Deformation of High Energy Symmetry in String Theory

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arXiv:0905.2322 [hep-th],JHEP 0911:081,2009.

Based on a series of collaboration with

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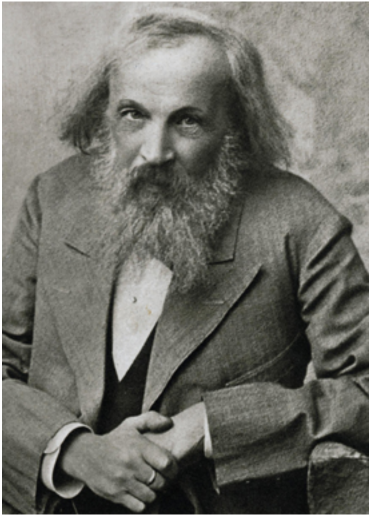
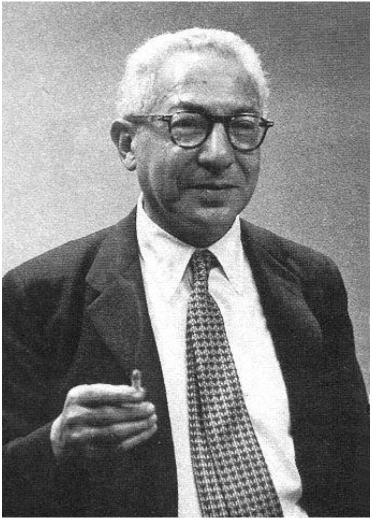
Lee,	Jen-Chi	(NCTU)
Teraguchi,	Shunsuke	(Nagoya)
Yang,	Yi	(NTU)
Yeh,	Chi-Hsien	(NTU)

