$B \to K_1 \ell\ell$ in the SM and Beyond

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Refs:

PRD78:074007(2008) with H. Hatanaka

PRD77:094023(2008) with H. Hatanaka

Eur.Phys.J.C67:149(2010) with H. Hatanaka

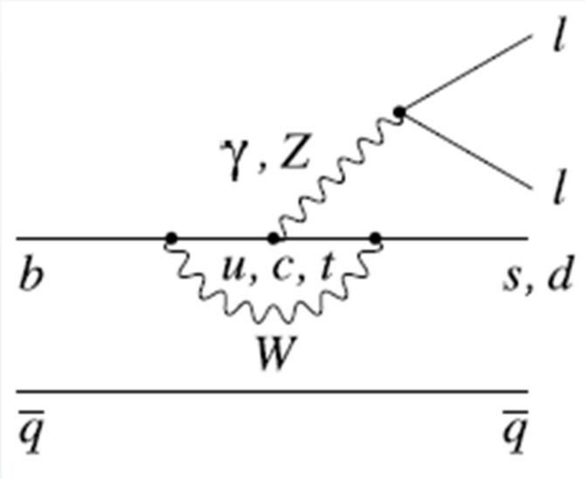
Phys.Rev.D78:034018,2008

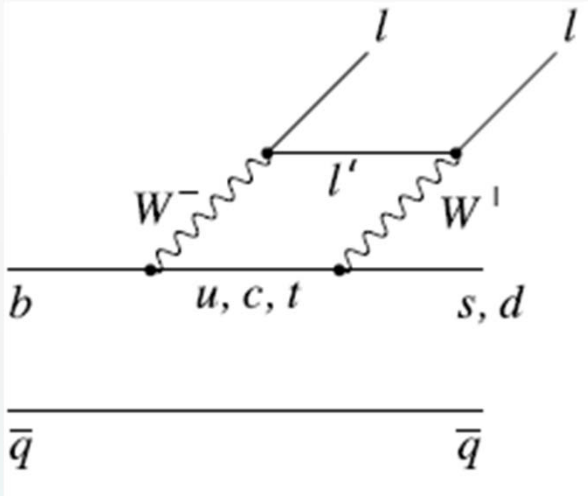
arXiv:1011.4661 with T.M. Aliev, M. Savci

Outline

- $b \to s(d)\, \ell\, \ell$: SM, new physics, data
- the K_{1A}-K_{1B} mixing angle, θ_{K_1}
- Results from SM and determination of θ_{K_1}
- Results for new physics
- Conclusion

In SM, flavor-changing neutral current (FCNC) transitions, $b \to s(d)\,\ell\,\ell$, are forbidden at the tree level





Electroweak penguin loops

Weak box diagram

sensitive to new physics

In SM, the $b \to s\,\ell^+\ell^-$ decay amplitude is given by

$$\mathcal{M}(b \to s\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{\rm em}}{\pi}V_{ts}^*V_{tb}\Big\{c_9^{\rm eff}(\hat{s})[\bar{s}\gamma_\mu Lb][\bar{\ell}\gamma^\mu\ell] + c_{10}[\bar{s}\gamma_\mu Lb][\bar{\ell}\gamma^\mu\gamma_5\ell] -2\hat{m}_b c_7^{\rm eff}\left[\bar{s}i\sigma_{\mu\nu}\frac{\hat{q}^\nu}{\hat{s}}Rb\right][\bar{\ell}\gamma^\mu\ell]\Big\},$$

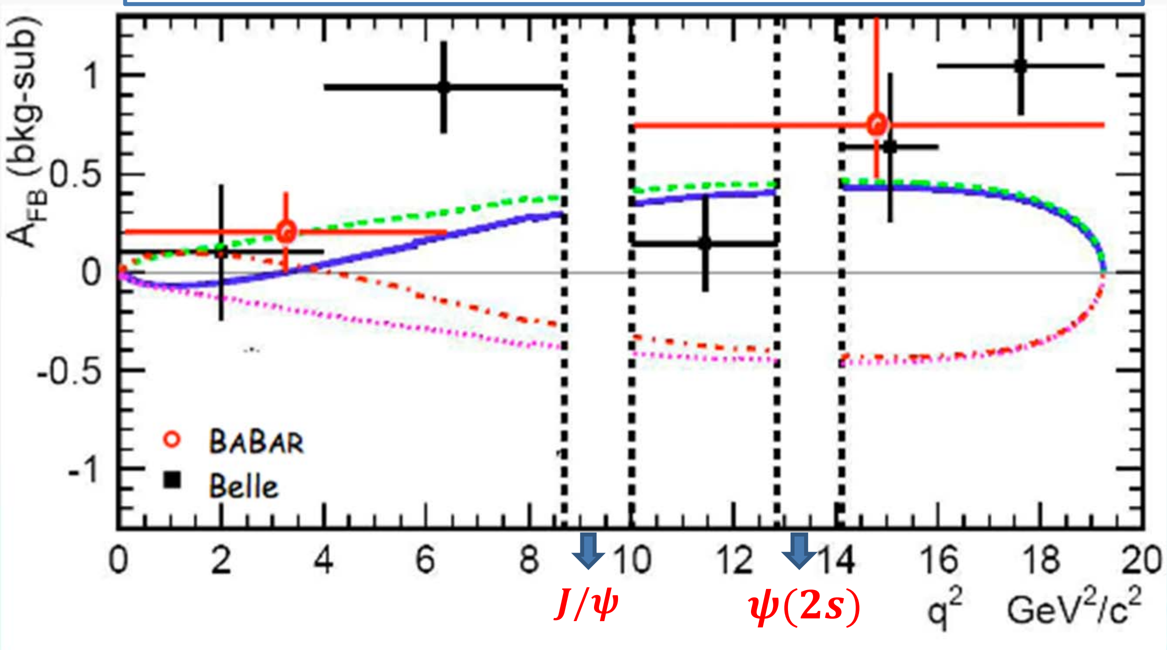
Dominant contributions are mainly from

- ♣ magnetic dipole operator O_7
- ♣ vector current operator O_9
- ♣ axial-vector current operator O_{10}

$|c_7^{eff}|$ is constrained by $\mathcal{B}(B \to X_s\,\gamma)$ data

Data: Forward-Backward Asymmetry of $B \to K^* \ell^+ \ell^-$

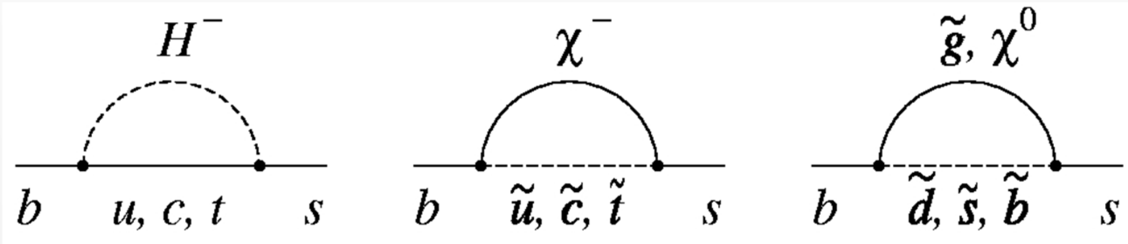
$$\frac{d\mathcal{A}_{FB}}{dq^2} \propto -\mathcal{R}e[(\tilde{C}_9(q^2))\tilde{C}_{10}(q^2)V(q^2)A_1(q^2) + \frac{M_B m_b}{q^2}\tilde{C}_7(q^2)\tilde{C}_{10}(q^2)(V(q^2)T_2(q^2)(1-\frac{m_{K^*}}{M_B}) + A_1(q^2)T_1(q^2)(1+\frac{m_{K^*}}{M_B}))].$$



New physics models

Dash: (1) sign-flipped c_7^{eff} model; or (2) sign flips for c_9 and c_{10} **(OK)**
Dotted dash: flipped sign(c_{10}) **(excluded)**
Dot: (1) sign flips for c_7 and c_{10}; or sign flip for c_9 alone **(excluded)**

SUSY phenomenology can enter at same order as SM contributions



Large $\tan\beta = \frac{v_u}{v_d} \geq 30, \ldots \implies$ sign flip in c_7^{eff}

