Vainshtein mechanism in the most general second-order scalar-tensor theories

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1. Motivation

Discovery of late-time cosmic acceleration

In 1998, the discovery of late-time cosmic acceleration based on Type Ia supernovae is reported. The source for this acceleration is named dark energy.

The equation of state defined below characterizes dark energy.

$$w \equiv P/\rho$$

Condition for acceleration : $w < -1/3$

Constraint of the equation of state from observations (SNe+BAO+CMB+H_0)

- Parameterization

$$w(a) = w_0 + w_a(1-a)$$

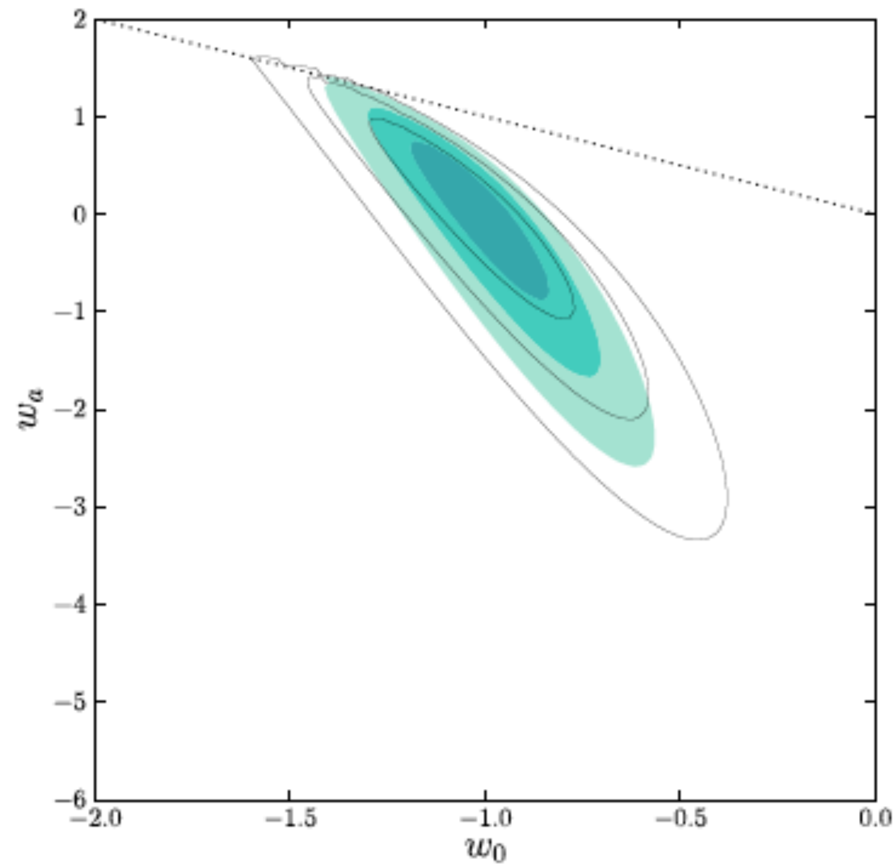
- Constraint on w_0

$$-1.216 < w_0 < -0.867$$

- Constraint on w_a

$$-0.53 < w_a < 0.56$$

1σ



N. SUZUKI *et al.* arXiv:1105.3470v1 [astro-ph.CO]

Candidates for dark energy

Cosmological constant (Λ)

$$w = -1$$

It corresponds to vacuum energy in particle physics. However, observational value is far smaller than theoretically predicted value!!

→ **Dark energy problem may imply some modification of gravity on large scales.**

Modified gravitational theories

$$w = w(t)$$

- There must be a stable accelerating solution which explains cosmic acceleration.
- **These models need to recover General Relativistic (GR) behavior at short distances to satisfy solar system constraints.**

f(R) gravity,
Brans–Dicke theory.

→ with potential term

DGP braneworld,
Galileon gravity.

→ without potential term

Recovery of GR behavior at short distances

1. Chameleon mechanism

f(R) gravity,
Brans−Dicke theory.

The potential term of a scalar field produces effective mass in the region of high density. The models recover GR if the effective mass is large enough.

→ **However a fine tuning of initial conditions is required to realize a viable cosmology.**

2. Vainshtein mechanism

DGP braneworld,
Galileon gravity.

In the DGP braneworld the non−linear effect of the field self−interaction term $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ allows the possibility to recover GR at short distances.

