Secret
Symmetry

Hidden
Symmetry

Spontaneous
Symmetry
Breakdown

Thematic Melodies of 20th Century Theoretical Physics:

Quantization

Symmetry

Phase Factor

20世紀理論物理的三个主旋律：

量子化

對稱

相位因子

C N Yang 2002

楊振寧

"The quantum numbers that designates the states of a system are often identical with those that represent the symmetries of the system."

Ex. 1) The general structure of the periodic table is essentially a direction consequence of the isotropy of Coulomb's law.

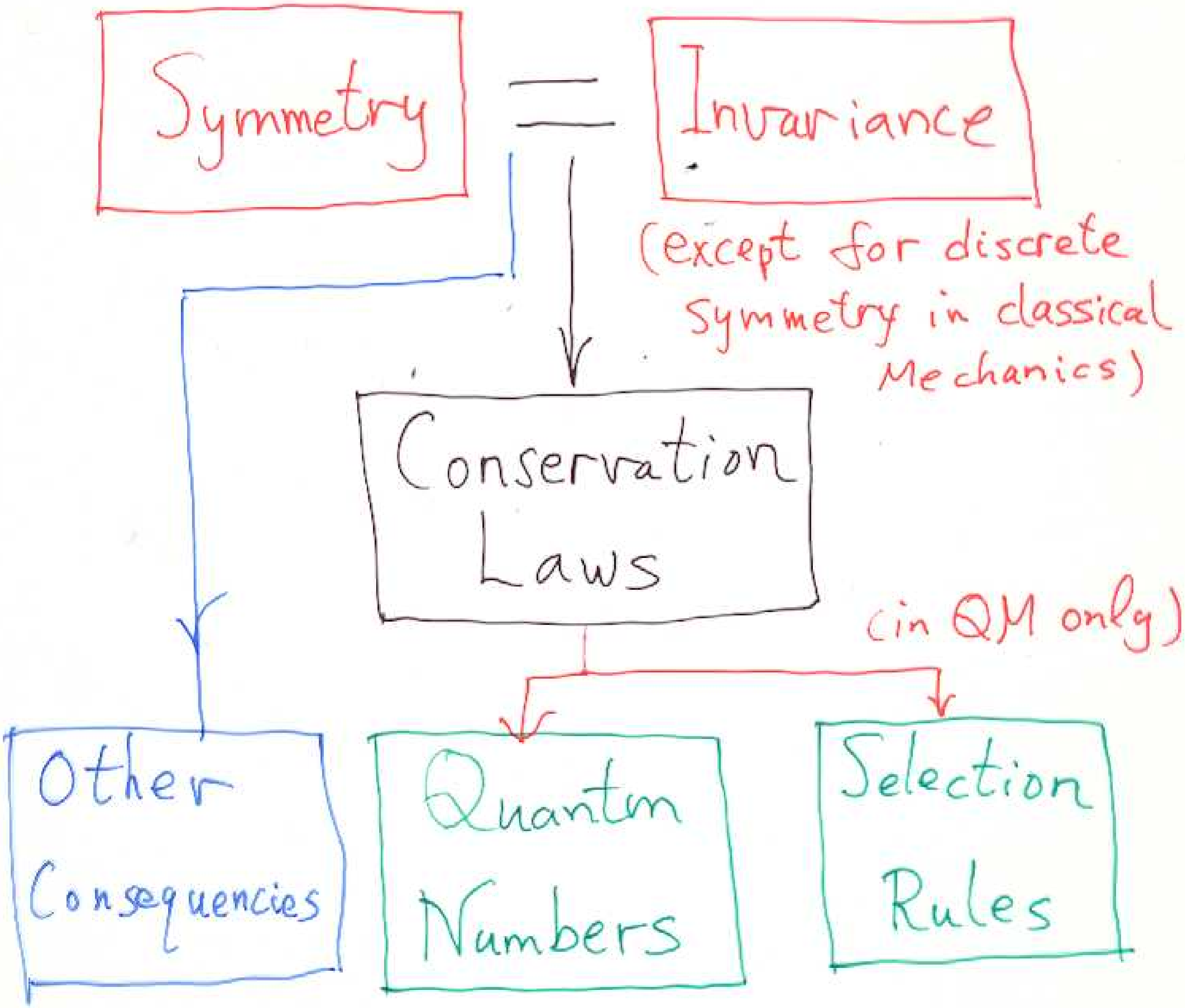
2) The existence of the anti-particles is a consequence of the Lorentz invariance.

R. P. Feynman (1942)

$$\langle f | i \rangle = \int d(\text{path})\, e^{\frac{i}{\hbar}(\text{action})}$$

$$\text{action} = S = \int dt\, L$$

$$e^{\frac{ie}{\hbar}\int A_\mu dx^\mu}$$



C. N. Yang

1957
Nobel Prize Lecture

Symmety dictates interaction

Ex. abelian gauge symmetry

→ electromagnetic interaction

non-abelian gauge symmetry

→ strong interaction

coordinate transformation invariance

→ gravity

general relativity

$$\mathcal{L}_{\text{Maxwell}} \propto F_{\mu\nu} F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

A_μ : photon → massless

$$\mathcal{L}_{\text{Yang-Mills}} \propto \operatorname{tr} \mathbb{F}_{\mu\nu} \mathbb{F}^{\mu\nu}$$

