# Spin Physics of the Nucleon With New Quantities 

Bo－Qiang Ma（马皌强）<br>PKU（北京大学）

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Collaborators：Enzo Barone，Stan Brodsky，Jacques Soffer，Andreas Schafer，Ivan Schmidt，Jian－Jun Yang，Qi－Ren Zhang，
and students：Zhun Lu，Bing Zhang，Jun She，Jiacai Zhu

## It has been 20 years of the proton "spin crisis" or "spin puzzle"

- Spin Structure:

$$
\begin{aligned}
& \Sigma=\Delta u+\Delta d+\Delta s \approx 0.020 \\
& \Sigma=\Delta u+\Delta d+\Delta s \approx 0.3
\end{aligned}
$$

spin "crisis" or "puzzle": where is the proton's missing spin?

## The Proton "Spin Crisis"

## $\Sigma=\Delta u+\Delta d+\Delta s \approx 0.3$

## In contradiction with the naïve quark model expectation:

Naive Quark Model:

$$
\begin{aligned}
& \Delta u=\frac{4}{3} ; \quad \Delta d=-\frac{1}{3} ; \quad \Delta s=0 \\
& \Sigma=\Delta u+\Delta d+\Delta s=1
\end{aligned}
$$

## The Ellis-Jaffe sum rule \& Its violation

$$
A_{1}^{p}=\int_{0}^{1} d x g_{1}^{p}(x)=\frac{1}{2}\left[\frac{4}{9} \Delta u+\frac{1}{9} \Delta d+\frac{1}{9} \Delta s\right]
$$

- Neutron beta decay and isospin symmetry

$$
\Delta u-\Delta d=\frac{G_{A}}{G_{V}}=1.261
$$

- Strangeness changing hyperon decay and $\mathrm{SU}(3)$ symmetry

$$
\Delta u+\Delta d-2 \Delta s=0.675
$$

- The assumption of zero strange spin constribution $\Delta s=0$

$$
\text { The Ellis-Jaffe sum } \quad A_{1}^{p}=\int_{0}^{1} d x g_{1}^{p}(x)=0.198
$$

However, what EMC measured $A_{1}^{p}=\int_{0}^{1} d x g_{1}^{p}(x)=0.126$

## The first stage of experiments

- Non-zero strange spin constribution

$$
\begin{gathered}
\Delta u=0.750 \\
\Delta d=-0.511 \\
\Delta s=-0.218 \\
\Sigma=\Delta u+\Delta d+\Delta s \approx 0.020 \\
\text { A large strange spin contribution? }
\end{gathered}
$$

## A previous global fit:

 $\mathbf{S U}(3)$ symmetry+measured $g_{1}^{p} g_{1}^{n}$$$
\begin{gathered}
\Delta u=0.83 \pm 0.03 \\
\Delta d=-0.43 \pm 0.03 \\
\Delta s=-0.10 \pm 0.03 \\
\Sigma=\Delta u+\Delta d+\Delta s \approx 0.3 \\
\text { The second stage of experiments. }
\end{gathered}
$$

## The third stage of experiments: $g_{1}^{p} g_{1}^{n}+$ semi-inclusive DIS process

$$
\begin{gathered}
\Delta u=0.599 \pm 0.022 \pm 0.065 \\
\Delta d=-0.280 \pm 0.026 \pm 0.057 \\
\Delta s=0.028 \pm 0.033 \pm 0.009 \\
\Sigma=\Delta u+\Delta d+\Delta s \approx 0.347 \pm 0.024 \pm 0.040
\end{gathered}
$$

HERMES Collaboration, PRL92 (2004) 012005.

## The strange contribution to the proton spin

# $\Delta s \approx-0.2 \rightarrow-0.1 \rightarrow 0.03$ 

$\Delta s \neq 0$, how large?

## Many Theoretical Explanantions

- The sea quarks of the proton are largely negatively polarized
- The gluons provide a significant contribution to the proton spin

It was though that the spin "crisis" cannot be understood within the quark model: " the lowest uud valence component of the proton is estimated to be of only a few percent." R.L. Jaffe and Lipkin, PLB266(1991)158

## The proton spin crisis <br> \& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity $\Delta \mathrm{q}$ measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.
B.-Q. Ma, J.Phys. G 17 (1991) L53
B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482


## Pion Spin Structure and Form Factor

## Based on collaborated works with T.Huang and Q.-X.Shen

[1] T. Huang, B.Q. MEA, and Q.X. Sherı, Ply̧s. Rex. D 49, 1490 (1994).
[2] B. Q. MEx, Z. Plyys. $\Lambda$ 345, 321 (1903).
[3] B.Q. MIa and T.Huang, J. Plỵs. G 21, (76a) (1905).

Fu-Guang Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 53 (1996) 6582-6585.
Fu-Guang Cao, Jun Cao, Tao Huang, and Bo-Qiang Ma, Phys.Rev.D 55 (1997) 7107-7113. Jun Cao, Fu-Guang Cao, Tao Huang, Bo-Qiang Ma, Phys. Rev. D 58 (1998) 113006.

Analysis of the pion wave function in the light-cone formalism<br>Tao Huang, Bo-Qiang Ma, and Qi-Xing Shen<br>Center of Theoretical Physics, China Center of Advanced Science and Technology (World Laboratory), Beijing, China and Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4), Beijing 100039, China*<br>(Received 22 January 1991; revised manuscript received 12 August 1993)


#### Abstract

We analyze several general constraints on the pionic valence-state wave function. It is found that the present model wave functions used in the light-cone formalism of perturbative quantum chromodynamics have failed to reproduce the Chernyak-Zhitnitsky (CZ) distribution amplitude which is required to fit the pionic form factor data and the reasonable valence-state structure function which does not exceed the pionic structure function data for $x \rightarrow 1$ simultaneously. A possible model wave function which can satisfy all the general constraints has been suggested and analyzed.


PACS númber(s): $12.38 .-\mathrm{t}, 12.39 .-\mathrm{x}, 13.60 .-\mathrm{r}$
calculation. Also, we have shown that there are two higher helicity $\left(\lambda_{1}+\lambda_{2}= \pm 1\right)$ components in the lightcone wave function for the pion as a natural consequence from the Melosh rotation and it is speculated that these components should be incorporated into the perturbative quantum chromodynamics. Some progress has been

## Pion Spin-Space Wave Function in Rest Frame

In the pion rest frame, the instant-form spin space wavefunction of pion is

$$
\chi_{T}=\left(\chi_{1}^{\dagger} \chi_{2}^{\downarrow}-\chi_{2}^{\dagger} \chi_{1}^{\downarrow}\right) / \sqrt{2},
$$

in which $\chi_{i}^{\dagger \cdot \downarrow}$ are the two-component Pauli spinors.

