

Testing modified gravity with cluster lensing

Tatsuya Narikawa
(RESCEU, U-Tokyo)



Based on

TN & K. Yamamoto, JCAP 1205, 016 (2012) [arXiv:1201.4037];
TN, T. Kobayashi, D. Yamauchi & R. Saito, work in progress.

3rd International Workshop on Dark Matter, Dark Energy, Matter-antimatter Asymmetry
暗物質、暗エネルギー及物質反物質不対称



Outline

$$\ddot{a} > 0$$

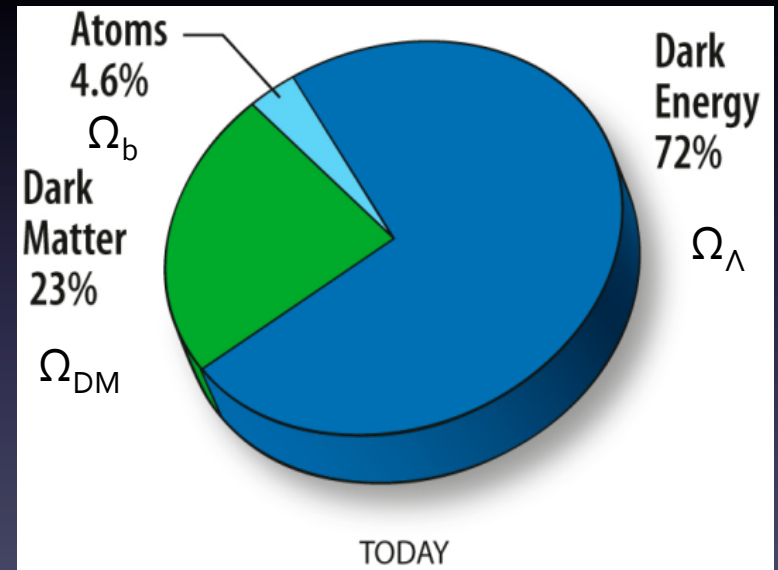
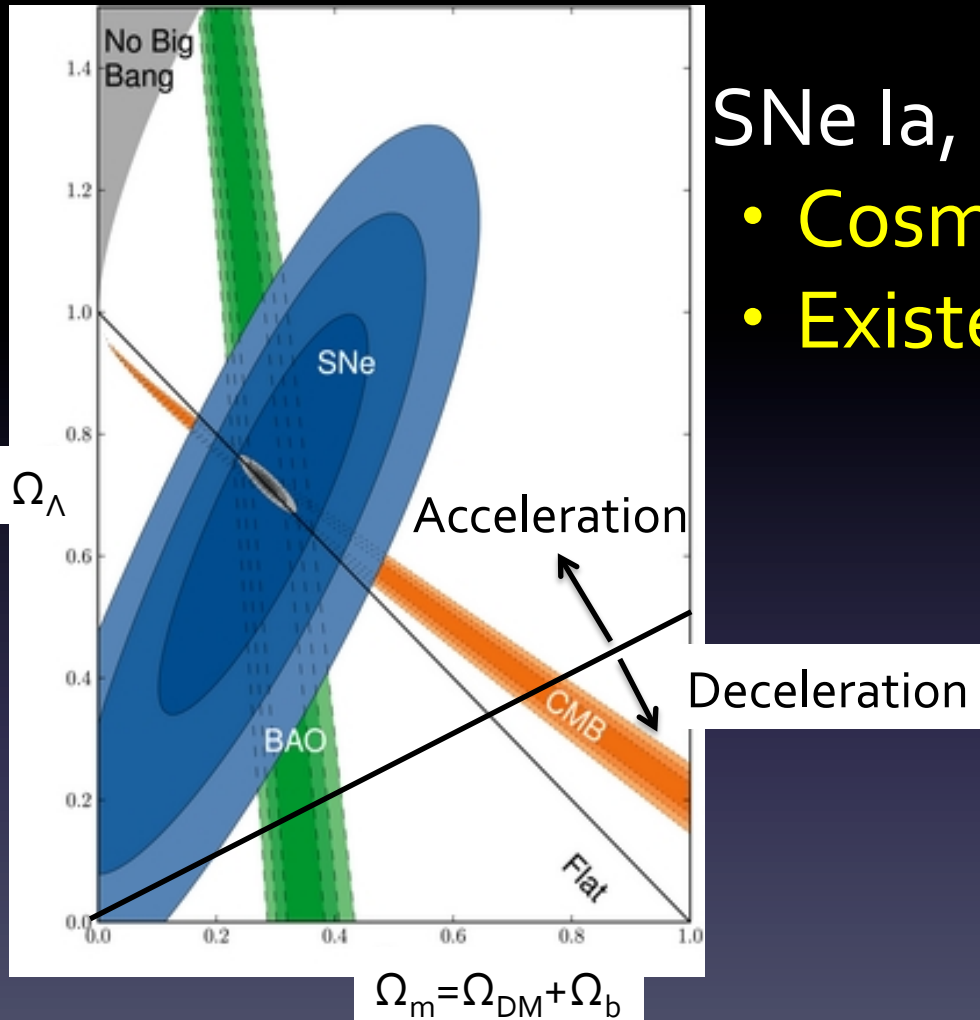
- Introduction & Motivation
 - Cosmic Acceleration [Sami's talk]
 - Recent Progress of Modified Gravity [*]
 - Theoretical: Screening/General Framework[Kase's talk]
 - Observational: Galaxy Cluster Surveys
- Spherical Symmetric Configuration in General Modification (Horndeski's Theory)
- Cluster Lensing Signal in Modified Gravity
- Summary & Conclusion

[*] X. Zhang; Bamba; Nojiri; Matsumoto; C.C. Lee; Saridakis; Ong; Wu; Ni; ...

Evidence of Cosmic Acceleration

SNe Ia, CMB, BAO are converging

- Cosmic acceleration
- Existence of Dark energy



[WMAP]

The Union2.1 compilation of 580 SNe
[Suzuki *et al.* (2012)]

How to Explain Cosmic Acceleration

- Cosmological constant: $w=-1$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}$$

How to Explain Cosmic Acceleration

- Dark energy: $w \neq -1$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu} + \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}^{(\phi)}$$

How to Explain Cosmic Acceleration

- Cosmological constant: $w=-1$
- Dark energy: $w \neq -1$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu} + \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}^{(\phi)}$$

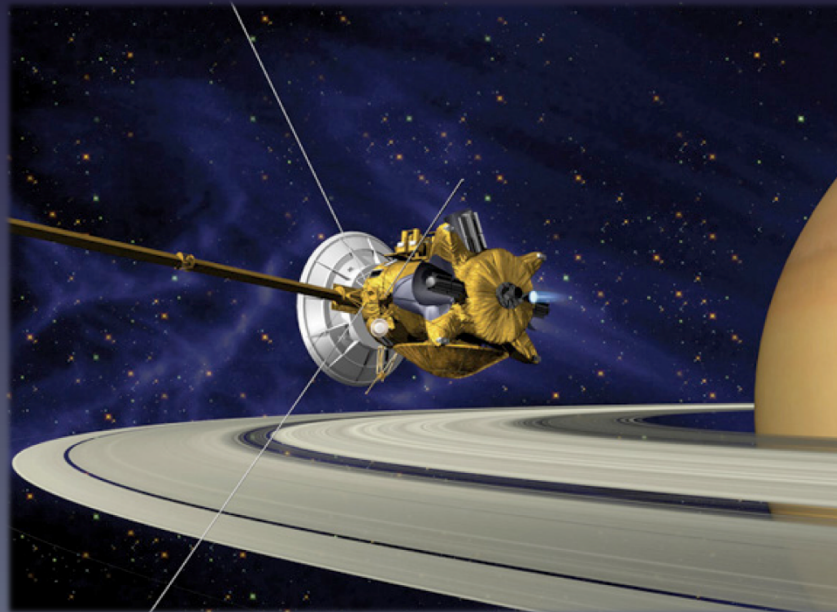
- Modification of gravity on cosmological scales

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + ? = \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}$$

The mystery of cosmic acceleration suggests that there is new gravitational physics on large scales.

