Helical magnetic fields produced by inflation

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Outline

- The generation of large-scale magnetic fields
- Generating PMF inside the horizon
- Generating PMF outside the horizon
- Summary and discussion

The origin of large-scale magnetic fields

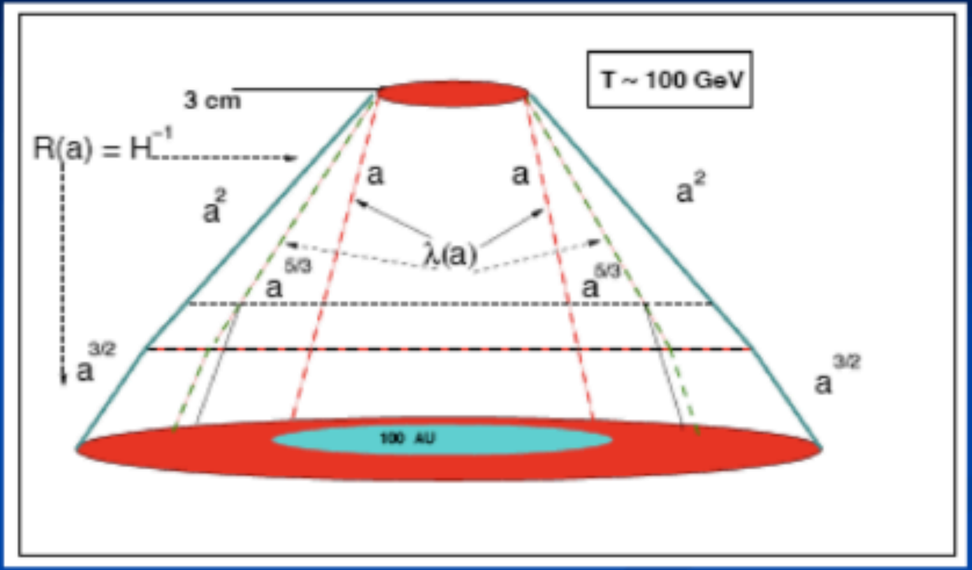
- The observed cluster and galactic magnetic fields of about a few μG may be resulted from the amplification of a seed field of $B_* \sim 10^{-23}$ G via the so-called galactic dynamo effect.

- Through most of its history, the universe has been a good conductor which preserves the magnetic flux:

$$r \equiv \frac{\rho_B}{\rho_\gamma} = \text{constant} \qquad \Rightarrow \qquad \rho_B \simeq 10^{-34} \rho_\gamma .$$

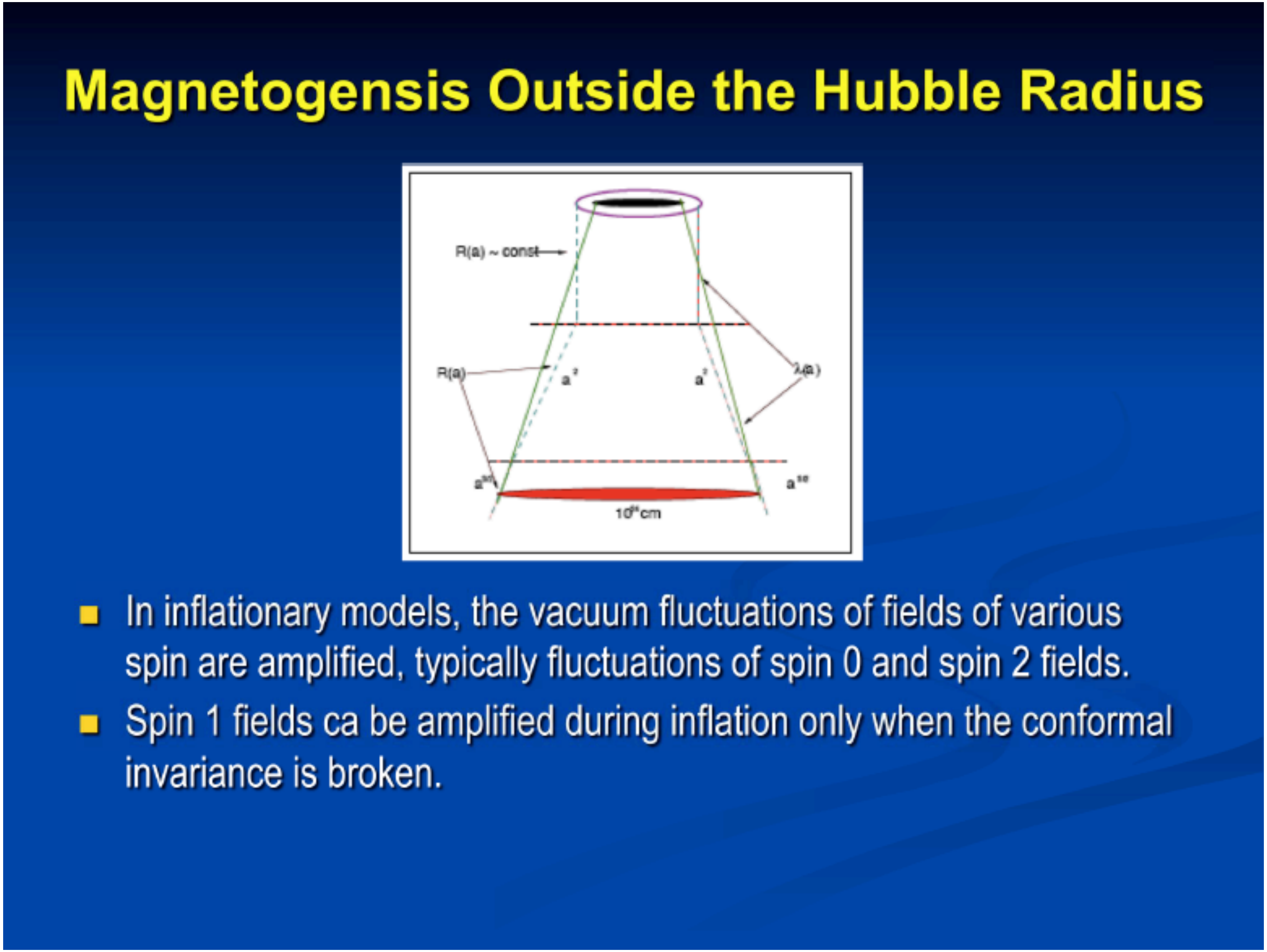
- The seed fields may be generated inside or outside the Hubble horizon.

Magnetogensis Inside the Hubble Radius



- Inverse cascade: many small-scale magnetic domains coalesce giving rise to a magnetic domain of larger size but of smaller energy
- If inverse cascade are invoked, the correlation scale may grow up to 100 AU
- But cosmic magnetic fields are coherent over much larger scales (i.e. Mpc and even larger)

M. Giovannini: astro-ph/0612378



Magnetogensis Outside the Hubble Radius

- In inflationary models, the vacuum fluctuations of fields of various spin are amplified, typically fluctuations of spin 0 and spin 2 fields.
- Spin 1 fields ca be amplified during inflation only when the conformal invariance is broken.

M. Giovannini: astro-ph/0612378

The Weyl (Conformal) Invariance

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)[d\tau^2 - dx^2]$$

$$S_{\rm em} = -\frac{1}{4}\int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \quad \partial_\mu (\sqrt{-g} F^{\mu\nu}) = 0$$

$$\sqrt{-g} F^{\mu\nu} = a^4(\tau) \frac{\eta^{\mu\alpha}}{a^2(\tau)} \frac{\eta^{\nu\beta}}{a^2(\tau)} F_{\alpha\beta} = F^{\mu\nu}$$

$\Rightarrow$ the evolution equations of Abelian gauge fields are the same in flat space-time and in a conformally flat FRW space-time

Breaking the Conformal Invariance

Turner & Widrow PRD (1988):

$$\frac{1}{m^2}F_{\mu\nu}F_{\alpha\beta}R^{\mu\nu\alpha\beta}, \qquad \frac{1}{m^2}R_{\mu\nu}F^{\mu\beta}F^{\nu\alpha}g_{\alpha\beta}, \qquad \frac{1}{m^2}F_{\alpha\beta}F^{\alpha\beta}R$$

Ratra ApJ (1992):

$$RA_\mu A^\mu, \qquad R_{\mu\nu}A^\mu A^\nu.$$

Carroll & Field (1990); Garretson et. al. (1992):

$$\sqrt{-g}c_{\psi\gamma}\alpha_{\rm em}\frac{\psi}{8\pi M}F_{\alpha\beta}\tilde{F}^{\alpha\beta}$$

Why the axionic PMF fail?