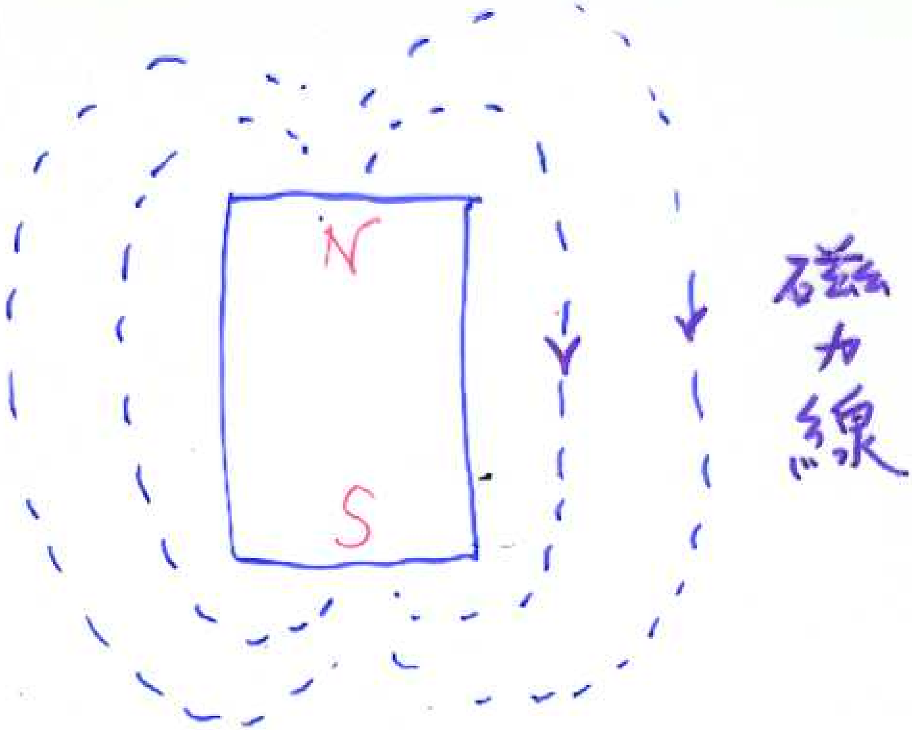
$$\mathbb{F}_{\mu\nu} = \partial_\mu \mathbb{B}_\nu - \partial_\nu \mathbb{B}_\mu + ig [\mathbb{B}_\mu, \mathbb{B}_\nu]$$

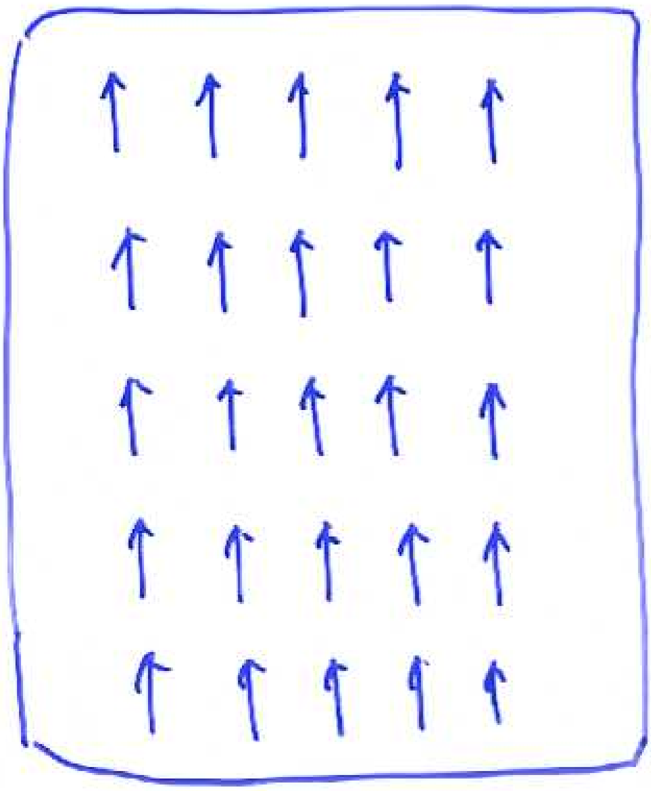
B is massless ???

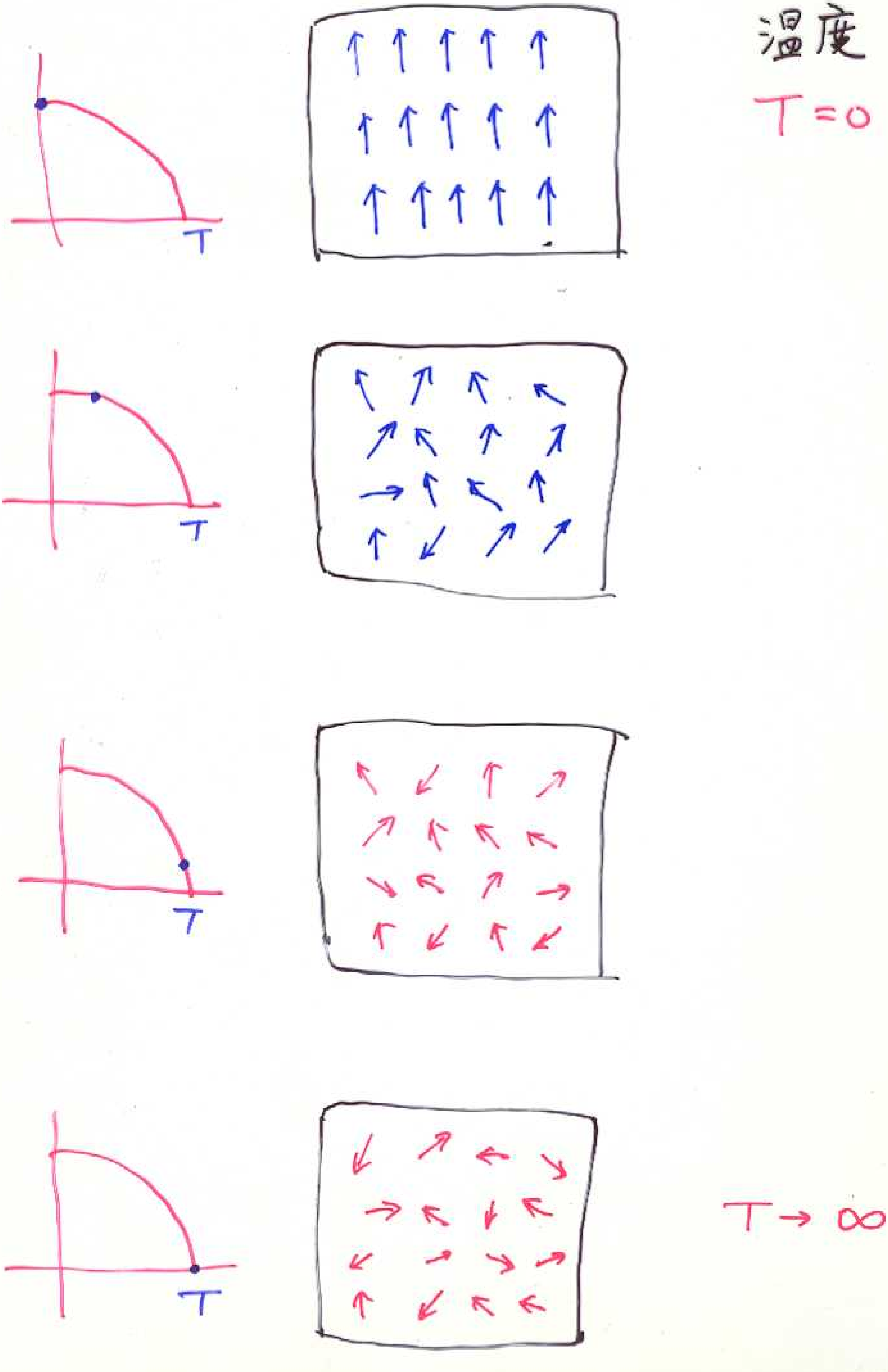
Pauli's question

How is the symmetry realized ?

磁鉄






温度
T=0
T
T
T
T
T→∞

對於鐵磁性來說,

在 T_c 之下,

旋轉對稱 不見了

自动對稱破缺（壞）

Spontaneous symmetry breakdown

對於超導現象而言,

在 T_c 之下,

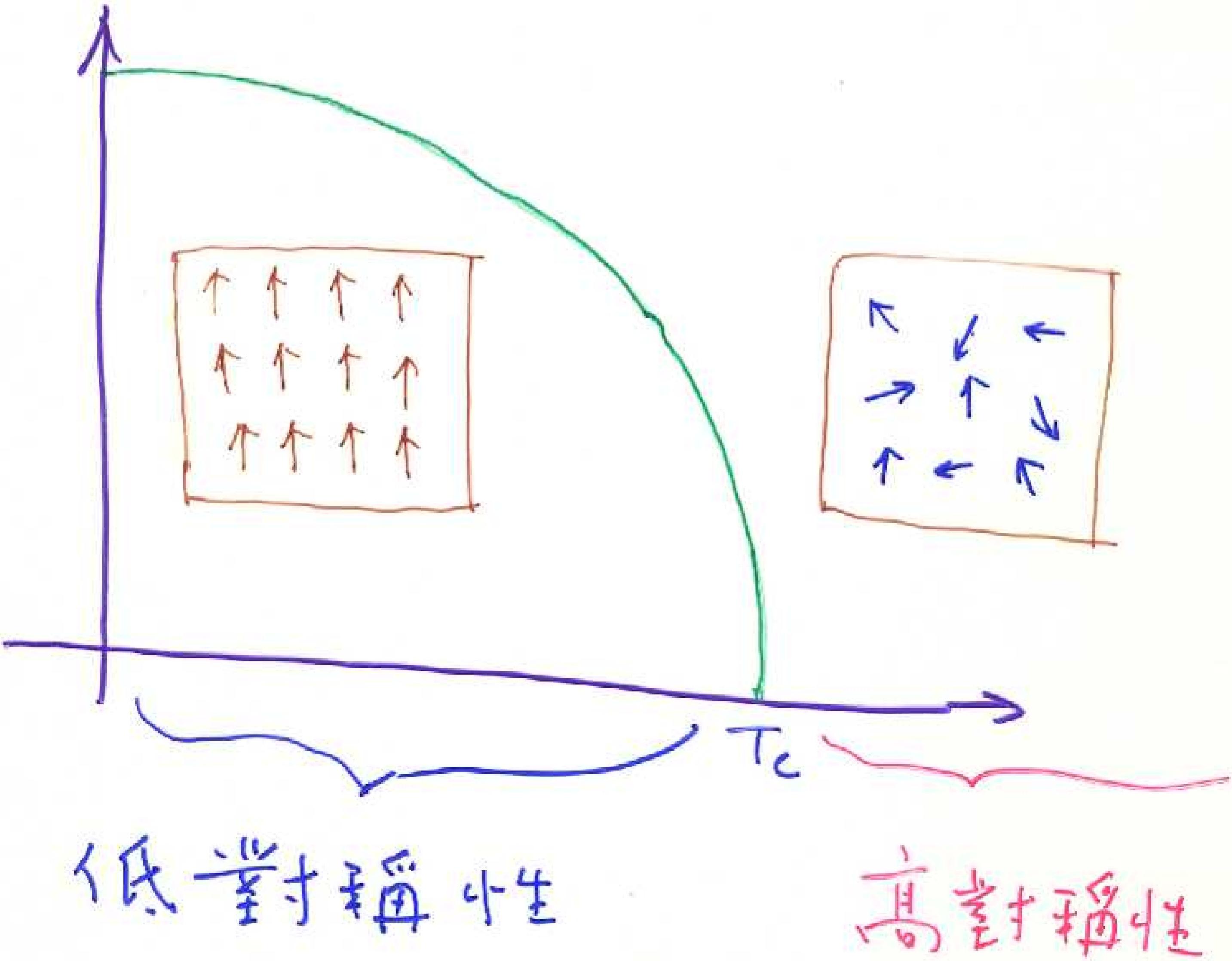
「規範對稱」不見了.