→ **However the DGP model suffers from a ghost, in addition to the difficulty of consistency with the combined data analysis.**

2. Field equation in a spherically symmetric background

We shall study how the Vainshtein mechanism works in the most general second-order scalar-tensor theories. Then the Lagrangian is described as

$$S = \int d^4x \sqrt{-g} \left[\sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_m \right] ,$$

$$\mathcal{L}_2 = K(\phi,\, X)\,, \mathcal{L}_3 = -G_3(\phi,\, X)\,\Box\phi\,,$$

$$\mathcal{L}_4 = G_4(\phi,\, X)\, R + G_{4,X} \times (\text{field derivatives})\ ,$$

$$\mathcal{L}_5 = G_5(\phi,\, X) G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \times (\text{field derivatives})\,.$$

from Horndeski's action

We want to check the consistency with solar system constraints. Then the line element is

$X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$

$G_{i,X} \equiv \partial G_i/\partial X$

$$ds^2 = -e^{2\Psi(r)}dt^2 + e^{2\Phi(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Equations of gravitational potential and scalar field derived from EOMs

For example, the (0,0) component of the full equation of motion is

$$\frac{2G_4}{r}\Phi' + \frac{G_4}{r^2}\left(e^{2\Phi}-1\right) - e^{2\Phi}\rho_m + \left[\left(G_4' - 2\phi' X G_{3,X} + \ldots\right)\Phi' - G_4'' - \frac{2G_4'}{r} + 2\phi'' X G_{3,X} + \ldots\right] = 0\,,$$

Dominant terms when the theory is almost GR.

$G_4' \equiv \partial G_4/\partial r$

$$\varepsilon_{G4\phi} = \frac{r\phi' G_{4,\phi}}{G_4}, \ \ldots$$

These quantities should be suppressed inside the Vainshtein radius. $\epsilon_i \ll 1$

Employing the approximation $\{|\Phi|, |\epsilon_i|\} \ll 1$ for the consistency with local gravity constraints, the combined EOMs give

$$\Box\Psi = \mu_1 \rho_m + \mu_2 \Box\phi + \mu_3$$

$$\Box\phi = \mu_4\, \rho_m + \mu_5$$

$\mu_i = \mu_i(K, G_3, G_4, G_5)$

The scalar field gives rise to the modification to Ψ .

The field equation

$$\Box\phi = \mu_4\,\rho_m + \mu_5\,,$$

$$\mu_4 \simeq -\frac{r}{4\,G_4\,\beta}\left[2\,G_{4,\phi} - 4\,XG_{4,\phi X} - 2\,XG_{3,X} - \phi'\beta - \frac{4\,X\,(G_{5,X} + XG_{5,XX})}{r^2}\right]$$

$$\beta \simeq \underbrace{(K_{,X} + 2\,XK_{,XX} - 2\,G_{3,\phi} - 2\,XG_{3,\phi X})}_{B(r)}\,r - 4\,\phi'\,(G_{3,X} + XG_{3,XX} + 3\,G_{4,\phi X} + 2\,XG_{4,\phi XX}) + \frac{4\,X\,(3\,G_{4,XX} + 2\,XG_{4,XXX} + 2\,G_{5,\phi X} + XG_{5,\phi XX})}{r}$$

In DGP model, the non-linear term $X\Box\phi$, i.e. $G_3(\phi, X) = X$, gives rise to the Vainshtein mechanism and local gravitational constraints are satisfied.

$$|B(r)\,r| \ll \left|4\,\phi'\,(G_{3,X} + XG_{3,XX} + 3\,G_{4,\phi X} + 2\,XG_{4,\phi XX}) - \frac{4\,X\,(3\,G_{4,XX} + 2\,XG_{4,XXX} + 2\,G_{5,\phi X} + XG_{5,\phi XX})}{r}\right|$$

In the regime satisfying this condition, the Vainshtein mechanism should be at work.

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$$|B(r_V)\,r_V| = \left|4\,\phi'(r_V)\,(G_{3,X} + XG_{3,XX} + 3\,G_{4,\phi X} + 2\,XG_{4,\phi XX})\,(r_V) - \frac{4\,X(r_V)\,(3\,G_{4,XX} + 2\,XG_{4,XXX} + 2\,G_{5,\phi X} + XG_{5,\phi XX})\,(r_V)}{r_V}\right|$$

In the regime satisfying this condition, the Vainshtein mechanism should be at work.

3. Vainshtein mechanism

Forms of the functions

In what follows, we need to specify the forms of the functions.
At first we shall consider the form of $G_4(\phi, X)$ as

$$G_4(\phi, X) = \frac{F(\phi)}{2} + \frac{f_4(\phi)}{M_4^{4k-2}} X^k,$$

✖

- $F(\phi) = \phi^p$ ($\phi \propto r^q$)
  $\varepsilon_{G4\phi} \simeq p\phi' r/\phi \to pq$

when p, q are $\mathcal{O}(1)$, $|\epsilon_{F\phi}| \ll 1$ breaks down.

