National Tsing Hua University, Sept 17, 2003

What is string theroy?

or

What have we learnt from String Theory in the last 30 years?

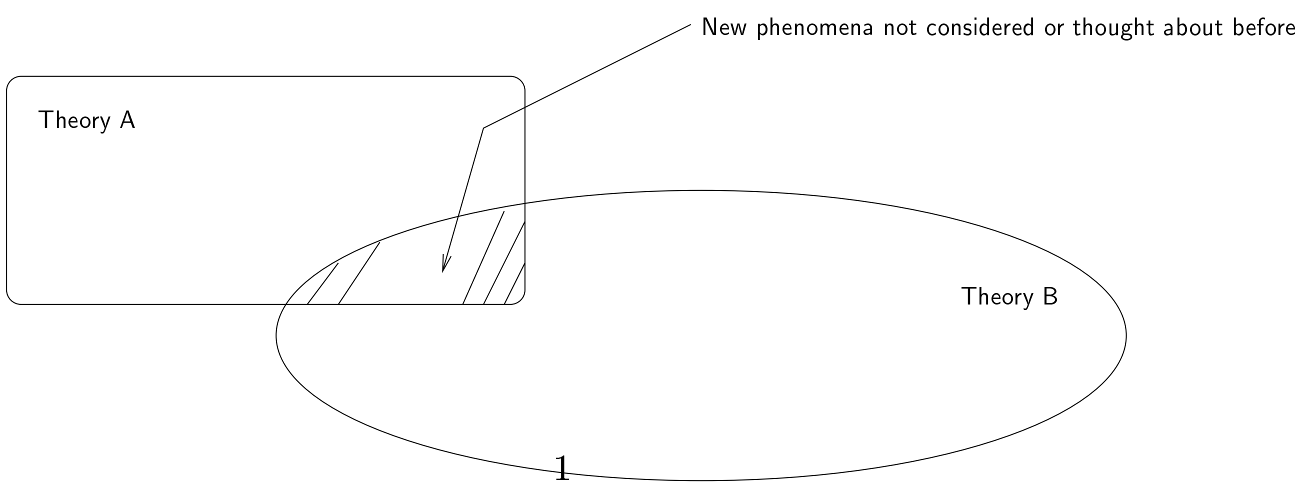
Chong-Sun Chu
National Tsing Hua University, Taiwan

Theoretical Physics and Limitation of theories

• Theoretical physicists use abstract concepts and notions to model the world. Often these concepts are not directly observable (e.g. wavefunction in Quantum Mechanics). The concept itself is confirmed through the experimental verification of the theory.