The coupled system of evolution equations to be solved in order to get the amplified field is

$$\begin{aligned} \mathbf{B}'' - \nabla^2\mathbf{B} - \frac{\alpha_{\rm em}}{2\pi M}\nabla\times\mathbf{B} &= 0, \\ \psi'' + 2\mathcal{H}\psi' + m^2a^2\psi &= 0, \end{aligned}$$

where $\mathbf{B} = a^2 B$. From the first equation, there is a maximally amplified physical frequency

$$\omega_{\rm max} \simeq \frac{\alpha_{\rm em}}{2\pi}\cdot m$$

However, the amplification of ω from the inflaton dynamics is at the order of

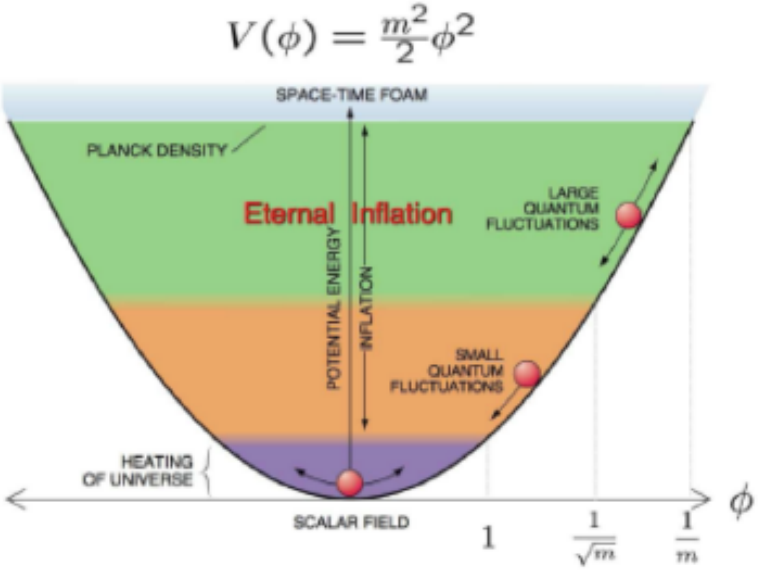
$$\exp\left\{\left(\frac{\alpha_{\rm em}}{2\pi}\right)\frac{m}{H}\right\}$$

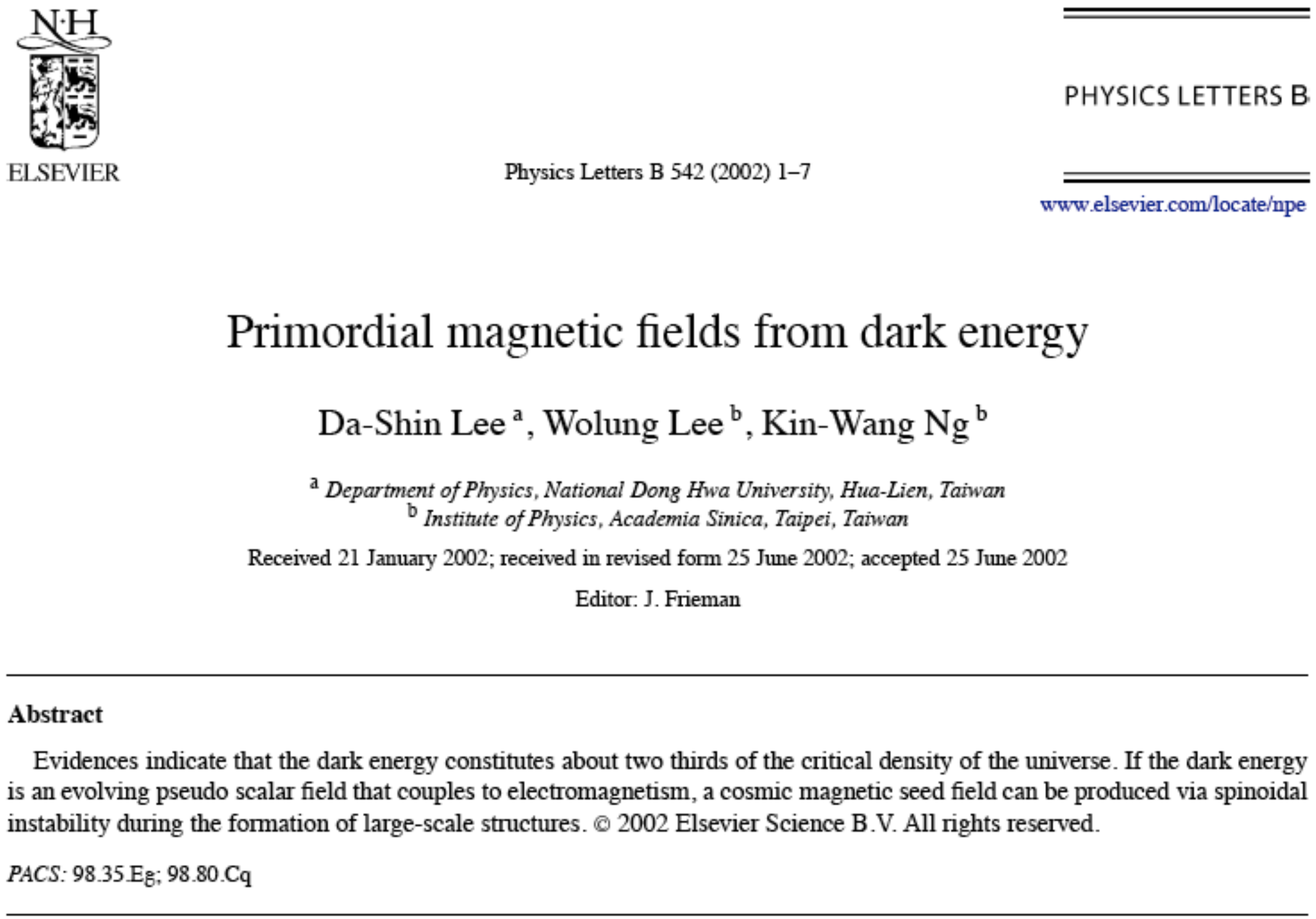
The modes which are substantially amplified are the ones for which $\omega_{\rm max} \gg H$, i.e. they are sub-horizon modes which could not lead to the large-scale magnetic fields we are interested in.

Helical PMF by inflation

Motivations:

- * Though the inverse cascade is not sufficient for helical fields generated at the electroweak phase transition, but it might work for magnetic fields from inflation.
- * Since the parity symmetry is violated, helical magnetic fields should give rise to observable effects, e.g. correlations between B-mode polarizations or in the E- and the B-polarizations in the CMB.





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Primordial magnetic fields from dark energy

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Abstract

Evidences indicate that the dark energy constitutes about two thirds of the critical density of the universe. If the dark energy is an evolving pseudo scalar field that couples to electromagnetism, a cosmic magnetic seed field can be produced via spinoidal instability during the formation of large-scale structures. © 2002 Elsevier Science B.V. All rights reserved.

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- As long as an ultralight ϕ field couples to photon where for a slow-roll condition, the mass m_ϕ is comparable to H_0, it is conceivable to have very long-wavelength electromagnetic fields generated via spinodal instabilities from the dynamics of ϕ as a possible source of seed magnetic fields for the galactic dynamo.

The ϕ-γ Coupling

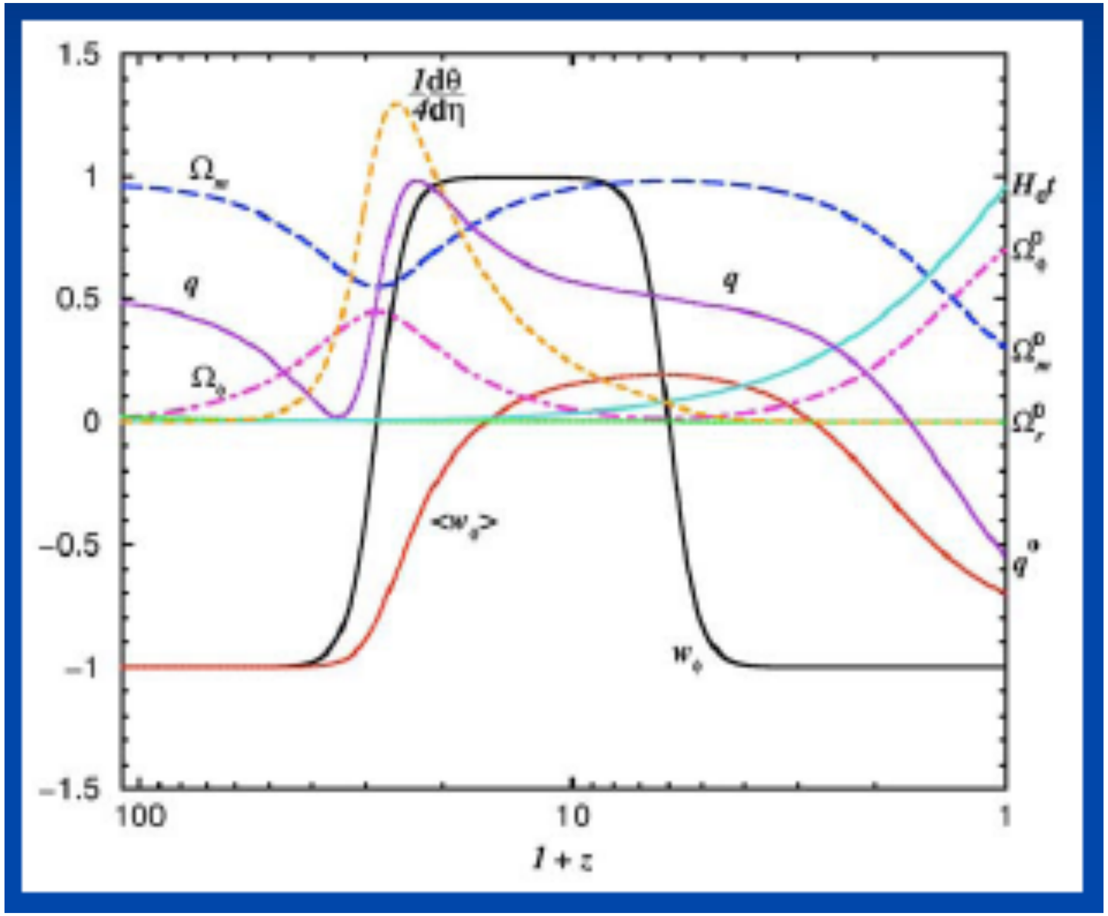
$$L_{\phi\gamma} = \frac{c}{f}\phi\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu},$$

$$S = S_M + \int d^4x\,\sqrt{g}\\ \times\left[-\frac{R}{16\pi G} - \frac{1}{4}g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}\right.\\ \left. -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \frac{1}{\sqrt{g}}L_{\phi\gamma}\right],$$

$$\frac{d^2\theta}{d\eta^2} + \frac{2}{a}\frac{da}{d\eta}\frac{d\theta}{d\eta} + \frac{a^2}{H_0^2M_p^2}\frac{\partial V(\theta)}{\partial\theta}\\ - \frac{c}{a^2H_0^2M_p^2}\langle F\widetilde{F}\rangle = 0,$$

As long as the energy density of the magnetic field is much less than that of photons, the back-action term <FF> can be ignored.