## Melosh Rotation for Spin-1/2 Particle

The connection between spin states in the rest frame and infinite momentum frame
Or between spin states in the conventional equal time dynamics and the light-front dynamics

$$
\begin{aligned}
& \chi^{\dagger}(T)=w\left[\left(q^{-}+m\right) \chi^{\dagger}(F)-q^{R} \chi^{\downarrow}(F)\right] ; \\
& \chi^{\downarrow}(T)=w\left[\left(q^{-}+m\right) \chi^{\downarrow}(F)+q^{I} \chi^{\dagger}(F)\right] .
\end{aligned}
$$

## The Wigner Rotation

for a rest particle $(m, \overrightarrow{0})=p^{\mu} \quad(0, \vec{s})=\mathrm{w}^{\mu}$
for a moving particle $\mathrm{L}(p) p=(m, \overrightarrow{0}) \quad(0, \vec{s})=\mathrm{L}(p) w / m$
$\mathrm{L}(p)=$ ratationless Lorentz boost
Wigner Rotation
$\vec{s}, p_{\mu} \rightarrow \overrightarrow{s^{\prime}}, p_{\mu}^{\prime}$
$\overrightarrow{s^{\prime}}=R_{w}(\Lambda, p) \vec{s} \quad p^{\prime}=\Lambda p$
$R_{w}(\Lambda, p)=\mathrm{L}\left(p^{\prime}\right) \Lambda \mathrm{L}^{-1}(p) \quad$ a pure rotation
E.Wigner, Ann.Math.40(1939)149

## The Lowest Valence State Wave Function in Light-Cone

$$
\begin{aligned}
\mid \psi_{4 \bar{q}}^{\pi}> & =\psi\left(x, \mathbf{k}_{-}, \uparrow, \downarrow\right)\left|\uparrow \downarrow>+\psi\left(x, \mathbf{k}_{-}, \downarrow, \uparrow\right)\right| \downarrow \uparrow> \\
& +\psi\left(x, \mathbf{k}_{-}, \uparrow, \uparrow\right)\left|\uparrow \uparrow>+\psi\left(x, \mathbf{k}_{-}, \downarrow, \downarrow\right)\right| \downarrow \downarrow>
\end{aligned}
$$

where

$$
\psi\left(x, \mathbf{k}_{-}, \lambda_{1}, \lambda_{2}\right)=C_{0}^{F}\left(x, \mathbf{k}_{-}, \lambda_{1}, \lambda_{2}\right) \varphi\left(x, \mathbf{k}_{-}\right) .
$$

Here $\varphi\left(\boldsymbol{x}, \mathbf{k}_{-}\right)$is the momentum space wave function in the
light-cone formalism.

## The Spin Component Coefficients

The spin component coefficients $C_{0}^{F}$ have the forms,

$$
C_{0}^{F}(x, q, \uparrow, \downarrow)=w_{1} w_{2}\left[\left(q_{1}^{-}+m\right)\left(q_{2}^{-}+m\right)-q_{-}^{2}\right] / \sqrt{ } 2 ;
$$

$$
C_{0}^{F}\left(x, q, \psi_{,} \uparrow\right)=-w_{1} u_{2}\left[\left(q_{1}^{-}+m\right)\left(q_{2}^{-}+m\right)-\mathrm{q}_{-}^{2}\right] / \sqrt{ } 2
$$

$$
C_{0}^{F}(x, q, \uparrow, \uparrow)=w_{1} w_{2}\left[\left(q_{1}^{-}+m\right) q_{2}^{L}-\left(q_{2}^{-}+m\right) q_{1}^{L}\right] / \sqrt{ } 2
$$

$$
C_{0}^{F}(x, q, \downarrow, \downarrow)=w_{1} w_{2}\left[\left(q_{1}^{-}+m\right) q_{2}^{R}-\left(q_{2}^{-}+m\right) q_{1}^{F}\right] / \sqrt{ } 2
$$

$C_{0}^{F}$ satisfy the relation
$\sum_{\lambda_{1}, \lambda_{2}} C_{0}^{F}\left(x, \mathbf{k}_{-}, \lambda_{1}, \lambda_{2}\right) C_{0}^{F}\left(x, \mathbf{k}_{-}, \lambda_{1}, \lambda_{2}\right)=1$.

## From field theory vertex calculation

$$
\begin{gathered}
\frac{\bar{v}\left(p_{2}^{+}, p_{2}^{-},-\mathbf{k}_{\perp}\right)}{\sqrt{p_{2}^{+}}} \gamma_{5} \frac{u\left(p_{1}^{+}, p_{1}^{-}, \mathbf{k}_{\perp}\right)}{\sqrt{p_{1}^{+}}} \\
\left\{\begin{array}{l}
\frac{\bar{v}_{\downarrow}}{\sqrt{p_{2}^{+}}} \gamma_{5} \frac{u_{\uparrow}}{\sqrt{p_{1}^{+}}}=-\frac{2 m P^{+}}{4 m x(1-x) P^{+2}} \\
\frac{\bar{v}_{\downarrow}}{\sqrt{p_{2}^{+}}} \gamma_{5} \frac{u_{\uparrow}}{\sqrt{p_{1}^{+}}}=+\frac{2 m P^{+}}{4 m x(1-x) P^{+2}} \\
\frac{\bar{v}_{\uparrow}}{\sqrt{p_{2}^{+}}} \gamma_{5} \frac{u_{\uparrow}}{\sqrt{p_{1}^{+}}}=+\frac{2\left(k_{1}+i k_{2}\right) P^{+}}{4 m x(1-x) P^{+2}} \\
\frac{\bar{v}_{\downarrow}}{\sqrt{p_{2}^{+}}} \gamma_{5} \frac{u_{\downarrow}}{\sqrt{p_{1}^{+}}}=+\frac{2\left(k_{1}-i k_{2}\right) P^{+}}{4 m x(1-x) P^{+2}}
\end{array}\right.
\end{gathered}
$$

Xiao \& Ma, PRD71(2005)014034

## The proton spin crisis <br> \& the Melosh-Wigner rotation

- It is shown that the proton "spin crisis" or "spin puzzle" can be understood by the relativistic effect of quark transversal motions due to the Melosh-Wigner rotation.
- The quark helicity $\Delta \mathrm{q}$ measured in polarized deep inelastic scattering is actually the quark spin in the infinite momentum frame or in the light-cone formalism, and it is different from the quark spin in the nucleon rest frame or in the quark model.
B.-Q. Ma, J.Phys. G 17 (1991) L53
B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482


## What is $\Delta q$ measured in DIS

- $\Delta \mathrm{q}$ is defined by $\Delta q \mathrm{~s}_{\mu}=\langle p, s| \bar{q} \gamma_{\mu} \gamma_{5} q|p, s\rangle$

$$
\Delta q=\langle p, s| \bar{q} \gamma^{+} \gamma_{5} q|p, s\rangle
$$

- Using light-cone Dirac spinors

$$
\Delta q=\int_{0}^{1} \mathrm{~d} x\left[q^{\uparrow}(x)-q^{\downarrow}(x)\right]
$$

- Using conventional Dirac spinors

$$
\begin{aligned}
& \Delta q=\int \mathrm{d}^{3} \vec{p} M_{q}\left[q^{\uparrow}(\vec{p})-q^{\downarrow}(\vec{p})\right] \\
& M_{q}=\frac{\left(p_{0}+p_{3}+m\right)^{2}-\vec{p}_{\perp}^{2}}{2\left(p_{0}+p_{3}\right)\left(p_{0}+m\right)}
\end{aligned}
$$

Thus $\Delta q$ is the light-cone quark spin or quark spin in the infinite momentum frame, not that in the rest frame of the proton

## Quark spin sum is not a Lorentz invariant quantity

Thus the quark spin sum equals to the proton in the rest frame does not mean that it equals to the proton spin in the infinite momentum frame
$\sum_{q} \overrightarrow{\boldsymbol{S}}_{q}=\vec{S}_{p}$ in the rest frame
does not mean that
$\sum_{q} \vec{s}_{q}=\vec{S}_{p}$ in the infinite momentum frame

Therefore it is not a surprise that the quark spin sum measured in DIS does not equal to the proton spin

## A general consensus

The quark helicity $\Delta q$ defined in the infinite momentum frame is generally not the same as the constituent quark spin component in the proton rest frame, just like that it is not sensible to compare apple with orange.

