



# High-brightness Electron Radiation

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# Outline



## 1. Relativistic Electron Radiation

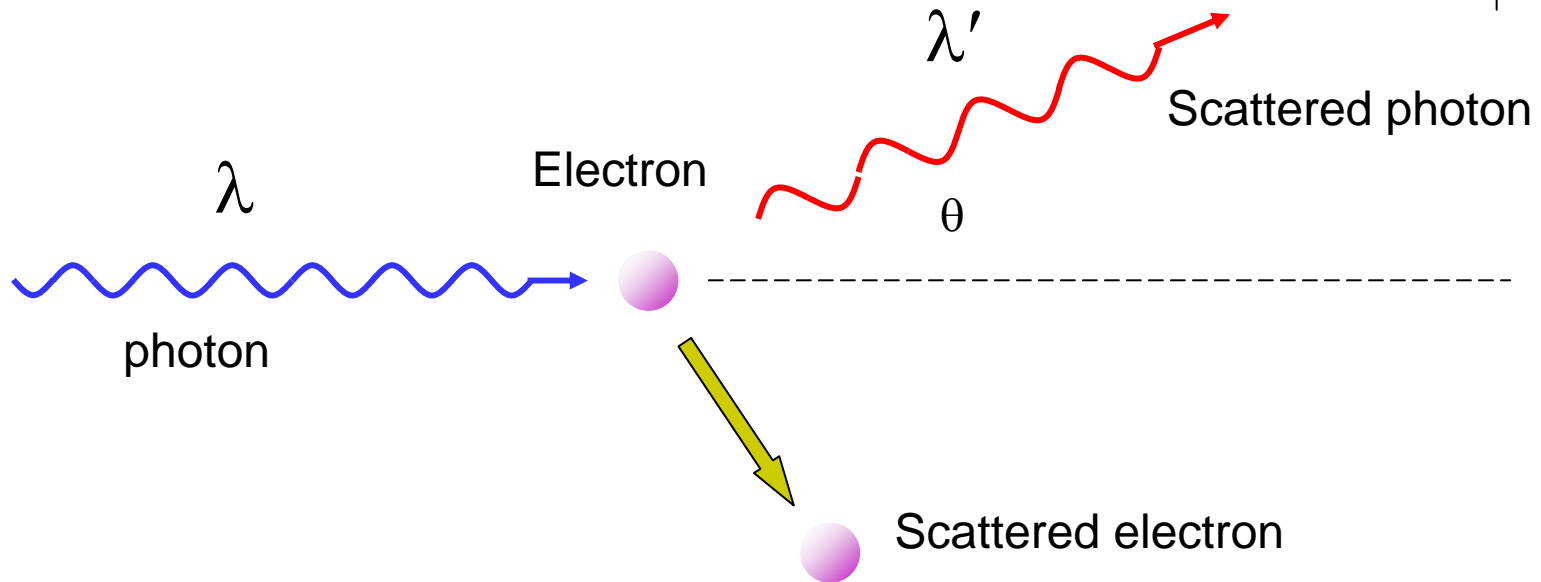
- Laser Synchrotron
- Undulator Radiation
- Smith-Purcell Radiation

## 2. Superradiance: Radiation from Bunched Electron Beam

## 3. High-brightness Radiation from x-ray to THz

## 4. Summary

# Compton Scattering



$\lambda$ : wavelength

$h$ : Planck's constant

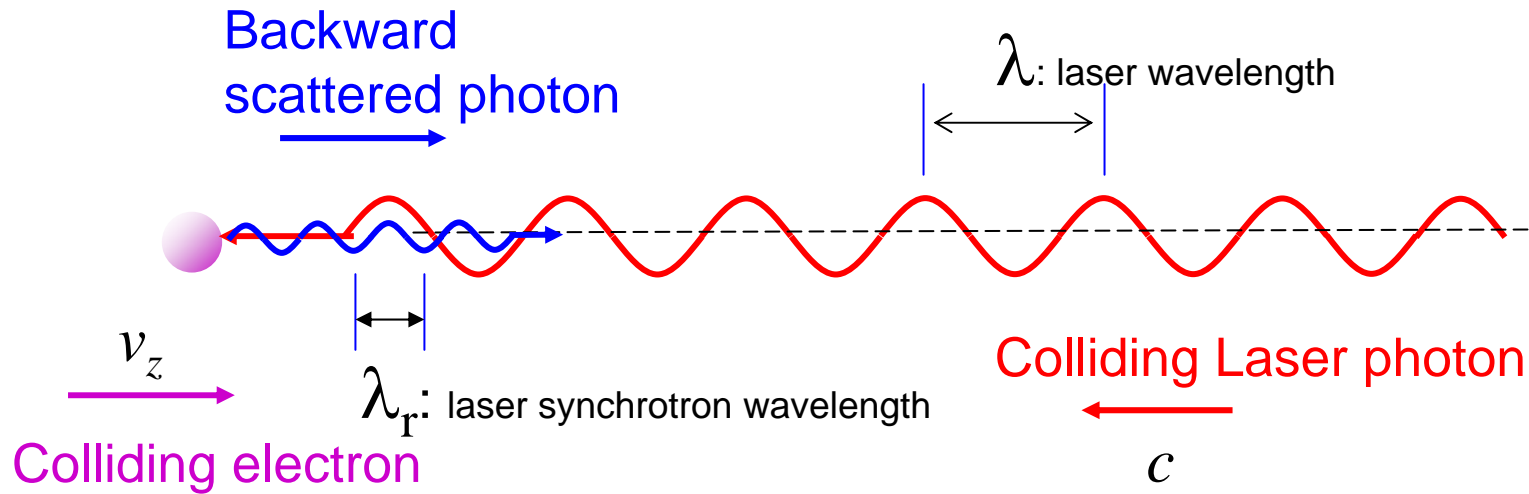
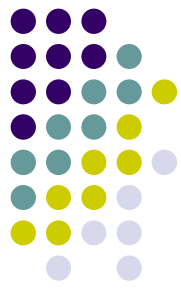
$m_0$ : electron rest mass

$c$ : vacuum wave speed

$p$ : momentum

**Compton Effect** 
$$\lambda' - \lambda = \frac{h}{m_0 c} (1 - \cos \theta)$$

# Thomson Scattering $\Rightarrow$ laser synchrotron



$$f = f_0 \sqrt{\frac{1 + \beta_z}{1 - \beta_z}}$$

Double Doppler shifted wavelength  $\lambda_r = \frac{\lambda}{4\gamma_z^2}$

Given  $\lambda = 800 \text{ nm}$  (Ti:sapphire laser),  $\gamma \sim \gamma_z = 45$  (23 MeV beam),  $\lambda_r = 1 \text{ \AA}$  (hard x-ray!)

Lorentz factor  $\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$

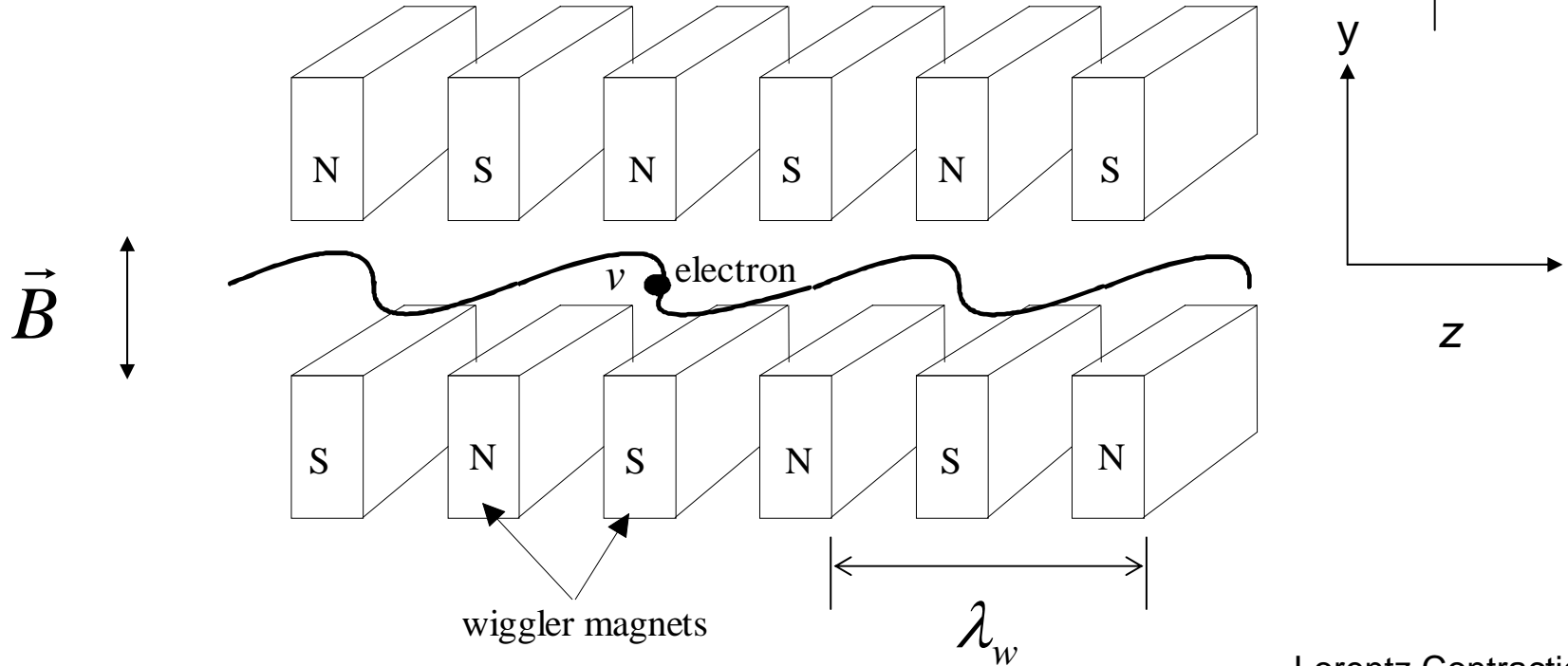
where  $\vec{\beta} \equiv \vec{v} / c$

Longitudinal Lorentz factor  $\gamma_z \equiv \frac{1}{\sqrt{1 - \beta_z^2}}$

where  $\beta_z \equiv v_z / c$

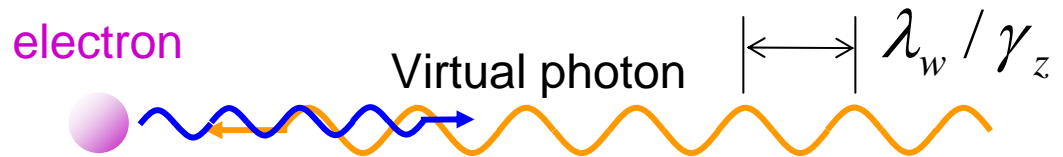
# Undulator/Wiggler Radiation

In laboratory frame

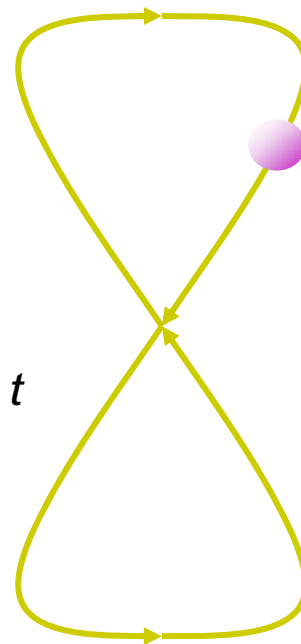
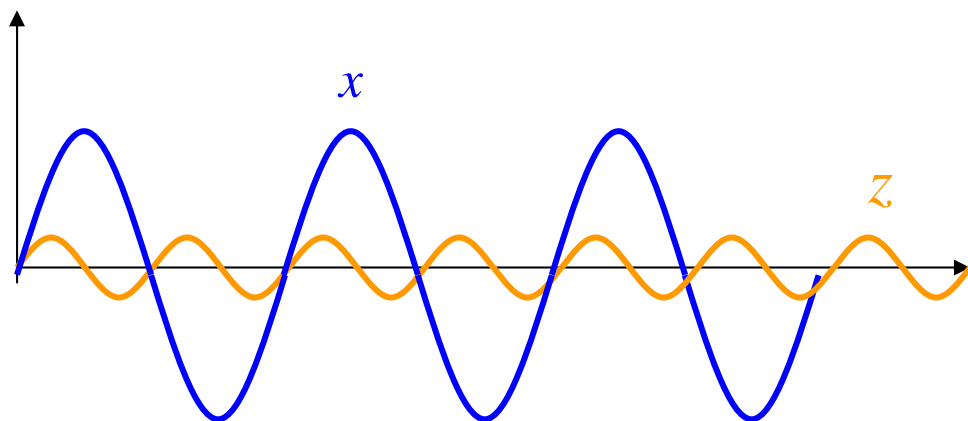


In electron rest frame

$$\vec{E}' = \gamma \vec{\beta} \times \vec{B}, \vec{B}' = \gamma \vec{B}$$



# In the Electron Rest Frame

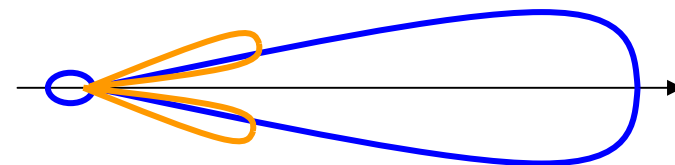
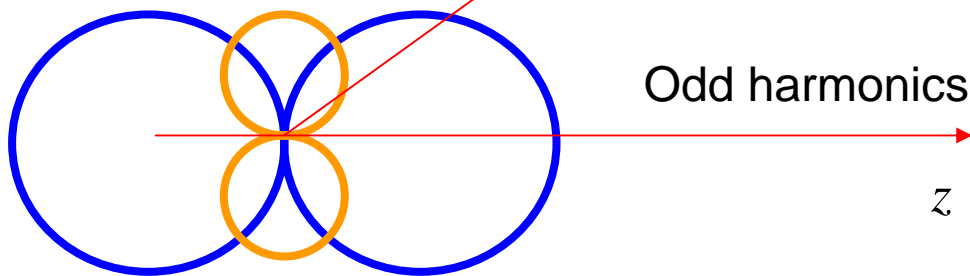


All harmonics

Figure-8 motion

Odd harmonics

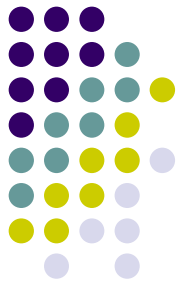
# In the laboratory frame ( $\gamma \gg 1$ )



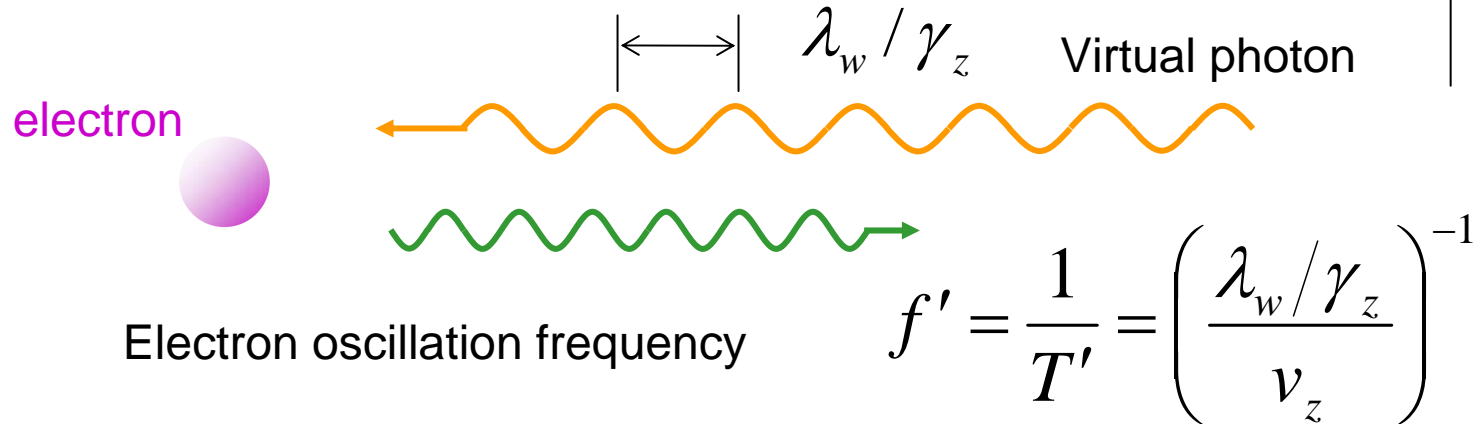
Dipole radiation pattern

Dipole radiation pattern<sup>6</sup>

# Spontaneous Compton Radiation



## Electron Rest Frame



## Laboratory Frame

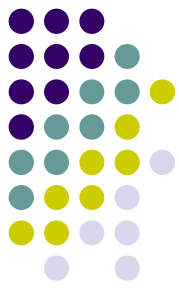
Doppler Shift

$$f = f' \sqrt{\frac{1 + \beta_z}{1 - \beta_z}} \quad \Rightarrow \quad \lambda = \lambda_w \left( \frac{1}{\beta_z} - 1 \right) \approx \frac{\lambda_w}{2\gamma_z^2}$$