The generic quintessence



$$\frac{d\bar{\rho}_\phi}{d\eta} = -3ah(1+w_\phi)\bar{\rho}_\phi,$$
$$\frac{dh}{d\eta} = -\frac{3}{2}ah^2 - \frac{1}{2}aw_\phi\bar{\rho}_\phi,$$

$$\dot{\phi}^2 = (1+w_\phi)\rho_\phi \text{ and } V(\phi) = (1-w_\phi)\rho_\phi/2.$$

$$\langle w_\phi \rangle = \int_{\eta_{ls}}^{\eta_0} \Omega_\phi(\eta) w_\phi(\eta)\, d\eta \left(\int_{\eta_{ls}}^{\eta_0} \Omega_\phi(\eta)\, d\eta \right)^{-1}$$

The Magnetic Field

- From the action, the comoving B field is given by

$$\left(\nabla^2 - \frac{\partial^2}{\partial\tau^2}\right)\mathbf{B} = \sigma a\left[\frac{\partial \mathbf{B}}{\partial\tau} - \nabla \times (\mathbf{v} \times \mathbf{B})\right] + 4c\frac{d\theta}{d\tau}\nabla \times \mathbf{B},$$

where $\tau = \eta / H_0$, σ is the plasma conductivity, and $\boldsymbol{v}$ is the peculiar velocity

- The magnetic fields are built up in regions about to collapse into galaxies, where the nonlinear turbulence effect plays an important role in generating and subsequently amplifying the created PMF which overcomes the damping mechanism due to the high plasma conductivity. This point is supported by the numerical simulations showed in the paper Kulsrud et. Al. [Astrophys. J. **480**, 481 (1997)]. Hence, we consider only

$$\left(\nabla^2 - \frac{\partial^2}{\partial\tau^2}\right)\mathbf{B} = 4c\frac{d\theta}{d\tau}\nabla \times \mathbf{B}$$

The Mode Equations

- Now use the transverse vector potential such that $\mathbf{B} = \nabla \times \mathbf{A}_T$ where

$$\mathbf{A}_T = \int \frac{d^3\mathbf{k}}{\sqrt{2(2\pi)^3 k}} \times \left[e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=\pm} b_{\lambda\mathbf{k}} V_{\lambda\mathbf{k}}(\tau) \epsilon_{\lambda\mathbf{k}} + \mathrm{h.c.} \right],$$

$$\frac{d^2}{d\eta^2} V_{\pm q} + \left(q^2 \mp 4cq \frac{d\theta}{d\eta} \right) V_{\pm q} = 0,$$

$$V_{\pm q} = 1, \qquad \frac{dV_{\pm q}}{d\eta} = -iq.$$

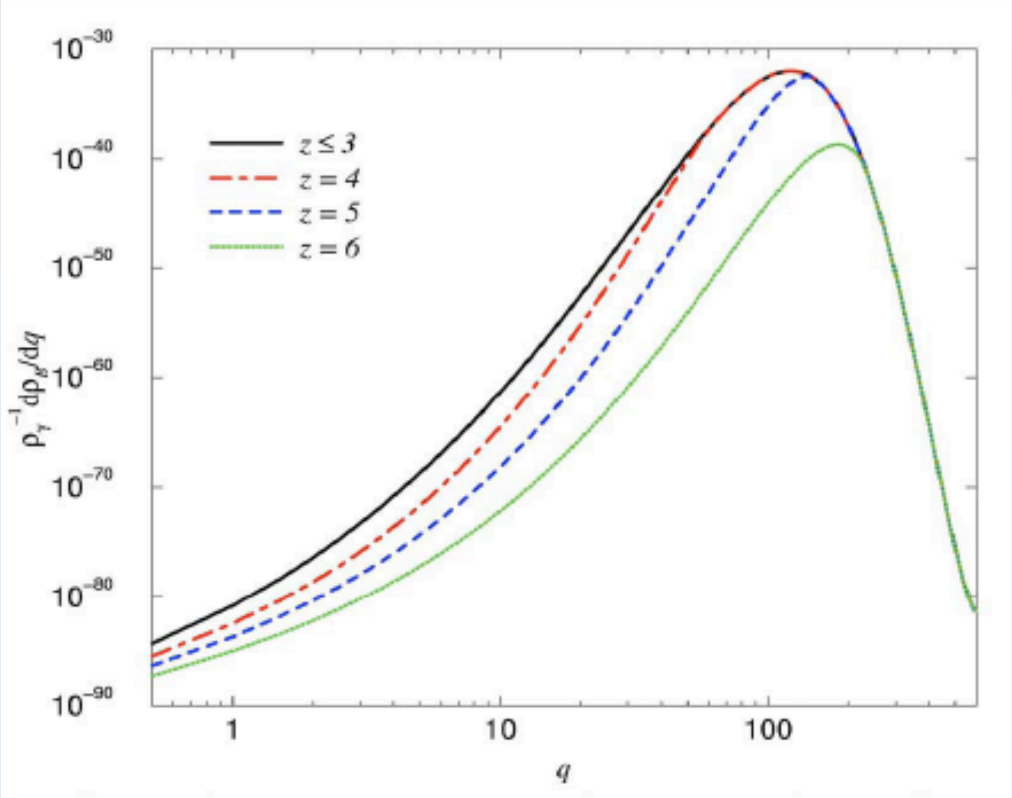
- The energy density of the magnetic field is given by

$$\rho_B = \langle B^2 \rangle / 8\pi = \int dq\, (d\rho_B/dq)$$

with

$$\frac{d\rho_B}{dq} = \frac{H_0^4}{32\pi^3} q^3 \coth\left[\frac{qH_0}{2T_0}\right] \sum_{\lambda=\pm} |V_{\lambda q}|^2,$$

The spectral magnetic energy density of PMF



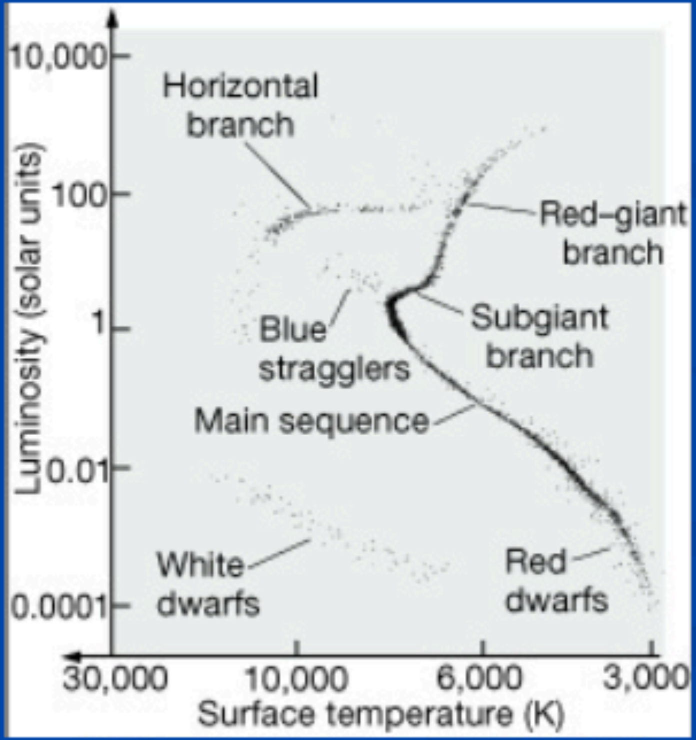
c ~ 130 ⇒ the amplitude fluctuations begin to grow up to a time at which the scalar field velocity approaches zero at redshift z ~ *4* and the scale of the resultant magnetic field is about 10 Mpc, i.e. a large scale PMF has been generated!

The Constraint on *c*

- The most stringent limit on the **ϕ-γ** coupling comes from the cooling via the Primakoff conversion of horizontal branch (HB) stars in globular clusters:

$$c/f < 1.5 \times 10^{-11}\ \mathrm{GeV}^{-1}$$

$$c < 3.7 \times 10^{7}$$



Magnetogenesis during inflation

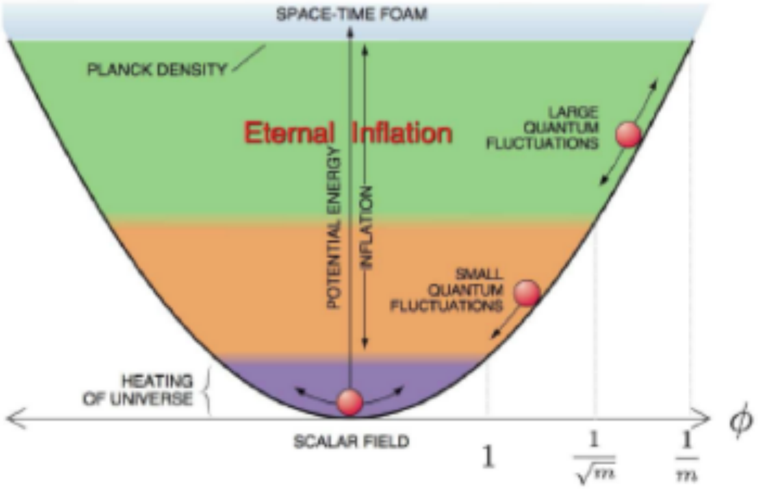
$$\frac{d^2}{d\eta^2}V_{\pm q} + \left(q^2 \mp 4cq\frac{d\theta}{d\eta}\right)V_{\pm q} = 0,$$

$$\frac{d^2V_\pm}{d\eta^2} = \left(-q^2 \pm 4cq \cdot \frac{d\theta}{d\eta}\right) \cdot V_\pm \Rightarrow V_\pm \propto \exp\left\{\left(\sqrt{\pm 4cq\frac{d\theta}{d\eta} - q^2}\right) \cdot \eta\right\}$$

