Photon Production from Nonequilibrium Disoriented Chiral Condensates

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Outline

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- Photon Production from Disoriented Chiral Condensates
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- Nonequilibrium Quantum Field Theory
- Concluding remark
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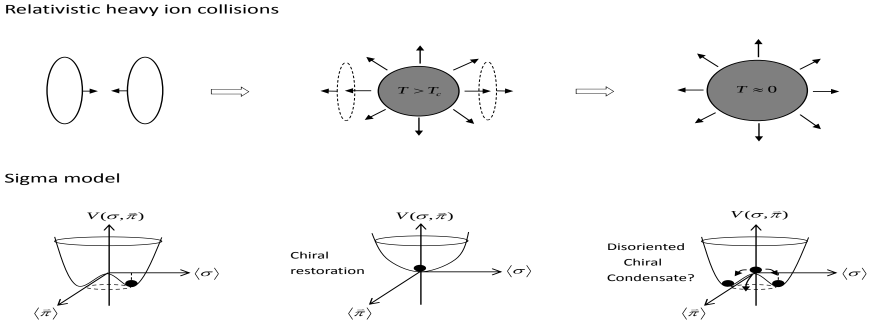
Introduction

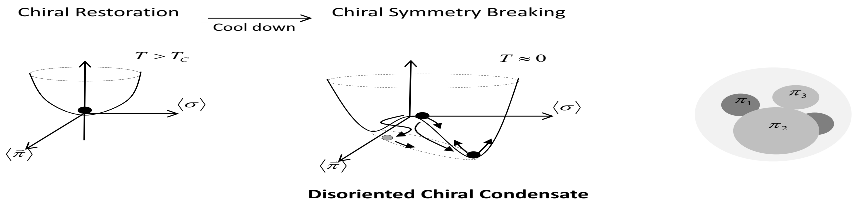
Bjorken idea of the possible formation of a disoriented chiral condensate in hadronic collisions:



The highly Lorentz contracted nuclei essentially pass through each other, leaving behind a hot plasma of baryon free in the central rapidity region where the chiral symmetry is restored.

This plasma then cools down via rapid hydrodynamic expansion to go through the NONEQUILIBRIUM chiral phase transition.

(For a review, see B. Mohantya, J. Serreau , Physics Reports 414 (2005) 263-358)



The DCCs are the correlated regions of space-time where the chiral order parameter of QCD is chirally rotated from its usual orientation in isospin space.

The growth of DCCs is triggered by the instabilities of long-wavelength fluctuations (spinodal instabilities) during the second order (or weakly first order) phase transition. Subsequent relaxation of such DCCs to the true ground state is expected to radiate copious soft pions.

Some theoretical models predict that DCC domain is of size $3-4$ fm in radius with a typical life time about 10 fm$/c$, and can emit $50-200$ pions.

The formation of DCC leads to large event-by-event fluctuation in the neutral pion fraction given by $f = N_{\pi^0}/N_{\pi^0} + N_{\pi^\pm}$.
For DCC, since $f = \cos^2\theta$ where θ is the angle between the direction of the isospin in a particular condensate and the third axis in isospin space, the event-by-event distribution on f is given by

$$\frac{dP(f)}{df} = \int \frac{d\Omega}{4\pi}\delta(f - \cos^2\theta) = \frac{1}{2\sqrt{f}}$$

assuming that there is no privileged isospin direction.
If pions are incoherently produced,the probability distribution of f will be a Gaussian, peaked at the value $f = 1/3$.
The event-by-event fluctuation in the neutral pion fraction is the most basic signature of DCC formation. But the pions decay into photons by the time they reach the detector. This study is equivalent to studying the fluctuation in the number of charged particles and photons.
We propose that to search for non-equilibrium photons in the direct photon measurements of heavy-ion collisions can be a potential test of the formation of a disoriented chiral condensate.

Let us cite Bjorken's words in his DCC trouble list on Existence of DCC:
Must it exist? NO
Should it exist? MAYBE
Might it exist? YES
Does it exist? IT'S WORTH HAVE A LOOK

Concluding remark:

We propose that to search for non-equilibrium photons in the direct photon measurements of heavy-ion collisions can be a potential test of the formation of disoriented chiral condensates. This has been stressed in ALICE: Physics performance report, volume II. by ALICE Collaboration (B. Alessandro et al. (ed.)), published in J. Phys.G32:1295-2040 (2006).

References:

- Y.-Y. Charng, K.-W. Ng, C.-Y. Lin and D.-S. Lee : Photon Production from Non-equilibrium Disoriented Chiral Condensates in a Spherical Expansion, Phys. Lett. B **548**, 175 (2002).
- D.-S. Lee and K. W. Ng: Out-of-Equilibrium Photon Production from Disoriented Chiral Condensates, Phys. Lett. B **492**, 303 (2000).
- D. Boyanovsky, D.-S. Lee and A. Singh: Phase Transitions out of Equilibrium: Domain Formation and Growth, Phys. Rev. D **48**, 800 (1993).
- Suggested review article for nonequilibrium formulation of quantum field theory: D. Boyanovsky, M. D'Attanasio, H.J. de Vega, R. Holman, D.-S. Lee, and A. Singh : Proceedings of International School of Astrophysics, D. Chalonge: 4th Course: String Gravity and Physics at the Planck Energy Scale, Erice, Italy (1995) , hep-ph/9511361 , or book: Nonequilibrium Quantum Field Theory by Esteban A. Calzetta and Bei-Lok Hu (Cambridge University Press).

Linear σ model + Sudden quench

The relevant phenomenological Lagrangian density is given by

$$\mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\gamma + \mathcal{L}_{\pi^0\gamma\gamma} + \mathcal{L}_V + \mathcal{L}_{V\pi\gamma},$$

where

$$\begin{aligned}
\mathcal{L}_\sigma &= \frac{1}{2}\partial^\mu\vec{\Phi}\cdot\partial_\mu\vec{\Phi} - \frac{1}{2}m^2(t)\vec{\Phi}\cdot\vec{\Phi} - \lambda\left(\vec{\Phi}\cdot\vec{\Phi}\right)^2 + h\sigma, \\
\mathcal{L}_\gamma + \mathcal{L}_{\pi^0\gamma\gamma} &= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{e^2}{32\pi^2}\frac{\pi^0}{f_\pi}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}, \\
\mathcal{L}_V + \mathcal{L}_{V\pi\gamma} &= -\frac{1}{4}V^{\mu\nu}V_{\mu\nu} - \frac{1}{2}m_V V^\mu V_\mu + \frac{e\lambda_V}{4m_\pi}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}V_{\mu\nu}\pi^0,
\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field-strength tensor, and $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is the field-strength tensor of the vector meson with mass m_V. In addition, $\vec{\Phi} = (\sigma, \pi^0, \vec{\pi})$ is an $O(N+1)$ vector with $\vec{\pi} = (\pi^1, \pi^2, ..., \pi^{N-1})$ representing the $N-1$ pions. Thus, we take

$$m^2(t) = \frac{m_\sigma^2}{2}\left[\frac{T_i^2}{T_c^2}\Theta(-t) - 1\right], \quad T_i > T_c.$$

The parameters can be determined by the low-energy pion physics as follows:

$$m_\sigma \approx 600 \text{ MeV}, \quad f_\pi \approx 93 \text{ MeV}, \quad \lambda \approx 4.5, \quad T_c \approx 200 \text{ MeV},$$
$$h \approx (120 \text{ MeV})^3, \quad m_V \approx 782 \text{ MeV}, \quad \lambda_V \approx 0.36,$$

where V is identified as the ω meson, and the coupling λ_V is obtained from the $\omega \to \pi^0\gamma$ decay width ((Davidson et al. PRD 43, 71 (1991)). Since we are only interested in the photon production, we can integrate out the vector meson to obtain the effective Lagrangian density that contains the relevant degrees of freedom given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\sigma + \mathcal{L}_\gamma + \mathcal{L}_{\pi^0\gamma\gamma} - \frac{e^2\lambda_V^2}{8m_\pi^2 m_V^2}\epsilon^{\mu\nu\lambda\delta}\epsilon^{\alpha\beta\gamma}{}_{\delta}\,\partial_\lambda\pi^0\partial_\gamma\pi^0 F_{\mu\nu}F_{\alpha\beta},$$

where the higher derivative terms are dropped out. At this point it must be noticed that here we have assumed the validity of the low-energy effective vertices.