Local Gravity Constraints

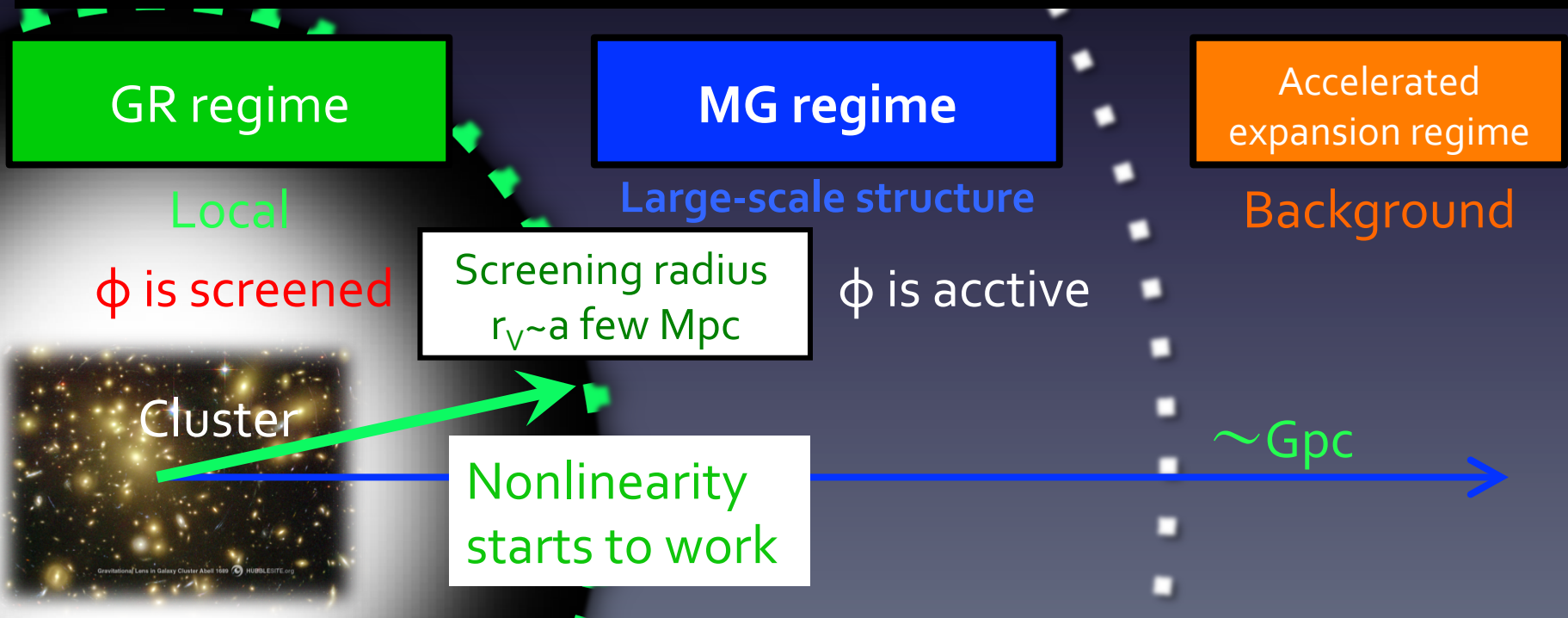
- In MG, an extra scalar field ϕ mediates fifth-force.
→ deviation from GR on large-scales.
- Local gravity tests strongly constrain gravity.
- e.g. $\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$ by time-delay [Bertotti et al. (2003)]
- Screening mechanisms for ϕ are needed to evade local gravity constraints in modified gravity!



[Cassini spacecraft]

Modified Gravity

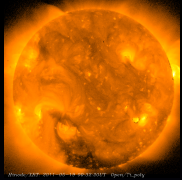
- Motivation: Cosmic acceleration
 - Introduce an additional ϕ
- {DGP, Galileon, Massive Gravity...} < **Horndeski's theory**
- This ϕ
 - cosmic accelerate,
 - mediate fifth force on large scales,
 - must be screened on small scales. [Vainshtein 1972]



Observational Tests of Modified Gravity

Especially Focus on Vainshtein Screening

Solar-system tests (time delay etc.)



Galaxy (Lensing & Velocity dispersion)



Clusters of galaxies



Observational Tests of Modified Gravity

Especially Focus on Vainshtein Screening

Solar-system tests (time delay etc.)

Strict constraint

[Nicolis, Rattazzi & Trincherini (2009)]

Screening radius

$$r_V \sim \text{kpc}$$

Galaxy (Lensing & Velocity dispersion)

Strong constraint

[Sjors & Mortsell (2012)]

$$r_V \sim \text{Mpc}$$

Clusters of galaxies

[Wyman (2011); TN & Yamamoto (2012)]

$$r_V \sim 10 \text{Mpc} \sim r_{\text{virial}}$$

$r_V \sim 10 \text{Mpc}$ that are very well measured by cluster surveys. Hence, it should be possible to discover or constrain this effect by cluster surveys.

Galaxy Cluster as a Probe of Modified Gravity

A variety of cluster observations

- X-ray
- Sunyaev-Zeldovich
- Gravitational lensing

Theor.> Lensing potential: $\Delta\Phi_+ \equiv \Delta(\Phi + \Psi)/2$

$\Delta\Phi_+ \propto r\phi'\phi'' \rightarrow$ deviation from GR at $r \sim r_v$

Obs.> Large amount of data for lensing profile will be provided by Subaru/Hyper Suprime-Cam (HSC).

Over a wide range of radius/ Small error of lensing profile data

Procedure

- Work in **General framework** (Horndeski's theory)
- Derive a condition that the Vainshtein screening work to study **spherically symmetric configuration**
- Demonstrate how an effect of ϕ appears on **lensing signal $\Delta\Phi_+$** in the case that the Vainshtein screening works in modified gravity models
- **Test** modified gravity models by comparing some model predictions with cluster lensing data

Top-down approach

→ Construction a viable model/ getting its hint

Outline

- Introduction & Motivation
 - Cosmic Acceleration
 - Recent Progress of Modified Gravity
 - Theoretical: Screening/General Framework
 - Observational: Galaxy Cluster Surveys
- ☑ Spherical Symmetric Configuration in General Modification (Horndeski's Theory)
- Cluster Lensing Signal in Modified Gravity
- Summary & Conclusion

Galileon-type Modified Gravity

- Motivation: Decoupling limit of DGP braneworld model

$$\boxed{\mathcal{L}_{\text{int}} \sim X \square \phi} : \text{higher-derivative term}$$

where ϕ : scalar field, $X \equiv -\frac{1}{2}(\partial\phi)^2$

enjoy Galileon shift symmetry: $\partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$

