Massive Scalar Field Quantum Cosmology

Sang Pyo Kim
Kunsan National University

3rd International Workshop on
Dark Matter, Dark Energy and Matter-Antimatter Asymmetry
December 29, 2012

Outline

- Motivations
  - Why Quantum Cosmology?
  - Why Massive Scalar Field Quantum Cosmology?
- Quantum Cosmology
- Quantum-Classical Transition
- Second Quantized Universes
- Third Quantization
- Conclusion

Why Quantum Cosmology?

Big Bang as an Ingredient of Cosmology

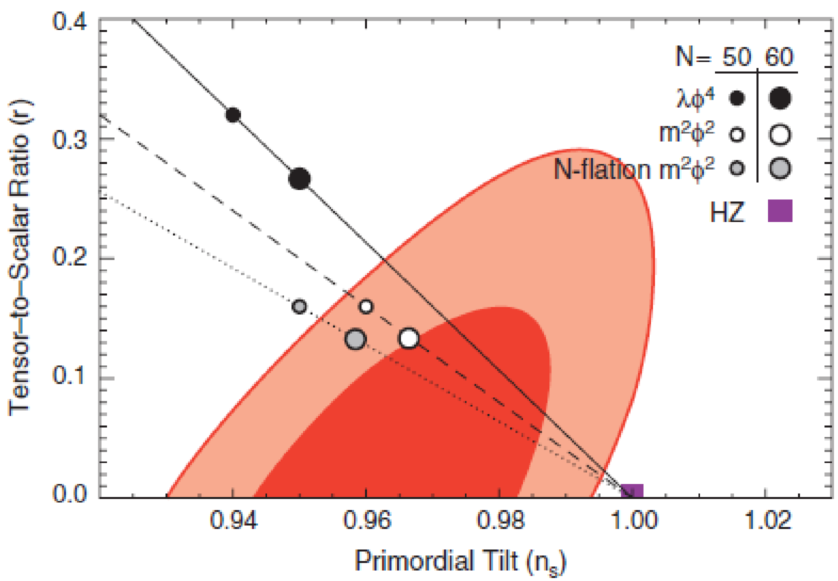
- The singularity theorem implies the Big Bang (BB) [Hawking, Penrose, Proc. R. Soc. Lond. A 314 (‘70)].
- Inflationary spacetimes have the singularity [Borde, Guth, Vilenkin, PRL 90 (‘03)].
- What is the spacetime geometry including the BB?
- How to quantize the spacetime as well as matter fields, that is, what is quantum gravity and quantum cosmology?
- How do a classical universe and the unitary quantum field theory emerge from quantum cosmology?

Why Massive Scalar Field Quantum Cosmology?

Single-Field Inflation Models

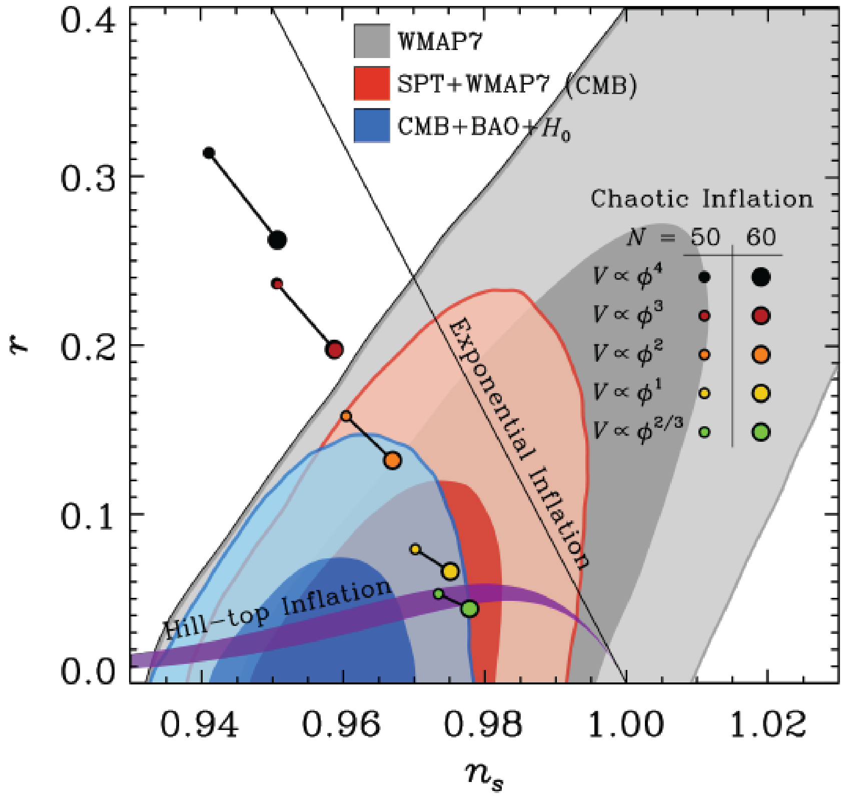
7-Year WMAP

[Astrophys. J. Suppl. 192 ('11)]



