Hawking-Moss instantons in non-linear Massive Gravity

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A review on Massive Gravity

"Can a graviton has mass ?"

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h| \ll 1$$

To the lowest order in h, one finds the Lagrangian:

$$L = L_{\rm EH}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} + \alpha h^2\right) \ ,$$

decompose $h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_{(\mu}A^{\perp}_{\nu)} + \partial_\mu\partial_\nu\chi,$

where $\partial^\mu h^{\perp}_{\mu\nu} = \partial^\mu A^{\perp}_\mu = 0,$

$$L \supset -\frac{m_g^2}{2}\left[(\partial_\mu\partial_\nu\chi)^2 + \alpha(\Box\chi)^2\right] \ ,$$

So to avoid higher-order derivatives, we impose

$\alpha = -1$ ⇨ Fierz-Pauli 1939

$$L = L_{\mathrm{EH}}(h) + \frac{m_g^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) \ ,$$

- The unique massive gravity theory in linear level without ghost in Minkowski background;
- Diffeomorphism invariance is broken due to mass term.

Problems of Fierz-Pauli theory

- vDVZ discontinuity (van Dam & Veltman ‘70, Zhakharov ’70)

Gravitational exchange amplitute between two conserved sources $T_{\mu\nu},\ T'_{\mu\nu}$

$$A_{\rm GR} = -\frac{2}{M_{\rm pl}}\int d^4x T'^{\mu\nu}\frac{1}{\Box}\left(T_{\mu\nu} - \frac{1}{2}T\eta_{\mu\nu}\right) \ ,$$

$$A_{\rm GR} = -\frac{2}{M_{\rm pl}}\int d^4x T'^{\mu\nu}\frac{1}{\Box - m_g^2}\left(T_{\mu\nu} - \frac{1}{3}T\eta_{\mu\nu}\right) \ ,$$

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$$A_{\rm MG} = -\frac{2}{M_{\rm pl}} \int d^4x T'^{\mu\nu} \frac{1}{\Box - m_g^2} \left(T_{\mu\nu} - \frac{1}{3} T \eta_{\mu\nu} \right) ,$$

GR cannot be recovered in the massless limit !

⇒ Introducing non-linear terms (Vainshtein '72)

- Boulware-Deser ghost (Boulware & Deser '72)

Once higher order terms enter, there appears a sixth mode which is a ghost

General Relativity (GR): $$S = \frac{1}{16\pi G}\int d^4x \, \sqrt{-g}R \, ,$$

In 3+1 dim, for symmetric tensor $g_{\mu\nu}$, the propagating degrees of freedom (dof) can be counted as:

$$6 \quad -4 \qquad = 2$$

Lagrangian multiplier

Helicity ± 2

Such situation changes in the Massive Gravity Theory.

In Massive Gravity (MG), the mass of graviton is non-vanishing, which breaks the gauge invariance

$$S = \frac{1}{16\pi G}\int d^4x\ \sqrt{-g}[R(g) - m^2 V(g)]$$
$$\supset -\frac{m^2}{16\pi G}\int d^4x \gamma N V(\gamma, N, N^i)$$

Generally speaking, the dof is

$$6 - 0 = 6$$

No Lagrangian multiplier…

Helicity $\pm 2,\ \ \pm 1,\ \ 0,$

In Ma shing,
which

Gene

Recently, a non-linear construction of massive gravity theory (dRGT) is proposed, where the BD ghost is removed by specially designed non-linear terms, so that the lapse function N becomes a Lagrangian Multiplier, which removes the ghost degree of freedom.

< A simple example >

physical $ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)\ ,$

reference $ds_f^2 = -dt^2 + dx^2\ ,$

set $N^i = 0\,,$

$$g^{-1}f = \begin{pmatrix} -1/N^2 & 0 \\ 0 & \gamma^{ij} \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & \delta_{ij} \end{pmatrix} = \begin{pmatrix} 1/N^2 & 0 \\ 0 & \gamma^{ik}\delta_{kj} \end{pmatrix}$$

define $\boxed{K^\mu_\nu \equiv \delta^\mu_\mu - \left(\sqrt{g^{-1}f}\right)^\mu_{\ \nu}}\,,$