Decays involving $K_1(1270)$ and/or $K_1(1400)$

From

$$Br(B^+ \to K_1^+(1270)\gamma) = (4.3 \pm 0.9 \pm 0.9) \times 10^{-5}$$

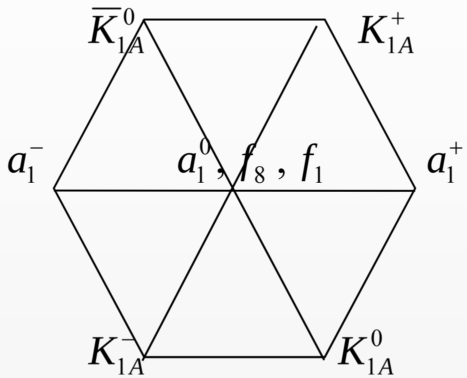
$$Br(B^+ \to K_1^+(1400)\gamma) < 1.5 \times 10^{-5}$$

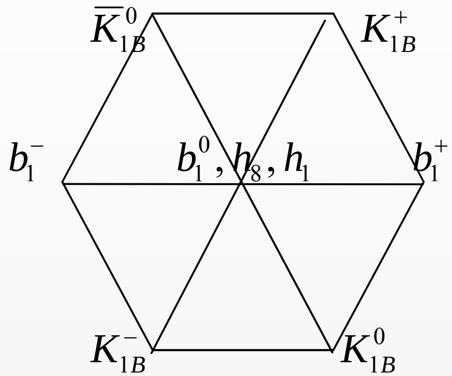
Belle

$B \to K_1\,\gamma,\ K_1\,\ell\ell$: New window for studying FCNC

Axial-vector mesons

$^3P_1$ (1^{++})





$^1P_1$ (1^{+-})

For $J^P=1^+$ light axial-vector mesons, two nonets have been observed:

- **$^3P_1$ nonet (S=1)**

  I=0: $f_1(1285)$, $f_1(1420)$, I=1/2: K_{1A}, I=1: $a_1(1260)$,

- **$^1P_1$ nonet (S=0)**

  I=0: $h_1(1170)$, $h_1(1380)$, I=1/2: K_{1B} , I=1: $b_1(1235)$

K_{1A}, $K_{1B} \rightarrow K_1(1270)$, $K_1(1400)$

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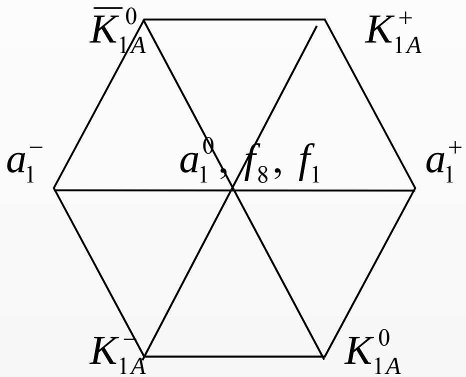
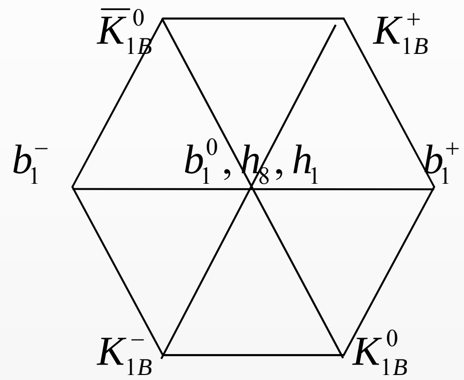
Mixing angles

$^3P_1$ (1^{++})





$^1P_1$ (1^{+-})

- **$^3P_1$ states $f_1(1285)$ & $f_1(1420)$ have mixing [so are $^1P_1$ states $h_1(1170)$ & $h_1(1380)$]**

$$|f_1(1285)\rangle = |f_1\rangle \cos\theta_{^3P_1} + |f_8\rangle \sin\theta_{^3P_1}, \quad |f_1(1420)\rangle = -|f_1\rangle \sin\theta_{^3P_1} + |f_8\rangle \cos\theta_{^3P_1}.$$

Using the Gell-Mann-Okubo mass formula

$$\cos^2\theta_{^3P_1} = \frac{4m^2_{K_{1A}} - m^2_{a_1} - 3m^2_{f_1(1285)}}{3\left(m^2_{f_1(1420)} - m^2_{f_1(1285)}\right)},$$

magnitude of θ

$$\tan\theta_{^3P_1} = \frac{4m^2_{K_{1A}} - m^2_{a_1} - 3m^2_{f_1(1420)}}{3m^2_{18}} > 0.$$

sign of θ

θ_{3P1} & θ_{1P1} depend on the K_{1A}-K_{1B} mixing angle

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■ **In SU(3) limit, K_{1A} & K_{1B} do not get mixed. However, they have admixture due to strange and light quark mass difference**

$$|\bar{K}_1(1270)\rangle = |\bar{K}_{1A}\rangle \sin\theta_{K_1} + |\bar{K}_{1B}\rangle \cos\theta_{K_1}$$
$$|\bar{K}_1(1400)\rangle = |\bar{K}_{1A}\rangle \cos\theta_{K_1} - |\bar{K}_{1B}\rangle \sin\theta_{K_1}$$

■ **PDG ⇒ $|\theta_K| \approx 45^o$**

■ **Strong decays $K_1(1270)$, $K_1(1400) \to K\rho, K^*\pi$ and their masses ⇒ $\theta_K = \pm 33^\circ, \pm 57^\circ$** (Suzuki,' 93)

$\tau \to K_1(1270)\nu_\tau, K_1(1400)\nu_\tau \Rightarrow \theta_K = \pm 37^\circ, \pm 58^\circ$ (HYC,'03)

From

$$Br(B^+ \to K_1^+(1270)\gamma) = (4.3 \pm 0.9 \pm 0.9) \times 10^{-5}$$
$$Br(B^+ \to K_1^+(1400)\gamma) < 1.5 \times 10^{-5}$$

Belle

$$Br(\tau^+ \to K_1^+(1270)\nu) = (4.7 \pm 1.1) \times 10^{-5}$$

ALEPH

⇒ **$\theta_K = -(34 \pm 13)^o$,** H. Hatanaka & KCY, Phys.Rev.D77:094023,2008

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For axial-vector mesons: Decay constants

$$\langle 3P_1(p,\varepsilon)|\bar{q}\gamma_\mu\gamma_5 q'|0\rangle = i f_{3P_1} m_{3P_1}\varepsilon^*_\mu, \qquad \langle 1P_1(p,\varepsilon)|\bar{q}\sigma_{\mu\nu}\gamma_5 q'|0\rangle = f^\perp_{1P_1}(\varepsilon^*_\mu p_\nu - \varepsilon^*_\nu p_\mu)$$

scale dependent

$^3P_1$	$a_1(1260)$	f_1	f_8	K_{1A}
$f_{^3P_1}$	238 ± 10	245 ± 13	239 ± 13	250 ± 13
$^1P_1$	$b_1(1235)$	h_1	h_8	K_{1B}
$f^\perp_{^1P_1}$	180 ± 8	180 ± 12	190 ± 10	190 ± 10

QCDSR by KCY, '07

Form factors for B → A

$$\langle A(p,\lambda)|A_\mu|\overline{B}_q(p_B)\rangle = i\frac{2}{m_{B_q}-m_A}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)}p^\alpha_B p^\beta A^{B_qA}(q^2),$$

$$\langle A(p,\lambda)|V_\mu|\overline{B}_q(p_B)\rangle = -\Big\{(m_{B_q}-m_A)\epsilon^{(\lambda)*}_\mu V_1^{B_qA}(q^2) - (\epsilon^{(\lambda)*}\cdot p_B)(p_B+p)_\mu\frac{V_2^{B_qA}(q^2)}{m_{B_q}-m_A} - 2m_A\frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2}q^\mu\left[V_3^{B_qA}(q^2)-V_0^{B_qA}(q^2)\right]\Big\},$$

By light cone sum rules (KCY,PRD78:034018,2008)

