QCD resummation for jet shapes

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Outlines

- Motivation
- Jet factorization
- Resummation
- Jet energy profile
- Summary

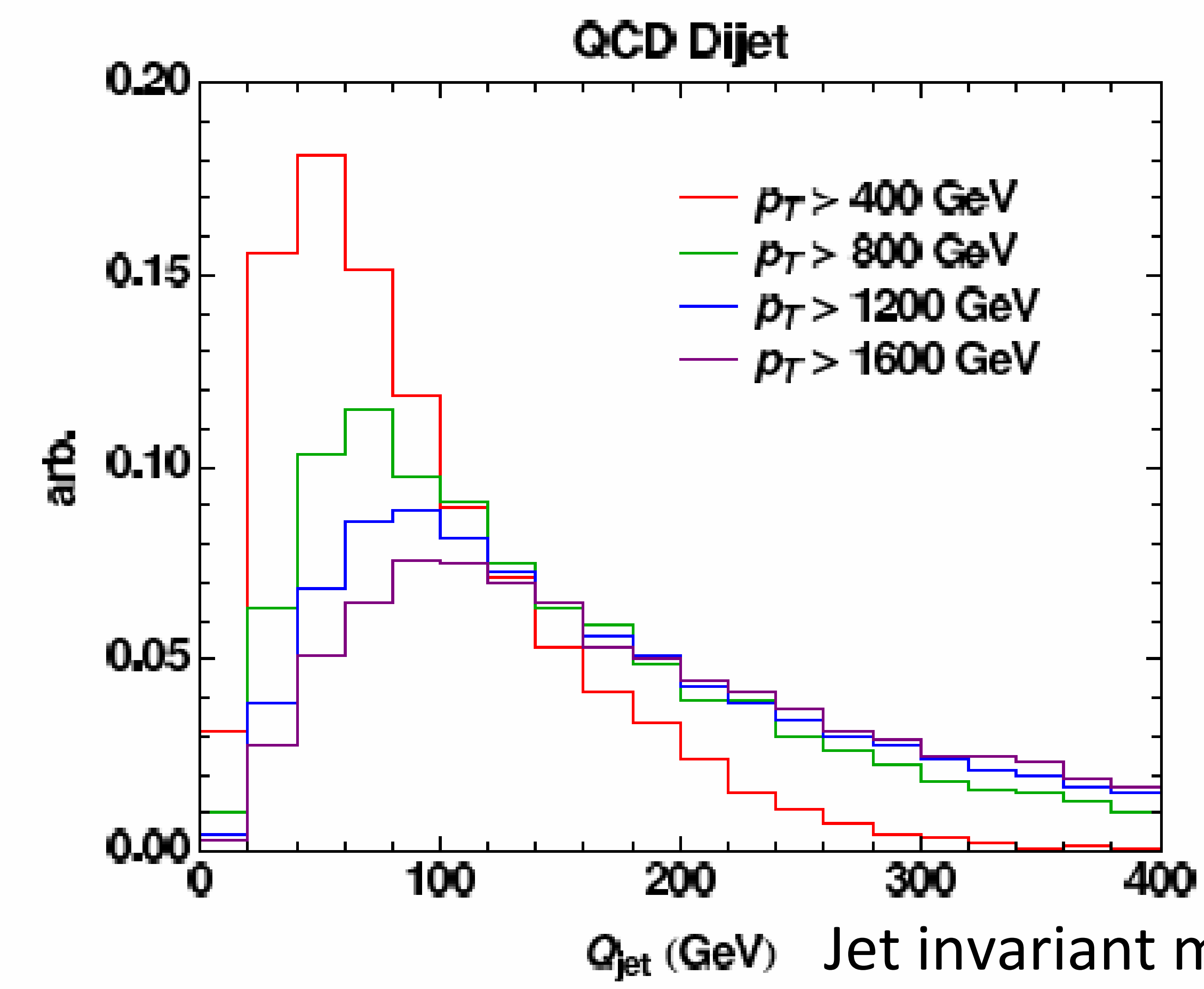
Motivation

Propose a theoretical framework for study of jet physics

Energetic heavy particles

- Large Hadron Collider (LHC) provide a chance to search new physics
- New physics involve heavy particles decaying possibly through cascade to SM light particles
- New particles, if not too heavy, may be produced with sufficient boost -> a single jet
- How to differentiate heavy-particle jets from QCD jets?
- Similar challenge of identifying energetic top quark at LHC

