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Vacuum Bubbles : revisited

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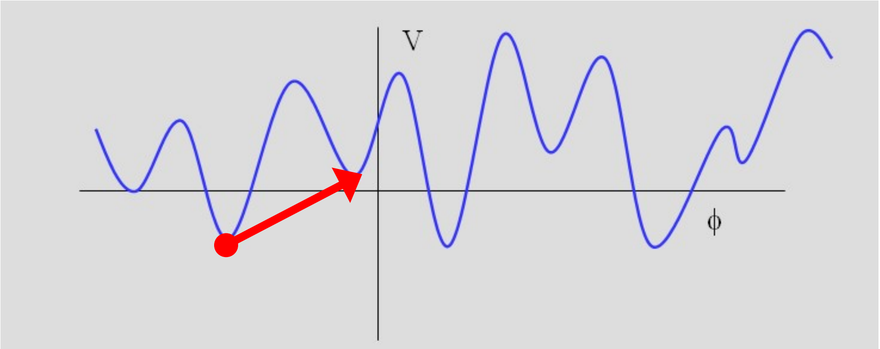
Contents

1. Motivations

◆ Observation : Our universe is expanding with acceleration
– needs positive cosmological constant

◆ The string theory landscape provides a vast number of metastable vacua.



◆ How can we be in the vacuum with positive cosmological constant ?

– KKLT(Kachru,Kallosh,Linde,Trivedi, PRD 2003))
– an alternative way ?
: Revisit the gravity effect in cosmological phase transitions

(*) In the early universe with or without cosmological constants,
Can we obtain the mechanism for the nucleation of a false vacuum bubble?
Can a false vacuum bubble expand within the true vacuum background?

(*) Or, the domain wall universe with charged BHs in the bulk spacetime?
(describe the dynamics of a domain wall universe with the charged BH.)

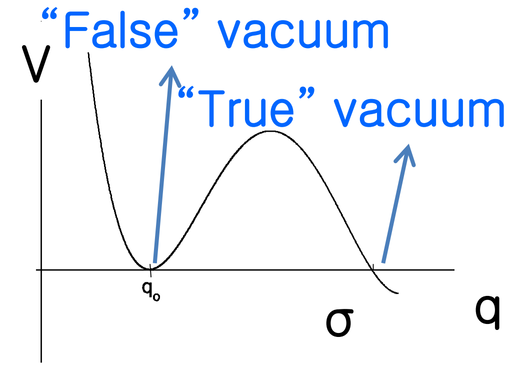
We will study and classify vacuum bubbles in various setup.

Basics : Bubble formation

– Nonperturbative Quantum Tunneling

Vacuum-to-vacuum phase transition rate

$$\Gamma / V = A e^{-B/\hbar}[1+\mathrm{O}(\hbar)]$$



B : Euclidean Action (semiclassical approx.)

S. Coleman, PRD 15, 2929 (1977)

S. Coleman and F. De Luccia, PRD21, 3305 (1980)

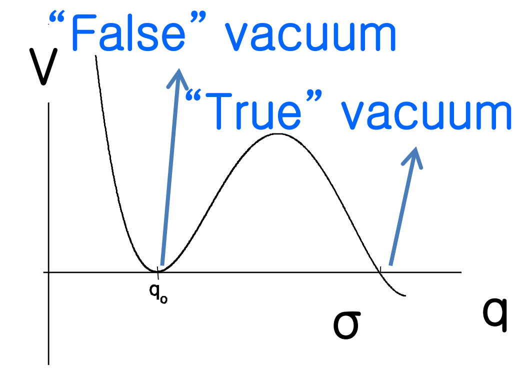
S. Parke, PLB121, 313 (1983)

A : determinant factor from the quantum correction

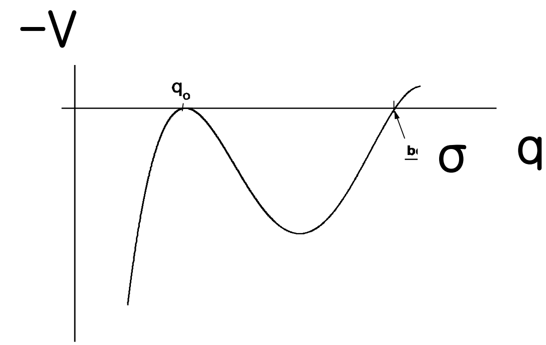
C. G. Callan and S. Coleman, PRD 16, 1762 (1977)

(1) Tunneling in Quantum Mechanics
- particle in one dim. with unit mass

-Lagrangian $L = \frac{1}{2}\left(\frac{dq}{dt}\right)^2 - V(q)$



● Quantum Tunneling:(Euclidean time $-\infty < \tau < 0$)
The particle (at "false vacuum" q_0) penetrates the potential barrier and materializes at the escape point, σ , with zero kinetic energy,



Tunneling probability

$$\Gamma / V = A e^{-B/\hbar}[1 + O(\hbar)]$$

where $\bar{S} = B = \int d\tau L_E(q(\tau)) = \int d\tau \left[\frac{1}{2}\left(\frac{dq}{d\tau}\right)^2 + V(q)\right]$ = Classical Euclidean action (difference)

Eq. of motion : $\tau = it$

$$\frac{d^2 q}{d\tau^2} + \left(-\frac{dV}{dq}\right) = 0$$

boundary conditions

$$x_i = x_f = 0 \qquad \lim_{\tau \to -\infty} q(\tau) = q_o, \frac{dq}{d\eta}\Big|_{\tau=0(\sigma)} = 0$$

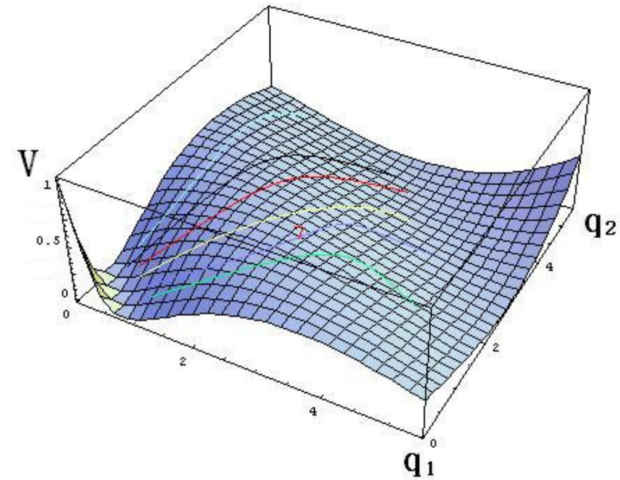
→ The bounce solution is a particle moving in the potential −V in time τ
is unstable (exists a mode w/ negative eigenvalue)

● Time evolution after tunneling : (back to Minkowski time, t > 0)
Classical Propagation after tunneling (at τ = 0)

(2) Tunneling in multidimension

Lagrangian

$$L = \frac{1}{2}\sum_{ij} \dot{q}_i \, \dot{q}_j - V(q) \qquad V(q_{oi}) = V(\sigma_i) = 0$$



The leading approx. to the tunneling rate is obtained from the path and endpoints σ_i that minimize the tunneling exponent B.

$$B = 2\int_{S1}^{S2} ds (2V)^{1/2} = \int_{-\infty}^{\infty} d\tau L_E = S_E$$

$$\delta \int d\tau L_E^b = 0 \qquad L_E = \frac{1}{2}\frac{d\vec{q}}{d\tau} \bullet \frac{d\vec{q}}{d\tau} + V \quad \Longrightarrow \quad \frac{d^2\vec{q}}{d\tau^2} = \frac{\partial V}{\partial \vec{q}}$$

Boundary conditions for the bounce

$$\lim_{\tau \to -\infty} q_i(\eta) = q_{oi}, \frac{dq_i}{d\tau}\Big|_{\tau=0(\sigma_i)} = 0$$

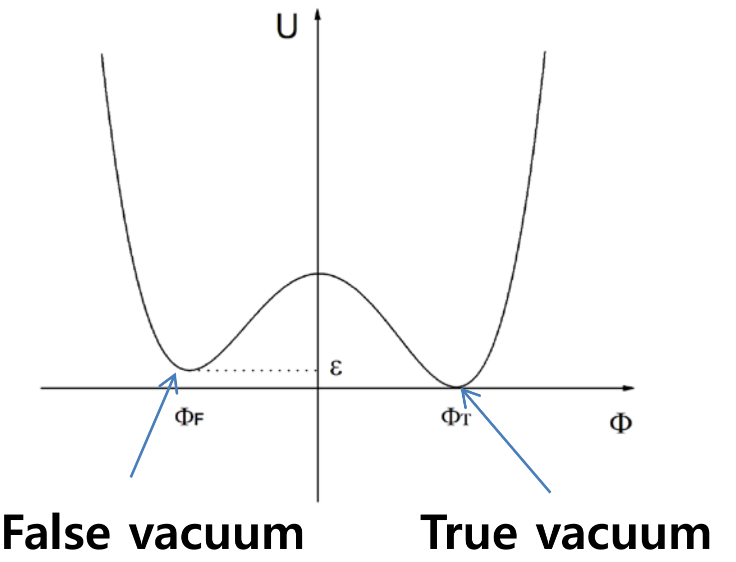
Time evolution after the tunneling is classical with the ordinary Minkowski time.

(3) Tunneling in field theory (in flat spacetime)

Theory with single scalar field

$$S = \int \sqrt{-\eta}\, d^4x \left[-\frac{1}{2} \nabla^{\alpha}\Phi \nabla_{\alpha}\Phi - U(\Phi) \right]$$

where $\eta = \det \eta_{\mu\nu}, \quad (-,+,+,+)$



Equation for the bounce from $\delta \int d\tau L_E^b = 0$

$$\left(\frac{\partial^2}{\partial \tau^2} + \nabla^2 \right) \Phi = U'(\Phi)$$

with boundary conditions

(Finite size true vacuum bubble)

$$\lim_{\tau \to \pm\infty} \Phi(\tau, \vec{x}) = \Phi_F \qquad \lim_{|\vec{x}| \to \infty} \Phi(\tau, \vec{x}) = \Phi_F \qquad \frac{\partial \Phi}{\partial \tau}(0, \vec{x}) = 0$$

Tunneling rate :

$$B = S_E = \int d\tau d^3x \left[\frac{1}{2} \left(\frac{\partial \Phi}{\partial \tau} \right)^2 + \underbrace{\frac{1}{2} (\vec{\nabla} \Phi)^2 + U(\Phi)}_{V} \right]$$

O(4)-symmetry : Rotationally invariant Euclidean metric

$$ds^2 = d\eta^2 + \eta^2[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2)] \qquad (\eta^2 = \tau^2 + r^2)$$

Tunneling probability factor

$$B = S_E = 2\pi^2 \int_0^\infty \eta^3 d\eta \left[\frac{1}{2}\left(\frac{\partial\Phi}{\partial\eta}\right)^2 + U(\Phi) \right]$$

The Euclidean field equations

$$\Phi'' + \frac{3\eta'}{\eta}\Phi' = \frac{dU}{d\Phi}$$

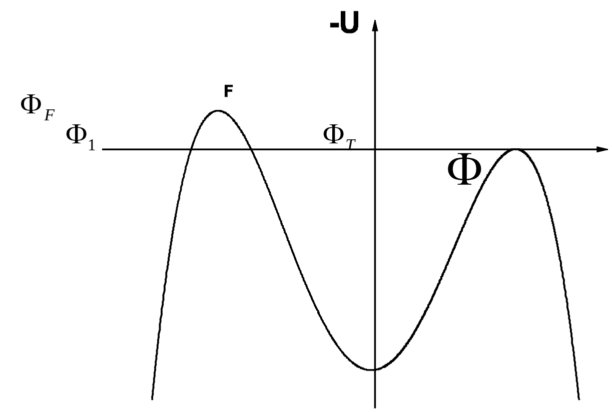
boundary conditions

$$\lim_{\eta\to\infty}\Phi(\eta) = \Phi_F, \frac{d\Phi}{d\eta}|_{\eta=0} = 0$$

"Particle" Analogy :

The motion of a particle located at position phi at time eta

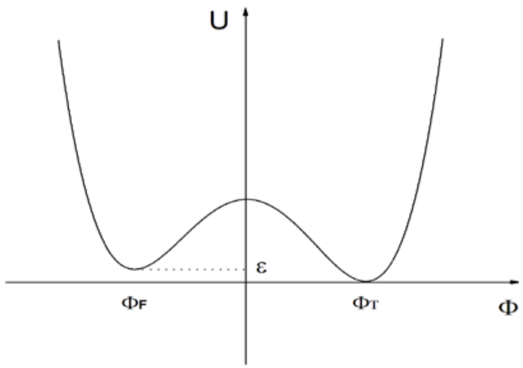
"Particle" moving in the potential –U, with the damping force inversely proportional to time

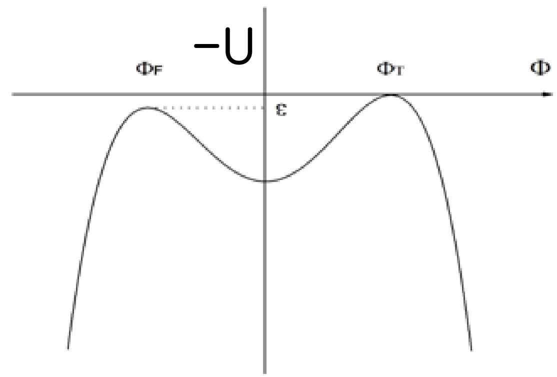


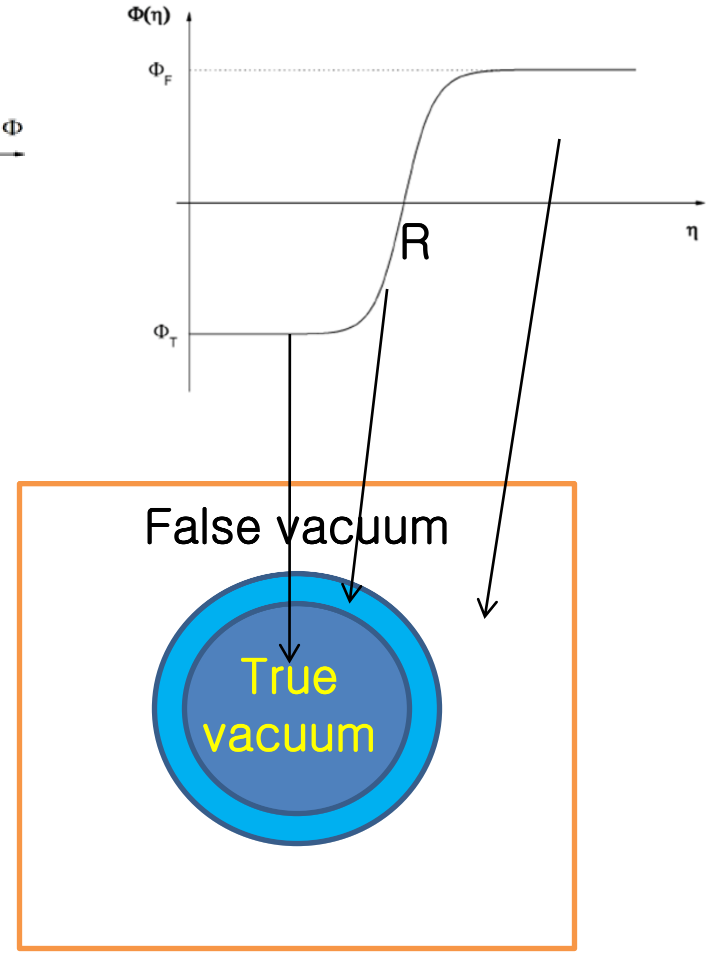
At time 0, the particle is released at rest
(The initial position should be chosen such that)
at time infinity, the particle will come to rest at Φ_F.

Thin-wall approximation

potential $$U(\Phi) = \frac{\lambda}{8}(\Phi^2 - b^2)^2 - \frac{\varepsilon}{2b}(\Phi - b)$$







(Epsilon : small parameter)

B is the difference $B = S_E^b - S_E^F$

In this approximation

$$B = B_{in} + B_{wall} + B_{out}$$

Outside the wall

$$B_{out} = S_E(\Phi_F) - S_E(\Phi_F) = 0$$

Large 4dim. spherical bubble
with radius R and thin wall

In the wall

$$B_{wall} = 2\pi^2\eta^3 S_o,$$

where $S_o = \int_{\Phi_T}^{\Phi_F} \sqrt{2[U(\Phi) - U(\Phi_T)]}\, d\Phi$

inside the wall $B_{in} = -\frac{1}{2}\pi^2\eta^4\varepsilon$

the radius of a true vacuum bubble

$$\eta = 3S_o / \varepsilon$$

the nucleation rate of a true vacuum bubble

$$B_o = S_E = \frac{27\pi^2 S_o^4}{2\varepsilon^3}$$

Evolution of the bubble

The false vacuum makes a quantum tunneling into a true vacuum bubble at time t = 0, described by

$$\Phi(t=0,\vec{x}) = \Phi(\tau=0,\vec{x})$$

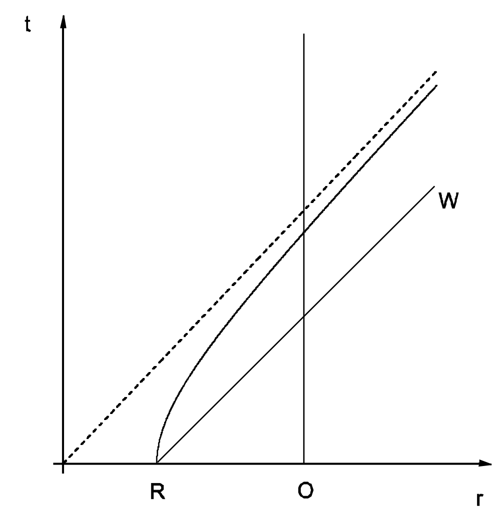
$$\frac{\partial}{\partial t}\Phi(t=0,\vec{x}) = 0.$$

Note that Φ is a function of η

$$(\eta^2 = \tau^2 + r^2)$$

Afterwards, it evolves according to the classical equation of motion

$$-\frac{\partial^2\Phi}{\partial^2 t} + \nabla^2\Phi = U'(\Phi)$$



The solution (by analytic continuation)

$$\Phi(t,\vec{x}) = \Phi(\eta = (|\vec{x}|^2 - t^2)^{1/2})$$

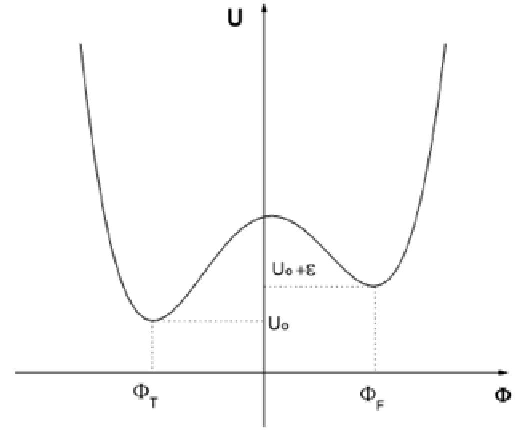
Note : 1) Analytic continuation in the presence of gravity is nontrivial. 2) The evolution of the (bubble or domain) wall can also be studied using the junction conditions

2. Bubble nucleation in the Einstein gravity

S. Coleman and F. De Luccia, PRD21, 3305 (1980)

Action

$$S = \int \sqrt{g}\, d^4x \left[\frac{R}{2\kappa} - \frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi - U(\Phi) \right] + S_{boundary}$$



Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \qquad T_{\mu\upsilon} = \left[\nabla_\mu \Phi \nabla_\upsilon \Phi - g_{\mu\upsilon} \left(\frac{1}{2} \nabla^\alpha \Phi \nabla_\alpha \Phi + U \right) \right]$$

Bubble nucleation rate

$$\Gamma / V = A \exp[-B/\hbar]$$

O(4)-symmetric Ansatz : Rotationally invariant Euclidean metric

$$ds^2 = d\eta^2 + \rho^2(\eta)[d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)]$$

The Euclidean field equations

$$\Phi'' + \frac{3\rho'}{\rho} \Phi' = \frac{dU}{d\Phi} \qquad \rho'^2 = 1 + \frac{\kappa \rho^2}{3} \left(\frac{1}{2} \Phi'^2 - U \right)$$

(Scalar eq. of motion & Einstein eq.)

boundary conditions

$$\lim_{\eta \to \eta(\max)} \Phi(\eta) = \Phi_F, \quad \frac{d\Phi}{d\eta} \Big|_{\eta=0} = 0$$

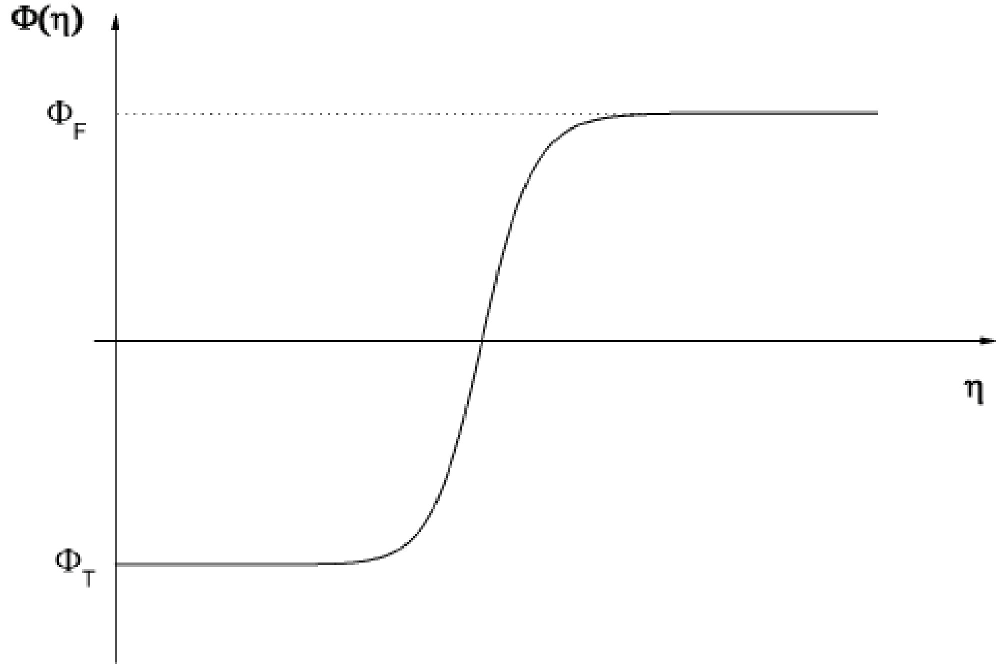


FIG. 2: The typical vacuum bubble profile with the wall in the middle. At $\eta = 0$ the starting point is somewhere near Φ_T, stays there for a time and goes to Φ_F and stays at that point from thereafter.

(i) From de Sitter to flat spacetime

the radius of a true vacuum bubble $\bar{\rho} = \dfrac{\bar{\eta}}{1+(\bar{\eta}/2\Lambda)^2}$

where $\Lambda = (\kappa\varepsilon/3)^{-1/2}$

the nucleation rate of a true vacuum bubble $B = \dfrac{B_o}{[1+(\bar{\eta}/2\Lambda)^2]^2}$

Note : 1) ρ bar less than η and less than or equal to Λ,
2) Transition probability increases. ($B < B_0$)

(ii) From flat to Anti-de Sitter spacetime

the radius of a true vacuum bubble $\bar{\rho} = \dfrac{\bar{\eta}}{1-(\bar{\eta}/2\Lambda)^2}$

the nucleation rate of a true vacuum bubble $B = \dfrac{B_o}{[1-(\bar{\eta}/2\Lambda)^2]^2}$

Note : ρ bar larger than η . Transition probability decreases. ($B > B_0$)
For small enough ϵ, false vacuum can be stable

(iii) the case of arbitrary vacuum energy
S. Parke, PLB121, 313 (1983)

$$\rho^2{}_p = \frac{\bar{\eta}^2}{\left[1+2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2+\left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right]}$$

$$\lambda_1^2 = [3/\kappa(U_F - U_T)]$$

$$\lambda_2^2 = [3/\kappa(U_F + U_T)]$$

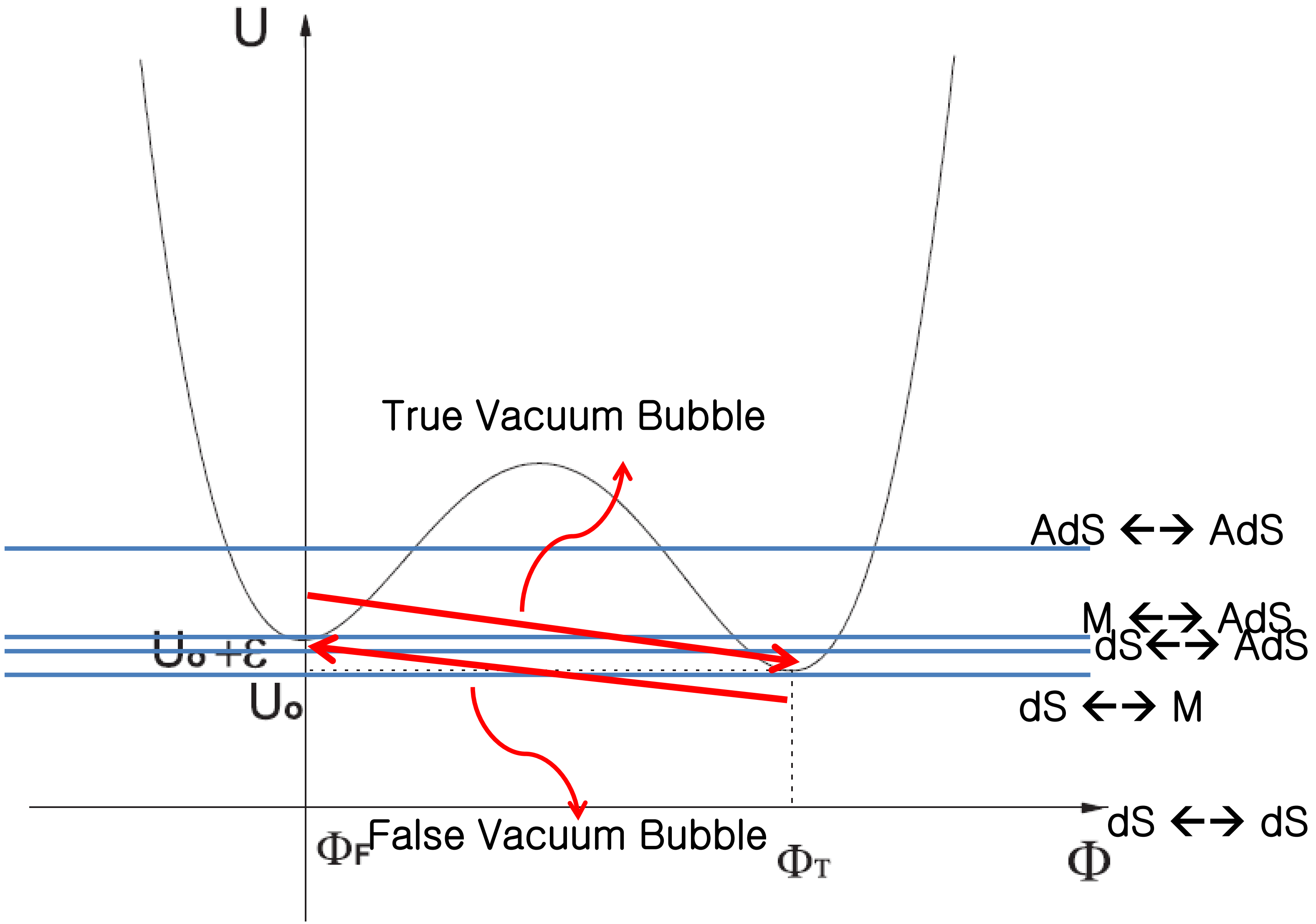
$$B_p = \frac{2B_o\left[\left\{1+\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2\right\}-\left\{1+2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2+\left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right\}^{1/2}\right]}{\left[\left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\left\{\left(\frac{\lambda_2}{\lambda_1}\right)^2-1\right\}\left\{1+2\left(\frac{\bar{\eta}}{2\lambda_1}\right)^2+\left(\frac{\bar{\eta}}{2\lambda_2}\right)^4\right\}^{1/2}\right]}$$

•Evolution of the bubble

→ via analytic continuation back to Lorentzian time

Ex) de Sitter -> de Sitter : A. Brown & E. Weinberg, PRD 2007

Cases for True & False Vacuum Bubble :



	False-to-true	True-to-false
De Sitter – De Sitter	O	O
De Sitter – Flat	O	?
De Sitter – Anti-de Sitter	O	?
Flat – Anti-de Sitter	O	?
Anti-de Sitter – Anti-de Sitter	O	?

True & False Vacuum Bubbles

(*)Lee, Weinberg, PRD

	False-to-true (True vac. Bubble)	True-to-false (*) **(False vac. Bubble)**
De Sitter – de Sitter	O	O (*)
Flat – de Sitter	O	O
Anti de Sitter – de Sitter	O	O
Anti de Sitter – flat	O	O
Anti de Sitter – Anti de Sitter	O	O

(*) exists in

(1)non-minimally coupled gravity

(W.Lee, BHL, C.H.Lee,C.Park, PRD(2006))

or in

(2)Brans-Dicke type theory

(H.Kim,BHL,W.Lee, Y.J. Lee, D.-H.Yoem, PRD(2011))

$$S = \int \sqrt{g} d^4x \left[\frac{R}{2\kappa} - \frac{1}{2}\nabla^\alpha \Phi \nabla_\alpha \Phi - \frac{1}{2}\xi R \Phi^2 - U(\Phi) \right] + S_b,$$

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' - \xi R_E \Phi = \frac{dU}{d\Phi},$$

$$\rho'^2 = 1 + \frac{\kappa \rho^2}{3(1 - \xi \Phi^2 \kappa)} \left(\frac{1}{2}\Phi'^2 - U \right),$$

Condition: $\xi R_E \Phi > \frac{3\rho'}{\rho}\Phi'.$

Anti de Sitter – de Sitter

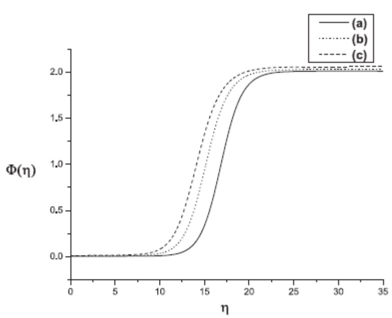


FIG. 8. The false vacuum bubble profiles for several values of $\tilde{\epsilon}$ and ξ in case 3. Here ξ is taken to be positive. The three curves are (a) solid curve: $\tilde{\epsilon} = 0.01$ and $\xi \simeq 0.328$; (b) dotted curve: $\tilde{\epsilon} = 0.02$ and $\xi \simeq 0.414$; (c) dashed curve: $\tilde{\epsilon} = 0.03$ and $\xi \simeq 0.508$.

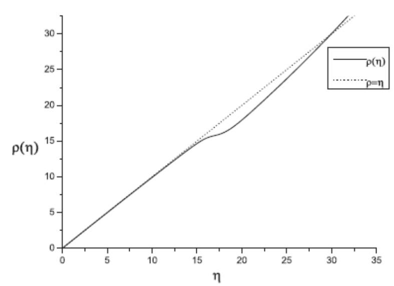


FIG. 9. The evolution of $\tilde{\rho}(\tilde{\eta})$ in case 3. The solid curve is the solution of $\tilde{\rho}$ with $\tilde{\epsilon} = 0.01$. In the region inside the bubble, $\rho = \Lambda \sin\frac{\eta}{\Lambda}$, and outside the bubble, $\rho = \Lambda_2 \sinh\frac{\eta}{\Lambda_2}$.

Dynamics of False Vacuum Bubble :

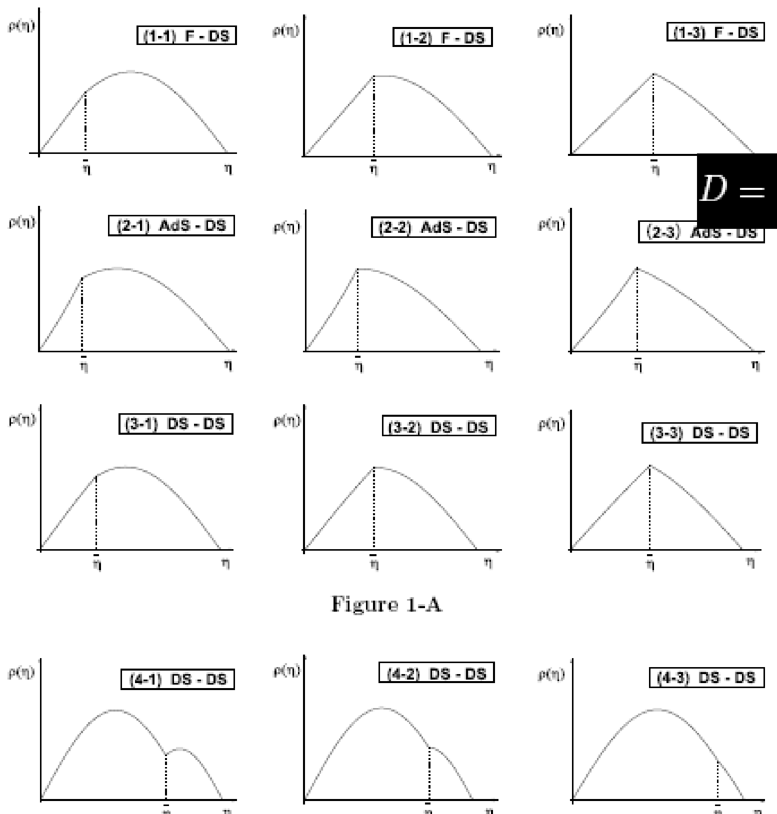
Can exists an expanding false vacuum bubble inside the true vacuum

BHL, C.H.Lee, W.Lee, S. Nam, C.Park, PRD(2008) (for nonminimal coupling)

BHL, W.Lee, D.-H. Yeom, JCAP(2011) (for Brans-Dicke)

2.1 True vacuum bubbles (with the de Sitter (dS) exterior geometry)

$U_F > 0$



BHL & W. Lee, CQG (2009)

Figure 1: The schematic diagram for 12 possible types of true vacuum bubbles or matching with the thin-wall approximation. The $\bar{\eta}$ indicates the location of the wall. All the nine cases in Figure 1-A are possible solutions. The cases (4-1) - (4-3) in Figure 1-B don't have the stationary point in the action, allowing no solutions.

Large background (Parke): $U_F - U_T > 3\kappa S_o^2/4.$

$$\bar{\rho}^2 = \frac{\bar{\rho}_o^2}{D}, \quad B = \frac{2B_o[\{1+(\frac{\bar{\rho}_o}{2\lambda_1})^2\} - D^{1/2}]}{[(\frac{\bar{\rho}_o}{2\lambda_2})^4\{(\frac{\lambda_2}{\lambda_1})^4 - 1\}D^{1/2}]},$$

$$D = \left[1 + 2(\frac{\bar{\rho}_o}{2\lambda_1})^2 + (\frac{\bar{\rho}_o}{2\lambda_2})^4\right], \ \lambda_1^2 = [3/\kappa(U_F + U_T)] \text{ and } \lambda_2^2 = [3/\kappa(U_F - U_T)]$$

$$\bar{\rho}_o = 3S_o/(U_F - U_T)$$

Half background : $U_F - U_T = 3\kappa S_o^2/4.$

$$\bar{\rho} = \frac{2}{\kappa\sqrt{S_o^2 + \frac{4}{3\kappa}U_T}}, \quad B = \frac{2B_o}{(\frac{\bar{\rho}_o}{2\lambda_2})^4[(\frac{\lambda_2}{\lambda_1})^4 - 1]D^{1/2}}\left[\frac{8U_F}{3\kappa S_o^2} - D^{1/2}\right]$$

$$\bar{\rho} = \rho_{max}(F) \qquad D = (S_o^2 + \frac{4}{3\kappa}U_T)(4/S_o^2)$$

Small background : $U_F - U_T < 3\kappa S_o^2/4.$

$$\bar{\rho}^2 = \frac{\bar{\rho}_o^2}{D}, \quad B = \frac{2B_o[\{1+(\frac{\bar{\rho}_o}{2\lambda_1})^2\} - D^{1/2}]}{[(\frac{\bar{\rho}_o}{2\lambda_2})^4\{(\frac{\lambda_2}{\lambda_1})^4 - 1\}D^{1/2}]},$$

the size of the background space (or bubble) will be called "large'' (or "small'') if its size is larger than half of the de Sitter space itself.

By redefining $\eta \rightarrow \eta_{max} - \eta$ we get False Vacuum Bubbles with finite geometry.

2-2. vacuum bubbles with finite geometry

BHL, C.H. Lee, W.Lee & C.Oh,

dS-dS

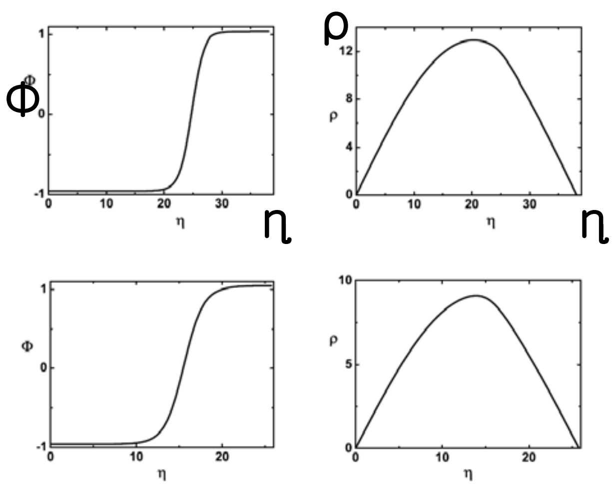


Figure 2: dS-dS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.1$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.2$, and $U_0 = 0.1$ for for bottom figure.

dS-AdS

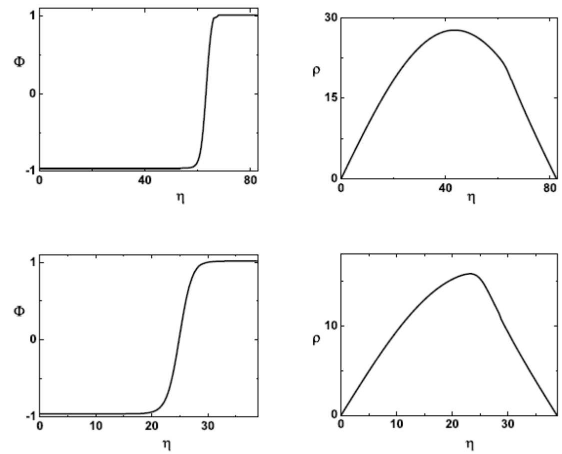


Figure 3: dS-AdS cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = -0.04$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = -0.04$ for for bottom figure.

dS-flat

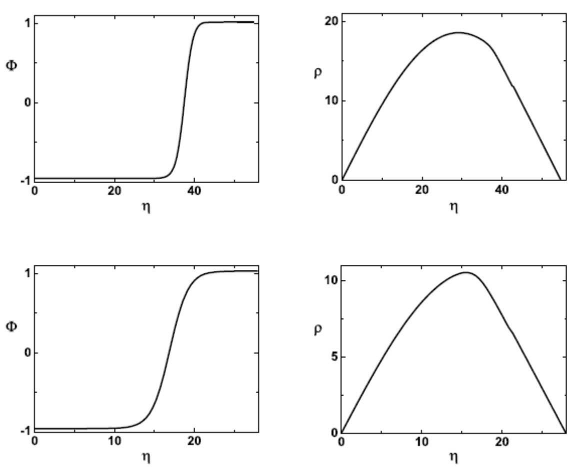


Figure 4: ds-flat cases. $\epsilon = 0.04$, $\kappa = 0.1$, and $U_0 = 0.0077$ for for top figure. $\epsilon = 0.04$, $\kappa = 0.3$, and $U_0 = 0.0077$ for for bottom figure.

flat-AdS and AdS-AdS

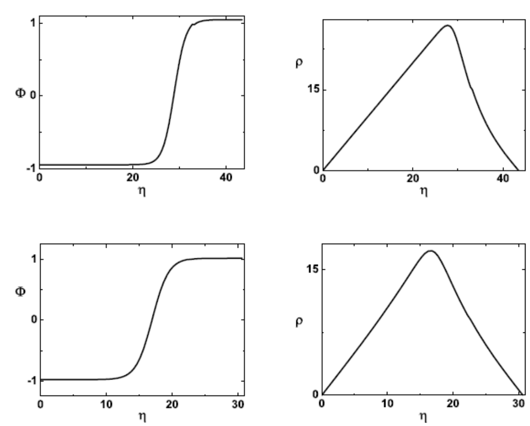


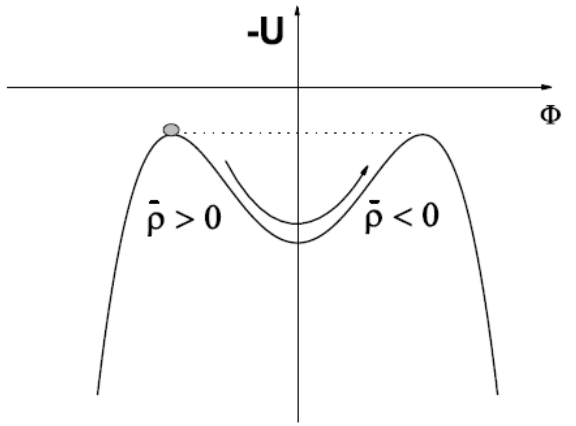
Figure 5: Flat-AdS and AdS-AdS cases. $\epsilon = 0.05$, $\kappa = 0.7$, and $U_0 = -0.09868$ for flat-AdS case. $\epsilon = 0.02$, $\kappa = 0.7$, and $U_0 = -0.05$ for AdS-AdS case.

3 Tunneling between the degenerate vacua

∃ Z2-symm. with finite geometry bubble

Boundary condition (consistent with Z2-sym.

$$\rho|_{\eta=0} = 0, \quad \rho|_{\eta=\eta_{max}} = 0, \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0, \quad \text{and} \quad \left.\frac{d\Phi}{d\eta}\right|_{\eta=\eta_{max}} = 0.$$



– in de Sitter space.

The numerical solution by Hackworth and Weinberg.

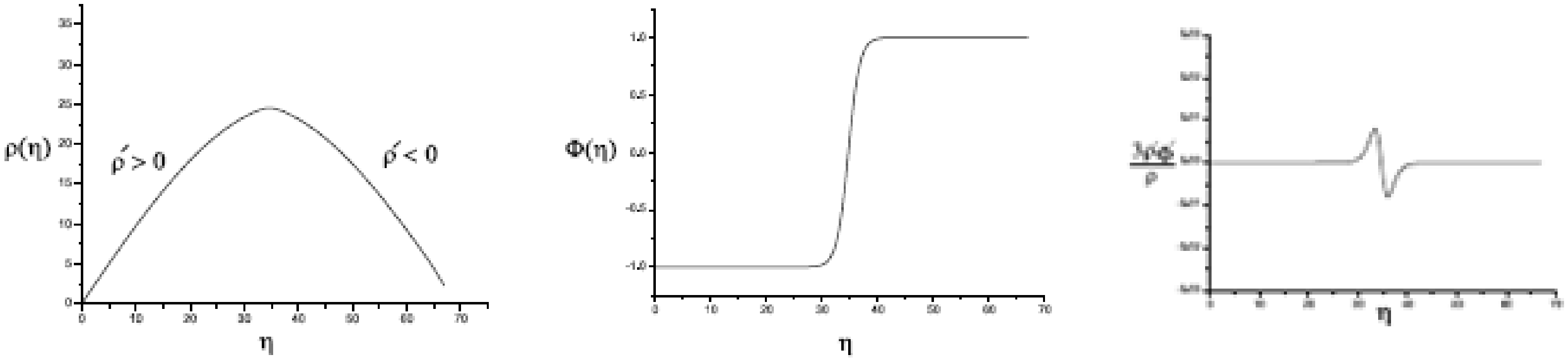
The analytic computation and interpretation :

(BHL & W. Lee, CQG (2009))

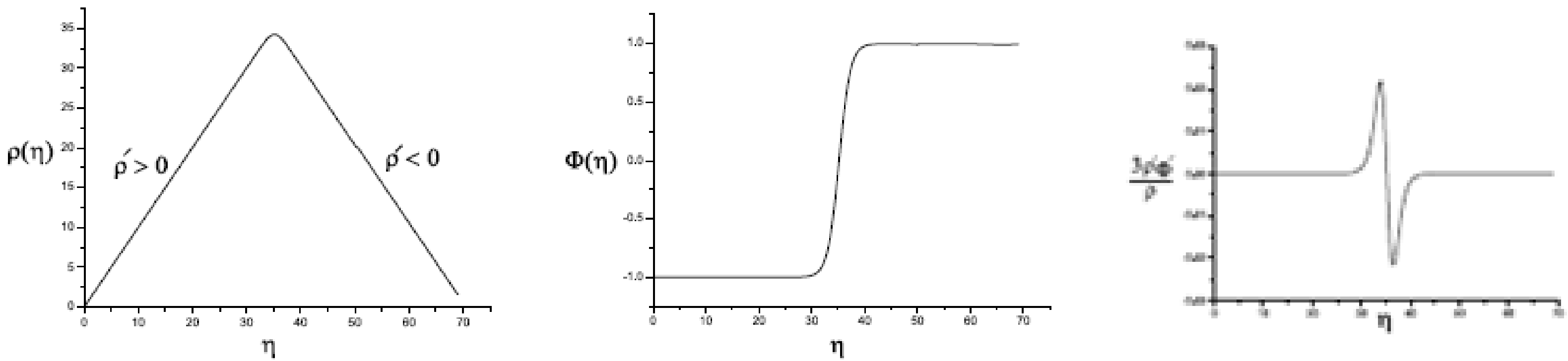
This tunneling is possible
due to the changing role of the second term in Eucildean equation from damping to accelerating during the phase transition.

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' = -\frac{d(-U)}{d\Phi}$$

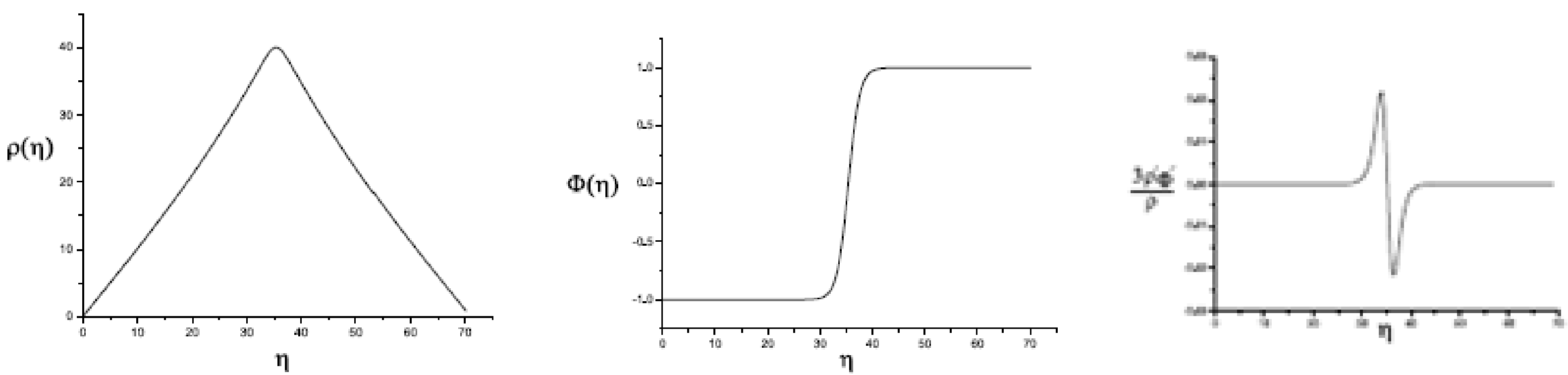
dS − dS



flat −flat



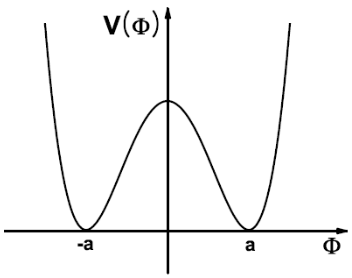
AdS−AdS



B.-H. L, C. H. Lee, W. Lee & C. Oh, PRD**82** (2010)

Euclidean Solutions

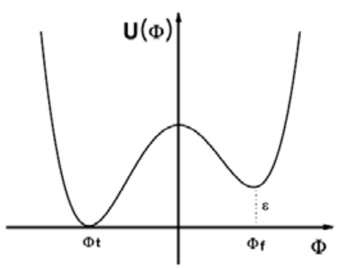
(1) Instanton solution (symmetric double-well potential)



The action has the same value as the action obtained in connection with the WKB calculation of the splitting in the energies of the two lowest levels for the double well potential.

$$E_{\pm} = \frac{\sqrt{\lambda}a}{2} \pm K e^{-S_E}$$

(2) Bounce solution (asymmetric double-well potential)

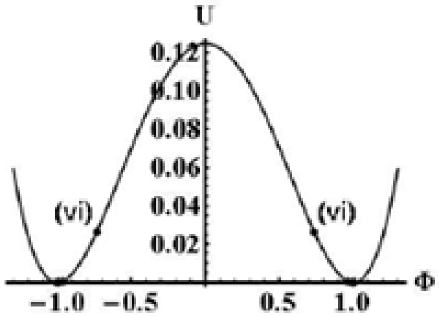


Decay of the background vacuum state or the nucleation of a vacuum bubble

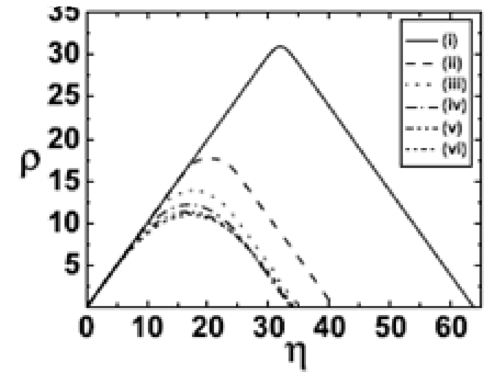
$$E_o = \frac{\sqrt{\lambda}a}{2} - \frac{i}{2}|K|e^{-S_E}$$

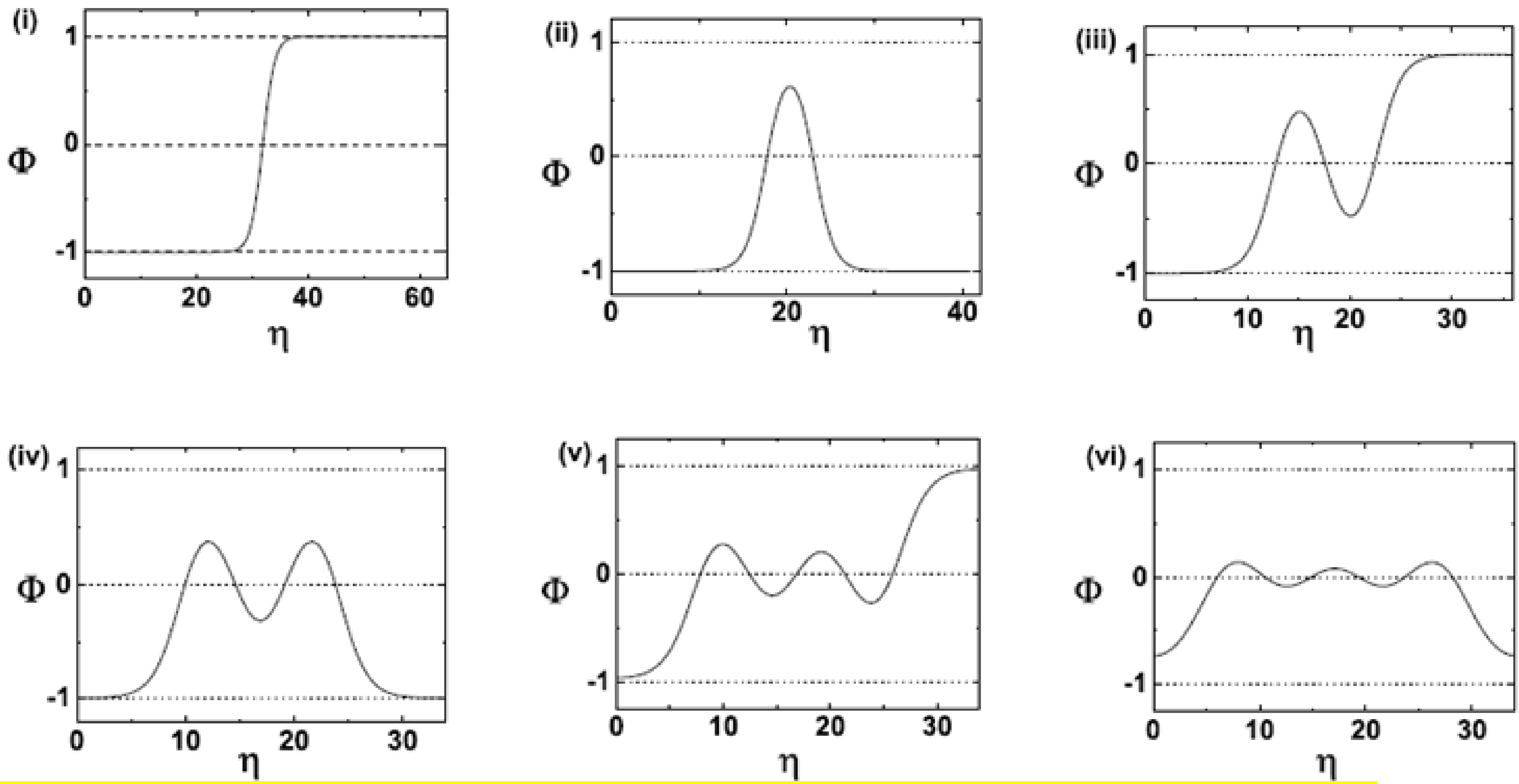
Oscillaing solutions - between flat-flat degenerate vacua ($\tilde{\kappa} = 0.2$)

B.-H. Lee, C. H. Lee, W. Lee & C. Oh, arXiv:1106.5865



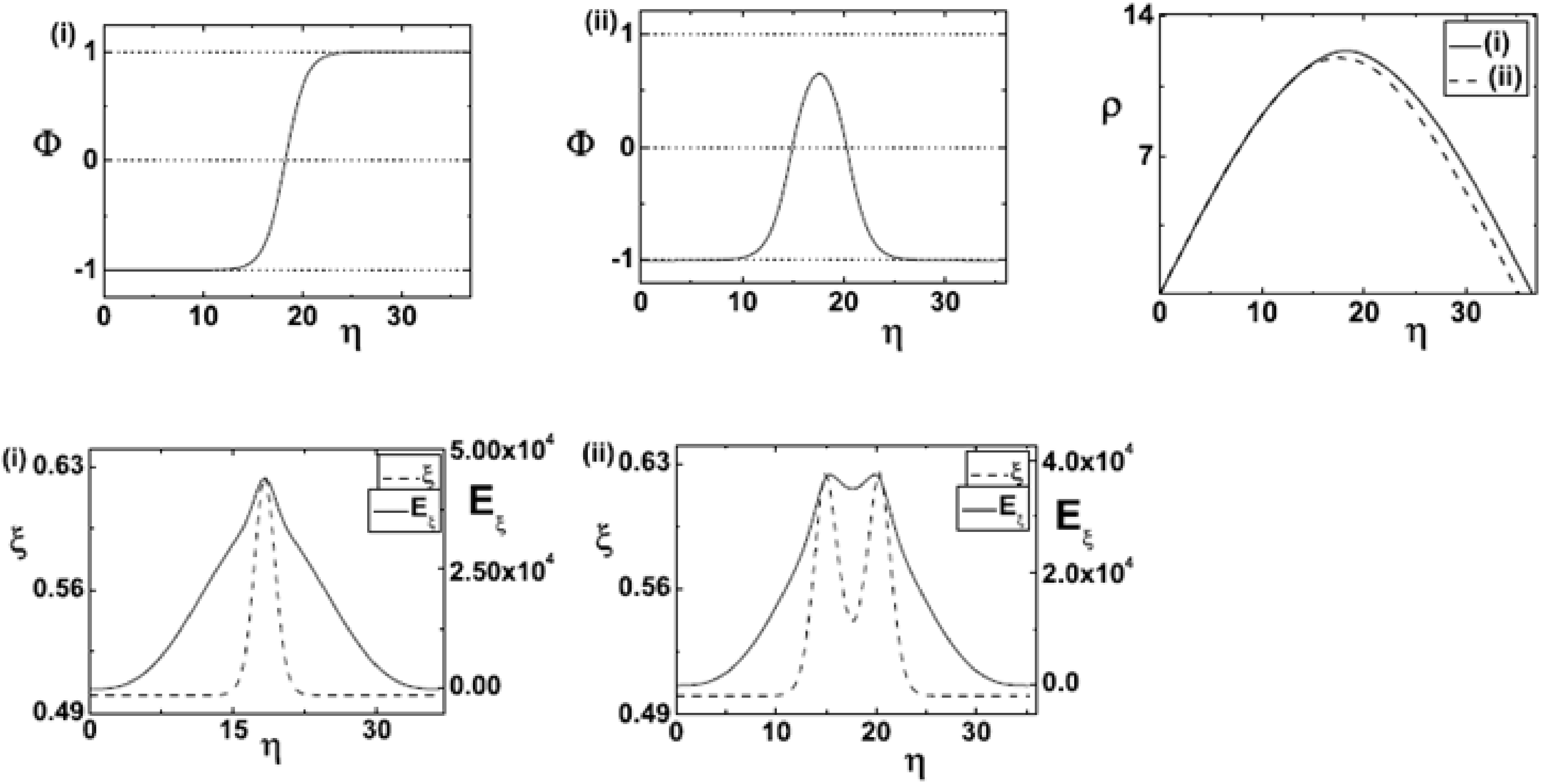
Number of Oscillation	Φ_0
1	-0.99999999999355985
2	-0.99999585754
3	-0.9995857315805
4	-0.994499
5	-0.9661682
6	-0.7348584





This type of solutions is possible only if gravity is taken into account.

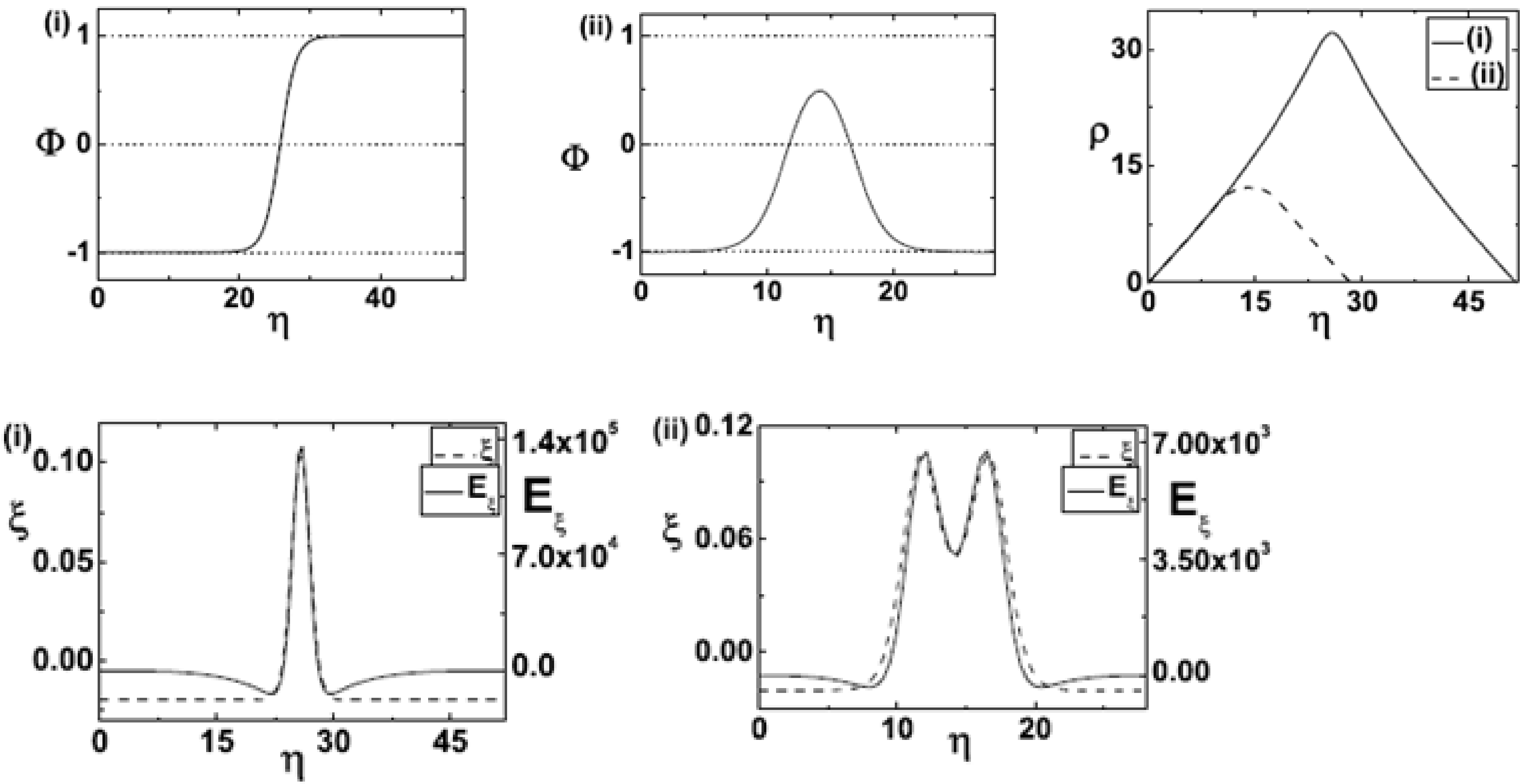
2.Oscillaing solutions - between dS-dS degenerate vacua



$$\tilde{U}_o = 0.5 \text{ and } \tilde{\kappa} = 0.04$$

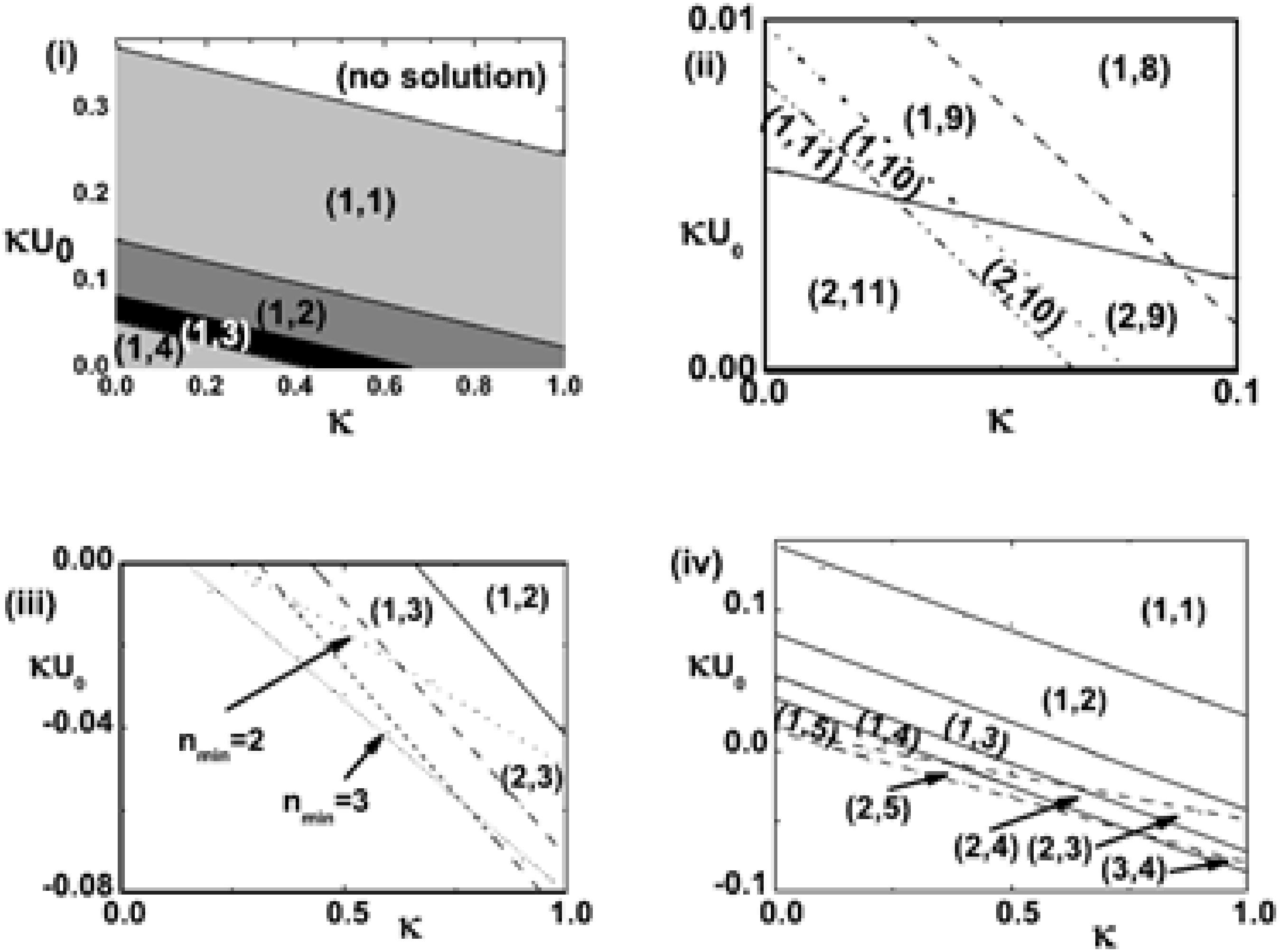
3.Oscillaing solutions - between AdS-AdS degenerate vacua

$\tilde{U}_o = -0$and $\tilde{\kappa} = 0.4$

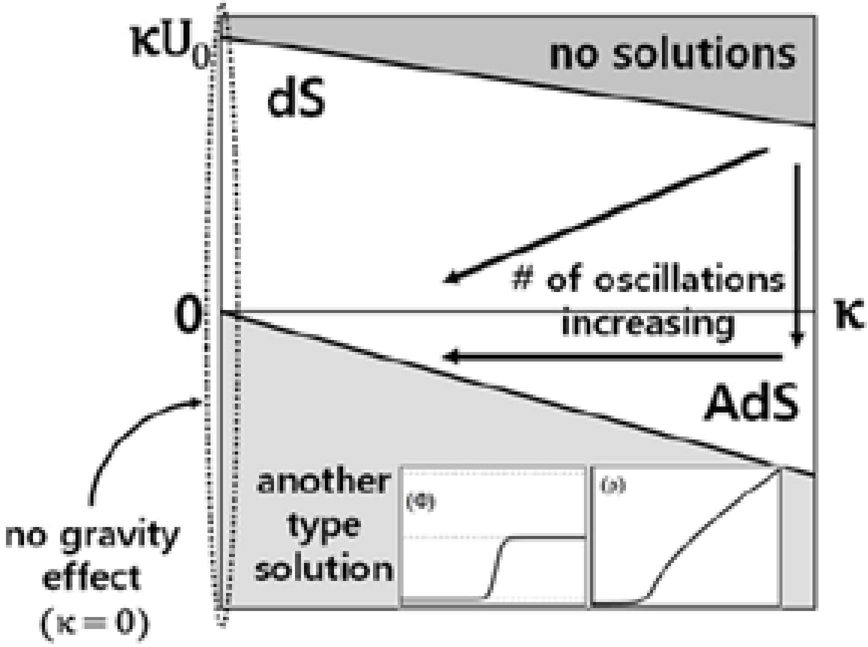


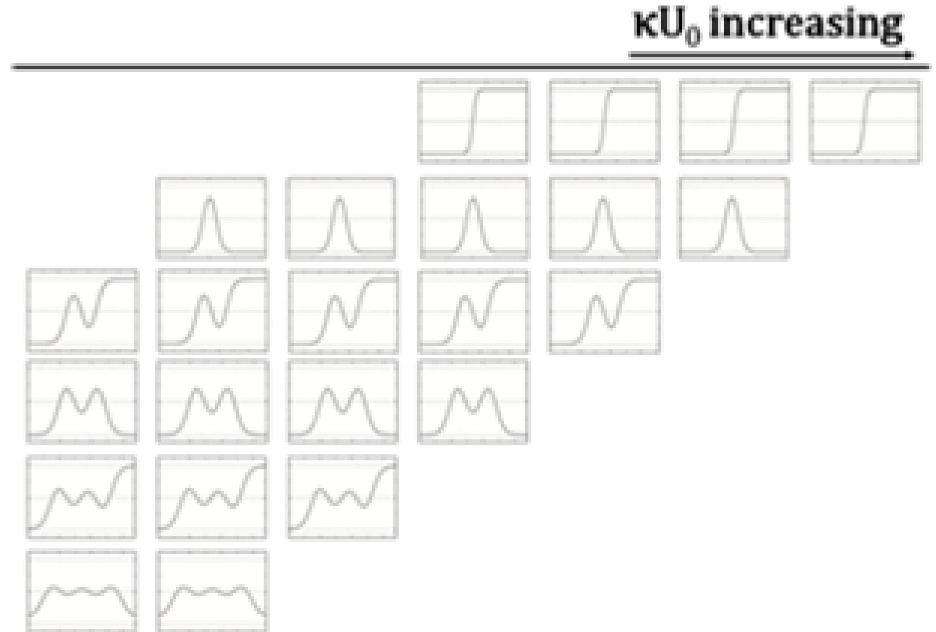
The phase space of solutions

$0 \leqq \tilde{\kappa} \leqq 1$



The y-axis represents no gravity.
the notation (n_min, n_max), where n_min means the minimum number of oscillations and n_max the maximum number of oscillation





the schematic diagram of the phase space of all solutions including another type solution and the number of oscillating solutions with different κs.

The left figure has κ = 0 line indicating no gravity effect. In the middle area including the flat case, n_min and n_max are increased as κ and κUo are decreased. The tendencies are indicated as the arrows. In the left lower region, there exist another type solution.

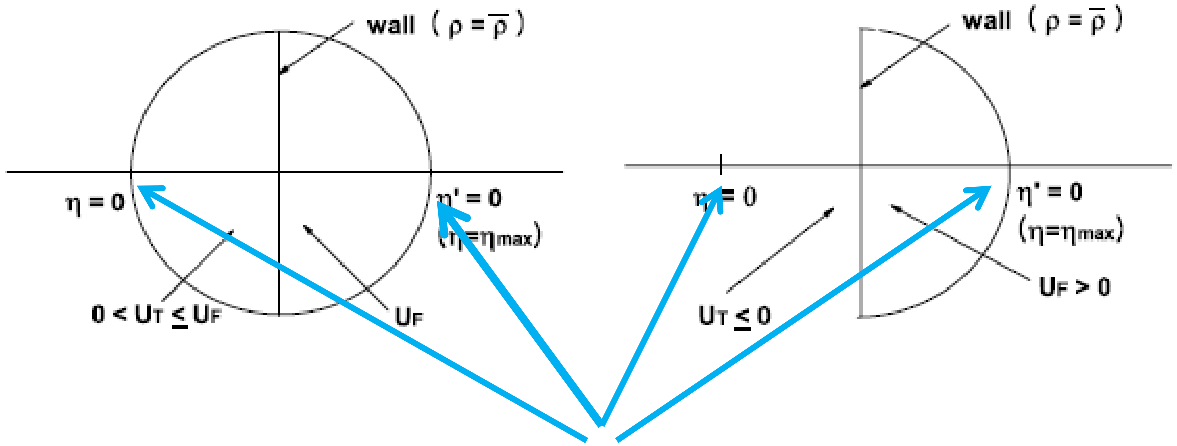
The right figure shows n_min and n_max are changed in terms of κUo and κ. As we can see from the figure, n_max and n_min are increased as κUo is decreased.

4. Black Hole Pair creation by a domain wall

Introduction

Gwak, B.-H. L., W. Lee, Minamitsuji, arXiv:1101.5748

The system with a domain wall with Z_2 symmetry by
(1) cut-and-paste method Caldwell, Chamblin and Gibbons,PRD(1996)
(2) by the instanton solution
Hackworth and E. J. Weinberg, PRD (2005);
B.-H. L. and W. Lee, CQG(2009);
B.-H. L, C. H. Lee, W. Lee, and C. Oh, PRD 82, 024019 (2010)].



the location of black holes

Application to the braneworld cosmology

After the nucleation, the domain wall (that may be interpreted as our braneworld universe) evolves in the radial direction of the bulk spacetime.

$$r = a(\tau), \quad \dot{a}^2 + V(a) = 0$$

The equation becomes

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{1}{3}\lambda + \frac{2m_*}{a^4} - \frac{q^2}{a^6},$$

$\lambda = 3A$: the effective cosmological constant.
mass term $\sim$ the radiation in the universe
charge term $\sim$ the stiff matter with a negative energy density.

Cosmological solutions

the expanding domain wall (universe) solution ($a > r*,+$).
approaching the de Sitter inflation with λ, since the contributions of the mass and charge terms are diluted.

contracting solution ($a < r*,+$) : the initially collapsing universe.
The domain wall does not run into the singularity & experiences a bounce since there is the barrier in $V(a)$ because of the charge q.

5. Summary and Discussions

- We reviewed the formulation of the bubble.
- False vacua exist e.g., in non-minimally coupled theory.
- Vacuum bubbles with finite geometry, with the radius & nucleation rate
- The tunneling of degenerate vacua in dS, flat, & AdS.
  Obtained the transition rate and the radius of a bubble.
- Exists Oscillating solutions; can make the thick domain wall.
- Similar analysis for the Fubini instanton under investigation
- Physical role and interpretation of many solutions are still not clear.
- Studied a magnetically charged BH pair separated by a domain wall in the 4 or 5-dimensional spacetime with a cosmological constant.
- The application to the braneworld cosmology has been discussed.
- Can it be another alternative model for the accelerating expanding universe?

Thank you!