

Flavor Models for the vMSM

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Some overlap with
T.A. & Li Yufeng(IHEP), PRD85, 065016.

Outline

- vMSM and its parameter space
 - dark matter abundance
 - neutrino masses (seesaw)
 - baryon asymmetry of the universe

based on [Canetti etal, arXiv: 1208.4607 [hep-ph]]

- Flavor Models
 - Flavor symmetries
 - model - I
 - model - II
- Summary

vMSM

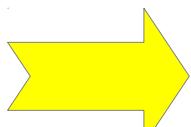
The vMSM

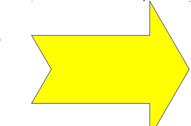
vMSM = neutrino **M**inimal **S**tandard **M**odel

The extended SM by three RHNs with masses smaller than the EW scale ($M_R < 100 \text{ GeV}$) :

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{SM}}$$

$$+ i \overline{N_R} \not{\partial} N_R + \left(-Y_{ij} \overline{L}_i \tilde{H} N_{Rj} - \frac{M_{Ri}}{2} \overline{N}_{Ri}^c N_{Ri} + h.c. \right)$$

$M_1 \sim \text{keV}$  (Warm) Dark Matter Candidate

$M_{2.3} \sim \text{GeV}$  Tiny Neutrino Masses and BAU

Three big mysteries, which cannot be explained within the SM, can be addressed **simultaneously!!**

The vMSM ~parameters~

The diagonal basis of the right-handed (sterile) neutrinos:

$$\mathcal{L}_{\nu \text{MSM}} = \mathcal{L}_{\text{SM}}$$

$$+ i \overline{N}_R \not{\partial} N_R + \left(- Y_{ij} \overline{L}_i \tilde{H} N_{Rj} - \frac{M_{Ri}}{2} \overline{N}_{Ri}^c N_{Ri} + h.c. \right)$$

Dirac

$$m_{ij} = v_{\text{ew}} Y_{ij}$$

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & m_{12} & m_{12} & m_{13} \\ 0 & 0 & 0 & m_{22} & m_{22} & m_{23} \\ 0 & 0 & 0 & m_{32} & m_{32} & m_{33} \\ M_1 & & & 0 & 0 & 0 \\ & & & M - \Delta M & & 0 \\ & & & & & M + \Delta M \end{pmatrix}$$

Majorana

The vMSM ~parameters~

The diagonal basis of the right-handed (sterile) neutrinos:

$$\mathcal{L}_{\nu \text{MSM}} = \mathcal{L}_{\text{SM}}$$

$$+ i \overline{N}_R \phi N_R + \left(-Y_{ij} \overline{L}_i \tilde{H} N_{Rj} - \frac{M_{Ri}}{2} \overline{N}_{Ri}^c N_{Ri} + h.c. \right)$$

Y_{i1} $\overline{\nu}_{Li}$ h N_{R1}

active – sterile mixing

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} m_{12} \\ m_{22} \\ m_{32} \end{pmatrix} \begin{pmatrix} m_{12} & m_{13} \\ m_{22} & m_{23} \\ m_{32} & m_{33} \\ 0 & 0 \\ M - \Delta M & 0 \\ M + \Delta M & 0 \end{pmatrix}$$

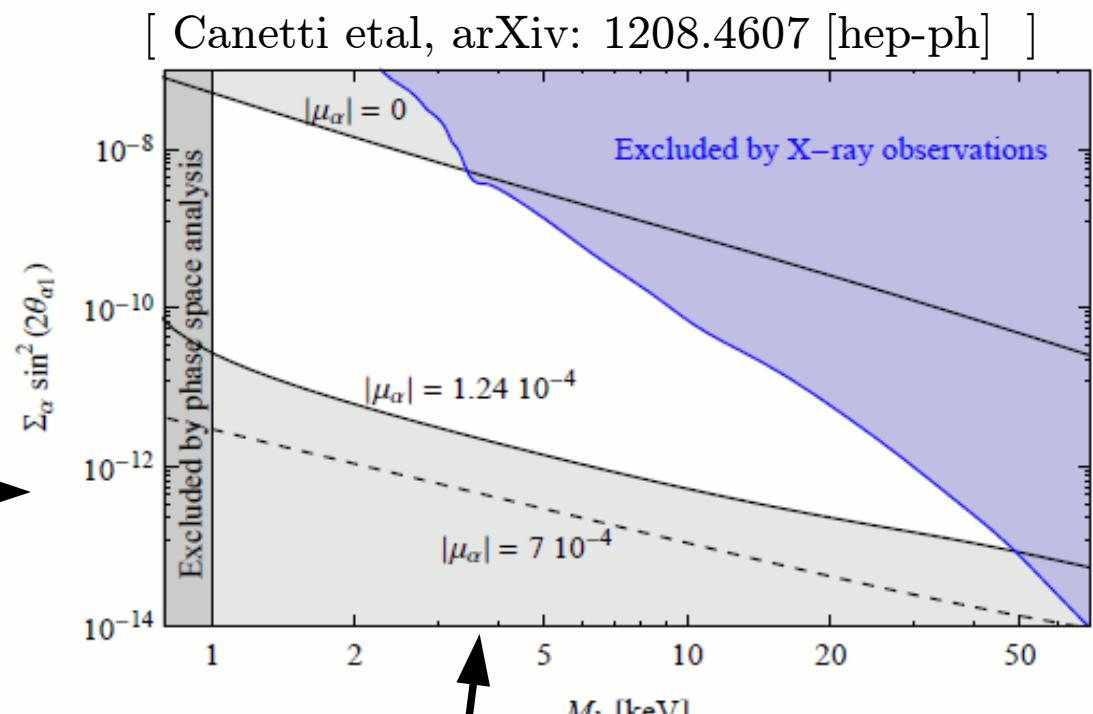
$m_{ij} = v_{\text{ew}} Y_{ij}$

(warm) DM candidate

The vMSM ~dark matter~

The N1 (wDM) abundance was generated via active-sterile mixing **with lepton number asymmetry**.

$$\begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}$$



$$y_{i1} \sim 10^{-12} \text{ for } M_1 = 1 \text{ keV}$$

$$y_{i1} \sim 10^{-13} \text{ for } M_1 = 30 \text{ keV}$$

A yellow arrow points upwards from the bottom left towards the matrix equation.

M_1

$$\begin{pmatrix} M_1 & 0 & 0 & 0 \\ 0 & M - \Delta M & 0 & 0 \\ 0 & 0 & M + \Delta M & 0 \end{pmatrix}$$