For  $\lambda_w \sim 1 \text{ cm}$ ,  $\gamma_z \sim 200$  (100 MeV),  $\Rightarrow \lambda = 125 \text{ nm}$  (deep UV)

- Relativistic undulator radiation is an effective means for producing expensive short-wavelength photons from economic long-wavelength virtual photons

# Wavelength Tuning



In a magnetic field,  $\gamma$  is a constant and  $\gamma_z$  is a function of  $B$  field

$$\gamma_z \equiv \frac{1}{\sqrt{1-\beta_z^2}} = \frac{1}{\sqrt{1-v_z^2/c^2}}, \text{ and } \lambda \approx \frac{\lambda_w}{2\gamma_z^2}$$

→  $\frac{1}{\gamma_z^2} = \frac{1+a_w^2}{\gamma^2}$  where  $a_w = 0.093 B_{rms} \text{ (kgauss)} \times \lambda_w \text{ (cm)}$

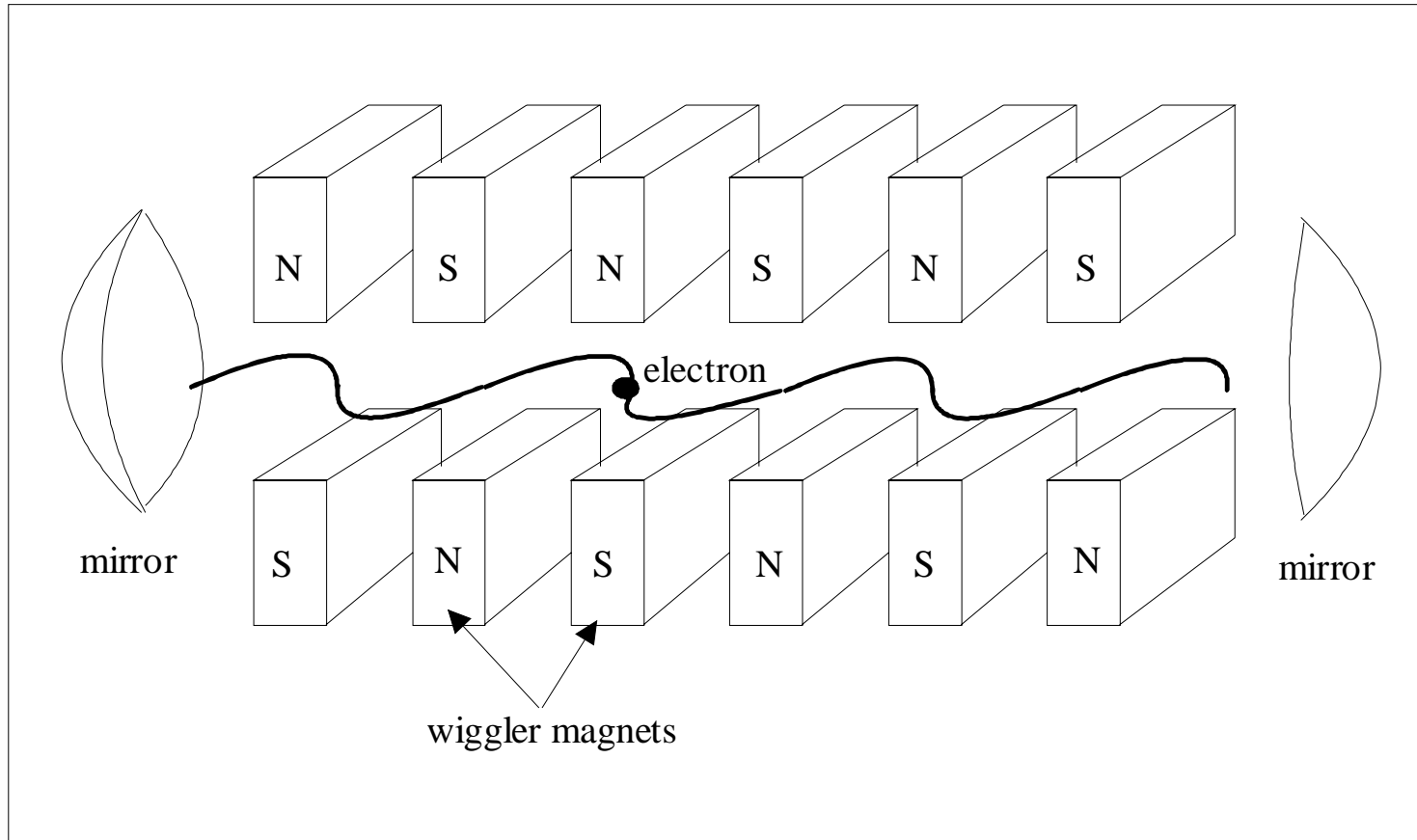
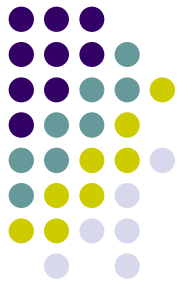
is called the *wiggler parameter*

→  $\lambda = \frac{1+a_w^2}{2\gamma^2} \lambda_w$  : FEL synchronism condition

Radiation wavelength can be tuned by magnetic field  $B$ , wiggler period  $\lambda_w$ , and electron energy  $\gamma$



# Free-electron Laser (Stimulated Compton Scattering)





Courtesy of 王逸群

China Times, Apr. 11, 2006

中華民國九十五年四月十一日/星期二

中國時報 CHINA TIMES http://news.chinatimes.com

# FEL 雷射光束加熱熔脂 革命性

## 嚴重粉刺、橘皮的剋星

### 自由電子雷射 比傳統更具威力

自由電子雷射比傳統雷射更具威力。科學家使用這種雷射時，能運用特定雷射波長對脂肪加熱。脂肪

自由電子雷射 (FEL) 機器，在不損及其他身體組織情況下，將皮膚表層下的脂肪加熱熔解。

英國《泰晤士報》十日報導，美國麻州綜合醫院一組研究人員利用能產生特定雷射光束的「自由電子雷射」(FEL) 機器，在不損及其他身體組織情況下，將皮膚表層下的脂肪加熱熔解。

治療特殊粉刺 還可預防  
醫學界為治療粉刺和橘皮，經歷過無數有效的粉刺治療劑 Isotretinoin (Accutane，國內品名「羅可坦」) 問題，如婦女在懷孕期間使用這種藥物，心臟等器官畸型。安德森對於雷射治療粉刺的潛力特別感興趣。他希望了解是否可使用特定波長的雷射。皮膚科會透過毛囊分泌皮脂，來潤滑皮膚。但皮脂分泌過多，一旦堆積起來，除了粉刺，安德森表示，自由電子雷射能將粉刺和其他身體表層脂肪相關用來對付動脈中的脂肪堆積，達到預防的目的。安德森目前正朝這個目標邁進。

### 消除脂肪

粉刺，如圖右，肇因於皮脂腺發炎，為青春期的常見毛病。患者會有面皰、黑頭粉刺或膿皰產生，且通常出現在臉部。粉刺有可能留下疤痕。

如圖左，因脂肪細胞間的薄層組織（通常只有數個細胞的厚度）變得更加緊密而形成。這個區域會圍繞著脂肪開始拉緊，造成小凹點和不規則狀，即典型的橘皮。

科學家發現，只要將自由電子雷射調到非常準確的頻率（波長1210奈米），雷射光束能穿透皮膚且不僅及皮膚。

### 自由電子雷射

循環利用的電子  
光在兩片光學鏡片間彈跳  
電子光束  
雷射光

雷射

表皮

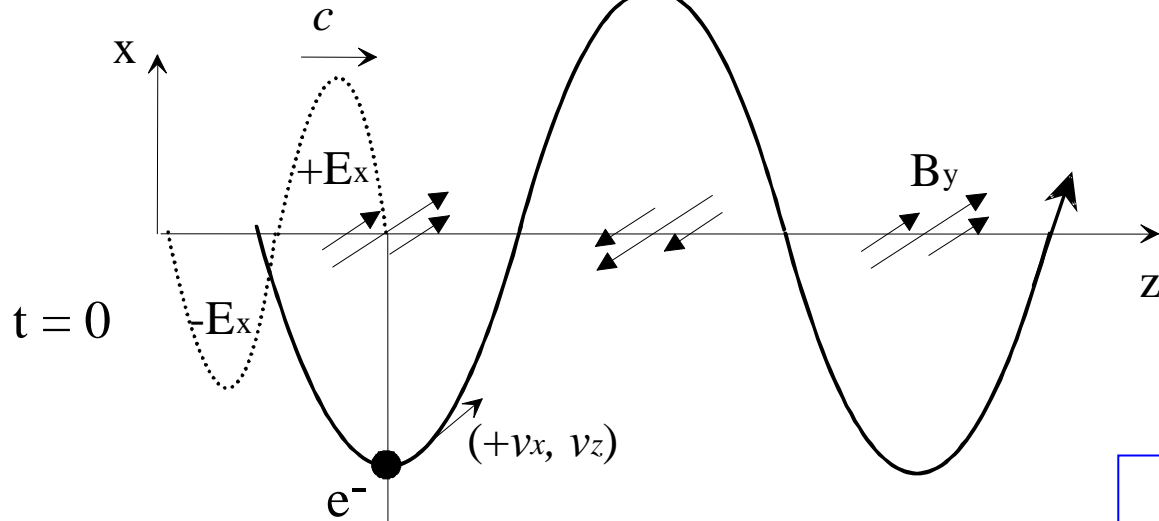
毛絲

毛孔

# Free-electron Laser



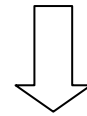
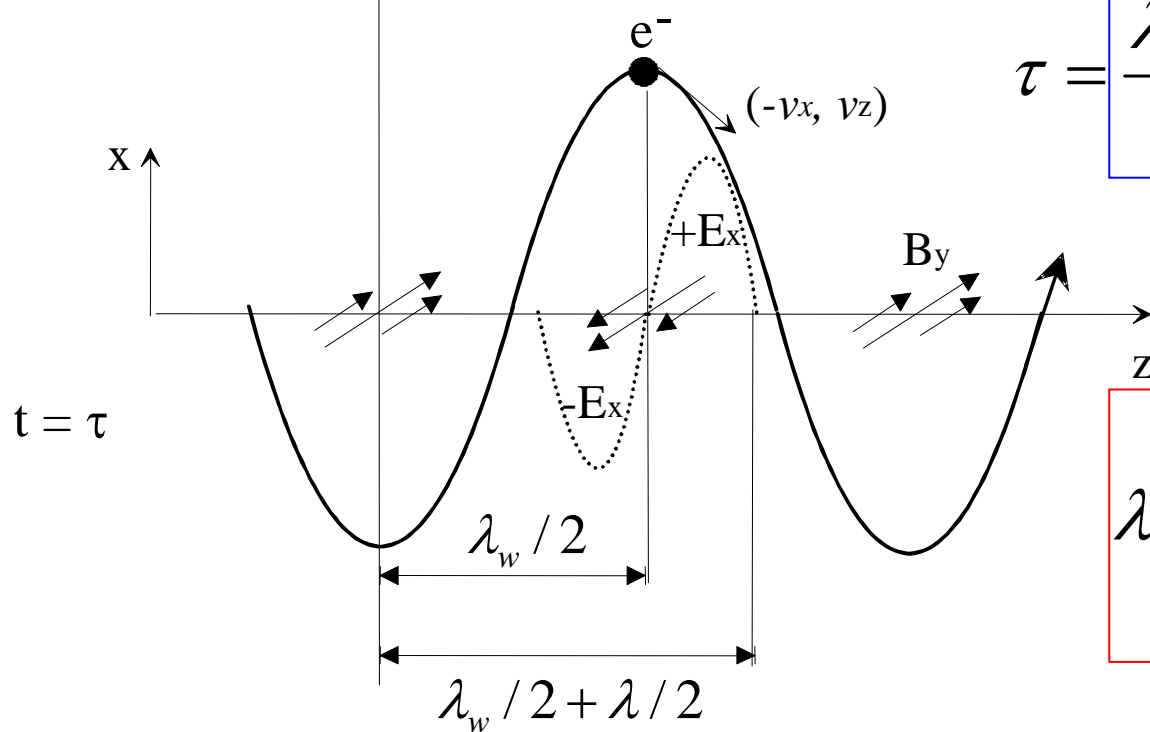
To have gain  $\Delta W = e \int_{\tau=L_w/v_z} \vec{E} \cdot \vec{v} dt < 0$



amplification

$$\vec{v} \cdot \vec{E} > 0$$

$$\tau = \frac{\lambda_w/2 + \lambda_z/2}{c} = \frac{\lambda_w/2}{v_z}$$



$$\lambda = \lambda_w \left( \frac{1}{\beta_z} - 1 \right) \approx \frac{\lambda_w}{2\gamma_z^2}$$

# Electron-Wave Energy Exchange



Wave Amplification  $\Delta W = e \int_{\tau=L/v_{//}} \vec{E} \cdot \vec{v} dt < 0$

Particle Acceleration  $\Delta W = e \int_{\tau=L/v_{//}} \vec{E} \cdot \vec{v} dt > 0$

Transverse Coupling  
(Eg. Compton/Thompson/undulator radiation etc.)

$$\Delta W = e \int_{\tau=L/v_{//}} \vec{E}_{\perp} \cdot \vec{v}_{\perp} dt$$

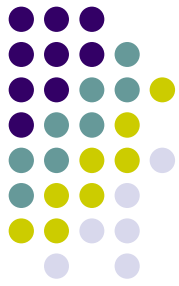
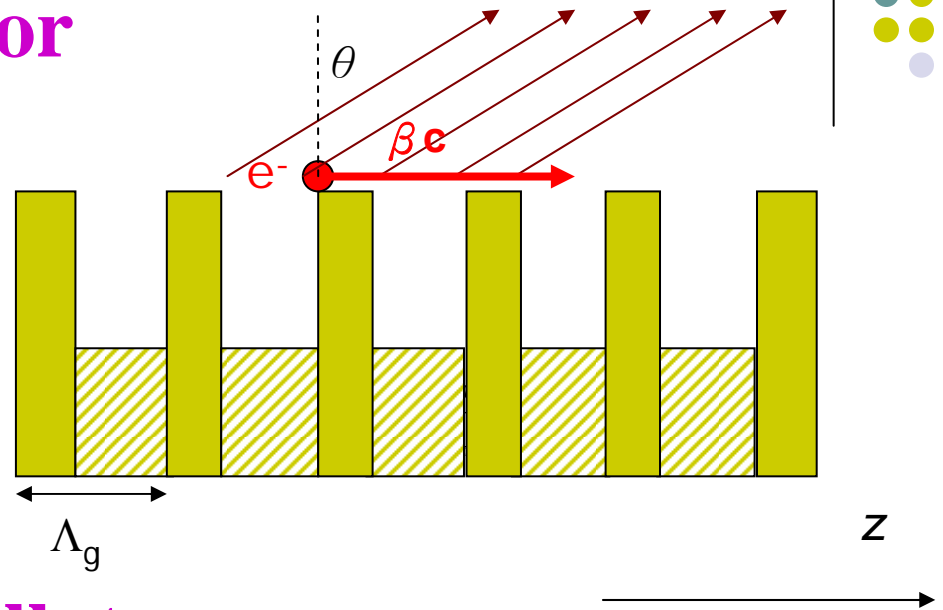
Longitudinal Coupling  
(Eg. Smith-Purcell radiator, backward-wave oscillator etc.)