- 「
- Cosmic acceleration
 - Up to 2nd order derivatives in EOM
 - Vainshtein screening
- 」

[DGP] Dvali, Gabadadze & Porrati 2000

[DLofDGP] Luty, Porrati, Rattazzi 2003; Nicolis, Rattazzi 2004

[Galileon] Nicolis, Rattazzi & Trincherini 2009

Vainshtein Screening in Cubic Galileon

$$\mathcal{L} = \mathcal{L}_{\text{GR}} - X - \frac{1}{\Lambda^3} X \square \phi + \mathcal{L}_m$$

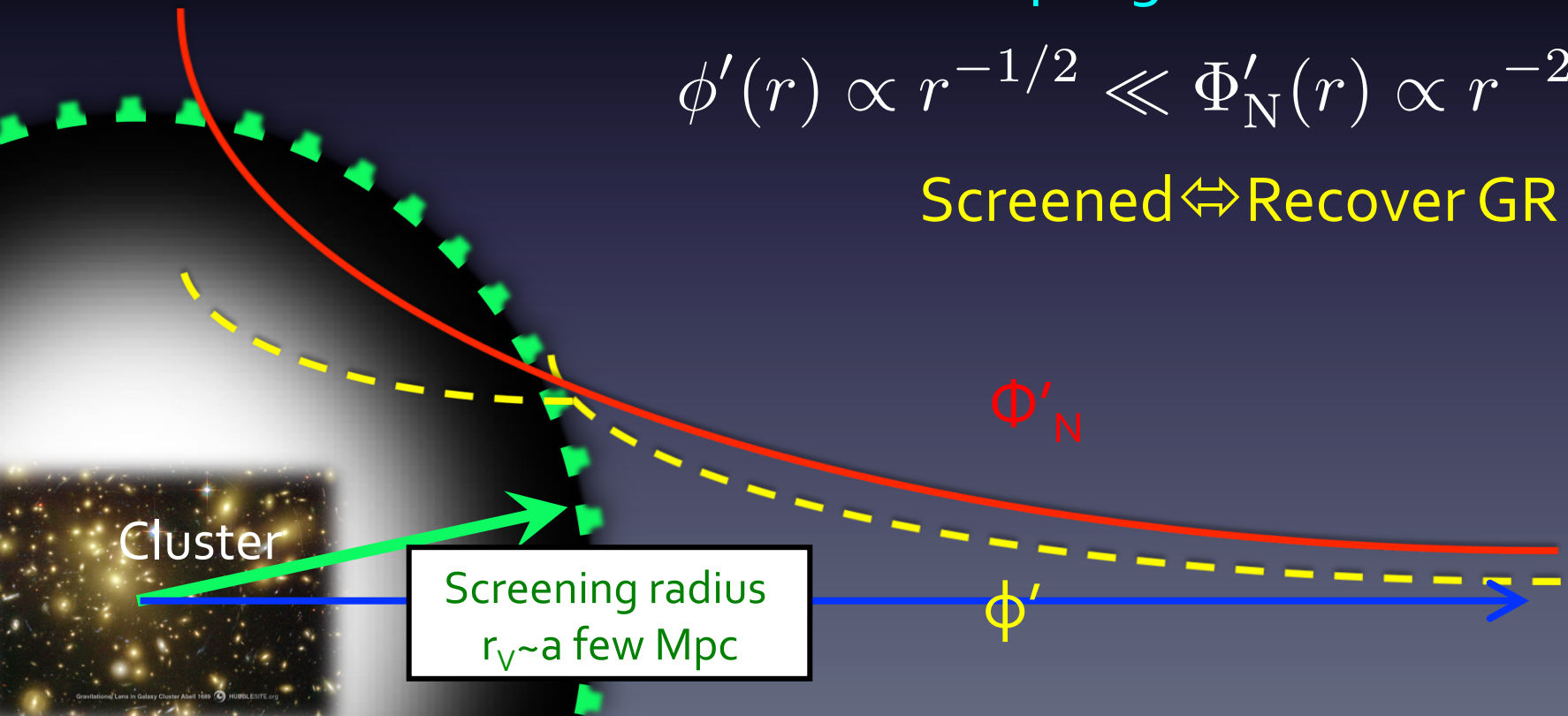
At $r \ll r_v$: Large kinetic term

$\Leftrightarrow$ Strong self-coupling

$\Leftrightarrow$ Weak coupling to matter

$$\phi'(r) \propto r^{-1/2} \ll \Phi'_N(r) \propto r^{-2}$$

Screened $\Leftrightarrow$ Recover GR !



Horndeski's Theory: General Modification in D=4 dim. Up to 2nd order derivatives in EOM

$$S = \int d^4x \sqrt{-g} [\mathcal{L} + \mathcal{L}_m] \quad \text{Minimally coupled to matter}$$

$$\begin{aligned} \mathcal{L} = & K(\phi, X) - G_3(\phi, X) \square \phi \\ & + G_4(\phi, X) R + G_{4X} \times (\text{field derivatives}) \\ & + G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi \\ & - \frac{1}{6} G_{5X} \times (\text{field derivatives}) \end{aligned}$$

where $G_{iX} \equiv \partial G_i / \partial X$ 4 arbitrary functions of ϕ, X

[Horndeski 1974] is equivalent to **the Generalized Galileon**
[Deffayet et al. 2011; Kobayashi, Yamaguchi, Yokoyama 2011]

Background Solution (Local Universe)

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\phi = \phi_0 = \text{const}, \quad X_0 = 0$$

Spherical symmetric perturbations produced by a nonrelativistic matter (Galaxy Cluster)

$$ds^2 = -[1 + 2\Phi(r)]dt^2 + [1 - 2\Psi(r)]\delta_{ij}dx^i dx^j$$

$$\phi = \phi_0 + \varphi(r)$$

All the coefficients are evaluated at the background.

Static-Spherically Symmetric Configurations

Metric EOM:

$$\frac{M_{\text{Pl}}^2}{2} \frac{(r^2 \Psi')'}{r^2} - M_{\text{Pl}} \xi \frac{(r^2 \varphi')'}{2r^2} - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r(\varphi')^2]'}{2r^2} - \frac{3M_{\text{Pl}}}{\Lambda^6} \beta \frac{[(\varphi')^3]'}{6r^2} = -\frac{1}{4} T_t^t$$

$$M_{\text{Pl}}^2 (\Psi' - \Phi') - 2M_{\text{Pl}} \xi \varphi' - \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{(\varphi')^2}{r} = 0$$

ϕ EOM:

$$\eta \frac{(r^2 \varphi')'}{r^2} - 2 \frac{\mu}{\Lambda^3} \frac{[r(\varphi')^2]'}{r^2} + 2M_{\text{Pl}} \xi \frac{[r^2 (2\Psi - \Phi)']'}{r^2} + 4 \frac{M_{\text{Pl}}}{\Lambda^3} \alpha \frac{[r\varphi'(\Psi' - \Phi)']'}{r^2} + 2 \frac{\nu}{\Lambda^6} \frac{[(\varphi')^3]'}{r^2} - \frac{6M_{\text{Pl}}}{\Lambda^6} \beta \frac{[(\varphi')^2 \Phi']'}{r^2} = 0$$

where six dimensionless parameters:

$\xi, \eta, \mu, \alpha, \nu, \beta$ are functions of $K_X, G_{3\phi}, G_{3X}, G_{4X}, G_{5\phi}, \dots$