Fat QCD jet looks like top jet at high pT



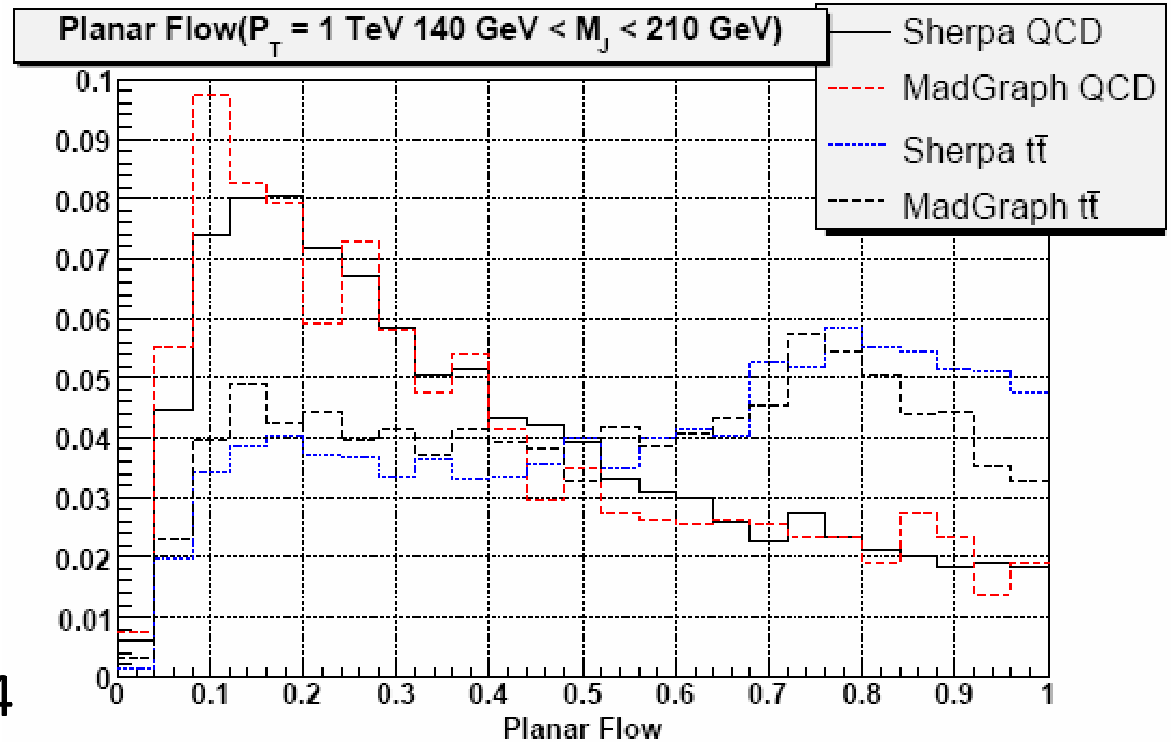
Jet invariant mass

Thaler & Wang 0806.0023 Pythia 8.108

Jet substructure

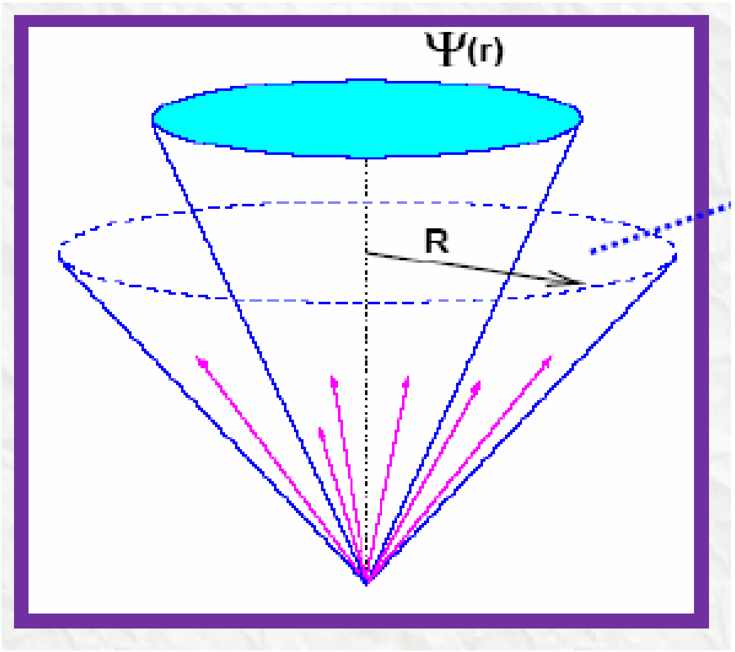
- Make use of differences in jet internal structure in addition to standard event selection criteria
- Example: planar flow
- QCD jets: 1 to 2 linear flow, linear energy deposition in detector
- Top jets: 1 to 3 planar flow

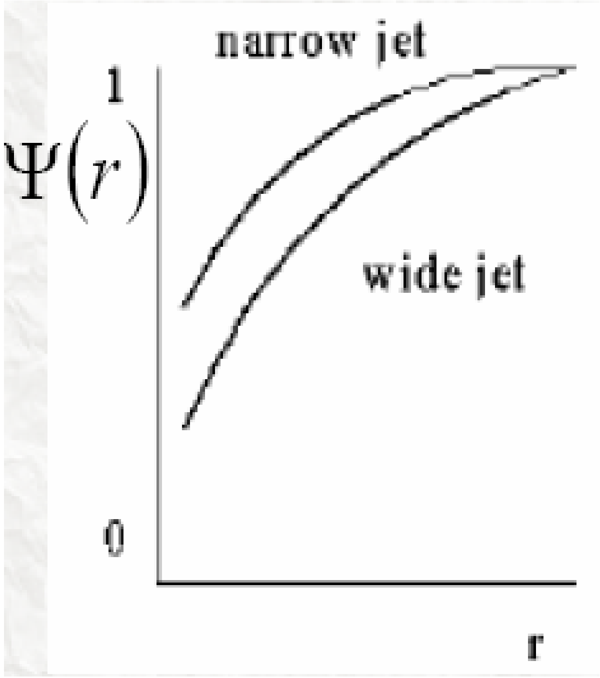
Almeida et al, 0807.0234



Our proposal: jet energy profile

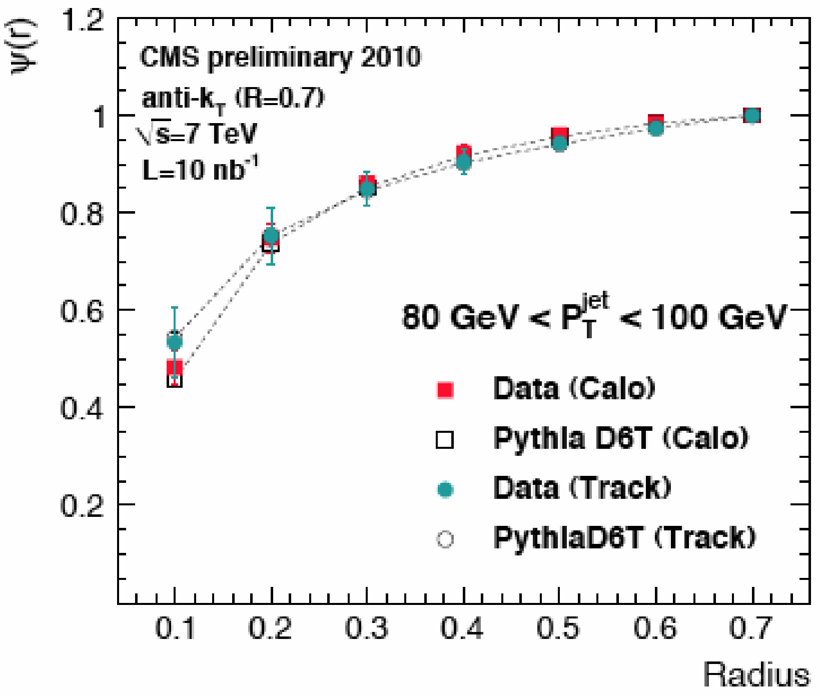
- Energy fraction in cone size of r, $\Psi(r)$, $\Psi(R) = 1$
- Quark jet is narrower than gluon jet
- Heavy quark jet energy profile should be different

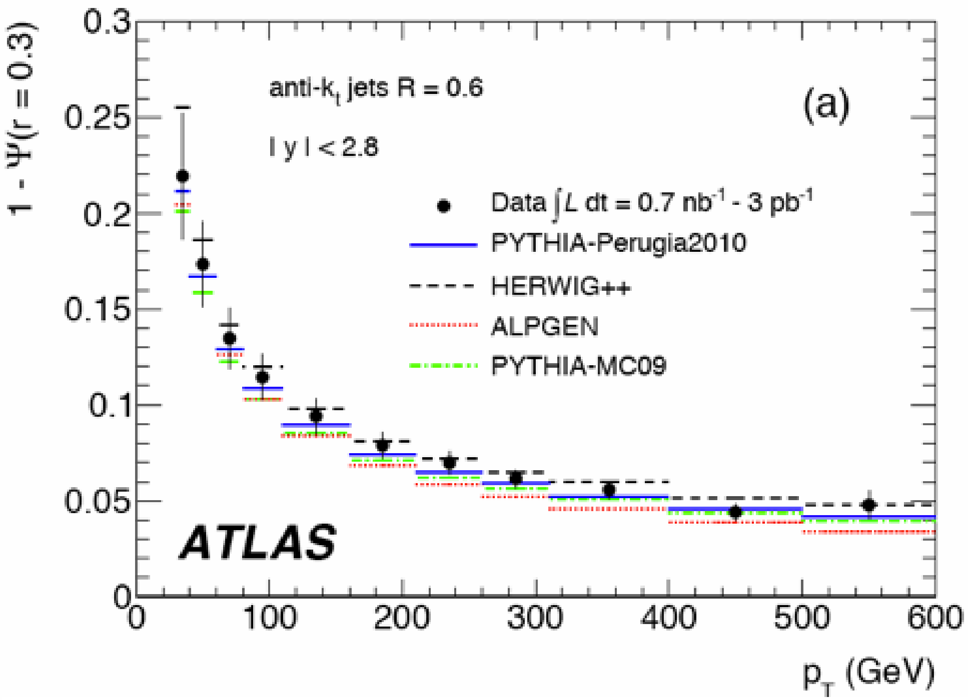




Preliminary data

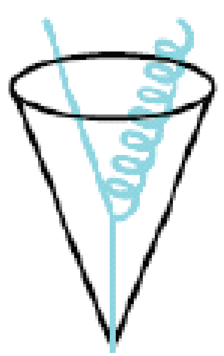
$$\Psi(r) = \frac{1}{\mathrm{N_{jet}}} \sum_{\mathrm{jets}} \frac{P_T(0,r)}{P_T(0,R)}, \qquad 0 \leq r \leq R$$