$$\Delta W = e \int_{\tau=L/v_{//}} \vec{E}_{//} \cdot \vec{v}_{//} dt$$

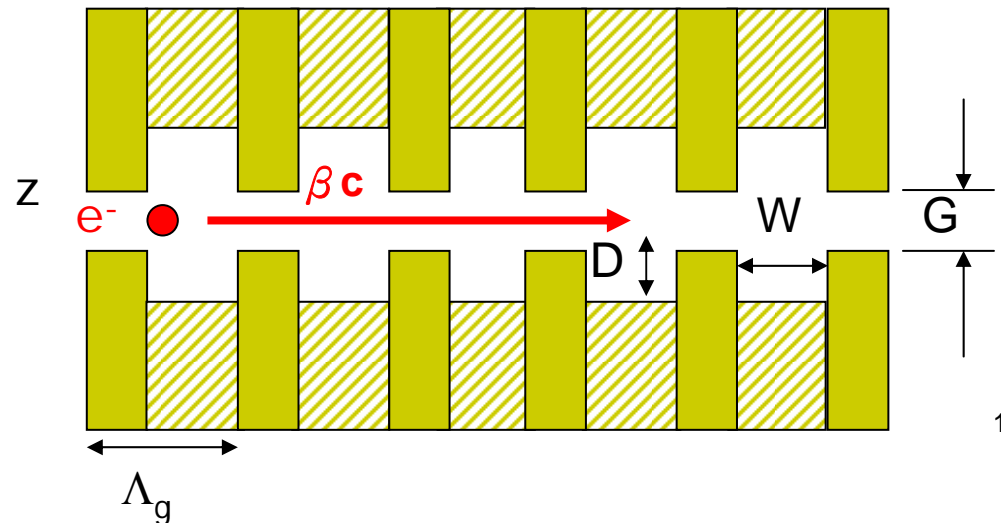
$$\Delta W = e \int_{\tau=L/v_z} \vec{E}_z \cdot \vec{v}_z dt < 0$$

## Smith-Purcell Radiator

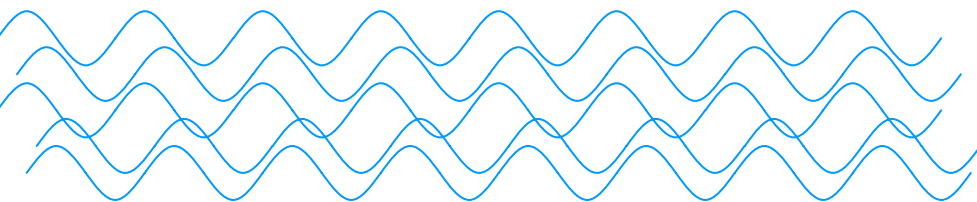
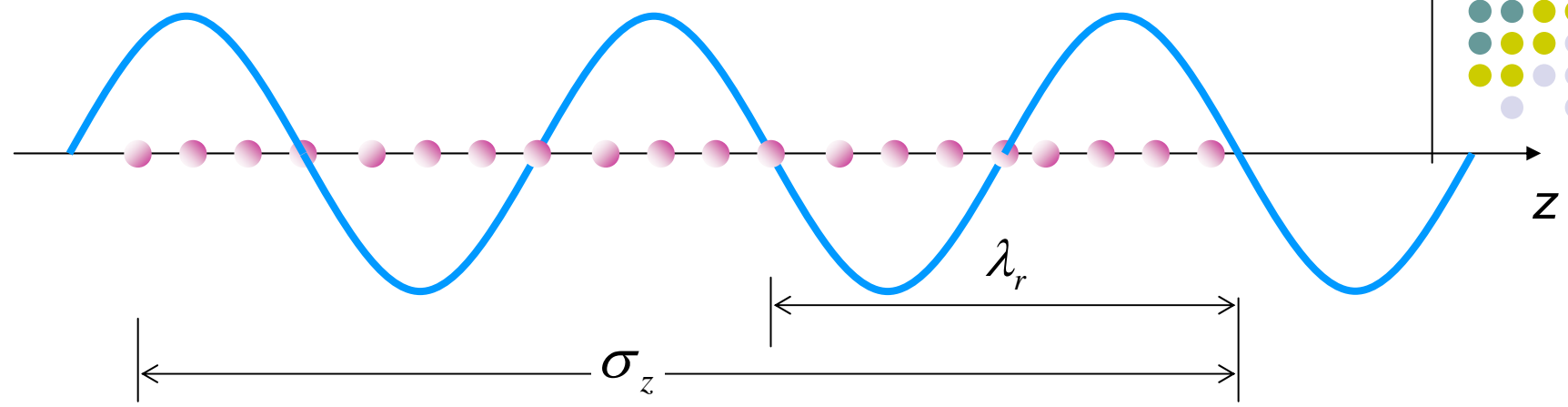
$$\lambda = \Lambda_g \left( \frac{1}{\beta} - \sin \theta \right)$$



## Backward-wave Oscillator



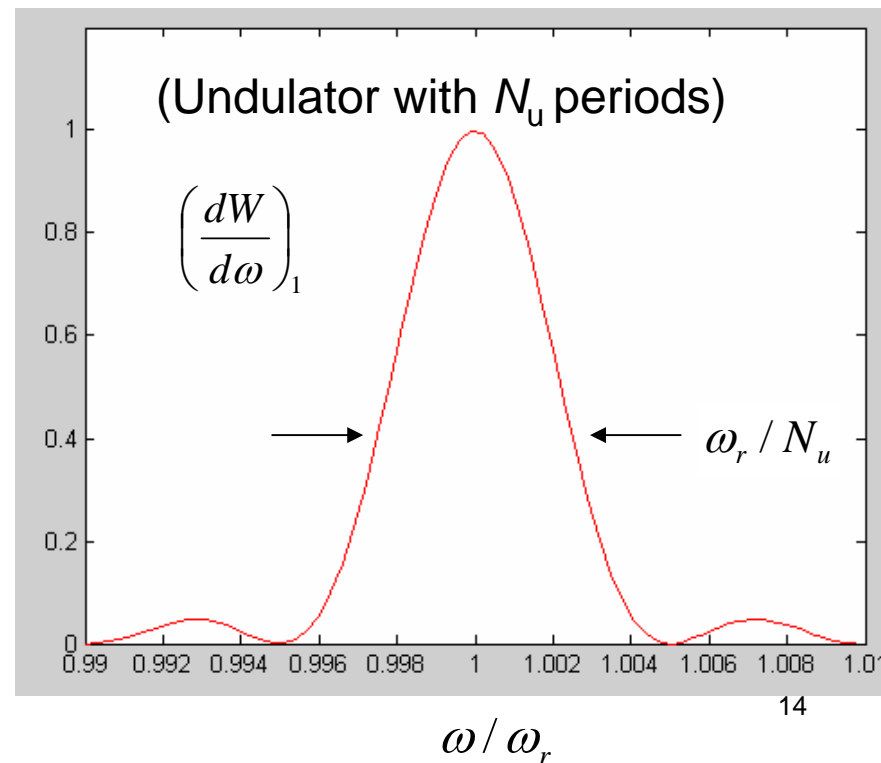
# Incoherence Radiation $\sigma_z \gg \lambda_r$



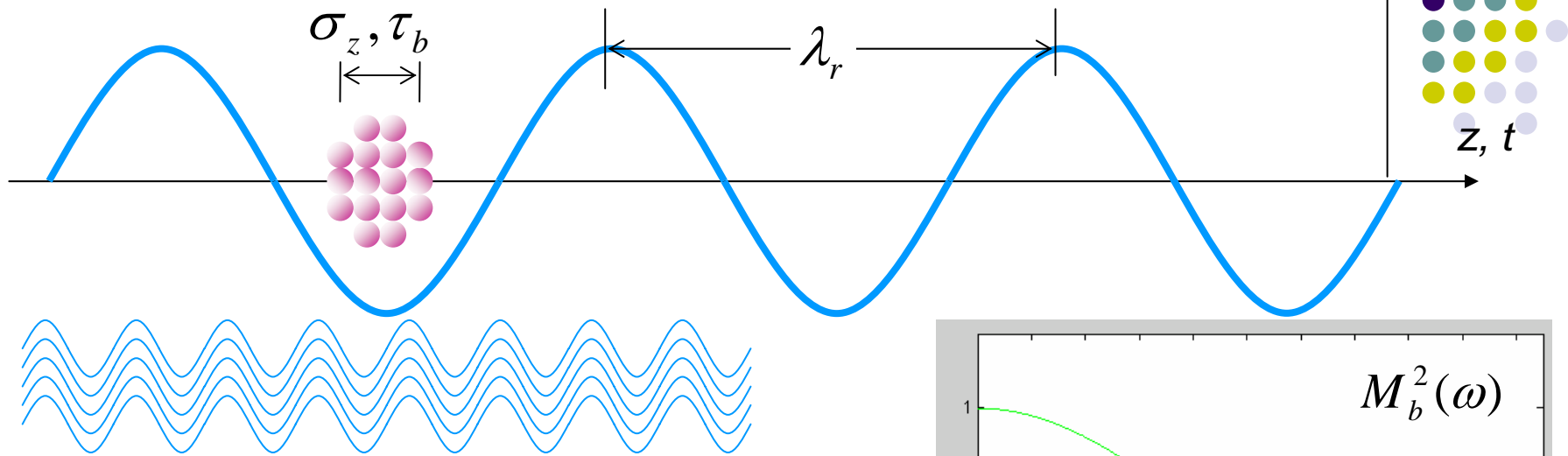
$$\left(\frac{dW}{d\omega}\right)_{inc,N} = \left(\frac{dW}{d\omega}\right)_1 \left| \sum_{j=1}^N e^{i\omega_j t} \right|^2$$

Spectral Energy  $\left(\frac{dW}{d\omega}\right)_{inc,N} = N \left(\frac{dW}{d\omega}\right)_1$

$N$ : number of electrons



# Superradiance: coherent radiation



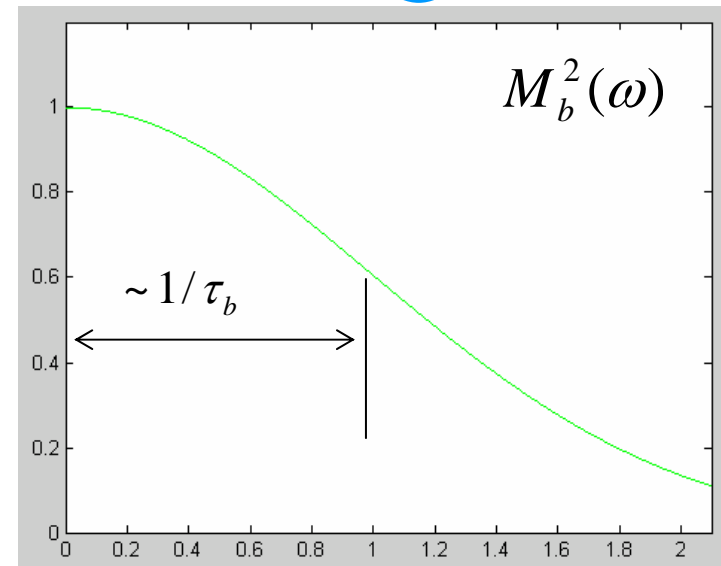
$$\text{Spectral Energy} \left( \frac{dW}{d\omega} \right)_{SR, N_b} = N_b^2 \left( \frac{dW}{d\omega} \right)_1 M_b^2(\omega)$$

$M_b(\omega)$  : Fourier transform of bunch shape

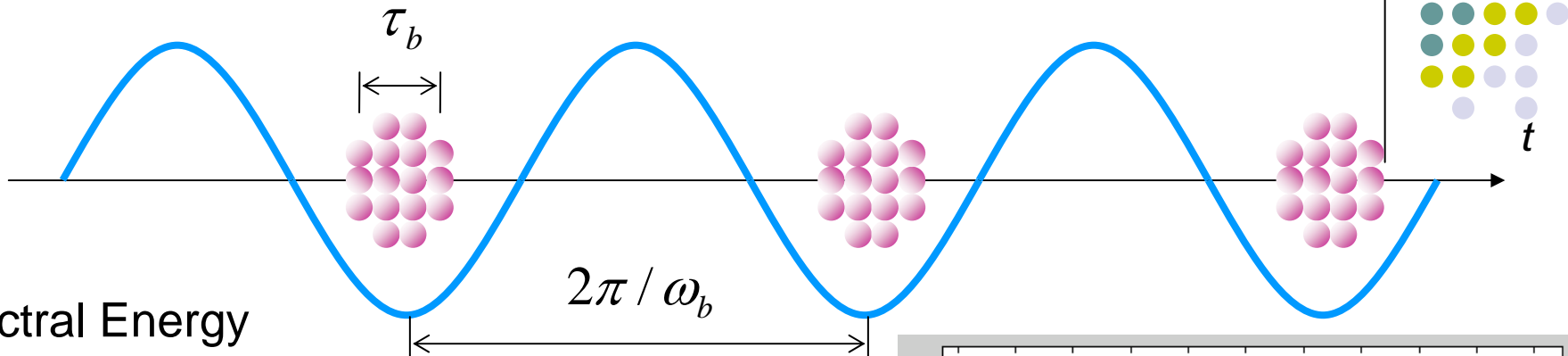
$N_b$ : number of bunched electrons

\* For 1 nC in 10 ps and  $\lambda_r = 1 \mu\text{m}$ ,  $N_b = 2 \times 10^6$ !

$$M_b(\omega) = \exp\left(-\frac{\omega^2 \tau_b^2}{4}\right) \text{ for Gaussian bunch shape function } f(t) = \frac{\exp(-t^2 / \tau_b^2)}{\sqrt{\pi \tau_b}}$$



# Superradiance from a Periodically Bunched Beam

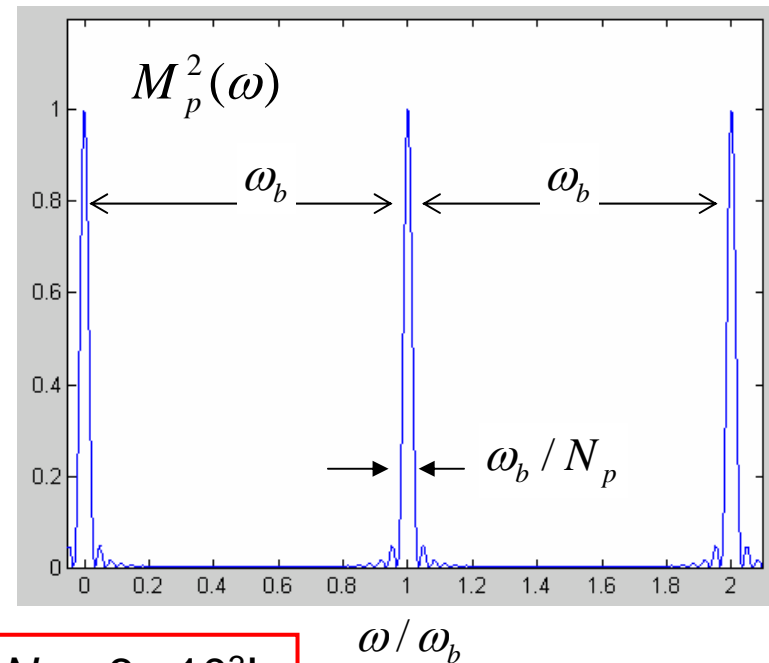


Spectral Energy

$$\left(\frac{dW}{d\omega}\right)_{SR, N_p \times N_b} = (N_p N_b)^2 \left(\frac{dW}{d\omega}\right)_1 M_b^2(\omega) M_p^2(\omega)$$

$$M_p(\omega) = \frac{\sin(N_p \pi \omega / \omega_b)}{N_p \sin(\pi \omega / \omega_b)}$$

Coherent sum of  $N_p$  bunches with bunching freq.  $\omega_b$



\* For 10-ps macro-bunch length and  $\lambda_r = 1 \mu\text{m}$ ,  $N_p = 3 \times 10^3!$