Nonequilibrium Quantum Field Theory

Suggested review article: D. Boyanovsky, M. D'Attanasio, H.J. de Vega, R. Holman, D.-S. Lee, and A. Singh : Proceedings of International School of Astrophysics, D. Chalonge: 4th Course: String Gravity and Physics at the Planck Energy Scale, Erice, Italy (1995) , hep-ph/9511361.

Initial valued problem in a nonequilibrium system

In the Schrödinger picture the evolution of the density matrix ρ is determined by

$$\rho(t) = U(t, t_0)\,\rho(t_0)\,U^{-1}(t, t_0),$$

where $U(t, t_0)$ is the time evolution operator:

$$U(t, t_0) = \mathcal{T} \exp\left[-i \int_{t_0}^{t} dt^{'}\, H(t')\right]$$

for a time-dependent Hamiltonian H. The symbol $\mathcal{T}$ means to take the time-ordered product.

The expectation value of an operator $\mathcal{O}$ in the Schrödinger picture is given by

$$\langle \mathcal{O} \rangle(t) = \frac{\mathrm{Tr}\,[U(t,t_0)\,\rho(t_0)\,U(t_0,t)\,\mathcal{O}]}{\mathrm{Tr}\,[\rho(t_0)]}\,,$$

Consider the case in which the initial density matrix describes a system in equilibrium. When a perturbation is switched on at time t_0, the resulting time-dependent Hamiltonian can drive the initial state out of equilibrium. Conveniently, we can model the dynamics by the following Hamiltonian:

$$H(t) = \Theta(t_0 - t)H_0(t_0) + \Theta(t - t_0)[H_0(t_0) + H_{\mathrm{int}}(t)].$$

where $\Theta(t - t_0)$ is the Heaviside step function.

The initial density matrix is assumed to be in equilibrium with respect to $H_0(t_0)$ at a temperature $T = 1/\beta$ given by

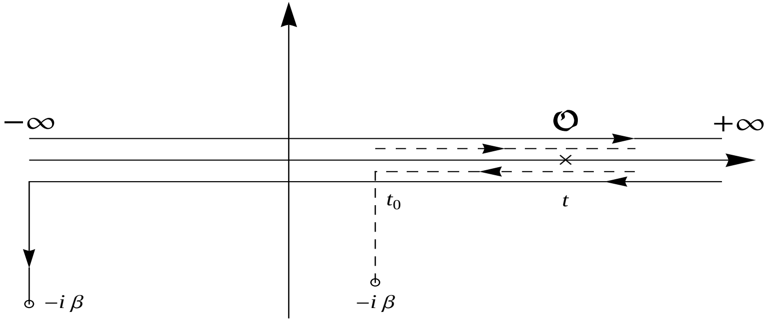
$$\rho(t_0) = \exp[-\beta H_0(t_0)]\,.$$

Notice that the initial density matrix can be expressed in terms of the time evolution operator along imaginary time in the distant past: $\rho(t_0) = U(-\infty - i\beta, -\infty)$. This allows us to write the expectation value as

$$\begin{aligned}\langle\mathcal{O}\rangle(t) &= \frac{\mathrm{Tr}\left[U(t,t_0)\,U(-\infty-i\beta,-\infty)\,U(t_0,-\infty)\,U(-\infty,t_0)\,U(t_0,t)\mathcal{O}\right]}{\mathrm{Tr}\left[U(-\infty-i\beta,-\infty)\right]}\\ &= \frac{\mathrm{Tr}\left[U(t,-\infty)\,U(-\infty-i\beta,-\infty)\,U(-\infty,t)\,\mathcal{O}\right]}{\mathrm{Tr}\left[U(-\infty-i\beta,-\infty)\right]}\end{aligned}$$

where we have inserted $U(t_0,-\infty)\,U^{-1}(t_0,-\infty) = U(t_0,-\infty)\,U(-\infty,t_0) = 1$ to the right of $\rho(t_0)$ in the numerator, commuted $U(t_0,-\infty)$ with $\rho(t_0)$, and used the composition property of the evolution operator: $U(t_a,t_0)\,U(t_0,t_b) = U(t_a,t_b)$. Another insertion of $1 = U(t,\infty)\,U(\infty,t)$ yields

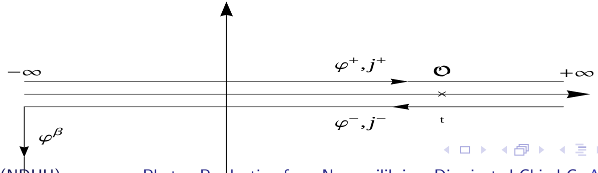
$$\langle\mathcal{O}\rangle(t) = \frac{\mathrm{Tr}\left[U(-\infty-i\beta,-\infty)\,U(-\infty,\infty)\,U(\infty,t)\,\mathcal{O}\,U(t,-\infty)\right]}{\mathrm{Tr}\left[U(-\infty-i\beta,-\infty)\right]}\,.$$


$-\infty$
$\mathcal{O}$
$+\infty$
t_0
t
$-i\,\beta$
$-i\,\beta$

Consider the case of a scalar field φ as an example. The generating functional above has the following path-integral representation

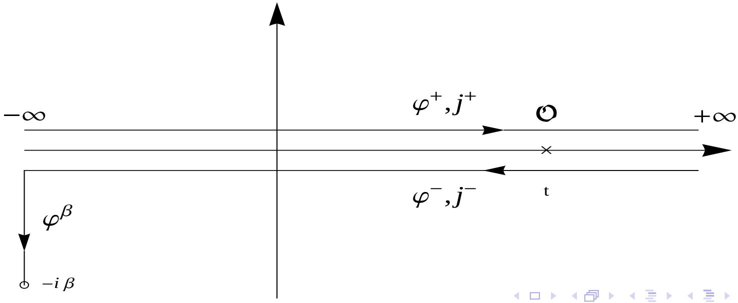
$$
\begin{aligned}
\mathcal{Z}[j^+, j^-] = & \int d\varphi d\varphi_1 d\varphi_2 \int \mathcal{D}\varphi^+ \mathcal{D}\varphi^- \mathcal{D}\varphi^\beta \\
& \exp\left(i \int_{-\infty}^{\infty} dt \int d^3\mathbf{x} \, \left\{ \mathcal{L}\left[\varphi^+\right] + j^+ \varphi^+ \right\} \right) \\
& \times \exp\left(-i \int_{-\infty}^{\infty} dt \int d^3\mathbf{x} \, \left\{ \mathcal{L}\left[\varphi^-\right] + j^- \varphi^- \right\} \right) \\
& \exp\left(i \int_{-\infty}^{-\infty - i\beta} dt \int d^3\mathbf{x} \, \left\{ \mathcal{L}_0\left[\varphi^\beta\right] \right\} \right)
\end{aligned}
$$

with the boundary conditions: $\varphi^+(\mathbf{x}, -\infty) = \varphi^\beta(\mathbf{x}, -\infty - i\beta) = \varphi(\mathbf{x})$, $\varphi^+(\mathbf{x}, \infty) = \varphi^-(\mathbf{x}, \infty) = \varphi_1(\mathbf{x})$, and $\varphi^-(\mathbf{x}, -\infty) = \varphi^\beta(\mathbf{x}, -\infty) = \varphi_2(\mathbf{x})$.