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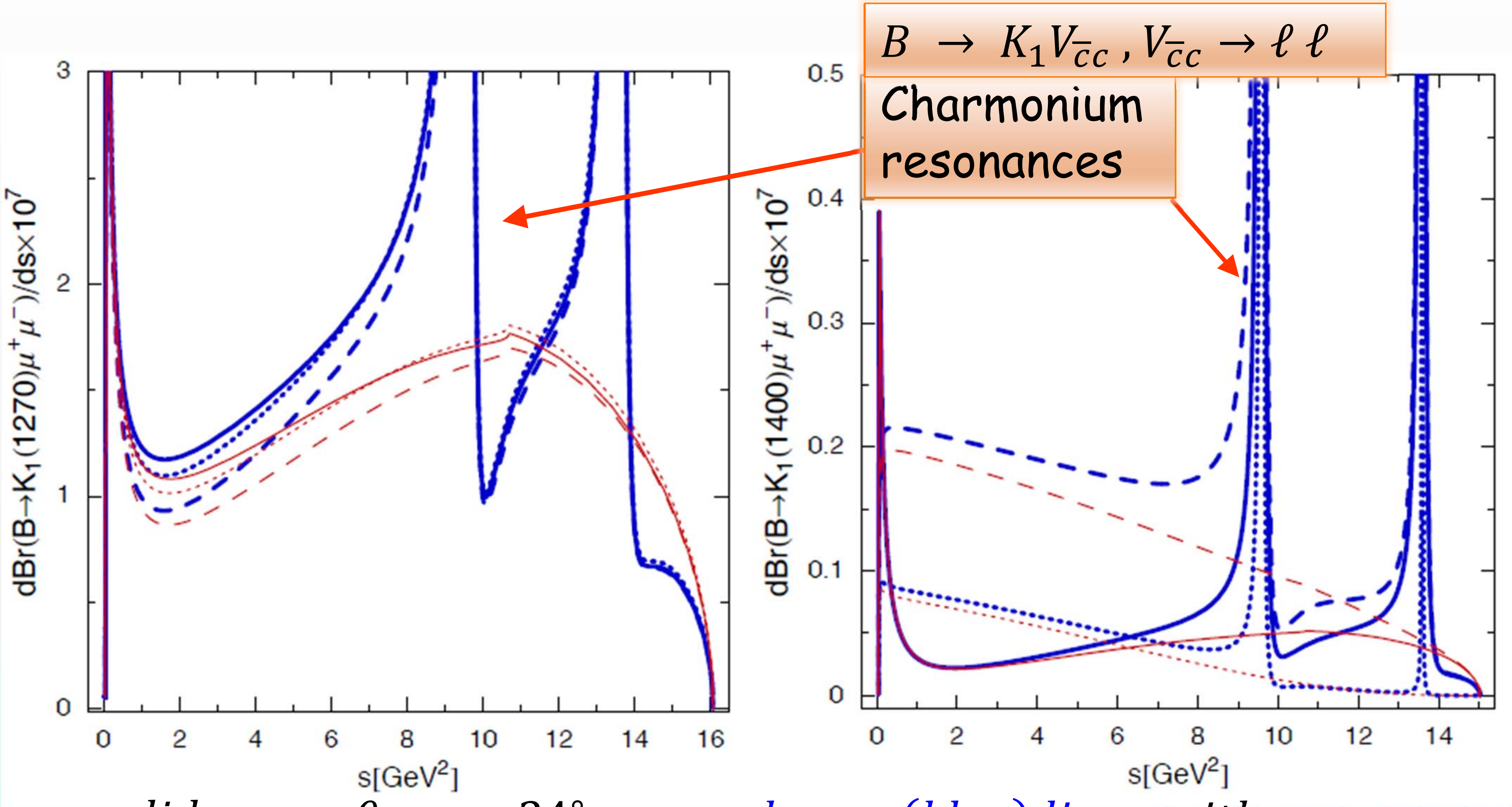
θ_{K_1} dependence of differential dilepton mass spectrum



solid curve: $\theta_{K_1} = -34°$
dotted curve: $\theta_{K_1} = -45°$
dashed curve: $\theta_{K_1} = -57°$

heavy (blue) lines: *with resonances*
light(red) lines: *without resonances*

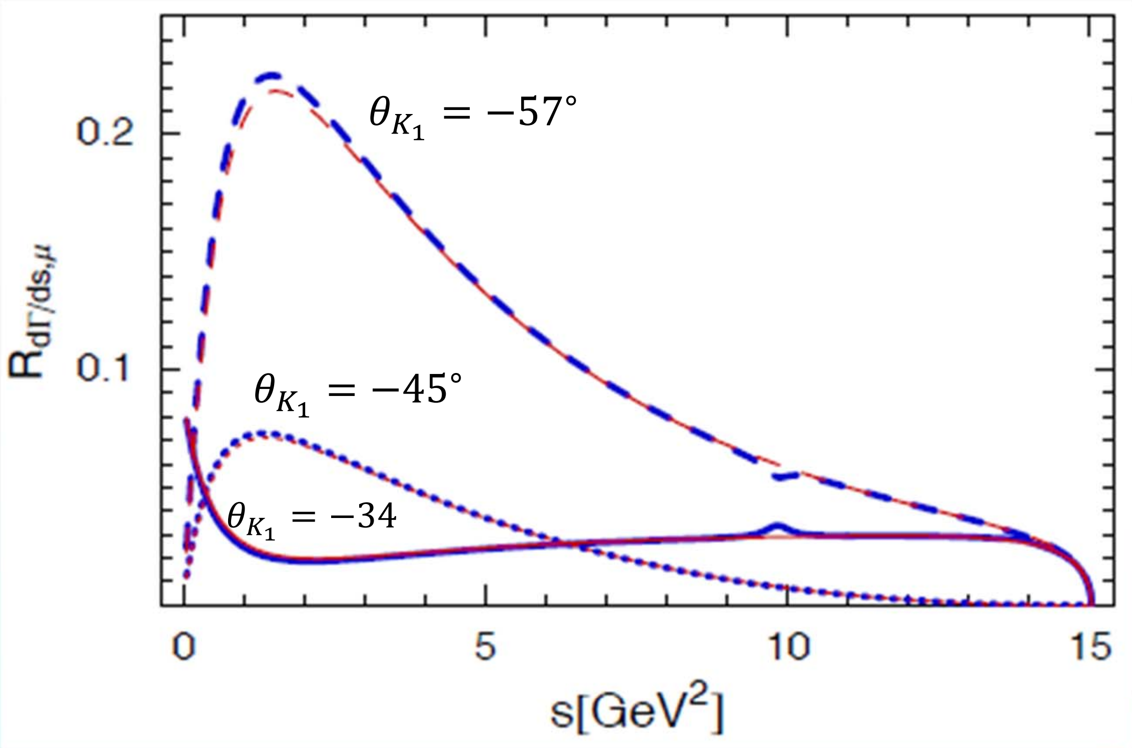
The effects of charmonium resonances become large for $s \geq 5\ \mathrm{GeV}^2$.

In the low s region, $\mathrm{s} \approx 2\ \mathrm{GeV}^2$, the differential decay rate for $B \to K_1(1400)\,\mu^+\mu^-$ with $\theta_{K_1} = -57°$ is enhanced by about 80% compared with that with $\theta_{K_1} = -34°$,

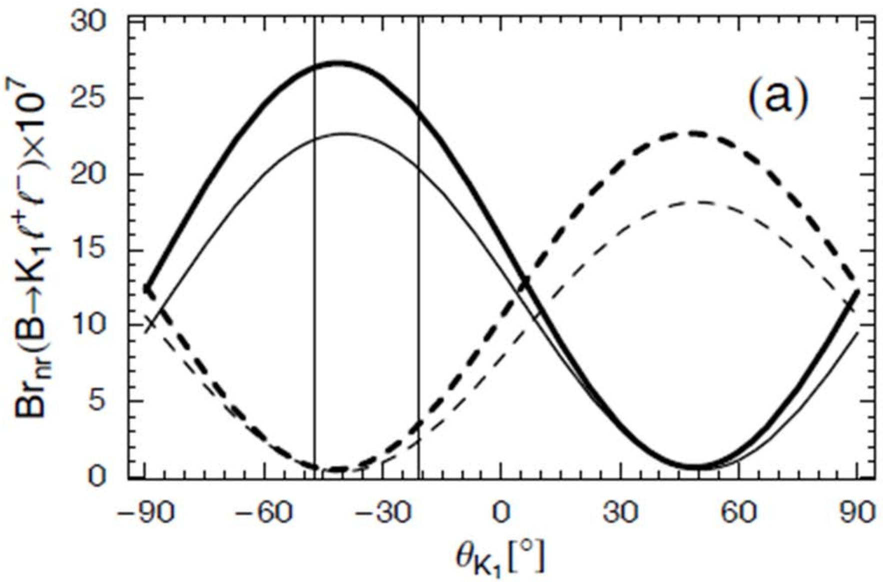
The rates for $B \to K_1(1270)\mu^+\mu^-$ is not so sensitive to variation of θ_{K_1}.

