Unusual Two Higgs Doublet Model from Warped Space

We-Fu Chang

National Tsing Hua University
(with John Ng, and A. Spray, PRD82,115022)

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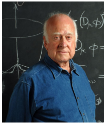

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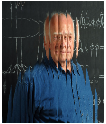


We are arrogant!

The ultimate dream of HEP theorist is to reduce the number of free parameters as many as possible.

- Prominent problems: gauge hierarchy? Electroweak symmetry breaking?
  For example, GUT makes three couplings to one
- Prominent problems: Why 3 generations? Why $m_t \gg m_q, m_l \gg m_\nu$? Why $\theta^{12}_{CKM} \gg \theta^{23}_{CKM} \gg \theta^{13}_{CKM}$?
  For example, flavor symmetry to reduce the 21(+2) flavor parameters to only few

All physical quantities in terms of Planck unit?

RS Model is one of the promising candidates

- Randall-Sundrum (PRL83, 3370) can explain the hierarchy between EW and M_{planck}

$$EW \sim ke^{-kr_c\pi}, \ kr_c \sim 11.7$$

where k is the 5D curvature $\sim M_{planck}$ and r_c is the radius of the compactified fifth dimension.

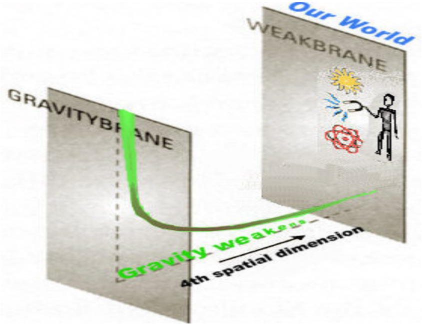
- Due to the special profile of bulk fermion in RS, the hierarchy among fermions can be achieved without fine tuning in Yukawa couplings.
- And the number of free parameters (in flavor sector) is smaller than in SM

Randall-Sundrum Model

- RS assumes a 1+4 dim with a warp or conformal metric, AdS.
- 5D interval (S_1/Z_2) is given by

$$ds^2 = G_{AB}dx^A dx^B = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu - r_c^2 d\phi^2, \ -\pi \leq \phi \leq \pi$$

- Two branes are localizes at $\phi = 0$(UV) and $\phi = \pi$(IR)
- Due to the metric, matters tend to stay near the IR brane.



Bulk Fermions in RS

- 5D action for fermions is

$$\int d^4x d\phi \sqrt{G} \left[E_a^A \bar{\Psi} \gamma^a D_A \Psi - c\, k\, \mathrm{sgn}(\phi) \bar{\Psi} \Psi \right]$$

  where E_a^A is the veilbien, and a dimensionless bulk mass c. The spectrum determined by B.C.'s (Neumann or Dirichlet).
- Desired chirality for zero mode set by orbifold parity.
- The coefficients $c_{L,R}$ control the zero modes peak at either UV or IR
- SM chiral zero modes localized near UV brane $\Rightarrow$ small overlap after SSB. No need to fine tune Yukawa's. Fermion masses are naturally small. (except 3rd generation quarks)

Fermion Masses in RS

- The fermion masses are given by

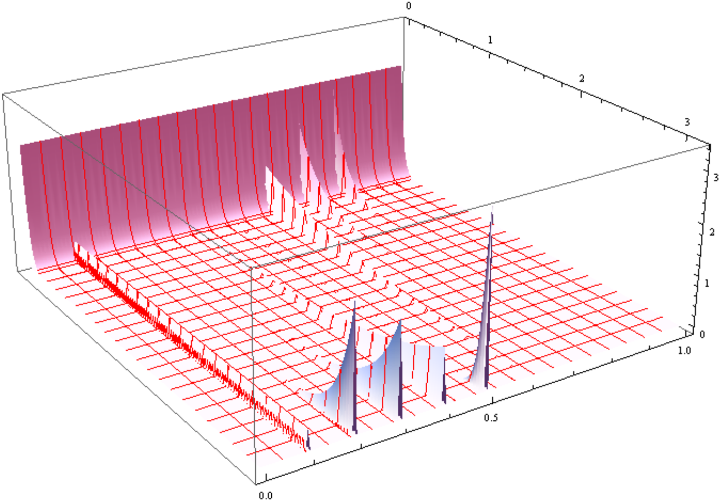
$$\left\langle M_{ij}^{f} \right\rangle = \frac{\lambda_{5,ij}^{f} v_W}{k r_c \pi} f_L^0(\pi, c_{f_i}^L) f_R^0(\pi, c_{f_j}^R)$$

where $v_W = 174$ GeV, and

$$f_{L,R}^0(\phi, c_{L,R}) \propto \exp\left[k r_c \phi (1/2 \mp c_{L,R})\right]$$