Thus, the relevant generating functional for computing real-time correlation functions can be obtained as

$$\mathcal{Z}[j^+,j^-] = \exp\left\{ i\int d^4x \left(\mathcal{L}_{\rm int}\left[-i\frac{\delta}{\delta j^+}\right] - \mathcal{L}_{\rm int}\left[i\frac{\delta}{\delta j^-}\right]\right)\right\}$$
$$\exp\left\{-\frac{1}{2}\int d^4x \int d^4x' \, \Big[j^+(x)\, G^{++}(x,x')\, j^+(x') - j^+(x)\, G^{+-}(x,x')\, j^-(x') - j^-(x)\, G^{-+}(x,x')\, j^+(x') + j^-(x)\, G^{--}(x,x')\, j^-(x')\Big]\right\} .$$



The nonequilibrium propagators are given by

$$\begin{aligned}
G^{++}(\mathbf{x},\mathbf{x}';t,t') &= \langle \varphi^{+}(\mathbf{x},t)\varphi^{+}(\mathbf{x}',t')\rangle \\
&= G^{>}(\mathbf{x},\mathbf{x}';t,t')\,\Theta(t-t') + G^{<}(\mathbf{x},\mathbf{x}';t,t')\,\Theta(t'-t)\,, \\
G^{--}(\mathbf{x},\mathbf{x}';t,t') &= \langle \varphi^{-}(\mathbf{x},t)\varphi^{-}(\mathbf{x}',t')\rangle \\
&= G^{>}(\mathbf{x},\mathbf{x}';t,t')\,\Theta(t'-t) + G^{<}(\mathbf{x},\mathbf{x}';t,t')\,\Theta(t-t')\,, \\
G^{+-}(\mathbf{x},\mathbf{x}';t,t') &= \langle \varphi^{+}(\mathbf{x},t)\varphi^{-}(\mathbf{x}',t')\rangle \\
&= G^{<}(\mathbf{x},\mathbf{x}';t,t') = \langle \varphi(\mathbf{x}',t')\varphi(\mathbf{x},t)\rangle\,, \\
G^{-+}(\mathbf{x},\mathbf{x}';t,t') &= \langle \varphi^{-}(\mathbf{x},t)\varphi^{+}(\mathbf{x}',t')\rangle \\
&= G^{>}(\mathbf{x},\mathbf{x}';t,t') = \langle \varphi(\mathbf{x},t)\varphi(\mathbf{x}',t')\rangle\,.
\end{aligned}$$

The assumed initial thermal state gives the boundary condition (the Kubo-Martin-Schwinger (KMS) condition)

$$G^{<}(\mathbf{x},\mathbf{x}';-\infty,t') = G^{>}(\mathbf{x},\mathbf{x}';-\infty-i\beta,t')\,.$$

In the perturbative expansion for weak couplings, there are four sets of nonequilibrium propagators. Two sets of interaction vertices are defined by the interaction Lagrangian $\mathcal{L}_{\text{int}}[\pm]$.

Nonequilibrium action+ equations of motion

Following the non-equilibrium quantum field theory that requires a path integral representation along the complex contour in time, the non-equilibrium Lagrangian density is given by

$$\mathcal{L}_{\rm neq} = \mathcal{L}_{\rm eff}[\Phi^+, A_\mu^+] - \mathcal{L}_{\rm eff}[\Phi^-, A_\mu^-],$$

where $+(-)$ denotes the forward (backward) time branches. As mentioned before, the situation of interest to us is a DCC in which both the σ and π^0 fields acquire the vacuum expectation values. We then shift σ and π^0 by their expectation values described by the initial non-equilibrium states specified later,

$$\begin{aligned} \sigma(\vec{x},t) &= \phi(t) + \chi(\vec{x},t), \quad \phi(t) = \langle \sigma(\vec{x},t) \rangle, \\ \pi^0(\vec{x},t) &= \zeta(t) + \psi(\vec{x},t), \quad \zeta(t) = \langle \pi^0(\vec{x},t) \rangle, \end{aligned}$$

with the tadpole conditions,

$$\langle \chi(\vec{x},t) \rangle = 0, \quad \langle \psi(\vec{x},t) \rangle = 0, \quad \langle \vec{\pi}(\vec{x},t) \rangle = 0.$$

Large-N approximation

To incorporate quantum fluctuation effects from the strong $\sigma - \pi$ interactions, we will use the large-N limit to provide a consistent, non-perturbative framework to study this dynamics. The corresponding Hartree factorizations are given by

$$
\begin{aligned}
\chi^4 &\rightarrow 6\langle\chi^2\rangle\chi^2 + \text{constant} \\
\chi^3 &\rightarrow 3\langle\chi^2\rangle\chi \\
(\vec{\pi}\cdot\vec{\pi})^2 &\rightarrow (2+\frac{4}{N})\langle\vec{\pi}^2\rangle\vec{\pi}^2 + \text{constant} \\
\vec{\pi}^2\chi^2 &\rightarrow \vec{\pi}^2\langle\chi^2\rangle + \langle\vec{\pi}^2\rangle\chi^2 \\
\vec{\pi}^2\chi &\rightarrow \langle\vec{\pi}^2\rangle\chi,
\end{aligned}
$$

To leading order in the $1/N$ expansion, the Lagrangian then becomes

$$
\begin{aligned}
&\mathcal{L}_{\rm eff}\left[\phi+\chi^{+},\zeta+\psi^{+},\vec{\pi}^{+},A_{\mu}^{+}\right]-\mathcal{L}_{\rm eff}\left[\phi+\chi^{-},\zeta+\psi^{-},\vec{\pi}^{-},A_{\mu}^{-}\right]\\
&=\left\{\frac{1}{2}(\partial\chi^{+})^{2}+\frac{1}{2}(\partial\psi^{+})^{2}+\frac{1}{2}(\partial\vec{\pi}^{+})^{2}-U_{1}(t)\chi^{+}-U_{2}(t)\psi^{+}\right.\\
&-\frac{1}{2}M_{\chi}^{2}(t)\chi^{+2}-\frac{1}{2}M_{\psi}^{2}(t)\psi^{+2}-\frac{1}{2}M_{\vec{\pi}}^{2}(t)\vec{\pi}^{+2}-\frac{1}{4}F_{\mu\nu}^{+}F^{+\mu\nu}\\
&+\frac{e^{2}}{32\pi^{2}f_{\pi}}\zeta(t)\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}^{+}F_{\mu\nu}^{+}+\frac{e^{2}}{32\pi^{2}f_{\pi}}\psi^{+}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}^{+}F_{\mu\nu}^{+}\\
&-\frac{e^{2}\lambda_{V}^{2}}{8m_{V}^{2}m_{\pi}^{2}}(\dot{\zeta}(t))^{2}\epsilon^{\mu\nu0\delta}\epsilon^{\alpha\beta0}{}_{\delta}\;F_{\mu\nu}^{+}F_{\alpha\beta}^{+}+\frac{e^{2}\lambda_{V}^{2}}{4m_{V}^{2}m_{\pi}^{2}}\ddot{\zeta}(t)\psi^{+}\epsilon^{\mu\nu0\delta}\epsilon^{\alpha\beta0}{}_{\delta}\;F_{\mu\nu}^{+}F_{\alpha\beta}^{+}\\
&+\frac{e^{2}\lambda_{V}^{2}}{4m_{V}^{2}m_{\pi}^{2}}\dot{\zeta}(t)\psi^{+}\epsilon^{\mu\nu0\delta}\epsilon^{\alpha\beta\sigma}{}_{\delta}\;\partial_{\sigma}F_{\mu\nu}^{+}F_{\alpha\beta}^{+}\\
&\left.-\frac{e^{2}\lambda_{V}^{2}}{8m_{V}^{2}m_{\pi}^{2}}\epsilon^{\mu\nu\lambda\delta}\epsilon^{\alpha\beta\gamma}{}_{\delta}\;\partial_{\lambda}\psi^{+}\partial_{\gamma}\psi^{+}F_{\mu\nu}^{+}F_{\alpha\beta}^{+}\right\}-\{+\rightarrow-\}\,.
\end{aligned}
$$

We treat the *linear* and non-linear terms in fields as interactions and impose the tadpole condition consistently in a perturbative expansion.

We define:

$$\begin{aligned} U_1(t) &= \ddot{\phi}(t) + \left[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t)\right]\phi(t) - h, \\ U_2(t) &= \ddot{\zeta}(t) + \left[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t)\right]\zeta(t), \\ M_\chi^2(t) &= m^2(t) + 12\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t) \\ M_\psi^2(t) &= m^2(t) + 4\lambda\phi^2(t) + 12\lambda\zeta^2(t) + 4\lambda\Sigma(t), \\ M_{\vec{\pi}}^2(t) &= m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t), \\ \Sigma(t) &= \langle\vec{\pi}^2\rangle(t) - \langle\vec{\pi}\rangle(0). \end{aligned} \quad (1)$$

For example, the tadpole condition reads $< \chi^{\pm}(\vec{x}, t) >= 0$. One obtains expressions of the form (here we quote the equation obtained from $< \chi^{+}(\vec{x}, t) >= 0$)

$$\int d\vec{x}' dt' \left[< \chi^{+}(\vec{x}, t)\chi^{+}(\vec{x}', t') > - < \chi^{+}(\vec{x}, t)\chi^{-}(\vec{x}', t') >\right] U_1(t') = 0$$

and similarly for $< \chi^{-}(\vec{x}, t) >= 0$. Because the Green's functions $< \chi^{+}(\vec{x}, t)\chi^{+}(\vec{x}', t') >$, etc. are all independent one obtains the equations of motion in the form $U_1(t) = 0$.