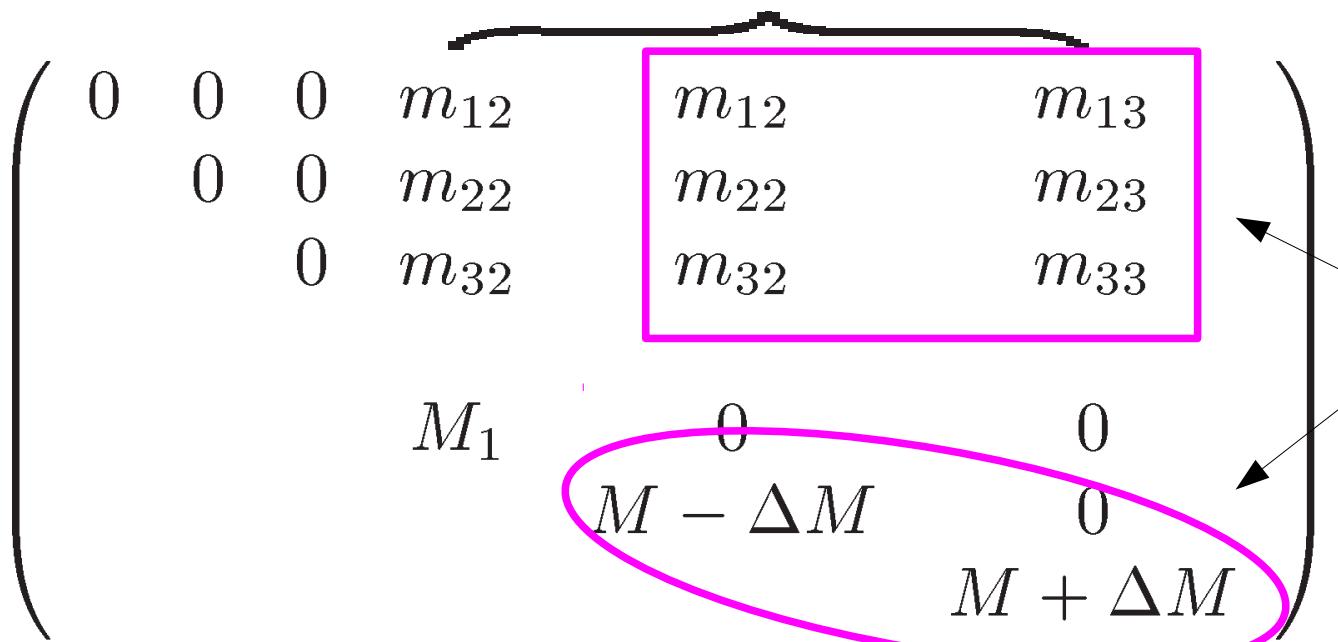
The vMSM ~parameters~

The diagonal basis of the right-handed (sterile) neutrinos:

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{SM}}$$

$$+ i \overline{N}_R \phi N_R + \left(-Y_{ij} \overline{L}_i \tilde{H} N_{Rj} - \frac{M_{Ri}}{2} \overline{N}_{Ri}^c N_{Ri} + h.c. \right)$$

$$m_{ij} = v_{\text{ew}} Y_{ij}$$



Masses of the active neutrinos via the seesaw mechanism.

The vMSM ~seesaw~

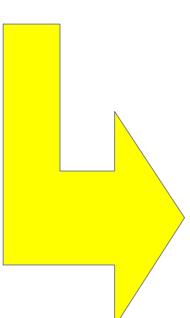
Consider the basis $M_R = \text{Diag}(M_1, M - \Delta M, M + \Delta M)$ and integrate out the right-handed neutrinos:

$$\mathcal{L}_{\nu\text{MSM}} = \mathcal{L}_{\text{SM}}$$

$$+ i \overline{N}_R \not{\partial} N_R + \left(-Y_{ij} \overline{L}_i \tilde{H} N_{Rj} - \frac{M_{Ri}}{2} \overline{N}_{Ri}^c N_{Ri} + h.c. \right)$$

$$m_D = v Y$$

$$\begin{pmatrix} 1 - \frac{1}{2}\xi\xi^\dagger & -\xi \\ \xi^\dagger & 1 - \frac{1}{2}\xi\xi^\dagger \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2}\xi^*\xi^T & \xi^* \\ -\xi^T & 1 - \frac{1}{2}\xi^T\xi^* \end{pmatrix}$$

$$\xi = m_D M_R^{-1} \ll 1$$


Active: $m_\nu = \xi M_R \xi^T \sim \mathcal{O}(\text{eV})$

Sterile: $M'_R = M_R + \frac{1}{2} (\xi^\dagger \xi M_R + M_R^T \xi^T \xi^*)$

The vMSM ~seesaw~

In the diagonal basis of the sterile neutrinos

$$M_R = \begin{pmatrix} M_1 & & \\ & M - \Delta M & \\ & & M + \Delta M \end{pmatrix} \quad Y_{ij} = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}$$

GeV

$m_\nu = v^2 Y M_R^{-1} Y^T \simeq 10^{-2}$ eV requires that

$$\underline{y_{i2}, y_{i3} \sim 10^{-7.5} \text{ for } M_{2,3} = 1 \text{ GeV.}}$$

The constraints $y_{i1} \sim 10^{-12}$ for $M_1 = 1$ keV result in
 $m_\nu^{\text{lightest}} \sim 10^{-5}$ eV.

The lightest active neutrino mass is negligibly-small !!

The vMSM ~seesaw~

The sterile neutrino mixing may be enhanced.

Sterile: $M'_R = M_R + \frac{1}{2} (\xi^\dagger \xi M_R + M_R^T \xi^T \xi^*)$

cannot be neglected!

$$U_N^\dagger M'_R U_N^* = \text{Diag}(M_1, M_2, M_3)$$

$$M_{2,3} = M + \frac{1}{2M} \text{Re} [\text{tr}(m_D^\dagger m_D)] \pm \delta M \equiv \tilde{M} \pm \delta M$$

$$(\delta M)^2 = \left\{ \frac{1}{2M} \left(\text{Re}[m_D^\dagger m_D]_{33} - \text{Re}[m_D^\dagger m_D]_{22} \right) + \Delta M \right\}^2 + \frac{1}{M^2} \text{Re}[m_D^\dagger m_D]_{23}^2$$

$\delta M \ll M$ and thus $M_2 \simeq M_3$, but U_N is not necessarily small because of the degeneracy.

$$U_N = \begin{pmatrix} 1 & & \\ 0 & & \\ 0 & \end{pmatrix}$$

Small

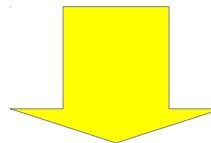
Large CP violating oscillation between N2 and N3.

The vMSM ~BAU~

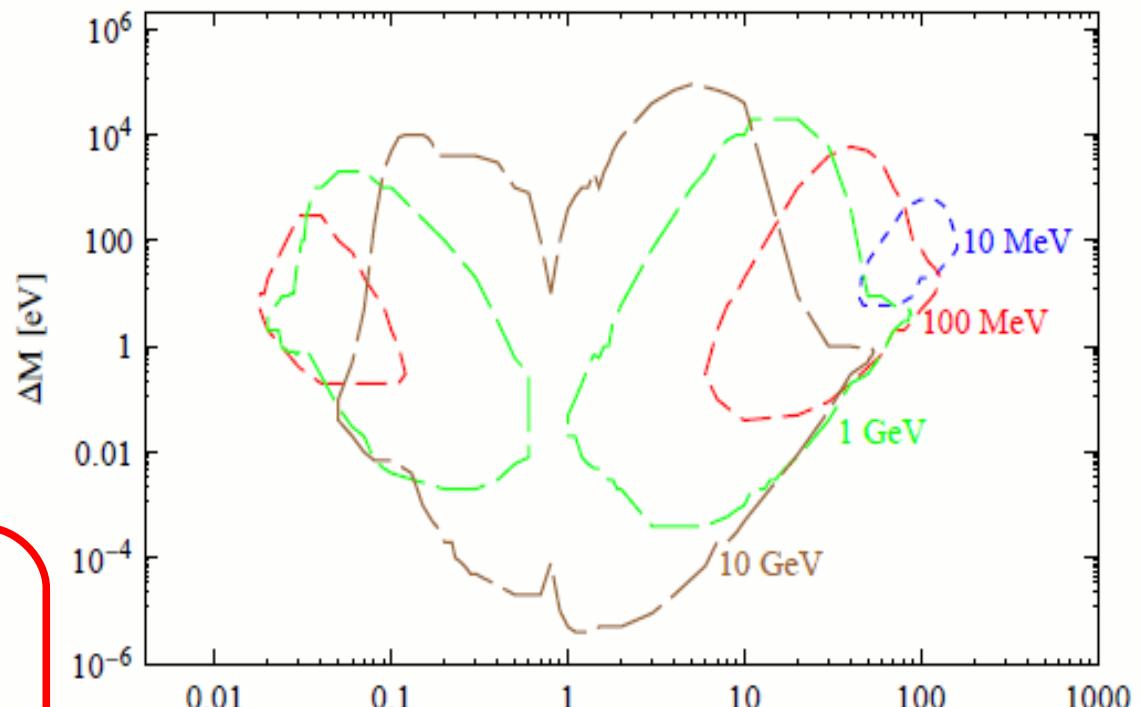
Lepton asymmetry was produced via N2-N3 oscillation, and it was converted into baryon asymmetry.