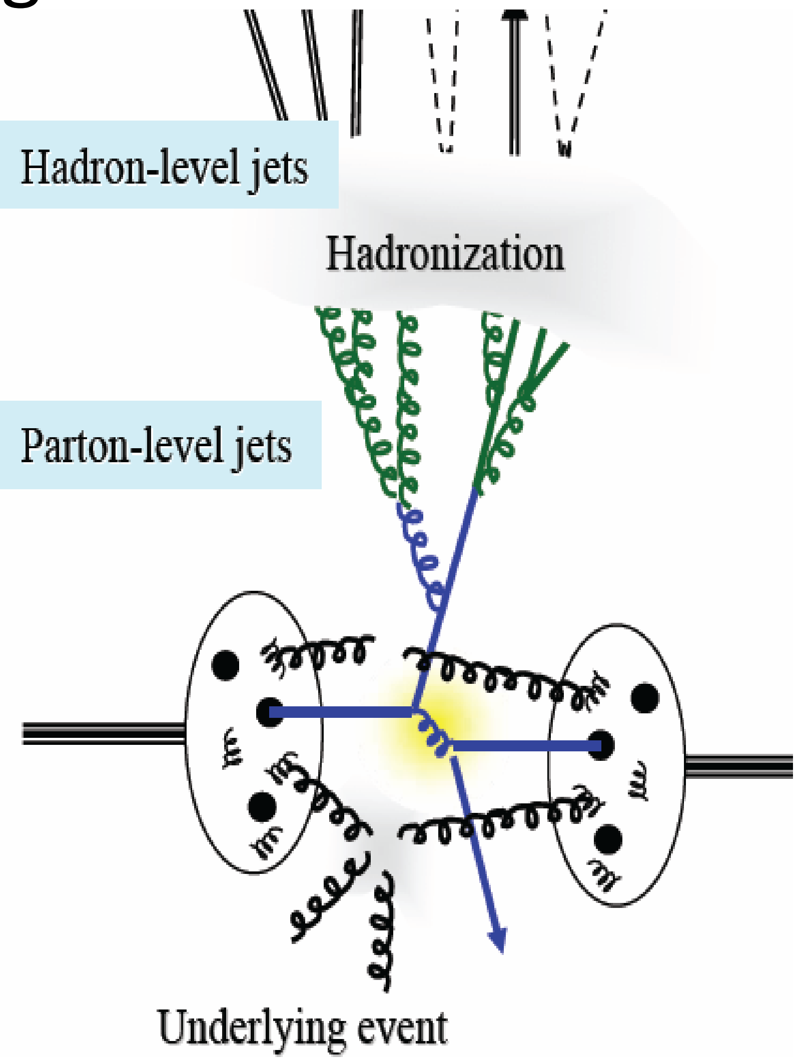




Various approaches

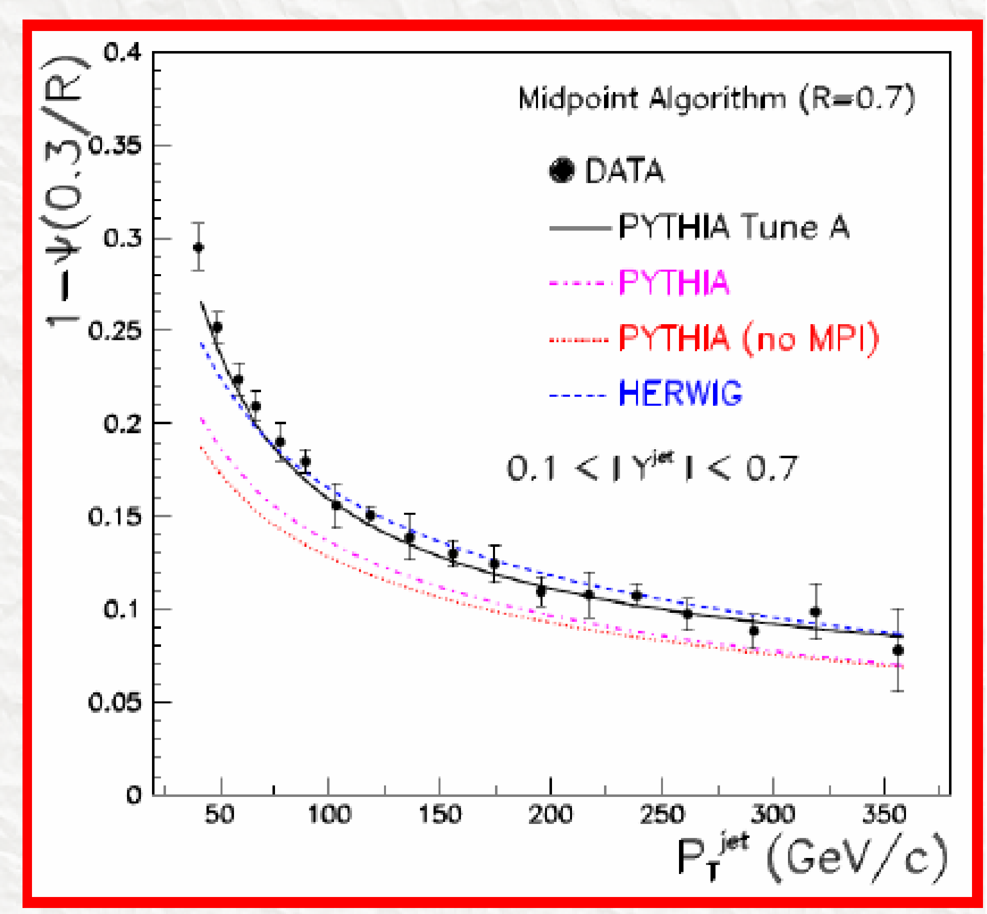
- Event generator: leading log radiation, hadronization, underlying events
- Fixed order: finite number of collinear/soft radiations
- Resummation: all-order collinear/soft radiations





Why resummation?

- Monte Carlo may have ambiguities from tuning scales for coupling constant
- NLO is not reliable at small jet mass
- Predictions from QCD resummation are necessary



Tevatron data vs MC predictions

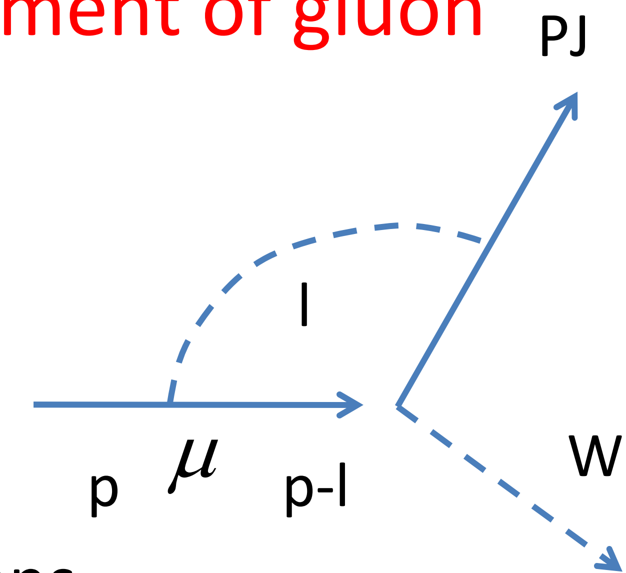
N. Varelas 2009

Jet factorization

Eikonalization

- Jet is dominated by collinear dynamics from loop momentum l to parallel jet momentum P_J^+
- For attachment of collinear gluon, eikonalization holds -> detachment of gluon
- For $l^- << l^+$

$$\frac{(p-l)_\alpha \gamma^\alpha + m_t}{(p-l)^2 - m_t^2}\gamma^\mu \approx \frac{\xi^\mu}{-\xi \cdot l}$$



eikonal vertex, eikonal propagator

-> Wilson line, collect collinear gluons

$$\Phi_\xi^{(f)}(\infty,0;0) = \mathcal{P}\left\{e^{-ig\int_0^\infty d\eta\, \xi\cdot A^{(f)}(\eta\,\xi^\mu)}\right\}$$