⇒ $$K^\mu_\nu = \begin{pmatrix} 1 - 1/N & 0 \\ 0 & \delta^i_j - \sqrt{\gamma^{ik}\delta_{kj}} \end{pmatrix}$$

$$\boxed{L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{-g}\,{\rm det}'\left(\delta^\mu_\nu + \beta K^\mu_\nu\right)}\,,$$

$$L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{-g}\,{\det}'\left(\delta^\mu_\nu + \beta K^\mu_\nu\right)\ ,$$

$$\sqrt{-g} \rightarrow N\sqrt{\gamma}$$

$${\det}'\left(\delta^\mu_\nu + \beta K^\mu_\nu\right) \rightarrow \left[1 + \beta\left(1 - \frac{1}{N}\right)\right] \det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right)$$

$$L = L_{\rm EH} - m_g^2 M_{\rm pl}^2 \sqrt{\gamma}\left[N(1+\beta) - \beta\right] \det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right)$$

Mass term is linear in lapse function ⟹ Langrangian multiplier

$$\frac{\partial L}{\partial N} = H - m_g^2 M_{\rm pl}^2 \sqrt{\gamma}(1+\beta) \det\left((1+\beta)\delta^i_j - \beta\sqrt{\gamma^{ik}\delta_{kj}}\right) = 0$$

Recover the Hamitonian constraint

For non-vanishing shift function case, the situation becomes more complicated, but we can still recover the Hamiltonian constraint by redefining a new shift function:

$$N^i = n^i + N m^i(\gamma_{ij},\ n^i)\,,$$

So that the corresponding mass term again is linear in lapse function:

$$N\sqrt{g^{-1}f} = A(\gamma_{ij}, n^i) + N B(\gamma_{ij},\ n^i)\,.$$

Non-linear Massive Gravity (dRGT)

C. de Rham, G. Gabadadze, Phys. Rev. D 82, 044020 (2010);
C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett 106, 231101 (2011);
S. F. Hassan and R. A. Rosen, JHEP 1107, 009 (2011)

$$S_{MG} = \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right],$$

where $[\mathcal{K}] = tr\left(K^\nu_\mu\right)$

$$\mathcal{L}_2 = \frac{1}{2}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),$$

$$\mathcal{L}_3 = \frac{1}{6}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),$$

$$\mathcal{L}_4 = \frac{1}{24}\left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]\right),$$

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \sqrt{g^{\mu\sigma} G_{ab}(\phi)\partial_\nu\phi^a\partial_\sigma\phi^b}.$$

fiducial metric

Stuckelberg field

Self-accelerating solution is found in context of non-linear massive gravity, where two branches exist with effective cosmological constant consists of a contribution from mass of graviton. A. E. Gumrukcuoglu et. al. JCAP 106, 231101(2011);

$$\Lambda_{\pm} = -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

There seems to be some hope to explain the current acceleration, but…

still there exists the Cosmological Constant Problem

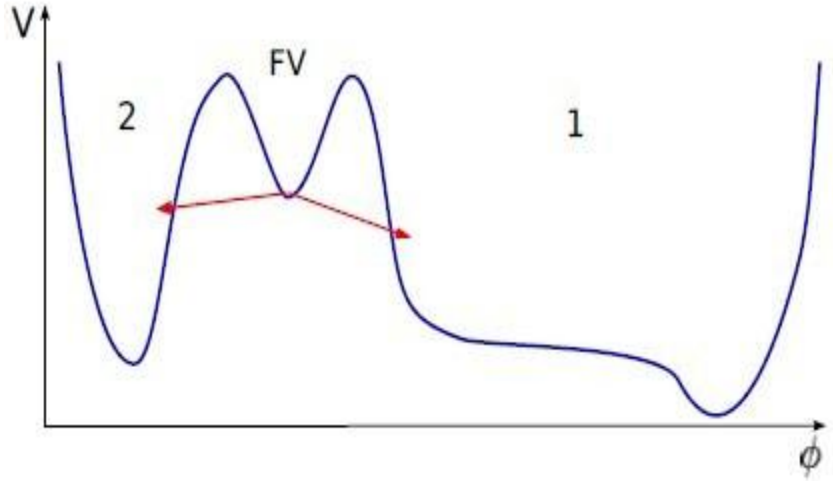
A possible resolution: Anthropic Landscape of Vacua

S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)

- The Landscape has several local minima;

- the fields can (and will) tunnel from a metastable minimum to a lower one;

- this process is driven by instanton.