# Spectral narrowing from periodic bunched beam

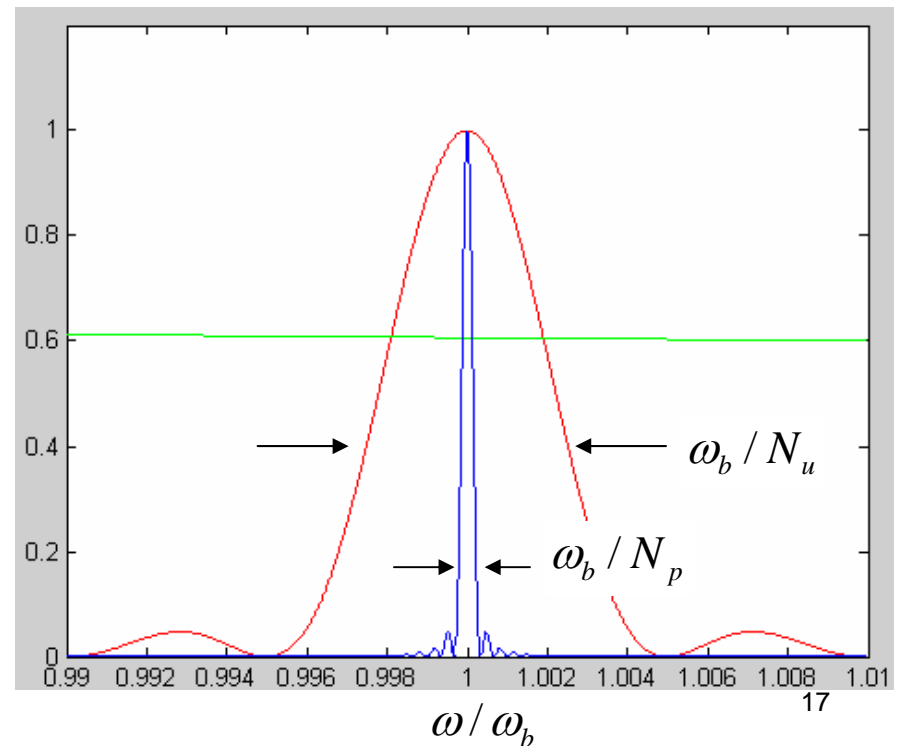
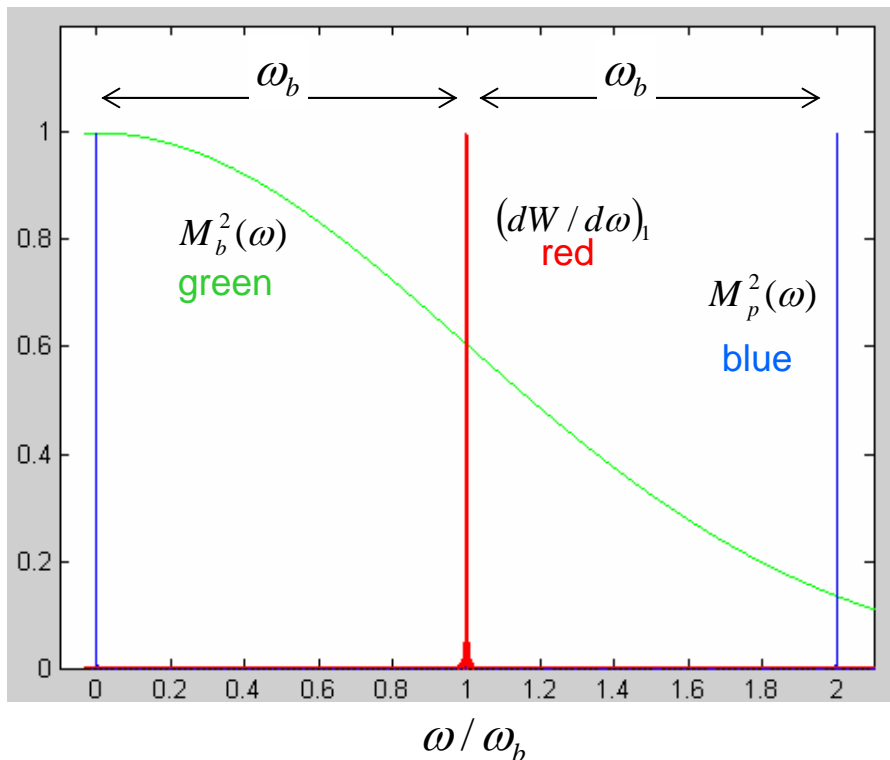


$$(dW / d\omega)_{SR, N_p \times N_b} = (N_p N_b)^2 (dW / d\omega)_1 M_b^2(\omega) M_p^2(\omega)$$

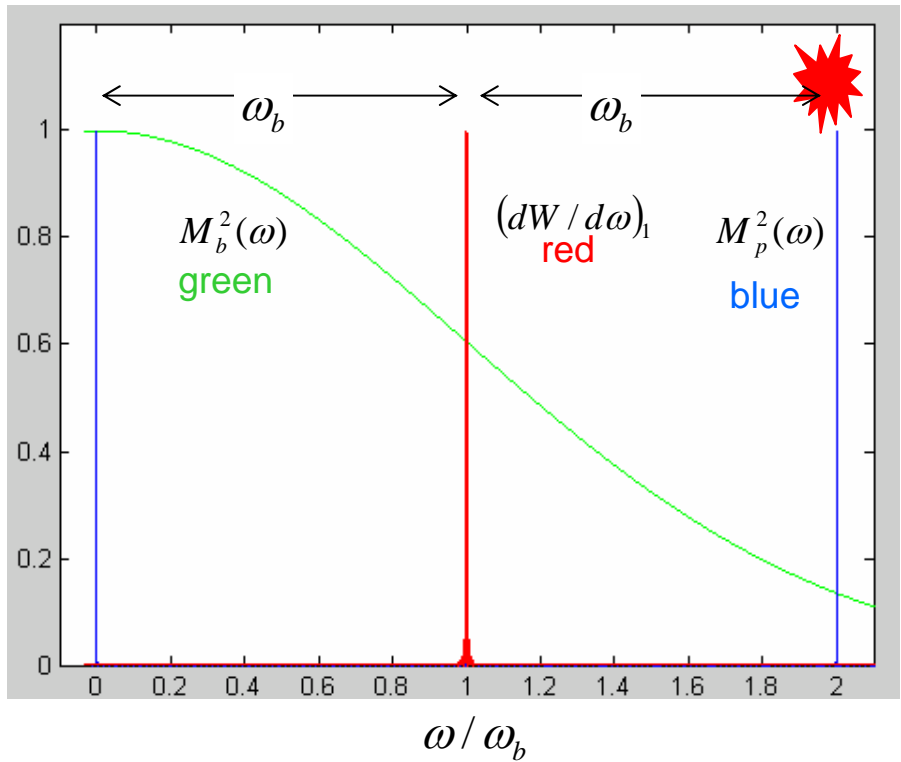
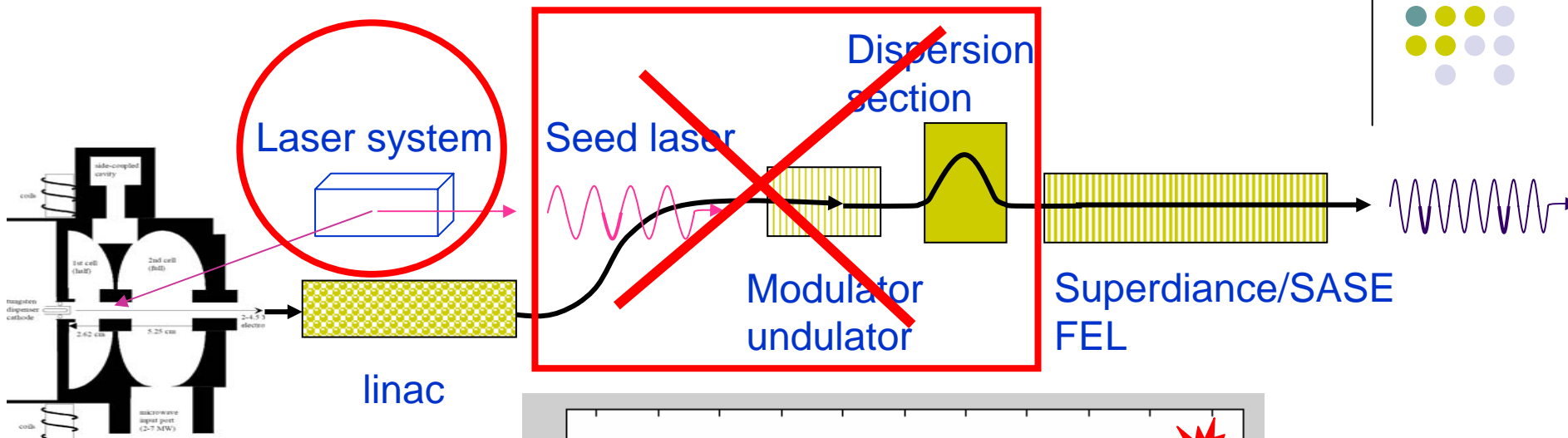
where  $M_p(\omega) = \sin(N_p \pi \omega / \omega_b) / \sin(\pi \omega / \omega_b) / N_p$

Assume undulator radiation  $\Rightarrow (dW / d\omega)_1 \propto \text{sinc}^2[2N_u(\omega / \omega_r - 1)]$   
for undulator with period  $N_u$

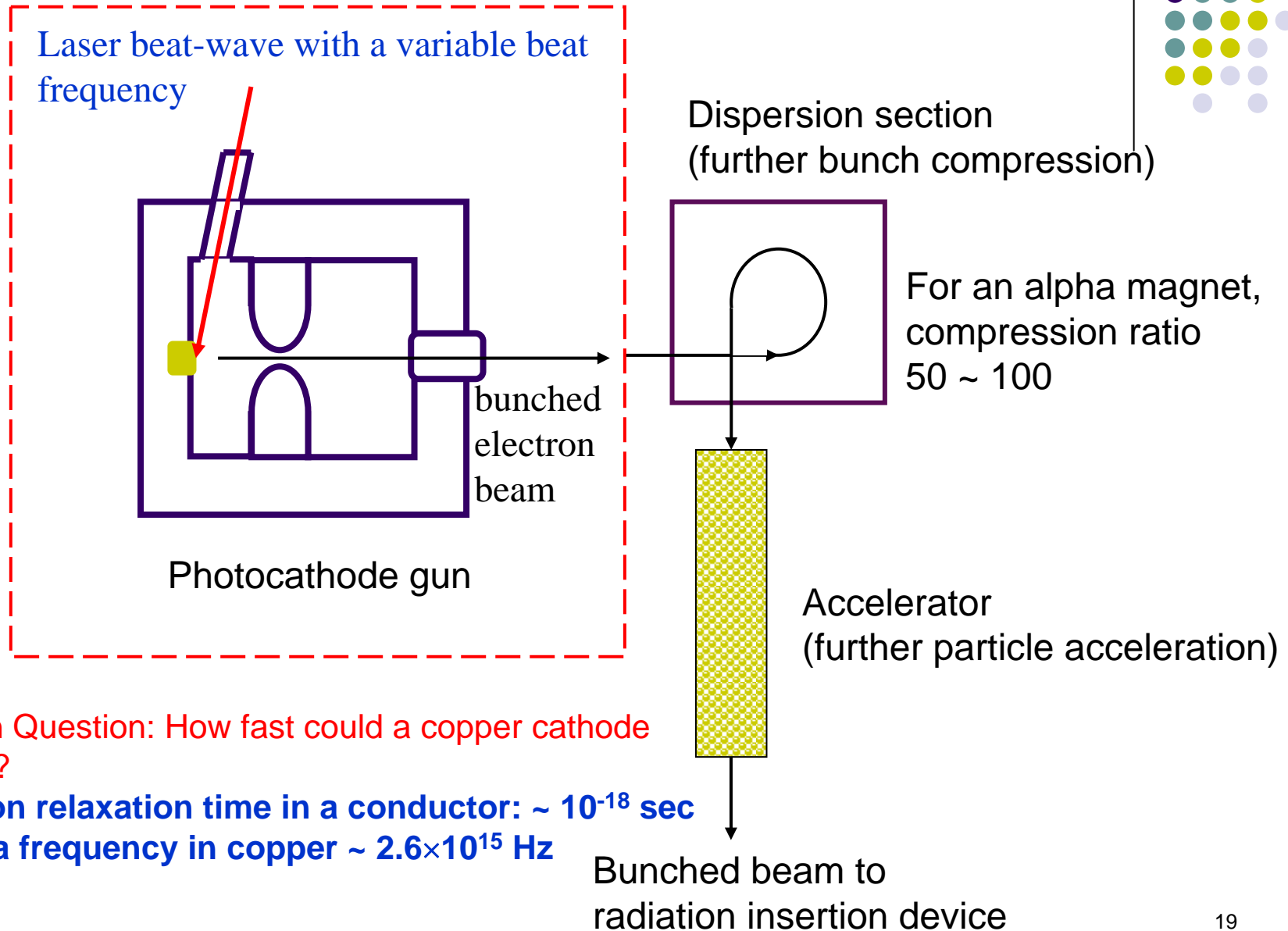
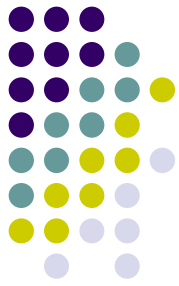
Assume  $\omega_b = 1 / \tau_b \Rightarrow M_b(\omega) = \exp(-\omega^2 / (4\omega_b^2))$  (3)  $\omega_r = \omega_b$



# Example: High Gain Harmonic Generation (HGFG, L. H. Yu et. @ BNL)

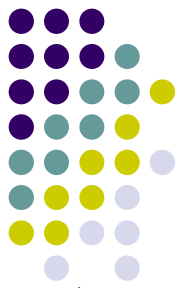


# Idea: Laser-beat-wave Bunched-beam Accelerator (B<sup>3</sup> technique)



An Open Question: How fast could a copper cathode respond?

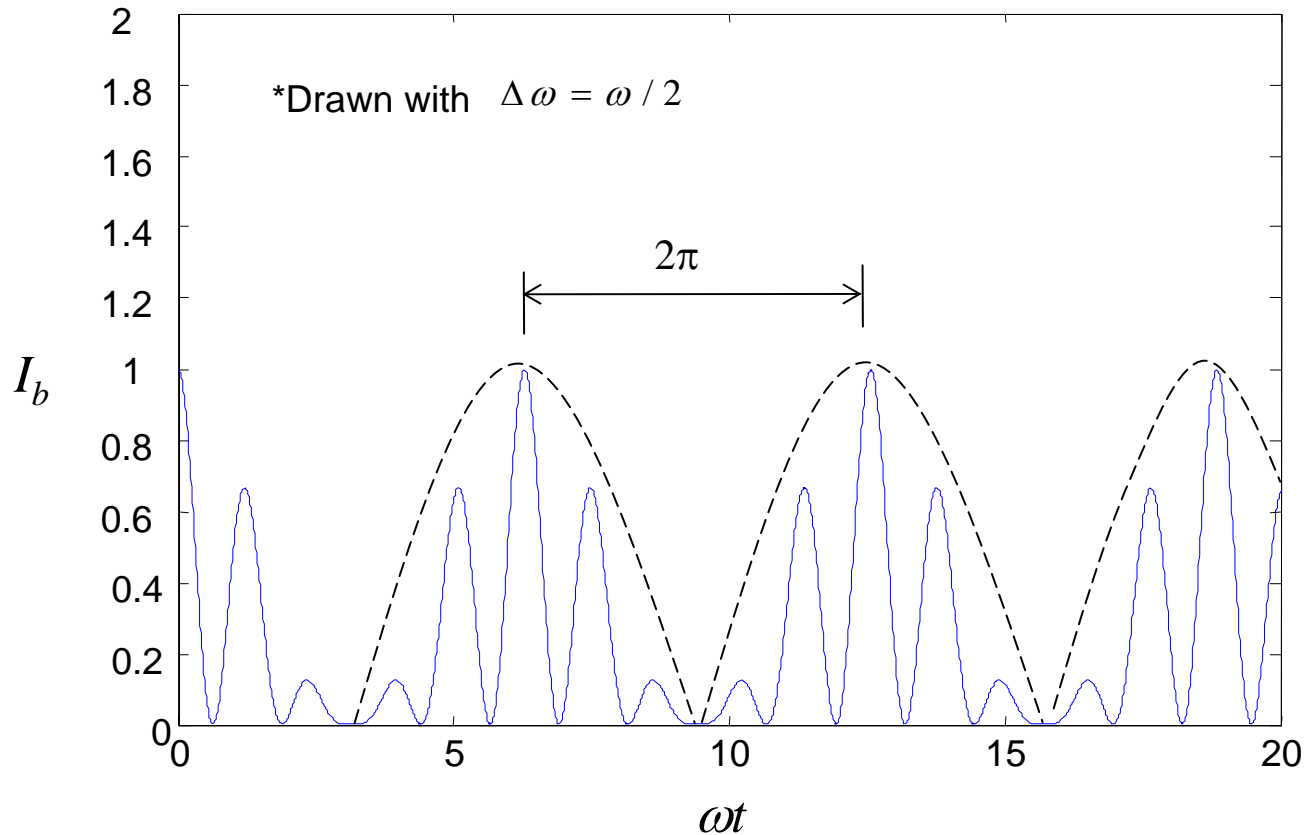
- Electron relaxation time in a conductor:  $\sim 10^{-18}$  sec
- Plasma frequency in copper  $\sim 2.6 \times 10^{15}$  Hz