With the above Hartree-factorized Lagrangian in the Coulomb gauge, following the tadpole conditions, we can obtain the full one-loop equations of motion while we treat the weak electromagnetic coupling perturbatively:

$$\ddot{\phi}(t) + \left[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t)\right]\phi(t) - h = 0,$$
$$\ddot{\zeta}(t) + \left[m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t)\right]\zeta(t)$$
$$-\frac{e^2}{32\pi^2 f_\pi}\epsilon^{\alpha\beta\mu\nu}\langle F_{\alpha\beta}F_{\mu\nu}\rangle(t) - \frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\frac{d}{dt}\left[\dot{\zeta}(t)\langle A^i_{\mathrm{T}}\vec{\nabla}^2 A^i_{\mathrm{T}}\rangle(t)\right] = 0.$$

Now we decompose the fields $\vec{\pi}$ and $\vec{A}_T$ into their Fourier mode functions $U_{\vec{k}}(t)$ and $V_{\lambda\vec{k}}(t)$ respectively,

$$\vec{\pi}(\vec{x},t) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\omega_{\pi\vec{k}}}}\left[\vec{a}_{\vec{k}}U_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right],$$
$$\vec{A}_T(\vec{x},t) = \sum_{\lambda=1,2}\int \frac{d^3k\ \vec{\epsilon}_{\lambda\vec{k}}}{\sqrt{2(2\pi)^3\omega_{A\vec{k}}}}\left[b_{\lambda\vec{k}}V_{\lambda\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right],$$

where $\vec{a}_{\vec{k}}$ and $b_{\lambda\vec{k}}$ are destruction operators, and $\vec{\epsilon}_{\lambda\vec{k}}$ are circular polarization unit vectors.

Then the mode equations can be read off from the quadratic part of the Lagrangian in the form

$$\left[\frac{d^2}{dt^2} + k^2 + m^2(t) + 4\lambda\phi^2(t) + 4\lambda\zeta^2(t) + 4\lambda\Sigma(t)\right] U_k(t) = 0,$$

$$\frac{d^2}{dt^2} V_{1k}(t) + \left[1 - \frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\dot{\zeta}^2(t)\right] k^2 V_{1k}(t) - k\frac{e^2}{2\pi^2 f_\pi}\dot{\zeta}(t) V_{1k}(t) = 0,$$

$$\frac{d^2}{dt^2} V_{2k}(t) + \left[1 - \frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\dot{\zeta}^2(t)\right] k^2 V_{2k}(t) + k\frac{e^2}{2\pi^2 f_\pi}\dot{\zeta}(t) V_{2k}(t) = 0.$$

The expectation values with respect to the initial states are given by

$$\Sigma(t) = (N-1)\int^{\Lambda} \frac{d^3k}{2(2\pi)^3\omega_{\pi k}} \left[|U_k(t)|^2 - 1\right] \coth\left[\frac{\omega_{\pi k}}{2T_i}\right],$$

$$\epsilon^{\alpha\beta\mu\nu}\langle F_{\alpha\beta}F_{\mu\nu}\rangle(t) = \int^{\Lambda} \frac{d^3k}{2(2\pi)^3\omega_{Ak}} (4k)\frac{d}{dt}\left[|V_{2k}(t)|^2 - |V_{1k}(t)|^2\right],$$

$$\langle A_{\mathrm{T}}^i \vec{\nabla}^2 A_{\mathrm{T}}^i\rangle(t) = \int^{\Lambda} \frac{d^3k}{2(2\pi)^3\omega_{Ak}} (-k^2)\left[|V_{1k}(t)|^2 + |V_{2k}(t)|^2\right],$$

where we set the cutoff scale $\Lambda \simeq m_V$ and $N = 3$.

The initial conditions for the mode functions are given by

$$U_k(0) = 1,\ \dot{U}_k(0) = -i\omega_{\pi k},\ \omega_{\pi k}^2 = k^2 + m^2(t<0) + 4\lambda[\phi^2(0) + \zeta^2(0)]$$
$$V_{\lambda k}(0) = 1,\ \dot{V}_{\lambda k}(0) = -i\omega_{Ak}.\ \omega_{Ak} = k,$$

We choose to represent the quench from an initial temperature set to be $T_i = 220\ \mathrm{MeV} = 1.1\ \mathrm{fm}^{-1}$ to zero temperature. The initial conditions for the mean fields are $\zeta(0) = 0.5\ \mathrm{fm}^{-1}$ and $1\ \mathrm{fm}^{-1}$, and $\dot{\zeta}(0) = \phi(0) = \dot{\phi}(0) = 0$.
The expectation value of the number operator for the asymptotic photons with momentum $\vec{k}$ is given by

$$\begin{aligned} \langle N_k(t)\rangle &= \frac{1}{2k}\left[\dot{\vec{A}}_T(\vec{k},t)\cdot\dot{\vec{A}}_T(-\vec{k},t) + k^2\vec{A}_T(\vec{k},t)\cdot\vec{A}_T(-\vec{k},t)\right] - 1 \\ &= \frac{1}{4k^2}\sum_\lambda\left[|\dot{V}_{\lambda k}(t)|^2 + k^2|V_{\lambda k}(t)|^2\right] - 1. \end{aligned}$$

This gives the spectral number density of the photons produced at time t, $dN(t)/d^3k$.

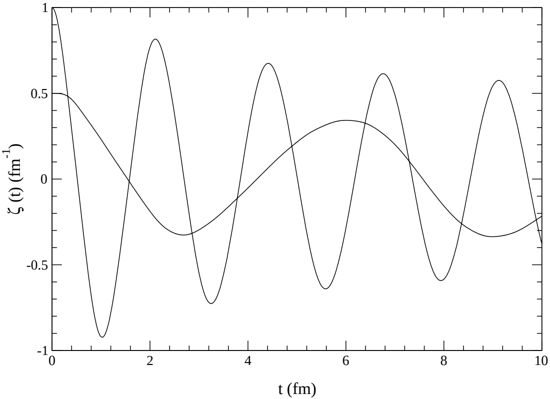
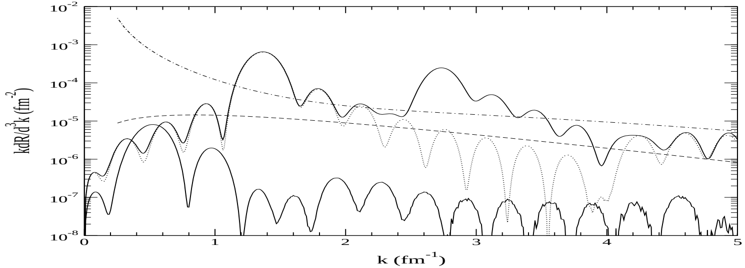


Fig1 shows the temporal evolution of the $\zeta(t)$ by choosing the initial conditions $\zeta(0) = 0.5\ \mathrm{fm}^{-1}$ and $1\ \mathrm{fm}^{-1}$, and $\dot{\zeta}(0) = \phi(0) = \dot{\phi}(0) = 0$. The $\zeta(t)$ evolves with damping due to the backreaction effects from the quantum fluctuations.

In Fig2, we present the time-averaged invariant photon production rate, kdR/d^3k, where

$$dR = \frac{1}{T}\int_0^T \frac{dN(t)}{dt}dt,$$

over a period from the initial time to time $T = 10\ \mathrm{fm}$.



Note that the peaks are located at $k = \omega_\zeta/2$ and $k = \omega_\zeta$ where ω_ζ is the oscillating frequency of the $\zeta(t)$ field with $\zeta(0) = 1\ \mathrm{fm}^{-1}$ in Fig1 Thus, the photon production mechanism is that of parametric amplification.