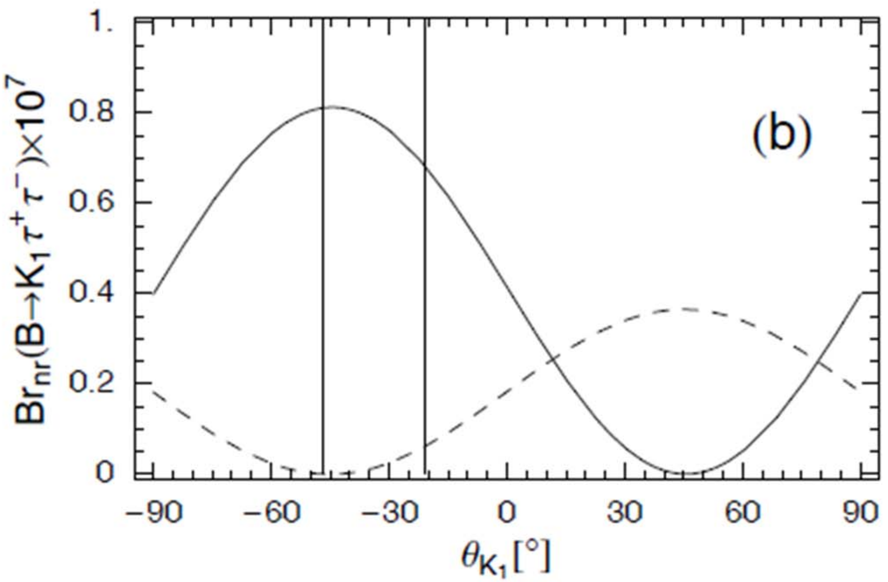
θ_{K_1} can be well determined by the ratio:

$$R_{d\Gamma/ds,\mu} \equiv \frac{d\Gamma(B^- \to K_1^-(1400)\mu^+\mu^-)/ds}{d\Gamma(B^- \to K_1^-(1270)\mu^+\mu^-)/ds}$$



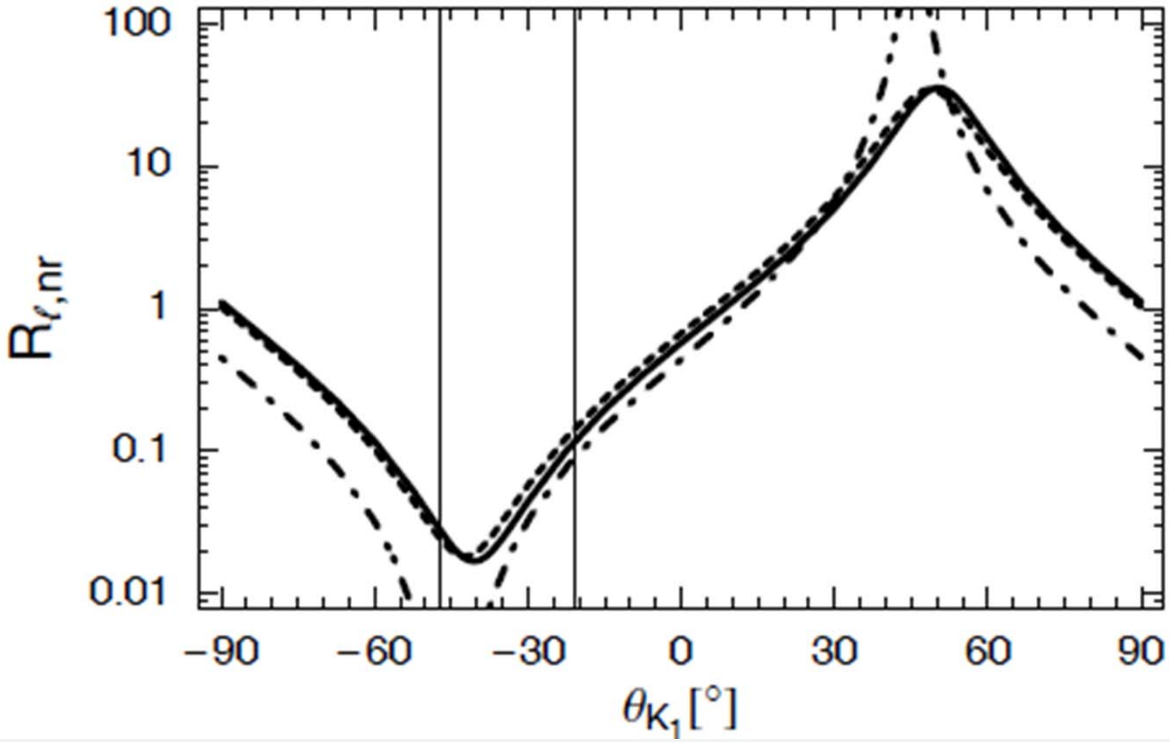
Insensitive to form factors and resonances





Non-resonant branching fractions $\mathcal{B}_{nr}(B^- \to K_1^- \ell^+ \ell^-)$ as functions of θ_{K_1}

Mode	$\mathcal{B}_{nr} \times 10^6$	Mode	$\mathcal{B}_{nr} \times 10^6$
$B^- \to K_1^-(1270)e^+e^-$	$2.7^{+1.5+0.0}_{-1.2-0.3}$	$\overline{B}^0 \to \overline{K}_1^0(1270)e^+e^-$	$2.5^{+1.4+0.0}_{-1.1-0.3}$
$B^- \to K_1^-(1270)\mu^+\mu^-$	$2.3^{+1.3+0.0}_{-1.0-0.2}$	$\overline{B}^0 \to \overline{K}_1^0(1270)\mu^+\mu^-$	$2.1^{+1.2+0.0}_{-0.9-0.2}$
$B^- \to K_1^-(1270)\tau^+\tau^-$	$0.08^{+0.04+0.00}_{-0.03-0.01}$	$\overline{B}^0 \to \overline{K}_1^0(1270)\tau^+\tau^-$	$0.08^{+0.04+0.00}_{-0.03-0.01}$
$B^- \to K_1^-(1400)e^+e^-$	$0.10^{+0.03+0.25}_{-0.03-0.05}$	$\overline{B}^0 \to \overline{K}_1^0(1400)e^+e^-$	$0.09^{+0.03+0.23}_{-0.03-0.04}$
$B^- \to K_1^-(1400)\mu^+\mu^-$	$0.06^{+0.02+0.18}_{-0.01-0.02}$	$\overline{B}^0 \to \overline{K}_1^0(1400)\mu^+\mu^-$	$0.06^{+0.02+0.18}_{-0.01-0.02}$
$B^- \to K_1^-(1400)\tau^+\tau^-$	$0.001^{+0.000+0.005}_{-0.000-0.001}$	$\overline{B}^0 \to \overline{K}_1^0(1400)\tau^+\tau^-$	$0.001^{+0.000+0.005}_{-0.000-0.001}$



Solid curve $R_{e,nr}$

Dashed curve $R_{\mu,nr}$

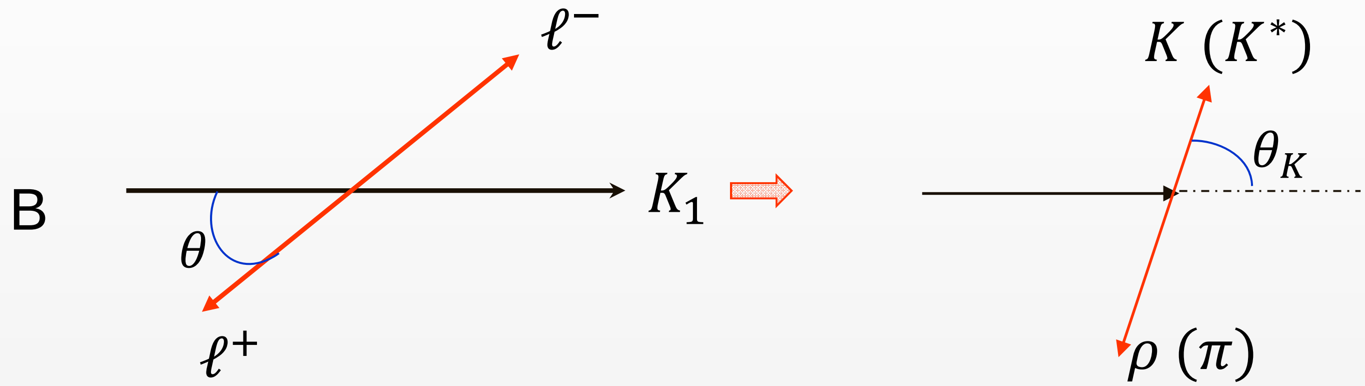
Dot-dashed curve $R_{\tau,nr}$

$$R_{\ell,\mathrm{nr}} \equiv \frac{\mathcal{B}_{\mathrm{nr}}(B \to K_1(1400)\ell^+\ell^-)}{\mathcal{B}_{\mathrm{nr}}(B \to K_1(1270)\ell^+\ell^-)}$$