> H.-Y.Cheng, hep-ph/0002157,
> Chin.J.Phys.38:753,2000

## A QED Example of Relativistic Spin Effect

S.J. Brodsky, D.S. Hwang, B.-Q. Ma, I. Schmidt, Nucl. Phys. B 593 (2001) 311

what are the heleicites of each porticle:

$\left(\frac{1}{2}\right\rangle \rightarrow\left|-\frac{1}{2}, 1\right\rangle$

$\psi_{\frac{1}{2}}^{*}+\left(x, k_{0}\right)=\sqrt{2} \frac{+k_{1}+k^{2}}{1-x} \varphi$
( $1 / 3 / 3\rangle \rightarrow\left|\frac{1}{3},-1\right\rangle$
$\longrightarrow \psi_{\left|-\frac{1}{2}\right\rangle \rightarrow\left|-\frac{1}{2},-1\right\rangle}^{r}\left(v, f_{0}\right)=0$

The lowest spin states of a composite system must contain the orbital angular momentum contribution.
$\Delta s_{\text {non-rel }}+L_{\text {non-rel }}=\Delta s_{\text {rel }}+L_{\text {rel }}$

## Other approaches with same conclusion

Contribution from the lower component of Dirac spinors in the rest frame:
B.-Q. Ma, Q.-R. Zhang, Z.Phys.C 58 (1993) 479-482
D.Qing, X.-S.Chen, F.Wang, Phys.Rev.D58:114032,1998.
P.Zavada, Phys.Rev.D65:054040,2002.

## The Spin Distributions in Quark Model

The spin distribution probabilities in the quark-diquark
model

$$
\begin{array}{cc}
u_{\top^{-}}^{\dagger}=\frac{1}{18} ; \quad u_{\uparrow}^{\downarrow}=\frac{2}{18} ; \quad d_{\uparrow^{-}}^{\uparrow}=\frac{2}{18} ; \quad d_{\uparrow}^{\downarrow}=\frac{t}{18} ; \\
u_{S}^{\dagger}=\frac{1}{2} ; \quad u_{S}^{\downarrow}=0 ; \quad d_{S}^{\dagger}=0 ; \quad d_{S}^{\downarrow}=0 . \tag{7}
\end{array}
$$

Naive Quark Model:

$$
\begin{aligned}
& \Delta u=\frac{4}{3} ; \quad \Delta d=-\frac{1}{3} ; \quad \Delta s=0 \\
& \Sigma=\Delta u+\Delta d+\Delta s=1
\end{aligned}
$$

## Relativistic Effect due to Melosh-Rotation

$$
\begin{gathered}
\Delta u_{\vartheta}(x)=u_{v}^{\dagger}(x)-u_{w}^{\downarrow}(x)=-\frac{1}{18} a_{\uparrow} \cdot(x) W_{T} \cdot(x)+\frac{1}{2} a_{S}(x) W_{S}(x) \\
\Delta d_{v}(x)=d_{v}^{\dagger}(x)-d_{v}^{\downarrow}(x)=-\frac{1}{9} a_{\uparrow} \cdot(x) W_{T} \cdot(x) \\
\text { from } \quad a_{S}(x)=2 u_{v}(x)-d_{v}(x) \\
a_{\uparrow}(x)=3 d_{v}(x)
\end{gathered}
$$

We obtain $\quad \Delta u_{v}(x)=\left[u_{v}(x)-\frac{1}{2} d_{v}(x)\right] W_{S}(x)-\frac{1}{6} d_{v}(x) W_{T}(x) ;$

$$
\Delta d_{w}(x)=-\frac{1}{3} d_{v}(x) W_{T}(x)
$$

## Relativistic SU(6) Quark Model

## Flavor Symmetric Case

Relativistic Correction: $\quad M_{q}=0.75$

$$
\begin{aligned}
& \Delta u=\frac{4}{3} M_{q}=1 ; \quad \Delta d=-\frac{1}{3} M_{q}=-0.25 ; \quad \Delta s=0 \\
& \Sigma=\Delta u+\Delta d+\Delta s=0.75 \\
& F_{2}^{p}(x) / F_{2}^{p}(x) \geq \frac{2}{3} \text { for all } x
\end{aligned}
$$

## Relativistic SU(6) Quark Model

## Flavor Asymmetric Case

Relativistic Correction: $\quad M_{u} \approx 0.6 ; \quad M_{d} \approx 0.9$
$\Delta u=\frac{4}{3} M_{t}=0.8 ; \quad \Delta d=-\frac{1}{3} M_{d}=-0.3 ; \quad \Delta s=0$
$\Sigma=\Delta u+\Delta d+\Delta s \approx 0 . \overline{2}$
$F_{2}^{n}(x) / F_{2}^{p}(x) \rightarrow \frac{1}{4}$ at large $x$
B.-Q.Man, Plys. Lett. B 375 (1996) 320.

## Relativistic SU(6) Quark Model

## Flavor Asymmetric Case + Intrinsic Sea

For Intrinsic $d \bar{d}$ Sea $(\sim 15 \%): \quad \Delta d_{\text {scal }} \approx-0.07$
For Intrinsic $s \bar{s} \operatorname{Sea}(\sim 5 \%): \quad \Delta s_{\text {sca }} \approx-0.03$
Thus: $\quad \Sigma=\Delta u+\Delta d+\Delta s+\Delta d_{\text {sca }}+\Delta s_{\text {sema }} \approx 0.4$
S. J. Brodklive and B.-Q.NEa, Pliys. Lett. B 381 (1906) 317.

More detailed discussions, see, B.-Q.Ma, J.-J.Yang, I.Schmidt, Eur.Phys.J.A12(2001)353
Understanding the Proton Spin "Puzzle" with a New "Minimal" Quark Model
Three quark valence component could be as large as $70 \%$ to account for the data

## A relativistic quark-diquark model



## A relativistic quark-diquark model

- The unpolarized distribution of quark $q$ in hadron $h$ can be written as

$$
q(x)=c_{q}^{S} a_{S}(x)+c_{q}^{V} a_{V}(x)
$$

where $a_{D}(x)$ is

$$
a_{D}(x) \propto \int\left[\mathrm{d}^{2} \mathbf{k}_{\perp}\right]\left|\phi\left(x, \mathbf{k}_{\perp}\right)\right|^{2} \quad(D=S \text { or } V),
$$

- BHL prescription of the light-cone momentum space wave function for quark-diquark

$$
\phi\left(x, \mathbf{k}_{\perp}\right)=A_{D} \exp \left\{-\frac{1}{8 \alpha_{D}^{2}}\left[\frac{m_{q}^{2}+\mathbf{k}_{\perp}^{2}}{x}+\frac{m_{D}^{2}+\mathbf{k}_{\perp}^{2}}{1-x}\right]\right\},
$$

## A relativistic quark-diquark model

- longitudinally polarized quark distribution

$$
\Delta q(x)=\tilde{c}_{q}^{S} \tilde{a}_{S}(x)+\tilde{c}_{q}^{V} \tilde{a}_{V}(x)
$$

where

$$
\tilde{a}_{D}(x)=\int\left[\mathrm{d}^{2} \mathbf{k}_{\perp}\right] W_{D}\left(x, \mathbf{k}_{\perp}\right)\left|\phi\left(x, \mathbf{k}_{\perp}\right)\right|^{2} \quad(D=S \text { or } V)
$$

- Melosh-Winger rotation factor

Longitudinally polarized
$W_{D}\left(x, \mathbf{k}_{\perp}\right)=\frac{\left(k^{+}+m_{q}\right)^{2}-\mathbf{k}_{\perp}^{2}}{\left(k^{+}+m_{q}\right)^{2}+\mathbf{k}_{\perp}^{2}}$
where $k^{+}=x \mathcal{M}, \mathcal{M}^{2}=\frac{m_{q}^{2}+\mathbf{k}_{\perp}^{2}}{x}+\frac{m_{D}^{2}+\mathbf{k}_{\perp}^{2}}{1-x}$.

## The Melosh-Wigner rotation

in $\mathrm{p} Q C D$ based parametrization of quark helicity distributions
"The helicity distributions measured on the light-cone are related by a Wigner rotation (Melosh transformation) to the ordinary spin $\mathrm{S}_{\mathrm{i}}{ }^{2}$ of the quarks in an equal-time rest-frame wavefunction description. Thus, due to the non-collinearity of the quarks, one cannot expect that the quark helicities will sum simply to the proton spin."
S.J.Brodsky, M.Burkardt, and I.Schmidt, Nucl.Phys.B441 (1995) 197-214, p. 202

## pQCD counting rule

$$
\begin{gathered}
q_{\mathrm{h}}^{ \pm} \propto(1-x)^{p} \\
p=2 n-1+2\left|\Delta s_{z}\right| \quad \Delta s_{z}=s_{q}-s_{N}
\end{gathered}
$$

- Based on the minimum connected tree graph of hard gluon exchanges.
- "Helicity retention" is predicted -- The helicity of a valence quark will match that of the parent nucleon.


## Parameters in pQCD counting rule analysis


B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

New Development: H. Avakian, S.J.Brodsky, D.Boer, F.Yuan, Phys.Rev.Lett.99:082001,2007.

## Two different sets of parton distributions

- $\mathrm{SU}(6)$ quark-diquark model

$$
\begin{aligned}
& \Delta u_{v}(x)=\left[u_{v}(x)-\frac{1}{2} d_{v}(x)\right] W_{S}(x)-\frac{1}{6} d_{v}(x) W_{V}(x) \\
& \Delta d_{v}(x)=-\frac{1}{3} d_{v}(x) W_{V}(x)
\end{aligned}
$$

- pQCD based counting rule analysis

$$
\begin{aligned}
u_{v}^{\mathrm{pQCD}}(x) & =u_{v}^{\text {para }}(x) \\
d_{v}^{\mathrm{pQCD}}(x) & =\frac{d_{v}^{\text {th }}(x)}{u_{v}^{\text {th }}(x)} u_{v}^{\text {para }}(x) \\
\Delta u_{v}^{\mathrm{pQCD}}(x) & =\frac{\Delta u_{v}^{\text {th }}(x)}{u_{v}^{\text {th }}(x)} u_{v}^{\text {para }}(x) \\
\Delta d_{v}^{\mathrm{pQCD}}(x) & =\frac{\Delta d_{v}^{\text {th }}(x)}{u_{v}^{\text {th }}(x)} u_{v}^{\text {para }}(x)
\end{aligned}
$$

- CTEQ5 set 3 as input.


## Different predictions in two models

- Helicity distribution
- $\mathrm{SU}(6)$ quark-diquark model:
$\Delta u(x) / u(x) \rightarrow 1$ as
$x \rightarrow 1$.
$\Delta d(x) / d(x) \rightarrow-\frac{1}{3}$ as $x \rightarrow 1$.
- pQCD based counting rule analysis:
$\Delta u(x) / u(x) \rightarrow 1$ as
$x \rightarrow 1$.
$\Delta d(x) / d(x) \rightarrow 1$ as
$x \rightarrow 1$.



## $W^{ \pm}$production at RHIC

- Parity-violating asymmetry

$$
A_{L}=-\frac{\sigma_{+}-\sigma_{-}}{\sigma_{+}+\sigma_{-}}, \quad A_{L}=-\frac{1}{P} \times \frac{N_{+}^{\prime}-N_{-}^{\prime}}{N_{+}^{\prime}+N_{-}^{\prime}},
$$

- The maximum parity violation of $W$ bosons.
- $u \bar{d} \rightarrow W^{+}$and $\bar{u} d \rightarrow W^{-}$.
- At LO, the parity-violating asymmetry will approach $\Delta q(x) / q(x)$ when the rapidity of $W^{ \pm}, y_{W}$, is large.
C. Bourrely, J. Soffer, Nucl. Phys. B423(1994) 329

One of the possible leading order production of $W^{+}$production.

Proton helicity $="+"$


$$
A_{L}^{W^{+}}=\frac{u_{-}^{-}\left(x_{1}\right) \bar{d}\left(x_{2}\right)-u_{+}^{-}\left(x_{1}\right) \bar{d}\left(x_{2}\right)}{u_{-}^{-}\left(x_{1}\right) \bar{d}\left(x_{2}\right)+u_{+}^{-}\left(x_{1}\right) \bar{d}\left(x_{2}\right)}=\frac{\Delta u\left(x_{1}\right)}{u\left(x_{1}\right)}
$$

$$
A_{L}^{W^{+}}=\frac{\Delta u\left(x_{1}\right) \bar{d}\left(x_{2}\right)-\Delta \bar{d}\left(x_{1}\right) u\left(x_{2}\right)}{u\left(x_{1}\right) \bar{d}\left(x_{2}\right)+\bar{d}\left(x_{1}\right) u\left(x_{2}\right)}
$$

$$
x_{1}=\frac{M_{W}}{\sqrt{s}} e^{y_{W}}
$$

$$
A_{L}^{W^{-}}=\frac{\Delta d\left(x_{1}\right) \bar{u}\left(x_{2}\right)-\Delta \bar{u}\left(x_{1}\right) d\left(x_{2}\right)}{d\left(x_{1}\right) \bar{u}\left(x_{2}\right)+\bar{u}\left(x_{1}\right) d\left(x_{2}\right)}
$$

$$
x_{2}=\frac{M_{W}}{\sqrt{s}} e^{-y_{W}}
$$

X. Chen, Y. Mao, B.-Q. Ma, NPA A759(2008)188


## It is possible to pin down flavor-dependence of spin distribution through polarized proton proton collider

- Quark-diquark model and pQCD have different predictions on flavor-dependence of quark helicity and transversity distributions.
- Such flavor dependence can be measured through polarized proton proton scattering in STAR.
- It is necessary to measure different combinations of polarization processes to extract flavor-dependent helicty and transversity quark distributions.


## The Melosh-Wigner rotation is not the whole story

- The role of sea is not addressed
- The role of gluon is not addressed

It is important to study the roles played by the sea quarks and gluons. Thus more theoretical and experimental researches can provide us a more completed picture of the nucleon spin structure.

## Chances: New Research Directions

- New quantities: Transversity, Generalized Parton Distributions, Collins Functions, Silver Functions, BoerMulders Functions, Pretzelosity
- Hyperon Physics: The spin structure of Lambda and Sigma hyperons
B.-Q. Ma, I. Schmidt, J.-J. Yang

Phys. Lett. B 477 (2000) 107
Phys. Rev. D 61 (2000) 034017

## The Melosh-Wigner Rotation in <br> Transversity

$$
\begin{gathered}
2 \delta q=\langle p, \uparrow| \bar{q}_{\lambda} \gamma^{\perp} \gamma^{+} q_{-\lambda}|p, \downarrow\rangle \\
\delta q(x)=\int\left[\mathrm{d}^{2} \boldsymbol{k}_{\perp}\right] \tilde{M}_{q}\left(x, \boldsymbol{k}_{\perp}\right) \Delta q_{\mathrm{RF}}\left(x, \boldsymbol{k}_{\perp}\right) \\
\tilde{M}_{q}\left(x, \boldsymbol{k}_{\perp}\right)=\frac{\left(k^{+}+m\right)^{2}}{\left(k^{+}+m\right)^{2}+\boldsymbol{k}_{\perp}^{2}}
\end{gathered}
$$

I.Schmidt\&J.Soffer, Phys.Lett.B 407 (1997) 331

## Transversity with Melosh-Wigner rotation in the quark-diquark model

$$
\begin{aligned}
& \delta u_{v}(x)=\left[u_{v}(x)-\frac{1}{2} d_{v}(x)\right] \hat{W}_{S}(x)-\frac{1}{6} d_{v}(x) \hat{W}_{V}(x), \\
& \delta d_{v}(x)=-\frac{1}{3} d_{v}(x) \hat{W}_{V}(x),
\end{aligned}
$$

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.

## The transversity in pQCD , in similar to helicity distributions

$$
\delta q(x)=\frac{\tilde{A}_{q}}{B_{3}} x^{(-1 / 2)}(1-x)^{3}-\frac{\tilde{C}_{q}}{B_{5}} x^{(-1 / 2)}(1-x)^{5}
$$

| Baryon | $q_{1}$ | $q_{2}$ | $\tilde{A}_{q_{1}}$ | $\tilde{C}_{q 1}$ | $\tilde{A}_{q 2}$ | $\tilde{X}_{q_{2}}$ | $\hat{A}_{q_{1}}$ | $\hat{C}_{q_{1}}$ | $\hat{A}_{q_{2}}$ | $\hat{\bar{C}}_{q_{2}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p | $\mathrm{q}_{1}$ | d | 1.375 | 0.625 | 0.275 | 0.725 | 1.52 | 0.48 | 0.305 | 0.695 |

B.-Q. Ma, I. Schmidt, J.-J. Yang, Phys.Rev.D63(2001) 037501.

## SU(6) quark- VS PQCD based diquark model analysis

 Ma, Schmidt and Yang, PRD 65, 034010 (2002)

solid curve for $\operatorname{SU}(6)$ and dashed curve for PQCD

## Collins asymmetry in semi-inclusive production

$$
\begin{aligned}
& A_{U T}^{\text {Colins }}=\frac{1}{\left|S_{\perp}\right|} \frac{d \sigma_{\sigma_{I}}^{\text {collins }}}{d \sigma_{U U}} \text { After integration over specific weighting functions } \\
& A_{T}(x, y, z)=-\frac{(1-y) \sum_{q} e_{q}^{2} \delta q(x) H_{1}^{\perp(1) q}(z)}{\left(1-y+y^{2} / 2\right) \sum_{q} e_{q}^{2} q(x) D_{1}^{q}(z)} \\
& q(x) \quad \text { unpolarized quark distrion } \\
& \delta q(x) \quad \text { transversity } \\
& D_{1}(x) \text { unpolarized fragmentation function } \\
& H_{1}^{\perp(1) q}(x) \quad \text { Collins function }
\end{aligned}
$$

## Including unfavored fragmentation in HERMES condition


Y. Huang, J. She, and B.-Q. Ma, Phys. Rev. D76 (2007) 034004.

## Prediction in JLab condition (proton target)



## Prediction in JLab condition (neutron target)

## 





## Transversity

from two pion interference fragmentation

$$
A_{u T}^{\left(2 \sin \left(\phi_{n}+\phi_{j}\right) / \sin \theta\right\rangle}=-\frac{\sum_{a} e_{a}^{2} \delta f^{a}(x) \int d \zeta \frac{|\vec{R}|}{M_{h}} H_{1}^{\square a}\left(z, \zeta, M_{h}^{2}\right)}{\sum_{a} e_{a}^{2} f^{a}(x) \int d \zeta D_{1}^{a}\left(z, \zeta, M_{h}^{2}\right)}
$$

New fragmentation functions are introduced: the dihadron FFs, including the chiral odd interference FF.

- Jaffe, Jin and Tang, PRL 80, 1166 (1998)
- Radici, Jakob and Bianconi, PRD, 65, 074031 (2002)
- Bacchetta and Radici, PRD 74, 114007 (2006)


## Prediction on the proton target




## Prediction on neutron target


J. She, Y.Huang, and B.-Q. Ma, Phys. Rev. D77 (2008) 014035.

## Comparison with HERMES Data

HERMES, JHEP 0806:017,2008

J. She, Y.Huang, and B.-Q. Ma, Phys. Rev. D77 (2008) 014035.

## Comparison with COMPASS Data

COMPASS Preliminary, arXiv:0907.0961



J. She, Y.Huang, and B.-Q. Ma, Phys. Rev. D77 (2008) 014035.

## Comparison with COMPASS Data

COMPASS, arXiv:1009.0819 [hep-ex]


Figure 1: Two-hadron asymmetry $A_{R S}$ as a function of $x, z$ and $M_{\text {inv }}$, compared to predictions of [16]. The lower bands indicate the systematic uncertainty of the measurement.
J. She, Y.Huang, and B.-Q. Ma, Phys. Rev. D77 (2008) 014035.

## The Melosh-Wigner Rotation in Quark Orbital Angular Moment

$$
\begin{gathered}
\hat{L}_{q}=-i\left(k_{1} \frac{\partial}{\partial k_{2}}-k_{2} \frac{\partial}{\partial k_{1}}\right) \\
L_{q}(x)=\int\left[d^{2} k_{\perp}\right] M_{L}\left(x, k_{\perp}\right) \Delta q_{Q M}\left(x, k_{\perp}\right) \\
M_{L}\left(x, k_{\perp}\right)=\frac{k_{\perp}^{2}}{\left(k^{+}+m\right)^{2}+k_{\perp}^{2}}
\end{gathered}
$$

Ma\&Schmidt, Phys.Rev.D 58 (1998) 096008

## Three QCD spin sums for the proton spin

$$
\begin{aligned}
& \vec{J}_{\text {CCD }}=\int d^{3} x \psi^{\dagger} \frac{\vec{t}}{2} \psi+\int d^{3} x \psi^{\dagger} \vec{x} \times(-i \nabla) \psi \\
& +\int d^{3} x \vec{E}^{a} \times \vec{A}^{a}+\int d^{3} x E^{a i} \vec{x} \times \nabla A^{a i} \\
& \equiv \vec{S}_{q}+\vec{L}_{q}+\vec{S}_{g}+\vec{L}_{g}, \\
& \vec{J}_{Q C D}=\int d^{3} x \psi^{\dagger} \frac{\vec{\rightharpoonup}}{2} \psi+\int d^{3} x \psi^{\dagger} \vec{x} \times(-i \vec{D}) \psi+\int d^{3} x \vec{x} \times(\vec{E} \times \vec{B}) \\
& \equiv \vec{S}_{q}+\vec{L}_{q}+\vec{J}_{g}, \\
& \vec{J}_{Q C D}=\int d^{3} x \psi \psi^{+} \frac{\vec{t}}{2} \psi+\int d^{3} x \psi^{\dagger} \vec{x} \times\left(-i \vec{D}_{\text {pure }}\right) \psi \\
& +\int d^{3} x \vec{E}^{a} \times \vec{A}_{p h y s}^{a}+\int d^{3} x E^{a i} \vec{x} \times \nabla A_{p h y s}^{a i} \\
& \equiv \vec{S}_{q}+\vec{L}_{q}+\vec{S}_{g}+\vec{L}_{g},
\end{aligned}
$$

X.-S.Chen, X.-F.Lu, W.-M.Sun, F.Wang, T.Goldman, PRL100(2008)232002

## Spin and orbital sum in light-cone formalism

$$
\begin{gathered}
\frac{1}{2} M_{q}+M_{L}=\frac{1}{2} \\
M_{q}\left(x, k_{\perp}\right)=\frac{\left(k^{+}+m\right)^{2}-k_{\perp}^{2}}{\left(k^{+}+m\right)^{2}+k_{\perp}^{2}} \quad M_{L}\left(x, k_{\perp}\right)=\frac{k_{\perp}^{2}}{\left(k^{+}+m\right)^{2}+k_{\perp}^{2}} \\
\frac{1}{2} \Delta q(x)+L_{q}(x)=\frac{1}{2} \Delta q_{Q M}(x)
\end{gathered}
$$

Ma\&Schmidt, Phys.Rev.D 58 (1998) 096008

## Relations of quark distributions

$$
\Delta q_{Q M}(x)+\Delta q(x)=2 \delta q(x)
$$

B.-Q. Ma, I. Schmidt, J. Soffer, Phys.Lett. B 441 (1998) 461-467.
$\frac{1}{2} \Delta q(x)+L_{q}(x)=\frac{1}{2} \Delta q_{Q M}(x)$,

$$
\Delta q(x)+L_{q}(x)=\delta q(x)
$$

Ma\&Schmidt, Phys.Rev.D 58 (1998) 096008

## The Melosh-Wigner Rotation in "Pretzelosity"

$$
\begin{aligned}
& g_{1}^{q}\left(x, k_{\perp}\right)-h_{1}^{q}\left(x, k_{\perp}\right)=h_{1 T}^{\perp(1) q}\left(x, k_{\perp}\right) . \\
& \frac{\left(k^{+}+m\right)^{2}-\mathbf{k}_{\perp}^{2}}{\left(k^{+}+m\right)^{2}+\mathbf{k}_{\perp}^{2}}-\frac{\left(k^{+}+m\right)^{2}}{\left(k^{+}+m\right)^{2}+\mathbf{k}_{\perp}^{2}}=-\frac{\mathbf{k}_{\perp}^{2}}{\left(k^{+}+m\right)^{2}+\mathbf{k}_{\perp}^{2}}
\end{aligned}
$$

$$
\text { Pretzelosity }=\Delta \mathrm{q}-\delta \mathrm{q}=-\mathrm{L}_{\mathrm{q}}
$$

$$
\text { Pretzelosity }=-\int\left[\mathrm{d}^{2} \mathbf{k}_{\perp}\right] \frac{\mathbf{k}_{\perp}^{2}}{\left(\mathrm{k}^{+}+\mathrm{m}\right)^{2}+\mathbf{k}_{\perp}^{2}} \Delta \mathrm{q}_{\mathrm{QM}}\left(\mathrm{x}, \mathbf{k}_{\perp}\right)
$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

## What is＂Pretzelosity＂？

－Pretzelosity：one of the eight leading twist transverse dependent parton distributions（TMDs）．
－The quark－quark correlator up to the leading twist

$$
\begin{aligned}
& \Phi\left(x, \mathbf{p}_{\perp}\right)=\frac{1}{2}\left\{f_{1} 巾_{+}-f_{1}^{\perp} \frac{\epsilon_{\perp}^{i j} p_{\perp}^{i} S_{\perp}^{j}}{M_{N}} 巾_{+}\right. \\
& +\left(S_{\|} g_{1 L}+\frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{\perp}}{M_{N}} g_{1 T}\right) \gamma_{5} \phi_{+}+h_{1 T} \frac{\left[\$_{\perp}, \phi_{+}\right] \gamma_{5}}{2} \\
& \left.+\left(S_{\|} h_{1 L}^{\perp}+\frac{\mathbf{p}_{\perp} \cdot \mathbf{S}_{\perp}}{M_{N}} h_{\uparrow}^{\perp}\right) \frac{\left[中_{\perp}, \phi_{+}\right] \gamma_{5}}{2 M_{N}}+i h_{1}^{\perp} \frac{\left[\phi_{\perp}, \phi_{+}\right]}{2 M_{N}}\right\} .(7) \\
& \text { P.J. Mulders and R.D. Tangerman, Nucl. Phys. B 461, } 197 \\
& \text { (1996), Erratum-ibid. B 484, } 538 \text { (1997). K. Goeke, A. } \\
& \text { Metz, and M. Schlegel, Phys. Lett. B 618, } 90 \text { (2005). }
\end{aligned}
$$

## Transverse Momentum Dependent Quark Distributions

$\rightarrow$ Nucleon Spin $\rightarrow$ Quark Spin

|  |  | Quark polarization |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Un-Polarized | Longitudinally Polarized | Transversely Polarized |
|  | U | $f_{1}=$ |  | $h_{1}^{\perp}=$ $\qquad$ i - <br> Boer-Mulder |
|  | L |  | $g_{1}=\underset{\text { Helicity }}{\rightarrow}$ | $h_{1 L}^{\perp}=\underset{\text { Worm Gear }}{\rightarrow}$ |
|  | T | $f_{1 T^{\perp}}^{\perp}=\bigodot_{\text {Sivers }}^{\perp}-$ | $g_{1 T}=$ <br> Worm Gear | $\begin{aligned} & h_{1 T}=\underbrace{t}_{\text {Transversity }}-( \\ & h_{1 T}{ }^{\perp}=\underbrace{t}_{\text {Pretzelosity }} \end{aligned}$ |

## What is "Pretzelosity"?

$$
\begin{align*}
\frac{p_{\perp}^{x} p_{\perp}^{y}}{M_{N}^{2}} h_{1 T}^{\perp}\left(x, p_{\perp}^{2}\right) & =\int \frac{d \xi^{-} d^{2} \boldsymbol{\xi}_{\perp}}{16 \pi^{3}} e^{i\left(x P^{+} \xi^{-}-\mathbf{p}_{\perp} \cdot \boldsymbol{\xi}_{\perp}\right)} \\
& \times\left\langle P S^{y}\right| \bar{\psi}(0) i \sigma^{1+} \gamma_{5} \psi\left(0, \xi^{-}, \xi_{\perp}\right)\left|P S^{y}\right\rangle, \tag{12}
\end{align*}
$$

$\left|P S^{y}\right\rangle$ : the hadronic state with a polarization in the $y$ direction.

- Some properties of pretzelosity:

1 It is chiral-odd, and needs a chiral-odd partner in the SIDIS.
2 There is no gluon analog of pretzelosity.
3 In a large class of models, it is the difference of helicity and transversity, and hence a measure for relativistic effects.
H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.

## A Simple Relation

- The difference of helicity and transversity is the first moment of pretzelosity.
$h_{1 T}^{\perp(1) q v}\left(x, \mathbf{p}_{\perp}\right) \equiv \frac{p_{\perp}^{2}}{2 M_{N}^{2}} h_{1 T}^{\perp q \nu}\left(x, \mathbf{p}_{\perp}\right)=g_{1}^{q \nu}\left(x, \mathbf{p}_{\perp}\right)-h_{1}^{q \nu}\left(x, \mathbf{p}_{\perp}\right)$,
- This relation has already been obtained in
H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355. B. Pasquini, S. Cazzaniga and S. Boffi, Phys. Rev. D 78, 034025 (2008).
- But this relation is not fully satisfied in A. Bacchetta, F. Conti, and M. Radici, Phys. Rev. D 78, 074010 (2008).


## Connection with Quark Orbital Angular Momentum

- The rotation factor for $\vec{x} \times-i \nabla$ is $\frac{p_{\perp}^{2}}{\left(\times M_{D}+m_{q}\right)^{2}+p_{\perp}^{2}}$ B.-Q. Ma, I. Schmidt, Phys. Rev. D 58, 096008 (1998).
- a simple relation between the pretzelosity and the quark orbital angular momentum
$L^{q v}\left(x, \mathbf{p}_{\perp}\right)=-h_{1 T}^{\perp(1) q v}\left(x, \mathbf{p}_{\perp}\right)=h_{1}^{q v}\left(x, \mathbf{p}_{\perp}\right)-g_{1}^{q v}\left(x, \mathbf{p}_{\perp}\right),(21)$
or at the integration level
$L^{q v}(x)=\int d^{2} \mathbf{p}_{\perp} L^{q v}\left(x, \mathbf{p}_{\perp}\right)=-h_{1 T}^{\perp(1) q v}(x)=h_{1}^{q v}(x)-g_{1}^{q v}(x)$.
- A measurement of pretzelosity may reveal the information on the quark orbital angular momentum.


## Pretzelosity in SIDIS

- Pretzelosity can be measured through $\sin \left(3 \phi_{h}-\phi_{S}\right)$ asymmetry in the SIDIS process, where the cross section can be written as

$$
\begin{aligned}
& \frac{d^{6} \sigma_{U T}}{d x d y d \phi_{S} d z d^{2} \mathbf{P}_{h \perp}}=\frac{2 \alpha^{2}}{s x y^{2}}\left\{\left(1-y+\frac{1}{2} y^{2}\right) F_{U U}\right. \\
& \left.\quad+S_{\perp} \sin \left(3 \phi_{h}-\phi_{S}\right)(1-y) F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)}+\ldots\right\},(23)
\end{aligned}
$$

with $F_{U U}=\mathcal{F}\left[\omega_{1} f_{1} D_{1}\right], \quad F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)}=\mathcal{F}\left[\omega_{2} h_{1 T}^{\perp} H_{1}^{\perp}\right]$

- The $\sin \left(3 \phi_{h}-\phi_{S}\right)$ asymmetry

$$
\begin{equation*}
A_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)}=\frac{\frac{2 \alpha^{2}}{s x y^{2}}(1-y) F_{U T}^{\sin \left(3 \phi_{h}-\phi_{S}\right)}}{\frac{2 \alpha^{2}}{s x y^{2}}\left(1-y+\frac{1}{2} y^{2}\right) F_{U U}} \tag{24}
\end{equation*}
$$

## Approach 0 to TMDs

- Starting with the equation

$$
\begin{align*}
& h_{1 T}^{\perp(u v)}(x)=\left[f_{1}^{(u v)}(x)-\frac{1}{2} f_{1}^{(d v)}(x)\right] \hat{W}_{S}(x)-\frac{1}{6} f_{1}^{(d v)}(x) \hat{W}_{v}(x), \\
& h_{1 T}^{\perp(d v)}(x)=-\frac{1}{3} f_{1}^{(d v)}(x) \hat{W}_{v}(x), \tag{25}
\end{align*}
$$

where $\hat{W}_{D}(x)=\int d^{2} \mathbf{p}_{\perp} \varphi^{2}\left(x, \mathbf{p}_{\perp}\right) W_{D}\left(x, \mathbf{p}_{\perp}\right) / \int d^{2} \mathbf{p}_{\perp} \varphi^{2}\left(x, \mathbf{p}_{\perp}\right)$

- $f_{1}(x)$ : CTEQ6L as an input. $h_{1 T}^{\perp}(x)$ : from Eq. 25
- Transverse momentum dependence: Gaussian form.
- How to fit the Gaussian width? $p_{a v} / k_{a v} \approx 2$ ?
H. Avakian, A.V. Efremov, P. Schweitzer, and F. Yuan, arXiv:0805.3355.


## Approach 1 to TMDs

- Model calculation.

$$
\begin{aligned}
f_{1}^{(u v)}\left(x, \mathbf{p}_{\perp}\right) & =\frac{1}{16 \pi^{3}} \times\left(\frac{1}{3} \sin ^{2} \theta \varphi_{V}^{2}+\cos ^{2} \theta \varphi_{S}^{2}\right), \\
f_{1}^{(d v)}\left(x, \mathbf{p}_{\perp}\right) & =\frac{1}{8 \pi^{3}} \times \frac{1}{3} \sin ^{2} \theta \varphi_{V}^{2} . \\
h_{1 T}^{\perp(u v)}\left(x, \mathbf{p}_{\perp}\right) & =-\frac{1}{16 \pi^{3}} \times\left(\frac{1}{9} \sin ^{2} \theta \varphi_{V}^{2} W_{V}-\cos ^{2} \theta \varphi_{S}^{2} W_{S}\right), \\
h_{1 T}^{\perp(d v)}\left(x, \mathbf{p}_{\perp}\right)= & -\frac{1}{8 \pi^{3}} \times \frac{1}{9} \sin ^{2} \theta \varphi_{V}^{2} W_{V} .
\end{aligned}
$$

- $\varphi_{D}\left(x, \mathbf{p}_{\perp}\right)$ : adopting the BHL form:

$$
\varphi_{D}\left(x, \mathbf{p}_{\perp}\right)=A_{D} \exp \left\{-\frac{1}{8 \alpha_{D}^{2}}\left[\frac{m_{q}^{2}+p_{\perp}^{2}}{x}+\frac{m_{D}^{2}+p_{\perp}^{2}}{1-x}\right]\right\}
$$

J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008

## Approach 2 to TMDs

- Staring with the equation (an unintegrated version)

$$
\begin{align*}
h_{1 T}^{\perp(u v)}\left(x, \mathbf{p}_{\perp}\right)= & {\left[f_{1}^{(u v)}\left(x, \mathbf{p}_{\perp}\right)-\frac{1}{2} f_{1}^{(d v)}\left(x, \mathbf{p}_{\perp}\right)\right] W_{S}\left(x, \mathbf{p}_{\perp}\right) } \\
& -\frac{1}{6} f_{1}^{(d v)}\left(x, \mathbf{p}_{\perp}\right) W_{V}\left(x, \mathbf{p}_{\perp}\right), \\
h_{1 T}^{\perp(d v)}\left(x, \mathbf{p}_{\perp}\right)= & -\frac{1}{3} f_{1}^{(d v)}\left(x, \mathbf{p}_{\perp}\right) W_{V}\left(x, \mathbf{p}_{\perp}\right) . \tag{27}
\end{align*}
$$

- $f_{1}\left(x, \mathbf{p}_{\perp}\right)$ : a Gaussian form

$$
\begin{equation*}
f_{1}\left(x, \mathbf{p}_{\perp}\right)=f_{1}(x) \frac{\exp \left(-p_{\perp}^{2} / p_{a v}^{2}\right)}{\pi p_{\mathrm{av}}^{2}} \tag{28}
\end{equation*}
$$

with CTEQ6L parametrization for $f_{1}(x)$.
J.She, J.Zhu, B.-Q.Ma, Phys.Rev.D79 (2009) 054008



Figure: The ratio $h_{1 T}^{\perp(1)(x)} / f_{1}(x)$. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the $u$ quark, and dashed curves for the $d$ quark. Only valence quarks are considered.

## Results at HERMES kinematics.




Figure: The results for HERMES kinematics with a proton target. Left panel for approach 0 and right panel for approach 1 (thin curves) and approach 2 (thick curves). Solid curves for the $\pi^{+}$production, and dashed curves for the $\pi^{-}$production.

## Results at COMPASS kinematics.



Figure: The results for COMPASS kinematics. a) proton target, b) neutron target, and c) deuteron target.

## Results at JLab kinematics.



Figure: The results for JLab kinematics. a) proton target and b) neutron target.

## Recent calculation to measure pretzelosity from polarized proton antiproton collider

The leading order differential cross section for the double transversely polarized Drell-Yan process reads [17]

$$
\begin{aligned}
& \frac{d \sigma}{d x_{a} d x_{b} d \boldsymbol{q}_{T} d \Omega}=\frac{\alpha_{e m}^{2}}{4 Q^{2}}\left\{F_{U U}^{1}+\left|\boldsymbol{S}_{a T}\right|\left|\boldsymbol{S}_{b T}\right| \sin ^{2} \theta\left[\cos \left(2 \phi-\phi_{a}-\phi_{b}\right) F_{T T}^{\cos \left(2 \phi-\phi_{a}-\phi_{b}\right)}+\cos \left(2 \phi+\phi_{a}-\phi_{b}\right) F_{T T}^{\cos \left(2 \phi+\phi_{a}-\phi_{b}\right)}\right]+\ldots\right\} . \\
& F_{U U}^{1}=\mathcal{C}\left[f_{1} \bar{f}_{1}\right], \quad F_{T T}^{\cos \left(2 \phi-\phi_{a}-\phi_{b}\right)}=\mathcal{C}\left[h_{1} \bar{h}_{1}\right], \quad F_{T T}^{\cos \left(2 \phi+\phi_{a}-\phi_{b}\right)}=\mathcal{C}\left[\frac{2\left(\boldsymbol{h} \cdot \boldsymbol{k}_{a T}\right)^{2}-k_{a T}^{2}}{2 M_{a}^{2}} h_{1 T}^{\perp} \bar{h}_{1}\right],
\end{aligned}
$$

J.Zhu, B.-Q.Ma, Phys.Rev.D82 (2011) 114022

## The asymmetries to measure pretzelosity

$$
\begin{aligned}
& A_{T T}^{\cos \left(2 \phi+\phi_{a}-\phi_{b}\right)}=\frac{\frac{\alpha_{e m}^{2}}{4 Q^{2}} \mathcal{C}\left[\frac{2\left(\boldsymbol{h} \cdot \boldsymbol{k}_{a T}\right)^{2}-k_{a T}^{2}}{2 M_{N}^{2}} h_{1 T}^{\perp} h_{1}\right]}{\frac{\alpha_{e m}^{2}}{4 Q^{2}} \mathcal{C}\left[f_{1} f_{1}\right]}, \\
& \quad=\frac{A_{T T}^{\frac{q_{T}^{2}}{2 M_{N}^{2}} \cos \left(2 \phi+\phi_{a}-\phi_{b}\right)}\left(x_{F}\right)}{\sum_{q} e_{q}^{2}\left[h_{1 T}^{\perp(2) q}\left(x_{a}\right) h_{1}^{q}\left(x_{b}\right)+h_{1 T}^{\perp(2) \bar{q}}\left(x_{a}\right) h_{1}^{\bar{q}}\left(x_{b}\right)\right]} \\
& \sum_{q} e_{q}^{2}\left[f_{1}^{q}\left(x_{a}\right) f_{1}^{q}\left(x_{b}\right)+f_{1}^{\bar{q}}\left(x_{a}\right) f_{1}^{\bar{q}}\left(x_{b}\right)\right]
\end{aligned},
$$

## The weighted asymmetry to measure pretzelosity



FIG. 2: The weighted $\cos \left(2 \phi+\phi_{a}-\phi_{b}\right)$ asymmetry as a function of $x_{F}$ for $s=45 \mathrm{GeV}^{2}$ and $Q^{2}=12 \mathrm{GeV}^{2}$. Solid curve corresponds to approach 1 , while dotted curve corresponds to approach 2 .

## The unweighted asymmetry to measure pretzelosity



FIG. 3: The unweighted $\cos \left(2 \phi+\phi_{a}-\phi_{b}\right)$ asymmetry as a function of $x_{F}$ for $s=45 \mathrm{GeV}^{2}$ and $Q^{2}=12 \mathrm{GeV}^{2}$. Solid curve corresponds to approach 1 , while dotted curve corresponds to approach 2.

## The Necessity of Polarized p pbar Collider

## The polarized proton antiproton Drell-Yan process

## is ideal to measure

## the pretzelosity distributions of the nucleon.

## PHYSICAL REVIEW D 82, 114022 (2010)

Probing the leading-twist transverse-momentum-dependent parton distribution function $h_{1 T}^{\perp}$ via the polarized proton-antiproton Drell-Yan process

[^0]
## Probing Pretzelosity in pion p Drell-Yan Process

## COMPASS pion p Drell-Yan process

## can also measure

## the pretzelosity distributions of the nucleon.

Physics Letters B 696 (2011) 513-517


Single spin asymmetry in $\pi p$ Drell-Yan process
Zhun Lu ${ }^{\mathrm{a}, \mathrm{b}}$, Bo-Qiang Ma ${ }^{\mathrm{c}, *}$, Jun She ${ }^{\mathrm{c}}$
${ }^{\text {a }}$ Department of Physics, Southeast University, Nanjing 211189, China
${ }^{\text {b }}$ Departamento de Fisica, Universidad Técnica Federico Santa María, and Centro Científico-Tecnológico de Valparaíso Casilla 110-V, Valparaíso, Chile
${ }^{\text {c }}$ School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

## Transverse Momentum Dependent Quark Distributions

$\rightarrow$ Nucleon Spin $\rightarrow$ Quark Spin

|  |  | Quark polarization |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Un-Polarized | Longitudinally Polarized | Transversely Polarized |
|  | U | $f_{1}=$ |  | $h_{1}^{\perp}=$ $\qquad$ i - <br> Boer-Mulder |
|  | L |  | $g_{1}=\underset{\text { Helicity }}{\rightarrow}$ | $h_{1 L}^{\perp}=\underset{\text { Worm Gear }}{\rightarrow}$ |
|  | T | $f_{1 T^{\perp}}^{\perp}=\bigodot_{\text {Sivers }}^{\perp}-$ | $g_{1 T}=$ <br> Worm Gear | $\begin{aligned} & h_{1 T}=\underbrace{t}_{\text {Transversity }}-( \\ & h_{1 T}{ }^{\perp}=\underbrace{t}_{\text {Pretzelosity }} \end{aligned}$ |

## Names for New (tmd) PDF: $g_{1 T}$ and $h_{1 L}^{\perp}$

$g_{1 T} \quad$ trans-helicity<br>$h_{1 L}^{\perp} \quad$ longi-transversity / heli-transversity

Physics Letters B 696 (2011) 246-251

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Proposal for measuring new transverse momentum dependent parton distributions $g_{1 T}$ and $h_{1 L}^{\perp}$ through semi-inclusive deep inelastic scattering
Jiacai Zhu ${ }^{\text {a }}$, Bo-Qiang Ma ${ }^{\text {a,b,* }}$
${ }^{\text {a }}$ School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
${ }^{\mathrm{b}}$ Center for High Energy Physics, Peking University, Beijing 100871, China

## Conclusions

- The relativistic effect of Melosh-Winger rotation is important in hadron spin physics.
- The pretzelosity is an important quantity for the spin-orbital correlation of the nucleon.
- It is necessary to push forward experimental measurements of new physical quantities of the nucleon.


[^0]:    Jiacai Zhu
    School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
    Bo-Qiang Ma*
    School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
    Center for High Energy Physics, Peking University, Beijing 100871, China (Received 10 October 2010; published 22 December 2010)

