Interacting holographic dark energy

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There is, possibly, some direct, non-gravitational interaction between dark energy and dark matter

Question: Why consider such an imaginary interaction in the case that we even know little about dark energy and dark matter

- From the theoretical perspective, the interaction can be used to alleviate the "cosmic coincidence" problem, and to avoid the "cosmic doomsday" caused by phantom, etc.
- On the other hand, more importantly, since this possibility exists, we have enough interests to have a look at its theoretical and observational consequences.

Interacting dark energy models

$$\dot{\rho}_{\rm dm} + 3H\rho_{\rm dm} = Q$$

$$\dot{\rho}_{\rm de} + 3H(1+w)\rho_{\rm de} = -Q$$

- $Q > 0$: **from DE to DM**
- $Q < 0$: **from DM to DE**
- Q **---forms:**

$$Q \propto H\rho \quad \text{or} \quad Q \propto \rho$$

$$\rho = \rho_{\rm de},\quad \rho_{\rm dm},\quad \rho_{\rm de} + \rho_{\rm dm}$$

***Q* changes sign?**

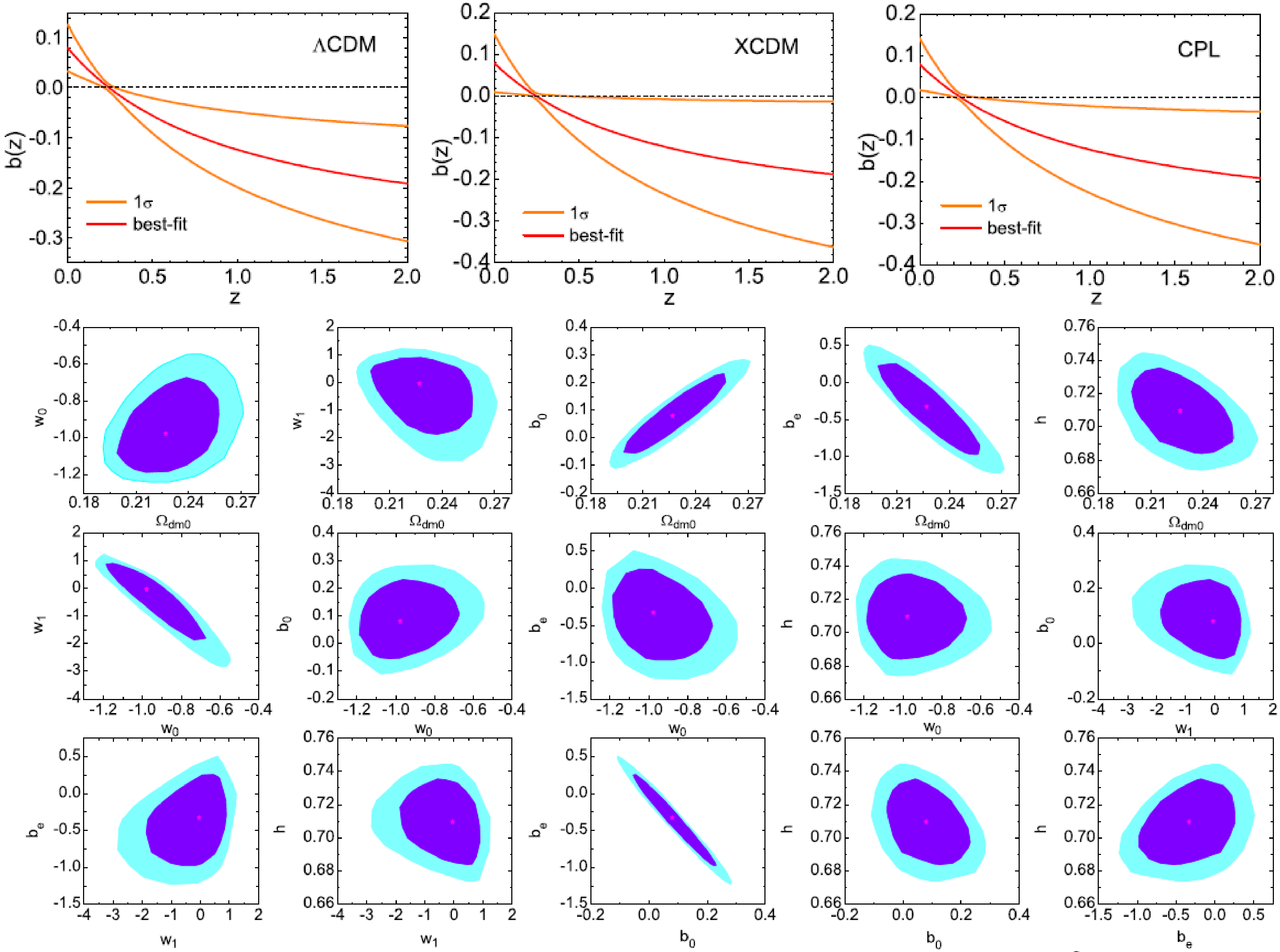
R.G. Cai, Q. Su, Phys. Rev. D **81**, 103514 (2010).

Y.H. Li, X. Zhang, Eur. Phys. J. C **71**, 1700 (2011).

Are there observable effects?

C. G. Boehmer, G. Caldera-Cabral, R. Lazkoz, and R. Maartens, Phys. Rev. D **78**, 023505 (2008).

B. M. Schaefer, G. A. Caldera-Cabral, and R. Maartens, arXiv:0803.2154.
J. Valiviita, E. Majerotto, and R. Maartens, J. Cosmol. Astropart. Phys. 07 (2008) 020.
G. Caldera-Cabral, R. Maartens, and L. A. Urena-Lopez, Phys. Rev. D **79**, 063518 (2009).
G. Caldera-Cabral, R. Maartens, and B. M. Schaefer, J. Cosmol. Astropart. Phys. 07 (2009) 027.
K. Koyama, R. Maartens, and Y.-S. Song, J. Cosmol. Astropart. Phys. 10 (2009) 017.
E. Majerotto, J. Valiviita, and R. Maartens, Mon. Not. R. Astron. Soc. **402**, 2344 (2010).
J. Valiviita, R. Maartens, and E. Majerotto, Mon. Not. R. Astron. Soc. **402**, 2355 (2010).
C. G. Boehmer, G. Caldera-Cabral, N. Chan, R. Lazkoz, and R. Maartens, Phys. Rev. D **81**, 083003 (2010).



Y.H. Li, X. Zhang, Eur. Phys. J. C **71**, 1700 (2011).

Holographic dark energy

- Cosmological constant (vacuum energy): incorrect calculation in QFT due to absence of gravity
- Holographic model: partly consider the gravity effect (by considering holographic principle to reduce DOFs) to construct an effective QFT.
- UV cutoff runs with IR cutoff: dynamical DE
- IR cutoff is taken to be event horizon

$$\rho_\Lambda = 3c^2 M_{\rm Pl}^2 L^{-2}$$

$$k_{\rm max} \propto L^{-1/2}$$

Holographic dark energy

- Causality trouble: the existence of the event horizon should be a consequence but should not be a premise of a dark energy model

  R. G. Cai, Phys. Lett. B **657**, 228 (2007)

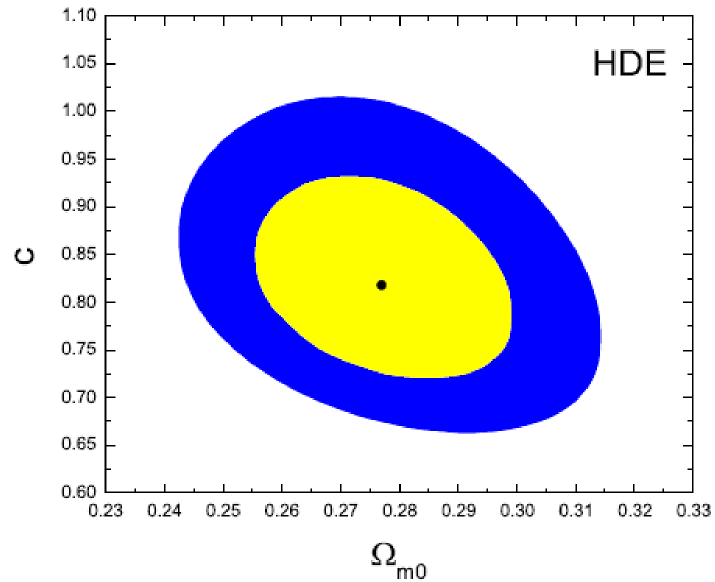
- Recent study on inflation and quantum mechanics: if inflation indeed happened in the early times, the quantum no-cloning theorem requires that the existence of event horizon is a must

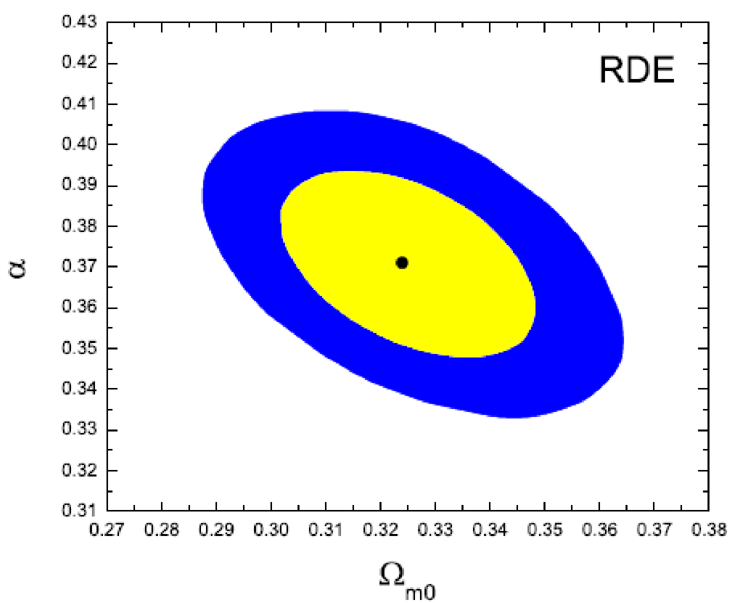
  Q. G. Huang and F. L. Lin, arXiv:1201.2443 [hep-th].