We now provide the analytical analysis to understand qualitatively the above numerical results.
Let us approximate the $\zeta(t)$ as $\zeta(t) \simeq \bar{\zeta} \sin(\omega_\zeta t)$. The $\bar{\zeta}$ is the average amplitude over a period from the initial time up to time of 10 fm. When $\lambda_V = 0$, we change the variable to $z = \omega_\zeta t/2$.Then, the photon mode equation becomes

$$\frac{d^2}{dz^2} V_{1k} + \frac{4k^2}{\omega_\zeta^2} V_{1k} - \frac{4ke^2\bar{\zeta}}{2\pi^2 f_\pi \omega_\zeta} \cos(2z) V_{1k} = 0. \tag{2}$$

This is the standard Mathieu equation. The widest and most important instability is the first parametric resonance that occurs at $k = \omega_\zeta/2$ with a narrow bandwidth $\delta \simeq e^2\bar{\zeta}/(2\pi^2 f_\pi)$. The instability leads to the exponential growth of photon modes with a growth factor $f = e^{2\mu z}$, where the growth index $\mu \simeq \delta/2$. This growth explains the peak at $k = \omega_\zeta/2$.

When the $\mathrm{U_A}(1)$ anomalous vertex is turned off ($f_\pi \to \infty$), we change the variable to $z' = \omega_\zeta t$. Then, the photon mode equation becomes

$$\frac{d^2}{dz'^2} V_{1k} + \frac{k^2}{\omega_\zeta^2} V_{1k} - \frac{k^2 e^2 \lambda_V^2 \bar{\zeta}^2}{2 m_\pi^2 m_V^2} \cos(2z') V_{1k} = 0. \quad (3)$$

Now, the parametric resonance occurs at $k = \omega_\zeta$ with a growth factor $f' = e^{2\mu' z'}$, where $\mu' \simeq e^2 \lambda_V^2 \bar{\zeta}^2 \omega_\zeta^2 / (8 m_\pi^2 m_V^2)$. This growth explains the peak at $k = \omega_\zeta$. Putting all values together, the ratio of their growth rates is given by $\dot{f}'/\dot{f} \simeq 0.5$ consistent with the numerical result.

Linear σ model+ Spherical expansion

In ultra-relativistic heavy-ion collisions, it is expected that the rapidity density of the particles produced in the hot central region has a plateau (an approximate Lorentz boost invariance along the longitudinal hydrodynamic expansion). However, at late times, a transverse flow can be generated due to the multi-scattering between the produced particles, so the expansion becomes three dimensional.

Here we will simply assume that the hydrodynamical flow is spherically symmetric, and that the boost is along the radial direction.
The natural coordinates for spherical boost invariant hydrodynamical flow are the proper time τ and the space-time rapidity η defined as

$$\tau \equiv (t^2 - r^2)^{\frac{1}{2}}, \quad \eta \equiv \frac{1}{2} \ln \left(\frac{t+r}{t-r} \right),$$

where $(t, \vec{r})$ are the coordinates in the laboratory,

$$t = \tau \cosh \eta, \quad r = \tau \sinh \eta.$$

The ranges of these coordinates are set to be $0 \le \tau < \infty$ and $0 \le \eta < \infty$, restricted to the forward light cone. In terms of spherical coordinates, the Minkowski line element is given by

$$\begin{aligned} ds^2 &= dt^2 - d\vec{r}^2 \\ &= d\tau^2 - \tau^2 (d\eta^2 + \sinh^2 \eta \, d\theta^2 + \sinh^2 \eta \, \sin^2 \theta \, d\phi^2) \\ &= d\tau^2 - \tau^2 d\vec{\eta}^2. \end{aligned}$$

This is a good first-order approximation since the boost invariance is in anyway applied only for small values of η.

The relevant phenomenological effective action in a general expanding space-time is given by

$$S = \int d^4x \sqrt{g} \left(L_\sigma + L_A + L_{\pi^0 A} \right),$$

where

$$\begin{aligned}
L_\sigma &= -\frac{1}{2} g^{\mu\nu} \partial_\mu \vec{\Phi} \cdot \partial_\nu \vec{\Phi} + \frac{1}{2} \frac{M_\sigma^2}{2} \vec{\Phi} \cdot \vec{\Phi} - \lambda \left(\vec{\Phi} \cdot \vec{\Phi} \right)^2 + h\sigma, \\
L_A &= -\frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu}, \\
L_{\pi^0 A} &= \frac{1}{\sqrt{g}} \frac{e^2}{32\pi^2} \frac{\pi^0}{f_\pi} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \\
&\quad + \frac{1}{(\sqrt{g})^2} \frac{e^2 \lambda_V^2}{8 m_\pi^2 m_V^2} \epsilon^{\mu\nu\lambda\sigma} \epsilon^{\alpha\beta\gamma\delta} g_{\sigma\delta} \partial_\lambda \pi^0 \partial_\gamma \pi^0 F_{\mu\nu} F_{\alpha\beta},
\end{aligned}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field, and $\vec{\Phi} = (\sigma, \pi^0, \vec{\pi})$ is an $O(N+1)$ vector of scalar fields with $\vec{\pi} = (\pi^1, \pi^2, ..., \pi^{N-1})$ representing the $N-1$ pions.

It is convenient to work with the conformal time defined by

$$du \equiv \tau_i \frac{d\tau}{\tau}$$

where τ_i is the initial proper time after which we expect that the quark-gluon plasma is formed and thermalized. Integration gives

$$u = \tau_i \ln\left(\frac{\tau}{\tau_i}\right).$$

Rewrite the metric as

$$ds^2 = -g_{\mu\nu} dx^\mu dx^\nu = a^2(u)(du^2 - d\vec{x}^2),$$

where $a(u) = \tau/\tau_i = e^{u/\tau_i}$ and $d\vec{x} = \tau_i d\vec{\eta}$.

Defining $\vec{\Phi} = \vec{\Phi}_a/a$, the action becomes

$$S = \int du\ d^3\vec{x}\ \ \mathcal{L} = \int du\ d^3\vec{x}\ \ \left(\mathcal{L}_\sigma + \mathcal{L}_A + \mathcal{L}_{\pi^0 A}\right),$$

where

$$\begin{aligned}
\mathcal{L}_\sigma &= -\frac{1}{2}\eta^{\mu\nu}\partial_\mu\vec{\Phi}_a\cdot\partial_\nu\vec{\Phi}_a + \frac{1}{2}\left[\frac{1}{2}a^2M_\sigma^2 + \frac{1}{a}\frac{d^2a}{du^2}\right]\vec{\Phi}_a\cdot\vec{\Phi}_a \\
&\qquad -\lambda\left(\vec{\Phi}_a\cdot\vec{\Phi}_a\right)^2 + a^3h\sigma_a, \\
\mathcal{L}_A &= -\frac{1}{4}\eta^{\alpha\mu}\eta^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}, \\
\mathcal{L}_{\pi A} &= \frac{e^2}{32\pi^2}\frac{\pi_a^0}{af_\pi}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \\
&\qquad + \frac{1}{a^2}\frac{e^2\lambda_V^2}{8m_\pi^2m_V^2}\epsilon^{\mu\nu\lambda\sigma}\epsilon^{\alpha\beta\gamma\delta}\eta_{\sigma\delta}\partial_\lambda\left(\frac{\pi_a^0}{a}\right)\partial_\gamma\left(\frac{\pi_a^0}{a}\right)F_{\mu\nu}F_{\alpha\beta},
\end{aligned}$$

where $\vec{\Phi}_a = (\sigma_a, \pi_a^0, \vec{\pi}_a)$ and $\eta^{\mu\nu}$ is the Minkowski metric. In terms of the conformal time, the effective action now has the time dependent mass term and interactions.