Outline of This Talk

Introduction





I. Introduction

Introduction

WebElements: the periodic table on the world-wide web

http://www.webelements.com/

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
hydrogen 1 **H** 1.00794(7)																	helium 2 **He** 4.002602(2)
lithium 3 **Li** 6.941(2)	beryllium 4 **Be** 9.012182(3)		Key: element name atomic number **symbol** 2003 atomic weight (mean relative mass)									boron 5 **B** 10.811(7)	carbon 6 **C** 12.0107(8)	nitrogen 7 **N** 14.0067(7)	oxygen 8 **O** 15.9994(3)	fluorine 9 **F** 18.9984032(5)	neon 10 **Ne** 20.1797(6)
sodium 11 **Na** 22.989770(2)	magnesium 12 **Mg** 24.3050(6)											aluminium 13 **Al** 26.981538(2)	silicon 14 **Si** 28.0855(3)	phosphorus 15 **P** 30.973761(2)	sulfur 16 **S** 32.065(5)	chlorine 17 **Cl** 35.453(2)	argon 18 **Ar** 39.948(1)
potassium 19 **K** 39.0983(1)	calcium 20 **Ca** 40.078(4)	scandium 21 **Sc** 44.955910(8)	titanium 22 **Ti** 47.867(1)	vanadium 23 **V** 50.9415(1)	chromium 24 **Cr** 51.9961(6)	manganese 25 **Mn** 54.938049(9)	iron 26 **Fe** 55.845(2)	cobalt 27 **Co** 58.933200(9)	nickel 28 **Ni** 58.6934(4)	copper 29 **Cu** 63.546(3)	zinc 30 **Zn** 65.38(2)	gallium 31 **Ga** 69.723(1)	germanium 32 **Ge** 72.64(1)	arsenic 33 **As** 74.92160(2)	selenium 34 **Se** 78.96(3)	bromine 35 **Br** 79.904(1)	krypton 36 **Kr** 83.798(2)
rubidium 37 **Rb** 85.4678(3)	strontium 38 **Sr** 87.62(1)	yttrium 39 **Y** 88.90585(2)	zirconium 40 **Zr** 91.224(2)	niobium 41 **Nb** 92.90638(2)	molybdenum 42 **Mo** 95.96(2)	technetium 43 **Tc** [98]	ruthenium 44 **Ru** 101.07(2)	rhodium 45 **Rh** 102.90550(2)	palladium 46 **Pd** 106.42(1)	silver 47 **Ag** 107.8682(2)	cadmium 48 **Cd** 112.411(8)	indium 49 **In** 114.818(3)	tin 50 **Sn** 118.710(7)	antimony 51 **Sb** 121.760(1)	tellurium 52 **Te** 127.60(3)	iodine 53 **I** 126.90447(3)	xenon 54 **Xe** 131.293(6)
caesium 55 **Cs** 132.90545(2)	barium 56 **Ba** 137.327(7)	lutetium 71 **Lu** 174.9668(1)	hafnium 72 **Hf** 178.49(2)	tantalum 73 **Ta** 180.9479(1)	tungsten 74 **W** 183.84(1)	rhenium 75 **Re** 186.207(1)	osmium 76 **Os** 190.23(3)	iridium 77 **Ir** 192.217(3)	platinum 78 **Pt** 195.078(2)	gold 79 **Au** 196.96655(2)	mercury 80 **Hg** 200.59(2)	thallium 81 **Tl** 204.3833(2)	lead 82 **Pb** 207.2(1)	bismuth 83 **Bi** 208.98038(2)	polonium 84 **Po** [209]	astatine 85 **At** [210]	radon 86 **Rn** [222]
francium 87 **Fr** [223]	radium 88 **Ra** [226]	lawrencium 103 **Lr** [262]	rutherfordium 104 **Rf** [267]	dubnium 105 **Db** [268]	seaborgium 106 **Sg** [271]	bohrium 107 **Bh** [272]	hassium 108 **Hs** [270]	meitnerium 109 **Mt** [276]	darmstadtium 110 **Ds** [281]	roentgenium 111 **Rg** [280]	ununbium 112 **Uub** [285]	ununtrium 113 **Uut** [284]	ununquadium 114 **Uuq** [289]	ununpentium 115 **Uup** [288]	ununhexium 116 **Uuh** [293]	ununseptium 117 **Uus** —	ununoctium 118 **Uuo** [294]

Lanthanoids	lanthanum 57 **La** 138.9055(2)	cerium 58 **Ce** 140.116(1)	praseodymium 59 **Pr** 140.90765(2)	neodymium 60 **Nd** 144.24(3)	promethium 61 **Pm** [145]	samarium 62 **Sm** 150.36(3)	europium 63 **Eu** 151.964(1)	gadolinium 64 **Gd** 157.25(3)	terbium 65 **Tb** 158.92534(2)	dysprosium 66 **Dy** 162.500(1)	holmium 67 **Ho** 164.93032(2)	erbium 68 **Er** 167.259(3)	thulium 69 **Tm** 168.93421(2)	ytterbium 70 **Yb** 173.054(5)
Actinoids	actinium 89 **Ac** [227]	thorium 90 **Th** 232.0381(1)	protactinium 91 **Pa** 231.03588(2)	uranium 92 **U** 238.02891(3)	neptunium 93 **Np** [237]	plutonium 94 **Pu** [244]	americium 95 **Am** [243]	curium 96 **Cm** [247]	berkelium 97 **Bk** [247]	californium 98 **Cf** [251]	einsteinium 99 **Es** [252]	fermium 100 **Fm** [257]	mendelevium 101 **Md** [258]	nobelium 102 **No** [259]

Element symbols and names: symbols, names, and spellings are recommended by IUPAC (http://www.iupac.org/). Names are not yet proposed for the elements beyond 111 - those used here are IUPAC's temporary systematic names (Pure & Appl. Chem., 1979, **51**, 381–384). In the USA and some other countries, the spellings **aluminum** and **cesium** are normal while in the UK and elsewhere the usual spelling is **sulphur**.