- So, as long as holographic model admits inflation, it seems that event horizon may be tolerable
- On the other hand, holographic model is the best in fitting data

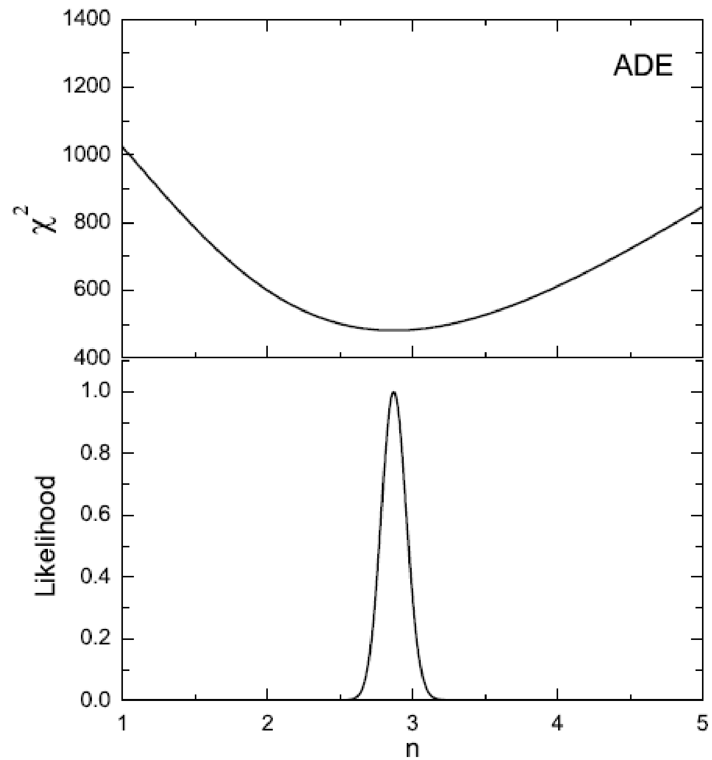
  M. Li, X. D. Li, S. Wang and X. Zhang, JCAP **0906** (2009) 036

A comparison of several holographic models





M. Li, X.D. Li, S. Wang & XZ, JCAP 06 (2009) 036



$$\mathrm{BE} = \int \mathcal{L}(\mathbf{d}|\theta, M)\mathbf{p}(\theta|M)d\theta$$

Model	HDE	ADE	RDE
$\Delta \ln \mathrm{BE}$	-0.86	-5.17	-8.14

M. Li, X.D. Li & XZ, Science China G (2010)

Model	χ^2_{min}	ΔAIC	ΔBIC
Λ	468.461	0	0
w	468.327	1.866	5.862
αDE	468.452	1.991	5.987
GCG	468.461	2	5.996
CPL	467.663	3.202	11.195
HDE	470.513	4.052	8.048
RDE	493.772	27.311	31.308
ADE	503.039	34.578	34.578
DGP	530.443	61.982	61.982

H. Wei, JCAP 08 (2010) 020

Model	ΛCDM	XCDM	CPL	DGP	NADE	HDE	RDE
χ^2_{min}	566.173	566.168	566.007	611.794	594.449	566.215	589.026
k	1	2	3	1	1	2	2
χ^2_{min}/dof	0.918	0.919	0.920	0.992	0.963	0.919	0.956
ΔBIC	0	6.421	12.687	45.621	28.276	6.468	29.280
ΔAIC	0	1.995	3.834	45.621	28.276	2.042	24.853
Rank	1	$2\sim3$	4	7	$5\sim6$	$2\sim3$	$5\sim6$

Why interaction?

- **Alleviate coincidence problem**

  B. Wang, G.Y. Gong & E. Abdalla, Phys. Lett. B 624 (2005) 141

- **Avoid big rip (doomsday)**

  Before the big rip, UV length scale (even IR scale) would become smaller than Planck scale, breaking down the effective QFT

  - Extra dimension

    X. Zhang, Phys. Lett. B 683 (2010) 81

  - Interaction between DE and DM

Spatial curvature

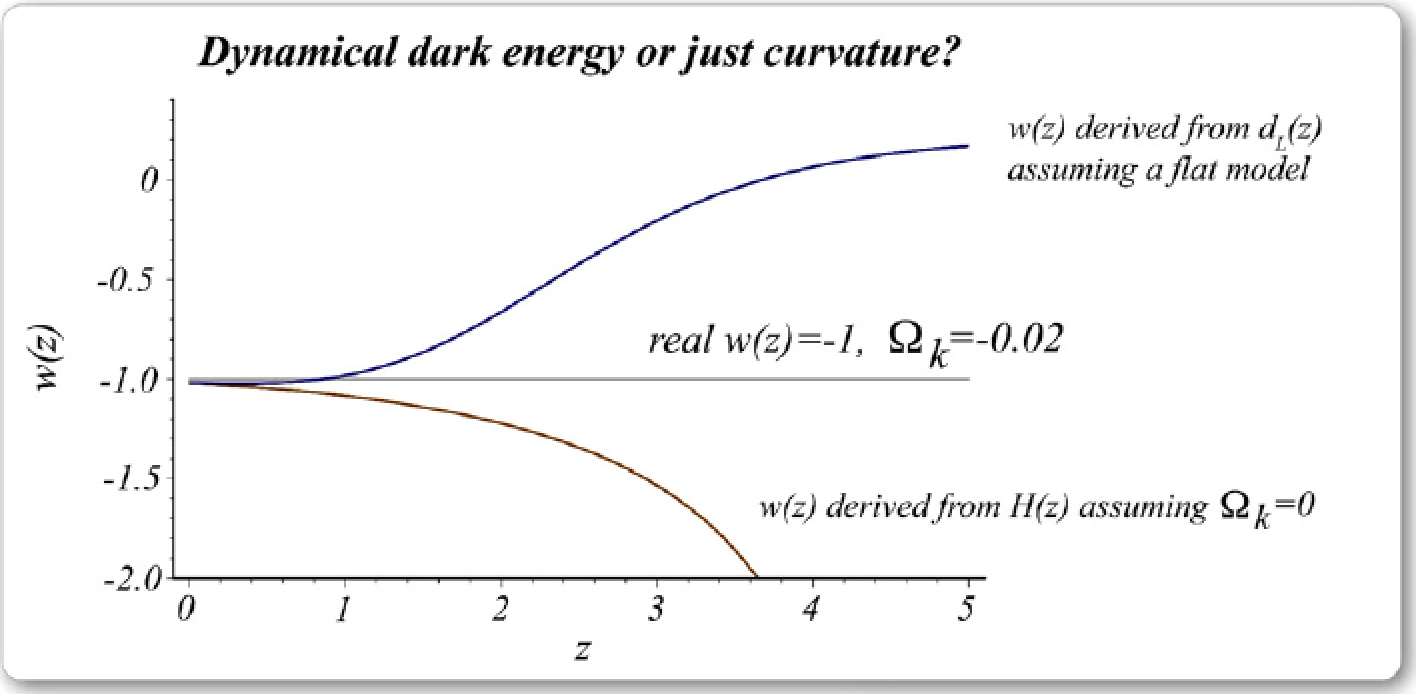
Clarkson, Cortes & Bassett, JCAP 08 (2007) 011

$$w(z) = -\frac{1}{3}\frac{\Omega_k H_0^2(1+z)^2 + 2(1+z)HH' - 3H^2}{H_0^2(1+z)^2[\Omega_\mathrm{m}(1+z)+\Omega_k] - H^2}$$

$$\begin{aligned} w(z) = \tfrac{2}{3}[(1+z)\{[\Omega_k D_L^2 + (1+z)^2]D_L'' - \tfrac{1}{2}(\Omega_k D_L'^2 + 1)[(1+z)D_L' - D_L]\}] \\ \times \{[(1+z)D_L' - D_L]\{(1+z)[\Omega_m(1+z)+\Omega_k]D_L'^2 - 2[\Omega_m(1+z)+\Omega_k] \\ \times D_L D_L' + \Omega_m D_L^2 - (1+z)\}\}^{-1}. \end{aligned}$$

$$d_L[\mathrm{flat}, w(z)] = d_L[\mathrm{curved}, w(z) = -1]$$

ArXiv ePrint: astro-ph/0702670



Interacting holographic DE model

Miao Li, Xiao-Dong Li, Shuang Wang, Yi Wang, & XZ, JCAP2009

$$\dot{\rho}_m + 3H\rho_m = Q,$$
$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q$$

$$Q \propto H\rho$$

$$Q_1 = -3bH\rho_\Lambda,$$
$$Q_2 = -3bH(\rho_\Lambda + \rho_m),$$
$$Q_3 = -3bH\rho_m.$$