S. Coleman and F. de Luccia, Phys.Rev. D21, 3305, (1980)

→ it is interesting to investigate how the stability of a vacuum is determined in the context of non-linear Massive Gravity

Setup of model

$$S = S_{MG} + S_m,$$

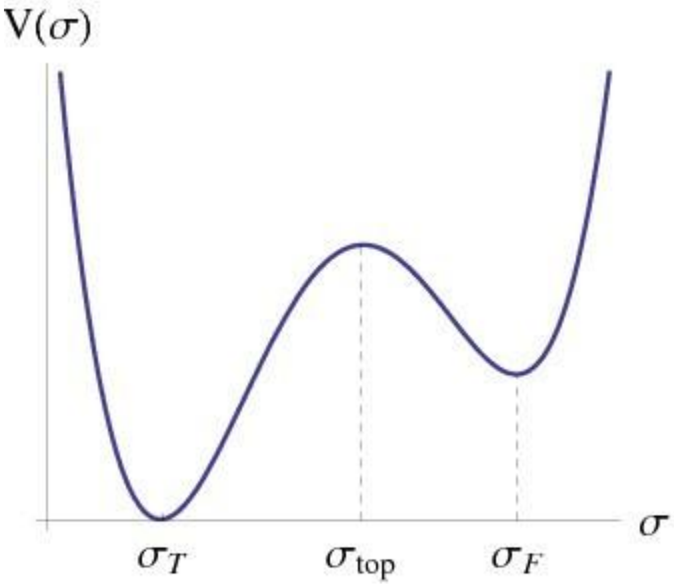
$$S_m \equiv -\int d^4x \, \sqrt{-g} \left[\frac{1}{2}(\partial\sigma)^2 + V(\sigma) \right],$$

- potential $V(\sigma)$

  local minima: σ_F

  global minima: σ_T

  local max: $\sigma_{\rm top}$



- tunneling probability per unit time per unit volume

$$\Gamma/V = Ae^{-B},$$

$$B = S_E[g_{\mu\nu,B}, \phi_B] - S_E[g_{\mu\nu,F}, \phi_F],$$

bounce solution ↑   'false vacuum' ↑   Lowest action ↓

usually, bounce solutions are explored by assuming an O(4) symmetry

- spacetime metric: Euclidean

$$g_{\mu\nu}dx^\mu dx^\nu = N(\xi)^2 d\xi^2 + a(\xi)^2 \Omega_{ij} dx^i dx^j,$$

$$\Omega_{ij} \equiv \delta_{ij} + \frac{K\delta_{il}\delta_{jm}x^l x^m}{1 - K\delta_{lm}x^l x^m}, \quad K > 0$$

Note: the fiducial metric may not respect the symmetry

- fiducial metric: deSitter

$$G_{ab}(\phi)d\phi^a d\phi^b \equiv -(d\phi^0)^2 + b(\phi^0)^2\Omega_{ij}d\phi^i d\phi^j,$$

$$b(\phi^0) \equiv F^{-1}\sqrt{K}\cosh(F\phi^0).$$

↑ fiducial Hubble parameter

⟶ the O(4)-symmetric solutions are obtained by setting

$$\phi^0 = f(\xi), \quad \phi^i = x^i.$$

Inserting these ansatz into the action, we obtain the constraint equation by varying with respect with f

$$(i\dot{a} + N b_{,f}) \left[\left(3 - \frac{2b}{a}\right) + \alpha_3 \left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4 \left(1 - \frac{b}{a}\right)^2 \right] = 0,$$

$$\dot{a} \equiv \frac{da}{d\xi}$$

$$b_{,f} \equiv \frac{db}{df} = \sqrt{K}\sinh(Ff)$$

$\longrightarrow$ Branch I $\quad N b_{,f} = -i\dot{a},$ (equivalent to branch II)

Branch II $\quad \left(3 - \frac{2b}{a}\right) + \alpha_3 \left(1 - \frac{b}{a}\right)\left(3 - \frac{b}{a}\right) + \alpha_4 \left(1 - \frac{b}{a}\right)^2 = 0.$

$$\longrightarrow \quad b = X_{\pm} a, \qquad X_{\pm} \equiv \frac{1 + 2\alpha_3 + \alpha_4 \pm \sqrt{1 + \alpha_3 + \alpha_3^2 - \alpha_4}}{\alpha_3 + \alpha_4}.$$