Atomic weights (mean relative masses): Apart from the heaviest elements, these are IUPAC 2007 values (Pure & Appl. Chem., 2007, **in press**). Elements with values given in brackets have no stable nuclides and are represented by integer values for the longest-lived isotope known at the time writing. The elements thorium, protactinium, and uranium have characteristic terrestrial abundances and these are the values quoted. The last significant figure of each value is considered reliable to ±1 except where a larger uncertainty is given in parentheses.

Periodic table organisation: for a justification of the positions of the elements La, Ac, Lu, and Lr in the WebElements periodic table see W.B. Jensen, "The positions of lanthanum (actinium) and lutetium (lawrencium) in the periodic table", J. Chem. Ed., 1982, **59**, 634–636.

Group labels: the numeric system (1–18) used here is the current IUPAC convention. For a discussion of this and other common systems see: W.C. Fernelius and W.H. Powell, "Confusion in the periodic table of the elements", J. Chem. Ed., 1982, **59**, 504–508.

Symmetry Dictates Physics!

The essence of the physics behind the periodic table is based on **THREE symmetry principles**:

- Dynamics: $U(1)$ gauge symmetry $\Rightarrow$ Coulomb Potential.
- Geometry: $SO(3)$ rotational symmetry $\Rightarrow$ angular momentum conservation.
- Statistics: Pauli exclusion principle $\Rightarrow$ atomic shell structure.

Symmetry and Dynamics

1.	General Covariance	$\Rightarrow$	Relativity (Gravity)
	(Equivalence principle)		(Hilbert-Einstein action)
2.	Gauge Invariance	$\Rightarrow$	Standard Model
	(local phase transformation)		(Yang-Mills action)
3.	?	$\Rightarrow$	Quantum Gravity

What is String Theory?

A perturbative prescription

1. Fundamental degrees of freedom: shape of the string ⇒ vibration frequency ⇒ mass, direction ⇒ spin
   internal charges can be engineered by
   D-brane (open channel) or compactification (closed channel)
2. Dynamics: area and topology of the world-sheet
   gluing of clothes and pants ⇒ Feynman Rules
3. Symmetries: irrelevant oscillations ⇒ gauge symmetry
   freedom of defining local coordinate ⇒ duality
   Global Symmetry?

A Hint from Supersymmetry:

- Unification of bosons and fermions

symmetry charges carry spin $1/2$. ($Q \sim \sqrt{P}$)

fermionic nature implies finite trucation.

$2 \leftrightarrow \frac{3}{2} \leftrightarrow 1 \leftrightarrow \frac{1}{2} \leftrightarrow 0$

True Unification of Space-time and Matter:

-Unification of graviton and gauge bosons (and fermions)

(basic) symmetry charges carry spin 1.

bosonic nature implies infinite multiplet!
$\Rightarrow \infty$ # of particles $+$ symmetry!

Infinite Gauge Symmetry of Open String Theory

(1) First Quantized String Theory: canonical transformation in phase space (G. Veneziano)

(2) World-sheet description: deformed conformal symmetry (M. Evans & B. Ovrut)

(3) First Quantized String Theory: old covariant quantization $\Rightarrow$ zero-norm states (J. C. Lee)

(4) Witten's Open String Field Theory: $\delta\Psi = Q \cdot \Lambda + g_s[\Psi, \Lambda]$ Chern-Simon's action in loop space

Introduction

Progress of Theoretical Physics, Vol. 91, No. 2, February 1994

Generalized On-Shell Ward Identities in String Theory

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(Received September 27, 1993)

It is demonstrated that an infinite set of string-tree level on-shell Ward identities, which are valid to all σ-model loop orders, can be systematically constructed without referring to the string field theory. As examples, bosonic massive scattering amplitudes are calculated explicitly up to the second massive excited states. Ward identities satisfied by these amplitudes are derived by using zero-norm states in the spectrum. In particular, the inter-particle Ward identity generated by the $D_2 \otimes D_{2'}$ zero-norm state at the second massive level is demonstrated. The four physical propagating states of this mass level are then shown to form a large gauge multiplet. This result justifies our previous consideration on higher inter-spin symmetry from the generalized worldsheet σ-model point of view.