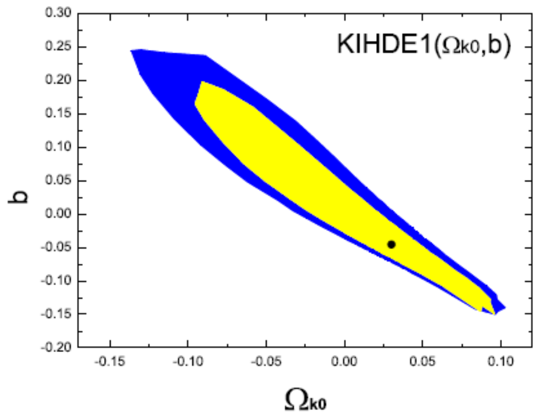
$$\frac{1}{H/H_0}\frac{d}{dz}\left(\frac{H}{H_0}\right) = -\frac{\Omega_\Lambda}{1+z}\left(\frac{3\Omega_\Lambda - \Omega_k - 3 - 3b\Omega_i}{2\Omega_\Lambda} - 1 + \sqrt{\frac{\Omega_\Lambda}{c^2} - \frac{\Omega_{k0}(1+z)^2}{(H/H_0)^2}}\right),$$

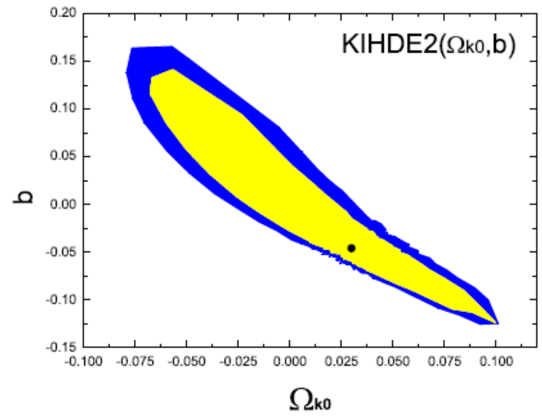
$$\frac{d\Omega_\Lambda}{dz} = -\frac{2\Omega_\Lambda(1-\Omega_\Lambda)}{1+z}\left(\sqrt{\frac{\Omega_\Lambda}{c^2} - \frac{\Omega_{k0}(1+z)^2}{(H/H_0)^2}} - 1 - \frac{3\Omega_\Lambda - \frac{(1+z)^2\Omega_{k0}}{(H/H_0)^2} - 3 - 3b\Omega_i}{2(1-\Omega_\Lambda)}\right)$$

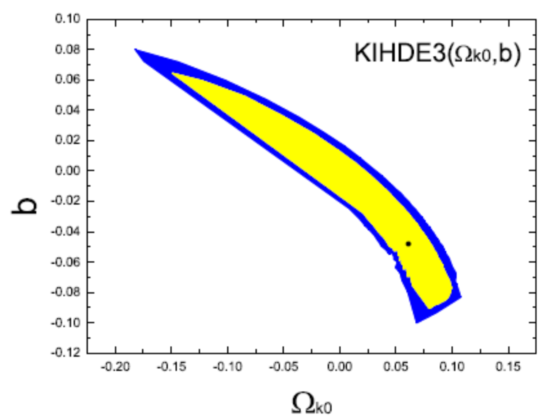
$\Omega_i = \Omega_\Lambda$, $\Omega_\Lambda + \Omega_k$, and $1 + \Omega_k - \Omega_\Lambda$, for $i = 1$, 2, and 3

Probing interaction and spatial curvature

Miao Li, Xiao-Dong Li, Shuang Wang, Yi Wang, & XZ, JCAP2009







$$Q \propto H\rho$$

$$Q_1 = -3bH\rho_\Lambda,$$
$$Q_2 = -3bH(\rho_\Lambda + \rho_m),$$
$$Q_3 = -3bH\rho_m.$$

When curvature and interaction are involved, parameter spaces are amplified --- models 1 & 2 can avoid big rip

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Revisit of the interaction between holographic dark energy and dark matter

ArXiv ePrint: 1204.6135

Zhenhui Zhang,[a,b] Song Li,[b,c,d] Xiao-Dong Li,[a,b,c,e] Xin Zhang[f,g] and Miao Li[b,c,d]

$$Q \propto \rho_{dm}^{\alpha}\rho_{de}^{\beta}$$

$$\alpha = 1 \text{ and } \beta = 0 \text{ — } Q \propto \rho_{dm}$$

$$\alpha = 0 \text{ and } \beta = 1 \text{ — } Q \propto \rho_{de}$$

$$\boxed{\begin{aligned}\frac{1}{E(z)}\frac{dE(z)}{dz} &= -\frac{\Omega_{de}}{1+z}\left(\frac{\Omega_k-\Omega_r-3+\Omega_I}{2\Omega_{de}}+\frac{1}{2}+\sqrt{\frac{\Omega_{de}}{c^2}+\Omega_k}\right)\\ \frac{d\Omega_{de}}{dz} &= -\frac{2\Omega_{de}(1-\Omega_{de})}{1+z}\left(\sqrt{\frac{\Omega_{de}}{c^2}+\Omega_k}+\frac{1}{2}-\frac{\Omega_k-\Omega_r+\Omega_I}{2(1-\Omega_{de})}\right)\end{aligned}}$$

$$\Omega_I \equiv \frac{Q}{H(z)\rho_c}$$

$$\Omega_k(z) = \frac{\Omega_{k0}(1+z)^2}{E(z)^2}, \quad \Omega_r(z) = \frac{\Omega_{r0}(1+z)^4}{E(z)^2}, \quad \Omega_b(z) = \frac{\Omega_{b0}(1+z)^3}{E(z)^2}$$

$$\Omega_{dm}(z) = 1-\Omega_k(z)-\Omega_{de}(z)-\Omega_r(z)-\Omega_b(z)$$

$$\Omega_{b0} = 0.02253h^{-2},$$

$$\Omega_{r0} = \Omega_{\gamma 0}(1+0.2271N_{\rm eff}), \quad \Omega_{\gamma 0} = 2.469\times 10^{-5}h^{-2}, \quad N_{\rm eff} = 3.04$$

$$Q = \Gamma H_0 \frac{\rho_{dm}^{\alpha}\,\rho_{de}^{\beta}}{\rho_{c0}^{\alpha+\beta-1}} \longrightarrow \begin{aligned} Q &= \Gamma H_0 \rho_{c0} E(z)^{2(\alpha+\beta)}\Omega_{dm}(z)^{\alpha}\Omega_{de}(z)^{\beta}\\ \Omega_I &= \Gamma E(z)^{2(\alpha+\beta)-3}\Omega_{dm}(z)^{\alpha}\Omega_{de}(z)^{\beta}\end{aligned}$$

$\alpha = 1$ and $\beta = 0$ — $Q \propto \rho_{dm}$ $\qquad Q = \Gamma H_0 \rho_{dm}, \quad \Omega_I = \Gamma \Omega_{dm}(z)/E(z)$