(cf. Vainshtein mechanism under considering background evolution [Kimura, Kobayashi, Yamamoto '12])

Dimensionless parameters

Let us introduce six dimensionless parameters:

$\xi, \eta, \mu, \alpha, \nu, \beta$

$$G_4 = \frac{M_{\text{Pl}}^2}{2}, \quad G_{4\phi} = M_{\text{Pl}}\xi,$$

$$K_X - 2G_{3\phi} = \eta,$$

$$-G_{3X} + 3G_{4\phi X} = \frac{\mu}{\Lambda^3},$$

$$G_{4X} - G_{5\phi} = \frac{M_{\text{Pl}}}{\Lambda^3}\alpha,$$

$$G_{4XX} - \frac{2}{3}G_{5\phi X} = \frac{\nu}{\Lambda^6},$$

$$G_{5X} = -\frac{3M_{\text{Pl}}}{\Lambda^6}\beta.$$

Quintic Scalar-Field Equation

Combining metric EOM and ϕ EOM, we arrive at

$$P(x, A) := \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2\right) x + [\mu + 6\alpha\xi - 3\beta A(r)] x^2 \\ + (\nu + 2\alpha^2 + 4\beta\xi) x^3 - 3\beta^2 x^5 = 0$$

where we define

$$x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3}$$

both of which are dimensionless.

$M(r)$ is the enclosed mass.

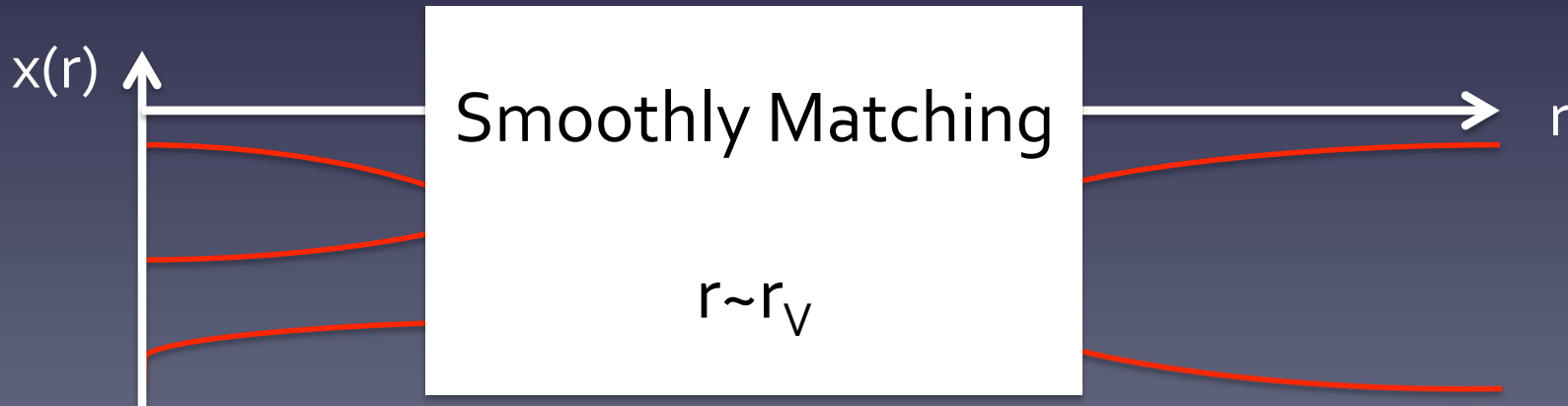
Scalar-Field Equation_[cf. Sbisa, Niz, Koyama, Tasinato '12]

• Solve $x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3}$

$$P(x, A) := \xi A(r) + \left(\frac{\eta}{2} + 3\xi^2 \right) x + [\mu + 6\alpha\xi - 3\beta A(r)] x^2 \\ + (\nu + 2\alpha^2 + 4\beta\xi) x^3 - 3\beta^2 x^5 = 0$$

for the inner region ($A \gg 1$) and the outer region ($A \ll 1$).

- Derive a condition under which the two solutions are smoothly matched in an intermediate region.



Outer Solution: Asymptotically Flat

- In the outer region ($A \ll 1$), there is always a decaying solution,

$$x \approx x_f := -\frac{2\xi A(r)}{\eta + 6\xi^2}$$

Inner Solution: Vainshtein

- In the inner region ($A \gg 1$), we have a solution ($\xi\beta > 0$):

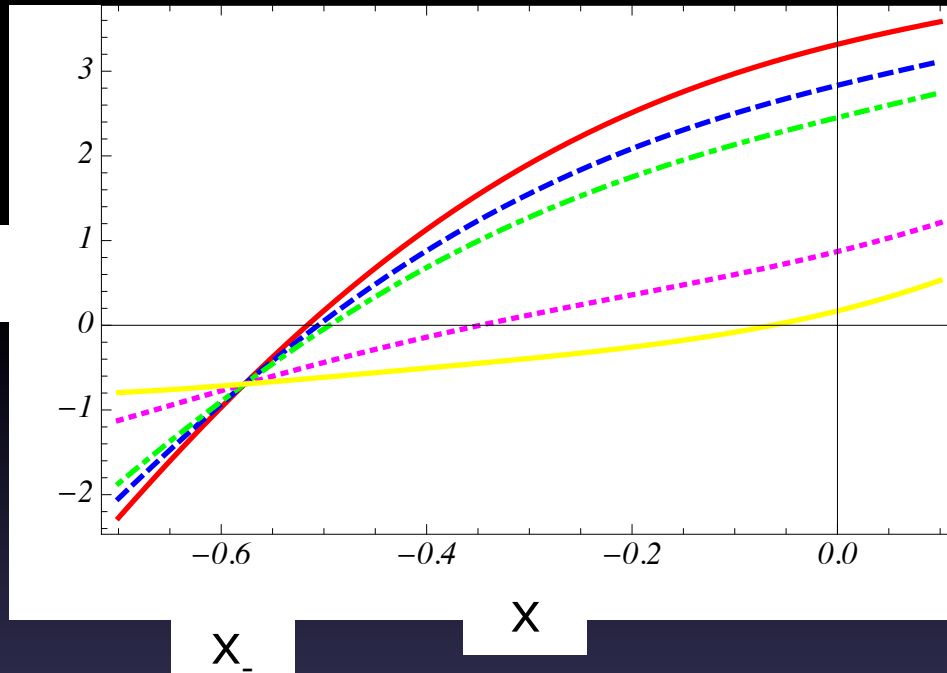
$$x \approx x_{\pm} := \pm \sqrt{\frac{\xi}{3\beta}} = \text{const}$$

($P(x_{\pm})$ does not depend on A .)

where
$$x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}, \quad A(r) = \frac{1}{M_{\text{Pl}} \Lambda^3} \frac{M(r)}{8\pi r^3}$$

Smoothly Matching of Two Solutions

$P(x,A)$

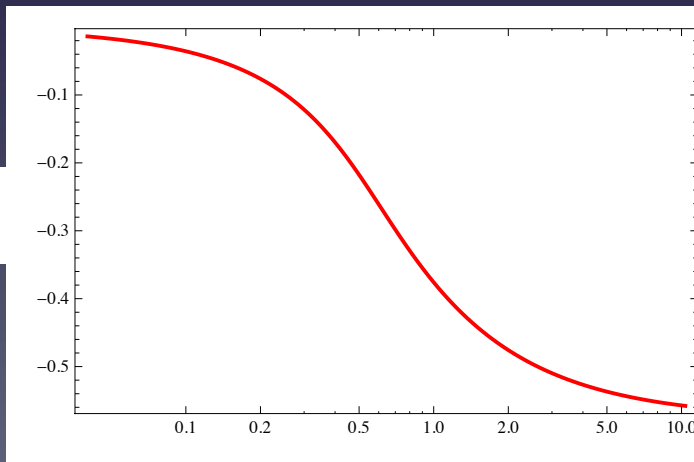


$\uparrow$
 A
 $\downarrow$

In this case, $P(x)=0$ has a single real root in $(x_-,0)$ for any $A>0$.