Here we first shift σ_a and π_a^0 by their expectation values with respect to an initial non-equilibrium states:

$$\sigma_a(\vec{x},u) = \phi_a(u) + \chi_a(\vec{x},u), \qquad \langle \sigma_a(\vec{x},u)\rangle = \phi_a(u); \langle \chi_a(\vec{x},u)\rangle = 0,$$
$$\pi_a^0(\vec{x},u) = \zeta_a(u) + \psi_a(\vec{x},u), \qquad \langle \pi_a^0(\vec{x},u)\rangle = \zeta_a(u); \langle \psi_a(\vec{x},u)\rangle = 0;$$
$$\langle \vec{\pi}_a(\vec{x},u)\rangle = 0.$$

Using the tadpole conditions, the large-N equations are as follows:

$$\left[\frac{d^2}{du^2} + m_a^2(u) + 4\lambda\phi_a^2(u) + 4\lambda\zeta_a^2(u) + 4\lambda\langle\vec{\pi}_a^2\rangle(u)\right]\phi_a - a^3 h = 0,$$
$$\left[\frac{d^2}{du^2} + m_a^2(u) + 4\lambda\phi_a^2(u) + 4\lambda\zeta_a^2(u) + 4\lambda\langle\vec{\pi}_a^2\rangle(u)\right]\zeta_a - \frac{e^2}{32\pi^2 f_\pi}\frac{1}{a}\langle F\tilde{F}\rangle$$
$$+\frac{e^2\lambda_V^2}{4m_\pi^2 m_V^2}\frac{1}{a}\frac{d}{du}\left[\frac{1}{a^2}\frac{d}{du}\left(\frac{\zeta_a}{a}\right)\right]\epsilon^{\mu\nu 0\sigma}\epsilon^{\alpha\beta 0\delta}\eta_{\sigma\delta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle$$
$$+\frac{e^2\lambda_V^2}{4m_\pi^2 m_V^2}\frac{1}{a^3}\frac{d}{du}\left(\frac{\zeta_a}{a}\right)\epsilon^{\mu\nu 0\sigma}\epsilon^{\alpha\beta\gamma\delta}\eta_{\sigma\delta}\langle\partial_\gamma F_{\mu\nu}F_{\alpha\beta}\rangle = 0,$$

where $m_a^2(u) = -(1/2)\, a^2 M_\sigma^2 - \ddot{a}/a$, $\zeta(u) = \zeta_a(u)/a$, $k = |\vec{k}|$, and the dot means d/du.

The Fourier mode functions $U_{a\vec{k}}(u)$ and $V_{\lambda\vec{k}}(u)$ respectively are defined by

$$\vec{\pi}_a(\vec{x},u) = \int \frac{d^3\vec{k}}{\sqrt{2(2\pi)^3\omega_{\vec{\pi}_a\vec{k},i}}} \left[\vec{a}_{\vec{k}} U_{a\vec{k}}(u) e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right],$$

$$\vec{A}_T(\vec{x},u) = \int \frac{d^3\vec{k}}{\sqrt{2(2\pi)^3\omega_{A\vec{k},i}}} \left\{\left[b_{+\vec{k}} V_{1\vec{k}}(u)\vec{\epsilon}_{+\vec{k}} + b_{-\vec{k}} V_{2\vec{k}}(u)\vec{\epsilon}_{-\vec{k}}\right] e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}\right\},$$

with the mode equations:

$$\left[\frac{d^2}{du^2} + k^2 + m_a^2(u) + 4\lambda\phi_a^2(u) + 4\lambda\zeta_a^2(u) + 4\lambda\langle\vec{\pi}_a^2\rangle(u)\right] U_{ak} = 0,$$

$$\left\{\frac{d^2}{du^2} + \left[1 - \frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\frac{\dot{\zeta}^2}{a^2}\right]k^2 - \frac{e^2}{2\pi^2 f_\pi}\dot{\zeta}k\right\} V_{1k} = 0,$$

$$\left\{\frac{d^2}{du^2} + \left[1 - \frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\frac{\dot{\zeta}^2}{a^2}\right]k^2 + \frac{e^2}{2\pi^2 f_\pi}\dot{\zeta}k\right\} V_{2k} = 0.$$

The frequencies $\omega_{\vec{\pi}_a\vec{k},i}$ and $\omega_{A\vec{k},i}$ can be determined from the initial states.

The expectation values with respective to the initial states are given by

$$\langle\vec{\pi}_a^2\rangle(u) = (N-1)\int^{\Lambda a(u)} \frac{d^3\vec{k}}{2(2\pi)^3\omega_{\vec{\pi}_a k}} \left[|U_{ak}(u)|^2\right] \coth\left[\frac{\omega_{\vec{\pi}_a k}}{2T_i}\right],$$

$$\epsilon^{\alpha\beta\mu\nu}\langle F_{\alpha\beta}F_{\mu\nu}\rangle(u) = \int^{\Lambda a(u)} \frac{d^3\vec{k}}{2(2\pi)^3\omega_{Ak}}(4k)\frac{d}{du}\left[|V_{2k}(u)|^2 - |V_{1k}(u)|^2\right],$$

$$\epsilon^{\mu\nu 0\sigma}\epsilon^{\alpha\beta 0\delta}\eta_{\sigma\delta}\langle F_{\mu\nu}F_{\alpha\beta}\rangle(u) = \int^{\Lambda a(u)} \frac{d^3\vec{k}}{2(2\pi)^3\omega_{Ak}}(4k^2) \left[|V_{1k}(u)|^2 + |V_{2k}(u)|^2\right],$$

$$\epsilon^{\mu\nu 0\sigma}\epsilon^{\alpha\beta\gamma\delta}\eta_{\sigma\delta}\langle\partial_\gamma F_{\mu\nu}F_{\alpha\beta}\rangle(u) = \int^{\Lambda a(u)} \frac{d^3\vec{k}}{2(2\pi)^3\omega_{Ak}}(4k^2) \frac{d}{du}\left[|V_{1k}(u)|^2 + |V_{2k}(u)|^2\right],$$

where $\langle\vec{\pi}_a^2\rangle(u)$ is self-consistently determined. In particular, when $u=0$, it becomes the gap equation,

$$\langle\vec{\pi}_a^2\rangle(0) = (N-1)\int^{\Lambda} \frac{d^3\vec{k}}{2(2\pi)^3\omega_{\vec{\pi}_a k,i}} \coth\left[\frac{\omega_{\vec{\pi}_a k,i}}{2T_i}\right]. \quad (4)$$

In this conformal frame, the momentum cutoff we choose depends linearly on a so as to keep the physical momentum cutoff Λ fixed in the laboratory frame. Here we choose

$$
\begin{aligned}
& U_{ak<\Lambda}(0) = 1,\ \dot{U}_{ak<\Lambda}(0) = -i\omega_{\vec{\pi}_a k,i}, \\
& \omega^2_{\vec{\pi}_a k,i} = k^2 + m_a^2(0) + 4\lambda\phi_a^2(0) + 4\lambda\zeta_a^2(0) + 4\lambda\langle\vec{\pi}_a^2\rangle(0); \\
& V_{1,2k<\Lambda}(0) = 1,\ \dot{V}_{1,2k<\Lambda}(0) = -i\omega_{Ak,i},\ \omega_{Ak,i} = k; \\
& U_{ak>\Lambda}(u_k) = 1,\ \dot{U}_{ak>\Lambda}(u_k) = -i\omega_{\vec{\pi}_a k,i}, \\
& \omega^2_{\vec{\pi}_a k,i} = k^2 + m_a^2(u_k) + 4\lambda\phi_a^2(u_k) + 4\lambda\zeta_a^2(u_k) + 4\lambda\langle\vec{\pi}_a^2\rangle(u_k); \\
& V_{1,2k>\Lambda}(u_k) = 1,\ \dot{V}_{1,2k>\Lambda}(u_k) = -i\omega_{Ak,i},\ \omega_{Ak,i} = k.
\end{aligned}
$$

Therefore, for the momentum lying below the momentum cutoff Λ at the initial conformal time, the initial condition for the mode function can be set at $u = 0$ as above. However, for the momentum above the cutoff momentum Λ at $u = 0$, it will become dynamical when $k \leq \Lambda a(u)$ after $u = u_k = \tau_i \ln(k/\Lambda)$, and the initial condition for this mode function must be set at that conformal time u_k.