# Laser Beat Wave: beating of two waves

Superposition of two fields with  $\Delta\omega$   $E = E_0 \cos \omega t + E_0 \cos(\omega - \Delta\omega)t$

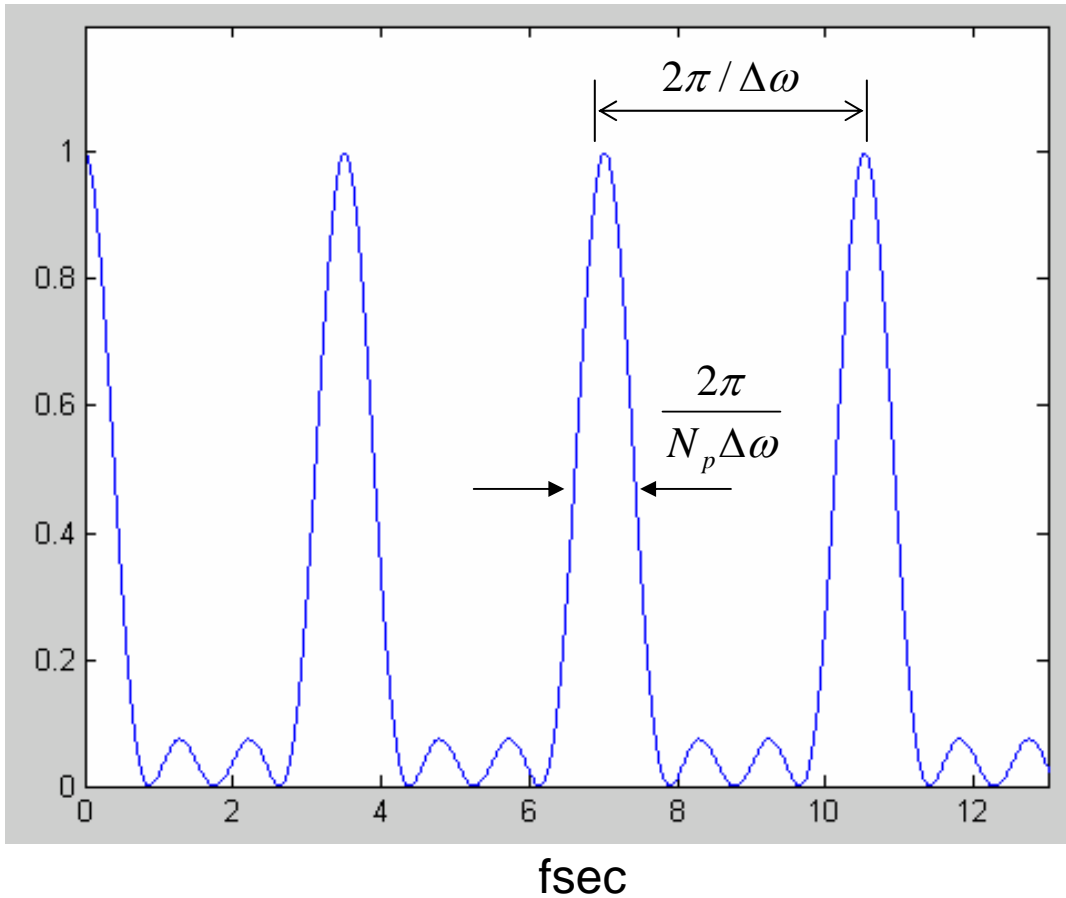
Instantaneous intensity  $I = I_0 \cos^2[(\omega - \frac{\Delta\omega}{2})t] \times \cos^2(\frac{\Delta\omega}{2}t)$  beating at frequency  $\Delta\omega$



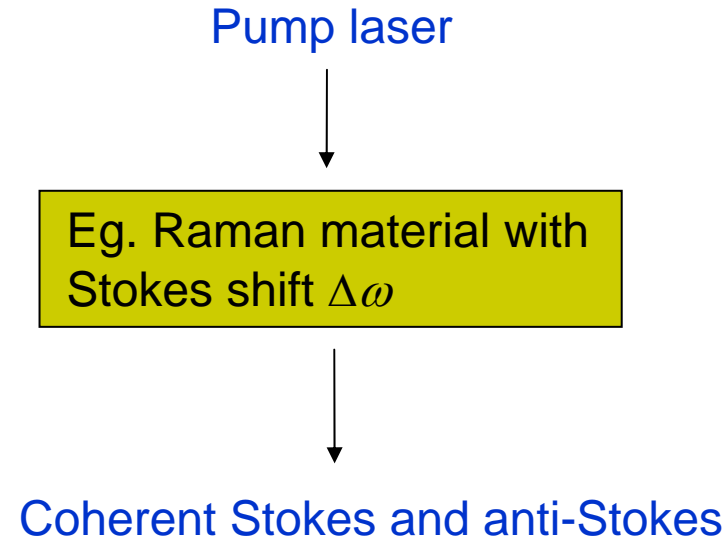


# Laser Beat Wave: multi-wave beating

$$I = I_0 \left| \exp(j\omega_0 t) \{1 + \exp(-j\Delta\omega t) + \dots + \exp[-j(N-1)\Delta\omega t]\} \right|^2 = I_0 \frac{\sin^2(N_p \Delta\omega t / 2)}{\sin^2(\Delta\omega t / 2)}$$



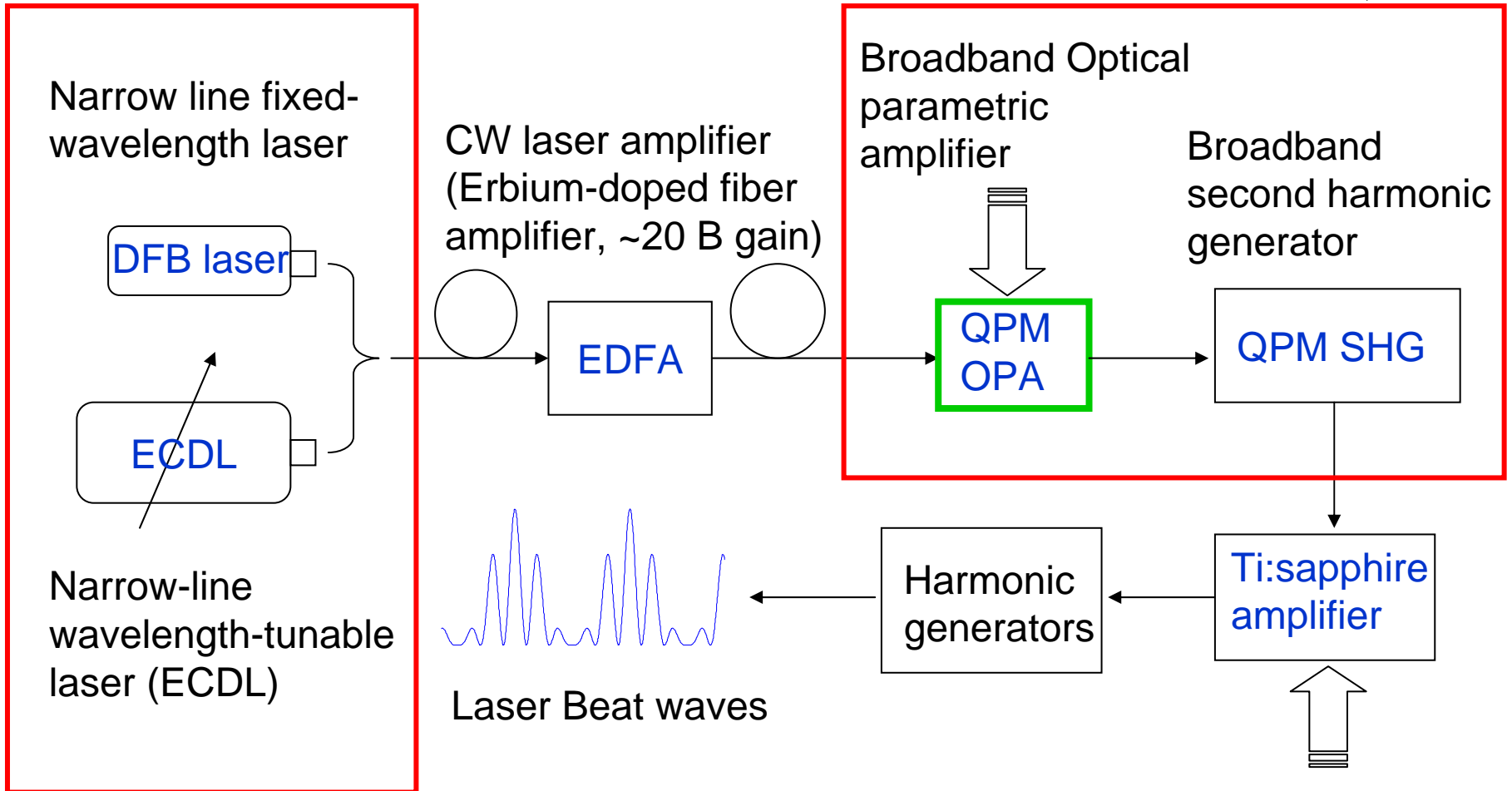
(1 fs  $\Leftrightarrow$  300 nm)



# Frequency Tunable Beat-wave Laser System



Quasi-phase-matched nonlinear optics

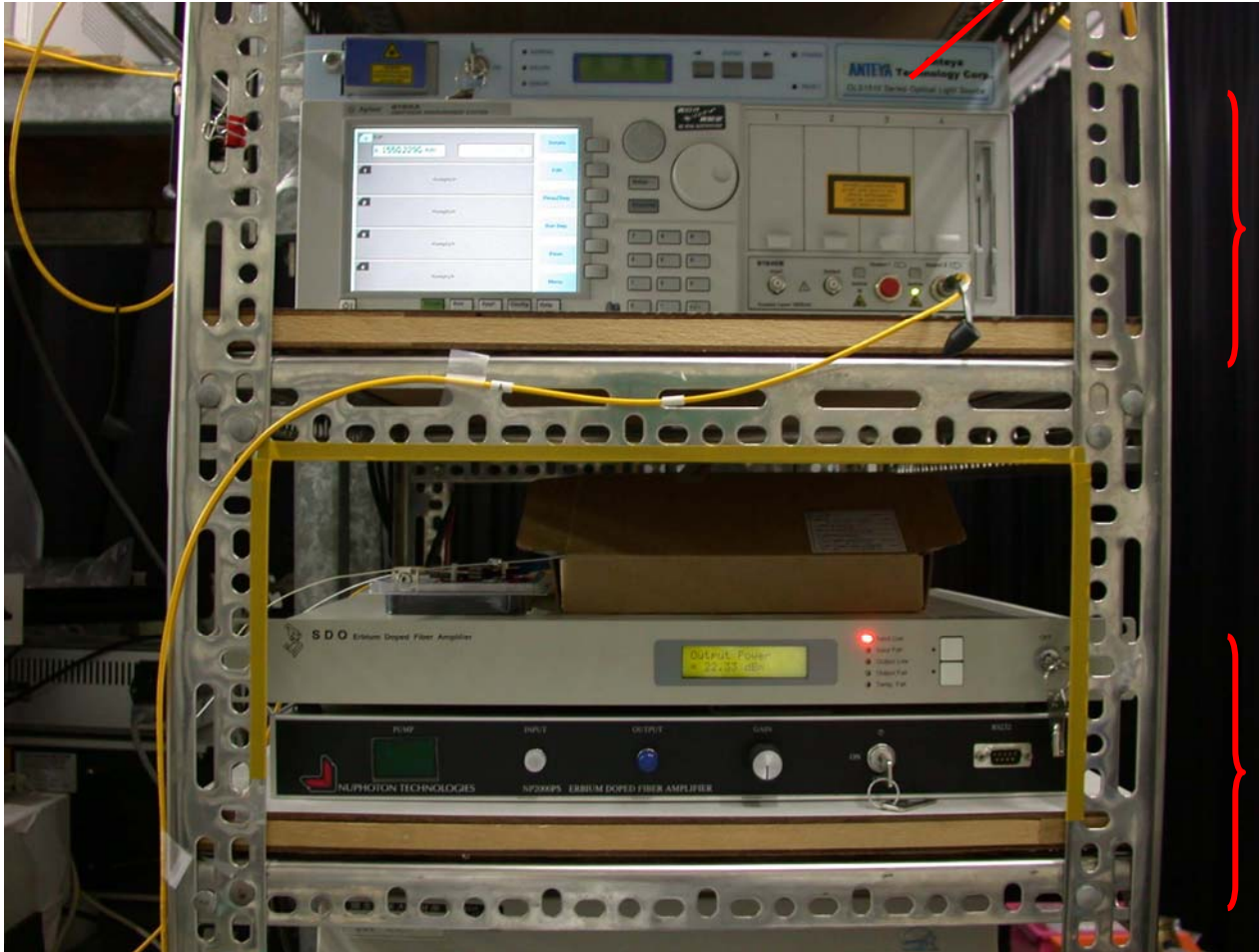
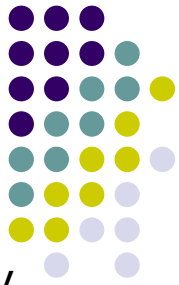


Telecom diode lasers

Laser amplifier

# Seed Laser System

Distributed-feedback diode laser with kHz linewidth

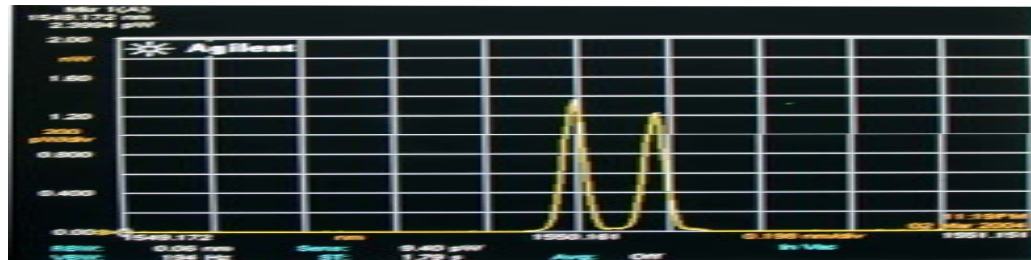


External-cavity tunable diode laser 1.4-1.6  $\mu\text{m}$  with MHz linewidth

Erbium doped fiber amplifier 1.48 ~1.62  $\mu\text{m}$  (20 THz bandwidth)