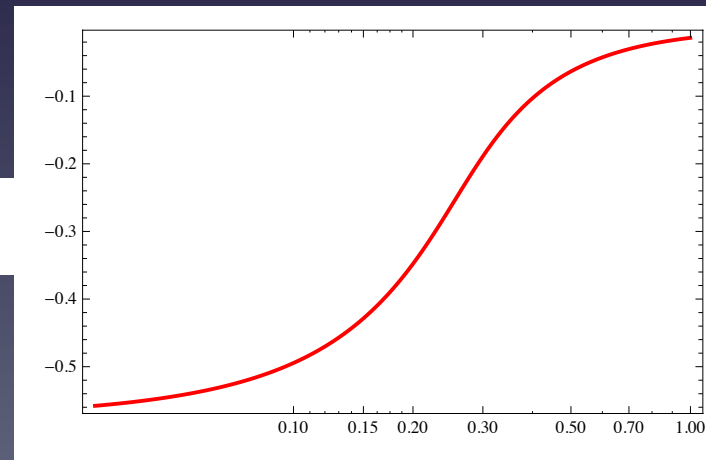
The two solutions are smoothly matched !

x



A

x



r

Parameters Corresponding to Decoupling Limit of Ghost-free Massive Gravity

$$\mathcal{L} = \mathcal{L}_{\text{GR}} + m_g^2 h_{\mu\nu}^2 + \mathcal{O}(h_{\mu\nu}^3)$$

Massive Gravity = A sort of Galileon

[de Rham, Gabadadze & Tolley 2010]

Decoupling limit: $M_{\text{pl}} \rightarrow \infty$, $m_g \rightarrow 0$, $\Lambda^3 = M_{\text{pl}} m_g^2 = \text{fixed}$.

Corresponding to

$$K = 0 = G_3, \quad G_4 = \frac{M_{\text{pl}}^2}{2} + M_{\text{Pl}} \phi + \frac{M_{\text{Pl}}}{\Lambda^3} \alpha X, \quad G_5 = -3 \frac{M_{\text{Pl}}}{\Lambda^6} \beta X$$

in Horndeski's theory.

$$G_{4X} - G_{5\phi} = \frac{M_{\text{Pl}}}{\Lambda^3} \alpha, \quad G_{5X} = -\frac{3M_{\text{Pl}}}{\Lambda^6} \beta.$$

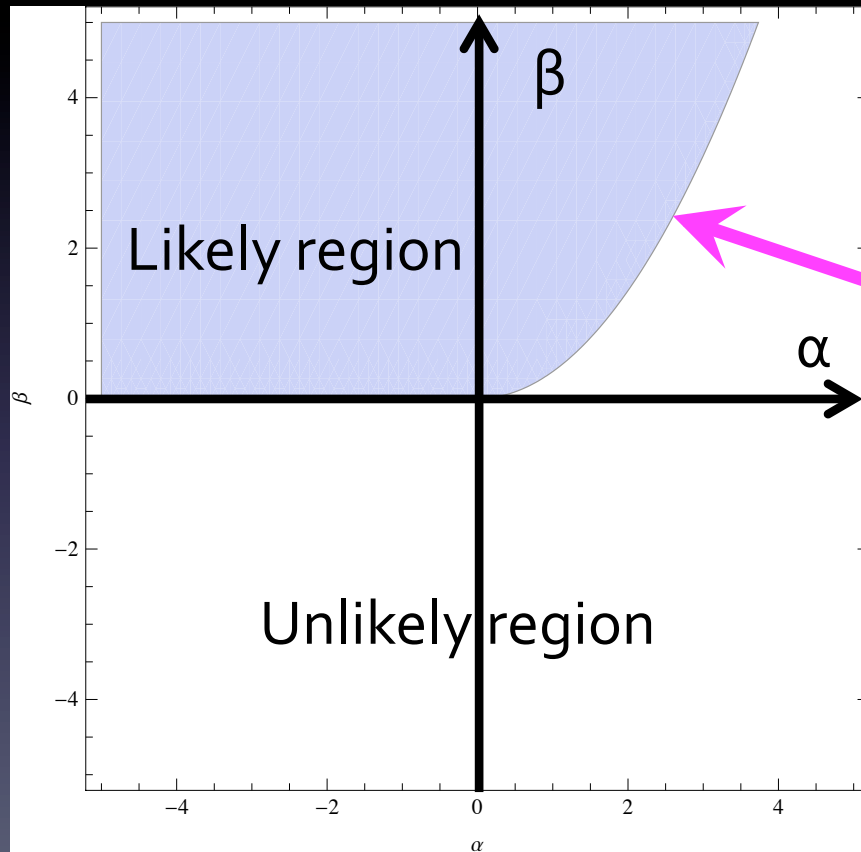
We found $\eta = \mu = \nu = 0$, $\xi = 1$, $\alpha \neq 0$, $\beta \neq 0$

(Proxy theory of massive gravity [de Rham & Heisenberg 2011])

The condition of smooth matching of the two solutions:

$$\alpha < 0 \text{ or } \frac{\sqrt{\beta}}{\alpha} \geq \sqrt{\frac{5 + \sqrt{13}}{24}} \sim 0.6$$

The region $\{\alpha, \beta\}$ which smoothly match the two solutions:



Boundary region
where the solutions
smoothly match.

Explore the observationally
allowed region $\{\alpha, \beta\}$ with
cluster lensing!

Outline

- Introduction & Motivation
 - Cosmic Acceleration
 - Recent Progress of Modified Gravity
 - Theoretical: Screening/General Framework
 - Observational: Galaxy Cluster Surveys
- Spherical Symmetric Configuration in General Modification (Horndeski's Theory)
- ☑ Cluster Lensing Signal in Modified Gravity
- Summary & Conclusion

Gravitational Lensing in Modified Gravity

- Geodesic equation

$$\frac{d^2}{d\chi^2}(\chi\theta^i) = 2\Phi_{+,i}, \quad i = 1, 2 \quad \Phi_+ \equiv (\Phi + \Psi)/2$$

- Surface mass density $\Sigma_S(\mathbf{r}_{\text{perp}}) \propto \kappa(\mathbf{r}_{\text{perp}})$

$$\Sigma_S \propto \int_0^\infty dZ \Delta^{(2D)} \Phi_+ \quad r = \sqrt{r_\perp^2 + Z^2}$$

where lensing potential in modified gravity:

$$\Delta\Phi_+ = \frac{\Lambda^3}{M_{\text{Pl}}} \frac{[(\alpha x^2 + 2\beta x^3 + 2A) r^3]'}{2r^2} \quad \Delta\Phi_+ \propto r\phi'\phi''$$