Gauge Symmetry

相當抽象的對稱 → 花了数十年工夫才了解

對稱性

Symmetry



Spontaneous Symmetry Breakdown

$$U|0\rangle \neq |0\rangle$$

$$U = e^{i\alpha Q} \qquad (1 + i\alpha Q + \cdots)|0\rangle \neq |0\rangle$$

$$Q|0\rangle \neq 0$$

$$U \phi_A U^+ = \phi_B$$

$$\langle 0|\phi_B|0\rangle \neq 0$$

$$\langle 0|[Q, \phi_A]|0\rangle \neq 0$$

$$\hookrightarrow \quad Q|0\rangle \neq 0$$

$$\langle 0 | S_z | 0 \rangle \neq 0$$

in Ferromagnetism

rotational invariance

$$\langle 0 | \Psi | 0 \rangle \neq 0$$

in G-L theory

$$\Psi \rightarrow e^{i\alpha} \Psi$$

gauge transformation

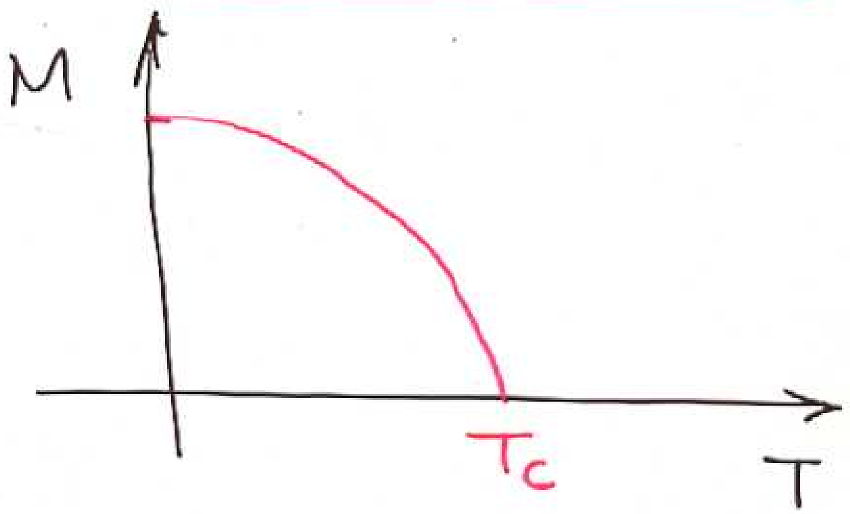
gauge invariance

Lev Landau

(1908 – 1968)

Phase Transition

Order Parameter 秩序参数



$$\langle S_z \rangle = M(T) \neq 0 \quad T<T_c$$

$$M(T) \underset{T \lesssim T_c}{\sim} |T - T_c|^{1/2}$$

$1/2$: critical exponent

2003 Nobel Prize

Ginsburg - Landau Theory

$$F = \alpha |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{1}{2m} \left| \left(\frac{\hbar}{i} \vec{\nabla} - \frac{2e\vec{A}}{c} \right) \psi \right|^2 + F_{EM}$$

$\beta > 0$

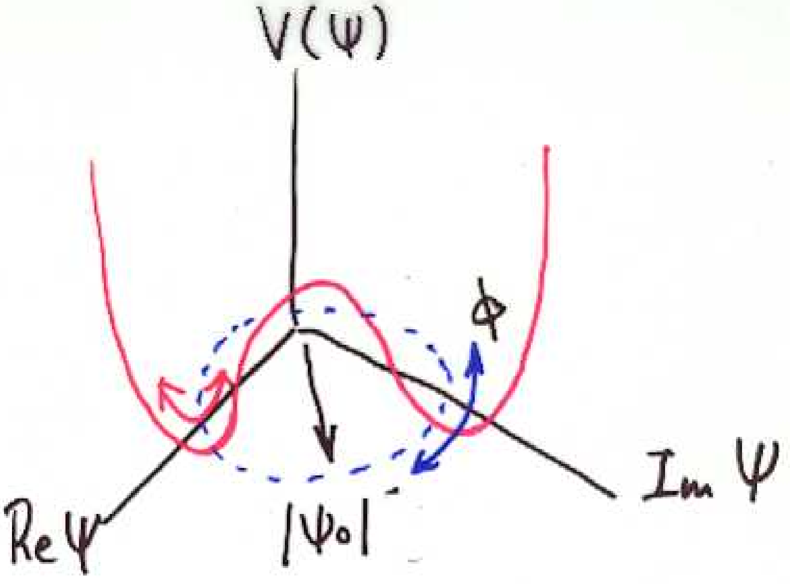
$\alpha < 0$ for $T < T_c$

$\Psi = \Psi_0 \qquad |\Psi_0|^2 = -\frac{\alpha}{\beta} > 0$

$$\vec{j} = i \frac{e\hbar}{m} \left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^* \right) - \frac{4e^2}{mc} \psi^* \psi \vec{A}$$

$$\vec{j} \propto |\psi_0|^2 \vec{A}$$

↳ Meissner effect if $|\psi_0|^2 \neq 0$.