[Canetti et al, arXiv: 1208.4607 [hep-ph]]

$$M_2 = M - \Delta M$$
$$M_3 = M + \Delta M$$



$\Delta M \sim 10^{-2} \text{ eV} - 10 \text{ keV}$
for
 $M = 1 \text{ GeV}$



CP phase in Yukawa couplings.

The vMSM ~thermal history~

Everything happened near or below the EW scale.

$$\mu_\ell \sim 10^{-10}$$

for Baryogenesis
at around the EW.

$$\mu_\ell > 10^{-7}$$

for DM production
at around 100 MeV.

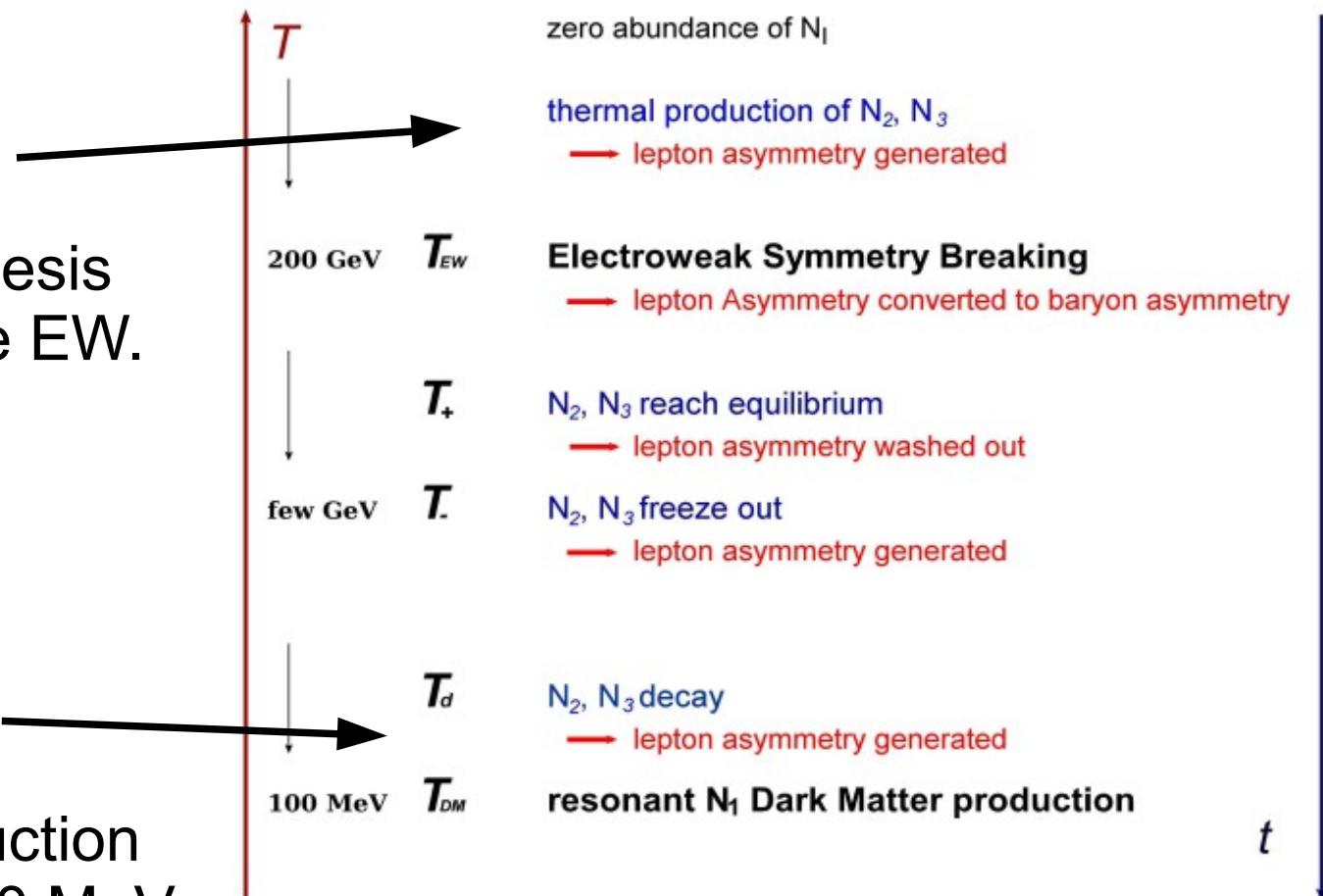


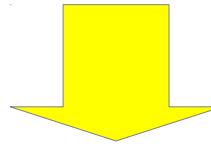
Figure 1: The thermal history of the universe in the ν MSM.

The vMSM ~lepton asym.~

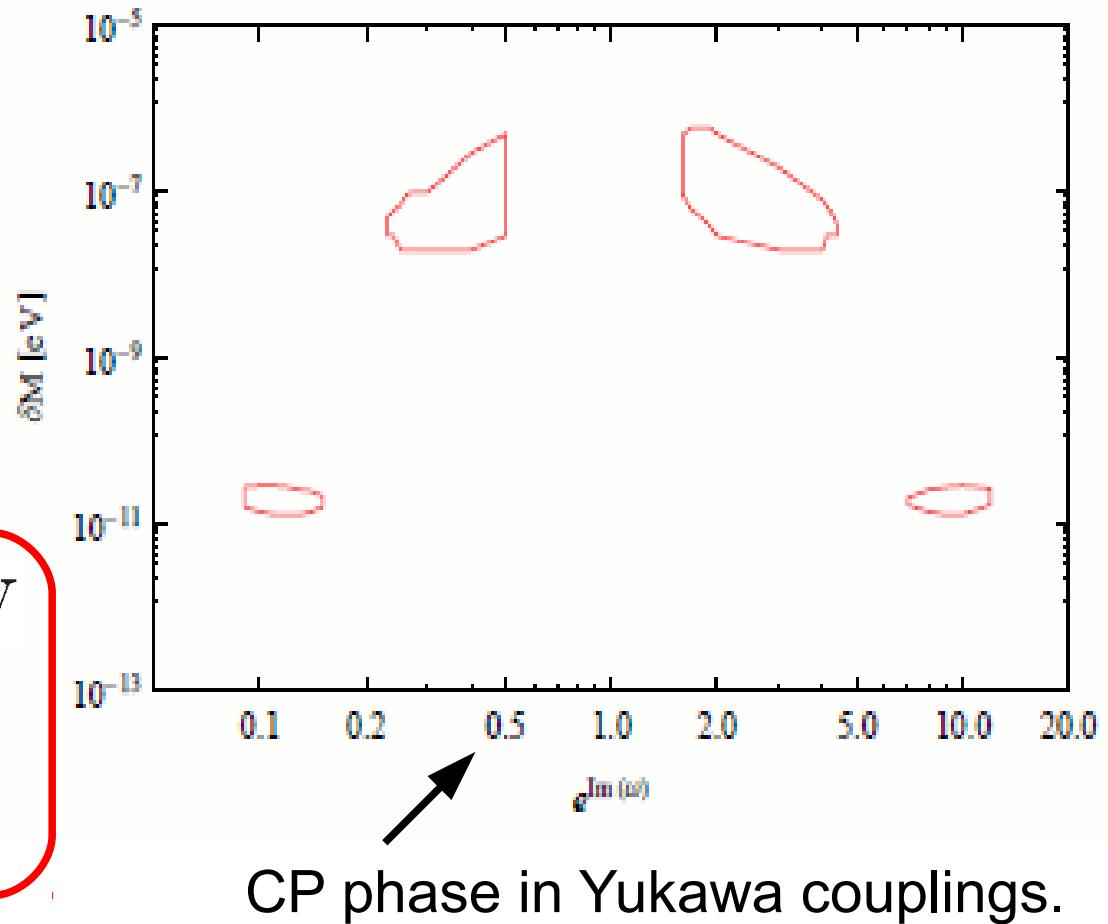
In order to have sufficient amount of baryon asymmetry, large lepton asymmetry is necessary.

[Canetti et al, arXiv: 1208.4607 [hep-ph]]

$$M_2 = M - \Delta M$$
$$M_3 = M + \Delta M$$

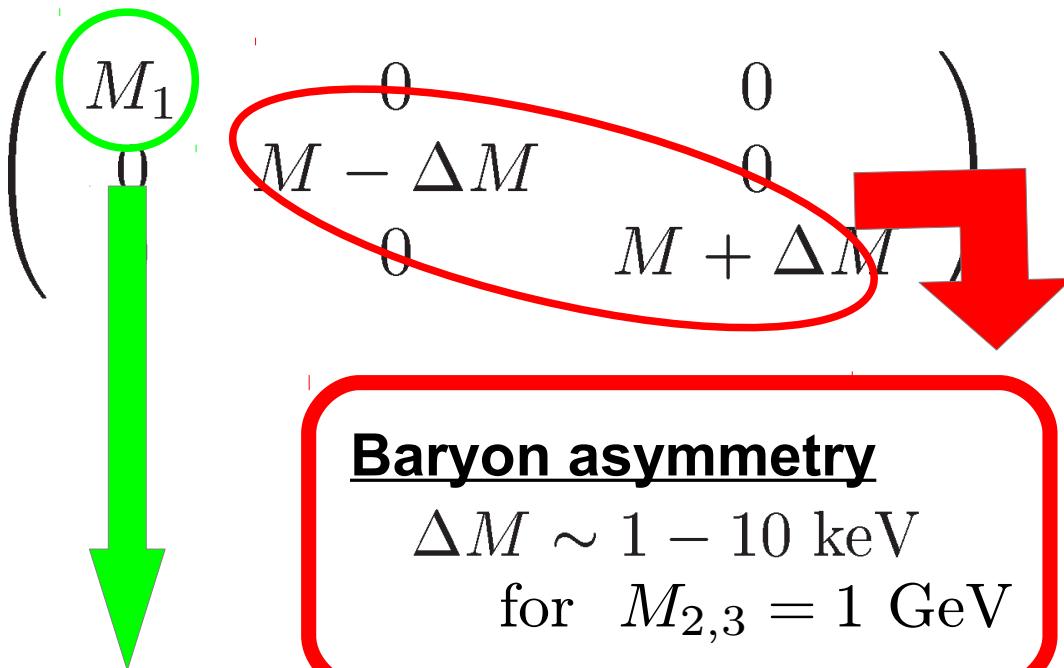


$\Delta M \sim 10^{-17} - 10^{-15} \text{ GeV}$
for
 $M = 1 \text{ GeV}$



Constraints for the vMSM

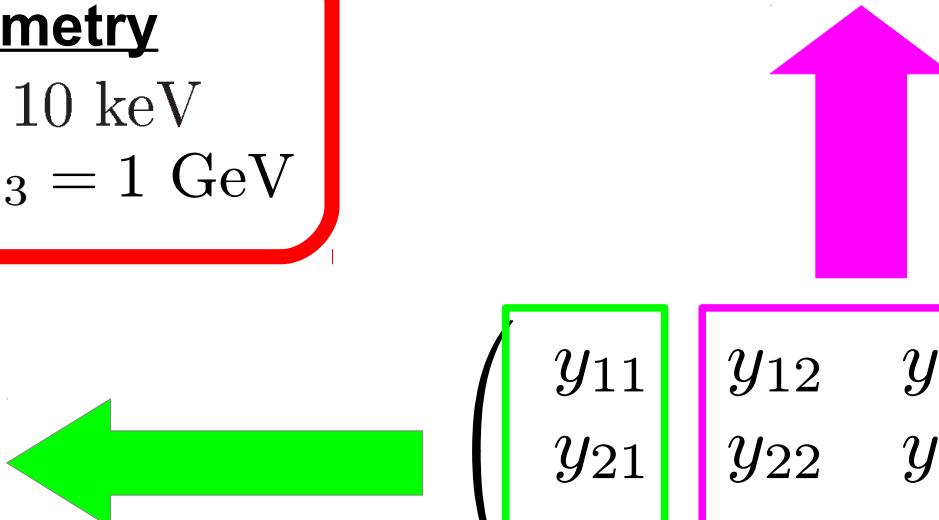
Suppose $M_{2,3} = 1 \text{ GeV}$



Seesaw

$y_{i2}, y_{i3} \sim 10^{-7.5}$
for $M_{2,3} = 1 \text{ GeV}$

DM production
 $M_1 \sim (1 - 10) \text{ keV}$
 $y_{i1} \sim 10^{-13} - 10^{-12}$



Constraints for the vMSM

Suppose $M_{2,3} = 1 \text{ GeV}$

$$\left(\begin{array}{ccc} M_1 & 0 & 0 \\ 0 & M - \Delta M & 0 \\ 0 & 0 & M + \Delta M \end{array} \right)$$

10^{-6} $\begin{array}{c} \nearrow \\ \nwarrow \end{array}$ 1 GeV

?

$10^{-13} \ll 10^{-7.5}$

$$\left(\begin{array}{cc} \boxed{y_{11}} & \boxed{\begin{array}{cc} y_{12} & y_{13} \\ y_{22} & y_{23} \\ y_{32} & y_{33} \end{array}} \end{array} \right)$$

Flavor Model

for vMSM

Flavor Symmetries

Q. What do $M_1 \ll M_{2,3}$ and $|M_2 - M_3| \ll 1$ suggest?

(1) The degeneracy seems to indicate that they serve as a doublet representation under a symmetry.

$$\begin{pmatrix} N_2 \\ N_3 \end{pmatrix} \stackrel{?}{\in} \text{doublet} \rightarrow N_1 \stackrel{?}{\in} 1, 1' \dots$$

(2) Furthermore, if the singlet is complex, one can prohibit N1 from having its bare (Majorana) mass term, leading to

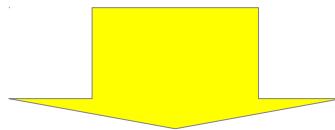
$$\rightarrow M_R = \text{Diag}(0, M, M).$$

A tiny mass splitting between N2 and N3 and the keV-scale mass of N1 may be obtained after taking higher dimensional operators into account.

Flavor Symmetries

Q. What kind of symmetries do we need?

- The symmetry should be non-abelian.
- The doublet should be real to give bare (Majorana) mass terms for N2 and N3.
- The singlet should be complex to forbid a bare mass term for N1.



The smallest non-abelian discrete group incorporating both of the complex singlet and real doublet is

Q_6 .

Flavor Symmetries

Q. What is Q_6 ?

Q_6 consists of four singlets and two doublet:

$$1, 1', \frac{1'', 1'''}{\text{complex}}, 2, \frac{2'}{\text{real}}$$

The multiplication rules are given by

$$1' \times 1' = 1, \quad 1'' \times 1'' = 1', \quad \begin{pmatrix} 2 \\ x_1 \\ x_2 \end{pmatrix} \times \begin{pmatrix} 2 \\ y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 \\ (x_1y_2 - x_2y_1) \\ (x_1y_1 + x_2y_2) \end{pmatrix} + \begin{pmatrix} 2' \\ -x_1y_2 - x_2y_1 \\ x_1y_1 - x_2y_2 \end{pmatrix}$$

$$1' \times 1''' = 1'', \quad 1' \times 1'' = 1''',$$

$$2 \times 1''' = 2', \quad 2' \times 1' = 2', \quad \begin{pmatrix} 2' \\ a_1 \\ a_2 \end{pmatrix} \times \begin{pmatrix} 2' \\ b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 1 \\ (a_1b_1 + a_2b_2) \\ (a_1b_2 - a_2b_1) \end{pmatrix} + \begin{pmatrix} 2' \\ -a_1b_1 + a_2b_2 \\ a_1b_2 + a_2b_1 \end{pmatrix}$$

$$1''' \times 1''' = 1', \quad 1'' \times 1''' = 1,$$

$$2 \times 1' = 2, \quad 2 \times 1'' = 2',$$

$$2' \times 1'' = 2, \quad 2' \times 1''' = 2.$$

$$\begin{pmatrix} 2 \\ x_1 \\ x_2 \end{pmatrix} \times \begin{pmatrix} 2' \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 1'' \\ (x_1a_2 + x_2a_1) \\ (x_1a_1 - x_2a_2) \end{pmatrix} + \begin{pmatrix} 2 \\ x_1a_1 + x_2a_2 \\ x_1a_2 - x_2a_1 \end{pmatrix}$$

Model - I

	\overline{L}_i	e_{Ri}	N_1	N_D	H	D	S
Q_6	1	1	1''	2'	1	2	1'
Z_2	+	+	+	+	+	-	-

Q_6 is complemented with an auxiliary Z_2 symmetry.

1. $N_D N_D \dagger (\mathbf{1}, +)$ (singlet by itself)
2. $N_D N_D \dagger (\mathbf{2}', +) \otimes D^2, |D|^2 \dagger (\mathbf{2}', +)$
3. $N_1 N_1 \dagger (\mathbf{1}', +) \otimes D^2 \dagger (\mathbf{1}', +)$
4. $N_1 N_D \dagger (\mathbf{2}, +) \otimes DS, D^* S \dagger (\mathbf{2}, +)$

Note

$$\langle D \rangle \equiv (d_1, d_2)$$

$$\langle S \rangle \equiv s$$

$$\lambda = \frac{d_i}{\Lambda} = \frac{s}{\Lambda}$$

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \textcolor{red}{M} & 0 \\ 0 & 0 & \textcolor{red}{M} \end{pmatrix} + \lambda^2 \begin{pmatrix} \textcolor{green}{m}_s & m_b & m_c \\ m_a & -m_d & m_e \\ m_c & m_e & m_d \end{pmatrix}$$

$m_i, M = \mathcal{O}(1\text{GeV})$

Model - I

Move on to the diagonal basis of M_R .

$$M_R = \begin{pmatrix} \lambda^2 m_s & & & \\ & M - \lambda^2 m_d & & \\ & & \lambda^2 m_e & \\ & & & M + \lambda^2 m_d \end{pmatrix}$$

Suppose λ is sufficiently small, then

$$R \simeq \begin{pmatrix} 1 & -\eta U_R \\ \eta^\dagger & U_R \end{pmatrix} \quad \rightarrow \quad \begin{aligned} M_1 &\simeq \lambda^2 m_s + \mathcal{O}(\lambda^4) \\ M_2 &\simeq M - \lambda^2 m_d + \mathcal{O}(\lambda^4) \\ M_3 &\simeq M + \lambda^2 m_d + \mathcal{O}(\lambda^4) \end{aligned}$$

$$\eta = \lambda^2(m_b, m_c) \begin{pmatrix} M - \lambda^2 m_d & \lambda^2 m_e \\ \lambda^2 m_e & M + \lambda^2 m_d \end{pmatrix}^{-1}$$

Note

$$\langle D \rangle \equiv (d_1, d_2)$$

$$\langle S \rangle \equiv s$$

$$\lambda = \frac{d_i}{\Lambda} = \frac{s}{\Lambda}$$

Given $m_i, M = \mathcal{O}(1\text{GeV})$, $\lambda = 10^{-3}$ results in

$$M_1 \sim 1 \text{ keV} \quad \text{and} \quad \Delta M \sim 1 \text{ keV}.$$

Model - I

	$\overline{L_i}$	e_{Ri}	N_1	N_D	H	D	S
Q_6	1	1	1''	2'	1	2	1'
Z_2	+	+	+	+	+	-	-

Q_6 is complemented with an auxiliary Z_2 symmetry.

1. $L_i \tilde{H} N_D \not\in (\mathbf{2}', +) \otimes D^2, |D|^2 \not\in (\mathbf{2}', +)$
2. $L_i \tilde{H} N_1 \not\in (\mathbf{1}'', +) \otimes D^3 S \not\in (\mathbf{1}''', +)$

Note

$$\langle D \rangle \equiv (d_1, d_2)$$

$$\langle S \rangle \equiv s$$

$$\lambda = \frac{d_i}{\Lambda} = \frac{s}{\Lambda}$$

$$Y_D = \lambda^2 \begin{pmatrix} 0 & y_{12} & y_{13} \\ 0 & y_{22} & y_{23} \\ 0 & y_{32} & y_{33} \end{pmatrix} + \lambda^4 \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} & 0 & 0 \\ y_{31} & 0 & 0 \end{pmatrix}$$

Model - I

In the diagonal basis of M_R

$$Y_D \rightarrow \begin{pmatrix} \lambda^4 y_{11} & \lambda^2 y_{12} & \lambda^2 y_{13} \\ \lambda^4 y_{21} & \lambda^2 y_{22} & \lambda^2 y_{23} \\ \lambda^4 y_{31} & \lambda^2 y_{32} & \lambda^2 y_{33} \end{pmatrix} \begin{pmatrix} 1 & -\eta U_R \\ \eta^\dagger & U_R \end{pmatrix}$$

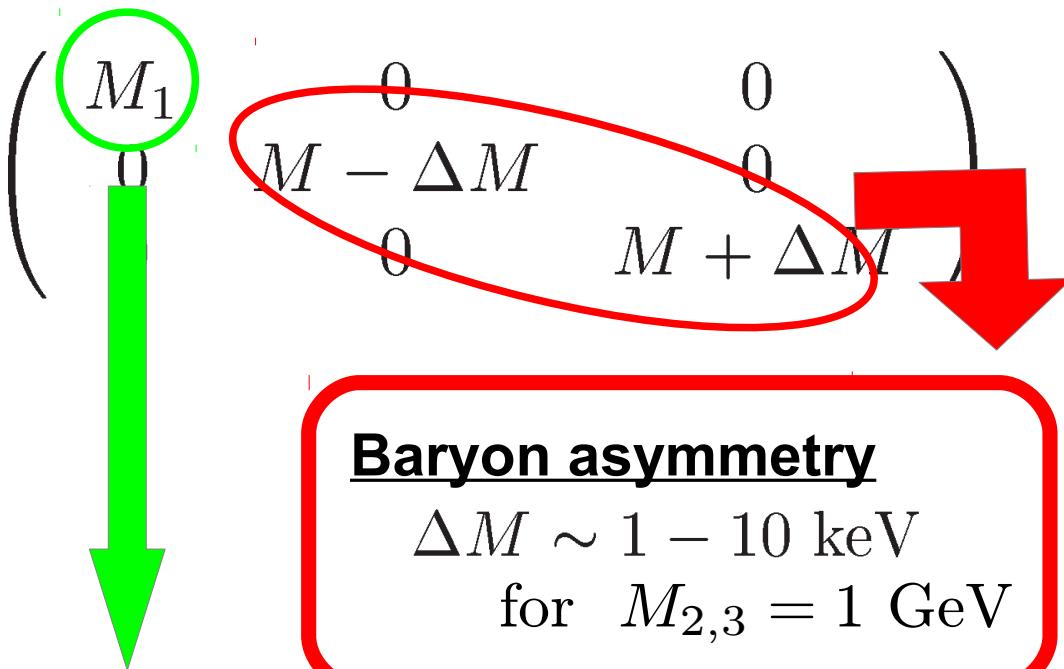
$$\eta = \mathcal{O}(\lambda^2) = \mathcal{O}(10^{-6}) \quad \equiv \quad \begin{pmatrix} \lambda^4 y_{11} + \mathcal{O}(\lambda^4) & \lambda^2 y'_{12} & \lambda^2 y'_{13} \\ \lambda^4 y_{21} + \mathcal{O}(\lambda^4) & \lambda^2 y'_{22} & \lambda^2 y'_{23} \\ \lambda^4 y_{31} + \mathcal{O}(\lambda^4) & \lambda^2 y'_{32} & \lambda^2 y'_{33} \end{pmatrix}$$

Given $y_{ij} \simeq y_\tau \simeq 10^{-2}$, we obtain 10^{-14} and 10^{-8} .

Note that U_R can be large but only mixes the second and third lines of Y_D .

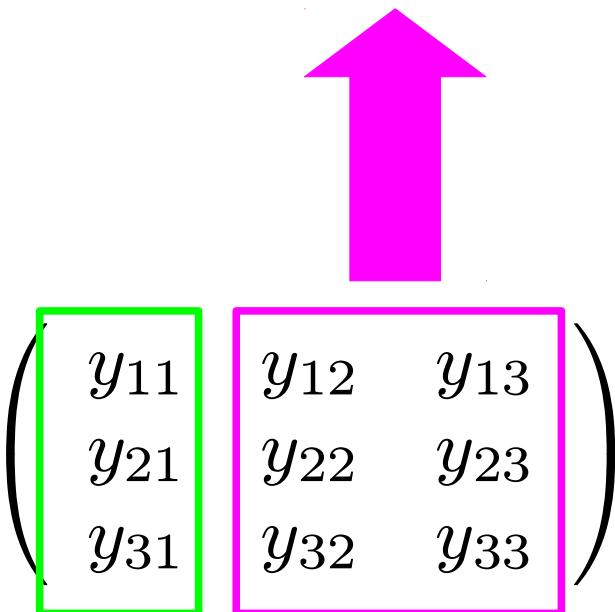
Constraints for the vMSM

Suppose $M_{2,3} = 1 \text{ GeV}$



Seesaw

$y_{i2}, y_{i3} \sim 10^{-7.5}$
for $M_{2,3} = 1 \text{ GeV}$



Model - II

$L_{1,2,3}$		$e_{R1,2,3}$	N_1	N_D	H	D	S_1	S_2
Q_6	$1, 1', 1'$	$1, 1', 1'$	$1''$	$2'$	1	2	$1'$	$1'''$
Z_{10}	$0, 0, 1$	$0, 0, -1$	4	0	0	1	3	0

Q_6 is complemented with an auxiliary Z_{10} symmetry.

1. $N_D N_D \ddagger (\mathbf{1}, 0)$ (singlet by itself)
2. $N_D N_D \ddagger (\mathbf{2}', 0) \otimes |D|^2 \ddagger (\mathbf{2}', 0)$
3. $N_1 N_1 \ddagger (\mathbf{1}', 8) \otimes D^2 \ddagger (\mathbf{1}', 2)$
4. $N_1 N_D \ddagger (\mathbf{2}, 4) \otimes (DS_1)^* \ddagger (\mathbf{2}, -4)$

Note
 $\langle D \rangle \equiv (d_1, d_2)$
 $\langle S_i \rangle \equiv s_i$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \textcolor{red}{M} & 0 \\ 0 & 0 & \textcolor{red}{M} \end{pmatrix} + \frac{1}{\Lambda^2} \begin{pmatrix} m_s(d_1^2 + d_2^2) & m_b d_2 s_1 & m_b d_1 s_1 \\ m_b d_2 s_1 & -m_d(d_1^2 - d_2^2) & 2m_d d_1 d_2 \\ m_b d_1 s_1 & 2m_d d_1 d_2 & m_d(d_1^2 - d_2^2) \end{pmatrix}$$

Model - II

	$L_{1,2,3}$	$e_{R1,2,3}$	N_1	N_D	H	D	S_1	S_2
Q_6	$1, 1', 1'$	$1, 1', 1'$	$1''$	$2'$	1	2	$1'$	$1'''$
Z_{10}	$0, 0, 1$	$0, 0, -1$	4	0	0	1	3	0

Q_6 is complemented with an auxiliary Z_{10} symmetry.

1. $L_{1,2} \tilde{H} N_D \not\propto (\mathbf{2}', 0) \otimes |D|^2 \not\propto (\mathbf{2}', 0)$
2. $L_3 \tilde{H} N_D \not\propto (\mathbf{2}', 1) \otimes D^* S_2 \not\propto (\mathbf{2}', -1)$
3. $L_{1,2} \tilde{H} N_1 \not\propto (\mathbf{1}'', 4) \otimes D^3 S_1 \not\propto (\mathbf{1}''', +)$
4. $L_3 \tilde{H} N_1 \not\propto (\mathbf{1}''', 5) \otimes D^5 \not\propto (\mathbf{1}''', 5)$

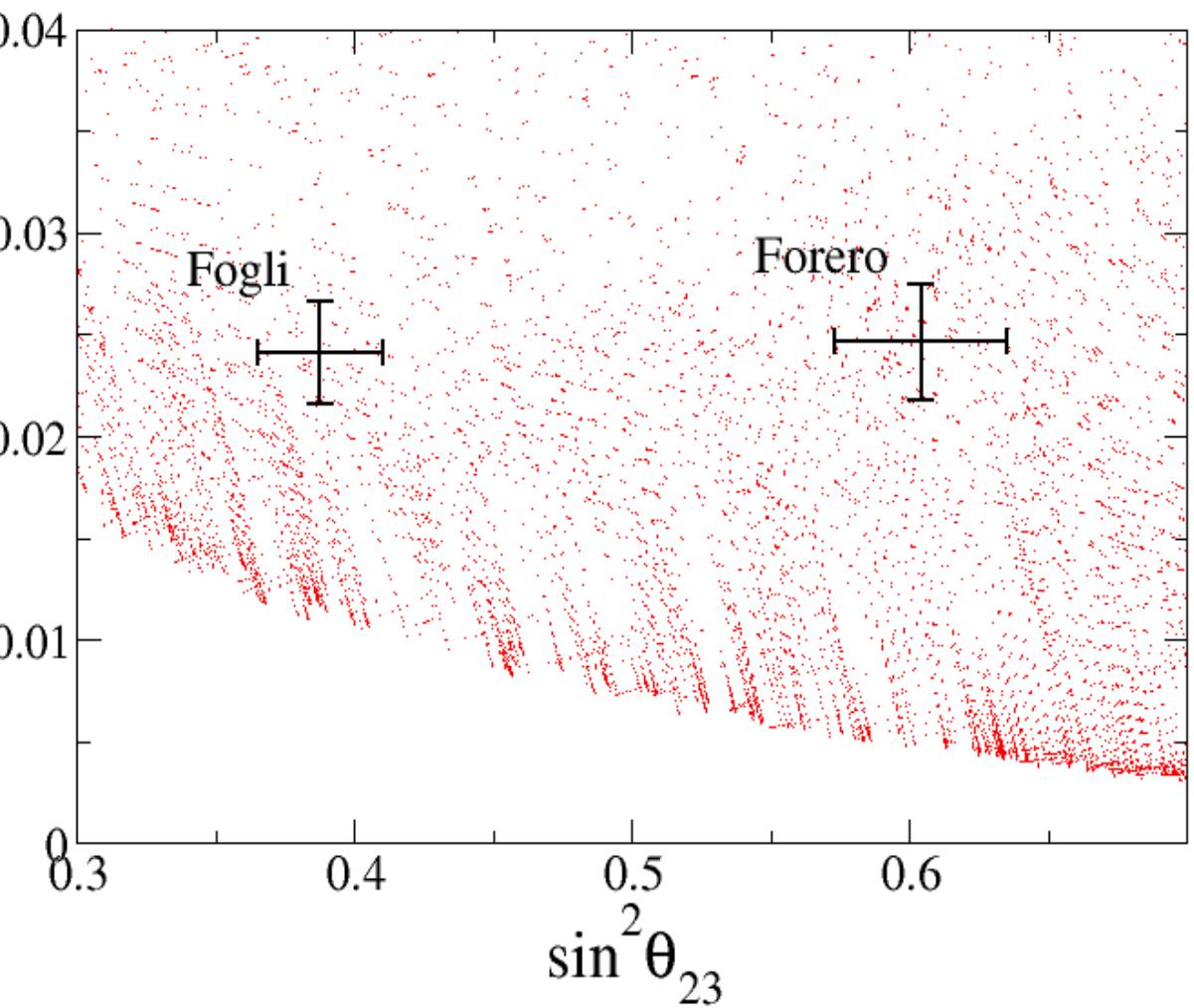
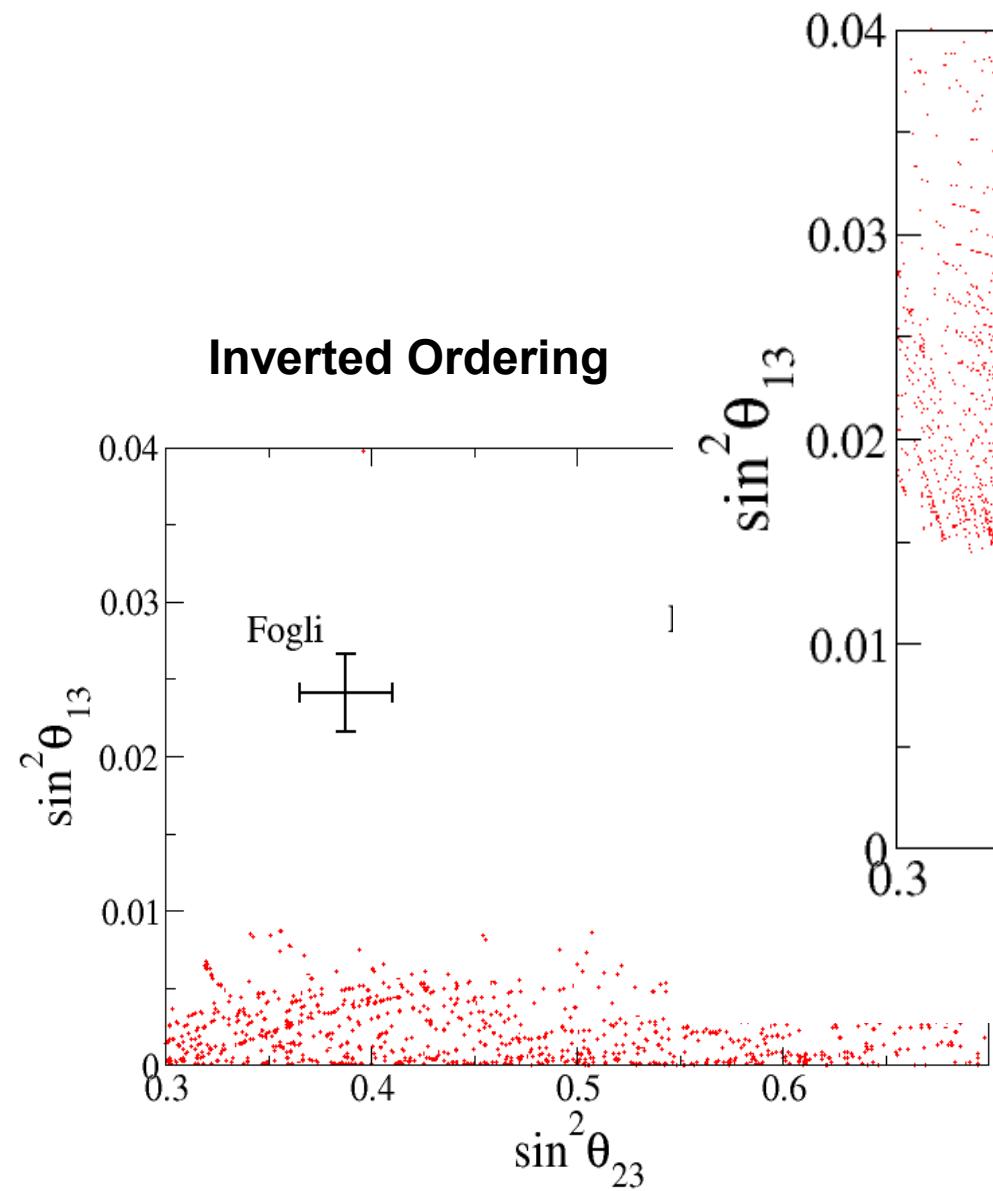
Note
 $\langle D \rangle \equiv (d_1, d_2)$
 $\langle S_i \rangle \equiv s_i$

$$\frac{1}{\Lambda^2} \begin{pmatrix} 0 & y_1(d_1^2 - d_2^2) & 2y_1 d_1 d_2 \\ 0 & 2y_2 d_1 d_2 & -y_2(d_1^2 - d_2^2) \\ 0 & y_3 d_2 s_2 & y_3 d_1 s_2 \end{pmatrix} + \frac{1}{\Lambda^4} \begin{pmatrix} y_{11} & 0 & 0 \\ y_{21} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Model - II

Normal Ordering

Inverted Ordering



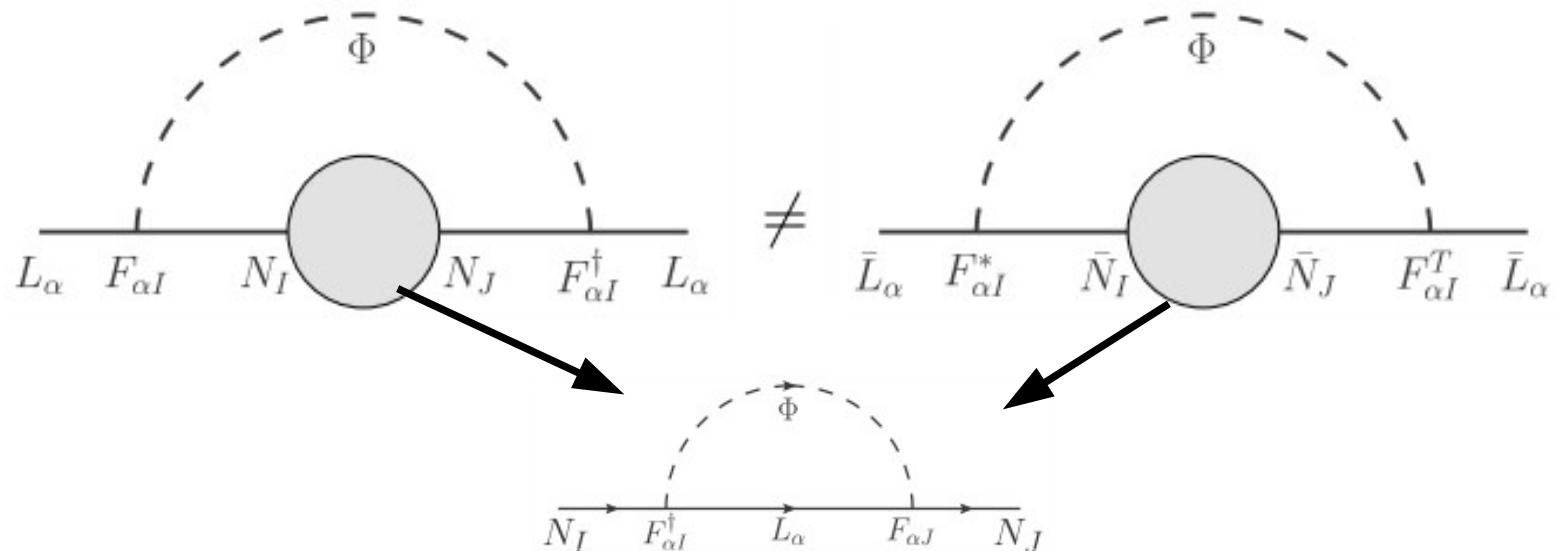
Summary

- The nMSM is one of the most attractive candidates for physics beyond the SM; it may be able to address the observed DM abundance, tiny neutrino masses and the BAU **at the same time**.
- Strong hierarchies and tunings among parameters, however, are necessary; the nMSM cannot explain these issues.
- We have proposed an extension of the nMSM by imposing flavor symmetries and succeeded in explaining some of them.
- Unfortunately, the tight fine-tuning still remains, so far no idea...

Backup Slides

The vMSM ~BAU~

Lepton asymmetry via CP violating N2-N3 oscillations:



$$\Delta L_\alpha = \frac{3^{\frac{2}{3}}\pi^{\frac{3}{2}}\sin^2\phi}{18\Gamma\left[\frac{5}{6}\right]} \frac{M_0^{\frac{4}{3}}}{(\Delta M_{32}^2)^{\frac{2}{3}}} A_{32}^\alpha \quad A_{32}^\alpha = \text{Im} \left[F_{\alpha I} [F^\dagger F]_{IJ} F_{\alpha J}^\dagger \right] \neq 0$$

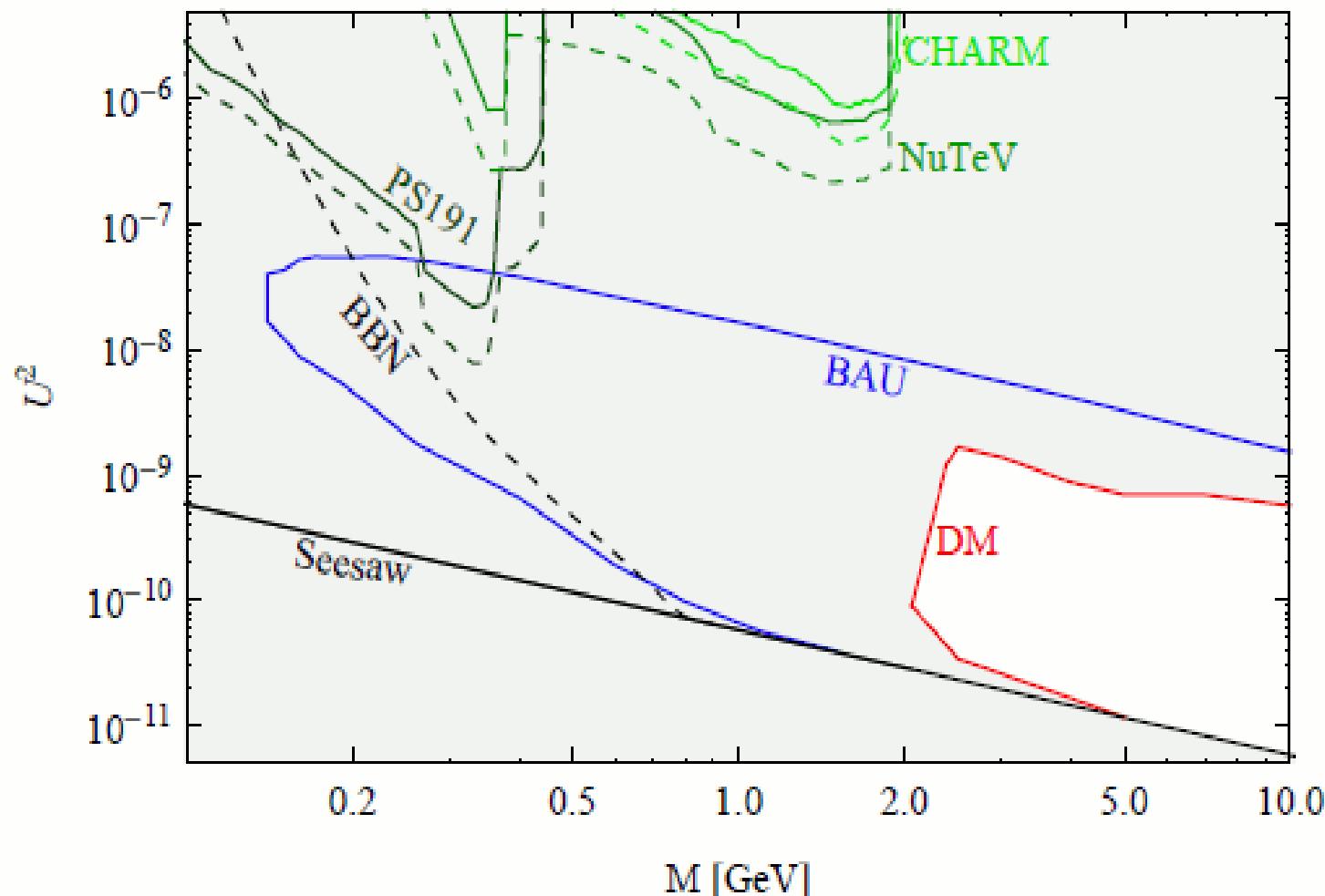
it was converted into baryon asymmetry via Sphalerons:

$$\Delta B = -\frac{28}{79} \sum_\alpha \Delta L_\alpha$$

The vMSM ~combined~

Combine all the constraints.

[Canetti etal, arXiv: 1208.4607 [hep-ph]]



Onbb