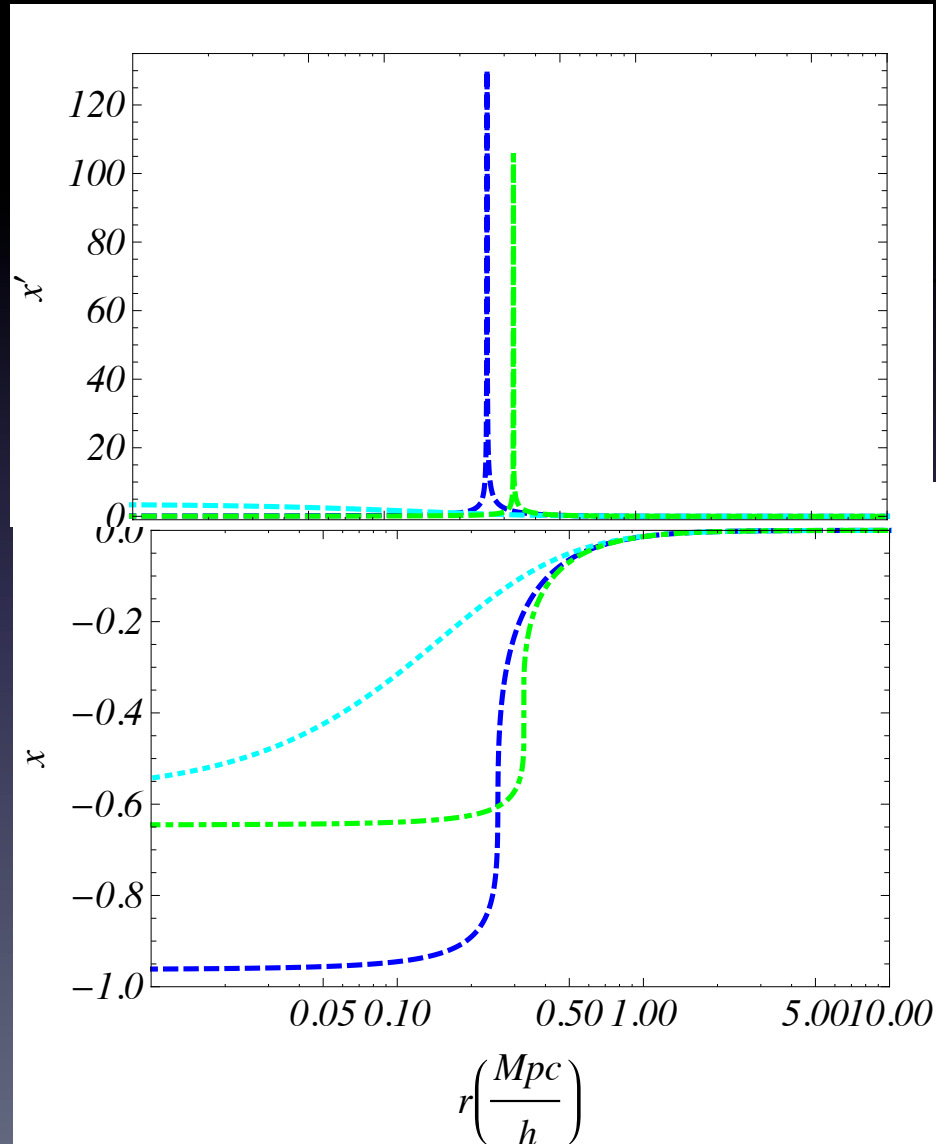
assuming $\delta\rho(r)$ as NFW profile.

Scalar field in modified gravity:

$$x = \frac{1}{\Lambda^3} \frac{\varphi'}{r}$$

$$\Delta\Phi_+ \propto r\phi'\phi''$$

- $\alpha=1, \beta=0.36$
- $\alpha=1.5, \beta=0.8$
- $\alpha=-1, \beta=1$



$\Psi)/2$
A dip appears.

Smoothly
matching

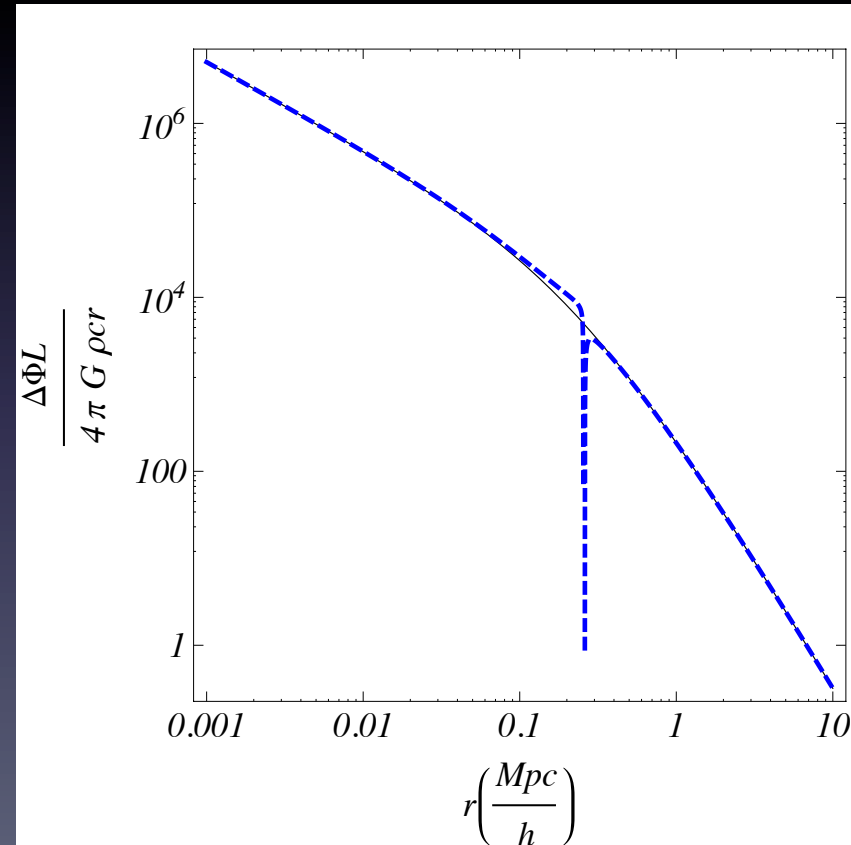
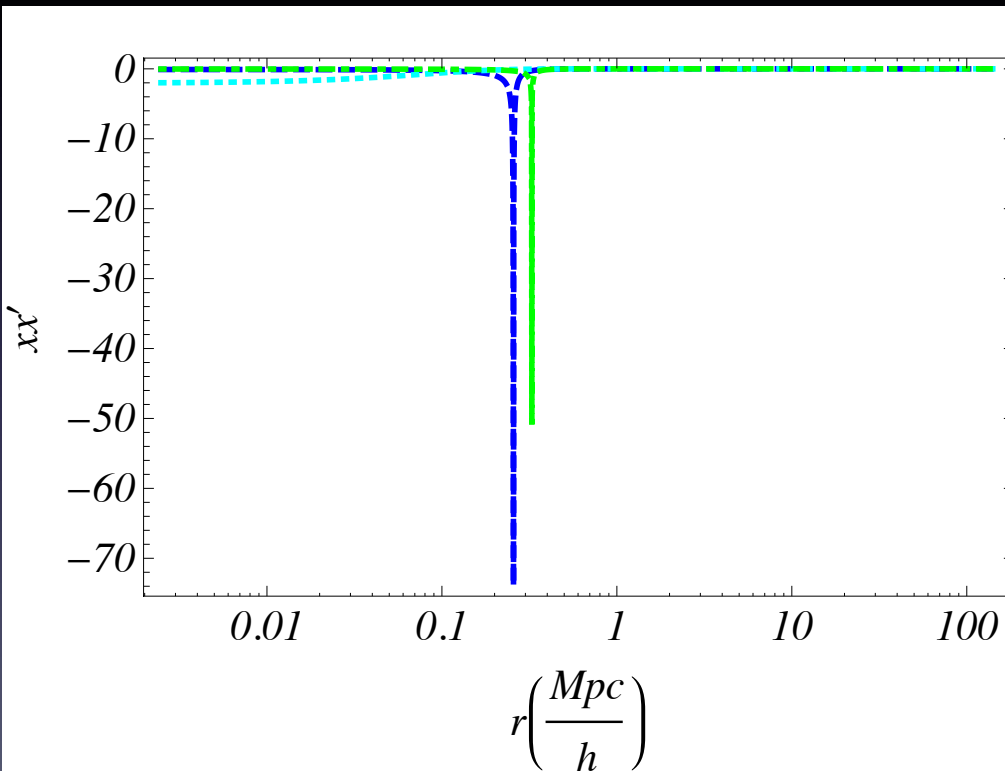
Lensing potential in modified gravity:

$$\Delta\Phi_+ = \frac{\Lambda^3}{M_{\text{Pl}}} \frac{[(\alpha x^2 + 2\beta x^3 + 2A) r^3]'}{2r^2}$$

$$\Delta\Phi_+ \propto r\phi'\phi''$$

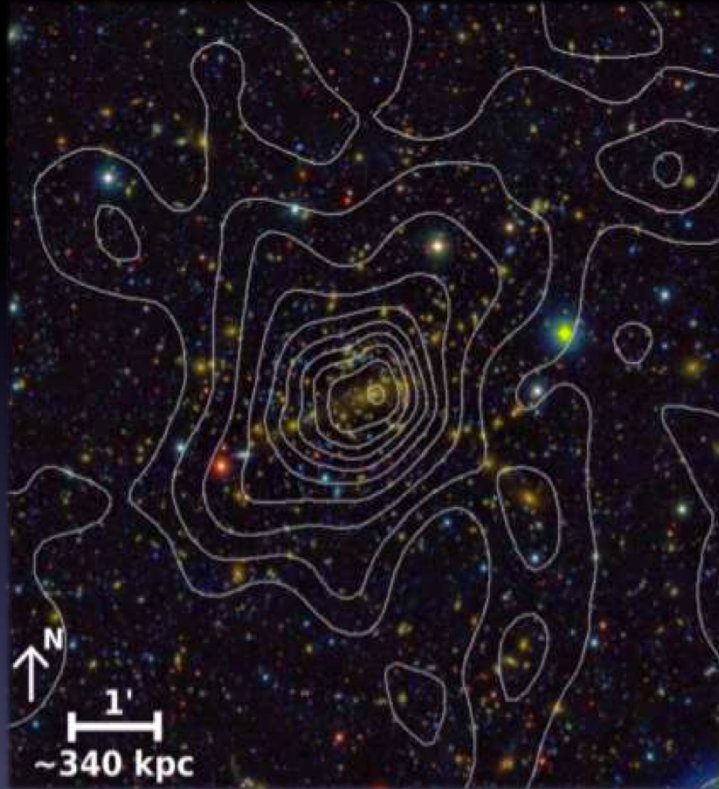
A dip appears.

- $\alpha=1, \beta=0.36$
- $\alpha=1.5, \beta=0.8$
- $\alpha=-1, \beta=1$

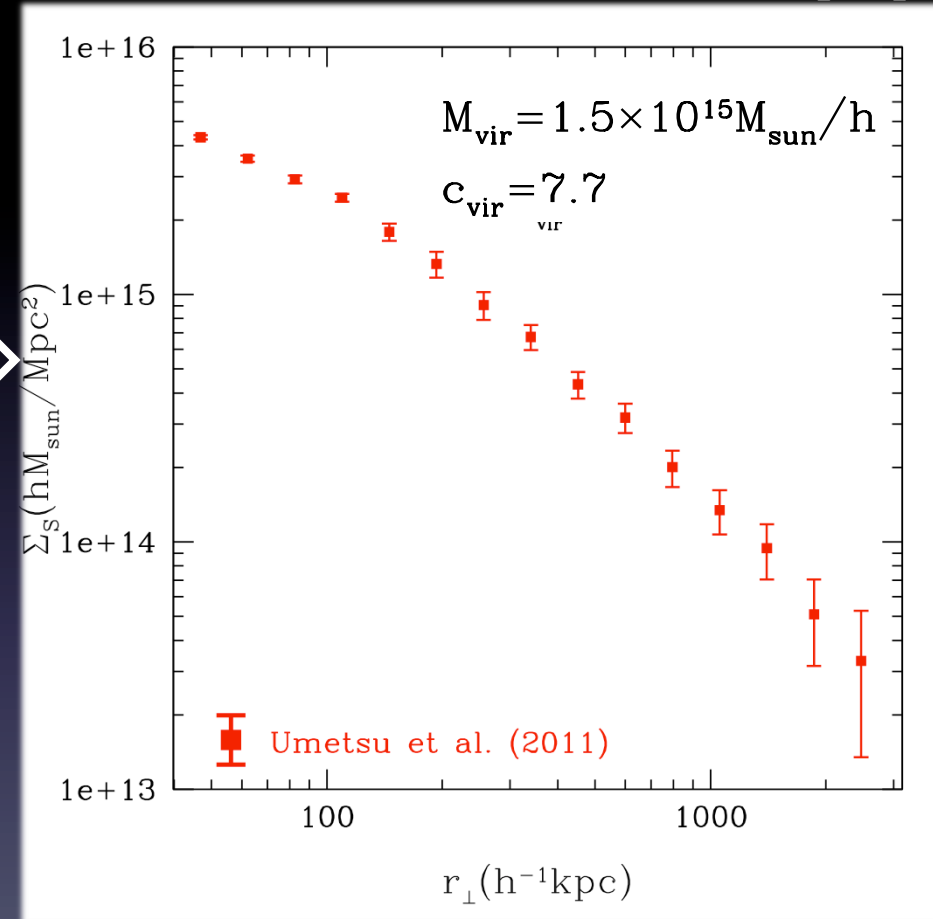


Lensing Potential of Galaxy Cluster

Mass map (shear)