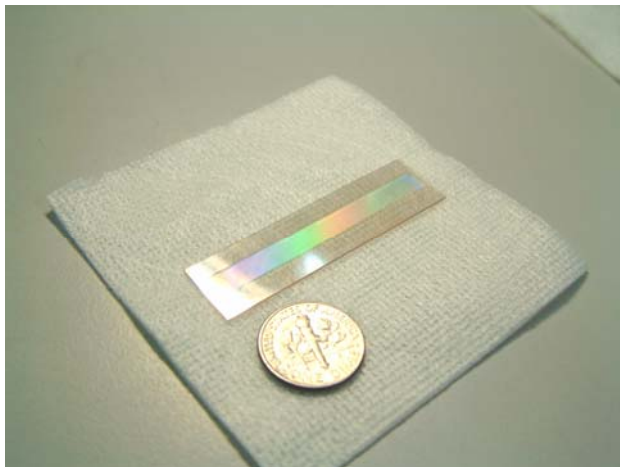
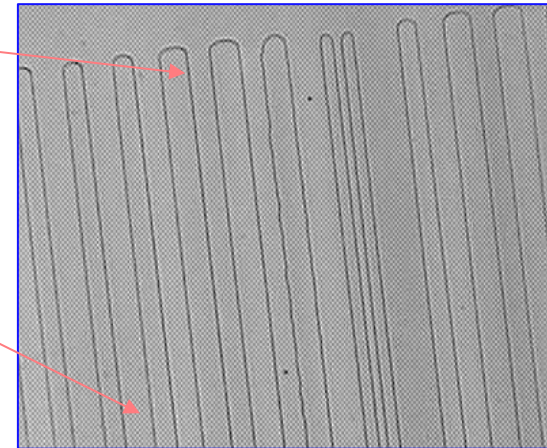
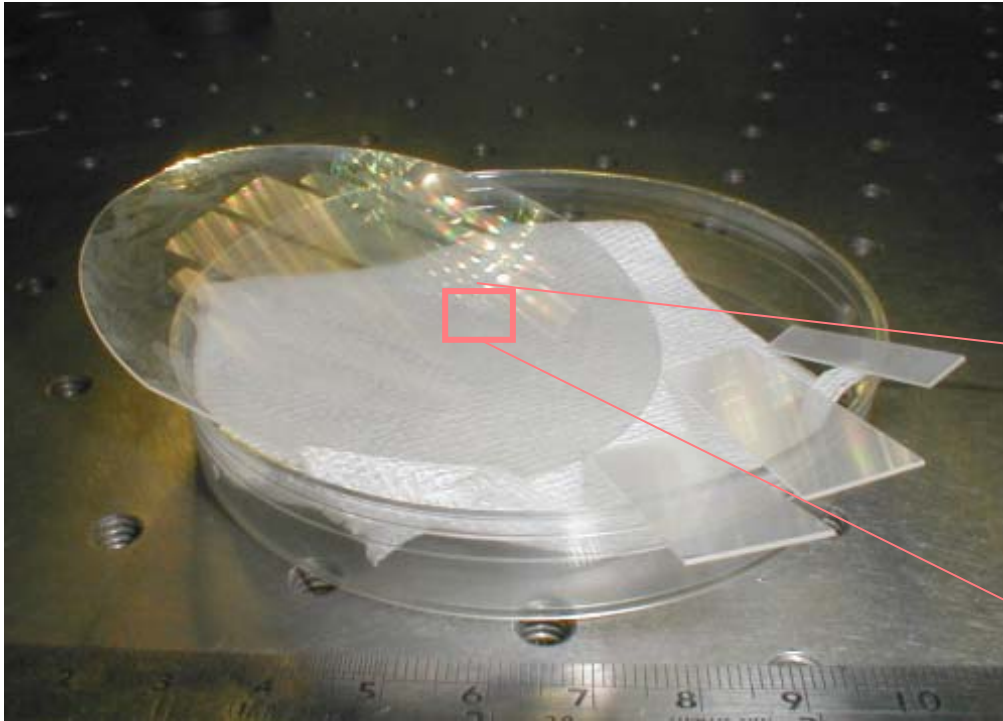
Seed laser spectrum



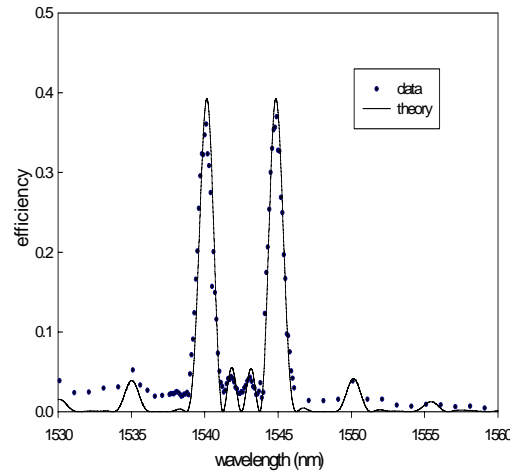


# Quasi-phase-matched (QPM) Nonlinear Optical Crystals

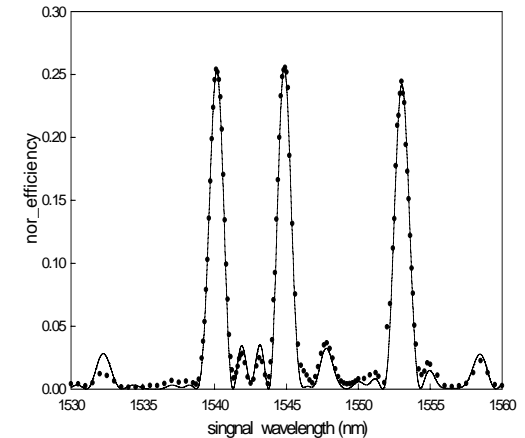
Phase-matching wavelengths can be engineered by using lithographic technology



AOS two peaks



Three channels – AOS



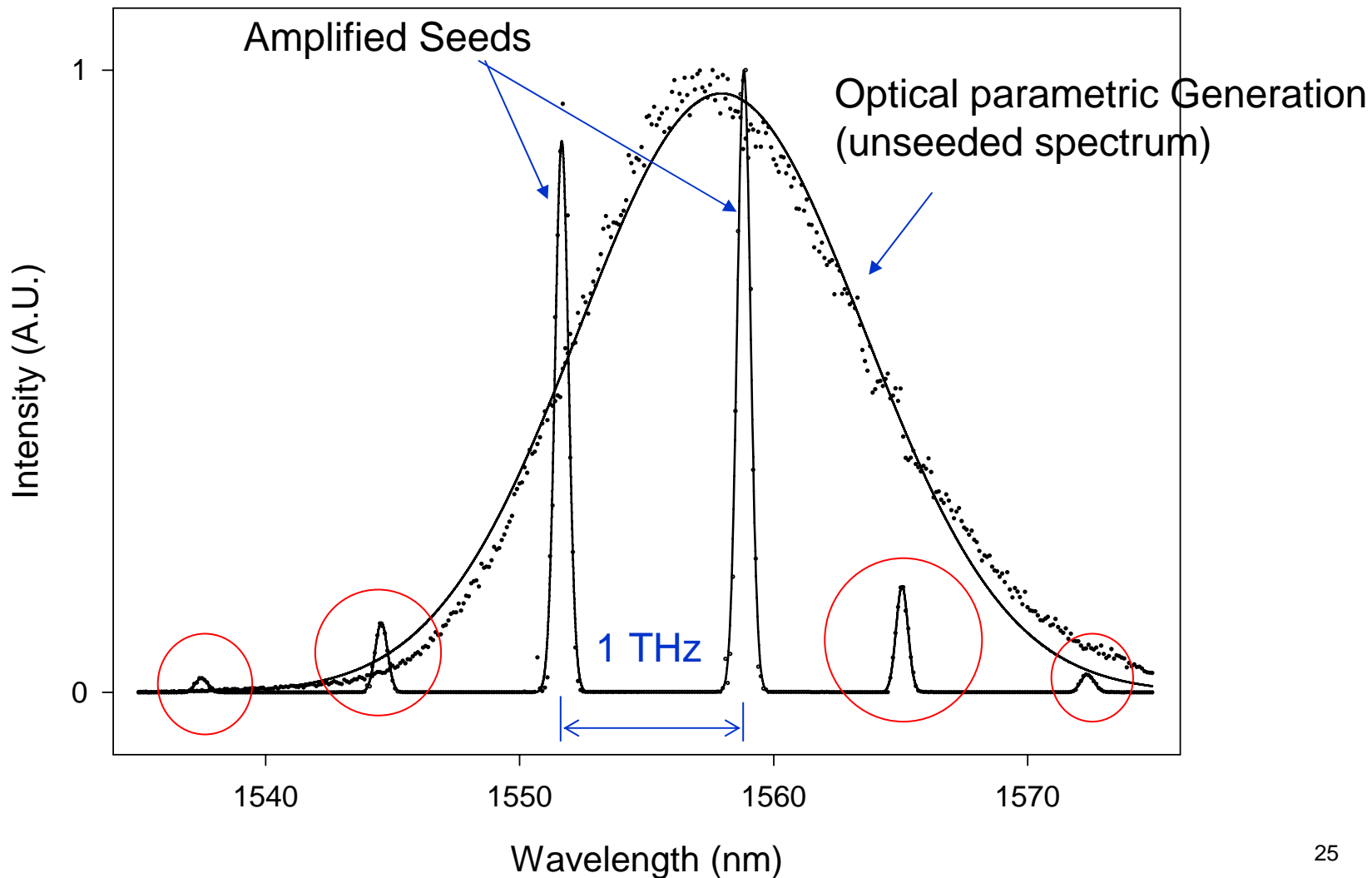


# Broadband Optical Parametric Amplification: Amplified Beat-wave Spectrum



Input: 60  $\mu\text{J}/\text{pulse}$  at 1064-nm

Output: 11  $\mu\text{J}/\text{pulse}$  at 1.55-6  $\mu\text{m}$



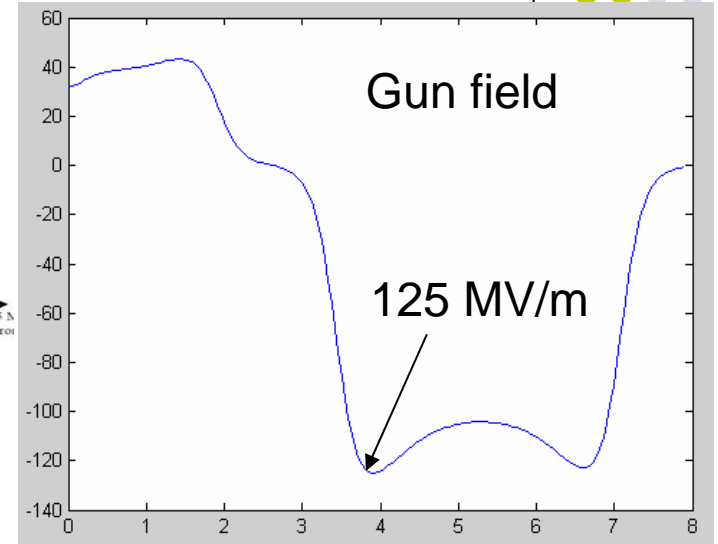
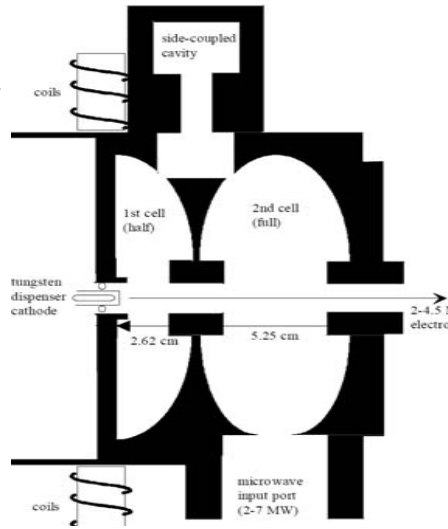
Question: Would particle acceleration smear out particle distribution?



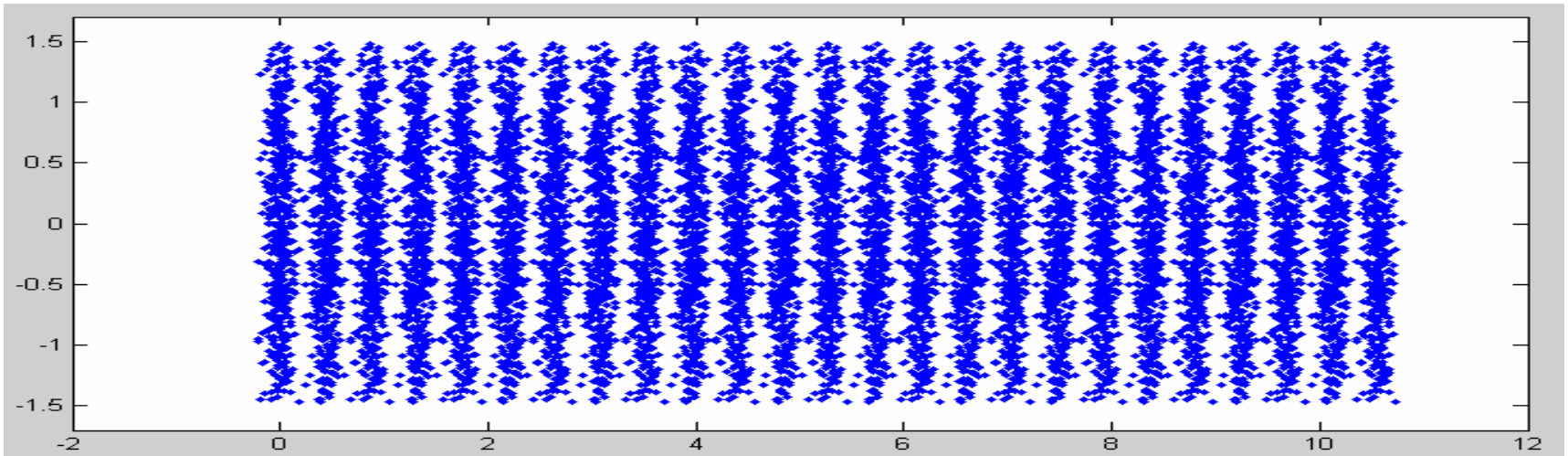
## Simulation (ASTRA) 1: 1-1/2 Cell S-band SSRL RF gun (input)

### Input

- 25 periodic bunches over 11 ps or 0.44 ps/bunch
- Gaussian bunch with rms bunch length = 72 fsec
- rms radius = 0.75 mm at cathode with radial distribution
- Total charge 0.1 nC or 4 pC/bunch
- Space charge force was considered.

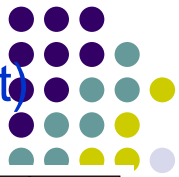


Cathode radius (mm)



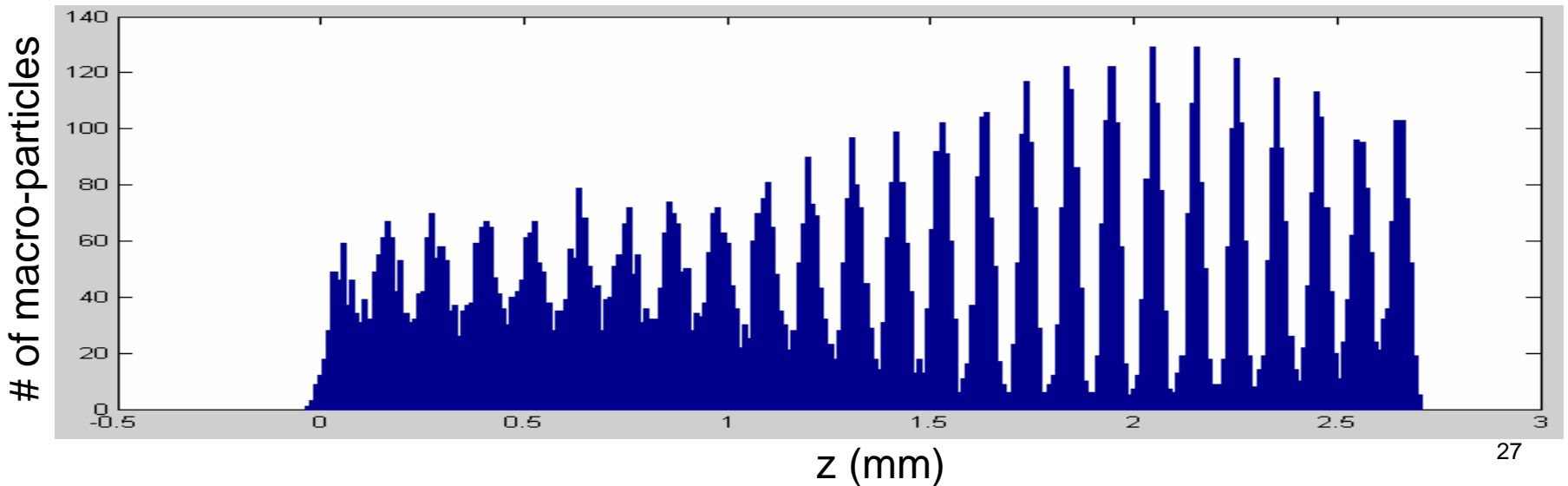
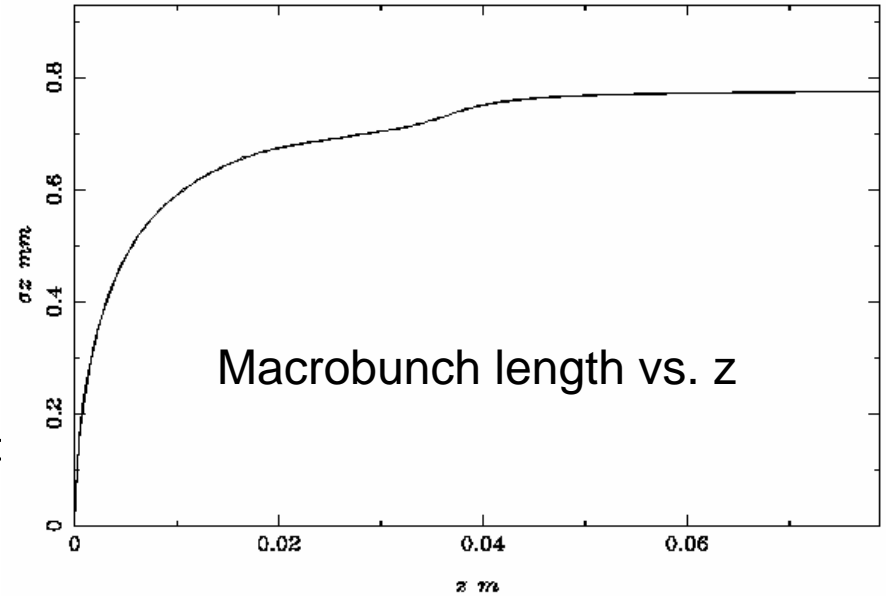
t (ps)