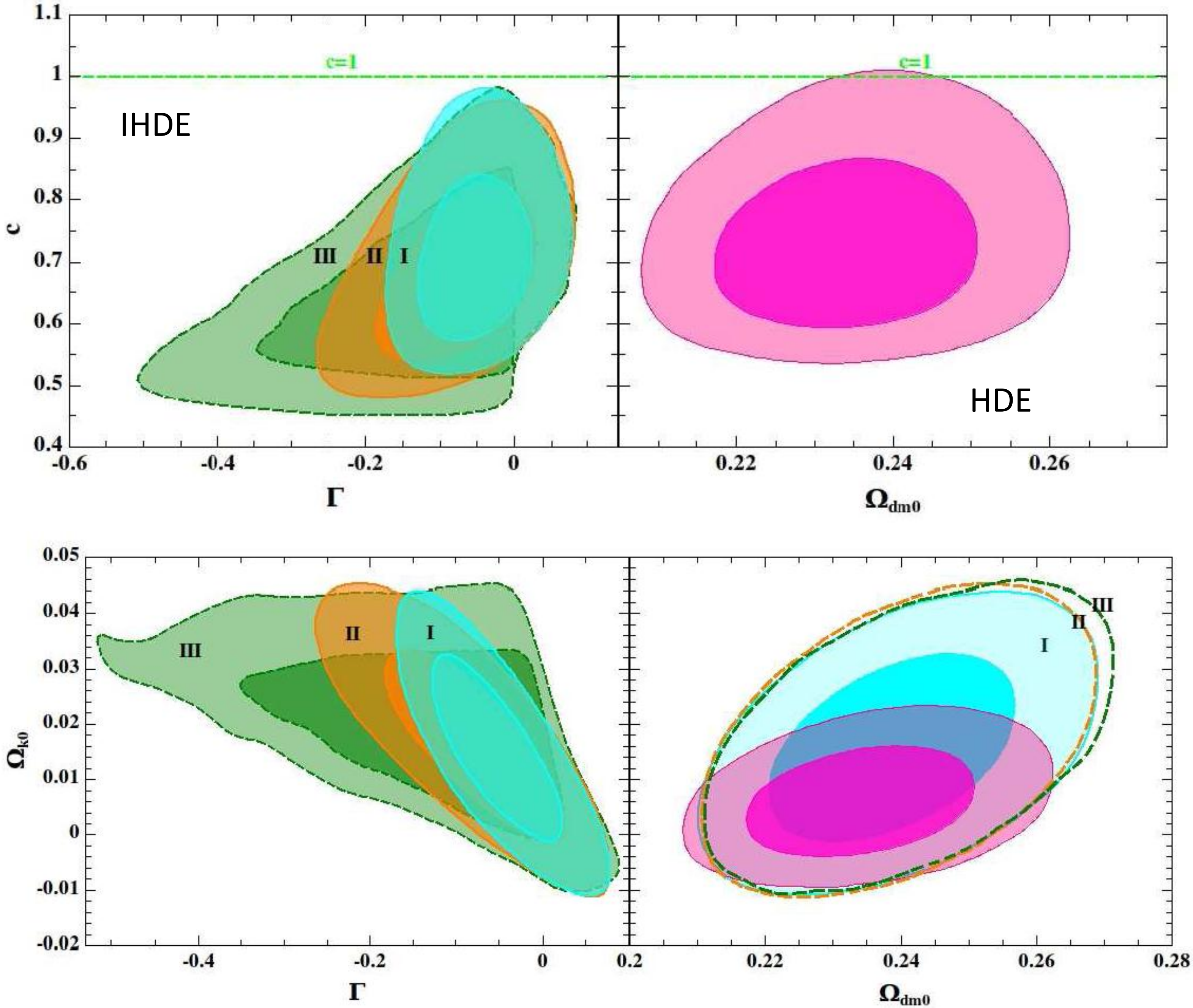
$\alpha = 0$ and $\beta = 1$ — $Q \propto \rho_{de}$ $\qquad Q = \Gamma H_0 \rho_{de}, \quad \Omega_I = \Gamma \Omega_{de}(z)/E(z)$

Data

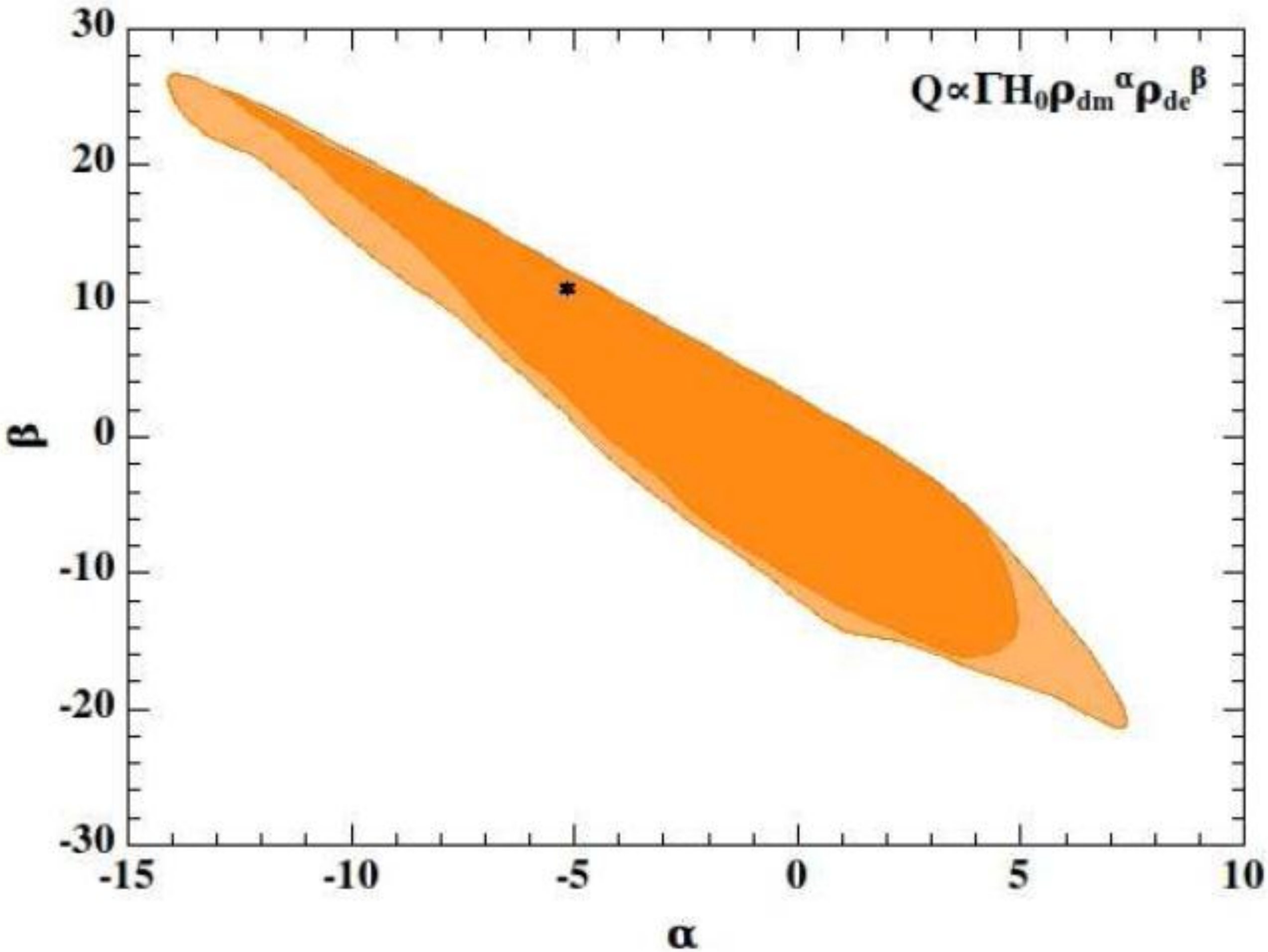
- Union2.1 SNIa data
- the CMB anisotropy data from the 7-yr WMAP observations
- The BAO measurements from the SDSS, 6dFGS and WiggleZ Dark Energy Survey
- the Hubble constant measurement from the WFC3 on the HST

Results

Model	Ω_{dm0}	c	c (95.4% range)	Γ	Ω_{k0}	$\chi^2_{\min}$
ΛCDM	$0.235^{+0.012}_{-0.011}$	–	–	0 (fixed)	$0.001^{+0.005}_{-0.005}$	550.354
HDE ($Q=0$)	$0.233^{+0.012}_{-0.011}$	$0.71^{+0.10}_{-0.08}$	$0.56 \leq c \leq 0.94$	0 (fixed)	$0.006^{+0.007}_{-0.007}$	549.461
$Q = \Gamma H_0 \rho_{dm}$	$0.238^{+0.013}_{-0.012}$	$0.69^{+0.10}_{-0.08}$	$0.54 \leq c \leq 0.91$	$-0.056^{+0.051}_{-0.051}$	$0.015^{+0.011}_{-0.011}$	548.352
$Q = \Gamma H_0 \rho_{de}$	$0.237^{+0.013}_{-0.011}$	$0.66^{+0.11}_{-0.09}$	$0.50 \leq c \leq 0.90$	$-0.073^{+0.070}_{-0.072}$	$0.016^{+0.012}_{-0.012}$	548.390
$Q = \Gamma H_0 \rho_{dm}^{\alpha} \rho_{de}^{\beta} / \rho_{c0}^{\alpha+\beta-1}$	$0.239^{+0.013}_{-0.013}$	$0.63^{+0.18}_{-0.08}$	$0.47 \leq c \leq 0.92$	$-0.014^{+0.014}_{-0.237}$	$0.017^{+0.011}_{-0.014}$	548.298