Jet definitions

Almeida et al. 08

- Eikonalization leads to factorization
- Quark jet

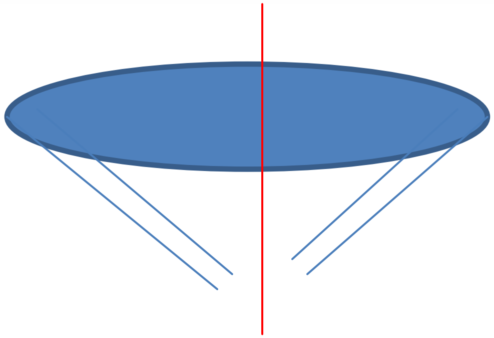
$$J_i^q(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2\sqrt{2}\,(p_{0,J_i})^2} \frac{\xi_\mu}{N_c} \sum_{N_{J_i}} \mathrm{Tr}\left\{ \gamma^\mu \langle 0|q(0)\Phi_\xi^{(\bar{q})\dagger}(\infty,0)|N_{J_i}\rangle\langle N_{J_i}|\Phi_\xi^{(\bar{q})}(\infty,0)\bar{q}(0)|0\rangle \right\}$$
$$\times\delta\left(m_J^2 - \tilde{m}_J^2(N_{J_i},R)\right)\delta^{(2)}(\hat{n}-\tilde{n}(N_{J_i}))\delta(p_{0,J_i}-\omega(N_{J_c})), \quad \text{(A.3)}$$

- Gluon jet

$$J_i^g(m_J^2, p_{0,J_i}, R) = \frac{(2\pi)^3}{2(p_{0,J_i})^3} \sum_{N_{J_i}} \langle 0|\xi_\sigma F^{\sigma\nu}(0)\Phi_\xi^{(g)\dagger}(0,\infty)|N_{J_i}\rangle\langle N_{J_i}|\Phi_\xi^{(g)}(0,\infty)F_\nu^{\rho}(0)\xi_\rho|0\rangle$$
$$\times\delta\left(m_J^2 - \tilde{m}_J^2(N_{J_i},R)\right)\delta^{(2)}(\hat{n}-\tilde{n}(N_{J_i}))\delta(p_{0,J_i}-\omega(N_{J_c})). \quad \text{(A.4)}$$

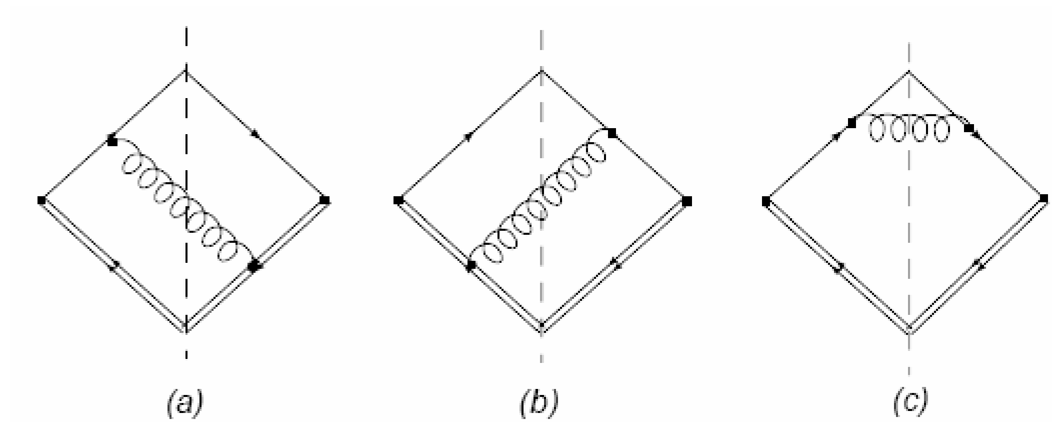
- LO jet

$$J_i^{(0)}(m_{J_i}^2, p_{0,J_i}, R) = \delta(m_{J_i}^2)$$

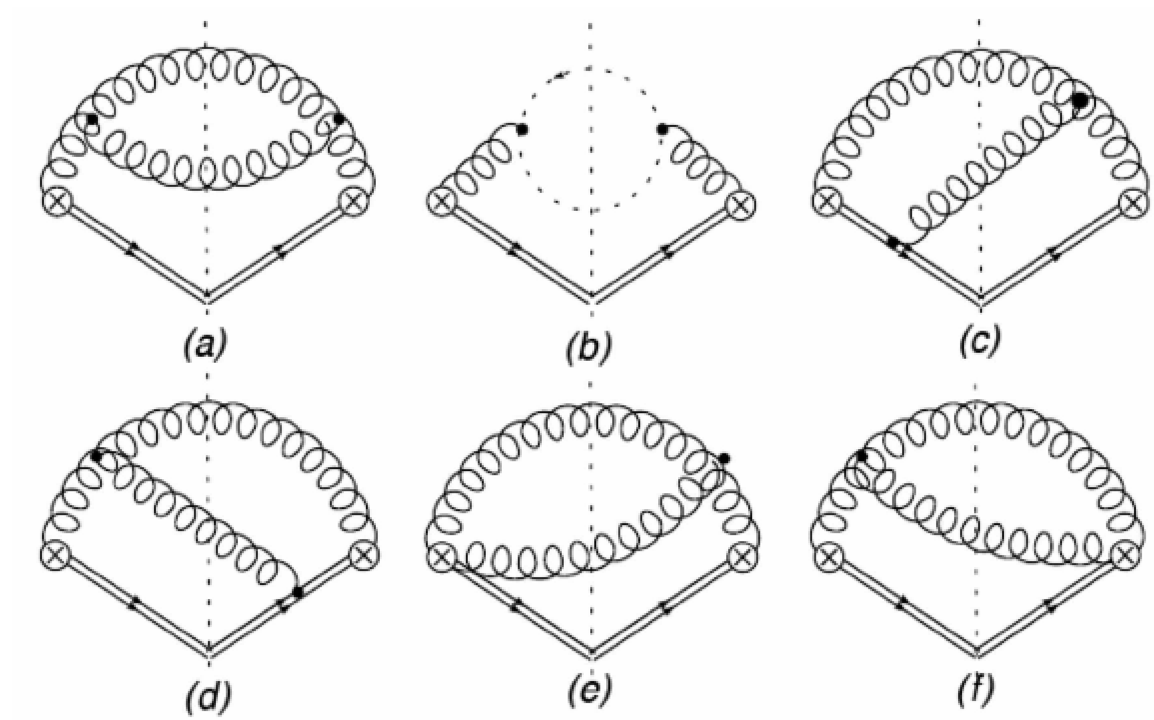


NLO diagrams

- NLO quark jet



- NLO gluon jet



Resummation

Technical part, ideas only

Key idea

- Key idea of resummation technique is to vary Wilson line vector to arbitrary n
- Collinear dynamics is independent of n
- Variation effect does not contain collinear dynamics, and can be factorized from jet
- Derive differential equation, whose solution resums important logs $\ln(P_J^0/m_J)$
- Study derivative $-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} J$