The ratio is less than 0.15 for $-47° \leq \theta_{K_1} \leq -21°$

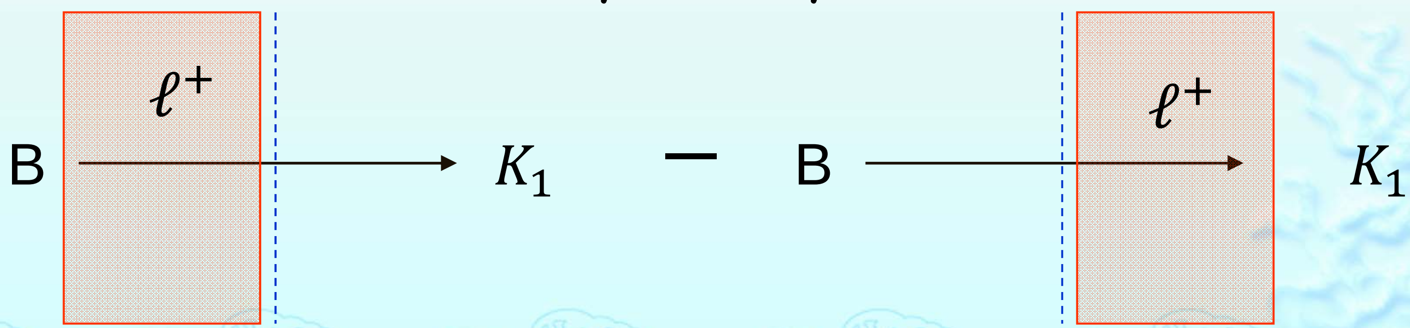
$$R_{e,\mathrm{nr}} = 0.04^{+0.01+0.11}_{-0.01-0.02}, \quad R_{\mu,\mathrm{nr}} = 0.03^{+0.01+0.09}_{-0.01-0.01}, \quad R_{\tau,\mathrm{nr}} = 0.02^{+0.01+0.07}_{-0.00-0.02}$$

$B \to K_1 \ell \ell$ decay



$$B \to \begin{cases} K_1(1270) \to K\rho \to K\pi\pi \\ K_1(1400) \to K^*\pi \to K\pi\pi \end{cases} + \ \ell\,\ell$$

Forward-Backward Asymmetry



Forward-Backward Asymmetry

$$\frac{dA_{\rm FB}}{d\hat{s}} \equiv \int_0^{\hat{u}(\hat{s})} d\hat{u}\frac{d^2\Gamma}{d\hat{u}d\hat{s}} - \int_{-\hat{u}(\hat{s})}^{0} d\hat{u}\frac{d^2\Gamma}{d\hat{u}d\hat{s}}$$

$0 \leq \theta < 90^\circ$ | $90^\circ \leq \theta < 180^\circ$

$$\frac{dA_{\rm FB}}{d\hat{s}} = -\frac{G_F^2\alpha_{\rm em}^2 m_B^5}{2^8\pi^5}|V_{ts}^*V_{tb}|^2\hat{s}\hat{u}(\hat{s})^2 \times c_{10}\Big[{\rm Re}(c_9^{\rm eff}(\hat{s}))A^{K_1}V_1^{K_1} + \frac{\hat{m}_b}{\hat{s}}c_7^{\rm eff}\Big\{A^{K_1}T_2^{K_1}(1-\hat{m}_{K_1}) + V_1^{K_1}T_1^{K_1}(1+\hat{m}_{K_1})\Big\} + \frac{\hat{m}_b}{\hat{s}}\Delta_{\rm HS}\Big]$$

with hard spectator correction

$$\Delta_{\rm HS} = \Big\{(1+\hat{m}_{K_1})V_1^{K_1} + (1-\hat{m}_{K_1})(1-\hat{s})A^{K_1}\Big\} \times \frac{\alpha_s(\mu_h)C_F}{4\pi}\frac{\pi^2}{N_c}\frac{f_B f_{K_1}^{\perp}}{\lambda_{B,+}m_B}\int_0^1 du\,\Phi_{K_1}^{\perp}(u)T_{\perp,+}^{(\rm nf)}(u)$$

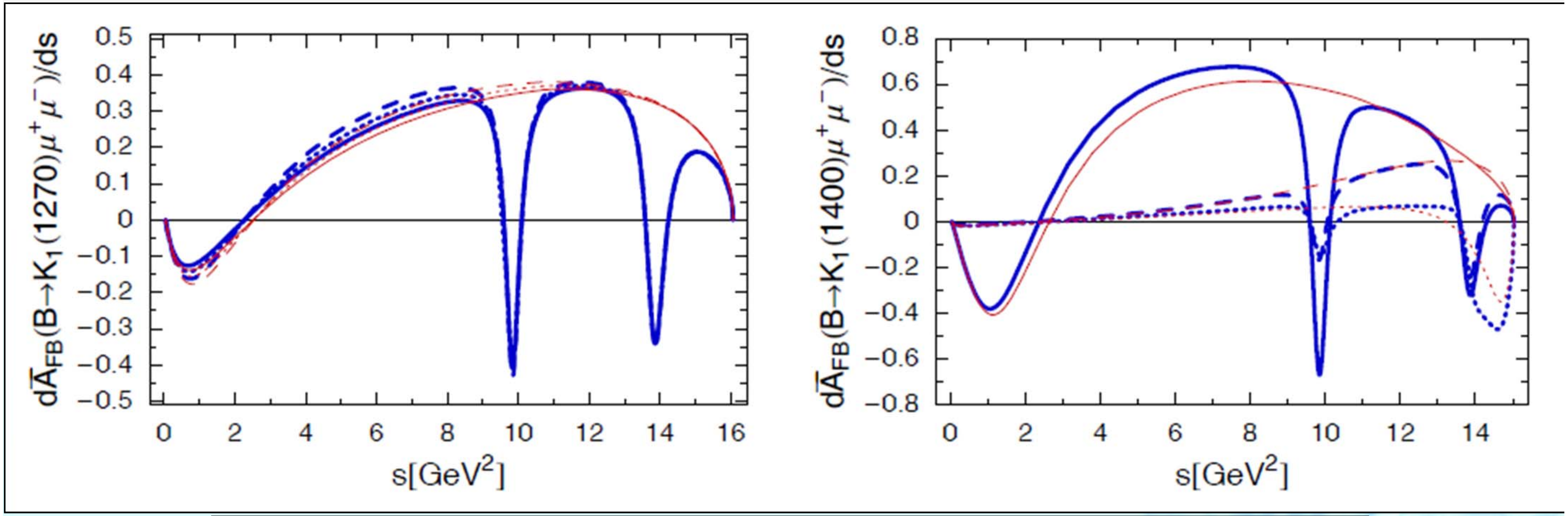
Non-factorizable contribution,
see Beneke, Feldmann & Seidel hep-ph/0106067

Forward-Backward Asymmetry (FBA)

Zero of FBA: $\frac{dA_{FB}}{d\hat{s}}\left(s_{0_1}^{K_1}\right) = 0$

$$\frac{\mathrm{Re}(c_9^{\mathrm{eff}}(\hat{s}_0^{K_1}))}{c_7^{\mathrm{eff,HS}}(\hat{s}_0^{K_1})} = -\frac{\hat{m}_b}{\hat{s}_0^{K_1}}\left\{\frac{T_2^{K_1}(\hat{s}_0^{K_1})}{V_1^{K_1}(\hat{s}_0^{K_1})}(1-\hat{m}_{K_1}) + \frac{T_1^{K_1}(\hat{s}_0^{K_1})}{A^{K_1}(\hat{s}_0^{K_1})}(1+\hat{m}_{K_1})\right\}$$