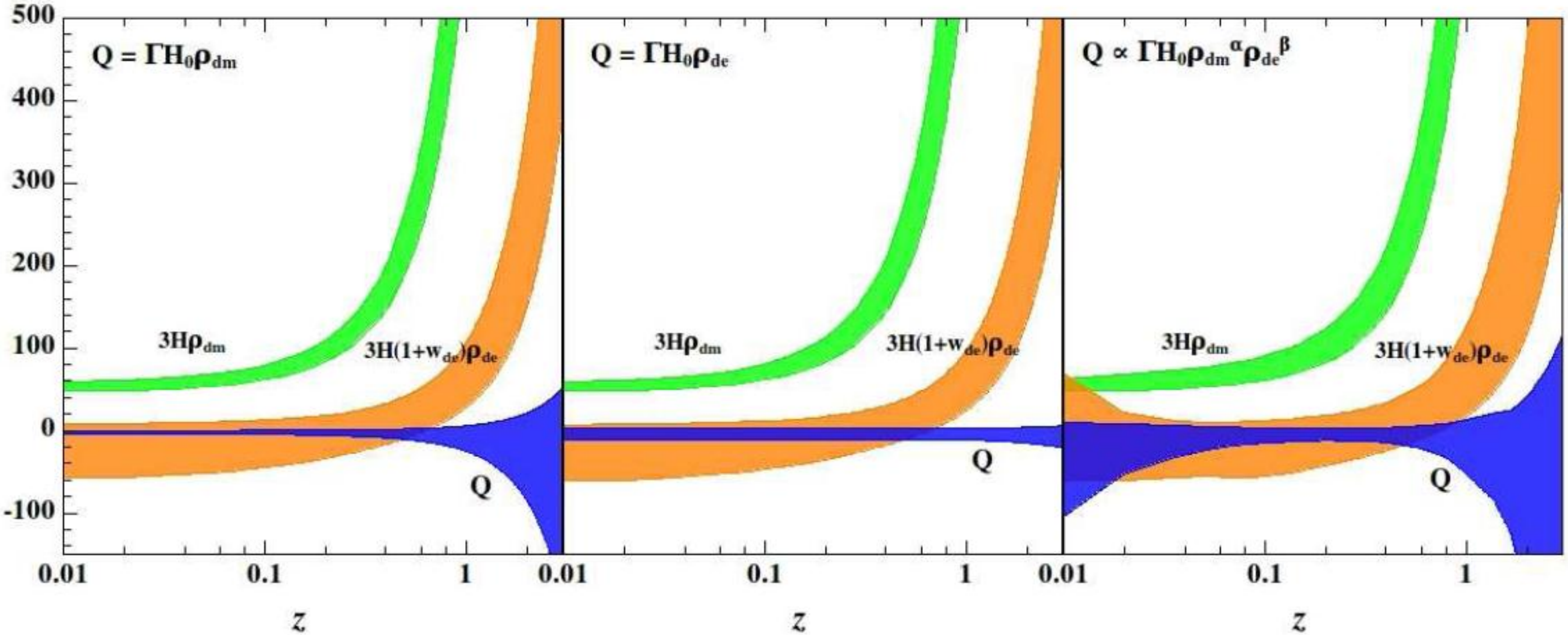
IHDE
HDE
c=1
c=1
III
II
I
c
Γ
Ω_{dm0}
III
II
I
III
II
I
Ω_{k0}
Γ
Ω_{dm0}



$$\alpha = -5.17^{+13.37}_{-9.59}, \quad \beta = 10.98^{+18.75}_{-33.42}$$

Constraints on the interaction term

$$
\begin{aligned}
-0.154 \le \Gamma \le 0.053 \quad & \text{for IHDE1,} \\
-0.229 \le \Gamma \le 0.059 \quad & \text{for IHDE2,} \\
-0.509 \le \Gamma \le 0.072 \quad & \text{for IHDE3.}
\end{aligned}
\qquad 95.4\%\ \text{CL}
$$



$$\Delta \equiv \frac{\rho_{dm} a^3\big|_{z=0} - \rho_{dm} a^3\big|_{ini}}{\rho_{dm} a^3\big|_{ini}}$$

$$\begin{aligned}
-15.22\% \leq \Delta \leq 8.83\% &\quad \text{for IHDE1},\\
-13.3\% \leq \Delta \leq 5.57\% &\quad \text{for IHDE2},\\
-14.49\% \leq \Delta \leq 6.57\% &\quad \text{for IHDE3}.
\end{aligned}$$

Fate of the universe

$c < 1$ —— can avoid future big rip?

$$p_{\mathrm{eff},de} = p_{de} + \frac{Q}{3H}, \qquad w_{\mathrm{eff},de} = w_{de} + \frac{Q}{3H\rho_{de}}$$

$$\begin{aligned} w_{\mathrm{eff},de} &= w_{de} + \frac{Q}{3H\rho_{de}} \\ &= -\frac{1}{3} - \frac{2}{3}\sqrt{\frac{\Omega_{de}}{c^2} + \Omega_k} \end{aligned} \qquad \begin{aligned} w_{\mathrm{eff},dm} &= -\frac{Q}{3H\rho_{dm}} = \frac{\Omega_I}{3\Omega_{dm}} \\ &= -\frac{1}{3}\Gamma E(z)^{2(\alpha+\beta)-3}\Omega_{dm}^{\alpha-1}\Omega_{de}^{\beta} \end{aligned}$$

$$w_{\mathrm{eff,tot}} = \sum_i w_{\mathrm{eff},i}\Omega_i$$

the simplified equations of motion in the region $z \to -1$

$$\frac{1}{E(z)}\frac{dE(z)}{dz} = -\frac{\Omega_{de}}{1+z}\left(\frac{\Omega_{de} - 3 + \Omega_I}{2\Omega_{de}} + \frac{\sqrt{\Omega_{de}}}{c}\right),$$

$$\frac{d\Omega_{de}}{dz} = -\frac{2\Omega_{de}(1-\Omega_{de})}{1+z}\left(\frac{\sqrt{\Omega_{de}}}{c} + \frac{1}{2} - \frac{\Omega_I}{2(1-\Omega_{de})}\right)$$

Stable solution (future)

$$\Omega_{de} = c^2, \quad \Omega_I = 3(1 - c^2)$$

which leads to

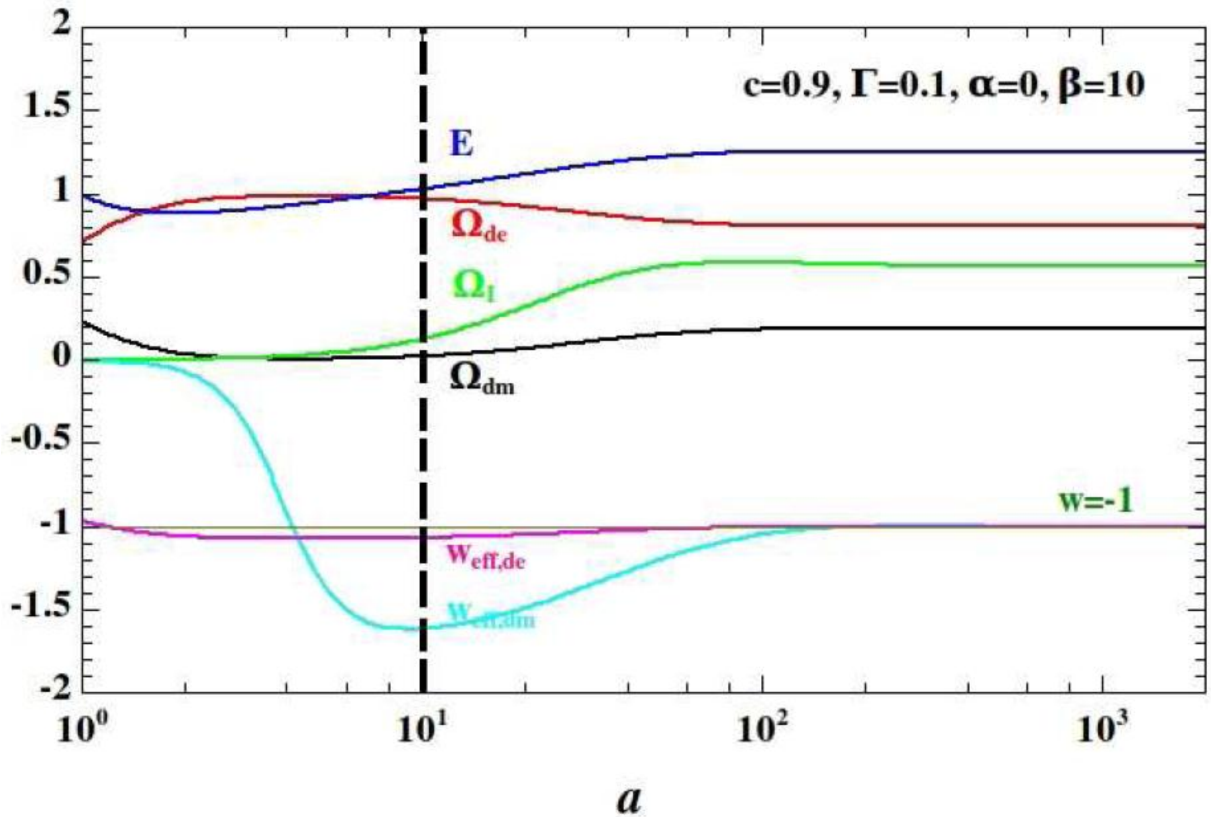
de Sitter!