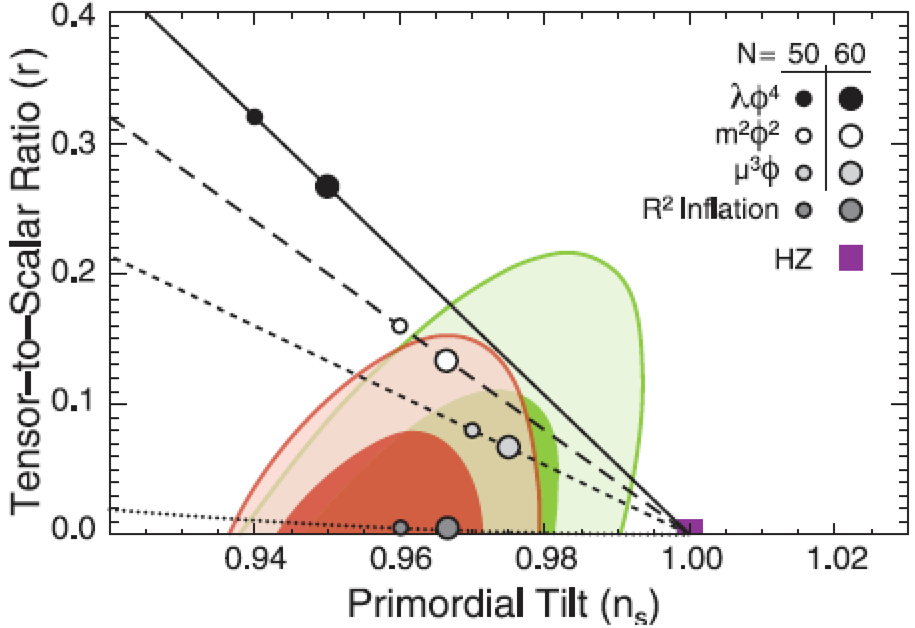
SPT

[arXiv:1210.7231]



9-Year WMAP

[arXiv:1212.5226]



R^2 Inflation Model

- Starobinsky's R^2 inflation model ($R+\alpha R^2$) [PLB 91 ('80)] : a de Sitter-type acceleration .
- Mukhanov and Chibisov [JETP Lett 33 ('81)]:
  - approximately scale-invariant quantum fluctuations

$$n_s - 1 = \left. \frac{d \ln \Delta_R^2(k)}{d \ln k} \right|_{\text{WMAP}} = -\frac{2}{N}$$

  - N = number of e-folds of expansion between the end of inflation and the epoch at which a fluctuation with k_{WMAP} left the horizon.
- Whitt [PLB 145 ('84)] and Maeda [PRD 37 ('87)]: equivalent to a scalar field under a conformal transformation

$$\tilde{g}_{\mu\nu} = (1+2\alpha R) g_{\mu\nu}, \qquad \Psi = \sqrt{3/2} \ln(1+2\alpha R)$$

Quantum Corrections to EH Action

- A charged scalar field in (D=d+1)-dimensional curved spacetime

$$H(x)\Phi = 0, \quad H(x) = -D^\mu D_\mu + m^2, \quad D_\mu = \partial_\mu - iqA_\mu(x)$$

- The effective action in Schwinger-DeWitt proper time integral

$$W = -\frac{i}{2}\int d^{d+1}x\sqrt{-g}\int_0^\infty d(is)\frac{1}{(is)}\left\langle x \,|\, e^{-isH} \,|\, x' \right\rangle$$

$$= \frac{1}{2}\int d^{d+1}x\sqrt{-g}\int_0^\infty d(is)\frac{e^{-im^2 s}}{(is)(4\pi s)^{(d+1)/2}}F(x, x'; is)$$

- The perturbative one-loop corrections to Einstein-Hilbert action

$$f_1 = R,\ f_2 = \frac{1}{30}R_{;\mu}{}^{;\mu} + \frac{1}{12}R^2 + \frac{1}{180}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - \frac{1}{180}R_{\mu\nu}R^{\mu\nu}$$

$$\text{Euler number} = \int\sqrt{-g}\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} + R^2 - 4R_{\mu\nu}R^{\mu\nu}\right)$$

One-Loop Action for dS

- de Sitter space with the metric

$$ds^2 = -dt^2 + \frac{\cosh^2(Ht)}{H^2} d\Omega_d^2$$

- Bogoliubov coefficients for a massive scalar field

$$\alpha_l = \frac{\Gamma(1-i\gamma)\Gamma(-i\gamma)}{\Gamma(l+d/2-i\gamma)\Gamma(1-l-d/2-i\gamma)}, \quad l \in Z^0$$

$$\beta_l = \frac{\Gamma(1-i\gamma)\Gamma(i\gamma)}{\Gamma(l+d/2)\Gamma(1-l-d/2)}, \quad \gamma = \sqrt{\frac{d(d+1)m^2}{R} - \frac{d^2}{4}}$$