○

- $F(\phi) = M_{\rm pl}^2 e^{-2Q\phi/M_{\rm pl}}$ ($\phi \propto r^q$)
  $\varepsilon_{G4\phi} \simeq -2Q\phi' r/M_{\rm pl} \to -2Qqr^q/M_{\rm pl}$

dilatonic coupling

Taking account of this condition, we define the forms of functions as the action covers a wide range of modified gravitational models.

$$K(\phi, X) = f_2(\phi) X, \quad G_3(\phi, X) = \frac{f_3(\phi)}{M_3^{4m-1}} X^m, \quad G_5(\phi, X) = \frac{f_5(\phi)}{M_5^{4n+1}} X^n,$$

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$|\epsilon_{F\phi}| \ll 1$ breaks down.

- Covariant Galileon
- Extended Galileon
- Brans−Dicke theories with dilatoinic coupling …

$/M_{\rm pl}$

$\phi \propto r^q$

$M_{\rm pl} \to -2Qqr^q/M_{\rm pl}$

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Vainshtein mechanism in the presence of $G_3(\phi, X)$

In our previous work, we studied how the Vainshtein mechanism works in the presence of the term $G_3(\phi, X)$. Here we briefly review that with substituting $f_4 = f_5 = 0$.

A. De Felice, R. Kase, S. Tsujikawa Phys. Rev. D 85 044059 (2012)

(1) $r \gg r_{V3}$ $\quad \phi'(r) \simeq \frac{QM_{\rm pl}}{B}\frac{r_g}{r^2}$ $\quad\rightarrow\quad$ $r_{V3} \simeq \left| \frac{4QM_{\rm pl} r_g}{B_V^2}(G_{3,X} + XG_{3,XX})(r_{V3}) \right|^{1/3}$

(2) $r_* \ll r \ll r_{V3}$ $\quad \phi'(r) \simeq \frac{QM_{\rm pl} r_g}{B_V r_{V3}^2}\left(\frac{r}{r_{V3}}\right)^{-1/2m}$ $\quad\rightarrow$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - \frac{2Q^2}{B_V}\left(\frac{r}{r_{V3}}\right)^{2-1/(2m)}\right]$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - \frac{4m}{2m-1}\frac{Q^2}{B_V}\left(\frac{r}{r_{V3}}\right)^{2-1/(2m)}\right]$$

$B \simeq f_2 \simeq \text{constant.}$

In the regime $r \ll r_*$ there exist the solution $\phi' \propto r$ and regularity condition is satisfied.

collection terms are suppressed in the regime $\mathbf{r \ll r_{V3}}$

When $G_3 = X/M_3^3$,

$$r_{V3} \approx \frac{(|Q|M_{\rm pl} r_g)^{1/3}}{M_3} \approx (|Q| r_g r_c^2)^{1/3}$$

$r_c \equiv H_0^{-1}$

when this solution is responsible for the cosmic acceleration today, $M_3^3 \approx M_{\rm pl} r_c^{-2}$.

which recovers the Vainshtein radius in the DGP model $r_{V\,DGP} \approx (r_g r_c^2)^{1/3}$, as long as Q is of the order of unity.

Vainshtein mechanism in the presence of $G_{4,X}(\phi, X)$

We show how the term $G_{4,X}(\phi, X)$ enables the Vainshtein mechanism to be at work. For simplicity we consider the case $f_3 = f_5 = 0$. Then the behavior of solutions are...

$$(1)\, r \gg r_{V4} \quad \phi'(r) \simeq \frac{Q M_{\rm pl}}{B} \frac{r_g}{r^2}$$

→

$$r_{V4} \simeq \left| \frac{2\, Q^2\, M_{\rm pl}^2\, r_g^2}{B_V^3} \left(3\, G_{4,XX} + 2\, X\, G_{4,XXX}\right)(r_{V4}) \right|^{1/6}$$

$$(2)\, r_* \ll r \ll r_{V4} \quad \phi'(r) \simeq \frac{Q M_{\rm pl}}{B_V} \frac{r_g}{r_{V4}^2} = \text{constant.}$$

→

$$\Phi \simeq \frac{r_g}{2r} \left[1 - \frac{2\, Q^2}{B_V} \left(\frac{r}{r_{V4}} \right)^2 + \frac{Q^2\, f_4}{2\,(2\,k-1)(k-1) B_V\, f_{4V}} \frac{{M_{\rm pl}}^2}{F} \left(\frac{r_g}{r_{V4}} \right) \left(\frac{r}{r_{V4}} \right) \right]$$

$$\Psi \simeq -\frac{r_g}{2r} \left[1 - \frac{2\, Q^2}{B_V} \left(\frac{r}{r_{V4}} \right)^2 + \frac{Q^2\, f_4}{(2\,k-1) B_V\, f_{4V}} \frac{{M_{\rm pl}}^2}{F} \left(\frac{r_g}{r_{V4}} \right) \left(\frac{r}{r_{V4}} \right) \ln r \right]$$