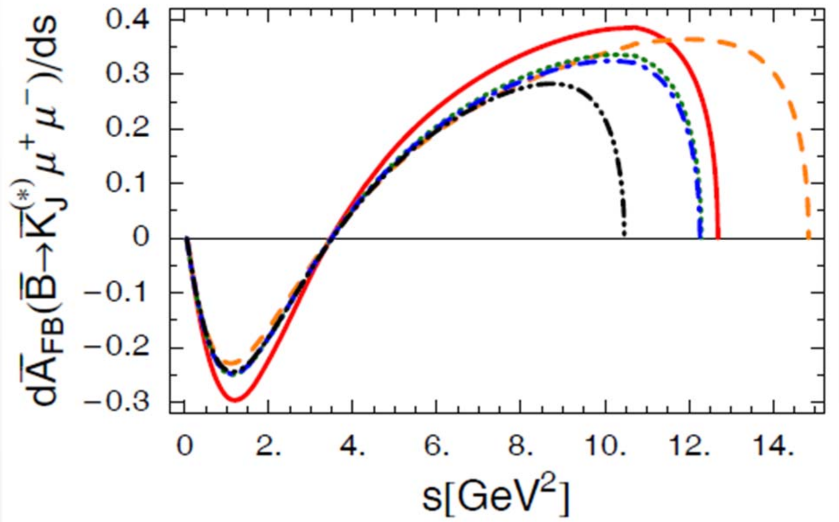
$$c_7^{\mathrm{eff,HS}}(\hat{s}) \equiv c_7^{\mathrm{eff}} + \frac{\Delta_{\mathrm{HS}}(\hat{s})}{A^{K_1}(\hat{s})T_2^{K_1}(\hat{s})(1-\hat{m}_{K_1}) + V_1^{K_1}(\hat{s})T_1^{K_1}(\hat{s})(1+\hat{m}_{K_1})}$$



$$s_0^{K_1(1270)} = 2.27^{+0.04+0.01}_{-0.07-0.01}\ \mathrm{GeV}^2 \quad \text{and} \quad s_0^{K_1(1400)} = 2.80^{+0.23+0.74}_{-0.29-0.07}\ \mathrm{GeV}^2$$

Form factor ← → (zero is insensitive to θ_{K_1})

FBA of $B \to K_n^* \ell \ell$ decay



$$\mathrm{Re}\left[c_9^{\mathrm{eff}}(\hat{s}_0)c_{10}\right] = -2\frac{\hat{m}_b}{\hat{s}_0}\mathrm{Re}(c_7^{\mathrm{eff}}c_{10})\frac{1-\hat{s}_0}{1+\hat{m}^2_{K_J^{(*)}}-\hat{s}_0}$$

s_0 is independent of the form factors but depends only on $m_{K_J^{(*)}}$

$$\delta s_0 \simeq -s_0 \cdot \frac{\delta m^2_{K_J^{(*)}}}{m_B^2}$$

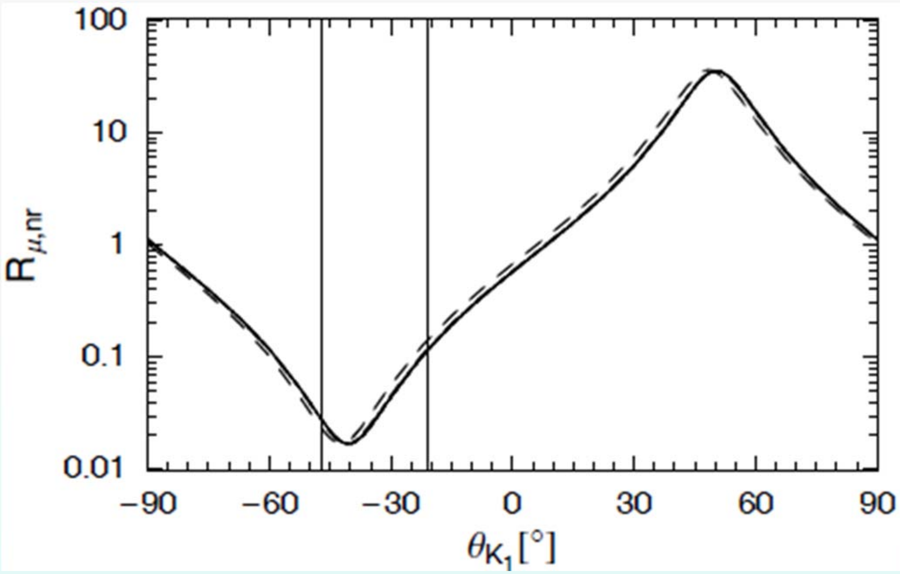
$$s_0^{K^*(980)} \approx 3.5\,\mathrm{GeV}^2 \gtrsim s_0^{K^*(1410)} \gtrsim s_0^{K_2^*(1430)} \gtrsim s_0^{K^*(1680)} \gtrsim s_0^{K_2(1770)} \gtrsim s_0^{K_3^*(1780)} \gtrsim s_0^{K_2(1820)} \gtrsim s_0^{K_2^*(1980)} \gtrsim s_0^{K_4^*(2045)} \gtrsim s_0^{K_2(2250)} \gtrsim s_0^{K_3(2320)} \gtrsim s_0^{K_5^*(2380)} \gtrsim s_0^{K_4(2500)} \gtrsim s_0^{K_5(2600?)}.$$

H. Hatanaka, KCY, Eur.Phys.J.C67:149-162,2010.

New Physics

The sign of $Re(C_7^{eff})$ can be flipped in susy models with non-minimal flavor violation via gluino-down-squark loops

In general flavor violating supersymmetric models the sign of c_9 and c_{10} can be flipped



$R_{\mu,nr}$ is highly insensitive to the NP effect
and thus is suitable for determining the value of θ_{K1}

R_i=1.0 (solid), 1.2 (dotted), 0.8 (dot-dashed) and –1.0 (dashed)

where $R_i \equiv R_7, or\ R_9\ or\ R_{10}$

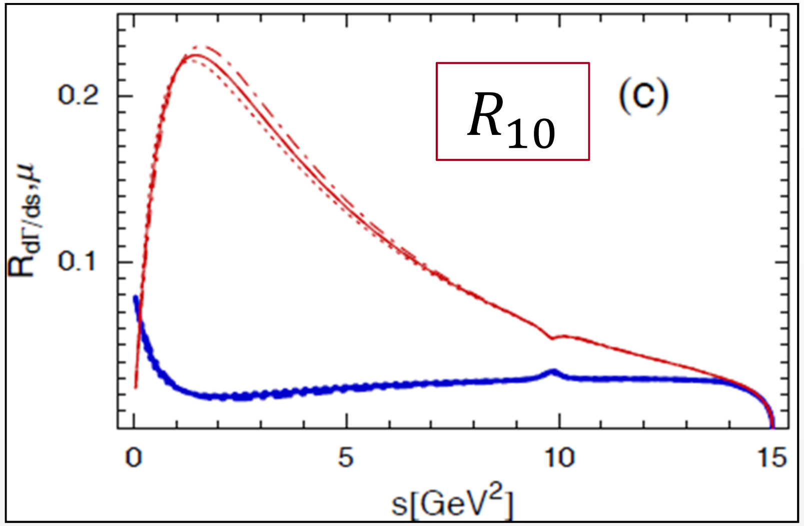
$$c_i \equiv c_i^{\rm SM} + c_i^{\rm NP} = R_i\, c_i^{\rm SM} \quad \text{for} \quad c_i = c_7^{\rm eff}, c_9, c_{10}.$$

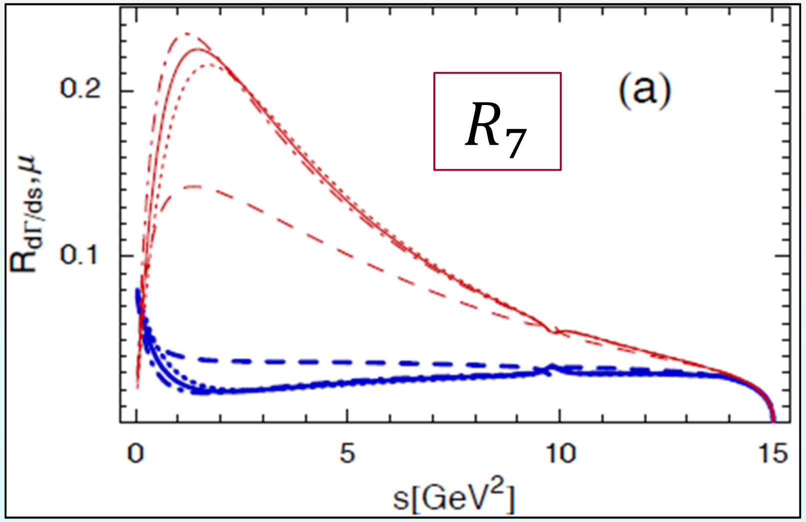
Ratio of differential widths as function of the dimuon invariant mass, s