# Simulation (ASTRA) 1: 1-1/2 Cell S-band SSRL RF gun (output)



## Output

1. 11 A over 9 ps
2. Average energy: 3.994 MeV
3. Rms energy spread = 0.37%
4. normalized emittance =  $1.19 \pi$ -mm-mrad
5. Micro-bunch length  $\sim 108 \mu\text{m}$



# Simulation (ASTRA) 2: UCLA/DULY Planewave Transformer Accelerator

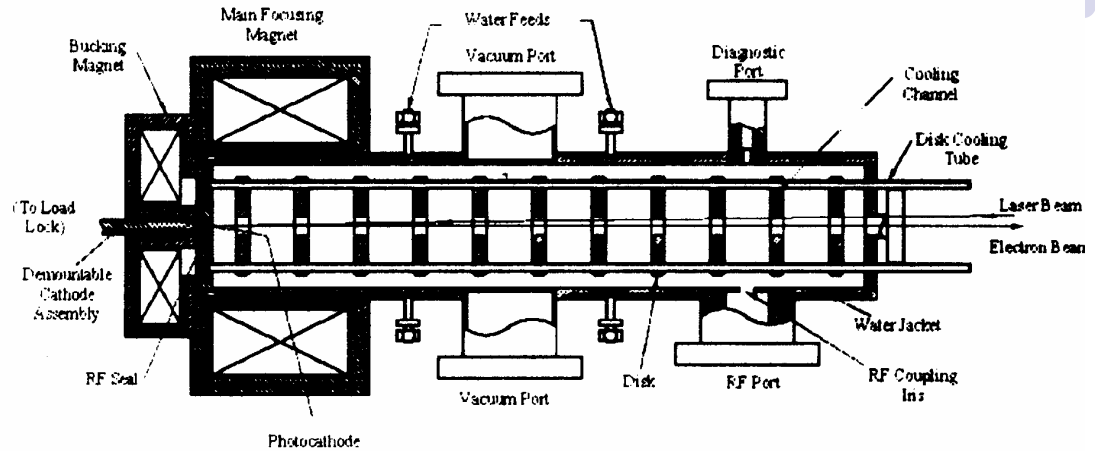


## Input

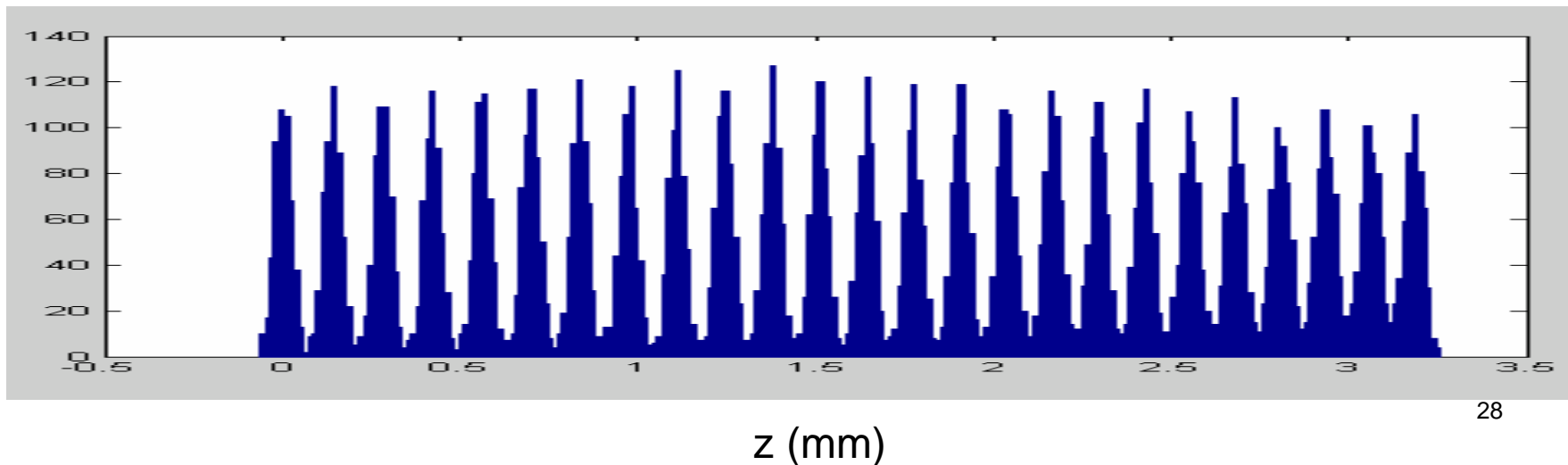
Same as in simulation 1

## Output

1. 9 A over 10.8 ps
2. Average energy: 19.15 MeV
3. Rms energy spread = 0.7%
4. normalized emittance =  $5.24 \pi$ -mm-mrad
5. Micro-bunch length  $\sim 130 \mu\text{m}$



# of macro-particles

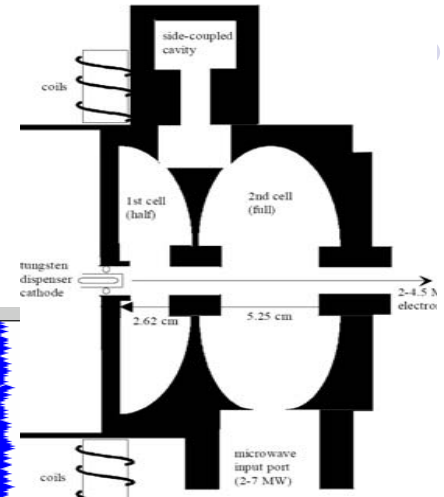


# Simulation (ASTRA) 3: Generation for 6 $\mu$ m bunch length

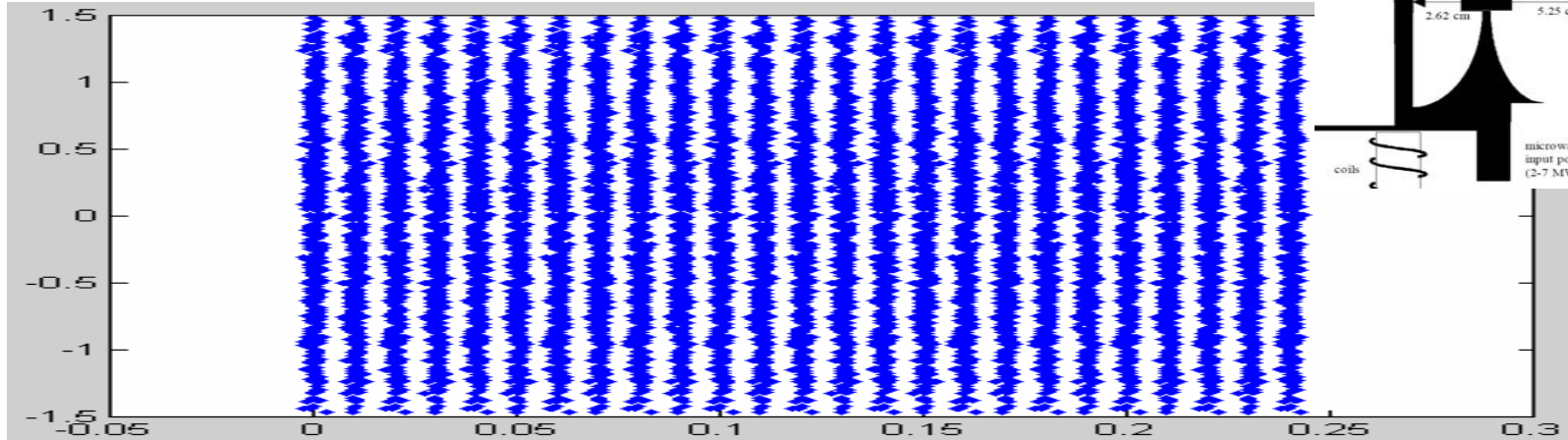


## Input

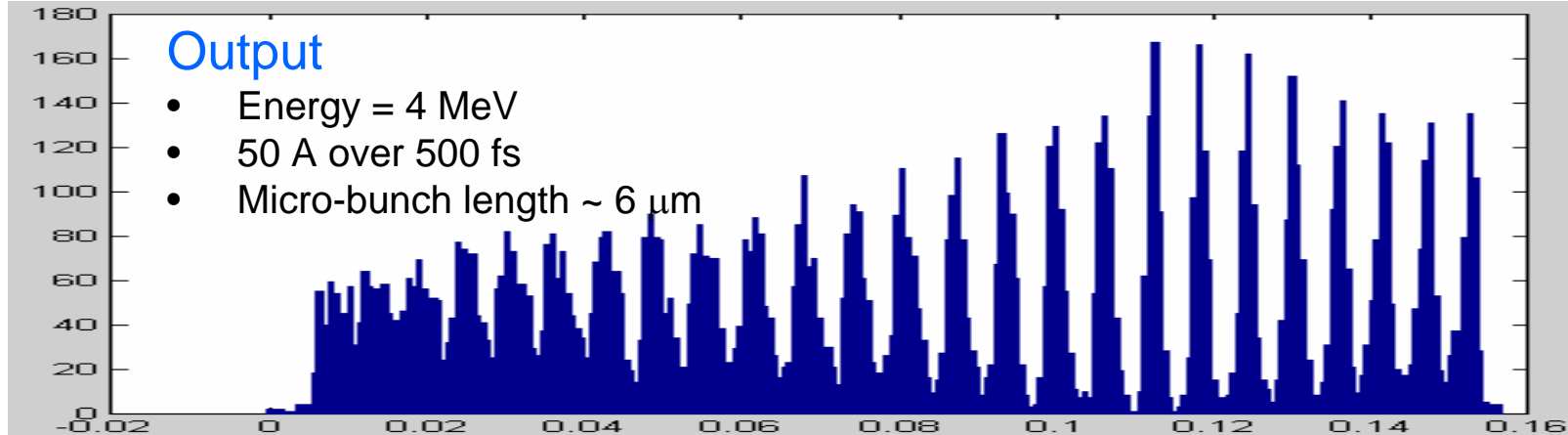
- 25 periodic bunches over 250 fs or 10 fs/bunch
- Gaussian bunch with rms bunch length = 1 fsec
- rms radius = 0.75 mm at cathode with radial distribution
- Total charge 25 pC or 1 pC/bunch



Cathode radius (mm)



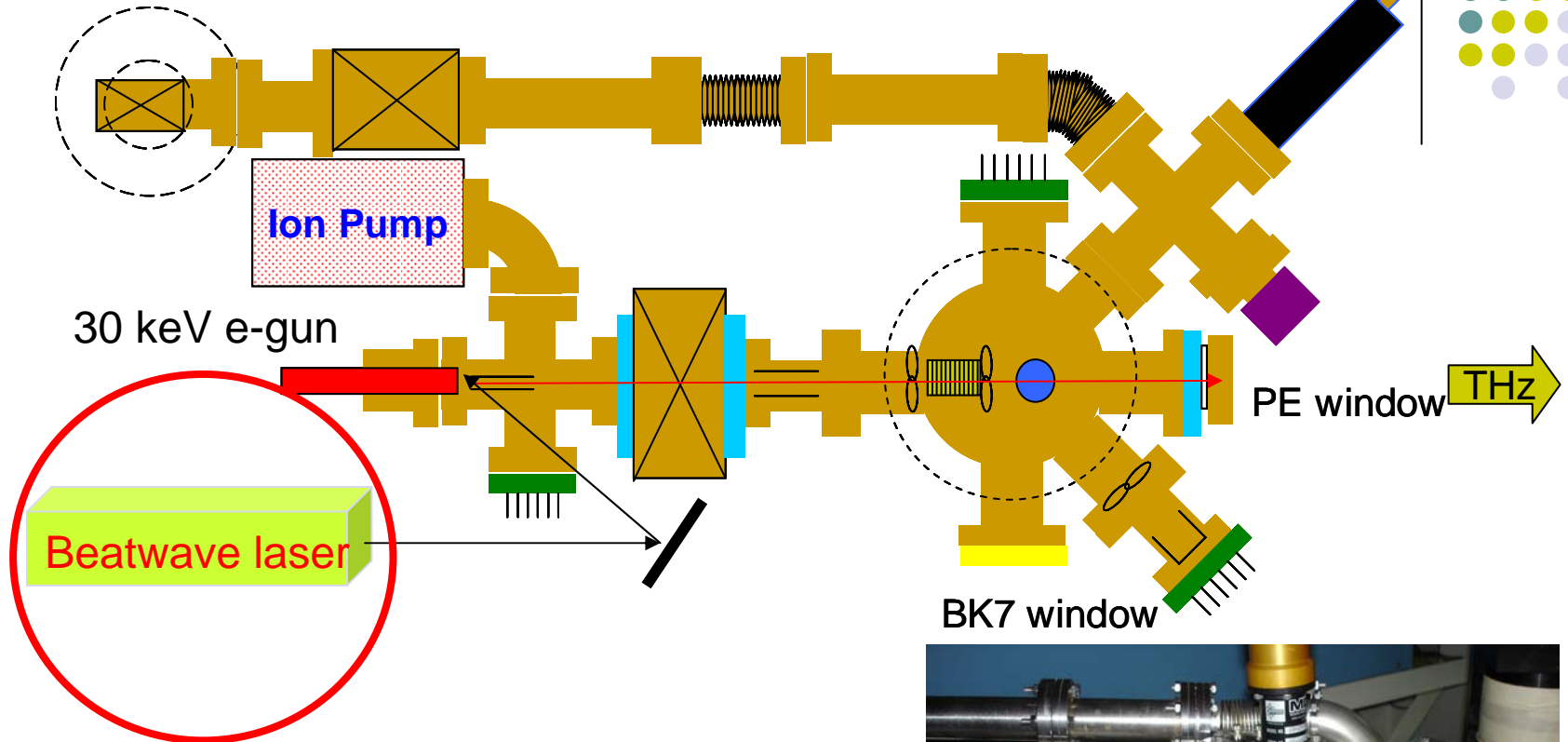
# of macro-particles



$z_2$  (mm)

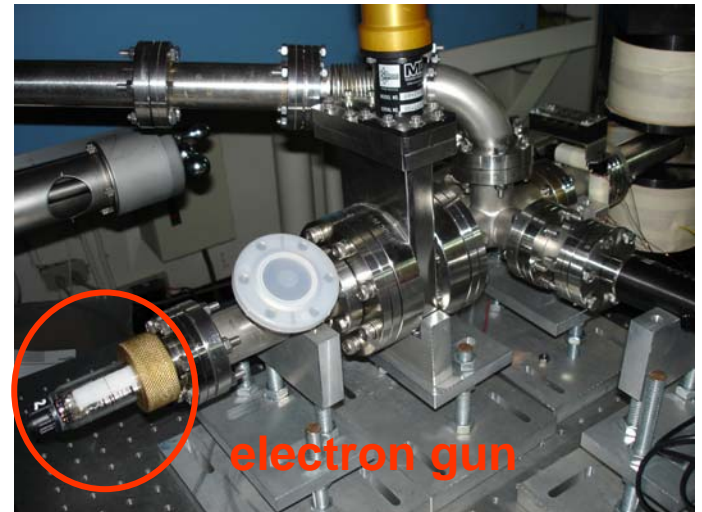
# Superradiance Smith-Purcell FEL: beam line