- The Yukawa couplings λ_{ij} are arbitrary complex numbers with $|\lambda| \sim \mathcal{O}(1)$.
- The task is find configurations that fit all the known fermion masses and the CKM/PMNS matrices.
(See our works: PRD78,096003, PRD79,056007, PRD80,113013)

the Bulk Wave Function Profiles



Nambu-Jona-Lasinio term

- In the gauge (weak) eigenbasis, the coupling of the nth level KK gluon, $G^{(n)}$, to zero-mode fermions is given by

$$G_\mu^{A(n)} \left[\sum_i (g_f^n)_{ii}^L \, \bar{f}_{iL} T^A \gamma^\mu f_{iL} + (L \to R) \right], \qquad f = u, \, d \, ,$$

where g_f^n is proportional to the fermion-KK gauge overlapping and can be determined by their profiles.

- For small exchanging momenta, tree-level exchange of G_{KK}^1 leads to 4-Fermi interactions between zero mode fermions given by

$$-\frac{g_i g_j}{M_{KK}^2} \left(\overline{Q_{iL}} T^A \gamma^\mu Q_{iL} \right) \left(\overline{f_{jR}} T^A \gamma_\mu f_{jR} \right)$$
$$= \frac{g_i g_j}{M_{KK}^2} \left(\overline{Q_{iL}} f_{jR} \right) \left(\overline{f_{jR}} Q_{iL} \right) + O(1/N_c)$$

2HDM at M_{KK}

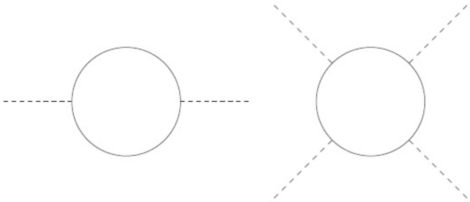
- In addition to the elementary scalar field H, below M_{KK}, the condensate can be viewed as a composite Higgs doublet.
- It has the same $SU(2)_L \times U(1)_Y$ quantum numbers as the SM Higgs. (ρ OK at tree level!)
- At M_{KK}, $\Phi \sim g_t < \bar{Q}t > /M_{KK}^2$ is a static auxiliary field.

$$\begin{aligned} \mathcal{L} &= |D_\mu H|^2 - m_0^2 H^\dagger H - \frac{1}{2}\lambda_0 (H^\dagger H)^2 \\ &\quad + \lambda_t \overline{Q_L} t_R \widetilde{H} + g_t \overline{Q_L} t_R \widetilde{\Phi} - M_{KK}^2 \Phi^\dagger \Phi + h.c. \end{aligned}$$

where $\widetilde{H} = i\sigma_2 H^*$, $\widetilde{\Phi} = i\sigma_2 \Phi^*$, $Q_L = (t, b)_L$, and m_0^2, λ_0 are the parameters in the brane Higgs scalar potential.

Bubble diagram

- At scales $\mu < M_{KK}$, quantum fluctuations generate a kinetic term for Φ as well as kinetic and mass term mixings between ϕ and H.
- Fermion bubble contribution to scalar (a) 2-point functions and (b) 4-point functions. The dashed lines can be either Φ or H fields.



(a) (b)

Effective 2HDM

- The transformations

$$H = \hat{H}\,,\ \Phi = -\frac{\lambda_t}{g_t}\hat{H} + \frac{1}{g_t\sqrt{\epsilon}}\hat{\Phi}$$

  will cast the kinetic terms into canonical diagonal form.