One-Loop Action for dS

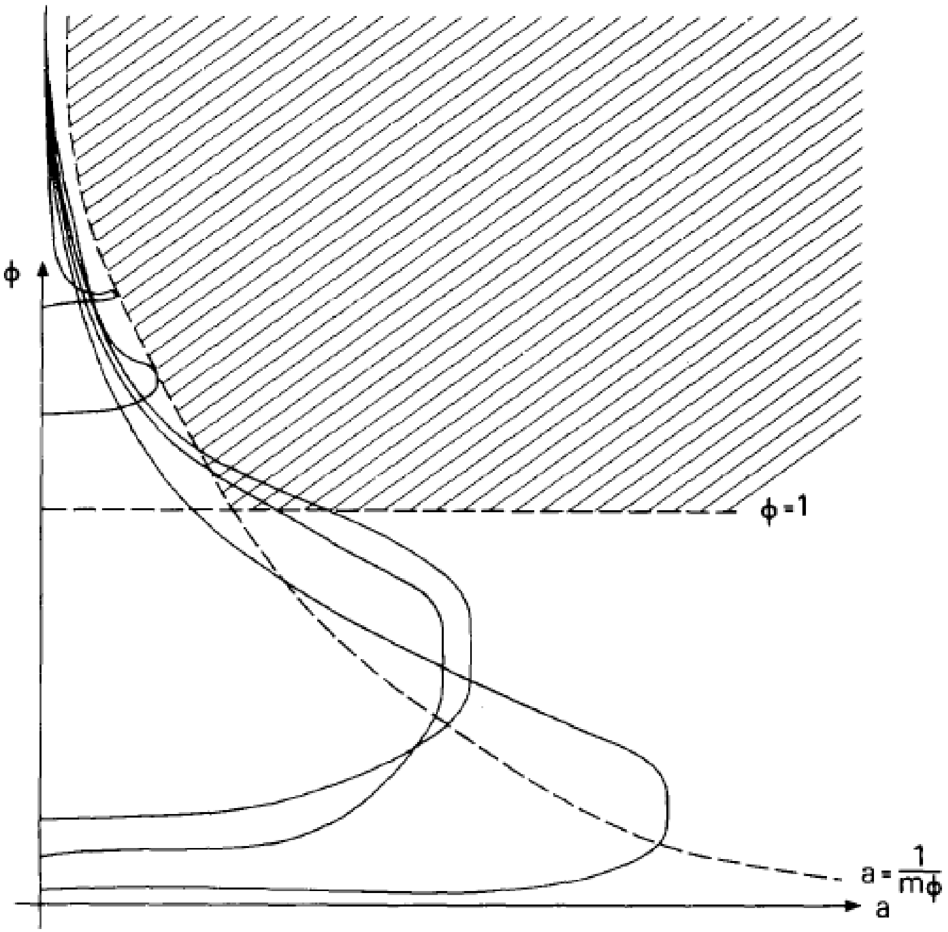
- Vacuum polarization and vacuum persistence from in-out formalism [SPK, arXiv:1008.0577]

$$\bar{N}_l^{\rm sc} = |\beta_l|^2 = \left(\frac{\sin \pi (l + d/2)}{\sinh(\pi\gamma)} \right)^2, \quad 2\,{\rm Im}\, L_{\rm eff}^{\rm sc}(R) = \ln\left(1 + \bar{N}_l^{\rm sc}\right)$$

$$W_{\rm eff}^{\rm sc}(R) = \sum_{\rm states} P \int_0^\infty ds \frac{e^{-\gamma s}}{s} \left[\frac{\cos((2l + d - 1)s/2) - \cos(s/2)}{\sin(s/2)} \right]$$

- One loop action is related to f(R)-gravity with all coefficients for higher curvature terms determined.

Hartle-Hawking No-Boundary Wave Function



Euclidean solutions for a FRW coupled to a massive field scalar [Hawking, NPB 239 (1984)]

Inflation with Negative Λ

[Hartle, Hawking, Hertog, arXiv:1205.3807,1207.6653]

- Negative Λ and massive scalar field with negative mass

$$\pi_a^2 = \underbrace{(2\Lambda + m^2\phi^2)}_{\text{negative}} a^4 - ka^2 + \frac{1}{a^2}\pi_\phi^2$$

- Phase transition from AdS to dS

$$\pi_a^2 \leq 0 \ \Rightarrow \pi_a^2 \geq 0$$

- The wave function is peaked around the classical trajectories.

What is Quantum Cosmology?

ADM Formalism

- Arnwitt-Deser-Misner formalism: foliate a globally hyperbolic spacetime manifold by spacelike 3-surfaces

$$ds^2 = -(N^2 - N_i N^i)dt^2 + 2N_i dt dx + h_{ij} dx^i dx^j$$

$$N = \text{lapse function} \;\&\; N_i = \text{shift vector}$$

- The Hamiltonian for gravity and matter fields

$$H = \int d^3x [N H_0 + N_i H^i]$$

$$H_0 = \int d^3x' \left[16\pi m_P^{-2} G_{ijkl}(x,x')\pi^{ij}(x)\pi^{kl}(x') + h^{1/2}\delta^3(x-x')\left(\frac{m_P^2}{16\pi}(-{}^3R + 2\Lambda) + T_{00}(\phi, \pi_\phi, h_{ij}) \right) \right]$$

$$G_{ijkl}(x,x') = \frac{1}{2h^{1/2}}\left[h_{ik}(x)h_{jl}(x') + h_{il}(x)h_{jk}(x') - h_{ij}(x)h_{kl}(x') \right]\delta^3(x-x')$$

$$H^i(x) = 2\pi^{ij}_{|j} - h^{1/2}T^{0i} \quad , \quad \pi^{ij} = -\frac{m_P^2}{16\pi} h^{1/2}\left(K^{ij} - h^{ij}K \right) (K^{ij} : \text{second fundamental form})$$

Wheeler-DeWitt Equation

- The WDW equation from the super-Hamiltonian constraint and/or the super-momentum constraints via Dirac quantization:

$$H_0 = \int d^3x' \left[-\frac{16\pi}{m_P^2} G_{ijkl}(x,x') \frac{\delta}{\delta h_{ij}(x)} \frac{\delta}{\delta h_{kl}(x')} \right.$$

$$\left. + h^{1/2}\delta^3(x-x') \left(\frac{m_P^2}{16\pi}(-{}^3R + 2\Lambda) + T_{00}(\phi, \frac{\delta}{i\delta\phi}, h_{ij}) \right) \right]$$

$$H^i(x) = 2\frac{\delta}{i\delta h_{ij}(x)}\Big|_j - h^{1/2} T^{0i}$$

- Quantum cosmology necessarily includes quantum fluctuations from spacetime and matter fields.