Special vertex

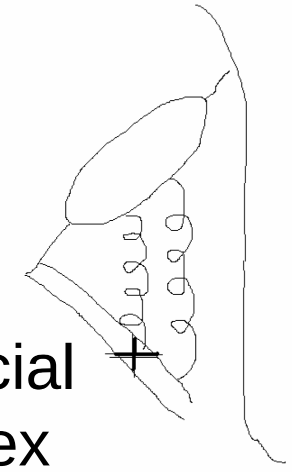
- Differentiation applies only to Wilson line

$$-\frac{n^2}{P_J\cdot n}P_{J\alpha}\frac{d}{dn_\alpha}\frac{n_\mu}{n\cdot l}=\frac{n^2}{P_J\cdot n}\left(\frac{P_J\cdot l}{n\cdot l}n_\mu-P_{J\mu}\right)\frac{1}{n\cdot l}=\frac{\hat{n}_\mu}{n\cdot l}$$

- Special vertex $\hat{n}_\mu$ kills collinear dynamics
- Differentiated gluon gives hard and soft contributions
- Differentiated gluon, carrying dynamics different from the jet, is factorizable

Soft factorization (virtual)

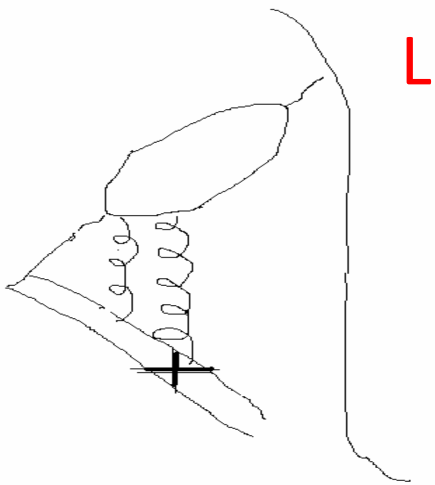
- If differentiated gluon is soft, special vertex locates at outer end of Wilson line



special vertex

- If it locates inside (see figure), both gluons are soft -> NLO soft kernel

LO soft kernel $K_v^{(1)}$



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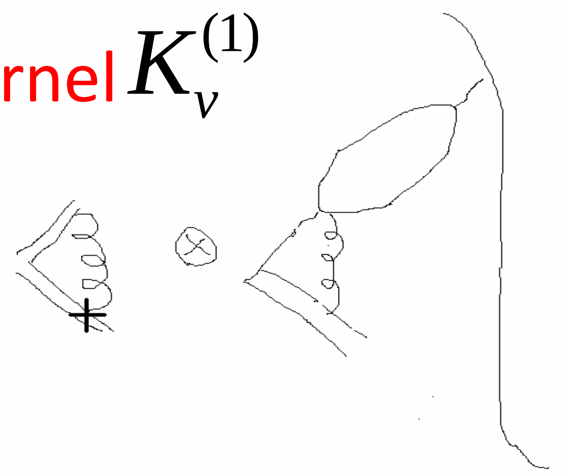
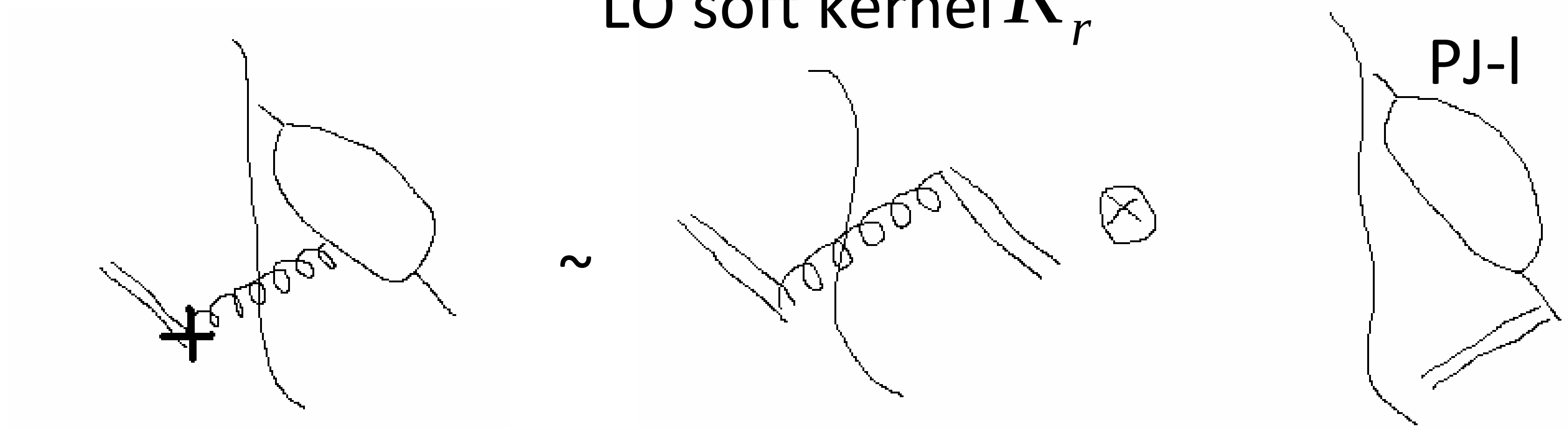


Soft factorization (real)

- Similar argument applies to factorization of differentiated soft real gluon

LO soft kernel $K_r^{(1)}$

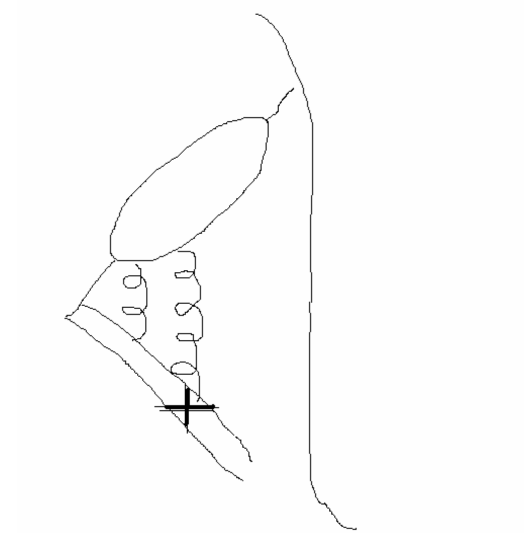


$$g^2 C_F \int \frac{d^4 l}{(2\pi)^4} \frac{\hat{n} \cdot P_J}{(n \cdot l + i\epsilon)(P_J \cdot l - i\epsilon)} 2\pi\delta(l^2 - a^2) J(m_J^2 - 2P_J \cdot l, P_J \cdot n, n^2, R)$$