- The resulting Lagrangian of the scalars is delightfully simple:

$$\mathcal{L} \supset |D_\mu \hat{H}|^2 + |D_\mu \hat{\Phi}|^2 - V(\hat{H}, \hat{\Phi})$$

  with the loop factors $\epsilon \sim O(0.1)$, $\Delta \sim O(0.3)M_{KK}$, and

$$V(\hat{H},\hat{\Phi}) = \left(m_0^2 + \frac{\lambda_t^2}{g_t^2}M_{KK}^2\right)\hat{H}^\dagger\hat{H} - \frac{\lambda_t}{g_t^2\sqrt{\epsilon}}M_{KK}^2\left(\hat{H}^\dagger\hat{\Phi} + \hat{\Phi}^\dagger\hat{H}\right) + \left(\frac{M_{KK}^2}{g_t^2\epsilon} - \frac{\Delta^2}{\epsilon}\right)\hat{\Phi}^\dagger\hat{\Phi} + \frac{1}{2}\lambda_0(\hat{H}^\dagger\hat{H})^2 + \frac{1}{\epsilon}(\hat{\Phi}^\dagger\hat{\Phi})^2$$

Electroweak Symmetry breaking of 2HDM

- Define $\tan\beta = v_H/v_\phi$ and minimizing the potential yields:

$$\left(m_0^2 + \frac{\lambda_t^2}{g_t^2} M_{KK}^2\right) v_H - \frac{\lambda_t}{g_t^2\sqrt{\epsilon}} M_{KK}^2 v_\phi + \frac{\lambda_0}{2} |v_H|^2 v_H = 0,$$

$$\left(\frac{M_{KK}^2}{g_t^2\epsilon} - \frac{\Delta^2}{\epsilon}\right) v_\phi - \frac{\lambda_t}{g_t^2\sqrt{\epsilon}} M_{KK}^2 v_H + \frac{2}{\epsilon} |v_\phi|^2 v_\phi = 0.$$

  We require that $v_H^2 + v_\phi^2 = (246\text{GeV})^2$.
- Above the cutoff, M_{KK}, the 4-Fermi condensate approximation is no longer valid.
- Here λ_0, m_0 are free parameters. (There are 14 free parameters in the general 2HDM with sever FCNC problems.)

Spectrum of Physical Scalars

- Charged and pseudoscalar sectors have the same mass matrix:

$$M_{\pm}^2 = M_A^2 = \begin{pmatrix} a + \frac{\lambda_0}{2} v_H^2 & c \\ c & b + \frac{1}{\epsilon} v_\phi^2 \end{pmatrix},$$

where

$$a = m_0^2 + \frac{\lambda_t^2}{g_t^2} M_{KK}^2 \, , b = \frac{1}{\epsilon}\left(\frac{M_{KK}^2}{g_t^2} - \Delta^2 \right), c = -\frac{\lambda_t}{g_t^2 \sqrt{\epsilon}} M_{KK}^2 .$$

- At tree level, we have ($H^{\pm} = c_\beta h^{\pm} - s_\beta \phi^{\pm}, A^0 = c_\beta h_I - s_\beta \phi_I$)

$$M_{A^0}^2 = M_{H^{\pm}}^2 = \frac{2\lambda_t}{g_t^2 \sqrt{\epsilon} \sin 2\beta} M_{KK}^2$$

and mixing angle $\beta = \tan^{-1}(v_H / v_\Phi)$.

Neutral Scalars

- The mass squared matrix for the two scalars is given by

$$M_0^2 = \begin{pmatrix} a + \frac{3}{2}\lambda_0 v_H^2 & c \\ c & b + \frac{3}{\epsilon} v_\phi^2 \end{pmatrix}$$

- $TrM_0^2 = M_H^2 + k_1 v^2$ and $\det M_0^2 = k_2 M_H^2 v^2$, $k_{1,2}$ are ratios of $\mathcal{O}(1)$ parameters. Therefore one of the scalars $M_H \sim \mathcal{O}(\mathrm{TeV})$, while the other has mass $\sim \mathcal{O}(v)$
- It can be diagonalized

$$\begin{pmatrix} H_0 \\ h_0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \hat{H}_R \\ \hat{\Phi}_R \end{pmatrix}.$$

and

$$\tan 2\alpha = -\frac{2c}{a + \frac{3}{2}\lambda_0 v_H^2 - b - \frac{3}{\epsilon} v_\phi^2},$$

