charge force was considered.
Simulation (ASTRA) 1: 1-1/2 Cell S-band SSRL RF gun (output)

Output

1. 11 A over 9 ps
2. Average energy: 3.994 MeV
3. Rms energy spread = 0.37%
4. Normalized emittance = 1.19 \( \pi \)-mm-mrad
5. Micro-bunch length \( \sim \) 108 \( \mu \)m

Macrobunch length vs. z

# of macro-particles vs. z (mm)
Simulation (ASTRA) 2: UCLA/DULY Planewave Transformer Accelerator

Input
Same as in simulation 1

Output
1. 9 A over 10.8 ps
2. Average energy: 19.15 MeV
3. Rms energy spread = 0.7%
4. normalized emittance = 5.24 $\pi$-mm-mrad
5. Micro-bunch length $\sim$ 130 $\mu$m

10+2x1/2 cell S-band RF gun with Peak accelerator gradient = 60 MV/m
**Simulation (ASTRA) 3: Generation for 6\(\mu\)m bunch length**

**Input**
- 25 periodic bunches over 250 fs or 10 fs/bunch
- Gaussian bunch with rms bunch length = 1 fsec
- rms radius = 0.75 mm at cathode with radial distribution
- Total charge 25 pC or 1 pC/bunch

**Output**
- Energy = 4 MeV
- 50 A over 500 fs
- Micro-bunch length ~ 6 \(\mu\)m
**Superradiance Smith-Purcell FEL: beam line**

Goal: ~100 mW CW power at THz frequencies

Power enhancement factor $2 \times 10^4$ (!) using $B^3$ technique

Superradiance Smith-Purcell FEL: components


Gold-coated silicon grating

Electron gun: 30-50 keV with 2-3 mA

Grating grooves (Microscope photo)
10-100 kW, THz Superradiance Free-electron Laser

Periodically bunched electrons

2-4 MeV RF
Photocathode Gun

Beatwave laser

undulator

Electron superradiance

THz medical imaging (NTHU, 馬偕醫院)
Hard x-ray Laser Synchrotron

TW laser \quad x-ray radiation \quad 25 \text{ MeV beam}

350 ps \quad 350 ps

X-ray wavelength 1 Å
$10^{10}$ photons/collision
$10^{17}$ photons/sec

3-4 MeV RF gun

25 MeV Linac

~10 MHz
X-ray, UV, Visible, IR, THz Sources

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Bio-molecular and Bio-medical Imaging Core Facility
(Courtesy of 奈米中心 齊正中主任) 楊尚達、楊士禮、林凡異、江安世、高甫仁
Thank you for your attention

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