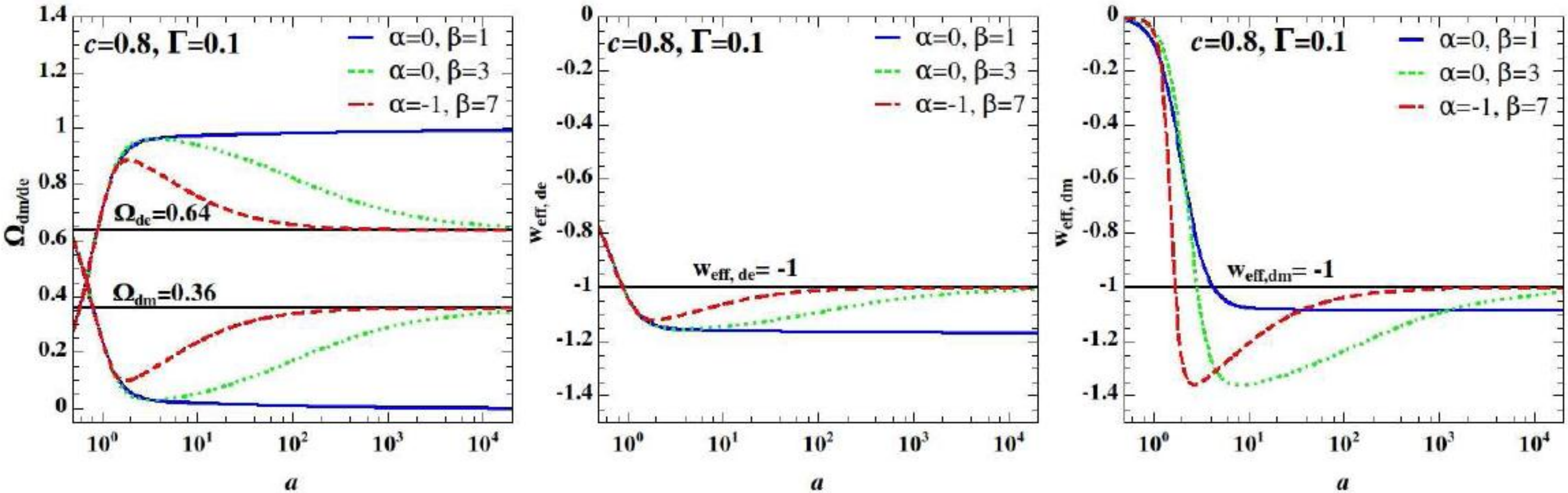
$$w_{\mathrm{eff},de} = -\frac{1}{3} - \frac{2\sqrt{\Omega_{de}}}{3c} = -1, \quad w_{\mathrm{eff},dm} = -\frac{\Omega_I}{3\Omega_{dm}} = -1,$$

and

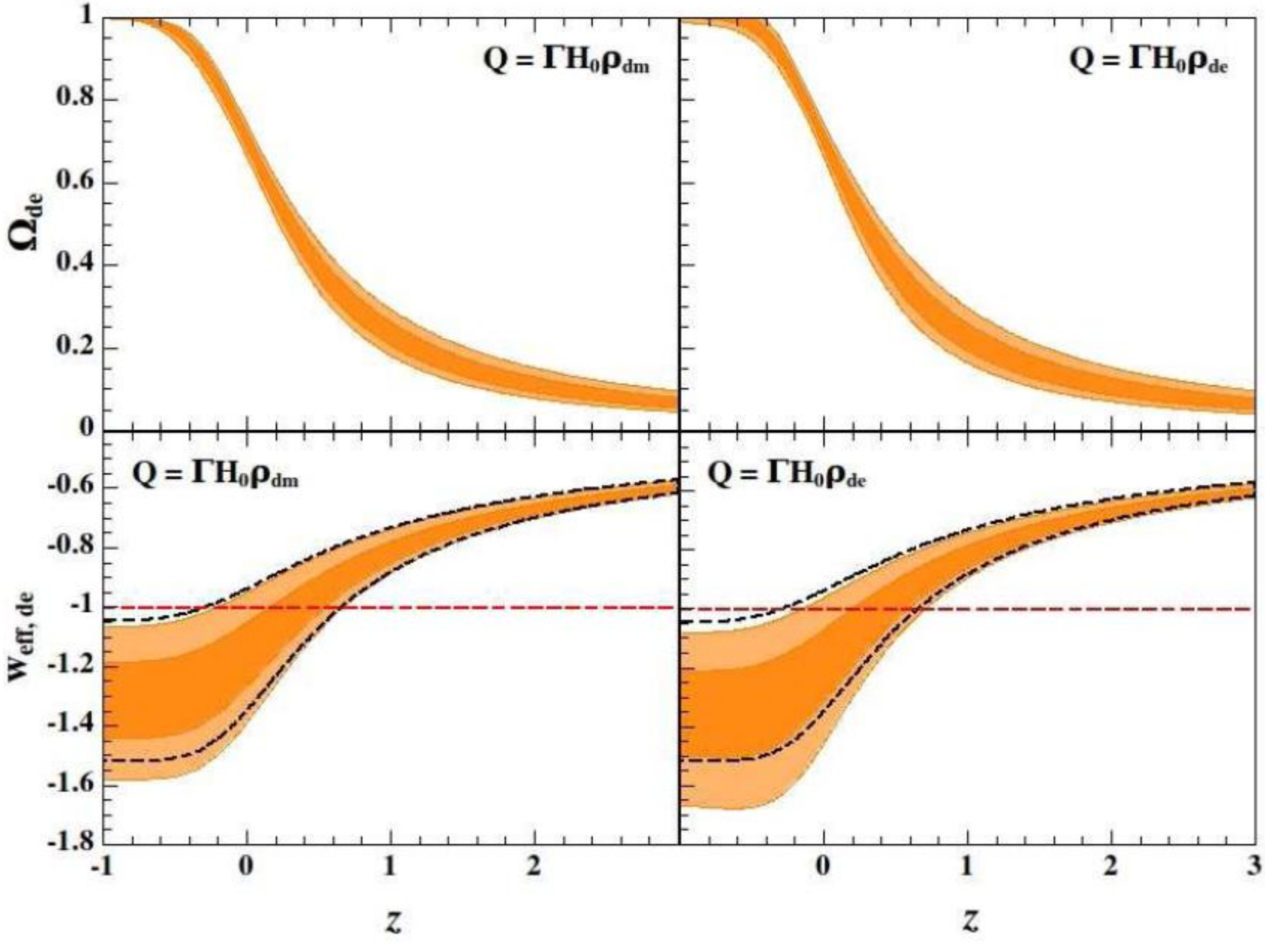
$$\frac{dE(z)}{dz} = 0, \quad \frac{d\Omega_{de}}{dz} = 0.$$

We find that this situation can really happen when β is large, and the interaction is strong enough to pull down Ω_{de} from 1