The post-Newtonian parameter $\gamma \equiv -\Phi/\Psi$ is strictly constrained by observations as $|\gamma - 1| < 2.3 \times 10^{-5}$. Then this tight constraint is translated as

$$\left| \frac{Q^2\, f_4}{2\,(2\,k-1)(k-1) B_V\, f_{4V}} \frac{{M_{\rm pl}}^2}{F} \left(\frac{r_g}{r_{V4}} \right) \left(\frac{r}{r_{V4}} \right) [1 + 2\,(k-1)\, \ln\, r] \right| < 2.3 \times 10^{-5}$$

Since the terms in l.h.s do not appear in linear level, this constraint can be satisfied in the entire region of $r_* \ll r \ll r_{V4}$.

Vainshtein mechanism in the presence of $G_5(\phi, X)$

In the presence of the term $G_5(\phi, X)$, we show that the Vainshtein mechanism can be disturbed by that term. For simplicity we consider the case $f_3 = f_4 = 0$.

$$(1)\, r \gg r_{V5} \quad \phi'(r) \simeq \frac{QM_{\rm pl}}{B}\frac{r_g}{r^2}$$

$$r_{V5} \simeq \left| \frac{2\,Q^2\,M_{\rm pl}^2\,r_g^2}{B_V^3}\left(2\,G_{5,\phi X} + XG_{5,\phi XX}\right)(r_{V5}) \right|^{1/6}$$

$$(2)\, \tilde{r}_{V5} \ll r \ll r_{V5} \quad \phi'(r) \simeq \frac{QM_{\rm pl}}{B_V}\frac{r_g}{r_{V5}^2} = \text{constant}.$$

This solution seems to trigger the Vainshtein mechanism in small scale as in the case of $G_{4,X}(\phi, X)$. However the second transition of solutions occurs in the smaller scale and the Vainshtein mechanism can be disturbed.

$$\Box\phi = \frac{\ldots - 4\,X\,(G_{5,X} + XG_{5,XX})/r^2}{\ldots + 4X\,(2\,G_{5,\phi X} + XG_{5,\phi XX})\,/r}\rho_m + \frac{\ldots - \,(2\,G_{5,\phi X} + XG_{5,\phi XX})/r}{\ldots + 4X\,(2\,G_{5,\phi X} + XG_{5,\phi XX})\,/r}$$

In the case of $G_5(\phi, X)$ the numerator can be much larger than denominator in smaller scale. Then this first term become the dominant contribution in this field equation.

In the case of $G_3(\phi, X)$ and $G_{4,X}(\phi, X)$, this second term gives the dominant contribution in small scale. Then the Vainshtein mechanism can be at work.

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$$(3)\ r_* \ll r \ll \tilde{r}_{V5} \quad \phi'(r) \simeq \frac{Q\,M_{\rm pl}}{B_V}\left(\frac{\tilde{r}_{V5}}{r_{V5}}\right)^2 \frac{r_g}{r^2}$$

$$\rho_m \simeq \rho_\odot (r/r_\odot)^{-3-b} + \bar{\rho}$$

$$\tilde{r}_{V5} \simeq \left| \left(\frac{n\,\rho_\odot\, B_V {r_{V5}}^2 r_\odot}{4\,(n+1)\,{M_{\rm pl}}^2 Q^2 r_g} \right)^{\frac{1}{2+b}} \right| r_\odot$$

Since this solution is proportional to r^{-2} as the exterior solution, the modification of gravitation can be very large in the smaller scale.

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$$(2)\, \tilde{r}_{V5} \ll r \ll r_{V5} \quad \phi'(r) \simeq \frac{QM_{\rm pl}}{B_V}\frac{r_g}{r_{V5}^2} = \text{constant.}$$

$$\Phi \simeq \frac{r_g}{2r}\left[1 - \frac{2\,Q^2}{B_V}\left(\frac{r}{r_{V5}}\right)^2 - \frac{2\,n\,Q\,f_5}{(n+1)\,(2+b)\,\eta_{5V}\,f_{5V}}\left(\frac{\tilde{r}_{V5}}{r}\right)^{2+b} + \frac{n^2\,B_V\,f_5\,\bar{\rho}\,r\,r_{V5}^2}{2(n+1)^2 f_{5V}\eta_{5V}\eta_5 M_{\rm pl}^2\,r_g}\right]$$

$$\Psi \simeq -\frac{r_g}{2r}\left[1 - \frac{2Q^2}{B_V}\left(\frac{r}{r_{V5}}\right)^2 - \frac{2\,n\,Q\,f_5}{(n+1)(2+b)(3+b)\eta_{5V}\,f_{5V}}\left(\frac{\tilde{r}_{V5}}{r}\right)^{2+b} + \frac{n^2\,B_V\,f_5\,\bar{\rho}\,r\,r_{V5}^2}{2(n+1)^2 f_{5V}\eta_{5V}\eta_5 M_{\rm pl}^2 r_g}\ln\left(\frac{\tilde{r}_{V5}}{r}\right)\right]$$

The tight constraint on the post-Newtonian parameter $|\gamma - 1| < 2.3 \times 10^{-5}$ is translated into...