Quantum-Classical Transition

From QG to SQG to CG

Quantum Gravity	
$\hat{G}_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu}$	WDW, HH wave function, tunneling wave function

$$G = 1/m_{\mathrm{P}}^2 \to 0$$

Semiclassical Quantum Gravity	
$G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left\langle \hat{T}_{\mu\nu} \right\rangle$	QFT in curved spacetime, Hawking radiation, pair production

$$\hbar \to 0$$

Classical Gravity	
$G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G \left(T^{\mathrm{C}}_{\mu\nu} + T^{\mathrm{Q}}_{\mu\nu}[\hbar] \right)$	Inflationary models

de Broglie-Bohm Pilot-Wave Theory

- In the causal interpretation, a particle has a definite (suitable) path that is affected by the wave function

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

- In a semiclassical regime, where $\psi = F\,e^{\frac{i}{\hbar}S}$

$$\text{Hamilton - Jacobi equation (real part of QM)}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\nabla S)^2 + V + V_{\mathrm{Q}} = 0, \quad \left(V_{\mathrm{Q}} = -\frac{\hbar^2}{2m}\frac{\nabla^2 F}{F}\right)$$

$$\text{continuity equation (imaginary part of QM)}$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v})^2 = 0, \quad \left(\rho = F^2, \quad \vec{v} = \frac{\nabla S}{m}\right)$$

Quantum FRW Universe
(minisuperspace model)

- The metric for Friedmann-Robertson-Walker universe

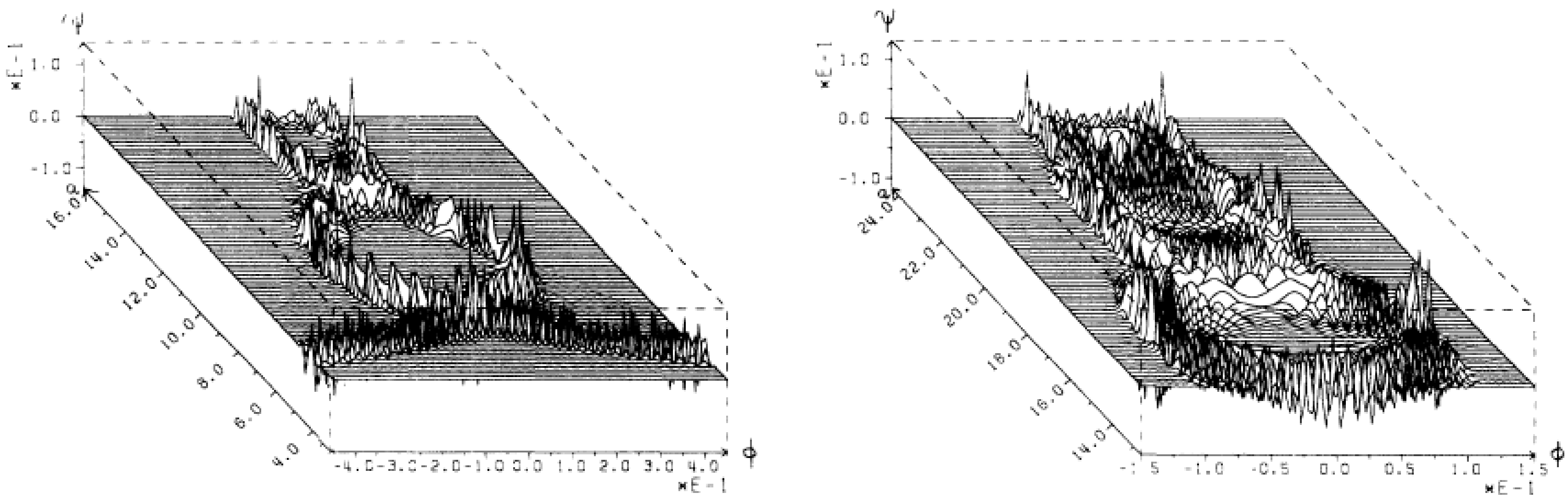
$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2$$

- The WDW equation for a FRW universe with a minimal scalar field (inflaton) up to a factor ordering

$$\left[-\frac{\hbar^2}{2M} \frac{\partial^2}{\partial a^2} - M V_G(a) + \hat{H}_m(\pi_\phi, \phi) \right] \Psi(a, \phi) = 0$$

$$V_G(a) = \frac{1}{2} k a^2 - \Lambda a^4 \ , \ \left(c = 1, \ M = \frac{3 m_P^2}{4\pi} \right)$$

Wave Packet for FRW with a Minimal Scalar



A closed universe (k=1), m = 6, and n =120 (harmonic quantum number)
[Fig. from Kiefer, PRD 38 ('88)]

Pilot-Wave Theory and Born-Oppenheimer Idea

- The wave functions are peaked around some trajectories (wave packets) and allow the pilot-wave theory

$$\left[-\frac{\hbar^2}{2M}\nabla^2 - MV_{\mathrm{G}}(h_a) + \hat{H}(\phi, -i\frac{\delta}{\delta\phi}, h_a)\right]\Psi(h_a,\phi) = 0\ , \quad (h_a = h_{ij})$$