Hence the growing mode can be identified with $V_{+q} \propto \exp(\omega\eta)$ with

$$\omega \equiv \sqrt{4cq\left|\frac{d\theta}{d\eta}\right| - q^2}$$

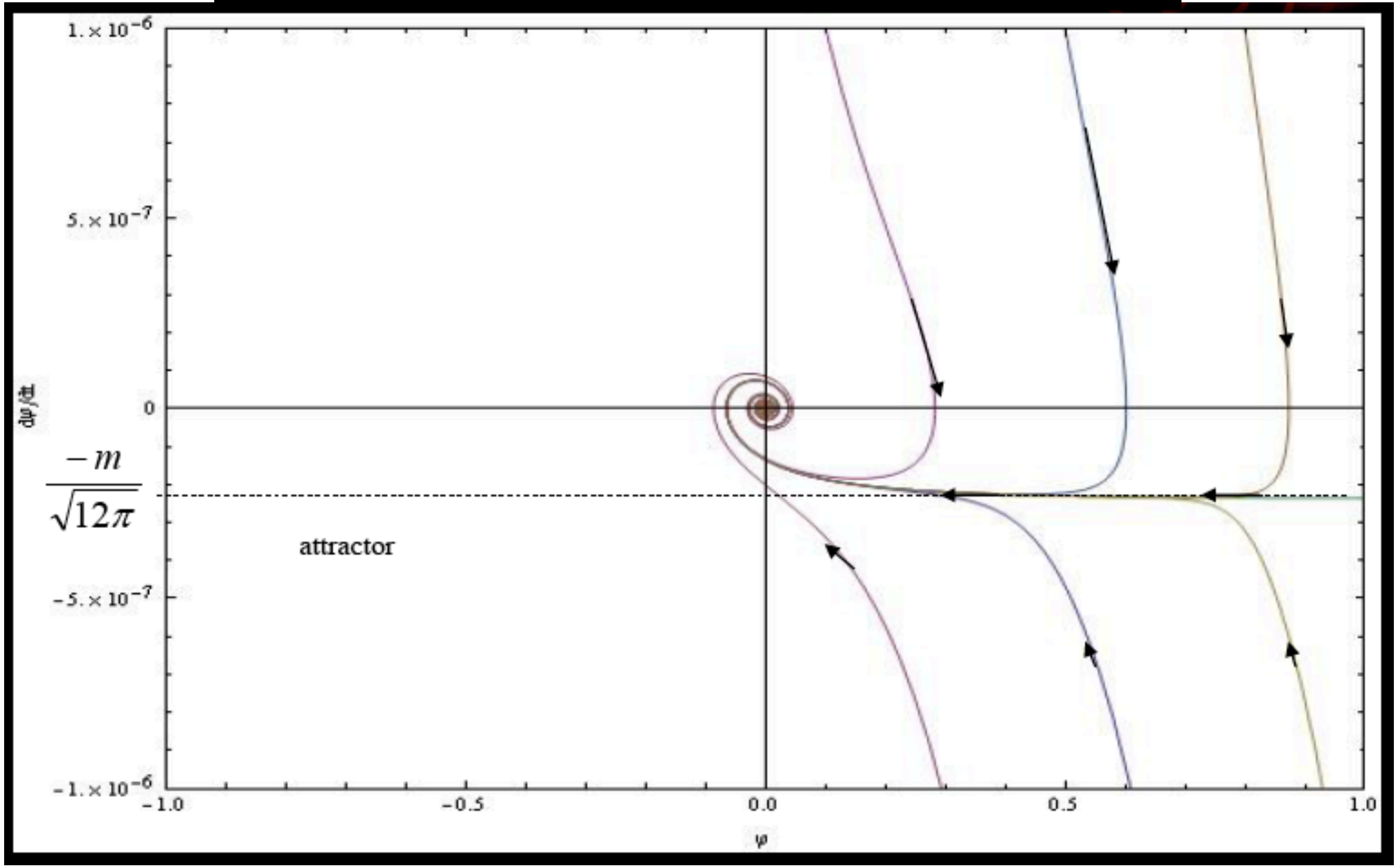
$$V(\phi) = \frac{m^2}{2}\phi^2$$



$$\ddot{\phi} + (3H + \gamma)\dot{\phi} + V'(\phi) = 0$$

$$\dot{\rho} + 4H\rho = \gamma\dot{\phi}^2$$

$$H^2 = \frac{8\pi}{3M_{Pl}^{\ 2}}\left(\frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2 + \rho\right)$$



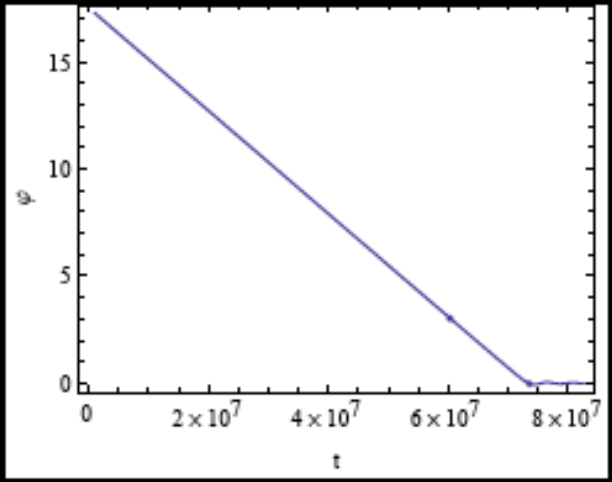
Magnetogenesis during inflation

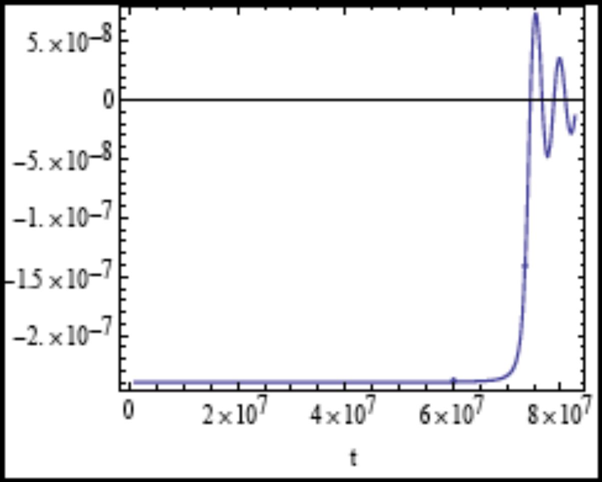
- 慢滚階段（slow-roll stage）

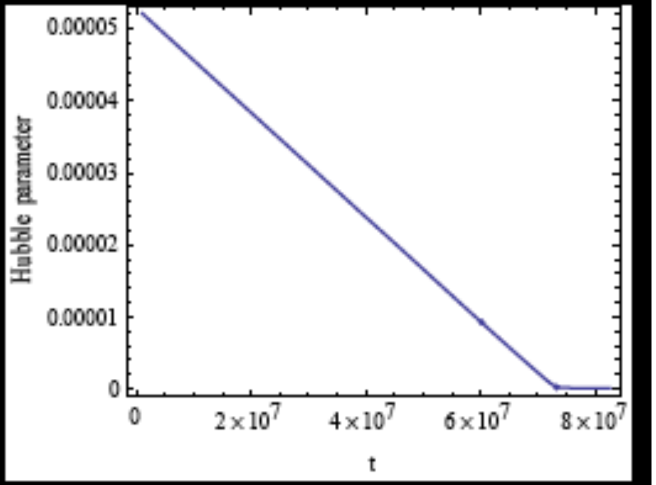
$$\frac{d^2}{d\eta^2}V_\pm + (q^2 \mp 4\cdot C\cdot q\cdot \frac{d\tilde{\phi}}{d\eta})V_\pm = 0 \longrightarrow V_\pm \propto Exp\left[\left(\sqrt{\pm 4\cdot C\cdot q\cdot \frac{d\tilde{\phi}}{d\eta} - q^2}\right)\cdot \eta\right]$$

$$\frac{a}{H}\cdot d\eta = dt$$

$$\Rightarrow \frac{a^2}{H^2}\left(\frac{\partial^2}{\partial \tilde{t}^2}V_\pm + \frac{1}{a}\frac{\partial a}{\partial \tilde{t}}\frac{\partial}{\partial \tilde{t}}V_\pm\right) + \left(q^2 \pm 4\cdot C\cdot q\cdot \frac{a}{H}\frac{d\tilde{\phi}}{d\tilde{t}}\right)V_\pm = 0$$