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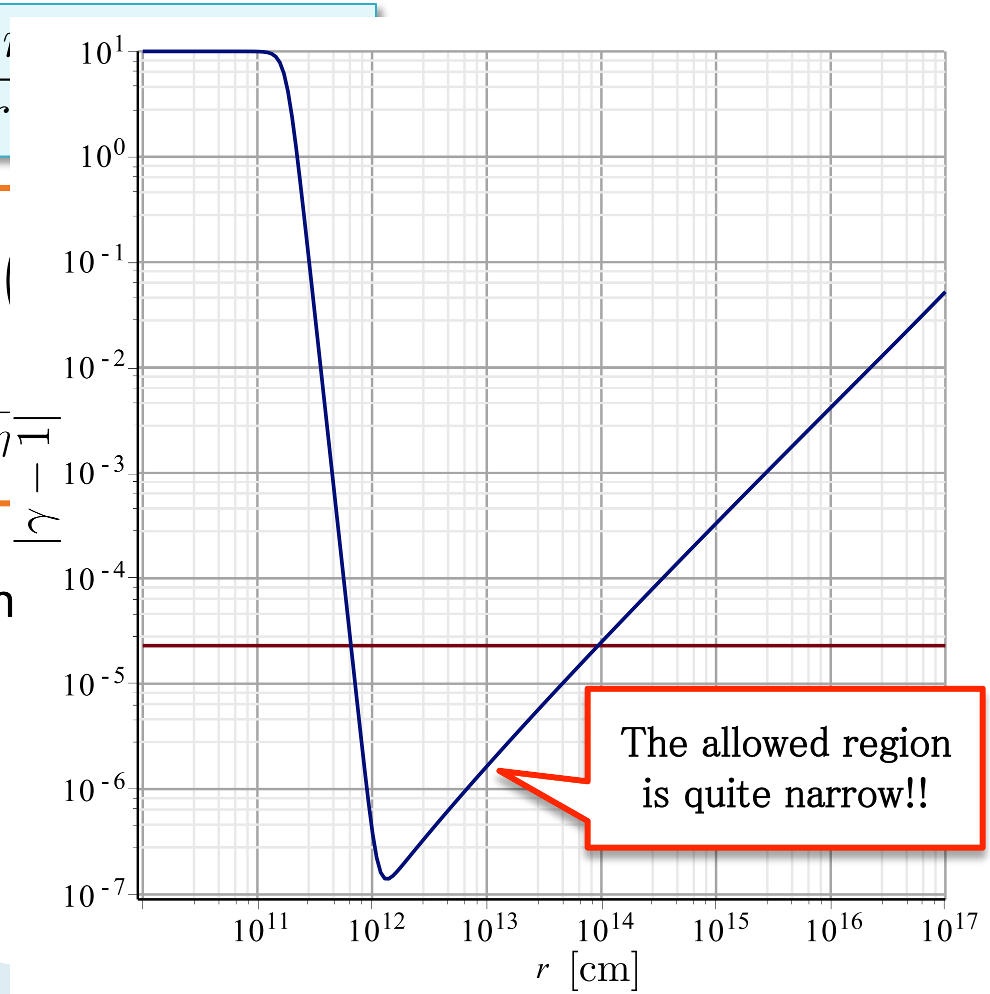