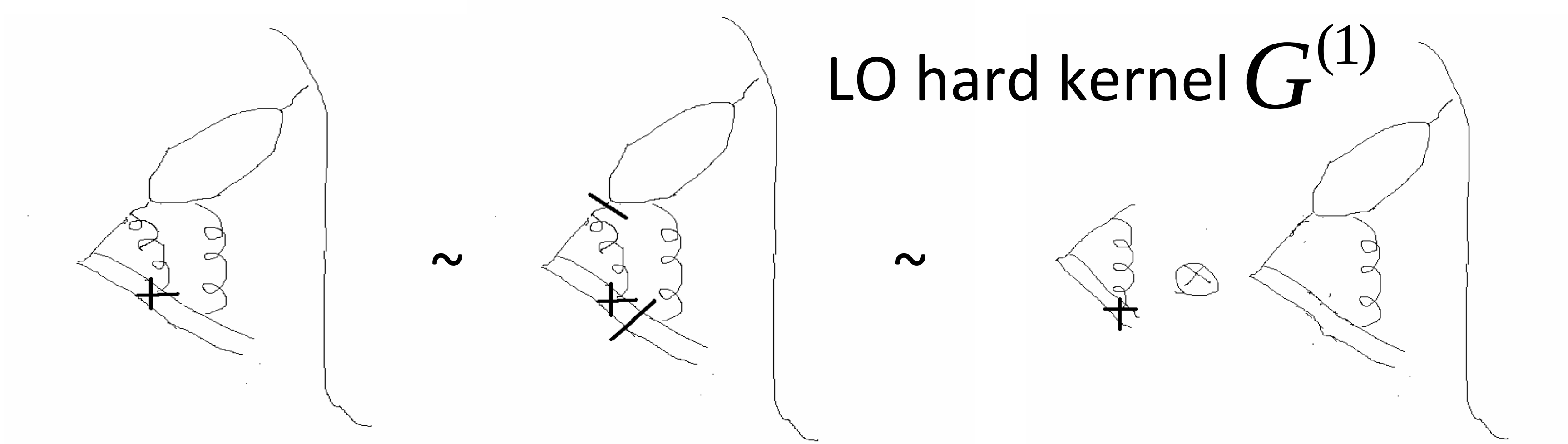
Jet invariant mass excluding soft momentum l, $(P_J - l)^2$

Hard factorization

- If differentiated gluon is hard, special vertex locates at inner end of Wilson line



- If it locates at outside, both gluons are hard -> NLO hard kernel



Resummation equation

- Up to leading logarithms, resummation equation is given by

$$-\frac{n^2}{P_J \cdot n} P_J^\alpha \frac{d}{dn^\alpha} J = [G^{(1)} + K_v^{(1)} + K_r^{(1)}] \otimes J$$

- For next-to-leading-logarithm accuracy, G and K are evaluated to two loops $\nu^2 \equiv (P_J \cdot n)^2/|n^2|$

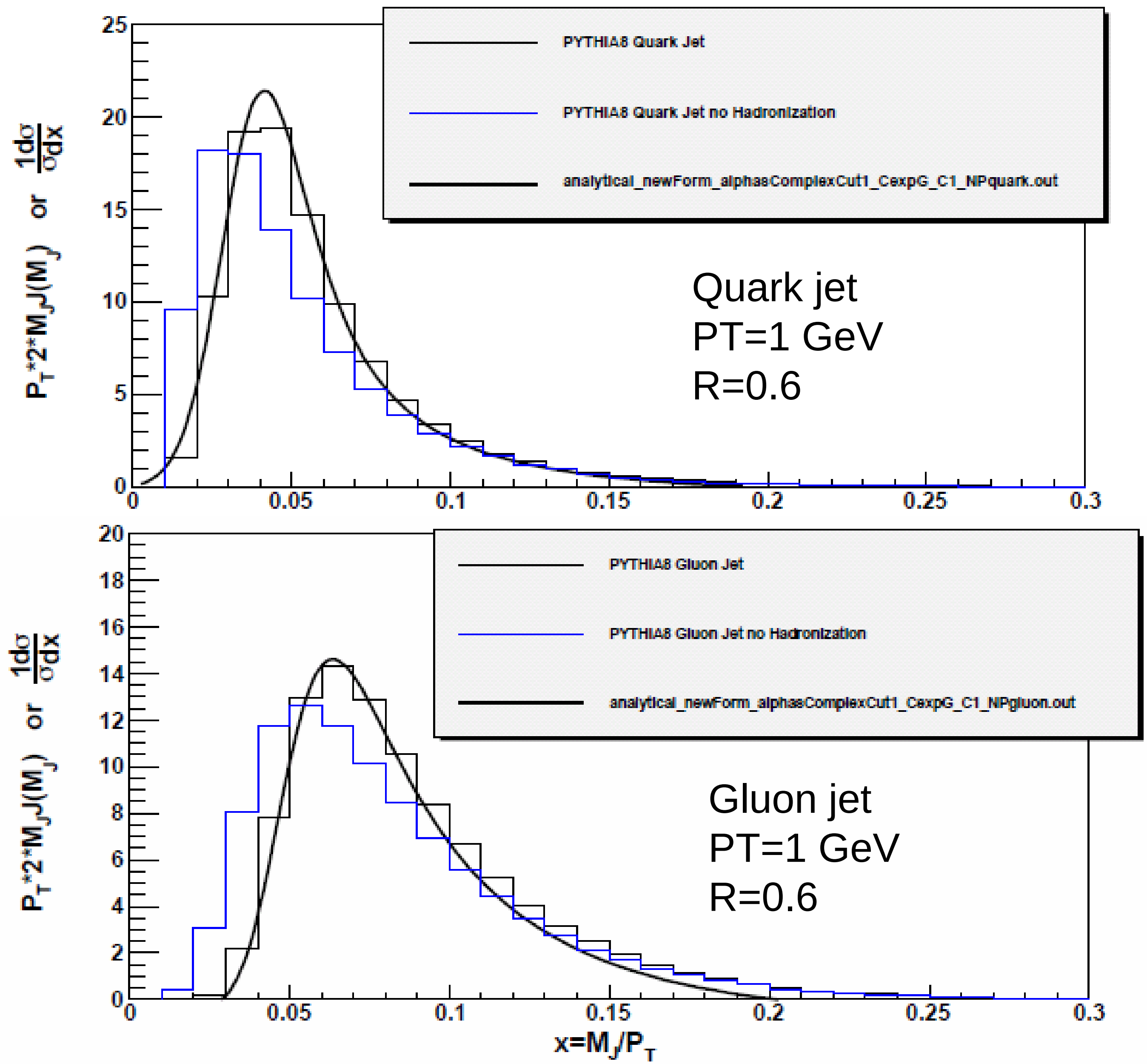
- Explicit expression in moment N space

$$\bar{J}(N, P_J \cdot n, n^2, R) \equiv \int_0^1 dx (1-x)^{N-1} J(x, P_J \cdot n, n^2, R) \qquad x \equiv m_J^2/(P_J^0)^2$$

$$-\frac{n^2}{P_J \cdot n} P_{J\alpha} \frac{d}{dn_\alpha} J(N, P_J \cdot n, n^2, R) = 2\nu^2 \frac{d}{d\nu^2} J(N, P_J \cdot n, n^2, R)$$

$$= -\left[\frac{\alpha_s(\nu^2)}{\pi} C_F (4 \ln 2 + 2\gamma_E - 1) + \int_{(P_J^0)^2/N^2}^{\nu^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \lambda_K(\alpha_s(\bar{\mu}^2))\right] J(N, P_J \cdot n, n^2, R)$$