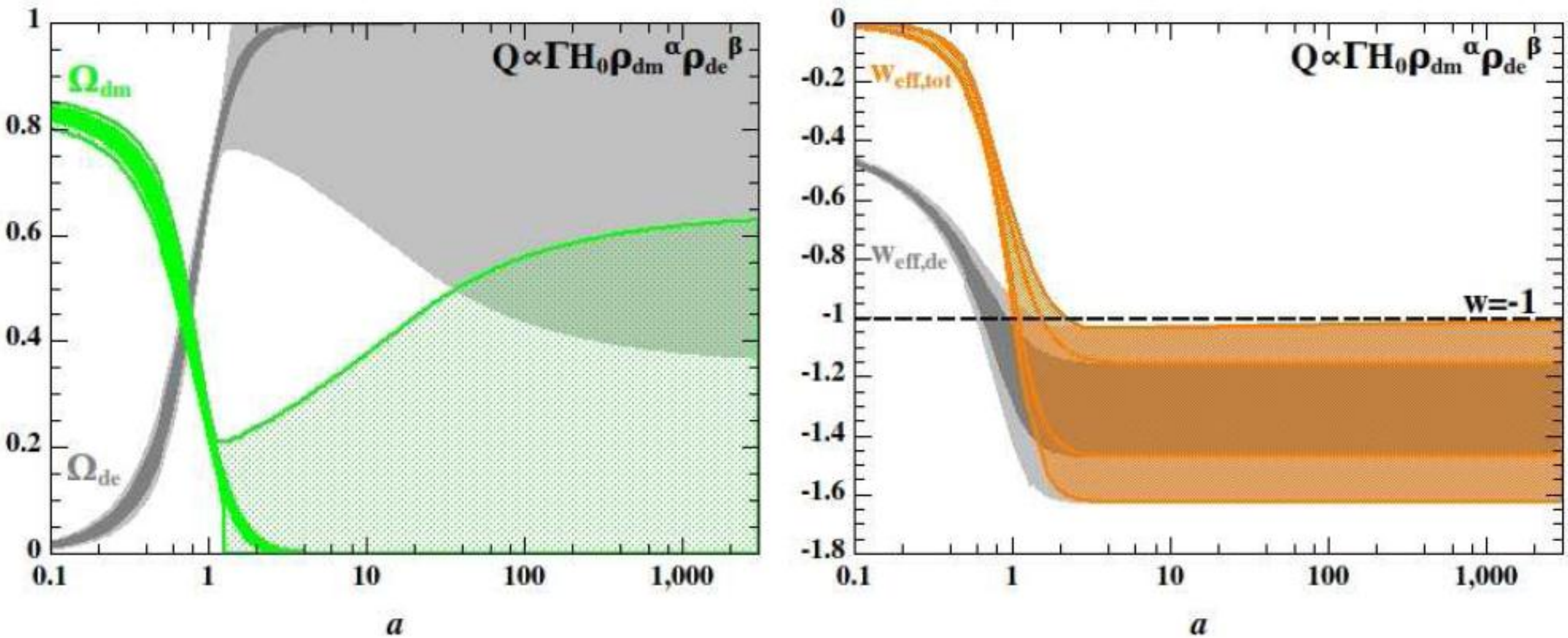




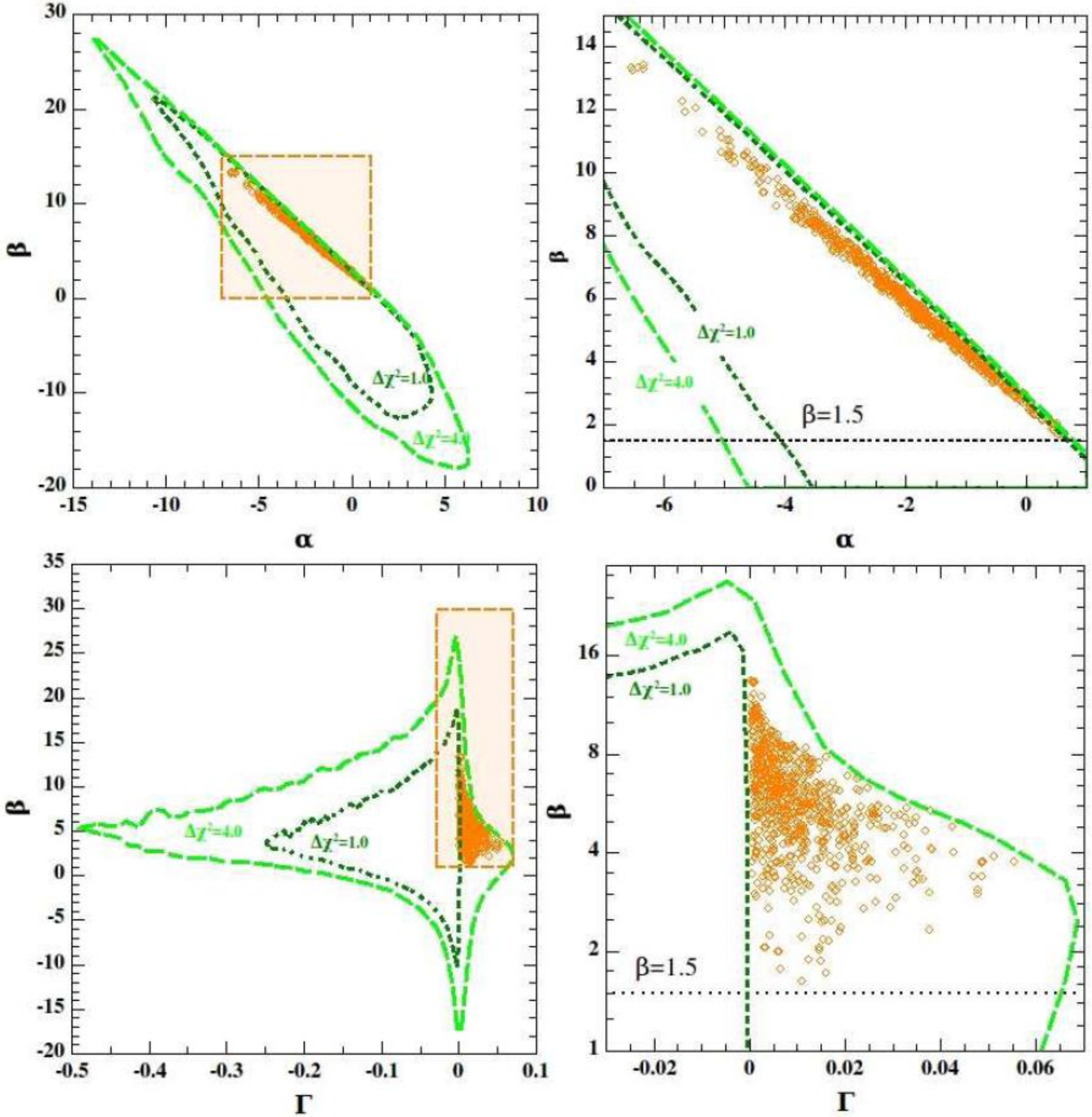
c=0.8, Γ=0.1
α=0, β=1
α=0, β=3
α=-1, β=7
Ω_de=0.64
Ω_dm=0.36
Ω_dm/de
a
w_eff, de= -1
w_eff, de
w_eff,dm= -1
w_eff, dm



$Q = \Gamma H_0 \rho_{dm}$
$Q = \Gamma H_0 \rho_{de}$
$Q = \Gamma H_0 \rho_{dm}$
$Q = \Gamma H_0 \rho_{de}$
Ω_{de}
$w_{eff,\,de}$
z
z



$Q \propto \Gamma H_0 \rho_{dm}{}^{\alpha} \rho_{de}{}^{\beta}$
Ω_{dm}
Ω_{de}
a
$Q \propto \Gamma H_0 \rho_{dm}{}^{\alpha} \rho_{de}{}^{\beta}$
$w_{eff,tot}$
$w_{eff,de}$
w=-1
a