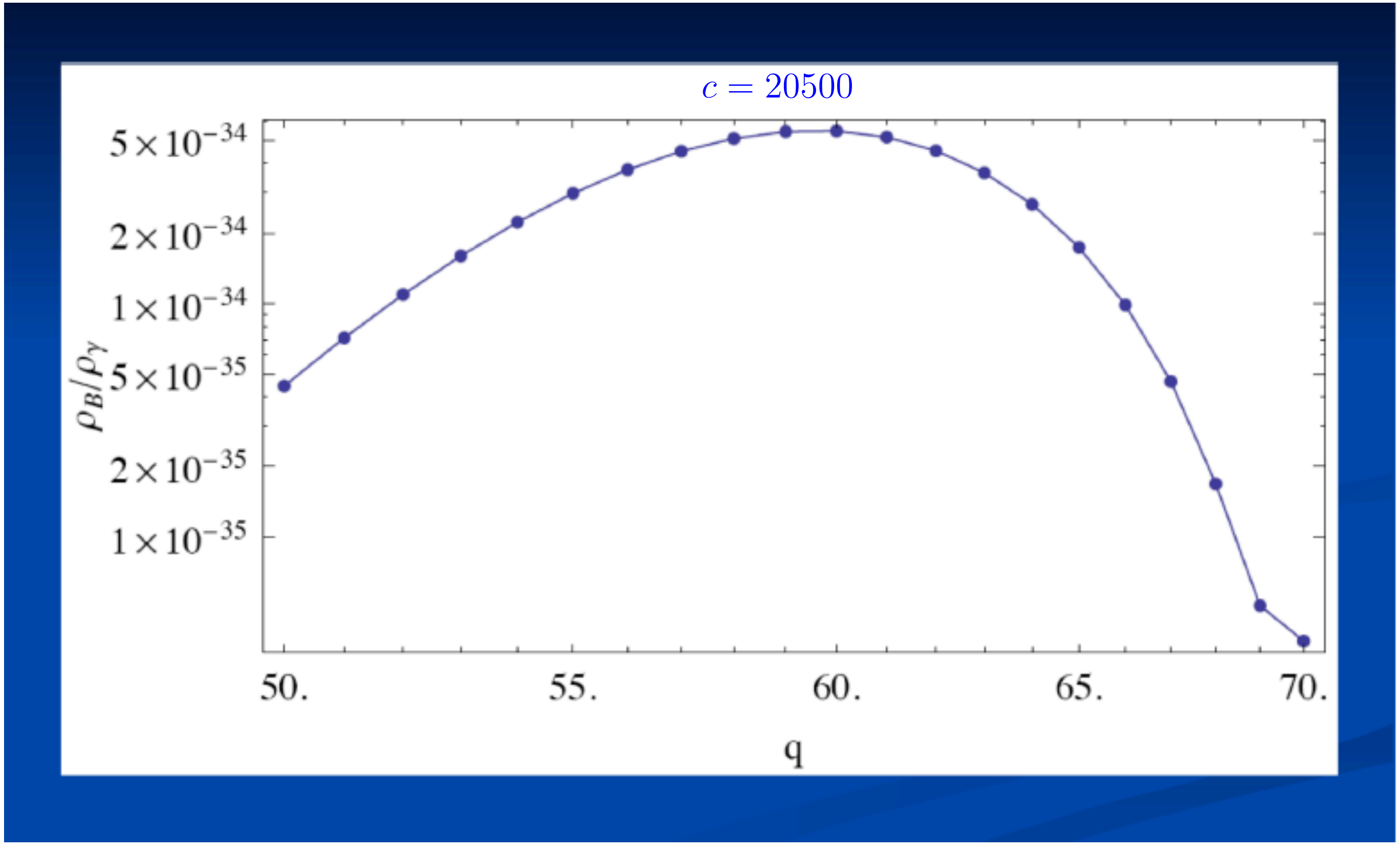




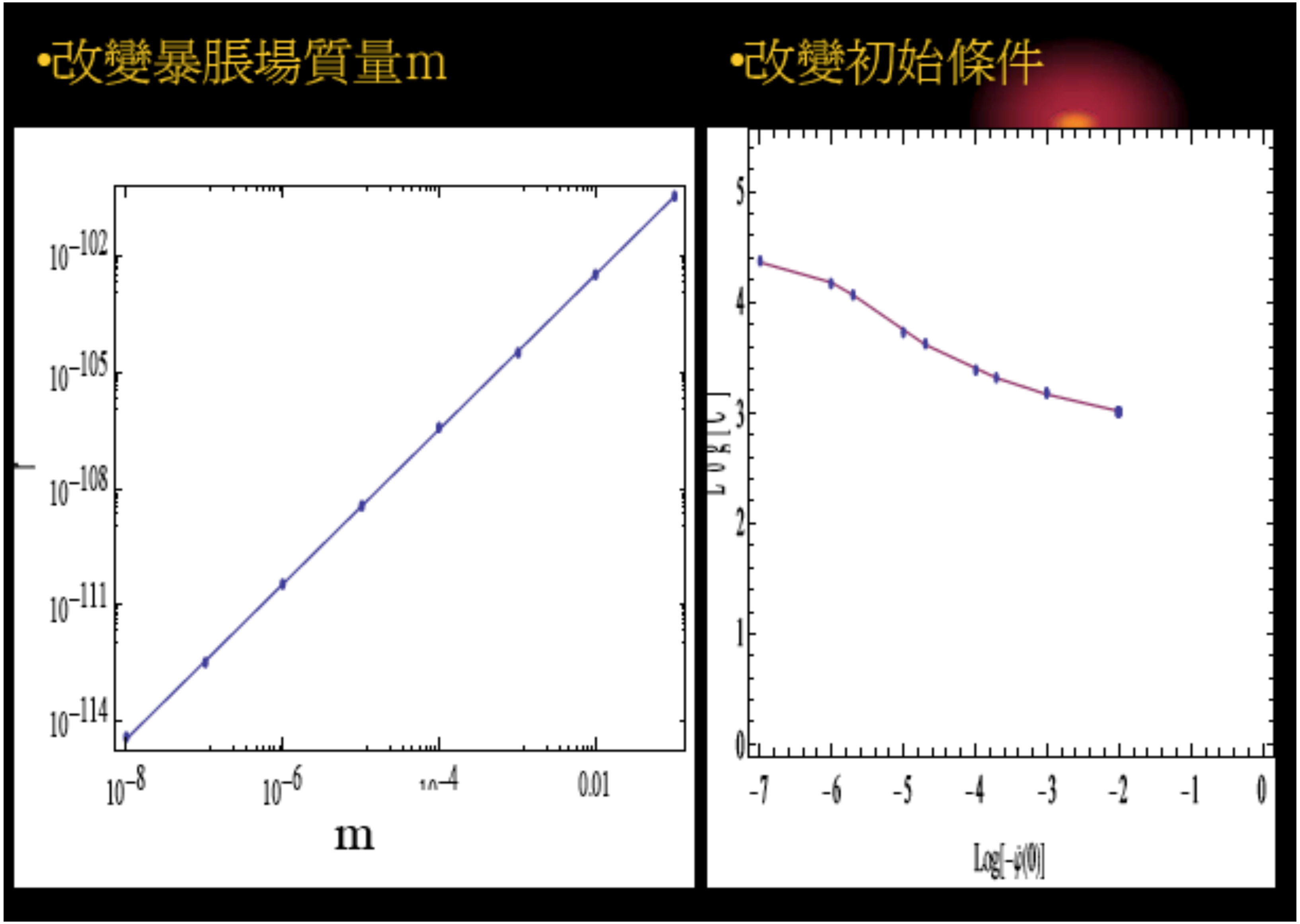


Inflationary PMF

$$\frac{d\rho_B}{dq} = \frac{q^3 H^4}{32\pi^3 a_{\mathrm{end}}^4} \sum_{i=\pm} |V_{iq}|^2$$



Slow-roll is not enough!



- 改變暴脹場質量m

- 改變初始條件

Slow-roll is not enough!

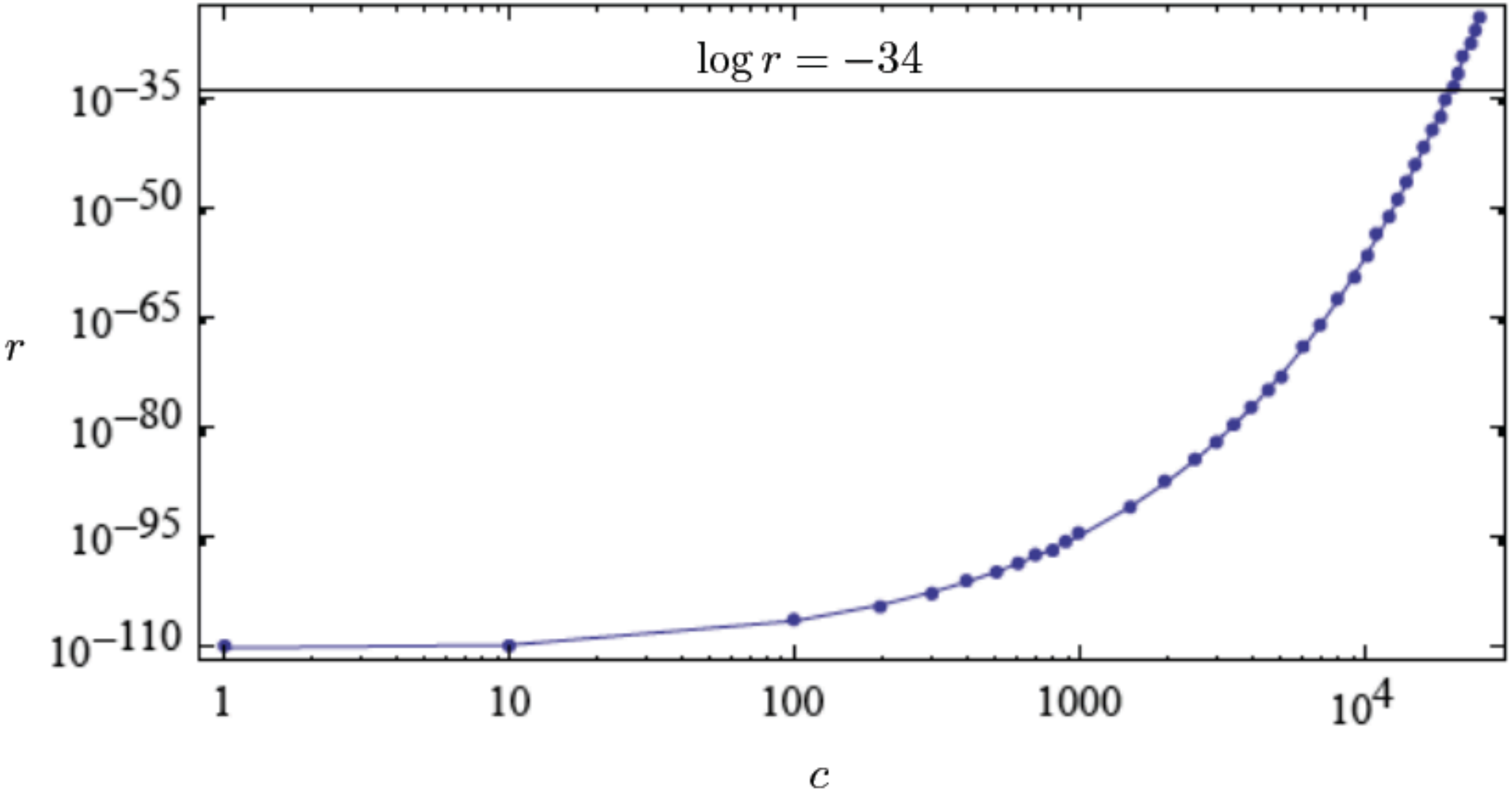


Fig. 1. The r vs c diagram suggests that the megnetogenesis induced by the spinodal instability in the slow roll regime is not efficient enough.

This is for the magnetic mode of horizon size $q = 1$. For magnetic modes of comoving size about 100 Mpc, $q \simeq 400$, we find that $c \simeq 800$ is required.