II. High Energy Symmetry of String Theory

High Energy Symmetry of String Theory

VOLUME 60, NUMBER 13 PHYSICAL REVIEW LETTERS 28 MARCH 1988

High-Energy Symmetries of String Theory

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(Received 12 February 1988)

By means of a recent analysis of the high-energy limit of string scattering, linear relations between string-scattering amplitudes are derived. These are shown to hold order by order in perturbation theory. If one assumes that they hold for the full theory, they suggest the existence of an enormous string-broken symmetry which is restored at high energies. Some speculations as to the nature of this symmetry are presented.

PACS numbers: 11.17.+y

It is often the case that spontaneously broken symmetries of a physical theory are hard to recognize at low energy, but become evident in the high-energy behavior of the theory. Thus, the broken $SU(2)\otimes U(1)$ symmetry of the electroweak interactions can be seen by examination of weak scattering amplitudes at energies high enough that the W and Z_0 masses can be neglected. String theory surely possesses a very rich symmetry, as suggested by its incredible degree of uniqueness; however, this symmetry is little understood. Presumably this is because most of the string symmetry is spontaneously broken in the known ground states, leaving only the familiar gauge symmetries unbroken and manifest. Perhaps all the string states are gauge particles, but most are massive because of spontaneous symmetry breaking. effective field theory. As stressed in the work of Gross and Mende,[1] the high-energy behavior of strings is very *stringy* and cannot be reproduced by an effective local-field theory. In ordinary general relativity this limit, which is the same as the strong-coupling limit of gravity, $G_{\text{Newton}}\propto 1/M^2_{\text{Planck}}$, is difficult to discuss since the theory breaks down in the ultraviolet. String theory does not necessarily suffer from this limitation. Finding this enlarged symmetry should help us to understand the structure of string physics at high energies. This is not just an academic issue; it is also crucial if we are to understand the Planckian-scale dynamics that determines the nature of the string ground state.

Recently, the high-energy behavior of string scattering was studied by saddle-point techniques.[1,2]

General Pattern of High-Energy Scattering Amplitudes (HESA) among Stringy Excitations (4-point functions)

$$\mathcal{A}(s,t,\alpha';n_i,\epsilon_i) = \mathcal{C}(n_i,\epsilon_i)\mathcal{R}(s,t,\alpha';n_i\epsilon_i)e^{-\alpha' f(s,t)}$$

- $\mathcal{C}(n_i,\epsilon_i)$ is a numerical coefficient, which gives rise to linear proportional relations among HESA.
- $\mathcal{R}(s,t,\alpha';n_i\epsilon_i)$ is a rational function of Mandelstam variables, which describes the subleading energy dependence and filter out the leading states.
- $\exp -\frac{\alpha'}{2}\left[-(s+t)\ln(s+t) + s\ln(s) - t\ln(-t)\right]$ is the universal tachyonic tail (surpression of high-energy divergence).

Main Results for HESS in flat space-time:

1. At any fixed mass level, only one independent high-energy scattering amplitude! $T^{n}_{TT\cdots T} \equiv < V_1 V_2^{(n)} (TT \cdots T) V_3 V_4 >$

2. All other leading amplitudes are proportional to $T^{n}_{TT\cdots T}$ and the proportional constants are independent of

   (i) scattering angle (ii) scattering process (iii) string loop-orderχ

   $$< V_1 V_2^{(n)} (J_2, k) V_3 V_4 >_\chi \propto < V_1 V_2^{(n)} \left(\tilde{J}_2, k\right) V_3 V_4 >_\chi$$

3. The proportional constants can be calculated by algebraic means and originate from zero-norm states.

4. Master formula for inter-level relation: $n = n_1 + n_2 + n_3 + n_4$

   $$T^{n_1 n_2 n_3 n_4}_{T_1\cdots T_1 T_2\cdots T_2 T_3\cdots T_3 T_4\cdots T_4} = \left(-2E^3 \sin\phi\right)^n \exp\left\{-\frac{\alpha'}{2}\left[s\ln(s) + t\ln(t) + u\ln(u)\right]\right\}$$