Top quark mass

- With the redefined scalar fields,

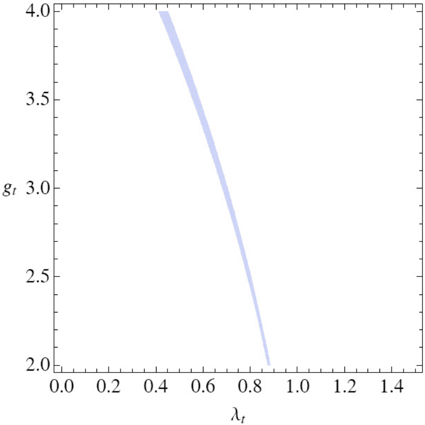
$$\mathcal{L}_Y = \lambda_t \overline{Q_L} t_R \widetilde{H} + g_t \overline{Q_L} t_R \widetilde{\Phi} + h.c. \rightarrow \frac{1}{\sqrt{\epsilon}} \overline{Q_L} t_R \widetilde{\hat{\Phi}} + h.c.$$

- Top quark gets its mass from coupling to $\hat{\Phi}$, which after symmetry breaking gives

$$m_t = \frac{v \cos\beta}{\sqrt{2\epsilon}}.$$

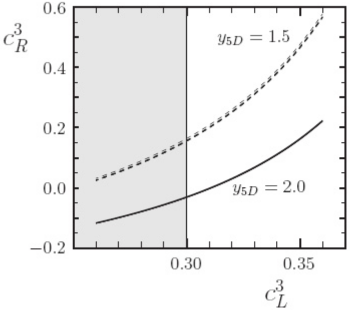
- $\tan\beta$ is determined by top mass!! $\cos\beta \sim \sqrt{\epsilon}$

$\{\lambda_t, g_t\}$



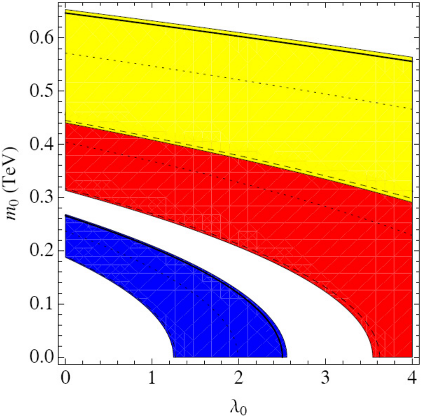
Allowed region in the $\{\lambda_t, g_t\}$ parameter space that satisfies m_t and 2nd Mini. Cond. M_{KK} lies between 1.5 to 4 TeV and m_t from 169.7 to 172.9 GeV.

location, location, location



The solution for bulk mass parameters c_L^3 and c_R^3 with two representative 5D Yukawa couplings. The KK mass is varied from 1.5 TeV to 4.0 TeV. The shaded areas are excluded by the $Z \to b_L \bar{b}_L$.

$\{\lambda_0, m_0\}$



Allowed region in the $\{\lambda_0, m_0\}$ parameter space that satisfies 1st Mini. Cond. The (blue, red, yellow) correspond to $M_{KK} = \{1.5, 2.5, 3.5\}$ TeV. The lines (solid, dotted, dash) correspond to $g_t = \{2, 3, 4\}$.

SM like Higgs mass

- The mass matrix for neutral scalar sector can be decomposed into

$$M_0^2 = M_\pm^2 + \begin{pmatrix} \lambda_0 v^2 \sin\beta & 0 \\ 0 & 4m_t^2 \end{pmatrix}.$$