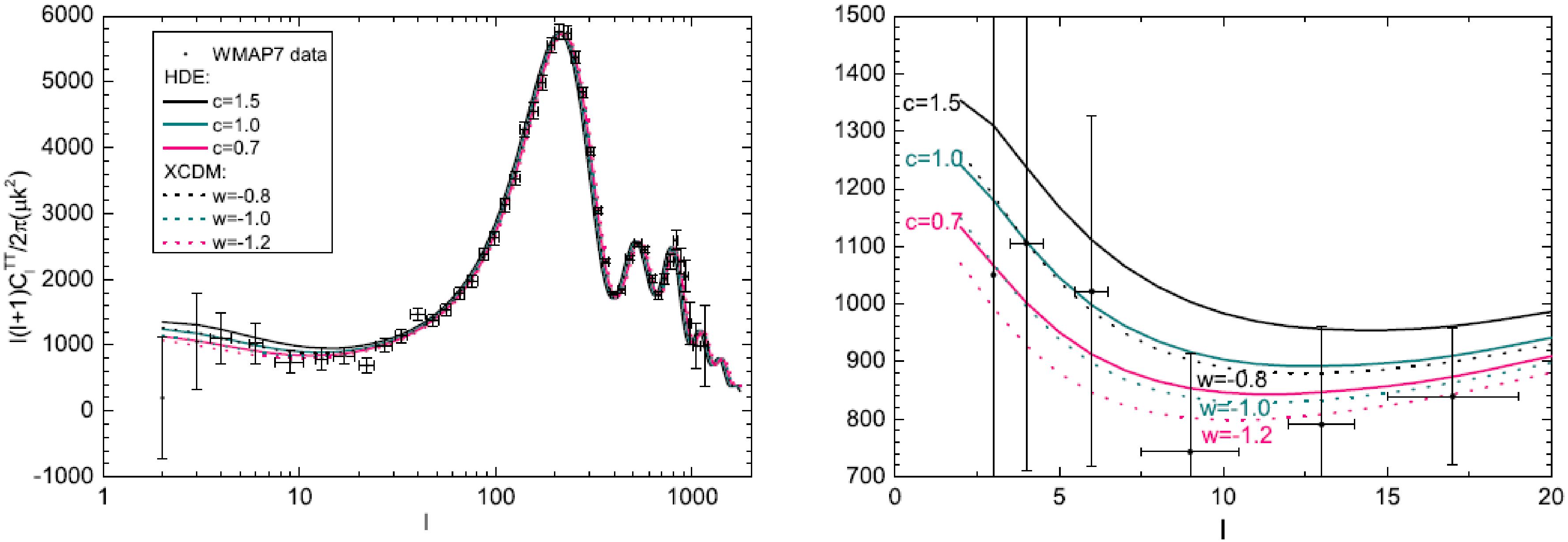


β
α
Δχ²=1.0
Δχ²=4.0
β=1.5
Γ

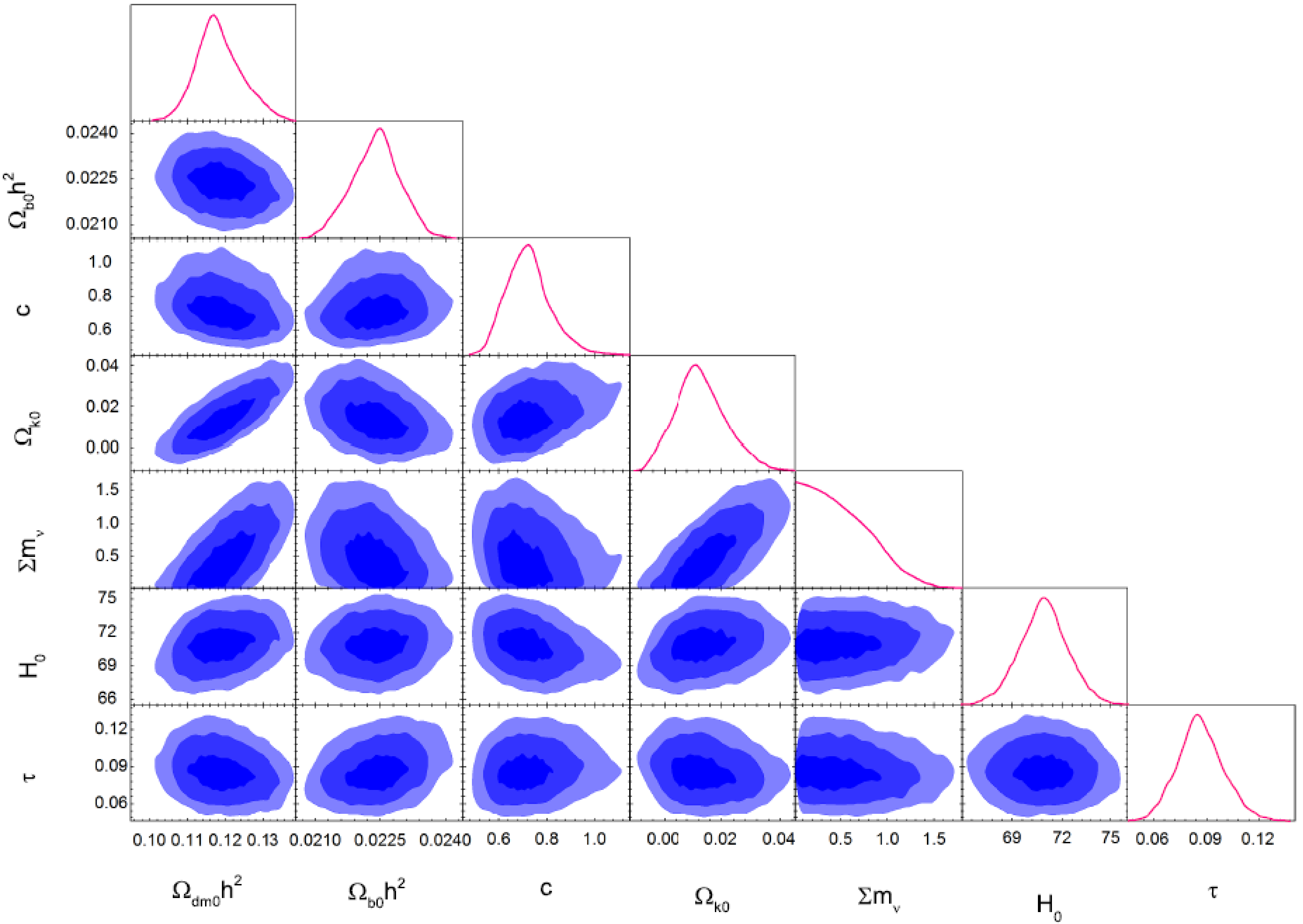
A global fit study on HDE

Y.H. Li, S. Wang, X.D. Li & XZ, 1207.6679

Parameters	HDE	KHDE	VHDE	KVHDE
$\Omega_{dm0}h^2$	$0.110^{+0.005+0.009+0.013}_{-0.002-0.006-0.010}$	$0.113^{+0.007+0.012+0.016}_{-0.002-0.007-0.012}$	$0.110^{+0.006+0.009+0.012}_{-0.002-0.005-0.009}$	$0.117^{+0.009+0.015+0.021}_{-0.004-0.009-0.016}$
$100\Omega_{b0}h^2$	$2.259^{+0.061+0.115+0.159}_{-0.048-0.100-0.152}$	$2.246^{+0.063+0.121+0.159}_{-0.045-0.098-0.142}$	$2.272^{+0.044+0.096+0.144}_{-0.069-0.105-0.155}$	$2.256^{+0.028+0.097+0.146}_{-0.066-0.128-0.173}$
c	$0.680^{+0.064+0.135+0.222}_{-0.066-0.119-0.159}$	$0.702^{+0.104+0.232+0.393}_{-0.063-0.102-0.176}$	$0.708^{+0.014+0.111+0.159}_{-0.099-0.153-0.215}$	$0.733^{+0.037+0.185+0.321}_{-0.107-0.170-0.230}$
Ω_{k0}	N/A	$0.004^{+0.009+0.016+0.023}_{-0.004-0.010-0.015}$	N/A	$0.010^{+0.010+0.020+0.032}_{-0.004-0.014-0.018}$
$\sum m_\nu$ (eV)	N/A	N/A	$\sum m_\nu < 0.48$ (2σ)	$\sum m_\nu < 1.17$ (2σ)
H_0 (km/s/Mpc)	$70.3^{+1.3+2.9+4.2}_{-1.3-2.8-4.0}$	$70.7^{+1.7+3.1+4.6}_{-1.2-2.6-4.2}$	$69.6^{+1.8+3.3+4.8}_{-0.9-2.1-3.5}$	$71.0^{+1.1+2.8+4.2}_{-1.2-2.8-4.1}$
τ	$0.087^{+0.015+0.030+0.047}_{-0.013-0.026-0.038}$	$0.087^{+0.014+0.029+0.046}_{-0.015-0.026-0.037}$	$0.087^{+0.015+0.031+0.047}_{-0.011-0.023-0.036}$	$0.085^{+0.012+0.029+0.044}_{-0.009-0.024-0.035}$
Θ	$1.039^{+0.003+0.006+0.008}_{-0.002-0.004-0.006}$	$1.039^{+0.003+0.005+0.008}_{-0.001-0.004-0.006}$	$1.040^{+0.002+0.005+0.007}_{-0.003-0.005-0.007}$	$1.040^{+0.001+0.004+0.007}_{-0.002-0.005-0.008}$
n_s	$0.966^{+0.018+0.029+0.041}_{-0.010-0.020-0.031}$	$0.968^{+0.009+0.024+0.035}_{-0.015-0.025-0.037}$	$0.969^{+0.012+0.025+0.039}_{-0.011-0.023-0.032}$	$0.969^{+0.008+0.020+0.034}_{-0.016-0.032-0.046}$
$\log[10^{10}A_s]$	$3.181^{+0.036+0.070+0.107}_{-0.037-0.078-0.112}$	$3.188^{+0.039+0.081+0.120}_{-0.026-0.071-0.108}$	$3.172^{+0.039+0.072+0.115}_{-0.029-0.068-0.106}$	$3.191^{+0.039+0.097+0.123}_{-0.022-0.065-0.118}$
Ω_{de0}	$0.731^{+0.009+0.022+0.032}_{-0.015-0.029-0.043}$	$0.725^{+0.012+0.023+0.036}_{-0.018-0.036-0.053}$	$0.726^{+0.015+0.026+0.035}_{-0.012-0.025-0.038}$	$0.714^{+0.011+0.030+0.042}_{-0.023-0.048-0.073}$
Age (Gyr)	$13.875^{+0.076+0.183+0.284}_{-0.123-0.230-0.328}$	$13.708^{+0.115+0.410+0.671}_{-0.422-0.594-0.870}$	$13.869^{+0.093+0.241+0.378}_{-0.129-0.209-0.289}$	$13.480^{+0.244+0.566+0.819}_{-0.176-0.507-0.804}$
Ω_{m0}	$0.269^{+0.015+0.029+0.043}_{-0.009-0.022-0.032}$	$0.271^{+0.014+0.029+0.045}_{-0.009-0.023-0.032}$	$0.274^{+0.012+0.025+0.038}_{-0.015-0.026-0.035}$	$0.276^{+0.015+0.031+0.049}_{-0.010-0.023-0.033}$
z_{re}	$10.580^{+1.184+2.312+3.552}_{-1.106-2.251-3.447}$	$10.607^{+1.128+2.275+3.533}_{-1.187-2.317-3.472}$	$10.504^{+1.235+2.439+3.674}_{-0.908-2.075-3.210}$	$10.526^{+1.143+2.532+3.455}_{-0.703-2.083-3.213}$
CL for $c < 1$	4.2σ	2.5σ	4.6σ	2.7σ



6000
5000
4000
3000
2000
1000
0
-1000
1
10
100
1000
l
$l(l+1)C_l^{TT}/2\pi(\mu k^2)$
WMAP7 data
HDE:
c=1.5
c=1.0
c=0.7
XCDM:
w=-0.8
w=-1.0
w=-1.2
1500
1400
1300
1200
1100
1000
900
800
700
0
5
10
15
20
l
c=1.5
c=1.0
c=0.7
w=-0.8
w=-1.0
w=-1.2



$\Omega_{dm0}h^2$
$\Omega_{b0}h^2$
c
Ω_{k0}
Σm_ν
H_0
τ

Interacting DE: in a perturbed universe

$$ds^2 = a^2\{-(1+2\phi)d\tau^2 + 2\partial_i B d\tau dx^i + [(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E]dx^i dx^j\}.$$

$$Q_A^\mu = Q_A u^\mu + F_A^\mu, \qquad Q_A = \bar{Q}_A + \delta Q_A, \qquad u_\mu F_A^\mu = 0, \qquad F_A^\mu = a^{-1}(0, \partial^i f_A),$$

$$Q_0^A = -a[\bar{Q}_A(1+\phi) + \delta Q_A],$$

$$Q_i^A = a\partial_i[\bar{Q}_A(v+B) + f_A]. \qquad \theta_A = -k^2(v_A + B),$$

$$\delta_A' + 3\mathcal{H}(c_{sA}^2 - w_A)\delta_A + (1+w_A)\theta_A + 9\mathcal{H}^2(1+w_A)(c_{sA}^2 - c_{aA}^2)\frac{\theta_A}{k^2} - 3(1+w_A)\psi' + (1+w_A)k^2(B-E') = \frac{a\bar{Q}_A}{\bar{\rho}_A}\left[\phi - \delta_A + 3\mathcal{H}(c_{sA}^2 - c_{aA}^2)\frac{\theta_A}{k^2}\right] + \frac{a}{\bar{\rho}_A}\delta Q_A,$$

$$\theta_A' + \mathcal{H}(1-3c_{sA}^2)\theta_A - \frac{c_{sA}^2}{(1+w_A)}k^2\delta_A - k^2\phi = \frac{a}{(1+w_A)\bar{\rho}_A}\{\bar{Q}_A[\theta - (1+c_{sA}^2)\theta_A] - k^2 f_A\}.$$

Next step

Add interaction, calculate perturbations, and do global fit analysis

Thanks!