III. Background (In)dependence of String Theory

Einstein-Hilbert action for pure gravity

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R, \qquad \text{expand } g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$\Rightarrow S_{EH} = \frac{1}{16\pi G}\int d^4x(\partial h\partial h + h\partial h\partial h + h^2\partial h\partial h + \cdots)$$

Similarly, in gauge theory with SSB

$$S_{gauge} = \int d^4x\left[-\frac{1}{4}F^2 + |D\phi|^2 - U\left(\phi;\mu^2,\lambda\right)\right], \text{ expand } \phi = \frac{v}{\sqrt{2}},$$

$$\text{we get } \mathcal{L}_{gauge} = -\frac{1}{4}F^2 + \frac{1}{2}M^2B^2 + e^2v\chi B^2 + \frac{1}{2}e^2\chi^2B^2$$
$$+\frac{1}{2}(\partial\chi)^2 - \mu^2\chi^2 - \sqrt{\lambda}\mu\chi^3 - \frac{\lambda}{4}\chi^4 + \frac{\mu^4}{4\lambda},$$

$$\text{where } \phi \equiv \frac{1}{\sqrt{2}}(v+\chi)\exp(i\theta), \; v = \sqrt{\frac{\mu^2}{\lambda}}, \; F_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu.$$

A non-perturbative formulation of string theory has to provide a starting point for expansion around arbitrary backgrounds.

Strategy: Push the theory to special limit and look for simplification!

⇒**High-energy expansion of string scattering amplitudes.**

String theory is a (infinitely) higher-spin gauge theory with spontaneous symmetry breaking.

Assuming that we are focusing on the transplanckian kinematic region (so that the vaccum expectation values can be ignored), the symmetry pattern among leading scattering amplitudes should reflect the global symmetry of Goldstone bosons.

⇒**A higher-spin generaliztion of equivalence theorem.**

IV. High Energy Scatterings of String States in Linear Dilaton Background

High Energy Scatterings of String States in Linear Dilaton Background

physical states / mass levels	POSITIVE NORM STATE	ZERO NORM STATE
$m^2 = -2$	• (singlet) tachyon	NA
$m^2 = 0$	$\square$ (vector) photon	• (singlet)
$m^2 = 2$	$\square\square$ (tensor)	$\square$ (vector) • (singlet) type II
$m^2 = 4$	$\square\square\square$ $\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\square\square$, $\square$, $\square$ type II, •
$m^2 = \frac{(n-1)}{\alpha'}$	open string $\alpha' = \frac{1}{2}$	

Special Features of String Theory in the Linear Dilaton Background

Deformation of the conformal symmetry

$$T = -\frac{1}{\alpha'}\partial X \cdot \partial X + V \cdot \partial^2 X, \qquad V = \partial \Phi.$$

$$\Rightarrow [L_m, L_n] = (m-n)L_{m+n} + \frac{D + 6\alpha' V^2}{12} m(m^2 - 1)\delta_{m+n}$$

Consequences:

- modified on-shell condition: e.g., “photon” $\alpha' k\,(k + iV) = 0, \epsilon\,(k + iV) = 0$.
- modified inner product for quantum state: $< k'|k >= (2\pi)^D \delta^{(D)}(k'^* - k - iV)$.
- modified vertex operators (extra factor).