The photon spectral number density at time u is given by the expectation value of the number operator for the asymptotic photons,

$$\begin{aligned} & \frac{dN}{d^3\vec{x}d^3\vec{k}} = \frac{dN_+}{d^3\vec{x}d^3\vec{k}} + \frac{dN_-}{d^3\vec{x}d^3\vec{k}} \\ = & \frac{1}{2k}\left[\dot{\vec{A}}_T(\vec{k},u)\cdot\dot{\vec{A}}_T(-\vec{k},u) + k^2\vec{A}_T(\vec{k},u)\cdot\vec{A}_T(-\vec{k},u)\right] - 1 \\ = & \frac{1}{4k^2}\left[|\dot{V}_{1k}(u)|^2 + k^2|V_{1k}(u)|^2\right] + \frac{1}{4k^2}\left[|\dot{V}_{2k}(u)|^2 + k^2|V_{2k}(u)|^2\right] - 1, \end{aligned}$$

and the invariant photon spectral production rate is given by

$$\begin{aligned} & \frac{kdN}{dud^3\vec{x}d^3\vec{k}} \\ & = \frac{1}{4}\left\{\frac{e^2\lambda_V^2}{m_\pi^2 m_V^2}\frac{\dot{\zeta}^2}{a^2}k\frac{d}{du}\left[|V_{1k}|^2 + |V_{2k}|^2\right] + \frac{e^2}{2\pi^2 f_\pi}\dot{\zeta}\frac{d}{du}\left[|V_{1k}|^2 - |V_{2k}|^2\right]\right\}. \end{aligned}$$

We now need to relate this spectral production rate to the invariant production rate measured in the laboratory frame. From the coordinate transformations, we find that

$$\begin{aligned} q &= p\cosh\eta + p_r\sinh\eta, \\ q_\eta &= p\tau\sinh\eta + p_r\tau\cosh\eta, \\ q_\theta &= p_\theta, \\ q_\phi &= p_\phi, \end{aligned}$$

and the zero-mass condition of the photon, i.e., $g^{\mu\nu}p_\mu p_\nu = 0$,

$$\begin{aligned} q &= \frac{|\vec{q}|}{\tau} = \frac{1}{\tau}\left(q_\eta^2 + \frac{1}{\sinh^2\eta}q_\theta^2 + \frac{1}{\sinh^2\eta\sin^2\theta}q_\phi^2\right)^{\frac{1}{2}}, \\ p &= |\vec{p}| = \left(p_r^2 + \frac{1}{r^2}p_\theta^2 + \frac{1}{r^2\sin^2\theta}p_\phi^2\right)^{\frac{1}{2}}, \end{aligned}$$

where $q_\mu = (q, \vec{q})$ is the photon four-momentum in the proper time and spatial rapidity coordinates and $p_\mu = (p, \vec{p})$ is that measured in the laboratory frame.

Because of the spherical symmetry of the problem, for convenience, we can choose $\vec{p}$ to be along the z-axis, which is also the polar axis. This gives $p_r = p\cos\theta$, and then

$$q = p(\cosh\eta + \cos\theta\sinh\eta).$$

We then find that

$$\frac{dN}{d\Gamma} = \frac{q\ dN}{dq_\eta dq_\theta dq_\phi d\tau d\eta d\theta d\phi} = \frac{p\ dN}{d^3\vec{p}\ \tau^3\sinh^2\eta\sin\theta d\tau d\eta d\theta d\phi},$$

where $d\Gamma$ stands for the invariant phase space element in the comoving frame. Therefore, the photon spectrum measured in the laboratory at the final time τ_f can be obtained as

$$\frac{p\ dN}{d^3\vec{p}} = \int_{\tau_i}^{\tau_f} d\tau \int_0^{\eta_{\max}} d\eta \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ \tau^3\sinh^2\eta\sin\theta\frac{dN}{d\Gamma}(\tau, q)\ ,$$

where $q = p(\cosh\eta + \cos\theta\sinh\eta)$.

Now, in the small η approximation, one can link the invariant rate in the comoving frame to that of the conformal frame, where $k = q\tau/\tau_i$ and $\vec{k} = \vec{q}/\tau_i$, in the following way:

$$\frac{dN}{d\Gamma} \simeq \frac{q\ dN}{d\tau d^3\vec{\eta} d^3\vec{q}} \simeq \left(\frac{\tau_i}{\tau}\right)^2 \frac{k\ dN}{du d^3\vec{x} d^3\vec{k}} \ ,$$

where $u = \tau_i \ln(\tau/\tau_i)$. Then, we can approximate

$$\begin{aligned}
&\frac{p\ dN}{d^3\vec{p}} \\
&\simeq \int_{\tau_i}^{\tau_f} d\tau \int_0^{\eta_{\max}} d\eta \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ \tau^2 \tau_i \sinh^2\eta \sin\theta \frac{k\ dN}{du d\vec{x} d\vec{k}}(u,k) \\
&\simeq \frac{2\pi\tau_i^2}{p} \int_{\tau_i}^{\tau_f} d\tau \int_{\tau p e^{-\eta_{\max}}/\tau_i}^{\tau p e^{\eta_{\max}}/\tau_i} dk \left[\cosh(\eta_{\max}) - \frac{1}{2}\left(\frac{k}{p}\frac{\tau_i}{\tau} + \frac{p}{k}\frac{\tau}{\tau_i}\right)\right] \\
&\qquad \times \frac{k\ dN}{du d^3\vec{x} d^3\vec{k}}(u,k) \ .
\end{aligned}$$

Here we consider the photon emission from the states of the local thermal equilibrium that are presumably produced in heavy-ion collisions.

FORMATION OF QUAKR-GLUON PLASMA:

After the ultra-relativistic heavy collisions, in the central rapidity regime, multi-scatterings among the constituents of nuclei cause the produced quarks and gluons to reach local thermal equilibrium at the time scale of τ_i.

The conservation law of entropy for a hydrodynamic expansion of spherical symmetry reads

$$s(T)\tau^3 = s(T_i)\tau_i^3.$$

In the QGP phase, the entropy density is dominated by the relativistic massless gas composed of quarks and gluons given by

$$s_Q = \frac{2\pi^2}{45} g_G T^3, \tag{5}$$

where the degeneracy $g_G = (2 \cdot 8 + 2 \cdot 2 \cdot 3 \cdot 2 \cdot \frac{7}{8})$ for the two-flavor quarks and the eight SU(3) gluons.

The cooling law in the QGP phase can be obtained as $T = (\tau_i/\tau)T_i$.

FIRST ORDER P.T. TO HADRONIC GAS

Subsequently, the plasma cools adiabatically down to the quark-hadron P.T. about T_c when the QGP starts to hadronize at $\tau_Q = (T_i/T_c)\tau_i$.
The entropy density of hadronic matter is taken as

$$s_H = \frac{2\pi^2}{45} g_H(T) T^3.$$

The effective degeneracy $g_H(T) \sim 13(T/\mathrm{fm}^{-1})^\delta$ where $\delta \simeq 3.4$ (the Walecka model (Alam et al.,Ann. of Phys.(2001)).
The entropy density of the mixed state is equal to the sum of the entropy densities of QGP and hadronic matter at T_c, weighted by their fractions:

$$s_{mix}(\tau) = f_Q(\tau) s_Q^c + f_H(\tau) s_H^c$$

The entropy conservation leads to

$$f_Q(\tau) = \frac{s_Q^c \left(\frac{\tau_Q}{\tau}\right)^3 - s_H^c}{s_Q^c - s_H^c}\,, \qquad f_H(\tau) = 1 - f_Q(\tau).$$

Therefore, the mixed phase ends at $\tau_H = (s_Q^c/s_H^c)^{1/3}\tau_Q$ determined from $f_Q(\tau_H) = 0$ above.

HADRONIC PHASE

As for the hadronic phase, the entropy conservation law gives the cooling law, $T = T_c(\tau_H/\tau)^{3/(\delta+3)}$. The freeze-out time can be obtained as $\tau_f = \tau_H(T_c/T_f)^{(\delta+3)/3}$ where the mean free path for all hadrons is of the order of the size of the plasma,, and after that all hadrons cease further interactions.

The *equilibrium* photon emission arising from an expanding quark-gluon plasma (QGP) as well as hadronic matter can be obtained by convoluting the equilibrium photon production rates with the expansion dynamic as

$$
\begin{aligned}
\frac{p\, dN}{d^3\vec{p}} &= \int_{\tau_i}^{\tau_Q} d\tau \int_0^{\eta_{\max}} d\eta \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ \tau^3 \sinh^2\eta \sin\theta \\
&\qquad \frac{dN}{d\Gamma}\,|_Q \left(q, T = T_i \frac{\tau_i}{\tau}\right) \\
&+ \int_{\tau_Q}^{\tau_H} d\tau \int_0^{\eta_{\max}} d\eta \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ \tau^3 \sinh^2\eta \sin\theta \\
&\qquad \left(f_Q(\tau) \frac{dN}{d\Gamma}\,|_Q\, (q, T_c) + f_H(\tau) \frac{dN}{d\Gamma}\,|_H\, (q, T_c) \right) \\
&+ \int_{\tau_H}^{\tau_f} d\tau \int_0^{\eta_{\max}} d\eta \int_0^{\pi} d\theta \int_0^{2\pi} d\phi\ \tau^3 \sinh^2\eta \sin\theta \\
&\qquad \frac{dN}{d\Gamma}\,|_H \left(q, T = T_c \left(\frac{\tau_H}{\tau}\right)^{\frac{3}{\delta+3}} \right),
\end{aligned}
$$

where again $q = p(\cosh\eta + \cos\theta \sinh\eta)$.