- Apply the Born-Oppenheimer idea that separates a slow moving massive particle (M=Planck mass squared) from a fast moving light particle (matter field) and then expand the quantum state for the fast moving variable by a certain basis to be determined

$$\left|\Psi(h_a,\phi)\right\rangle = \psi(h_a)\left|\Phi(\phi,h_a)\right\rangle$$

$$\left|\Phi(\phi,h_a)\right\rangle = \sum_k c_k(h_a)\left|\Phi_k(\phi,h_a)\right\rangle$$

Semiclassical Quantum Gravity

[SPK, PRD 52 ('95); CQG 13 ('96); PRD 55 ('97)]

- Apply the de Broglie-Bohm pilot-wave theory to the gravity part only

$$\psi(h_a) = F(h_a) e^{iS(h_a)/\hbar}$$

- Then, in a semiclassical regime, the WDW equation is equivalent to

$$\frac{1}{2M}(\nabla S)^2 - MV_G(h_a) + H_{nn} - \frac{\hbar^2}{2M}\frac{\nabla^2 F}{F} - \frac{\hbar^2}{M}\mathrm{Re}(Q_{nn}) = 0$$

$$\frac{1}{2}\nabla^2 S + \frac{\nabla F}{F}\cdot\nabla S + \mathrm{Im}(Q_{nn}) = 0$$

$$H_{nk}(h_a) := \langle \Phi_n(\phi, h_a) | \hat{H} | \Phi_k(\phi, h_a) \rangle \; ; \; \vec{A}_{nk}(h_a) := i\langle \Phi_n(\phi, h_a) | \nabla | \Phi_k(\phi, h_a) \rangle$$

$$Q_{nn}(h_a) := \frac{\nabla F}{F}\cdot\left(\frac{\nabla c_n}{c_n} - i\sum_k \vec{A}_{nk}\frac{c_k}{c_n}\right)$$

Semiclassical Quantum Gravity

- In quantum gravity, time is NOT *a priori* given since the WDW equation is a constraint equation (problem of time). Thus, time should be defined from the wave function itself.
- In the semiclassical quantum gravity, time emerges from the wave packet and the cosmological time is defined as the directional derivative of the action, not necessarily a classical one, along the trajectory

$$\frac{\delta}{\delta\tau} := \frac{1}{M}\nabla S(h_a)\cdot\nabla$$

Semiclassical Quantum Gravity

- The matter field obeys the Heisenberg (matrix) equation

$$i\hbar \frac{\delta}{\delta\tau} c_n = \sum_{k\neq n} H_{nk} c_k - \frac{\hbar}{M} \nabla S \cdot \sum_{k} \vec{A}_{nk} c_k - \frac{\hbar^2}{2M} \sum_{k\neq n} \Omega_{nk} c_k$$

$$H_{nk}(h_a) := \left\langle \Phi_n(\phi, h_a) \middle| \hat{H} \middle| \Phi_k(\phi, h_a) \right\rangle \; ; \; \vec{A}_{nk}(h_a) := i \left\langle \Phi_n(\phi, h_a) \middle| \nabla \middle| \Phi_k(\phi, h_a) \right\rangle$$

$$\Omega_{nk}(h_a) := \nabla^2 \delta_{nk} - 2i\vec{A}_{nk} \cdot \nabla + \left\langle \Phi_n(\phi, h_a) \middle| \nabla^2 \middle| \Phi_k(\phi, h_a) \right\rangle$$

- The unitarity of the quantum state of matter field is preserved.

$$C^{+}(\tau) \cdot C(\tau) = 1, \quad C(\tau) = \begin{pmatrix} c_1(\tau) \\ c_2(\tau) \\ \vdots \end{pmatrix}$$

Scalar Field Cosmology

- The extended superspace for a FRW with a minimal scalar and the cosmological time:

$$ds^2 = -a da^2 + a^3 d\phi^2$$

$$\frac{\partial}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a}\frac{\partial}{\partial a}, \left(\frac{\partial a(\tau)}{\partial \tau} = -\frac{1}{Ma}\frac{\partial S(a)}{\partial a} \right)$$

- The Heisenberg matrix equation for the scalar field

$$i\hbar \frac{\partial c_n}{\partial \tau} = \sum_{k \neq n} H_{nk} c_k - \hbar \sum_k B_{nk} c_k - \frac{\hbar^2}{2Ma} \sum_{k \neq n} \Omega_{nk} c_k$$

$$H_{nk}(a(\tau)) := \left\langle \Phi_n \middle| \hat{H} \middle| \Phi_k \right\rangle \ ; \ B_{nk}(a(\tau)) := i \left\langle \Phi_n \middle| \frac{\partial}{\partial \tau} \middle| \Phi_k \right\rangle$$

$$\Omega_{nk}(a(\tau)) := -\frac{1}{\dot{a}^2} \left[\left(\frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}} \frac{\partial}{\partial \tau} \right) \delta_{nk} - 2iB_{nk} \frac{\partial}{\partial \tau} + \left\langle \Phi_n \middle| \frac{\partial^2}{\partial \tau^2} - \frac{\ddot{a}}{\dot{a}} \frac{\partial}{\partial \tau} \middle| \Phi_k \right\rangle \right]$$