Vertex operators for low-lying stringy excitations:

$$|T(k)\rangle : e^{-\alpha' k\cdot V\tau} e^{ik\cdot X} :$$

$$|P(\zeta,k)\rangle : \frac{\zeta\cdot\left(\dot{X}+i\alpha' V\right)}{\sqrt{2\alpha'}} e^{-\alpha' k\cdot V\tau} e^{ik\cdot X} :$$

$$|M(\epsilon_{\mu\nu},k)\rangle : \left[\frac{\epsilon_{\mu\nu}}{2\alpha'}\left(\dot{X}^{\mu}+i\alpha' V^{\mu}\right)\left(\dot{X}^{\nu}+i\alpha' V^{\nu}\right)-\frac{i\epsilon}{\sqrt{2\alpha'}}\cdot\ddot{X}\right] e^{-\alpha' k\cdot V\tau} e^{ik\cdot X} :$$

Oscillator basis for low-lying stringy excitations:

$$|T(k)\rangle \equiv |0,k\rangle, \quad \text{with} \quad \alpha' k\cdot\left(k+iV\right)=1.$$

$$|P(\zeta,k)\rangle \equiv \zeta_{\mu}\alpha_{-1}^{\mu}|0,k\rangle, \quad \text{with} \quad \alpha' k\cdot\left(k+iV\right)=0, \quad \text{and} \quad \zeta\cdot\left(k+iV\right)=0.$$

$$|M(\epsilon_{\mu\nu},k)\rangle = \left(\epsilon_{\mu\nu}\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}+\epsilon_{\mu}\alpha_{-2}^{\mu}\right)|0,k\rangle, \quad \text{with} \quad \alpha' k\cdot\left(k+iV\right)=-1.$$

$$\mathcal{L}_1|M(\epsilon_{\mu\nu},k)\rangle = 0, \quad \text{gives} \quad \sqrt{2\alpha'}\epsilon_{\mu\nu}\left(k^{\nu}+iV^{\nu}\right)+\epsilon_{\mu}=0,$$

$$\mathcal{L}_2|M(\epsilon_{\mu\nu},k)\rangle = 0, \quad \text{gives} \quad \epsilon_{\mu\nu}\eta^{\mu\nu}+\sqrt{2\alpha'}\epsilon_{\mu}\left(2k^{\mu}+3iV^{\mu}\right)=0.$$

Spectrum of Zero-norm States at $\acute{\alpha} m^2 = 1$:

$$|ZNS_I(L)\rangle = \left[\begin{array}{l} \left(-iv - \frac{v^2}{2}\right)\alpha^P_{-1}\alpha^P_{-1} + \left(iv - \frac{v^2}{2}\right)\alpha^L_{-1}\alpha^L_{-1} \\ +(2+v^2)\alpha^P_{-1}\alpha^L_{-1} - \frac{iv}{\sqrt{2}}\alpha^P_{-2} + \left(\sqrt{2} + \frac{iv}{\sqrt{2}}\right)\alpha^L_{-2} \end{array}\right]|0,k\rangle$$

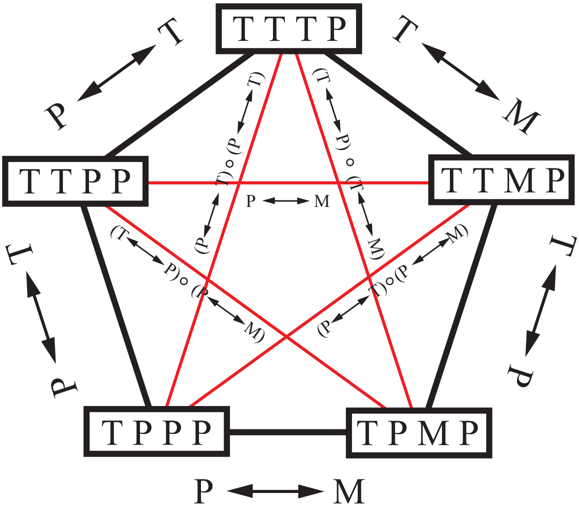
$$|ZNS_I(T_i)\rangle = \sum_{i=1}^{24} u_{PT_i}\left[2\alpha^P_{-1}\alpha^{T_i}_{-1} + \frac{iv}{2-iv}\alpha^L_{-1}\alpha^{T_i}_{-1} + \frac{2\sqrt{2}}{2-iv}\alpha^{T_i}_{-2}\right]|0,k\rangle$$