We now turn into the discussions of the photon production processes from the QGP and the hadronic phase (Peitzmann & Thoma, Phys. Rep. (2002)).
The thermal photon production rates in the high energy regime ($E >> T$) from the QGP take account of the following important processes:

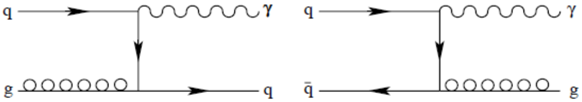


Fig. 1. Lowest order contributions to photon production from the QGP: Compton scattering (left) and quark–antiquark annihilation (right).

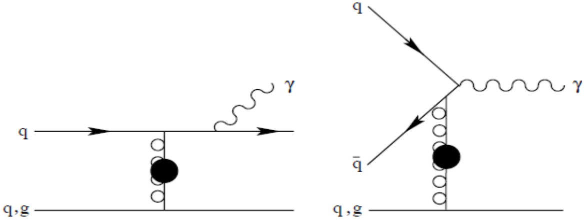


Fig. 5. Photon production processes corresponding to the 2-loop HTL contribution: bremsstrahlung (left) and annihilation with scattering (right). The filled circles indicate HTL resummed gluon propagators. The lower line indicates either a quark or a gluon.

As for the photon production processes from the hadronic phase, the dominant processes in the energy regime of our interest are those of the reactions:

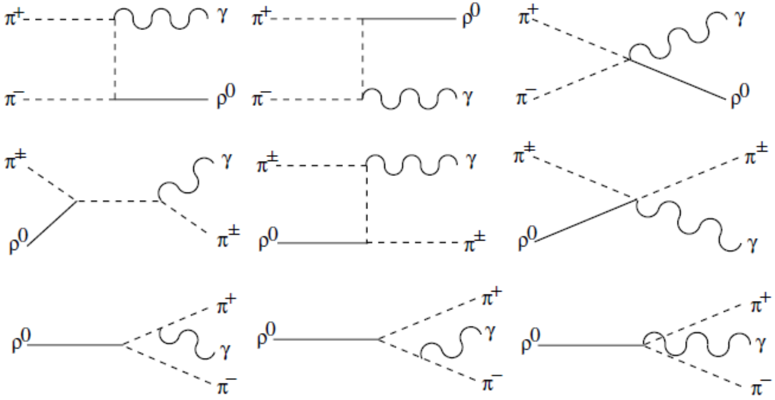
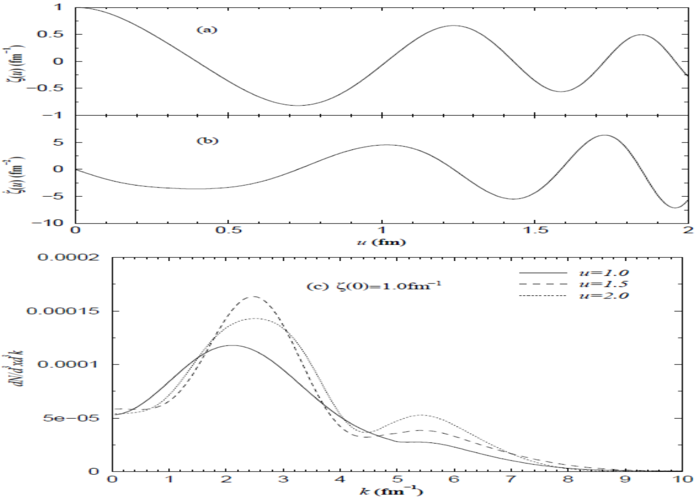


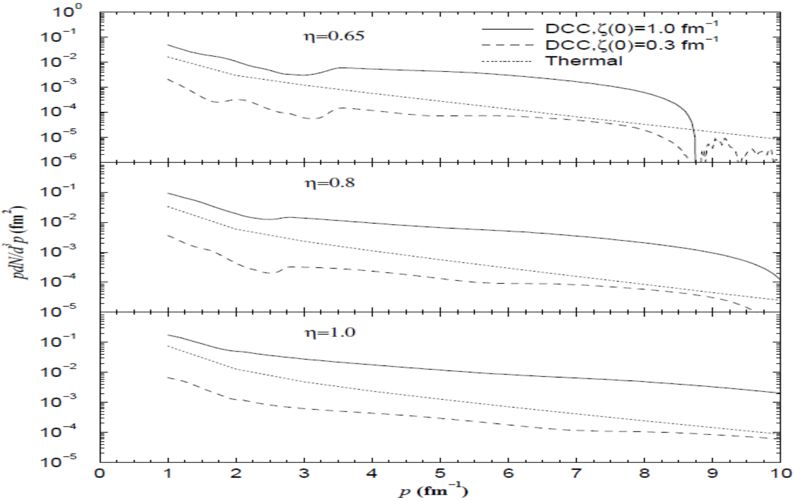
Fig. 9. Photon production from pions and rhos.

The reaction for producing photons that involves the intermediary axial vector meson a_1 is also important, for example, $\pi\rho \to a_1 \to \pi\gamma$.

We assume that the QGP is formed with temperature $T_i = 1.0\ \mathrm{fm}^{-1}$ at the formation time $\tau_i = 1.0\ \mathrm{fm}.$
In addition, the cooling law in the QGP phase gives the time scale $\tau_Q = 1.1\ \mathrm{fm}$ when the QGP begins hadronizing and the subsequent quark-hadron mixed state is formed at the critical temperature assumed to be $T_c = 0.9\ \mathrm{fm}^{-1}$. One can also estimate the time when the mixed state ends, $\tau_H = (s^c_Q/s^c_H)^{1/3} = 1.77\ \mathrm{fm}.$
In the final stage, the system turns into the hadronic phase starting at τ_H till the freeze-out time $\tau_f = 5.6\ \mathrm{fm}$ estimated from the cooling law of the hadronic phase and the freeze-out temperature chosen as $T_f = 0.5\ \mathrm{fm}^{-1}$.



(a) Temporal evolution of the π^0 mean field $\zeta(u)$. (b) Evolution of the derivative $\dot{\zeta}(u)$. (c) Photon spectral number density dN/d^3xd^3k for $\zeta(0) = 1.0\ \mathrm{fm}^{-1}$ at different times, where u is in units of fm.



Photon energy spectrum measured in laboratory. The solid line denotes the initial mean field $\zeta(0) = 1.0\ \mathrm{fm}^{-1}$, while the dashed line denotes $\zeta(0) = 0.3\ \mathrm{fm}^{-1}$. The dotted line is the thermal photons emitted from quark-gluon phase transition.

Concluding remark:

We propose that to search for non-equilibrium photons in the direct photon measurements of heavy-ion collisions can be a potential test of the formation of disoriented chiral condensates. This has been stressed in ALICE: Physics performance report, volume II. by ALICE Collaboration (B. Alessandro et al. (ed.)), published in J. Phys.G32:1295-2040 (2006).

Longitudinal hydrodynamic expansion (in progress)

In the longitudinal hydrodynamic expansion, for example along the z direction, the natural coordinates for describing the boost invariant phenomena can be the proper time and spatial rapidity given by

$$\tau = (t^2 - z^2)^{1/2} \ , \eta = \frac{1}{2} \ln \left(\frac{t-z}{t+z} \right) . \tag{6}$$

Then, the Minkowski metric can be reexpressed as:

$$ds^2 = d\tau^2 - dx^2 - dy^2 - \tau^2 d\eta^2 \, , \tag{7}$$

which lies within the category of the metric of the Bianchi-type I cosmologies of this type:

$$ds^2 = C_0^2(\tau) d\tau^2 - C_1^2(\tau) dx^2 - C_2^2(\tau) dy^2 - C_3^2(\tau) dz^2 \, . \tag{8}$$

1. Quantizing gauge potentials of electromagnetic fields under this metric can be done by a proper choice of the gauge conditions. (DONE).
2. Numerical study of photon creation that arises via the couplings to nonequilibrium relaxation of DCCs is in progress.

Thank you for your attention!!