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Fast-roll inflation

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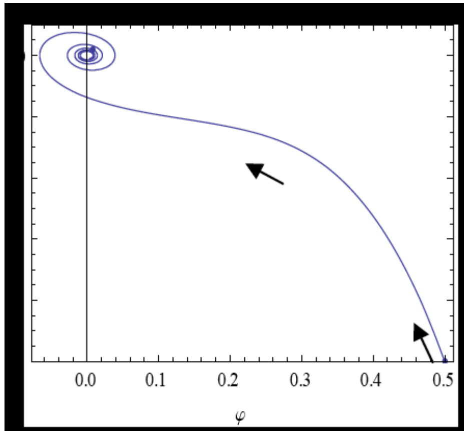
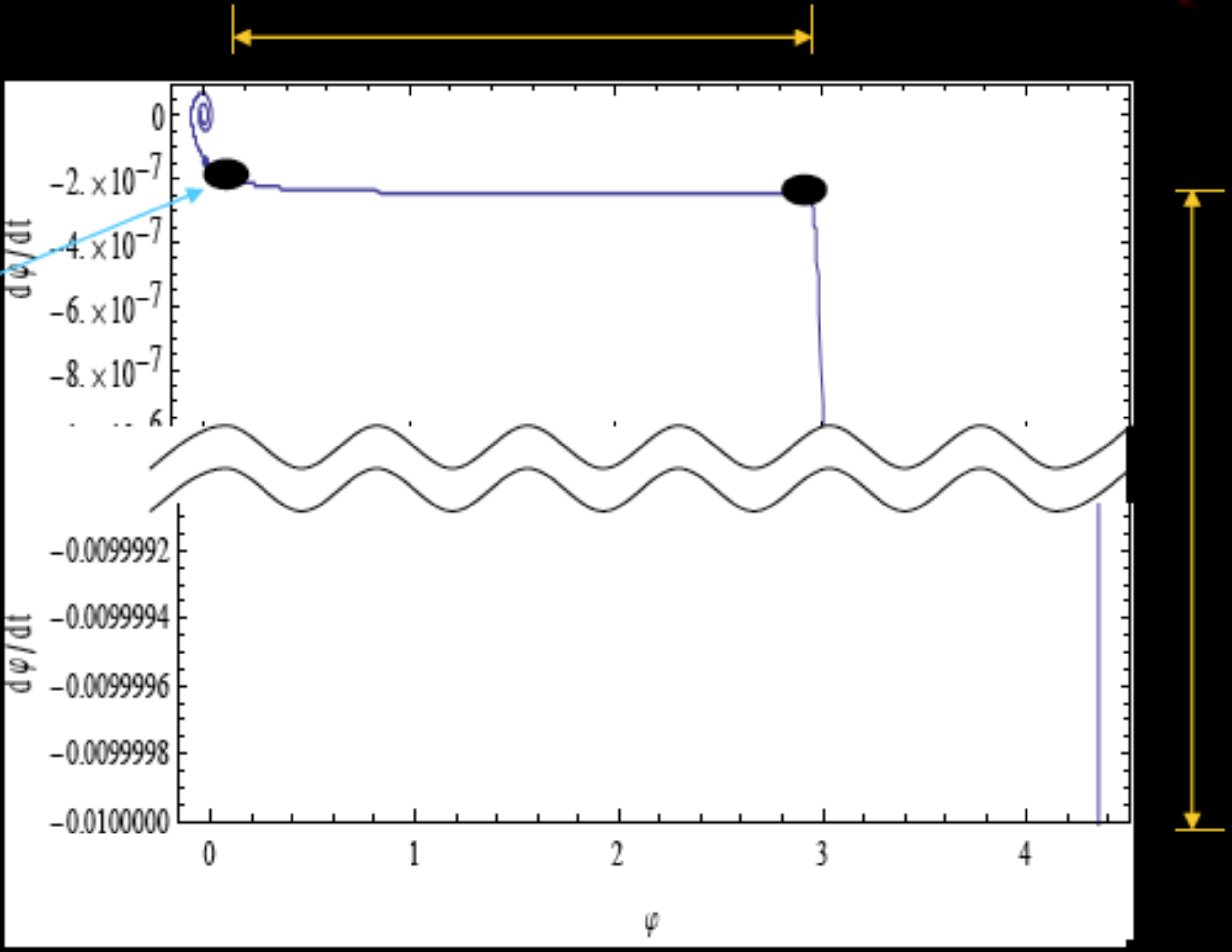
Abstract: We show that in the simplest theories of spontaneous symmetry breaking one can have a stage of a *fast-roll inflation*. In this regime the standard slow-roll condition $|m^2| \ll H^2$ is violated. Nevertheless, this stage can be rather long if $|m|$ is sufficiently small. Fast-roll inflation can be useful for generating proper initial conditions for the subsequent stage of slow-roll inflation in the very early universe. It may also be responsible for the present stage of accelerated expansion of the universe. We also make two observations of a more general nature. First of all, the universe after a long stage of inflation (either slow-roll or fast-roll) cannot reach anti-de Sitter regime even if the cosmological constant is negative. Secondly, the theories with the potentials with a "stable" minimum at $V(\phi) < 0$ in the cosmological background exhibit the same instability as the theories with potentials unbounded from below. This instability leads to the development of singularity with the properties practically independent of $V(\phi)$. However, the development of the instability in some cases may be so slow that the theories with the potentials unbounded from below can describe the present stage of cosmic acceleration even if this acceleration occurs due to the fast-roll inflation.

Keywords: Cosmology of Theories beyond the SM, Cosmological Phase Transitions, Physics of the Early Universe.