Jet mass distribution



Jet energy profile

Prescription

- Define jet shape function $J_E(r)$ as follows:
- Associate $k_i^0\Theta(r-\theta_i)$ with each final-state particle i within jet cone r, r<R
- Separation $\sum_i k_i^0\Theta(r-\theta_i) = \sum_i k_i^0\Theta(r-\theta_i) + l^0\Theta(r-\theta)$

negligible in soft region

- Still vary Wilson direction n.
- First term gives

$$[G^{(1)} + K_v^{(1)} + K_r^{(1)}]\otimes J_E$$

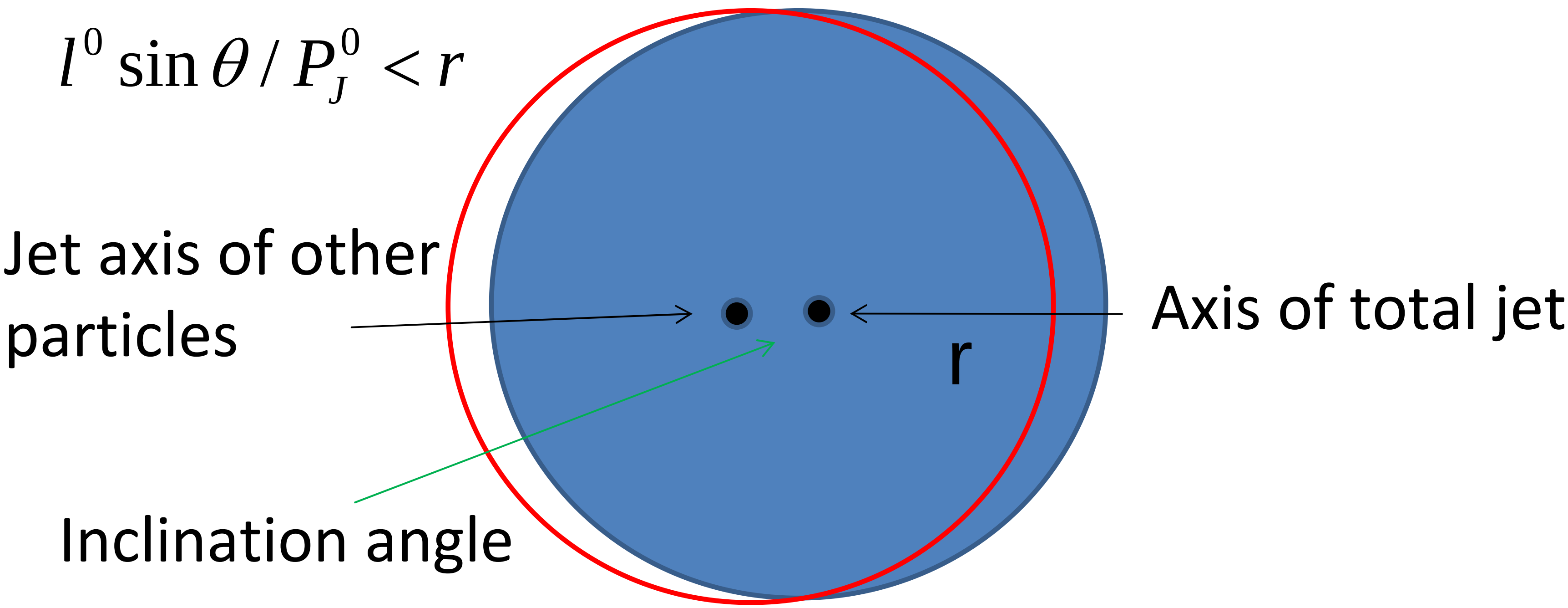
$$\downarrow$$

$$g^2 C_F \int \frac{d^4 l}{(2\pi)^4}\frac{\hat{n}\cdot P_J}{(n\cdot l + i\epsilon)(P_J\cdot l - i\epsilon)}2\pi\delta(l^2)J^E(m_J^2 - 2P_J\cdot l, P_J, \nu^2, R, r)$$

Soft gluon effect

- Differentiated soft real gluon renders jet axis of other particles inclined by small ar $l^0 \sin\theta / P_J^0$
- This jet axis can not go outside of the subcone

$$l^0 \sin\theta / P_J^0 < r$$



Resummation equation

- Resummation equation for jet profile

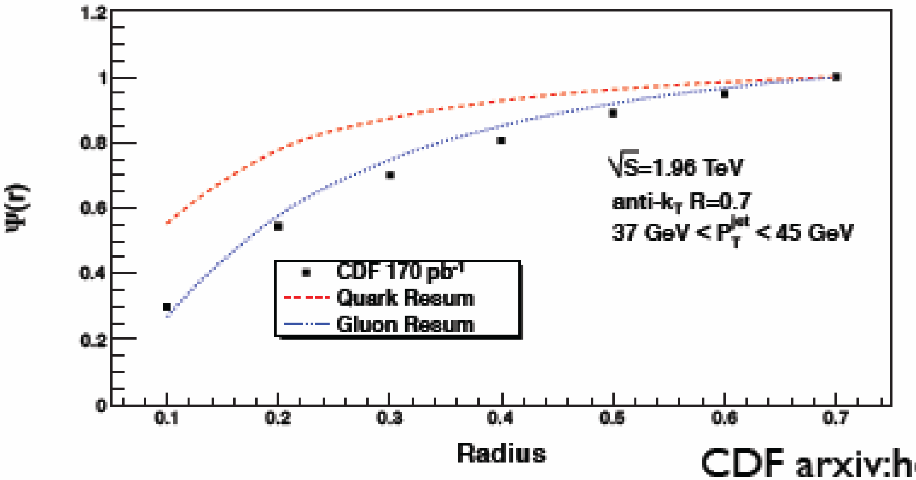
$$\bar{K}_r^{(1)}(1) = g^2 C_F \int \frac{d^4 l}{(2\pi)^3} \frac{n^2}{(n\cdot l + i\epsilon)^2} \delta(l^2 - a^2) \Theta\left(r - \frac{|\mathbf{l}|\sin\theta}{P_J^0}\right)$$

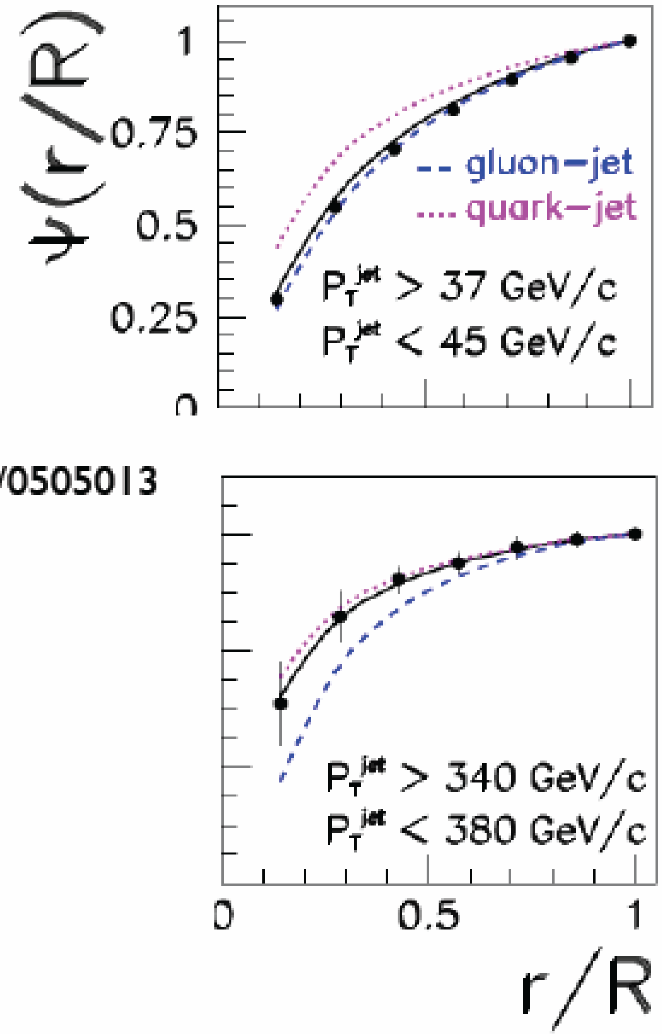
$$-\frac{n^2}{v\cdot n} v_\alpha \frac{d}{dn_\alpha} \bar{J}_E(N, P_J \cdot n, n^2, R, r)$$