$$|ZNS_{II}\rangle = \left[\begin{array}{l} \left(5 - 6iv - \frac{3v^2}{2}\right)\alpha^P_{-1}\alpha^P_{-1} + \left(1 - \frac{3v^2}{2}\right)\alpha^L_{-1}\alpha^L_{-1} \\ +\sum_{i=1}^{24}\alpha^{T_i}_{-1}\alpha^{T_i}_{-1} + \left(6iv + 3v^2\right)\alpha^P_{-1}\alpha^L_{-1} \\ +\left(\frac{10}{\sqrt{2}} - \frac{7iv}{\sqrt{2}}\right)\alpha^P_{-2} + \frac{7iv}{\sqrt{2}}\alpha^L_{-2} \end{array}\right]|0,k\rangle$$

Main Results in our calculations (hep-th/0905.2322)

1. We solved the Virasoro constraints up to the first massive level and obtained the spectrum as a continuous function of moduli parameter (V_0).
2. We checked the on-shell stringy Ward identities hold true up to the first massive level with the generalized on-shell conditions and modified energy-momentum conservation law.
3. We examined the deformation of the high-energy stringy symmetry from $V_0 = 0$ to $V_0 \to \infty$ limit.

High Energy Scatterings of String States in Linear Dilaton Background



High Energy Scatterings of String States in Linear Dilaton Background

	Flat Background ($V_0 = 0$, $E \to \infty$)	Linear Dilaton Background ($V_0/E \to \infty$, $E \to \infty$)
$T \to P$	$2^{\frac{3}{2}} \sqrt{\alpha'} E \tan \frac{\phi}{2}$	$2^{\frac{3}{2}} \sqrt{\alpha'} E \tan \frac{\phi}{2}$
$P \to M_{LL}$	$2^{-\frac{1}{2}} \sqrt{\alpha'} E \tan \frac{\phi}{2}$	$2^{-\frac{1}{2}} \alpha'^{\frac{3}{2}} E V_0^2 \cot \frac{\phi}{2}$
$P \to M_{LT}$	$2^{-\frac{1}{2}} \frac{\cos \phi}{\cos^2 \frac{\phi}{2}}$	$2^{\frac{1}{2}} \alpha' E V_0$
$P \to M_{TT}$	$2^{\frac{3}{2}} \sqrt{\alpha'} E \tan \frac{\phi}{2}$	$2^{\frac{3}{2}} \sqrt{\alpha'} E \tan \frac{\phi}{2}$
$P \to M_L$	0	$\frac{2i\sqrt{\alpha'} V_0 (1 + \sin^2 \frac{\phi}{2})}{\sin \phi}$
$P \to M_T$	0	$2^{-\frac{1}{2}} \sec^2 \frac{\phi}{2}$
$T \to M_{LL}$	$2\alpha' E^2 \tan^2 \frac{\phi}{2}$	$2\alpha'^2 E^2 V_0^2$
$T \to M_{LT}$	$2\sqrt{\alpha'} E \frac{\cos \phi}{\cos^2 \frac{\phi}{2}} \tan \frac{\phi}{2}$	$2^2 \alpha'^{\frac{3}{2}} E^2 V_0 \tan \frac{\phi}{2}$
$T \to M_{TT}$	$2^3 \alpha' E^2 \tan^2 \frac{\phi}{2}$	$2^3 \alpha' E^2 \tan^2 \frac{\phi}{2}$
$T \to M_L$	0	$\frac{2^{\frac{3}{2}} i\alpha' E V_0 (1 + \sin^2 \frac{\phi}{2})}{\cos^2 \frac{\phi}{2}}$
$T \to M_T$	0	$2\sqrt{\alpha'} E \sec^2 \frac{\phi}{2} \tan \frac{\phi}{2}$