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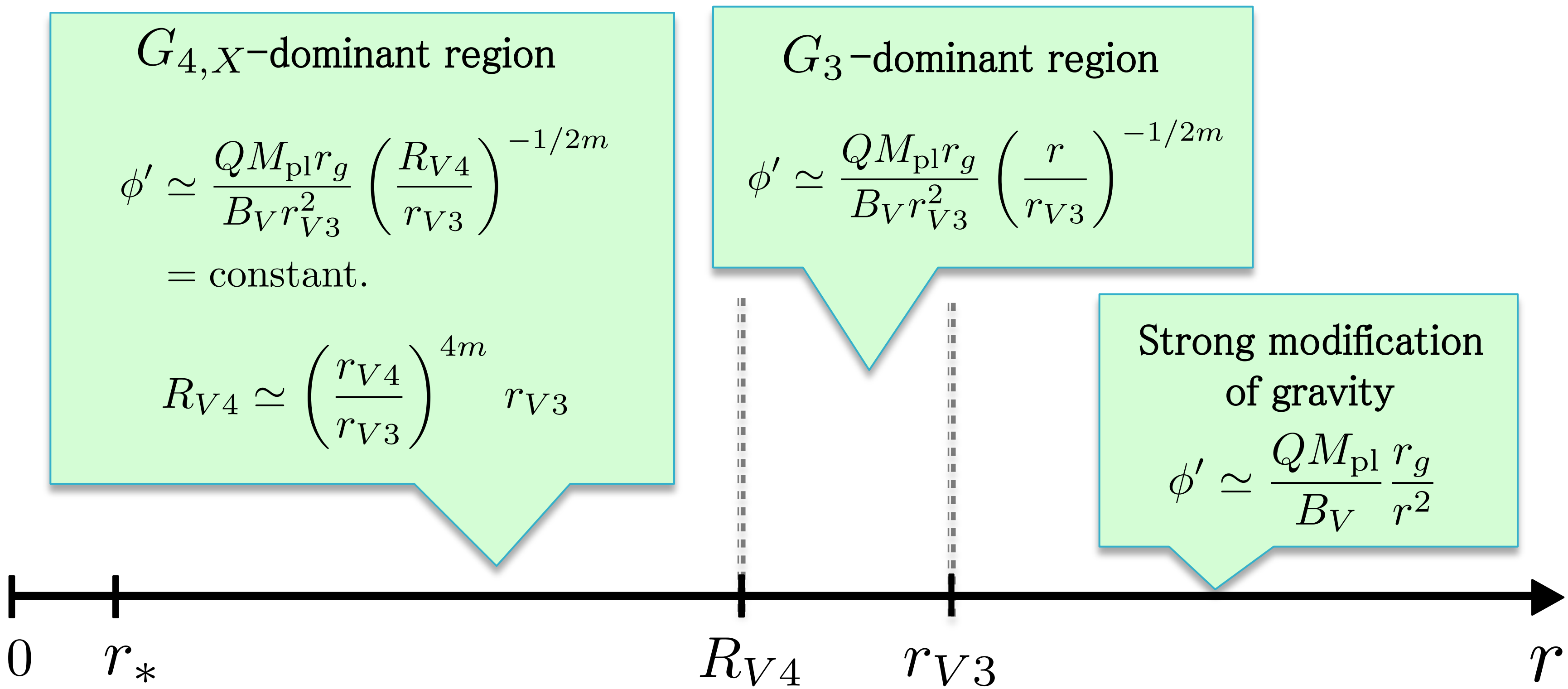
$$\rho_m \simeq \rho_\odot (r / r_\odot)^{-3-b} + \bar{\rho}$$

f_2, f_5 and Q are of the order of unity.
$b = 8,\ n = 2.$

In the presence of $G_3(\phi, X)$, $G_{4,X}(\phi, X)$ and $G_5(\phi, X)$

In the context of cosmology, the coefficients f_i tends to be same order, moreover m , k and n tends to be of the order of unity in viable models. In this case the magnitude relation among r_{V3}, r_{V4} and r_{V5} is inclined to be $r_{V5} \ll r_{V4} \lesssim r_{V3}$.



The schematic view of the Vainshtein mechanism.