$$= 2[G^{(1)} + K^{(1)}(N)] \bar{J}_E(N, P_J, \nu^2, R, r)$$

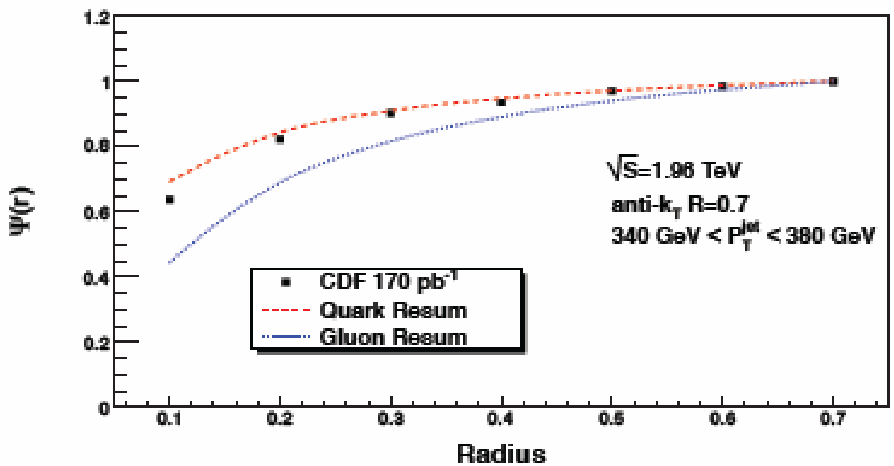
- Consider N=1 here, corresponding to integration over jet mass (insensitive to nonperturbative physics)
- Resum ln(angle r) from phase space constraint for real gluons

Jet energy profile @ CDF



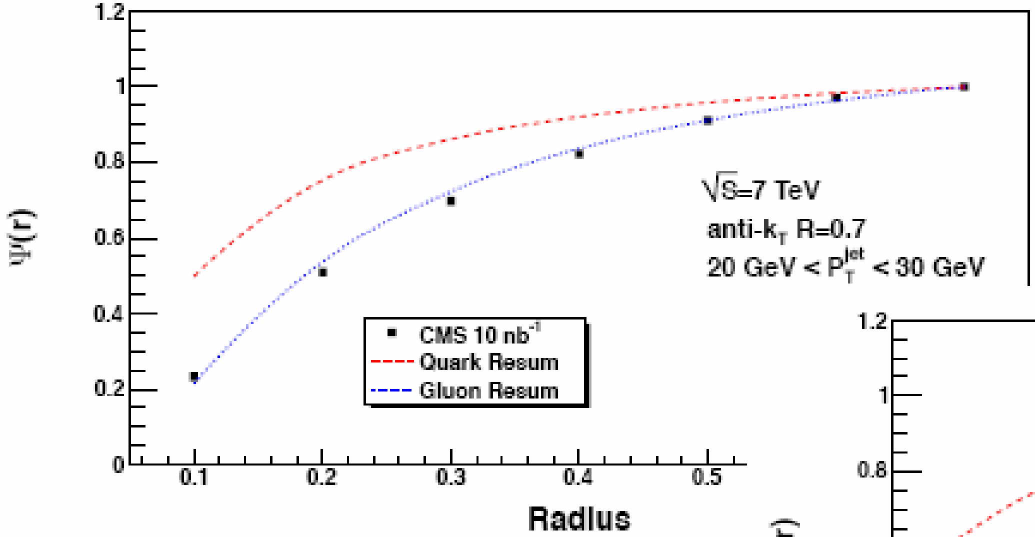


CDF arxiv:hep-ex/0505013

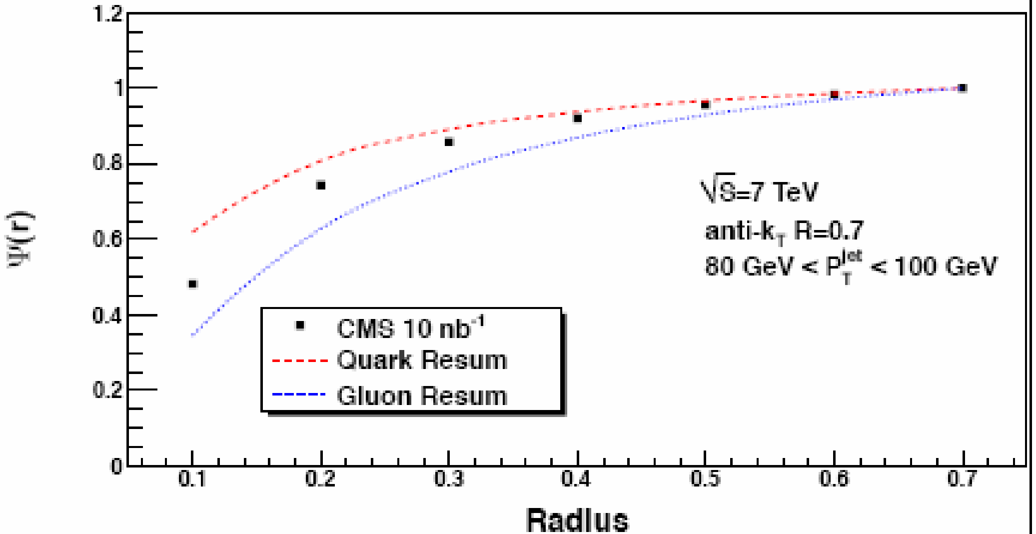


Gluon jet and quark jet dominates in low and high pT region, respectively.

Jet energy profile @ CMS



agree well with
Resummation calculation



Need to convolute quark and gluon jet energy function with hard scattering amplitudes in order to compared to data directly. This calculation is in progress.

Summary

- Jet substructure improves jet identification
- Perturbative calculation is not reliable in extreme kinematic region (e.g. small jet invariant mass)
- Event generators may have ambiguities (from tuning scales for coupling constant)
- QCD resummation provides reliable prediction, independent check, and alternative approach
- Analyzed jet function and jet profile for light quark. Results are consistent with event generators qualitatively
- Numerical work on heavy-quark jet in progress

Back-up slides

Clustering algorithms

Algorithm description :

- Define a distance

$$D_{ij} = \min(P_{Ti}^2, P_{Tj}^2)\frac{\Delta R_{ij}^2}{R^2} \qquad D_i = P_{Ti}^2$$

- Compute all $\{D_{ij}, D_i\}$ and $d = \min(\{D_{ij}, D_i\})$
  - if $d = D_{ij}$: combine jet i with jet j
  - if $d = D_i$define jet i as a final jet
- Exhaust all proto-jets

Variants : Anti-kt and Cambridge. In distance formula replace $P_T^2 \longrightarrow P_T^{2p}$

- p=1 : standard Kt
- p=0 : cambridge
- p=-1 : Antit-kt

Factorization theorem

- Different dynamics (characterized by E and by hadronic scale Λ) factorize.
- The former into hard kernels H, and the latter into distribution amplitudes ϕ.
- Factorization theorem holds up to all order in α_s, but to certain power in 1/E.
- H is process-dependent, but calculable.
- Inputs ϕ are universal (process-independent) $\Rightarrow$ predictive power of factorization theorem

Solution in Mellin space

- Jet function

$$\bar{J}_{\mathrm{in}}(N, P_J^0, R)\left[-\int_{1/N}^{1}\frac{dy}{y}\lambda_q(\alpha_s(\sqrt{y}P_J^0))\right]$$
$$\times\exp\left\{-\int_{1/N}^{C}\frac{dy}{y}\left[\frac{\alpha_s\left(yP_J^0\right)}{2\pi}C_F\left(2\gamma_E-1\right)+\int_{1/N}^{y}\frac{dz}{z}\lambda_K(\alpha_s(zP_J^0))\right]\right\}$$

- Initial condition $\bar{J}_{in}$ without large logarithm is evaluated up to NLO
- Inverse Mellin transform to get distribution in jet mass