Friedmann equation & EOM for tunneling field

$$\begin{cases} \dfrac{3}{a^2}\left(\dfrac{da}{d\tau}\right)^2 - \dfrac{3K}{a^2} = \dfrac{1}{2}\left(\dfrac{d\sigma}{d\tau}\right)^2 - V(\sigma) - \Lambda_\pm, \\ \dfrac{d^2\sigma}{d\tau^2} + 3\left(\dfrac{da}{d\tau}\right)\dfrac{d\sigma}{d\tau} - V_{,\sigma}(\sigma) = 0 \end{cases}$$

where $\mathrm{d}\tau \equiv N\mathrm{d}t,$

$$\Lambda_\pm \equiv -\frac{m_g^2}{(\alpha_3+\alpha_4)^2}\left[(1+\alpha_3)\left(2+\alpha_3+2\,\alpha_3^2-3\,\alpha_4\right) \pm 2\,\left(1+\alpha_3+\alpha_3^2-\alpha_4\right)^{3/2}\right],$$

Hawking-Moss(HM) solutions

- HM solutions can be found at the local maximum of the potential

$\sigma = \sigma_{\rm top}$

$$a_{\rm HM}(\tau) = H_{\rm HM}^{-1}\sqrt{K}\cos\left(H_{\rm HM}\tau\right) \ ,$$

$d\tau \equiv N d\xi$

$H_{\rm HM} \equiv \sqrt{\frac{\Lambda_{\pm} + V(\sigma_{top})}{3}}$

- inserting this result into the Euclidean action and evaluate by integrating in the range $H_{\rm HM}\tau = -\pi/2 \longrightarrow \pi/2$, we finally express the HM action

$$Y_\pm \equiv 3(1 - X_\pm) + 3\alpha_3(1 - X_\pm)^2 + \alpha_4(1 - X_\pm)^3,$$

$$S_E[a_{\rm HM}, \sigma_{\rm top}] = -\frac{8\pi^2}{H_{\rm HM}^2}\left[1 - \frac{Y_\pm X_\pm}{6\alpha^4}\left(\frac{m_g}{H_{\rm HM}}\right)^2\left(2 - \sqrt{1-\alpha^2}(2+\alpha^2)\right)\right]$$