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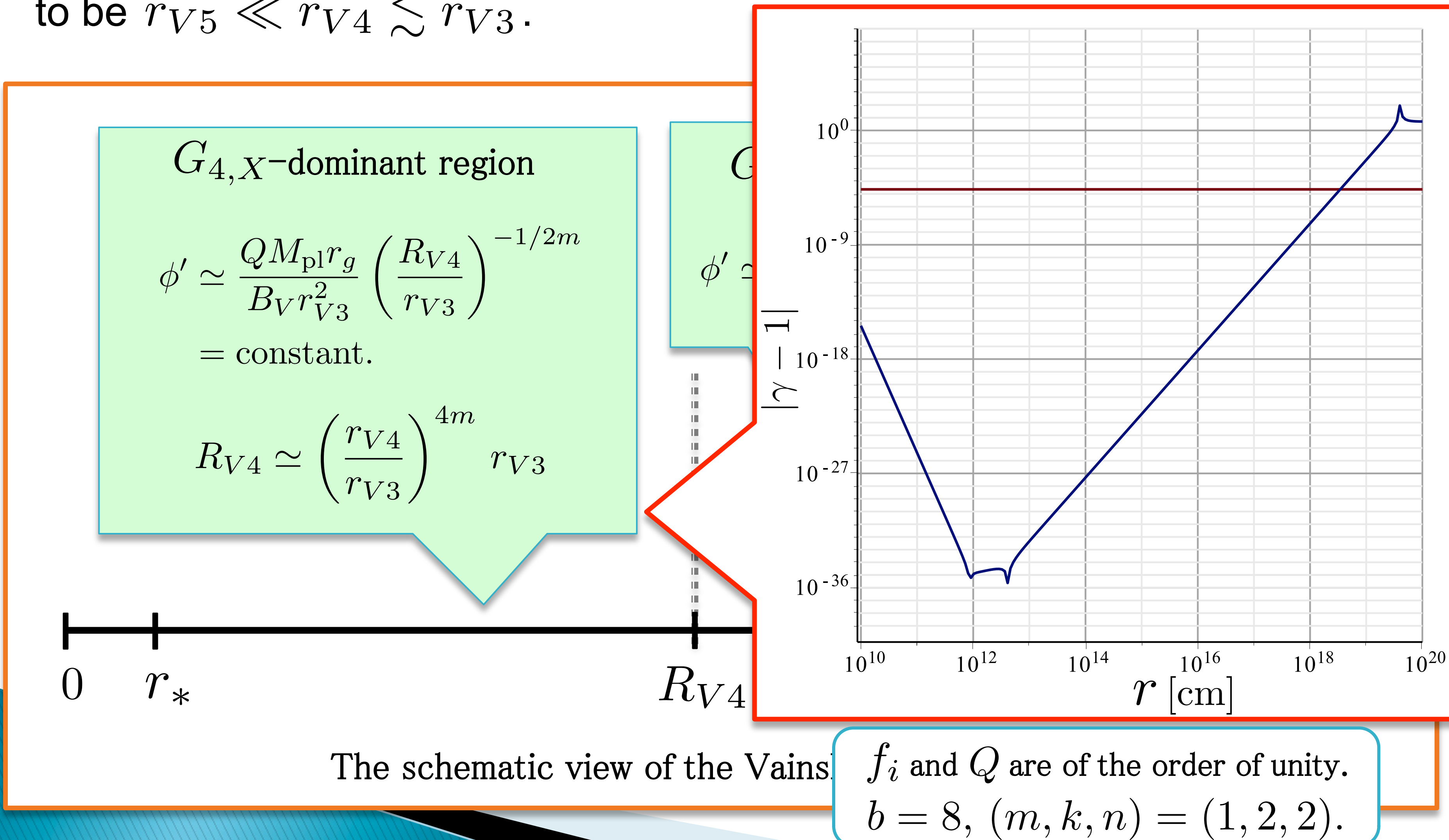

4. Application to concrete models

The Vainshtein radius in the models responsible for dark energy around the sun is...

• extended Galileon with dilatonic coupling

$$(m,k,n) = (1+(2q-1)/2, 1+2q, 1+(6q-1)/2)$$

$$f_i(\phi) = c_i e^{-2Q\phi/M_{\rm pl}}, \quad c_i : \text{constants of the order of unity}$$

When this model is responsible for the cosmic acceleration today,

$$M_3^{1-4m} \approx M_{\rm pl}^{1-2m} H_0^{-2m}, \quad M_4^{2-4k} \approx M{\rm pl}^{2-2k} H_0^{-2k}$$

$$r_{V3} \simeq (r_g^{2m-1} r_c^{2m})^{1/(4m-1)}$$

$$r_{V4} \simeq (r_g^{k-1} r_c^{k})^{1/(2k-1)}$$

$r_c \equiv H_0^{-1}$

$q = 1/2$

This model corresponds to covariant Galileon.

$$r_{V3} \simeq r_{V4} \simeq 10^{20}\ \text{cm}$$

$q = 5/2$

$$r_{V3} \simeq r_{V4} \simeq 10^{17}\ \text{cm}$$

aphelion distance of Pluto : $r \simeq 10^{14}\text{cm}$

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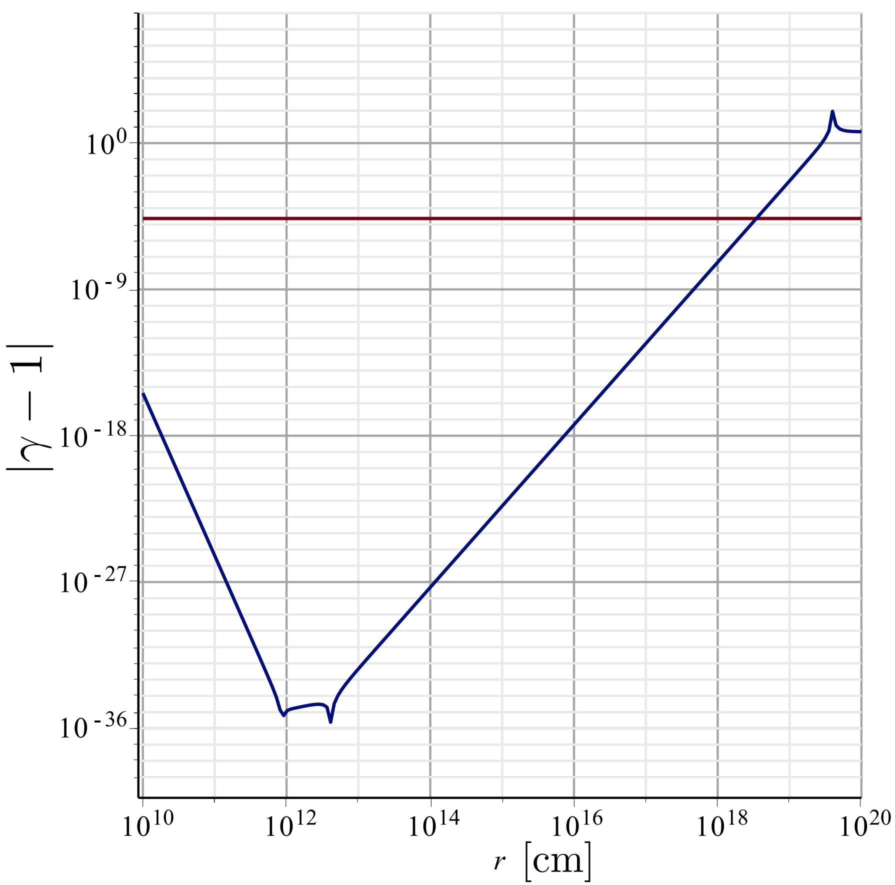