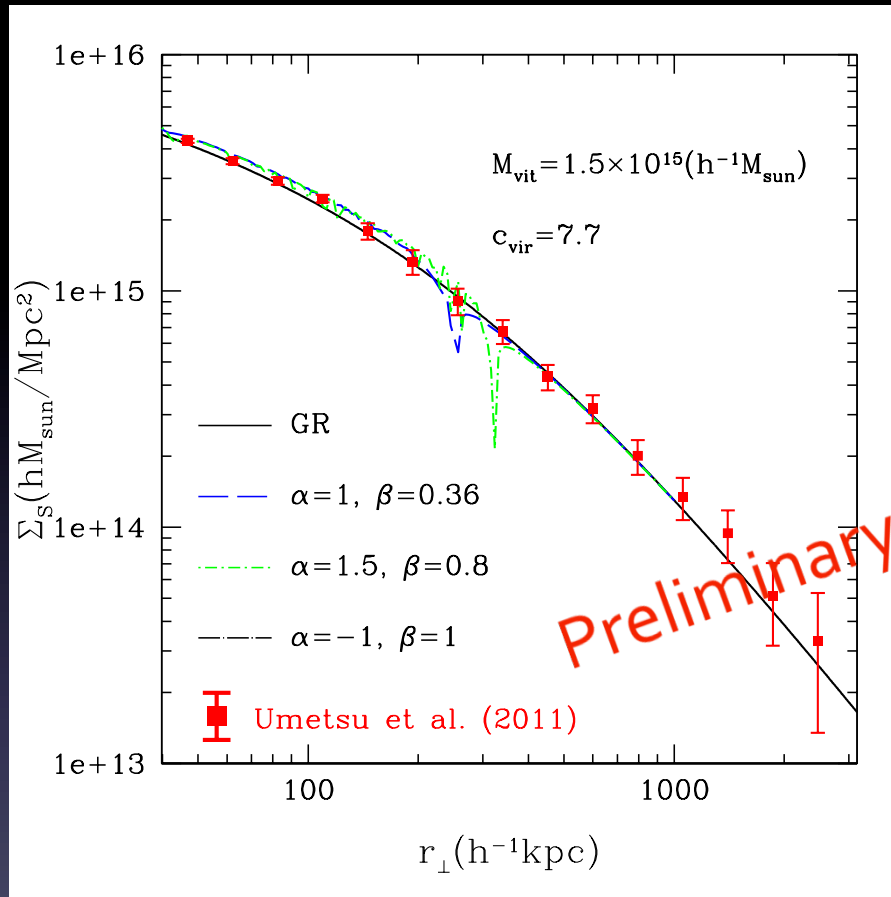
Surface mass density $\Sigma_S(r_{\text{perp}})$



Over a wide range of radius
Small error of the lensing profile data

[Umetzu et al. '11,
cf. Oguri et al. '12]

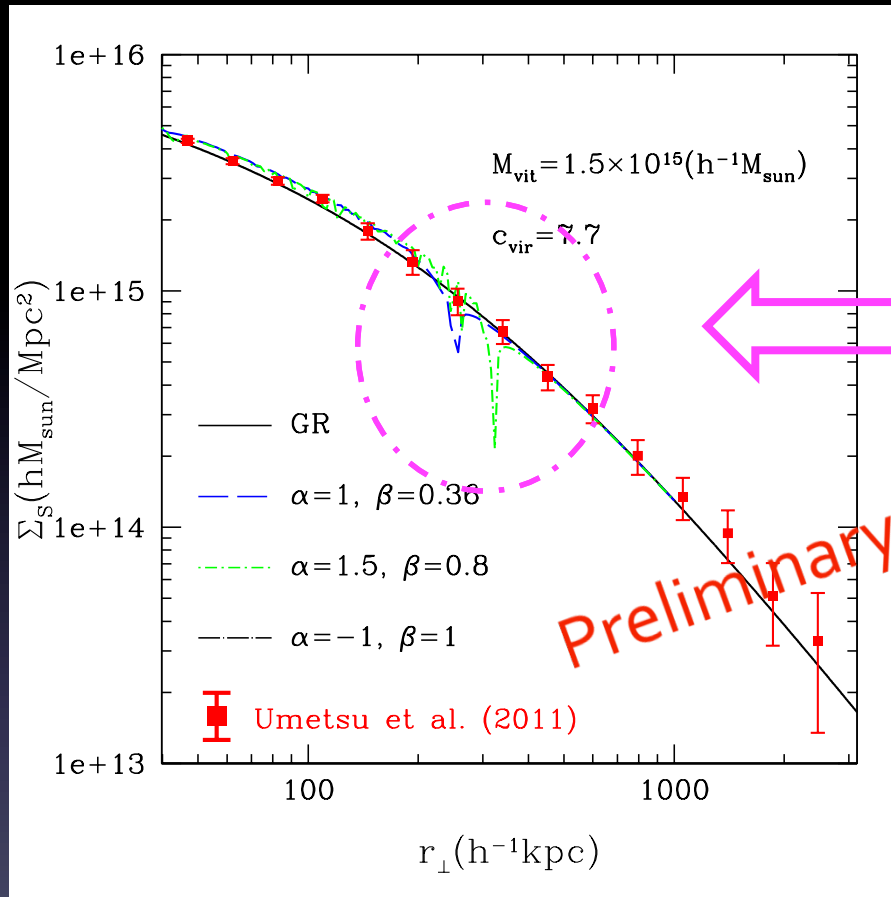
Surface Mass Density in Modified Gravity



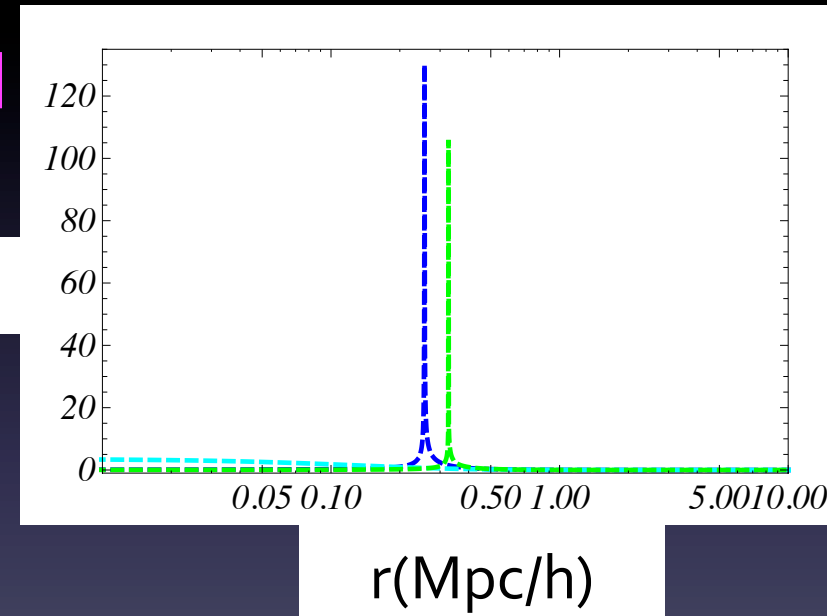
Parameters: $\{\alpha, \beta, \Lambda; \rho_s, r_s\}$
 $(\Lambda^3 = M_{\text{Pl}} / (0.01/H_0)^2, \rho_s, r_s \text{ fixed})$

A dip appears at $r \sim r_V := (r_s M_{\text{Pl}} / \Lambda^3)^{1/3}$ in a typical case.
 → This allows us to put constraint.

Surface Mass Density in Modified Gravity



Parameters: $\{\alpha, \beta, \Lambda; \rho_s, r_s\}$
 $(\Lambda^3 = M_{\text{Pl}} / (0.01/H_0)^2, \rho_s, r_s \text{ fixed})$



An origin of the dip: $x' (\Leftrightarrow \phi'')$

A dip appears at $r \sim r_V := (r_s M_{\text{Pl}} / \Lambda^3)^{1/3}$ in a typical case.

→ This allows us to put constraint.

Summary & Conclusion

- Modified gravity by an ϕ
 - Motivation: Cosmic acceleration
 - With the screening mechanism
- We demonstrate how an effect of ϕ on $\Delta\Phi_+$ appears in General framework (Horndeski's theory). A dip appears!
- This allows us to put a constraint on modified gravity model with lensing profile of galaxy cluster.

Thank you for your attention