Scalar Field Cosmology

- The semiclassical Friedmann equation

$$\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}-\Lambda=\frac{8\pi}{3m_P^2a^3}\left[H_{nn}-\frac{4\pi\hbar^2}{3m_P^2a\dot a}U_{nn}\,\mathrm{Re}(R_{nn})+\frac{2\pi\hbar^2}{3m_P^2a}\left(U_{nn}^2+\frac{1}{\dot a}\dot U_{nn}\right)\right]$$

$$R_{nn}=\frac{\dot c_n}{c_n}-i\sum_k B_{nk}\frac{c_k}{c_n}$$

$$U_{nn}:=\frac{\partial F/\partial a}{F}=-\frac{1}{2}\frac{(a\dot a)^{\bullet}}{a\dot a^2+(4\pi\hbar/3m_P^2)\,\mathrm{Im}(R_{nn})}$$

- The effective energy density

$$\rho_{nn}=H_{nn}-\frac{4\pi\hbar^2}{3m_P^2a\dot a}U_{nn}\,\mathrm{Re}(R_{nn})+\frac{2\pi\hbar^2}{3m_P^2a}\left(U_{nn}^2+\frac{1}{\dot a}\dot U_{nn}\right)$$

Massive Scalar Field Cosmology

- The semiclassical Friedmann equation with a massive scalar field at the lowest order of $\hbar / M$

$$\hat{H} = -\frac{\hbar^2}{2a^3}\frac{\partial^2}{\partial\phi^2} + \frac{m^2 a^3}{2}\phi^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} - \Lambda = \frac{8\pi}{3m_P^2 a^3}\left[H_{nn} + \frac{2\pi\hbar^2}{3m_P^2 a}\left(U_{nn}^{(0)2} + \frac{1}{\dot{a}}\dot{U}_{nn}^{(0)}\right)\right]$$

$$R_{nn}^{(0)} = 0 \;\; ; \;\; U_{nn}^{(0)} = -\frac{1}{2}\frac{(a\dot{a})^{\bullet}}{a\dot{a}^2}$$

$$H_{nn} = \hbar a^3\left(n + \frac{1}{2}\right)\left[\dot{\varphi}^*\dot{\varphi} + m^2\varphi^*\varphi\right] \;\; ; \;\; \ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} + m^2\varphi = 0$$

Back-reaction from Quantum Gravity

- Explain the back-reaction of de Sitter radiation [J-A. Gu, SPK, C-M. Shen, arXiv:1210.7902]

$$G^{\mathrm{C}}_{\mu\nu} + G^{\mathrm{Q}}_{\mu\nu}[G] = 8\pi G\left(T^{\mathrm{C}}_{\mu\nu} + T^{\mathrm{Q}}_{\mu\nu}[\hbar]\right)$$

  - dS temperature increased by the ratio of the dS scale to the Planck scale.
  - Equivalent to the back-reaction of Hawking radiation in a de Sitter Schwarzschild black hole [Greene et al, JHEP04 (‘06)057].

Second Quantized Universes

Scalar Field Quantum Cosmology

- The ADM formalism for a FRW geometry

$$ds^2 = -N^2 dt^2 + a^2(t) d\Omega_3^2$$

- The WDW equation (Dirac quantization of Hamiltonian constraint) for a FRW universe minimally coupled to a single-field inflaton (scalar field); the WDW equation is already second quantized

$$\left[\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2} + 2a^4 V(\phi) - V_G(a)\right]\Psi(a,\phi) = 0$$

$$V_G(a) = ka^2 - 2\Lambda a^4 \ , \ \left(\hbar = c = l_P^2 = \frac{16\pi}{m_P^2} = 1\right)$$

Quantum Universes in the Superspace

- The supermetric for FRW geometry and a minimal scalar

$$ds^2 = -da^2 + a^2 d\phi^2$$

- The Hamiltonian constraint and the WDW equation

$$H(a,\phi) = \underbrace{-\left(\pi_a^2 + V_G(a)\right)}_{\text{gravity part } H_G} + \frac{1}{a^2}\underbrace{\left(\pi_\phi^2 + 2a^6 V(\phi)\right)}_{\text{scalar field part } H_M} = 0$$

$$\left[-\nabla^2 - V_G(a) + 2a^4 V(\phi)\right]\Psi(a,\phi) = 0$$

$$\nabla^2 = -\frac{\partial^2}{\partial a^2} + \frac{1}{a^2}\frac{\partial^2}{\partial \phi^2}, \quad V_G(a) = ka^2 - 2\Lambda a^4$$

- A Cauchy initial value problem w.r.t. the scale factor a and a prescription for the boundary condition.