standard HM solution

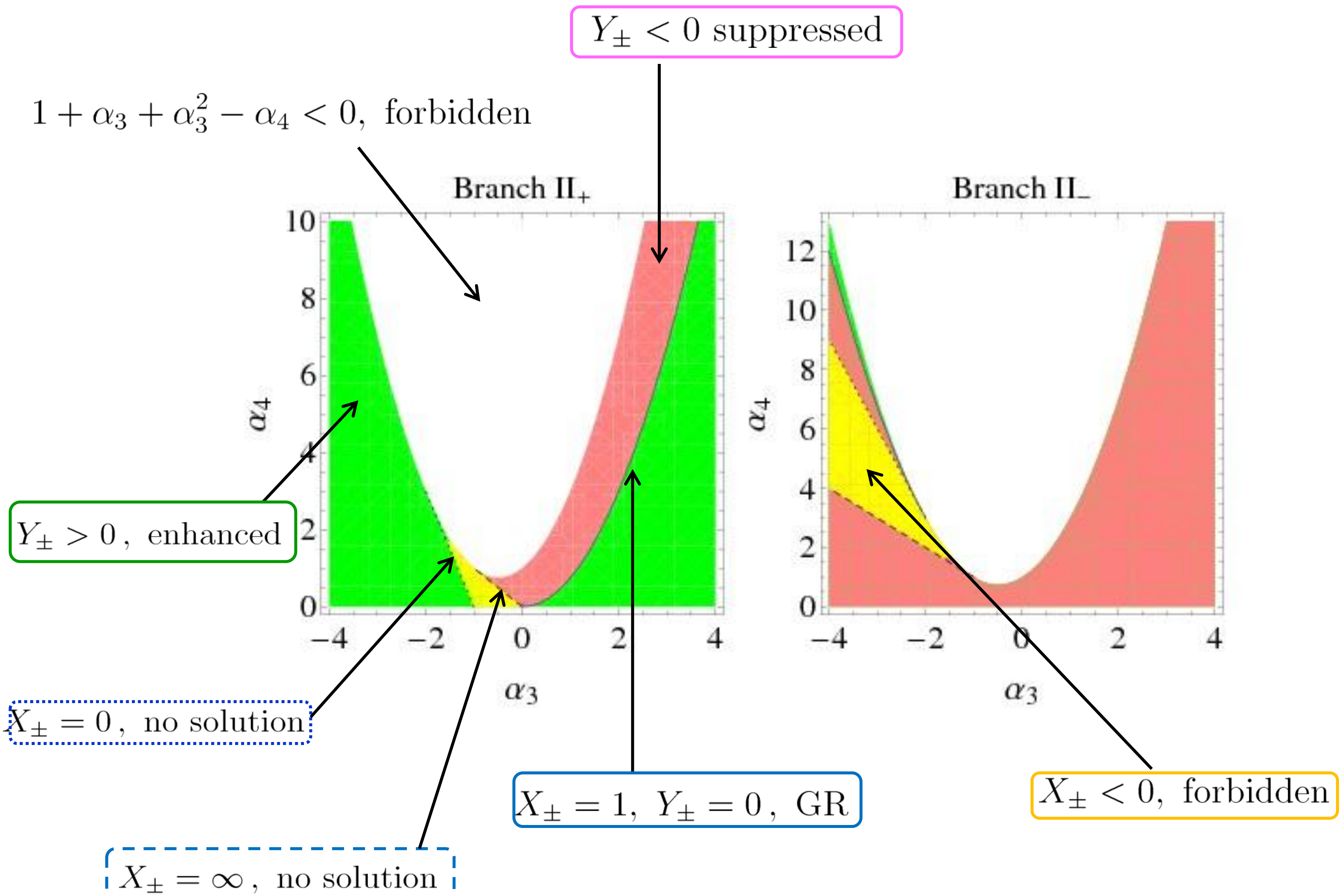
Correction due to the mass of graviton

$$\alpha \equiv X_\pm \frac{F}{H_{\rm HM}}$$

Comparing with GR case, recalling the tunneling probability $\Gamma/V = Ce^{-B}$, we obtains:

$$\Delta B \equiv B^{(\rm MG)} - B^{(\rm GR)} = AY_\pm\,, \quad A < 0$$

Tunneling rate is enhanced for $Y_\pm > 0$,
suppressed for $Y_\pm < 0$.


$Y_\pm < 0$ suppressed
$1+\alpha_3+\alpha_3^2-\alpha_4<0$, forbidden
Branch II$_+$
Branch II$_-$
α_4
α_3
$Y_\pm > 0$, enhanced
$X_\pm = 0$, no solution
$X_\pm = \infty$, no solution
$X_\pm = 1$, $Y_\pm = 0$, GR
$X_\pm < 0$, forbidden

Summary and future work

- We constructed a model in which the tunneling field minimally couples to the non-linear massive gravity;
- corrections of HM solution from mass term is found, which implies suppression or enhancement of tunneling rate, depending on the choices of parameters;
- analysis of Colemann-DeLuccia solutions is under work;
- it would be interesting to investigate the case where the tunneling field couples to the non-linear massive gravity non-minimally.

$$Y_\pm \equiv 3(1-X_\pm) + 3\alpha_3(1-X_\pm)^2 + \alpha_4(1-X_\pm)^3,$$

$$S_E[a_{\rm HM},\sigma_{\rm top}] = \int {\rm d}^3x\sqrt{\Omega}\int_{-\pi/2H_{\rm HM}}^{\pi/2H_{\rm HM}} {\rm d}\tau\ a_{HM}^3\left(2\Lambda_{\pm,{\rm eff}} - \frac{6K}{a_{\rm HM}^2} + m_g^2 Y_\pm\sqrt{-\left(\frac{df_{\rm HM}}{d\tau}\right)^2}\right)$$

$$\Lambda_{\pm,{\rm eff}} \equiv \Lambda_\pm + V(\sigma_{\rm top})$$

$$b_{\rm HM} = F^{-1}\sqrt{K}\cosh(F f_{\rm HM}) = X_\pm a_{\rm HM} \Longrightarrow \left(\frac{df_{\rm HM}}{d\tau}\right)^2 = \frac{X_\pm^2\sin^2(H_{\rm HM}\tau)}{\alpha_{\rm HM}^2\cos^2(H_{\rm HM}\tau) - 1}$$

$$\alpha_{\rm HM} \equiv X_\pm\frac{F}{H_{\rm HM}} \in [0,1]$$

- Note: for the Minkowski fiducial metric, $b_{\rm HM} = \sqrt{K} f_E$, by setting $f_E = -if$

$$\left(\frac{df_{E,{\rm HM}}}{d\tau}\right)^2 = -X_\pm\sin(H_{\rm HM}\tau)$$

so we recover the Minkowski one by setting $\alpha_{\rm HM} = 0$.