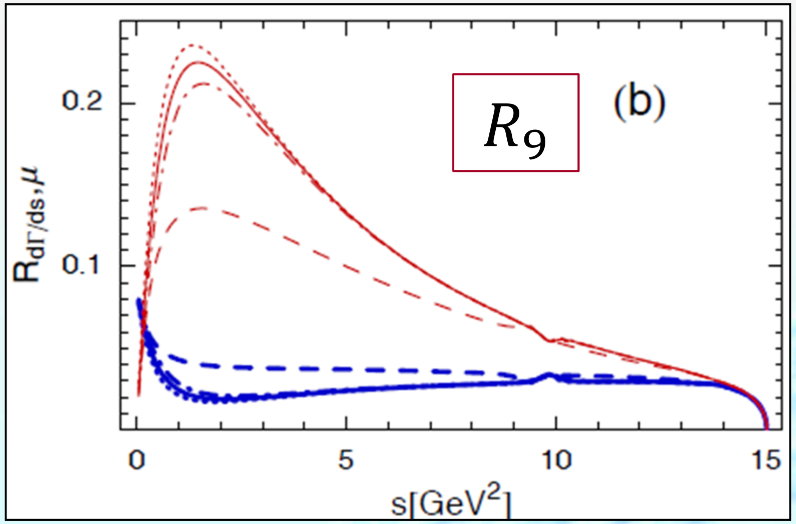
The blue and red curves correspond to $\theta_{K1} = -34^\circ$ and -57°

R_i=1.0 (solid), 1.2 (dotted), 0.8 (dot-dashed) and −1.0 (dashed) where $R_i \equiv R_7, or\ R_9\ or\ R_{10}$

$R_{\frac{d\Gamma}{ds},\mu}$ is insensitive to variation of R_{10}

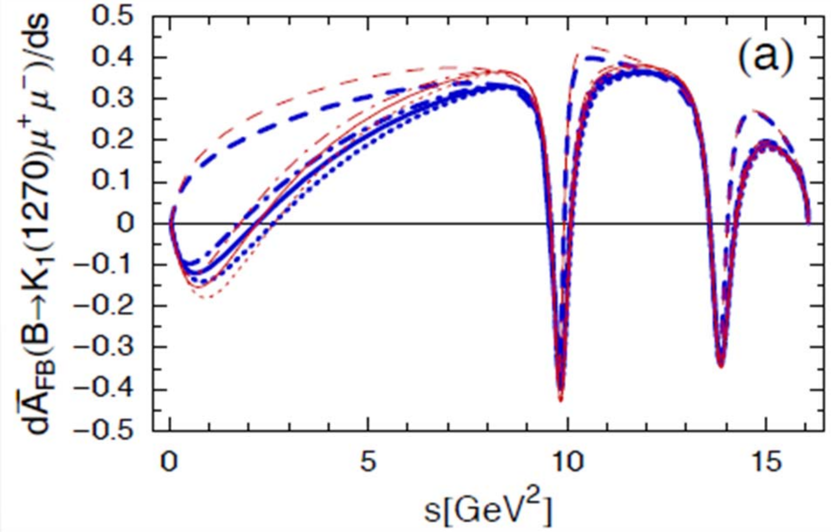




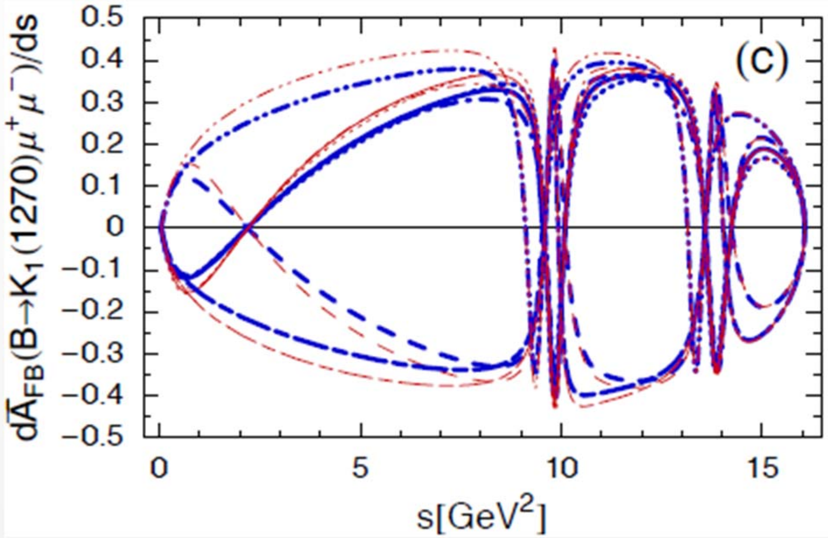


The ratio is increased (decreased) by about 100% (40%) at about s = 1.5 GeV^2 corresponding to $\theta_{K1} = -34^\circ$ (-57°) when R_7 or R_9 equals to −1 (sign flip)

Forward-Backward Asymmetry of $B \to K_1(1270)\,\mu^+\,\mu^-$



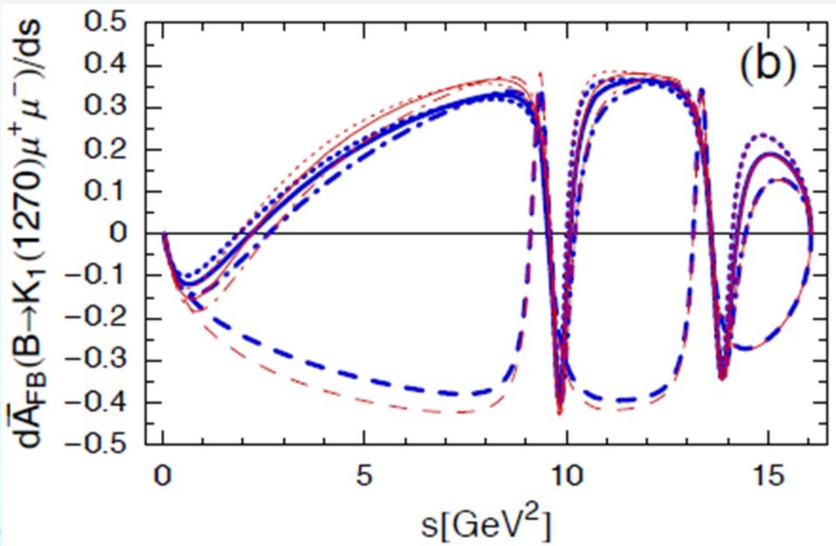
Dash: flipped sign of c_7^{eff}



Double-dotted dash: sign flips for c_9 and c_{10}

Dash: flipped sign(c_{10})

long-short dash: sign flips for c_7 and c_{10}



Dash: flipped sign(c_9)

Conclusions

- the K_{1A}-K_{1B} mixing angle, θ_{K_1} can be more precisely determined
- It is accessible for new physics signals
- The mode is accessible in LHCb and future B-factories

- 謝謝

BACK UP

For axial-vecotr mesons: Decay constants

$$\langle 3P_1(p,\varepsilon)|\bar{q}\gamma_\mu\gamma_5 q'|0\rangle = i f_{3P_1} m_{3P_1}\varepsilon^*_\mu, \qquad \langle 1P_1(p,\varepsilon)|\bar{q}\sigma_{\mu\nu}\gamma_5 q'|0\rangle = f^\perp_{1P_1}(\varepsilon^*_\mu p_\nu - \varepsilon^*_\nu p_\mu)$$

scale dependent

$^3P_1$	$a_1(1260)$	f_1	f_8	K_{1A}
$f_{^3P_1}$	238 ± 10	245 ± 13	239 ± 13	250 ± 13

$^1P_1$	$b_1(1235)$	h_1	h_8	K_{1B}
$f^\perp_{^1P_1}$	180 ± 8	180 ± 12	190 ± 10	190 ± 10