Invoking a fast-roll stage





慢滾階段（slow-roll stage）

再熱化
（reheating）

快滾階段
（fast-roll stage）

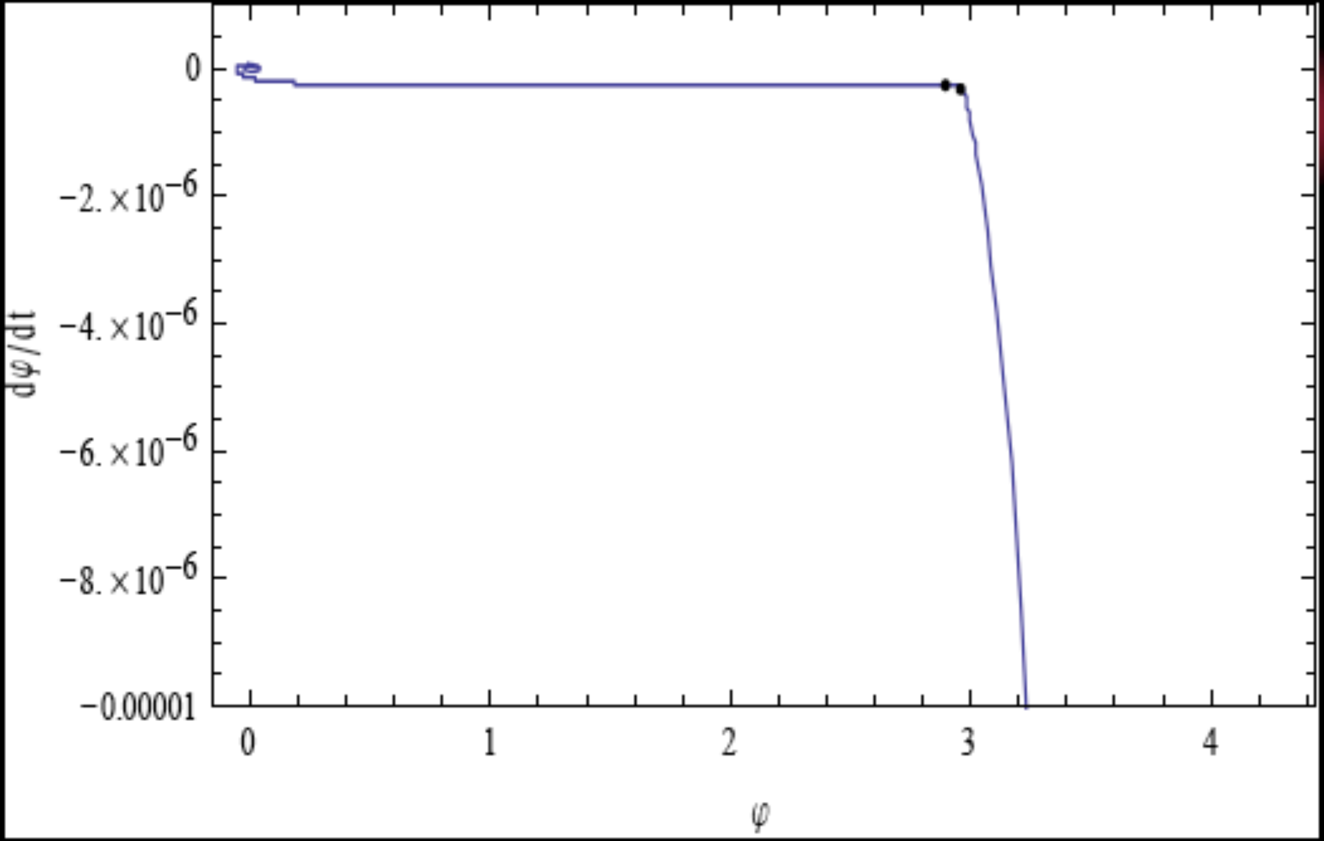
Magnetogenesis during inflation

$\tilde{\phi}_i = 4.35025$

$\dot{\tilde{\phi}}_i = -1.00289 \times 10^{-2}$

最大值位在

q=54



Invoking a fast-roll stage

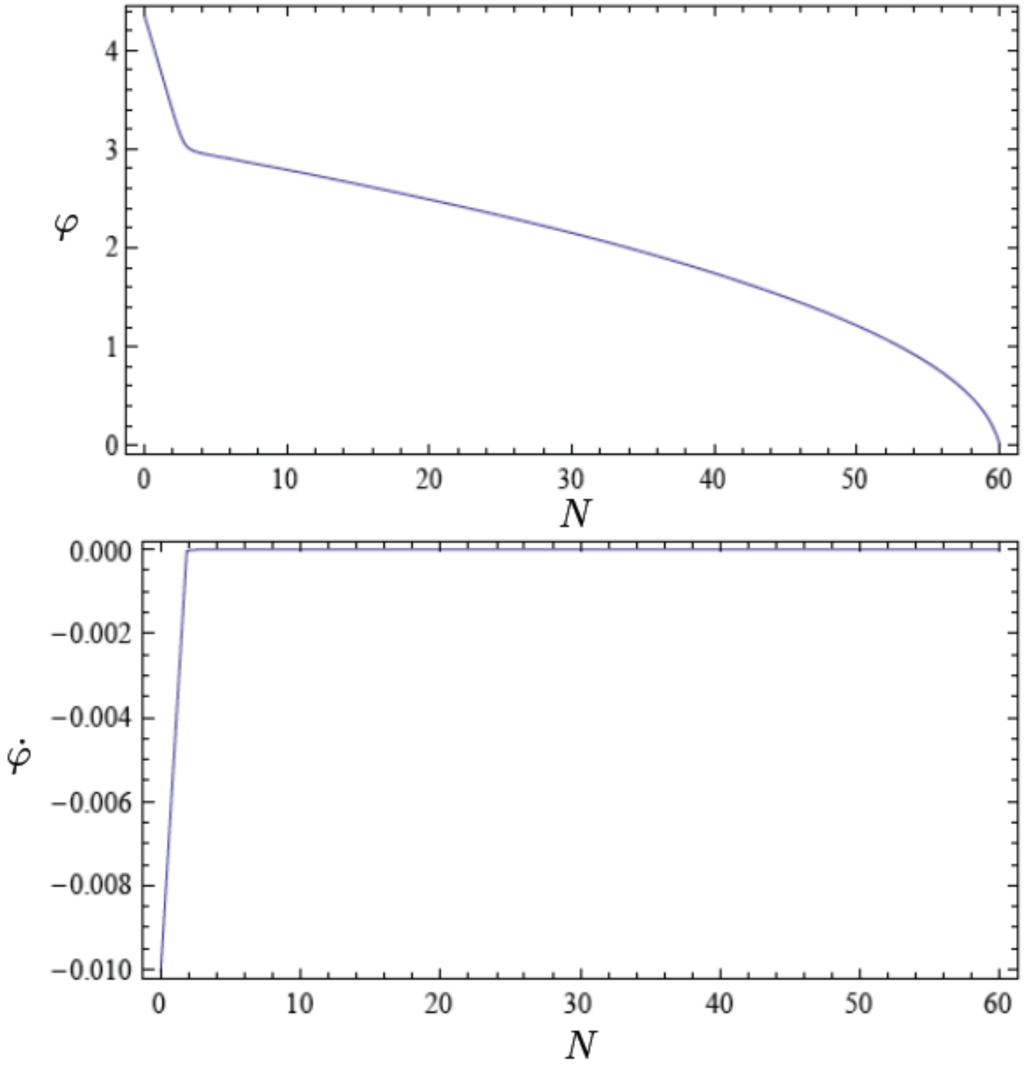


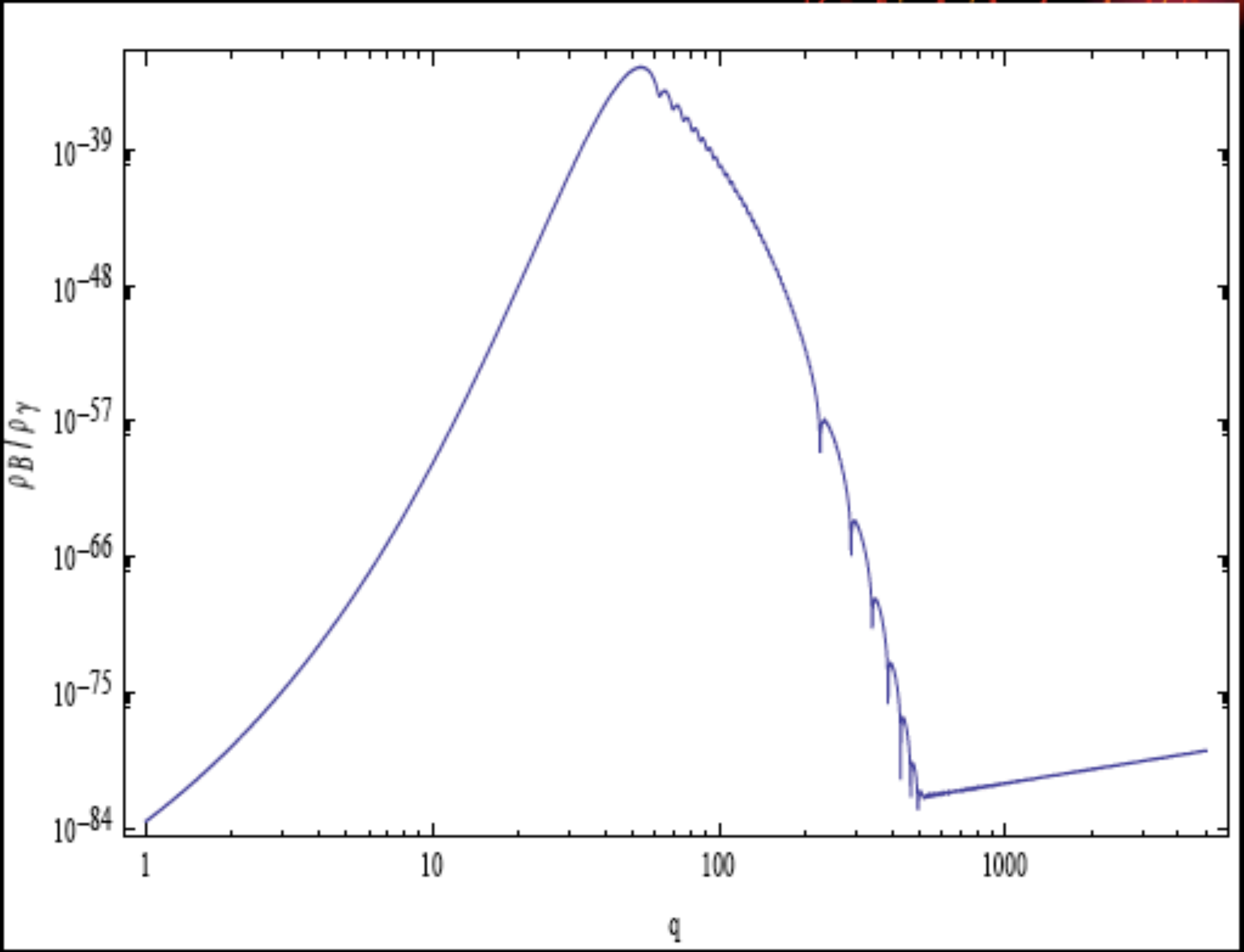
Fig. 3. Changes in φ and $\dot{\varphi}$ against the inflationary e-folds. As the inflaton being placed initially at a position high enough, the magnitude of its velocity $|\dot{\varphi}|$ will decrease rapidly before the inflaton entering the slow roll regime.

Magnetogenesis during inflation

$c = 34.5$

$\tilde{\phi}_i = 4.35025$

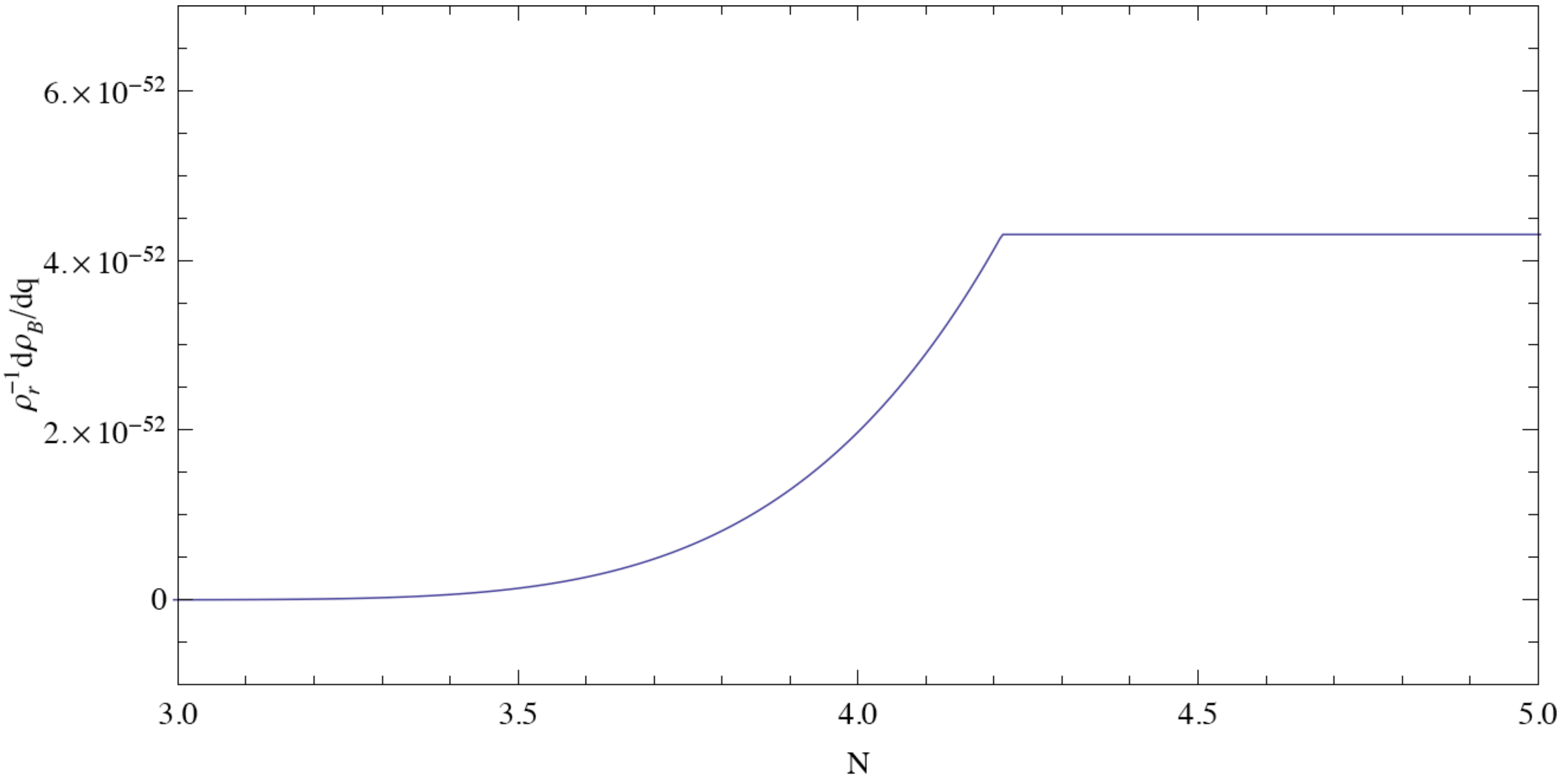
$\dot{\tilde{\phi}}_i = -1.00289 \times 10^{-2}$