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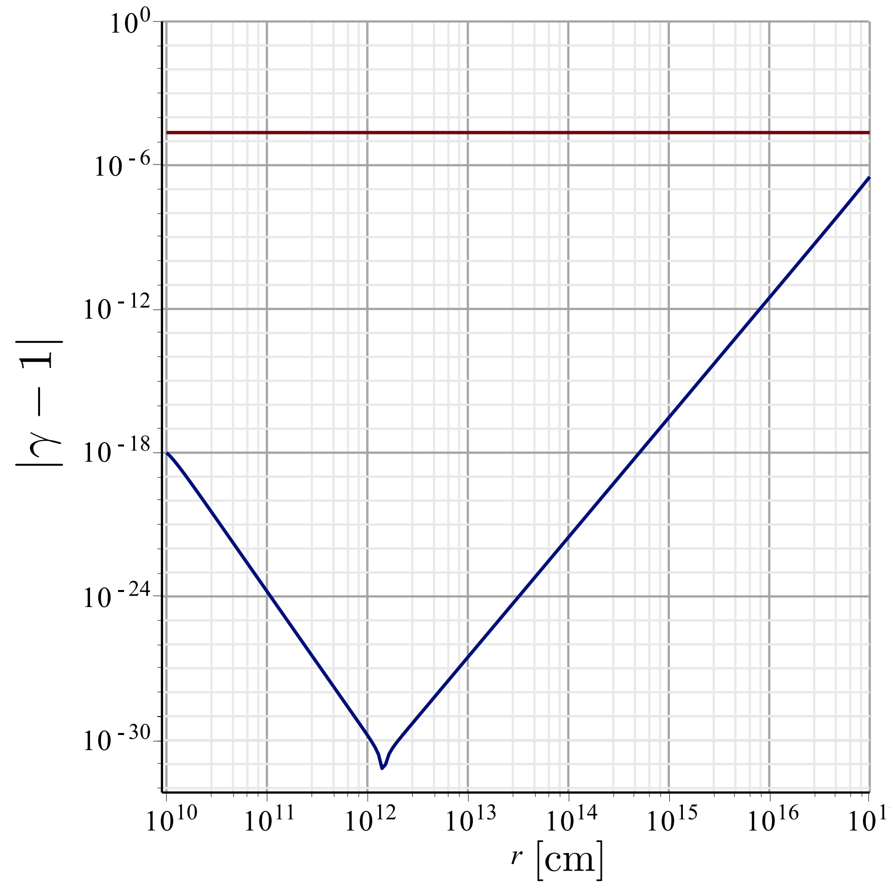

aphelion distance of Pluto : $r \simeq 10^{14}\mathrm{cm}$

5. Conclusions

- In the most general second-order scalar-tensor theories we have studied how the Vainshtein mechanism works in a spherically symmetric background with a matter source.
- We derived the full equation of motion, and general formula for the Vainshtein radius r_V in the presence of non-minimal coupling $F(\phi) = M_{\rm pl}^2 e^{-2Q\phi/M_{\rm pl}}$.
- In the presence of $G_3(\phi, X)$, $G_{4,X}(\phi, X)$ and $G_5(\phi, X)$, we derived analytic exterior and interior solutions. We also estimated the post-Newtonian parameter.
- In the presence of the term $G_5(\phi, X)$ the Vainshtein mechanism can be disturbed. However if there coexist the term $G_{4,X}(\phi, X)$, this puzzling effect can be suppressed and the Vainshtein mechanism works sufficiently.
- We applied our general results to concrete models that are responsible for the late-time cosmic acceleration.