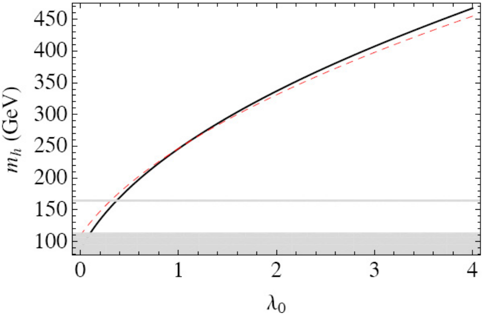
- Since the second term is much smaller than the first one, one expects that the heavier $M_H \sim M_{H^\pm}$, and $\alpha \sim \beta$.
- By using $\alpha \sim \beta$, it can be derived that

$$M_{h_0}^2 \simeq \lambda_0 v^2 \sin^4\beta + 2\epsilon\, m_t^2.$$

- Also, the h^0 is very SM like. For example,

$$h^0 Z^0 Z^0 \,\text{coupling} \,:\, \cos(\beta - \alpha)$$

SM like Higgs mass (Numerical)



The mass of the lighter Higgs boson v.s. λ_0. The black line is for $M_{KK} = 1.5$ TeV and the red line is for $M_{KK} = 4$ TeV. The shaded regions are the LEP and Tevatron exclusions for the Higgs mass.

Flavor Changing Neutral Current -1

- The full Yukawa sector, including light quarks,

$$\mathcal{L}_Y = \lambda^d_{ij}\overline{Q_{Li}}d_{jR}H + g_t\overline{Q_{3L}}t_R\widetilde{\Phi} + \lambda^u_{ij}\overline{Q_{iL}}u_{jR}\widetilde{H} + h.c.$$

- After the rotation to go to canonical kinetic term, we have

$$\mathcal{L}_Y = \lambda^d_{ij}\overline{Q_{Li}}d_{jR}\hat{H} + \left(\lambda^u_{ij} - \lambda^u_{33}\right)\overline{Q_{iL}}u_{jR}\widetilde{\hat{H}} + \frac{1}{\sqrt{\epsilon}}\overline{Q_{3L}}t_R\widetilde{\hat{\Phi}} + h.c.$$

- After SSB, the up quark mass matrix is

$$\mathcal{M}^u_{ij} = -\frac{1}{\sqrt{2}}\begin{pmatrix} \lambda^u_{11}v_H & \lambda^u_{12}v_H & \lambda^u_{13}v_H \\ \lambda^u_{21}v_H & \lambda^u_{22}v_H & \lambda^u_{23}v_H \\ \lambda^u_{31}v_H & \lambda^u_{32}v_H & \frac{1}{\sqrt{\epsilon}}v_\phi \end{pmatrix}$$

Flavor Changing Neutral Current -2

- After little algebra, we can rewrite the Yukawa sector as,

$$\begin{aligned}\mathcal{L}_Y &= -\frac{\sqrt{2}\mathcal{M}^d_{ij}}{v\sin\beta}\overline{Q_{Li}}d_{jR}\hat{H} - \frac{\sqrt{2}\mathcal{M}^u_{ij}}{v\sin\beta}\overline{Q_{iL}}u_{jR}\widetilde{\hat{H}} \\ &\quad + \frac{1}{\sqrt{\epsilon}\cos\beta}\overline{Q_{3L}}t_R\left(\widetilde{\hat{\Phi}}\cos\beta - \widetilde{\hat{H}}\sin\beta\right) + h.c.\end{aligned}$$

- It's clear that FCNC comes solely from the last term (no VEV, physical $H^{\pm}$ or A^0). And because $\alpha \sim \beta$, it is mainly H_0 in the combination.
- The light quark FCNCs are suppressed by
  - M_{KK} suppression if through H_0, $H^{\pm}$, and A^0
  - $\sin(\beta - \alpha)$ suppression if through h_0
  - Flavor structure of RS.
- From the first two terms, the h^0 Yukawa coupling is $-\sqrt{2}(M_{ij}/v)(\sin\alpha/\sin\beta)$, very close to the SM.

Conclusions

- RS model provides an interesting framework to address both the gauge hierarchy and flavor problems.
- Light(heavy) fermion results from the UV(IR) peaking profiles. KK gauge boson peaks near IR.
- $\mathcal{O}(10)$ enhancement for the $SU(3)_c$ coupling between the first KK gluon, t_R, and Q_3.
- A composite Higgs could emerge from the $Q_3 t_R$ condensation below M_{KK}.
- The 2HDM is very predictive: $\tan\beta \sim 3$, close to the decoupled limit, no(suppressed) FCNC in down(up) sector.