High Energy Scatterings of String States in Linear Dilaton Background

$SO(3)$ algebra$= \vec{L} \equiv \vec{r} \times \vec{P}$(set $\hbar = 1$)

$$\begin{cases} [L_x, L_y] = iL_z \\ [L_y, L_z] = iL_x \\ [L_z, L_x] = iL_y \end{cases}, \begin{cases} [L_x, L_y] = yP_z - zP_y \\ [L_y, L_z] = zP_x - xP_z \\ [L_z, L_x] = xP_y - yP_x \end{cases} \Rightarrow \begin{cases} [P_x, P_y] = 0 \\ [P_x, L_z] = -iP_y \\ [P_y, L_z] = iP_x \end{cases}$$

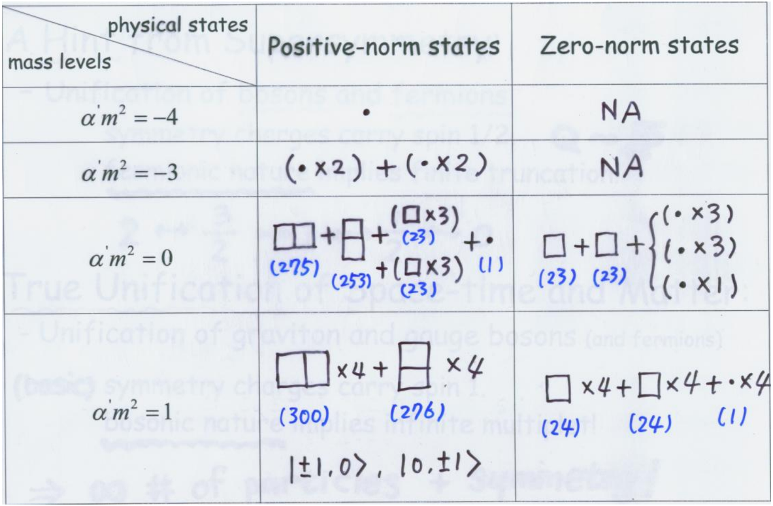
Rescale $z \to \xi z'$, and take $\xi \to \infty$ limit.

Squash the sphere! $\frac{x^2}{R^2} + \frac{y^2}{R^2} + \frac{z^2}{R^2} = 1 \Rightarrow \frac{x^2}{R^2} + \frac{y^2}{R^2} + \frac{z'^2}{\left(\frac{R}{\xi}\right)^2} = 1.$

$L_x \to -P_y$, $L_y \to P_x$, $L_{z'} = L_z$.

$SO(3)$: isometry $\to E(2)$ Euclidean Group!

High Energy Scatterings of String States in Linear Dilaton Background



physical states / mass levels	Positive-norm states	Zero-norm states
$\alpha' m^2 = -4$	$\cdot$	NA
$\alpha' m^2 = -3$	$(\cdot \times 2) + (\cdot \times 2)$	NA
$\alpha' m^2 = 0$	$\underset{(275)}{\square\square} + \underset{(253)}{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + \underset{(23)}{(\square \times 3)} + \underset{(23)}{(\square \times 3)} + \underset{(1)}{\cdot}$	$\underset{(23)}{\square} + \underset{(23)}{\square} + \begin{cases}(\cdot \times 3)\\(\cdot \times 3)\\(\cdot \times 1)\end{cases}$
$\alpha' m^2 = 1$	$\underset{(300)}{\square\square \times 4} + \underset{(276)}{\begin{smallmatrix}\square\\\square\end{smallmatrix} \times 4}$ $\lvert \pm 1, 0 \rangle, \lvert 0, \pm 1 \rangle$	$\underset{(24)}{\square \times 4} + \underset{(24)}{\square \times 4} + \underset{(1)}{\cdot \times 4}$

V. Conclusion

Summary of This Talk

- Concrete Realization of Gross' Conjectures on the High-Energy Symmetry of String Theory.
- Connection between High-Energy Stringy Symmetry and infinite stringy gauge symmetry.
- Deformation of High-Energy Stringy Symmetry as a function of moduli parameters.
- Stringy Generalization of Equivalence Theorem for SSB?

Thank You!!