Goal: ~100 mW CW power at THz frequencies

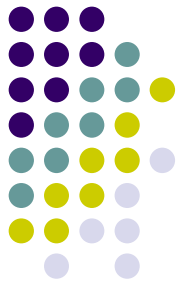


Power enhancement factor  $2 \times 10^4$   
(!)using  $B^3$  technique

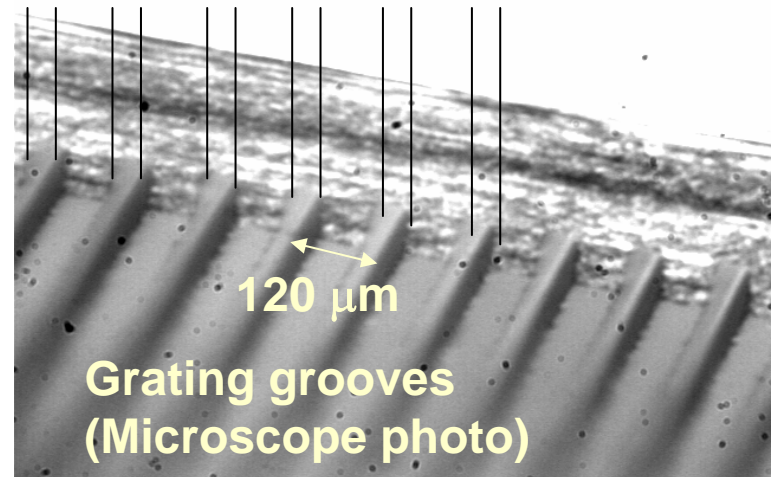
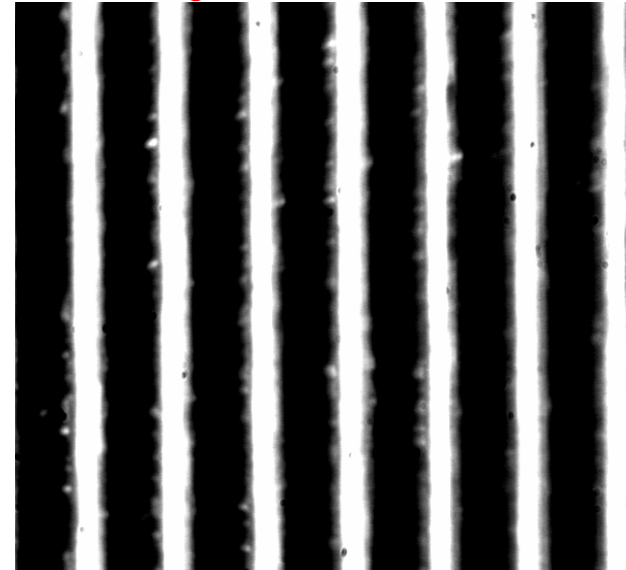
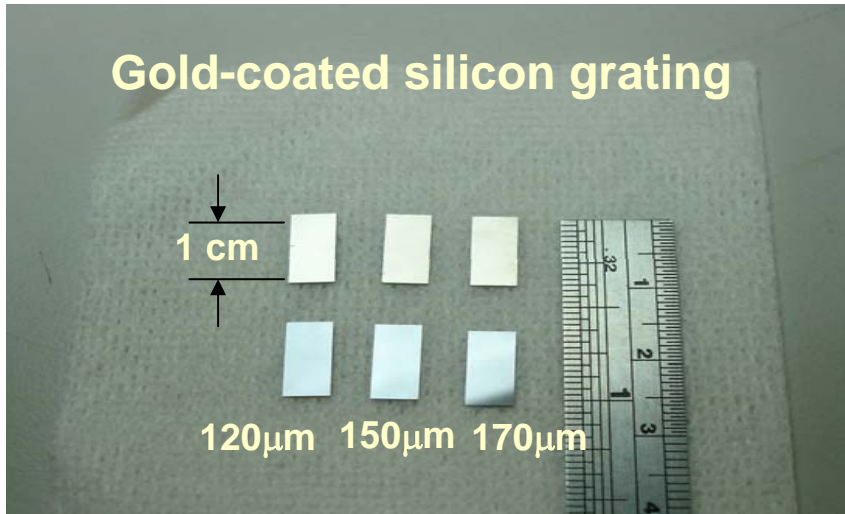
Y. Y. Lin et al., Proceedings 2005 FEL  
Conference, Stanford U., Aug. 22-26, 2005.



# Superradiance Smith-Purcell FEL: components

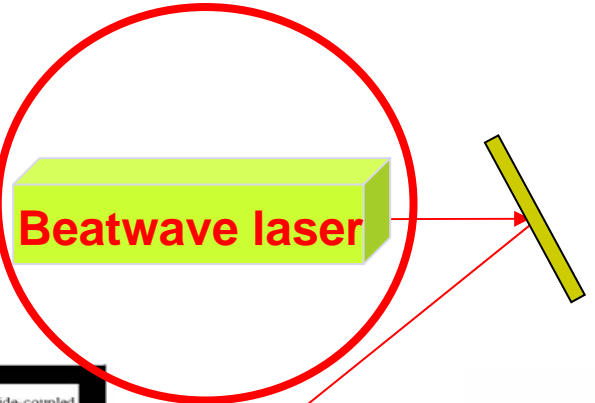


Y. Y. Lin et al., Proceedings 2005 FEL Conference, Stanford, Aug. 22-26, 2005.

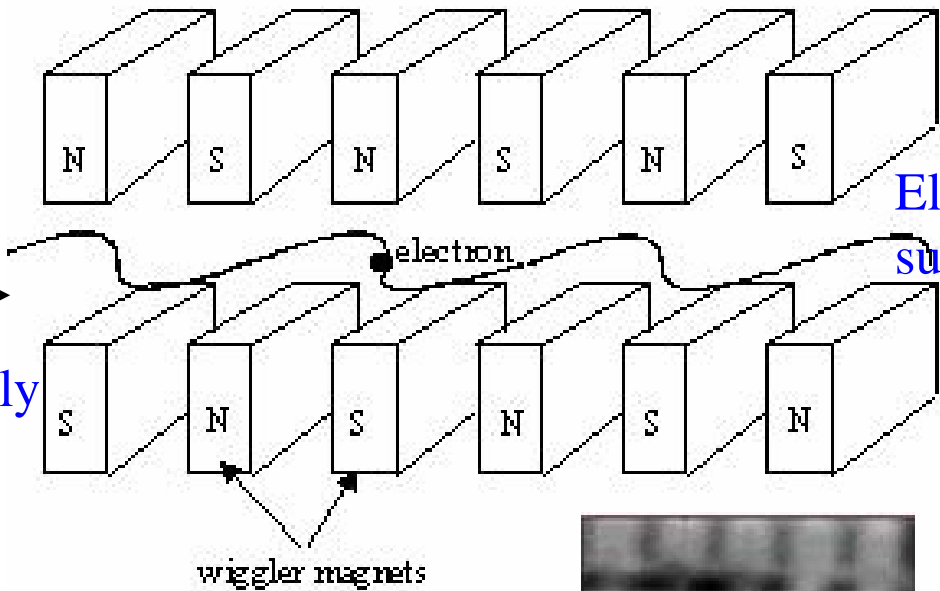
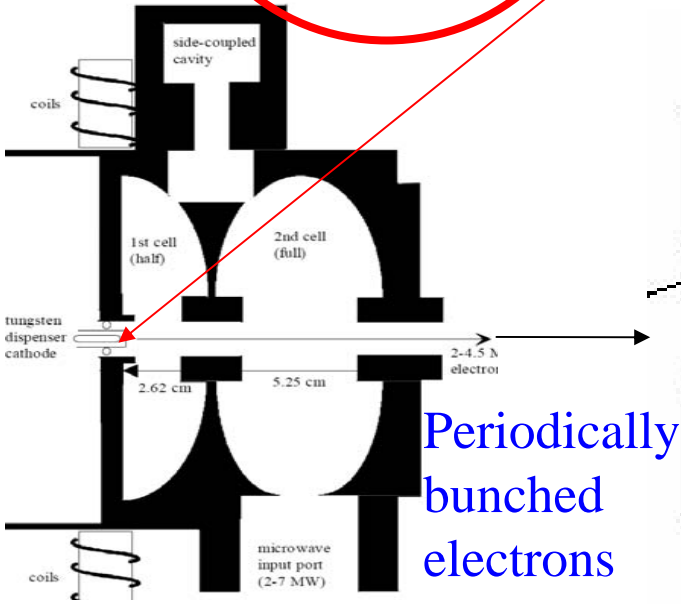




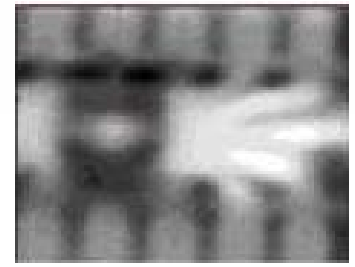
# 10-100 kW, THz Superradiance Free-electron Laser



undulator



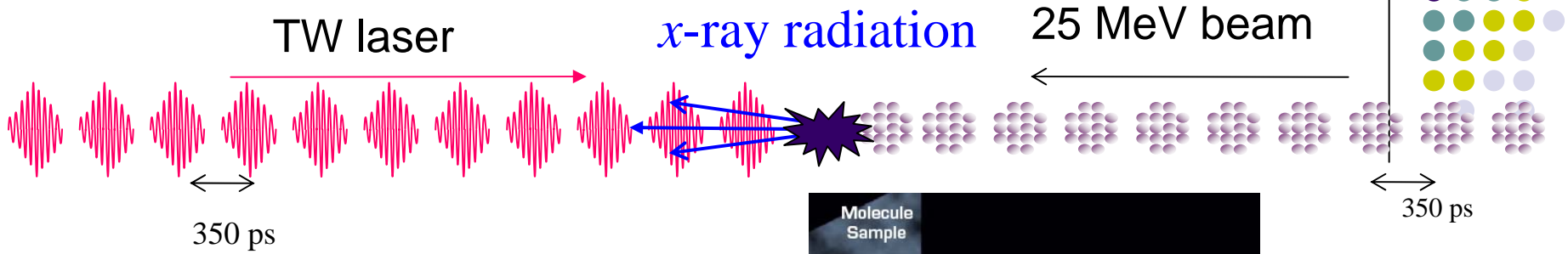
2-4 MeV RF  
Photocathode Gun



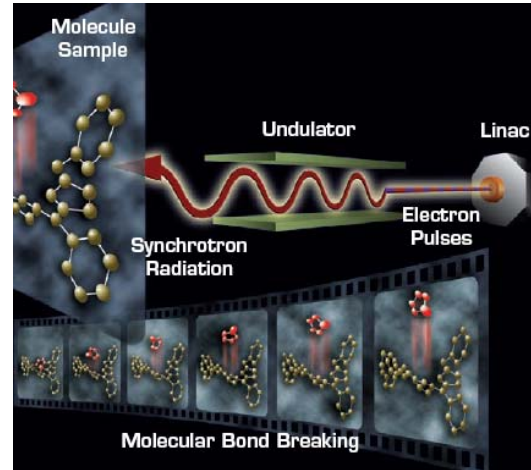
THz medical imaging (NTHU, 馬偕醫院)<sub>32</sub>



# Hard x-ray Laser Synchrotron

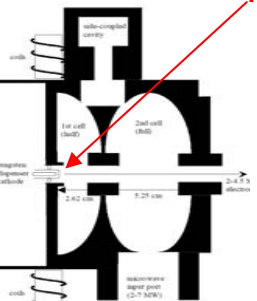


X-ray wavelength 1 Å  
 $10^{10}$  photons/collision  
 $10^{17}$  photons/sec

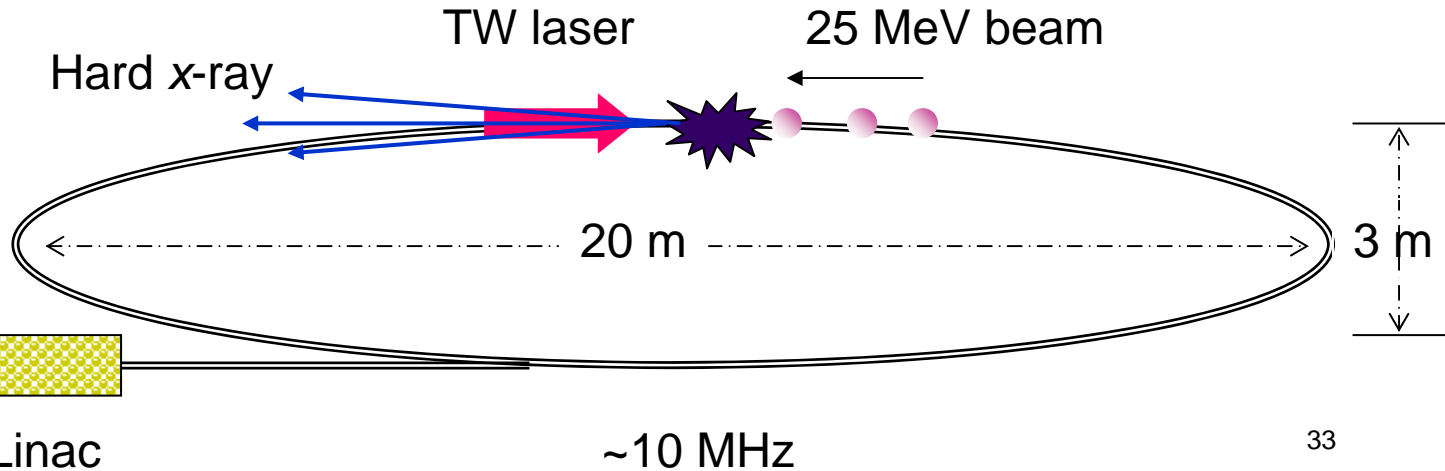


3-4 MeV  
 RF gun

laser



25 MeV Linac



# NTHU Relativistic Photonics Research Laboratory



X-ray, UV, Visible, IR, THz Sources

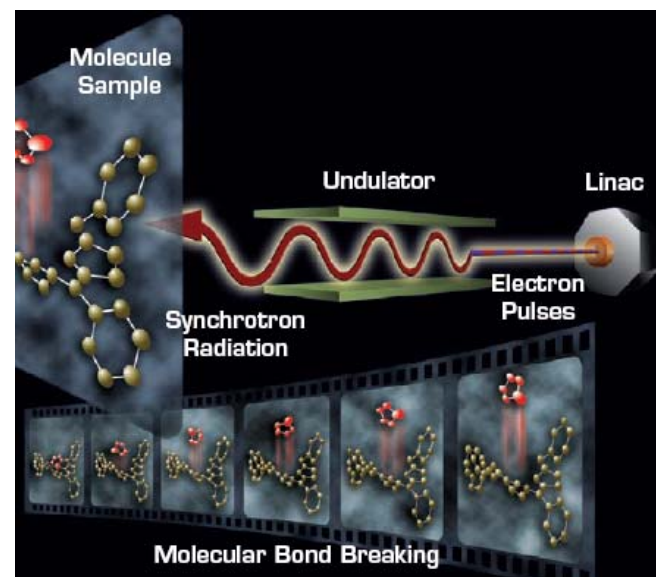
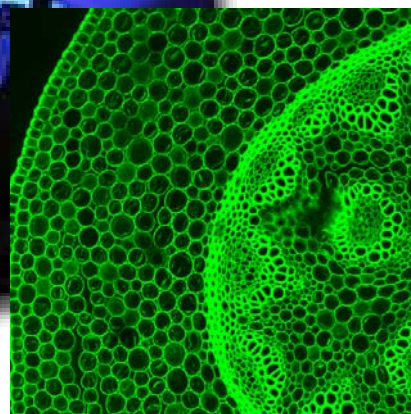
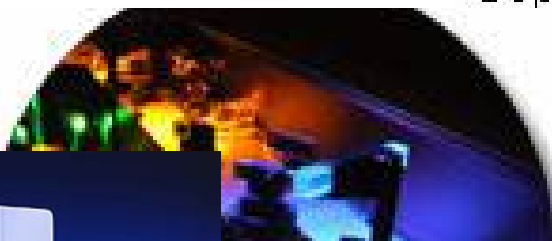
Jan. 2005



Sep. 2005

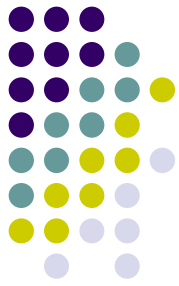


Mar. 2006



**Bio-molecular and Bio-medical Imaging Core Facility**

(Courtesy of 奈米中心 齊正中主任) 楊尚達、楊士禮、林凡異、江安世、高甫仁



*Thank you for your attention*

歡迎加入跨領域研究

雷射光電、加速器物理、相對論電子輻射、  
生物醫學影像