Quantum Universes in the Superspace
[SPK, Page, PRD 45 ('92); SPK, PRD 46 ('92)]

- The scalar field for single-field inflation model

$$V(\phi) = \lambda_{2p}\phi^{2p}/(2p)$$

- The eigenfunctions and the Symanzik scaling law

$$H_M(\phi, a)\Phi_n(\phi, a) = E_n(a)\Phi_n(\phi, a)$$

$$E_n(a) = \left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\varepsilon_n$$

$$\Phi_n(\phi, a) = \left(\lambda_{2p}a^6/p\right)^{1/4(p+1)}F_n\left(\left(\lambda_{2p}a^6/p\right)^{1/(p+1)}\phi\right)$$

- The coupling matrix among the energy eigenfunctions

$$\frac{\partial}{\partial a}\vec{\Phi}(\phi, a) = \Omega(a)\vec{\Phi}(\phi, a)$$

$$\Omega_{mn}(a) = \left(3/4(p+1)a\right)\left(\varepsilon_m - \varepsilon_n\right)\int d\varsigma F_m(\varsigma)F_n(\varsigma)\varsigma^2$$

Quantum Universes in the Superspace
[SPK, Page, PRD 45 ('92); SPK, PRD 46 ('92)]

- The gravitational part of the WDW equation

$$\Psi(a,\phi) = \vec{\Phi}^T(\phi,a)\cdot\vec{\psi}(a)$$

$$\left[\frac{d^2}{da^2} - V_G(a) + \frac{E(a)}{a^2} - \left(2\Omega(a)\frac{d}{da} - \Omega^2(a) - \frac{1}{a}\Omega(a)\right)\right]\vec{\psi}(a) = 0$$

- The transition matrix and the Cauchy problem

$$\Psi(a,\phi) = \vec{\Phi}^T(\phi,a)T(a)\vec{\psi}(a);\; T(a) = \exp\left[\int^a da'\Omega(a')\right]$$

$$\frac{d}{da}\begin{pmatrix}\vec{\psi}(a) \\ d\vec{\psi}(a)/da\end{pmatrix} = \begin{pmatrix} 0 & I \\ T^{-1}\left(V_G - \frac{E}{a^2}\right)T & 0\end{pmatrix}\begin{pmatrix}\vec{\psi}(a) \\ d\vec{\psi}(a)/da\end{pmatrix}$$

Quantum Universes in the Superspace
[SPK, Page, PRD 45 ('92); SPK, PRD 46 ('92)]

- The two-component wave function

$$\begin{pmatrix} \Psi(a,\phi) \\ \partial\Psi(a,\phi)/\partial a \end{pmatrix} = \begin{pmatrix} \vec{\Phi}^T(\phi,a) & 0 \\ 0 & \vec{\Phi}^T(\phi,a) \end{pmatrix}$$

$$\times T\exp\left[\int \begin{pmatrix} \Omega(a') & I \\ V_G(a') - E/a'^2 & \Omega(a') \end{pmatrix} da'\right] \begin{pmatrix} \vec{\psi}(a_0) \\ d\vec{\psi}(a_0)/da_0 \end{pmatrix}$$

- The off-diagonal components are the gravitational part equation only with $V_G(a) - E/a^2$.
- The continuous transitions among energy eigenfunctions.

Third Quantization

Third Quantization in 3+1 Dimensions

Second quantization	Third quantization
Particle	Universe
Interaction Vertex	Topology Change
Field	Third Quantized Field
Spacetime	Superspace of Three Geometries
Free Laplacian	Wheeler-DeWitt Operator
Vacuum	Void

[Strominger, "Baby Universes," in *Quantum Cosmology and Baby Universes* edited by S. Coleman et al (World Scientific, 1991)]

Hilbert Space for Quantum Universes

- The inner product for the WDW equation [SPK, J. Kim, K.S. Soh, NPB 406 ('93); SPK, Y.H. Lee, JKPS 26 ('93)]

$$\left\langle \Psi_\alpha \middle| \Psi_\beta \right\rangle = \int_{\Sigma_a} d\phi\, \Psi_\alpha^*(a,\phi) \overleftrightarrow{\partial}_a \Psi_\beta(a,\phi)$$

$$\left\langle \Psi_{I\alpha} \middle| \Psi_{I\beta} \right\rangle = \delta_{\alpha\beta}\,, \left\langle \Psi_{II\alpha} \middle| \Psi_{II\beta} \right\rangle = -\delta_{\alpha\beta}\,, \left\langle \Psi_{I\alpha} \middle| \Psi_{II\beta} \right\rangle = 0$$

- The Feshbach-Villars' first order equation for the WDW equation guarantees the inner product [Mostafazadeh, JMP 39 ('98)].

$$\frac{\partial}{\partial a}\begin{pmatrix} \Psi + i\dfrac{\partial \Psi}{\partial a} \\ \Psi - i\dfrac{\partial \Psi}{\partial a} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+D & -(1-D) \\ 1-D & -(1+D) \end{pmatrix}\begin{pmatrix} \Psi + i\dfrac{\partial \Psi}{\partial a} \\ \Psi - i\dfrac{\partial \Psi}{\partial a} \end{pmatrix}$$

Hilbert Space for Quantum Universes

- The wave function $\Psi_{I\alpha} / \Psi^*_{I\alpha}$ is positive/negative w.r.t. $i\partial_a$ so that the universe is quantized [Banks, NPB 309 ('88); McGuigan, PRD 38 ('88); Giddings, Strominger, NPB 321 ('89); Peleg, CQG 8 ('91); Castagnino et al, 10 ('93), SPK et al ('93)]

$$\Psi(a,\phi) = \sum_{\alpha} \Psi_{I\alpha}(a,\phi) A_{I\alpha} + \Psi^*_{I\alpha}(a,\phi) A^+_{I\alpha}$$

- The operator $A^+_{I\alpha}$ creates a universe in the wave function $\Psi^*_{I\alpha}$, and so on.
- The Hilbert space consists of the void of no-universe, one-universe H and multi-universes.

$$F(H) = C \oplus H \oplus (H \otimes H) \oplus \cdots$$

Third Quantization

- The WDW equation from the third quantized Hamiltonian

$$S = \int da\, d\phi \left[-\left(\frac{\partial \Psi}{\partial a} \right)^2 + \frac{1}{a^2} \left(\frac{\partial \Psi}{\partial \phi} \right)^2 + \left(2a^4 V(\phi) - V_G(a) \right) \Psi^2 \right]$$

- A massless field is a sum of a-dependent oscillators [Hosoya, Morikawa, PRD 39 ('89); Abe, PRD 47 ('93); Horiguchi, 48 ('93)] and is a tachyonic state for the closed universe [SPK, arXiv:1212.5355].
- The third quantization of a massive field is analogous to the second quantized charged KG in a time-dependent, homogeneous, magnetic field $(\vec{A}(t, \vec{r}) = \vec{B}(t) \times \vec{r} / 2)$.