QCDSR by KCY, '07

Form factors for B → A

$$\langle A(p,\lambda)|A_\mu|\overline{B}_q(p_B)\rangle = i\frac{2}{m_{B_q}-m_A}\varepsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}_{(\lambda)}p_B^\alpha p^\beta A^{B_qA}(q^2),$$

$$\langle A(p,\lambda)|V_\mu|\overline{B}_q(p_B)\rangle = -\Bigg\{(m_{B_q}-m_A)\epsilon^{(\lambda)*}_\mu V_1^{B_qA}(q^2) - (\epsilon^{(\lambda)*}\cdot p_B)(p_B+p)_\mu\frac{V_2^{B_qA}(q^2)}{m_{B_q}-m_A} -2m_A\frac{\epsilon^{(\lambda)*}\cdot p_B}{q^2}q^\mu\left[V_3^{B_qA}(q^2)-V_0^{B_qA}(q^2)\right]\Bigg\},$$

By light cone sum rules (KCY,PRD78:034018,2008)

Light-cone distribution amplitudes (LCDAs)

chiral-even

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_\mu\gamma_5 q_2(x)|0\rangle = i f_A m_A \int_0^1 du\, e^{i(u\,py+\bar{u}px)} \left\{ p_\mu \frac{\epsilon^{*(\lambda)} z}{pz} \Phi_\parallel(u) + \varepsilon^{*(\lambda)}_{\perp\mu} g_\perp^{(a)}(u) \right\}$$

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_\mu q_2(x)|0\rangle = -i f_A m_A\, \varepsilon_{\mu\nu\rho\sigma}\, \epsilon^{*\nu}_{(\lambda)} p^\rho z^\sigma \int_0^1 du\, e^{i(u\,py+\bar{u}\,px)} \frac{g_\perp^{(v)}(u)}{4}$$

chiral-odd

$$\langle A(P,\lambda)|\bar{q}_1(y)\sigma_{\mu\nu}\gamma_5 q_2(x)|0\rangle = f_A^\perp \int_0^1 du\, e^{i(u\,py+\bar{u}\,px)} \left\{ (\varepsilon^{*(\lambda)}_{\perp\mu} p_\nu - \varepsilon^{*(\lambda)}_{\perp\nu} p_\mu) \Phi_\perp(u) + \frac{m_A^2\, \epsilon^{*(\lambda)} z}{(pz)^2} (p_\mu z_\nu - p_\nu z_\mu) h_\parallel^{(t)}(u) \right\}$$

$$\langle A(P,\lambda)|\bar{q}_1(y)\gamma_5 q_2(x)|0\rangle = f_A^\perp m_A^2 (\epsilon^{*(\lambda)} z) \int_0^1 du\, e^{i(u\,py+\bar{u}\,px)} \frac{h_\parallel^{(p)}(u)}{2},$$

twist-2: $\Phi_\parallel, \Phi_\perp$

twist-3: $g_\perp^{(v)}, g_\perp^{(a)}, h_\perp^{(t)}, h_\parallel^{(p)}$ related to twist-2 ones via Wandzura-Wilczek relations (neglecting 3-parton distributions)

$^1P_1$ meson

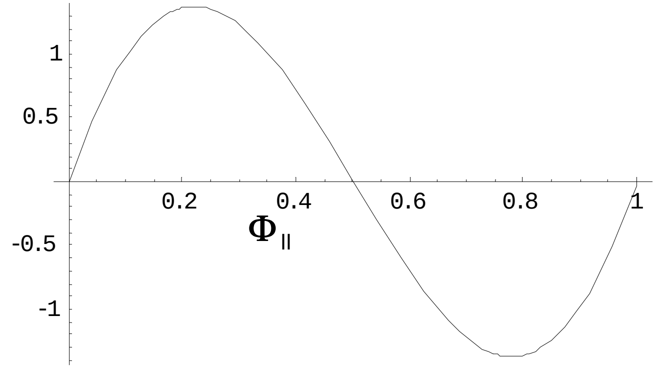
Due to G-parity, $\Phi_\perp$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$ are symmetric under u→1-u, while $\Phi_\parallel$, $g_\perp^{(v)}$, $g_\perp^{(a)}$ are antisymmetric with the replacement u→1-u in SU(3) limit

$$\int_0^1 du\Phi_\parallel(u) = \int_0^1 dug_\perp^{(a)}(u) = \int_0^1 dug_\perp^{(v)}(u) = \int_0^1 dug_3(u) = 0$$

Convention:

$$\int_0^1 du\Phi_\perp^{^1P_1}(u) = 1 \qquad f_{^1P_1} = f_{^1P_1}^\perp(\mu = 1\ \mathrm{GeV})$$

$$\begin{aligned}\Phi_\parallel(u) &= 6u\bar{u}\left[a_0^\parallel + 3a_1^\parallel\,\xi + a_2^\parallel\,\frac{3}{2}(5\xi^2-1)\right]\\ \Phi_\perp(u) &= 6u\bar{u}\left[1 + 3a_1^\perp\,\xi + a_2^\perp\,\frac{3}{2}(5\xi^2-1)\right]\end{aligned}$$



$^3P_1$ meson

Due to G-parity, $\Phi_\perp$, $h_\parallel^{(t)}$, $h_\parallel^{(p)}$ are anti-symmetric under u→1-u, while $\Phi_\parallel$, $g_\perp^{(v)}$, $g_\perp^{(a)}$ are symmetric with the replacement u→1-u in SU(3) limit

$$\int_0^1 du\Phi_\perp(u) = \int_0^1 duh_\parallel^{(t)}(u) = \int_0^1 duh_\parallel^{(p)}(u) = \int_0^1 duh_3(u) = 0$$

Convention:

$$\int_0^1 du\Phi_\parallel^{^3P_1}(u) = 1 \qquad f_{^3P_1}^\perp = f_{^3P_1}$$

Gegenbauer moments

KCY, Nucl. Phys. B776, 187 (2007)

μ	$a_2^{\parallel,a_1(1260)}$	$a_2^{\parallel,f_1^{3P_1}}$	$a_2^{\parallel,f_8^{3P_1}}$	$a_2^{\parallel,K_{1A}}$	$a_1^{\parallel,K_{1A}}$
1 GeV	-0.02 ± 0.02	-0.04 ± 0.03	-0.07 ± 0.04	-0.05 ± 0.03	$-0.30^{+0.26}_{-0.00}$
2.2 GeV	-0.01 ± 0.01	-0.03 ± 0.02	-0.05 ± 0.03	-0.04 ± 0.02	$-0.24^{+0.21}_{-0.00}$

μ	$a_1^{\perp,a_1(1260)}$	$a_1^{\perp,f_1^{3P_1}}$	$a_1^{\perp,f_8^{3P_1}}$	$a_1^{\perp,K_{1A}}$	$a_0^{\perp,K_{1A}}$	$a_2^{\perp,K_{1A}}$
1 GeV	-1.04 ± 0.34	-1.06 ± 0.36	-1.11 ± 0.31	-1.08 ± 0.48	$0.26^{+0.03}_{-0.22}$	0.02 ± 0.21
2.2 GeV	-0.81 ± 0.26	-0.82 ± 0.28	-0.86 ± 0.24	-0.84 ± 0.37	$0.24^{+0.03}_{-0.21}$	0.01 ± 0.15

μ	$a_1^{\parallel,b_1(1235)}$	$a_1^{\parallel,h_1^{1P_1}}$	$a_1^{\parallel,h_8^{1P_1}}$	$a_1^{\parallel,K_{1B}}$	$a_0^{\parallel,K_{1B}}$	$a_2^{\parallel,K_{1B}}$
1 GeV	-1.95 ± 0.35	-2.00 ± 0.35	-1.95 ± 0.35	-1.95 ± 0.45	-0.15 ± 0.15	$0.09^{+0.16}_{-0.18}$
2.2 GeV	-1.56 ± 0.28	-1.60 ± 0.28	-1.56 ± 0.28	-1.56 ± 0.36	-0.15 ± 0.15	$0.06^{+0.11}_{-0.13}$

μ	$a_2^{\perp,b_1(1235)}$	$a_2^{\perp,h_1^{1P_1}}$	$a_2^{\perp,h_8^{1P_1}}$	$a_2^{\perp,K_{1B}}$	$a_1^{\perp,K_{1B}}$
1 GeV	0.03 ± 0.19	0.18 ± 0.22	0.14 ± 0.22	-0.02 ± 0.22	$0.30^{+0.00}_{-0.31}$
2.2 GeV	0.02 ± 0.14	0.14 ± 0.17	0.11 ± 0.17	-0.02 ± 0.17	$0.25^{+0.00}_{-0.26}$