$$\Psi \sim (|\Psi_0| + \delta\rho)\, e^{i\phi}$$

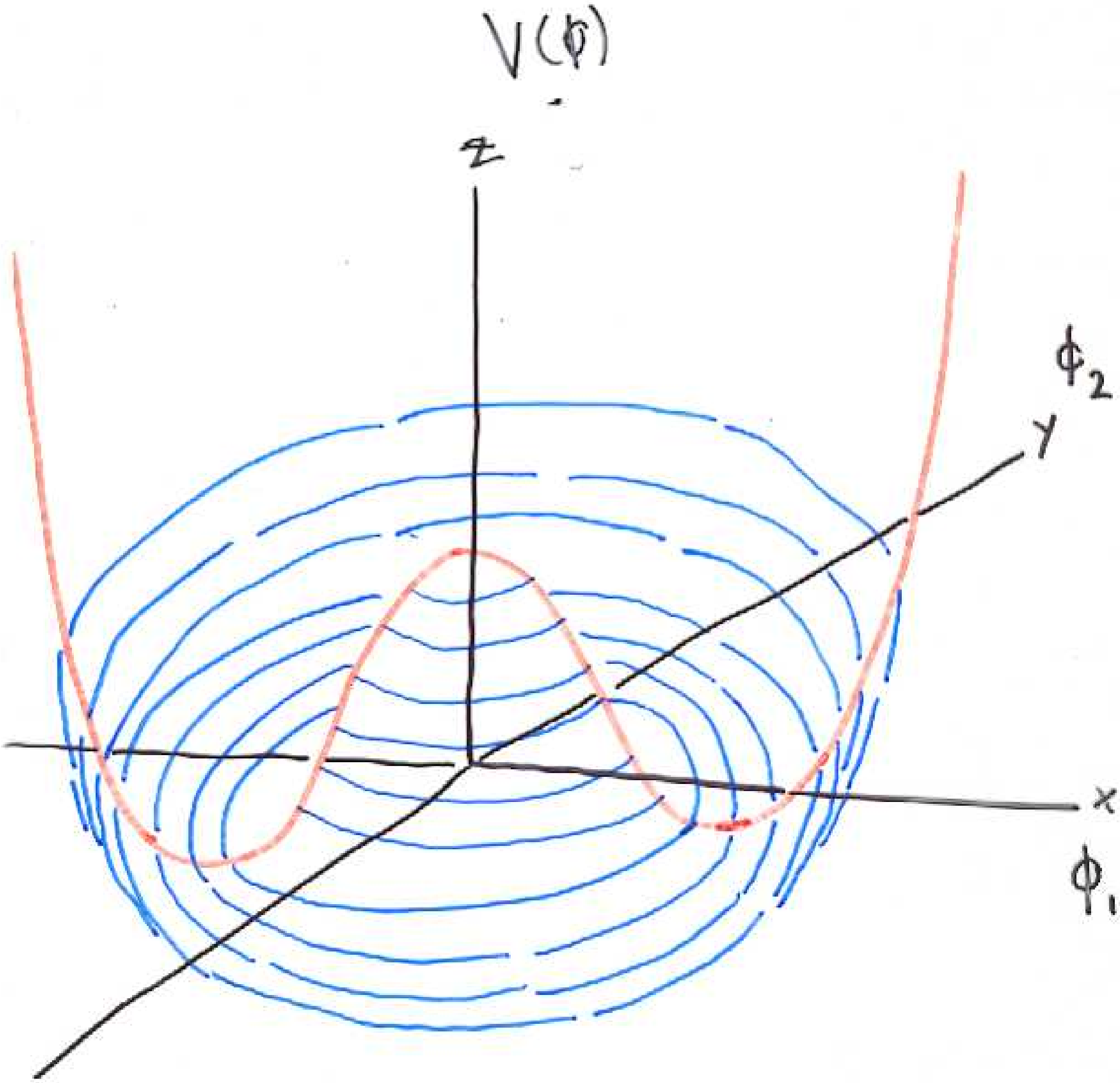
gapless Goldstone mode

Bardeen - Cooper - Schrieffer Theory

BCS - - - - - → GL

microscopic - - - - → effective theory (phenomenological)

$\langle \psi^\dagger \psi^\dagger \rangle \neq 0$ $\langle \Psi_{GL} \rangle \neq 0$



$$\phi = \phi_1 + i\phi_2 = \rho e^{i\theta}$$

$$V(\phi) = V(\phi_1 + i\phi_2)$$

$\vec{J} = \frac{\hbar}{m} \vec{\nabla} \theta$

in superfluid

∴ θ is angle variable

→ flux quantization

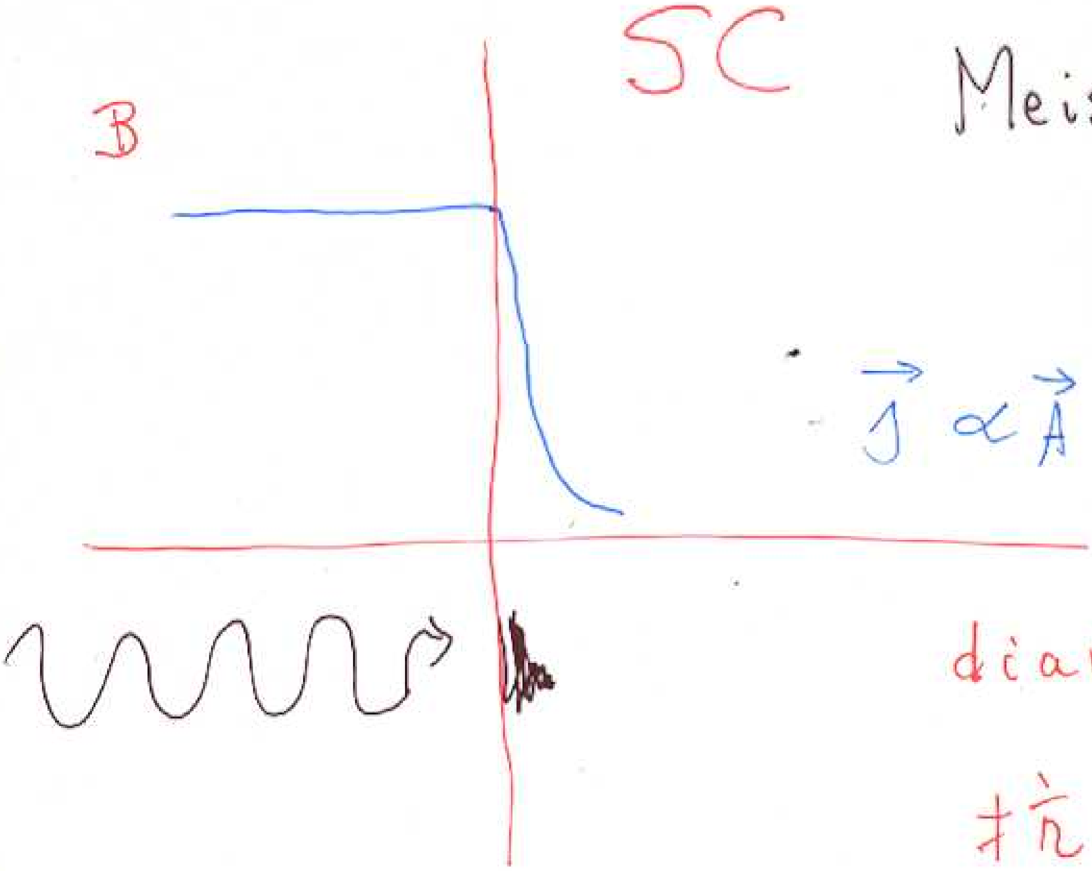
$$\langle 0 | \Psi_e^{\dagger} \Psi_e^{\dagger} | 0 \rangle \neq 0$$

$|0\rangle$ does not have definite electron number

— Y. Nambu

$|\text{Ground State}\rangle \sim |\text{"Vacuum"}\rangle$

is strange
complicated
interesting



Meissner effect

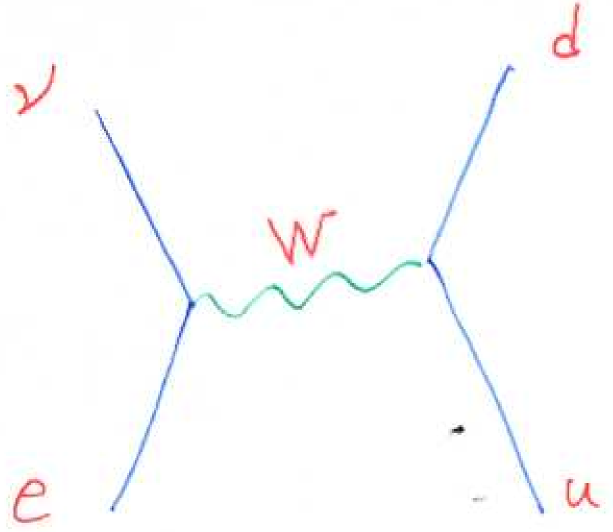
$\vec{J} \propto \vec{A}$

diamagnetism

抗磁性

宇宙是一抽象
的超導体

促成了基本粒子理論
標準模型的成立



W is massive

⟷ Meissner effect

$SU(2) \times U(1)$ is hidden only $U(1)_{EM}$ is explicit

Superconductivity: $U(1)_{gauge}$ symmetry is hidden

Weinberg - Salam proposed

in 1967/8 that W^+ W^- Z particles

are massive

because of the Higgs mechanism.

$$SU(2) \times U(1)$$

$$\downarrow$$

$$U_{EM}(1)$$

Unification of EM and weak interactions.

Spontaneous Symmetry Breakdown

Lagrangian is invariant

but ground state is not.

(degenerate ground states)

→ There exist Goldstone particles, if there is no long range force

Inspired by BCS, Nambu proposed that the chiral symmetry is broken spontaneously, and the pions are the GB

Higgs Mechanism

GL - Anderson - Schwinger - Higgs - Kibble - Hagen - Guralnik - Brout - Englert - ......

$$\mathcal{L} = |(\partial_\mu + ieA_\mu)\phi|^2 + \mu^2|\phi|^2 - \frac{\lambda}{2}(|\phi|^2)^2 + \mathcal{L}_{EM}$$

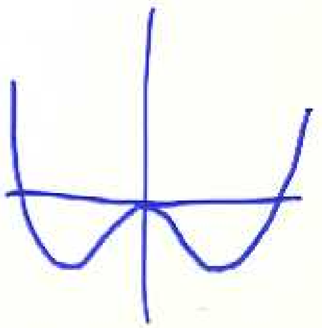
$$\langle \phi \rangle = \phi_0$$

$$|\phi_0|^2 = \frac{\mu^2}{\lambda} \qquad \mu^2 > 0, \ \lambda > 0$$

$$V(\phi) = -\mu^2 \phi^* \phi + \frac{\lambda}{2}(\phi^*\phi)^2$$

↳ Meissner effect

↳ photon is massive



ϕ

$$\boxed{m_{photon}^2 \propto e^2|\phi_0|^2}$$

Actually, these two subjects have a lot in common!

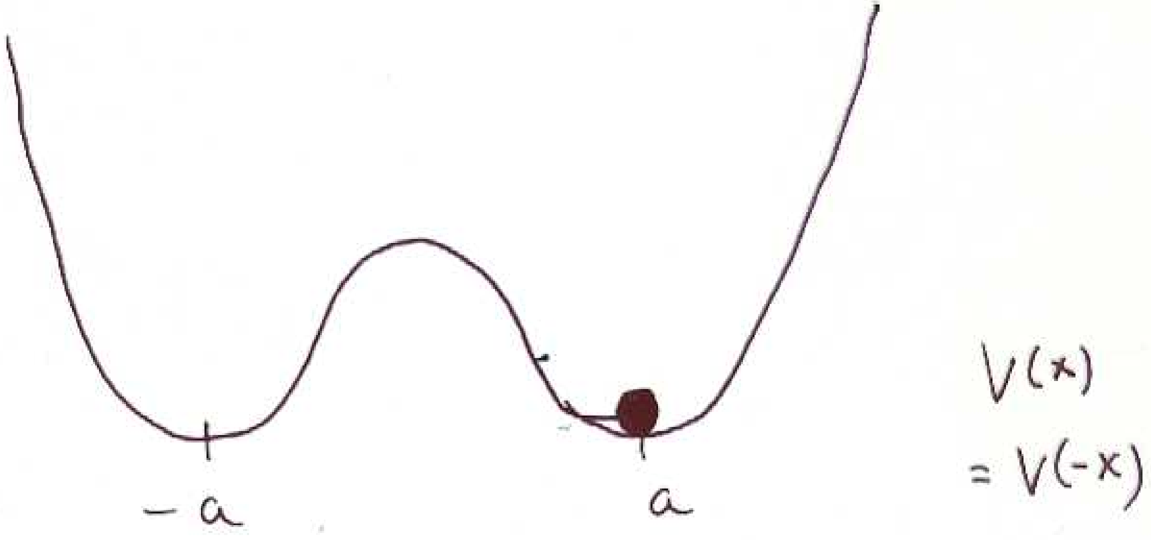
Particle Physics		Condensed Matter Physics
Quantum Field Theory	$\approx$	Quantum Many Body Theory Statistical Mechanics