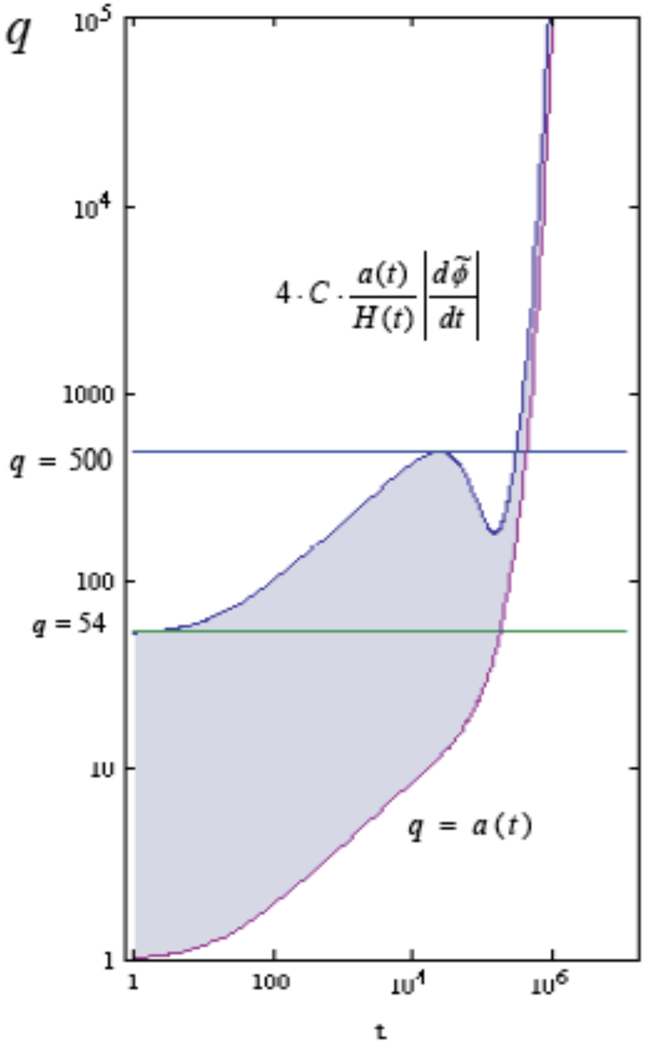
最大值位在

q=54

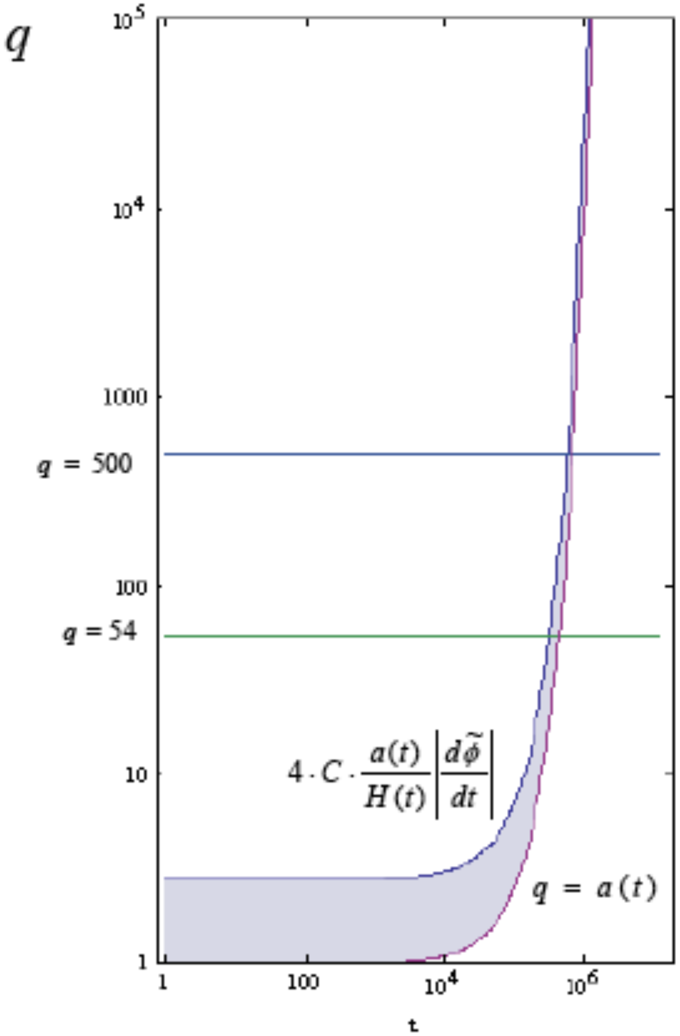
The evolution of the peak mode



Fast vs. slow

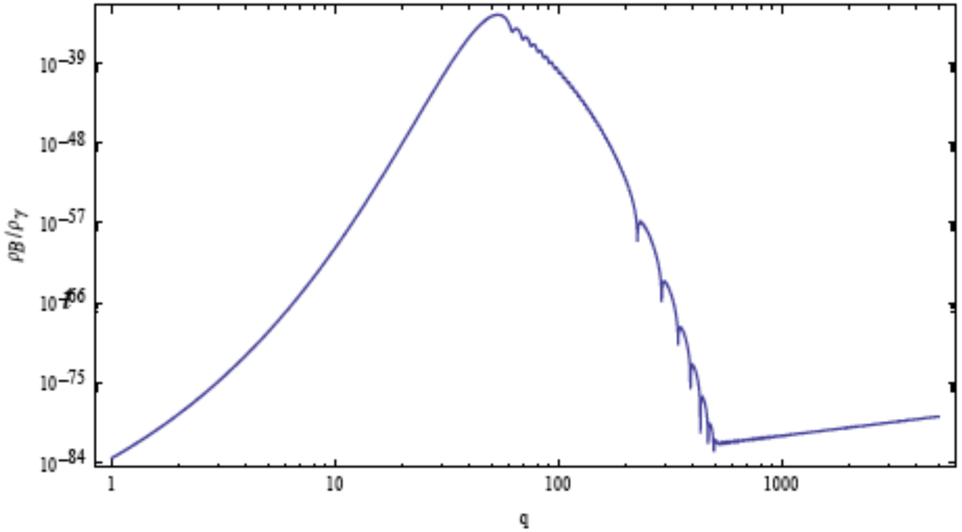


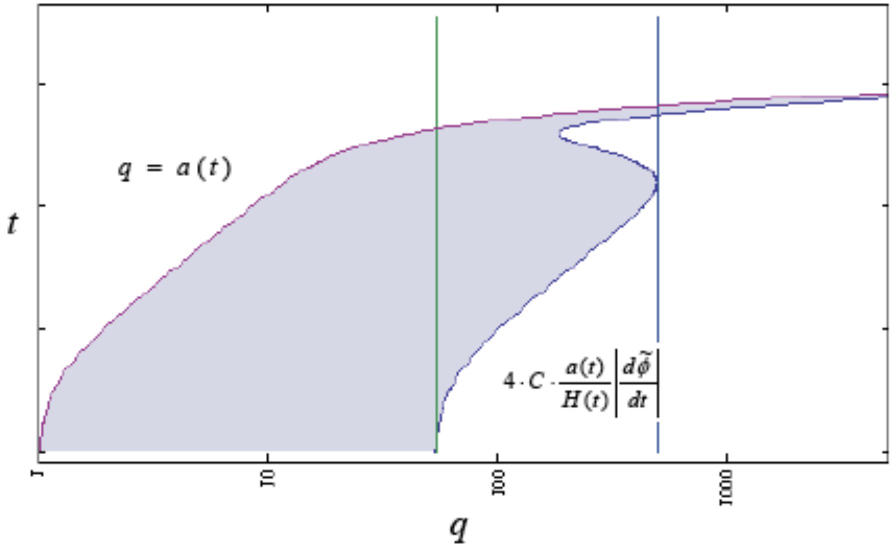
(a)



(b)

The spectrum vs. growing times





Summary

- Spinodal instability is a robust mechanism to generate PMF both inside and outside the Hubble horizon.
- By virtue of the generic quintessence scenario, we have successfully connected the generation of PMF and the late time acceleration of the Universe.
- Contrary to the previous studies, it is plausible to generate a large enough PMF through the coupling of the EM field to the inflaton provided that a fast-roll stage is involved.
- Due to the conservation of helicity, the B field produced by spinodal instability naturally avoids from any possible destruction along the evolution.

Discussion

- Due to the inclusion of a fast-roll phase, the generation of PMF may be related to the low quadrupole moment in the CMB anisotropy power spectrum.
- The magnetized cosmological perturbations add incoherently with the "vacuum" perturbations, and are highly nongaussian.
- The WMAP bound on nongaussianity implies that the coupling c of the pseudo-scalar inflaton to any gauge field must be smaller than about 100 [Barnaby et. al. arXiv: 1102.4333]
- The backreaction of <FF> may need to be rechecked!