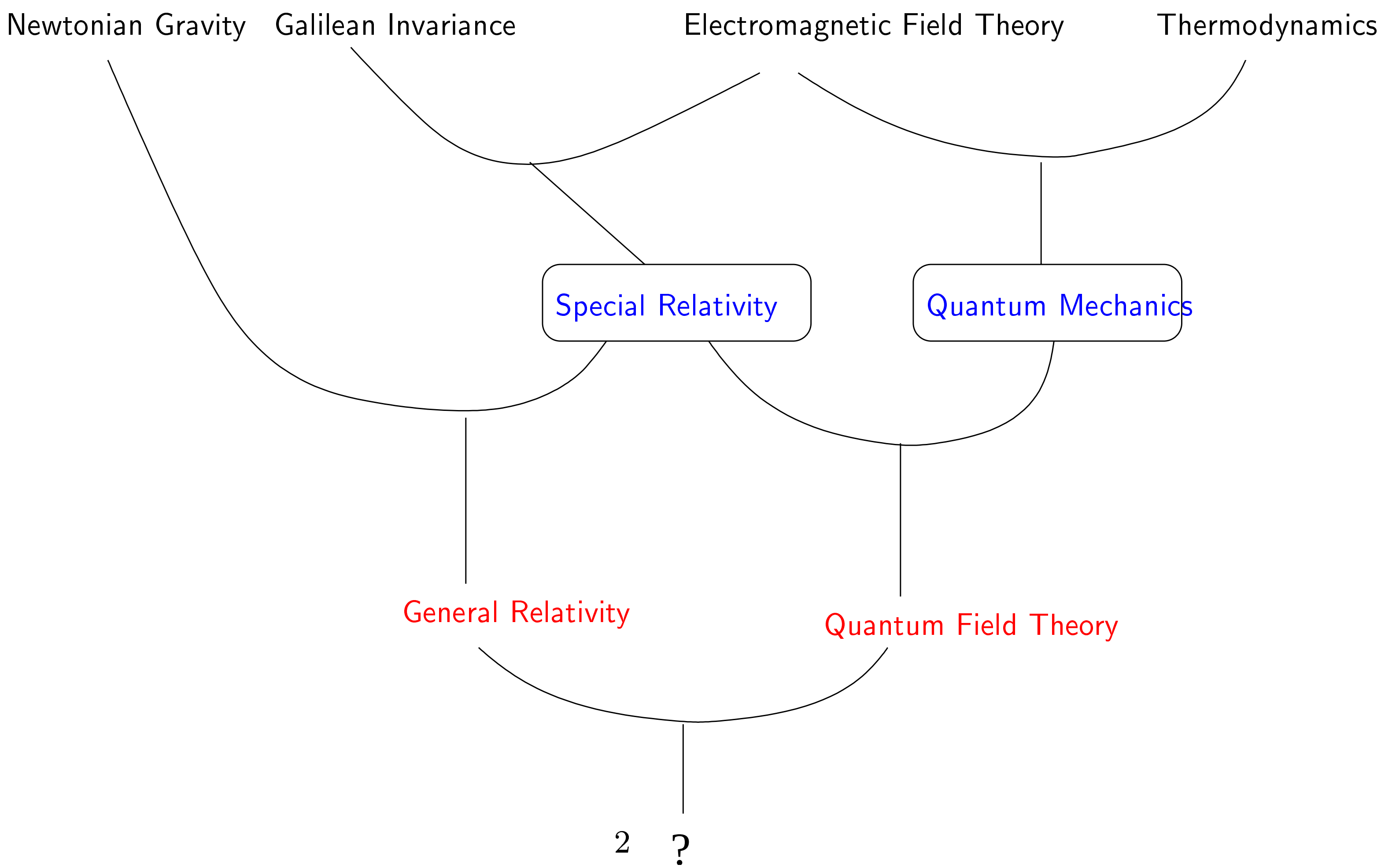
• When a concept reaches its limitation and is unable to explain new phenomena, one need to replace or extend it.

• Sometime the source of incompatibility come from experiment. Sometime it come from pure thought: if exist internal inconsistencies among different theories



Inconsistencies lead to new theories

New theories are developed to resolve contradictions from old theories. Their new predictions are confirmed by experiments.



What are the inconsistencies?

• Quantum field theory (QFT) is based on the assumptions:
special relativity + quantum mechanics + point-like particles

• Einstein gravity is governed by the Einstein equation

$$G_{\mu\nu} = T_{\mu\nu}.$$

Geometry of spacetime (LHS of the equation) = distribution of matter-energy (RHS of the equation).

• Standard model of elementary particles and Einstein theory of gravity describe the world pretty well. However, the two are actually inconsistent with each other.

• Standard model is treated using the framework of Quantum field theory. However Einstein gravity is a classical theory and cannot be quantized using the framework of QFT.

Challenges!

• We need a better framework that allows us to put standard model and gravity together in a fully quantum mechanical manner.

• Challanges:

1. to formulate an internal consistent theory of quantum gravity;
2. that incoporate standard model so that at the low enery limit, the theory leads the observed pattern of gauge interactions, spectrum of chiral fermions etc.;
3. and explain parameters of standard models, big bang etc.

String theory provides a solution to 1, and can in principle achieve 2 and 3.

String = Theory of Everything (TOE)?

String theory

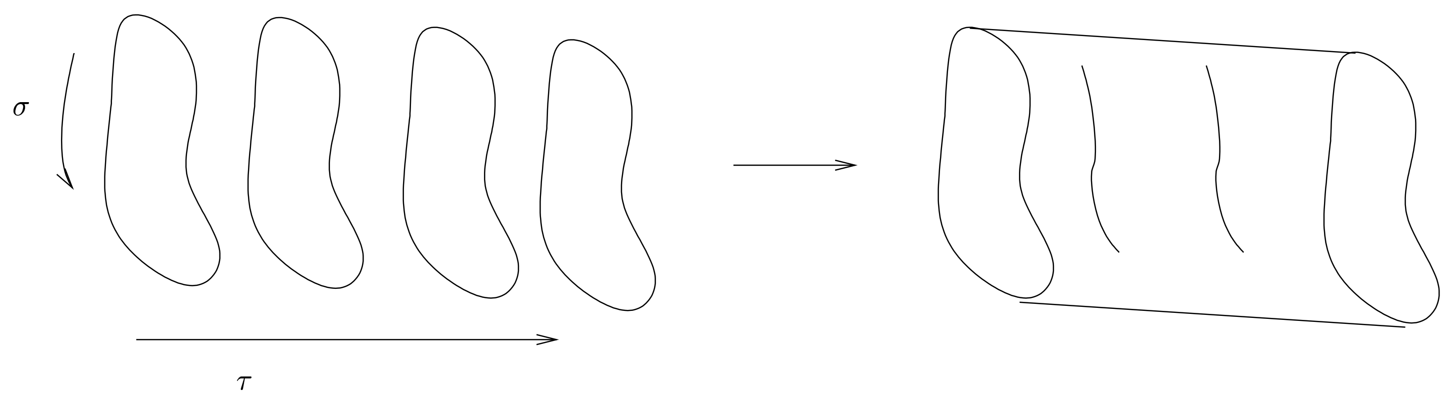
String theory is a theory still under construction and active develoment. It is more than 30 years old. In the following, I will discuss:

1. (basic ideas) what is string theory
2. what have we achieved in the past
3. what are the big issues

Basic ideas of string theory

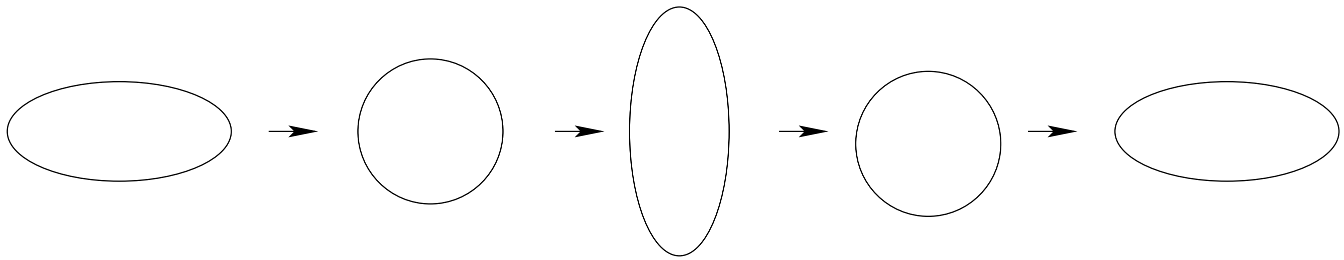
• In a nutshell, String theory = special relativity + quantum mechanics + strings

• As a string move through the spacetime, it wrap out a two dimensional surface call the worldsheet.



String moves in such a way trying to minimize the area of worldsheet.

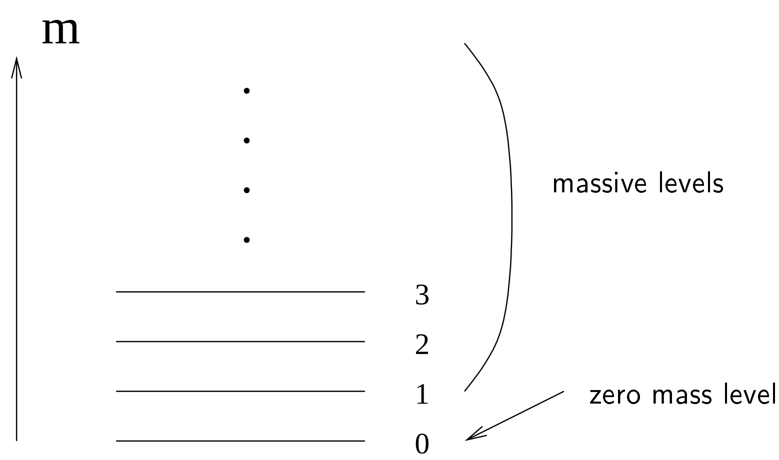
• In addition to translation, string is also capable of vibration



• Different vibrational mode of the elementary string give rises to particle excitation that has mass and spin depending on the vibrational mode.

These give rises to the elementary particles we observe in particle accelerators.

There are infinite number of them, with mass of order $m \sim l_s^{-1}$ and different spin.

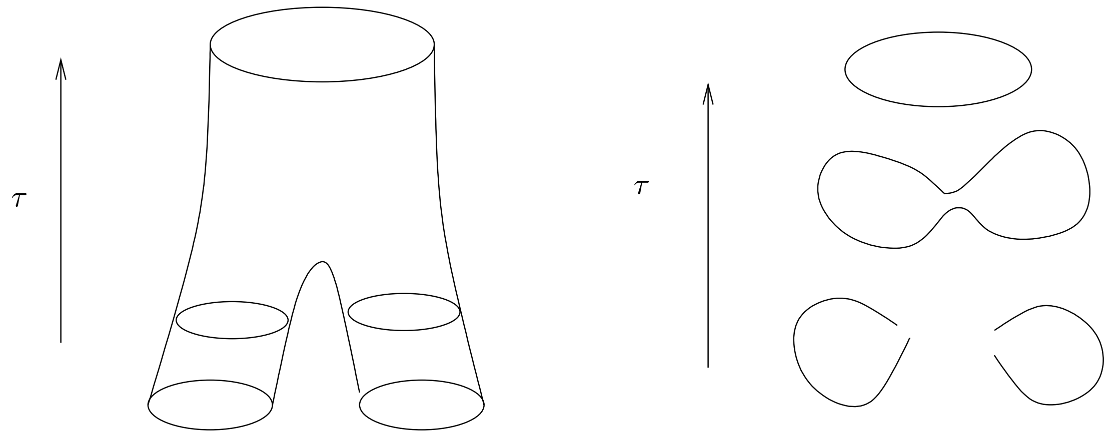


• Particularly one of the string excitations is a particle with zero mass and two units of spin. This has the same quantum number as that of a graviton, the elementary quanta of gravity.

• Is it the graviton? .... $\rightarrow$ need to know how these particles (or strings) interact.

How do strings interact?

• Two strings can split and join to form a new string



• The string interaction is a cubic interaction

$$L_{\text{int}} = g_s \Phi^3,$$

where g_s is the string coupling constant. This interaction fixes unambiguously the interaction among the string modes. In particular, one find that, at the classical level, the interaction for the massless spin two excitations is precisely that of the Einstein gravity!!

.

So far we have discussed some basic aspects of the simplest kind of string theory: bosonic string theory.

This is what people knew up until around early 70's.

But ... there are a number of problems with this string theory

———————————————

1. it contains only bosons. Where are the fermions??
2. in order to be consistent, it requires a 26 dimensional spacetime

.

First string revolution (1984)

Crucial concepts introduced:

- supersymmetry and superstring
- compactification

Superstring (1981-1983)

• In order to include fermions in string theory, there must be a special kind of symmetry called **supersymmetry**

• Supersymmetry is a symmetry relating bosons and fermions.

$$Q|b\rangle = |f\rangle, \quad Q|f\rangle = |b\rangle.$$

• So far we do not have any direct evidence of supersymmetry. Supersymmetric partners to known particles have not been observed in particle accelerators.

However we may be at the verge of finding it!

• Supersymmetry in string theory transform force mediating bosonic particles with matter field fermions. So supersymmetry relates the particles that transmit forces to the particles that make up matter. Unification!.

• With supersymmetry built in, there is only five consistent superstring theories:

I, IIA, IIB, HO, HE.

Each of these superstring theory respects different supersymmetries and gauge symmetries.

Why so many? which one is the one explaining our world? TOE(s)?

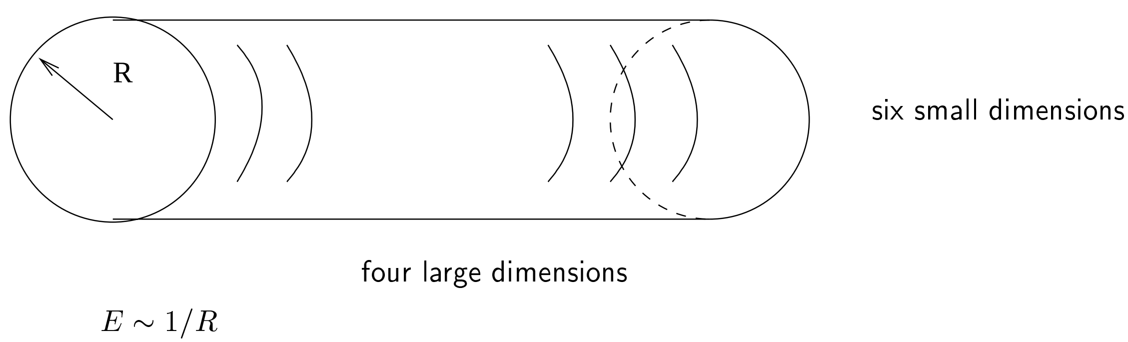
Compactification (1984)

• Superstring theory provides a possibility to unify all forces and particles. Both the force mediating particles and the matter particles are part of the string oscillation spectrum.

• Superstring live in a 10 dimensional spacetime instead of 26.

• The observable world is four dimensional. The extra six dimensions must curl up into a tiny geometrical space. This is called string **compactification**. (compact = small)

• To observe the small dimensions, one needs at least energy of order $E \sim 1/R$. This is a lot of energy if R is small. Thus in the low energy regime, only the large $3 + 1$ dimensional spacetime is observed.



• There are many possible ways to make six dimensions much much smaller than the other four in string theory. The precise nature of the four dimensional physics (e.g. chiral fermion spectrum, gauge group) depends on how these extra dimensions curl up.

• For example the HE superstring theory compactified on a particular kind of six-dimensional space – Calabi-Yau space gives physics like the Standard Model of particle physics at low energies.

• However there are millions of Calabi-Yau space and why is a particular one choosen?

This is the problem of string vaccum ... still poorly understood.

.

Old problems solved. New problems arised....

Why so many string theories?

Why so many string vacua?

Part of these were answered in the second string revolution.

.

Second string revolution (1995-)

key achievements:

- string duality
- nonperturbative aspect of string theory: D-brane
- M-theory

String duality

• It was believed that there were five distinct superstring theories:

I, IIA, IIB, HO, HE.

But only one of them is the actual correct Theory of Everything. The other four would be mathematically consistent, but not used by nature. (similar to gauge theory)

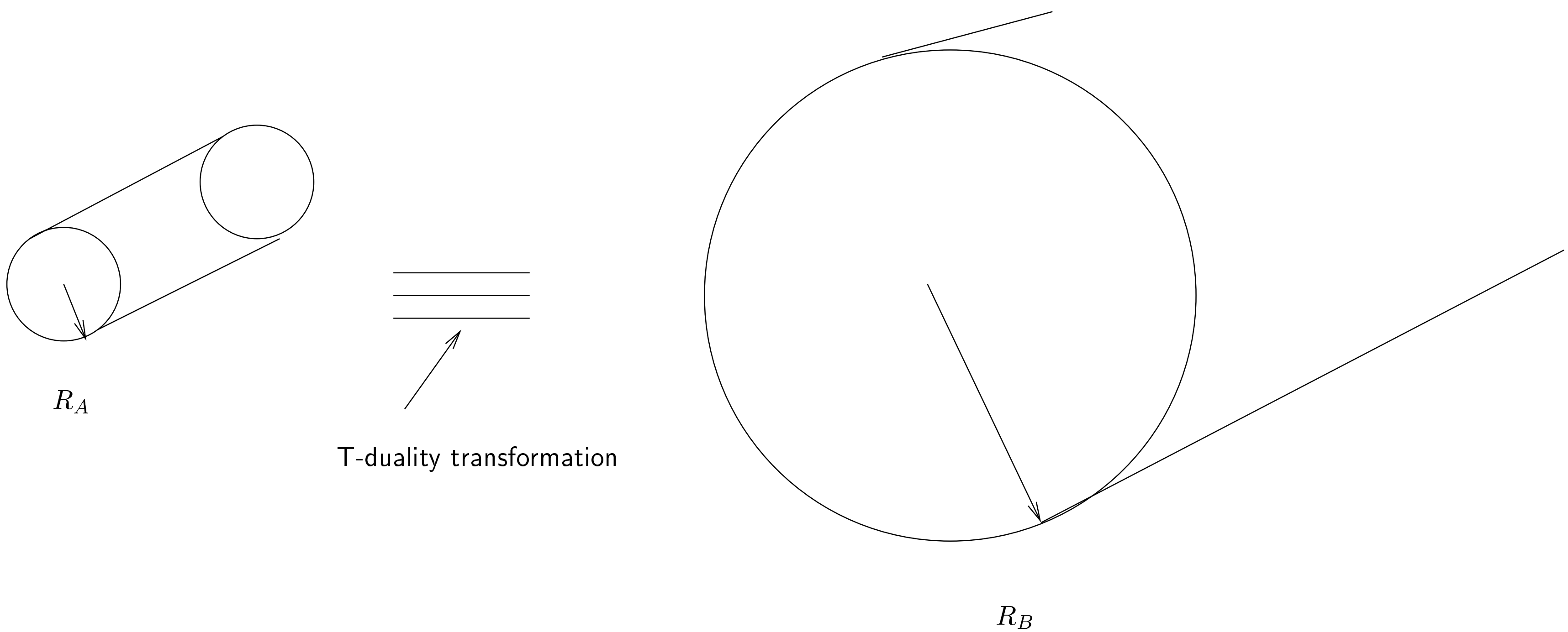
• Now it is known that this naive picture is wrong, The five superstring theories are connected to one another by transformations called duality transformations.

• There are two main types of duality transformations: **S, T**-duality.

T duality

• Sometime string theory A with large compact dimensions is equivalent to another string theory B with a small compact dimensions. Typically

$$R_A = \frac{l_s^2}{R_B}.$$



• “T” stands for target space. It is a duality between target space geometry of string theory.

• String probes spacetime geometry in a very different way from particle!

• Because of this noval property of string dynamics, string theory implies a generalized uncertainity principle

$$\Delta X \geq \frac{1}{\Delta P} + l_s^2 \, \Delta P$$

S duality

(...... to be skipped)

• Sometime string theory A at strong coupling is equivalent to a string theory B at weak coupling.

• "S" stands for strong coupling. It is a duality that applies only when coupling is very strong (or very weak).

• Under S-duality, perturbative effects in string theory A are mapped to non-perturbative effects in string theory B.

• In particular the elementary string of one theory is mapped to soltionic string in the dual theory.

• So if two theories are related by S-duality, then we just need to understand the weakly coupled theory, since anyting we want to learn about for the dual theory can be answered in terms of the weakly coupled theory.

Eleven dimensions and M-theory

• One of the most intriguing discovery during the second string revolution is Witten discovery (1995) that string actually lives in eleven dimensions!

• This extra eleventh dimension is not visible to weakly coupled string theory. But is seen when the coupling is very strong.

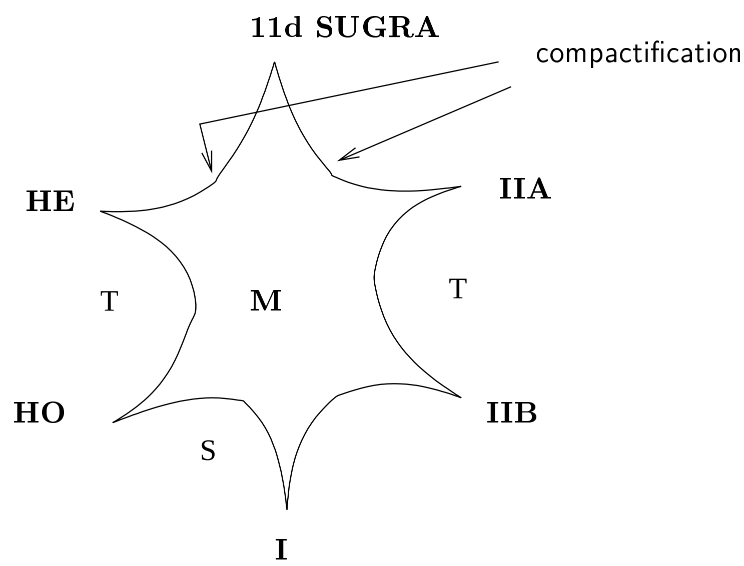
• It turns out that the strongly coupled string theory is better described in terms of an eleven dimensional theory called M-theory (Mother, "W"itten).

• The full quantum mechanical formulation of M-theory is still not known. Few things we are sure are:

1. M-theory has a low energy limit which is the eleven dimensional supergravity theory (supersymmetrization of Einstein gravity).

2. M-theory has membrane excitations.

• Moreover, the previously known five superstring theories can all be considered as different perturbative limits of M-theory.

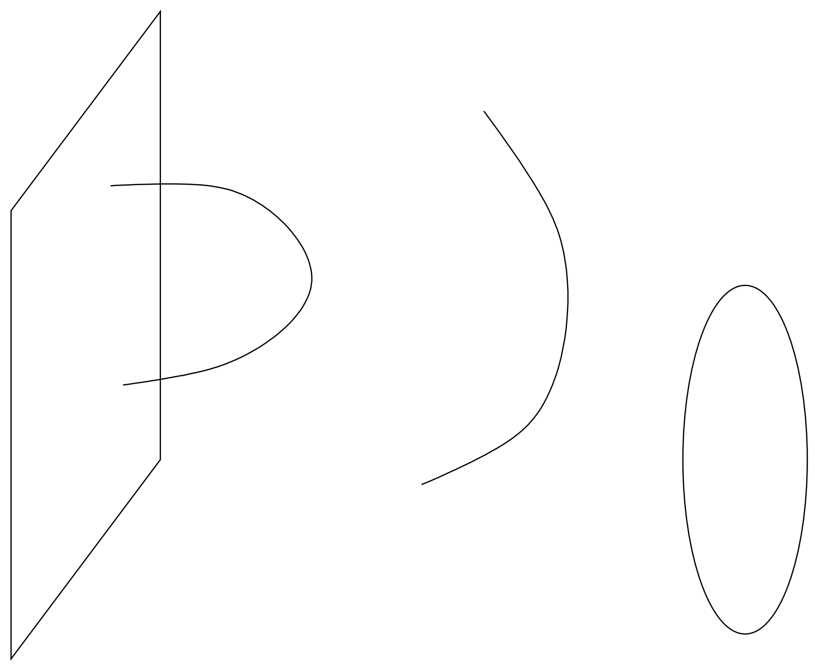


• D-branes play important and essential role in this web of duality.

So, what is a D-brane?

D-branes (1995)

• Open string has endpoints and can end on a hyper-plane.



• Since the open string is dynamical, the vibrational modes of the open string will give rises to nontrivial dynamics of the hyper-plane, making it dynamical. This dynamical hyper-plane is called a D-brane.

• If the hyper-plane is of dimensions p, then the dynamical D-brane is a $(p+1)$-dimensional spacetime object. It is called a Dp-brane.

• D0-brane = D-particle, D2-brane= D-membrane, etc.

• D-brane has tension (mass per unit volume)

$$T = \frac{1}{g_s {l_s}^{p+1}}$$

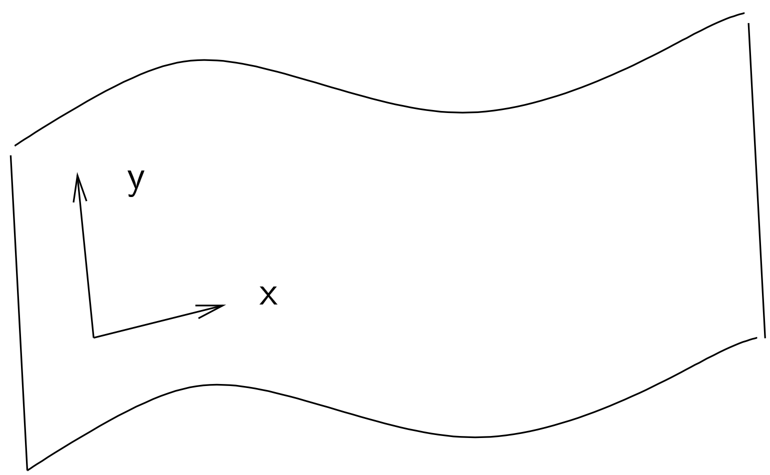
and is thus nonperturbative.

Non-commutative geometry

• The discovery of D-brane opens up a new channel to study new geometrical effects in string theory.

→ Quantum Geometry

One of the surprise is the discovery of noncommutative geometry on the D-brane worldvolume at short distance.



$$[x, y] = i\theta \neq 0$$

A noncommutative D-brane

More questions

- Why is spacetime 3+1 dimensional?
- Why standard model?
- How to choose the string vaccum?
- How is M-theory defined?
- What are the fundamental symmetries that underpin string theory?
- Can we explain big bang?
- Relation to inflation? dark matter? (string cosmology)
- ...