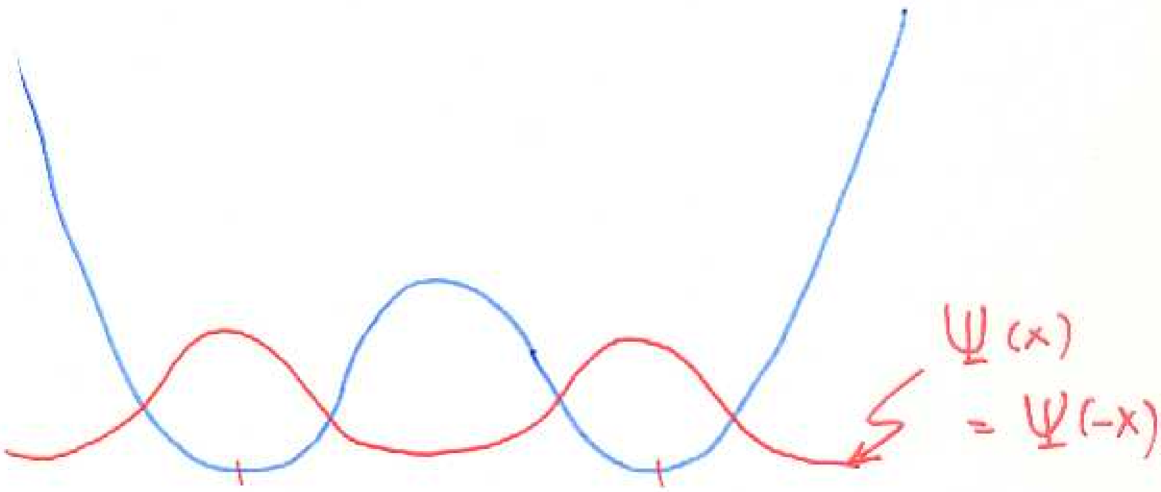
"∞" # deg of freedom

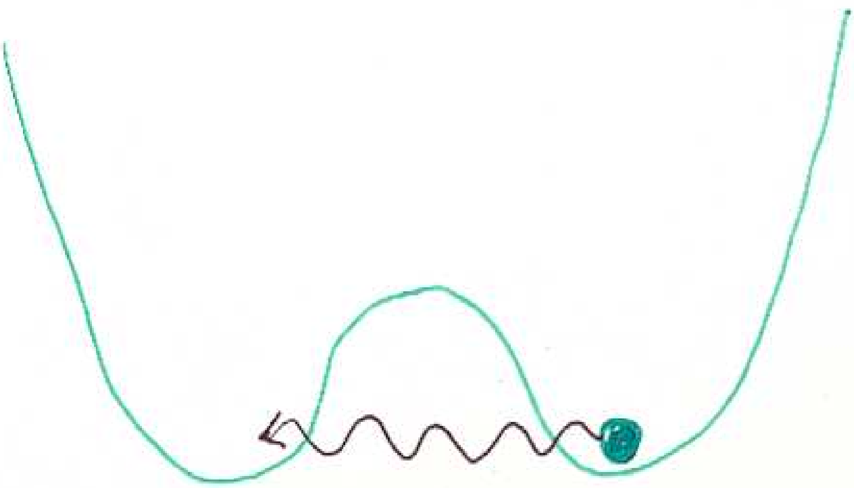
<----- renormalization --->

<--- Symmetry --->

<--- Spontaneous Symmetry breakdown ---







quantum tunneling

Supersymmetry

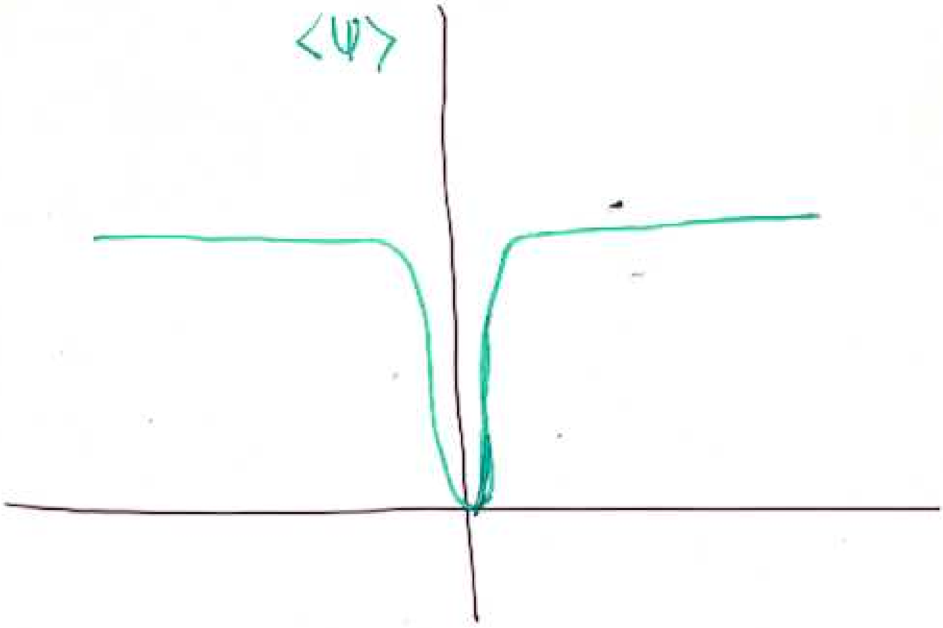
Fermion $\leftrightarrow$ Boson

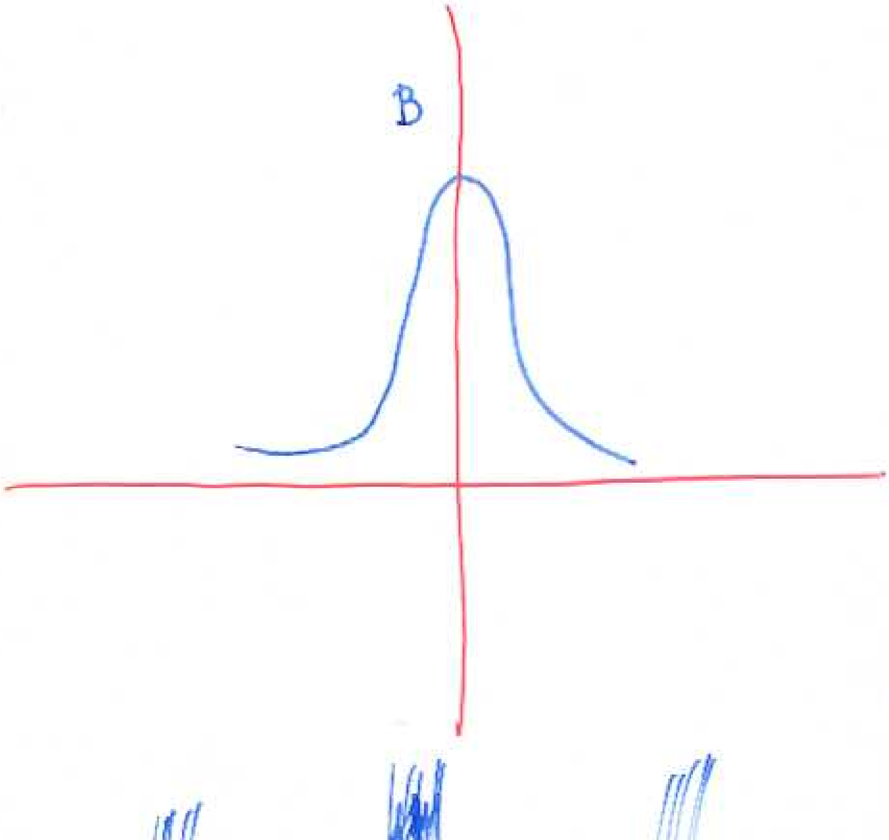
Hidden ?

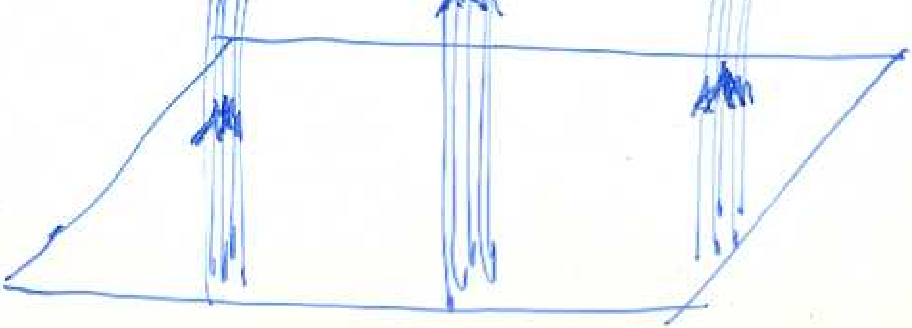
How ?

Vortex

2003 Nobel Prize







Solitons

But ferromagnetism is unique.

$$S_z|0\rangle = \sum_i S_{iz}|0\rangle = M|0\rangle$$

$|0\rangle$ is an eigenstate of S_z

In BCS theory

$|0\rangle$ is not an eigenstate of $\psi^\dagger\psi^\dagger$