Third Quantization

- Expand the wave function by the energy eigenfunctions of Hamiltonian, $\Psi(a,\phi)=\vec{\Phi}^T(\phi,a)\cdot\vec{\psi}(a)$, for the scalar field to obtain the third quantized Hamiltonian

$$H(a)=\frac{1}{2}\vec{\pi}^T\cdot\vec{\pi}-\underbrace{\vec{\pi}^T\Omega(a)\vec{\psi}}_{\text{very early universe: O(1/a)}}+\underbrace{\frac{1}{2a^2}\vec{\psi}^T E(a)\vec{\psi}}_{\text{late universe: O}(a^{(4-2p)/(p+1)})}$$

$$\vec{\pi}=\partial\vec{\psi}(a)/\partial a+\Omega(a)\vec{\psi}(a)$$

- The massive scalar quantum cosmology can be solved in the sense that the coupling matrix Ω and the energy-eigenvalue matrix E are explicitly known.

WDW Equation vs KG Equation in B

- The WDW equation for the FRW universe with a massive scalar field

$$\left[-\left(\pi_a^2 + V_G(a)\right) + \frac{1}{a^2}\left(\pi_\phi^2 + a^6 m^2 \phi^2\right)\right]\Psi(a,\phi) = 0$$

- The transverse motion of a charged scalar in a temporal, homogeneous, magnetic field $\vec{A}(t,\vec{r}) = \vec{B}(t)\times\vec{r}/2$

$$\left[\frac{\partial^2}{\partial t^2} + \left(\vec{p}_\perp^2 + \left(\frac{qB(t)}{2}\right)^2 \vec{x}_\perp^2 - qB(t)L_z\right) + \left(m^2 + k_z^2\right)\right]\Phi_\perp(t,\vec{x}_\perp) = 0$$

Conclusion

- Quantum cosmology may be a consistent framework for studying quantum fluctuations of spacetime and matter fields.
- Semiclassical and classical cosmology with the back-reaction can be derived from quantum cosmology.
- The ΛCDM model with a massive scalar field seems to be viable with the current CMB data (WMAP 7, SPT, etc.).
- Can massive scalar field quantum cosmology predict all observational data (WMAP 9)?

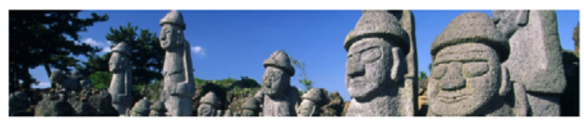
Invited Lectures

- **Martin Bucher** (Univ. of Paris): **Overview of CMB physics: from COBE to Planck and beyond**
- **Chul Hoon Lee** (Hanyang Univ.): **Einstein-Maxwell theory in astrophysics**
- **Hong Lu** (Beijing Normal Univ.): **Rotating black holes in (gauged) supergravities**
- **Viatcheslav Mukhanov** (Munich Univ.): **Quantum origin of universe structure**
- **James M. Nester** (National Central Univ.): **Covariant Hamiltonian formalism for geometric gravity theories**
- **Takahiro Tanaka** (YITP/Kyoto Univ.): **Infrared phenomena of field theory in de Sitter space**

Topics for Workshop*

General relativity and modified gravity, Gravitational waves, Numerical relativity, Black hole physics, Cosmic microwave backgrounds, Inflationary universe and cosmological models, Dark matter and dark energy, Quantum gravity and quantum cosmology, String cosmology

★ **There will be EPL (Europhysics Letters) awards for excellent presentations by students and postdocs**



IAC & LOC

- **IAC:** Rong-Gen Cai (ITP), Sean Hayward (Shanghai Normal Univ.), Gungwon Kang (KISTI), Sang Pyo Kim (Kunsan National Univ.), Kei-ichi Maeda (Waseda Univ.), James M. Nester (National Central Univ.), Misao Sasaki (YITP/Kyoto Univ.), Hoi-Lai Yu (Academia Sinica)
- **LOC:** Inyong Cho (SeoulTech), Jinn-Ouk Gong (APCTP), Gungwon Kang (KISTI), Sang Pyo Kim (Kunsan National Univ., Chair), Seoktae Koh (Jeju National Univ.), Mu-In Park (Kunsan National Univ., Secretary)

Homepage, Deadline & Contact

- **Homepage:** http://www.apctp.org/plan.php/AP2013Jeju
- **Deadline:** 31 Dec. 2012 (registration and abstract submission)
- **Contact:** sec@apctp.org (APCTP), muinpark@gmail.com (Mu-In Park), sangkim@kunsan.ac.kr (Sang Pyo Kim)

Sponsors

APCTP Asia Pacific Center for Theoretical Physics · NCTS · YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS