# Cosmological models with dynamical scalar torsion

James M. Nester
Dept. of Physics, Inst. of Astronomy and
Center for Mathematics and Theoretical Physics,
National Central University, Chungli Taiwan
based on work with **H. Chen, F.H. Ho**, P. Baekler, F.W. Hehl, C.H. Wang, **H.J. Yo**, K.F. Shie

2012-12-30 LeCosPa, Taipei

# *Abstract, Outline*

For the Poincaré gauge theory of gravity (PG, aka PGT)

two good dynamical torsion modes with spin 0 have been found.

An effective Lagrangian analysis of cosmological models
reveals some of the dynamical possibilities
including damped oscillations in the expansion rate.

- Foundations, the PG theory
- good dynamic modes, the scalar mode model
- PG cosmology, kinematics
- PG scalar cosmology, dynamics
- 2nd order and 1st order eqns
- effective Lagrangian & Hamiltonian
- constant curvature solutions
- linearized theory, late time normal modes
- numerical evolution
- summary

# GEOMETRY: connection, torsion, curvature, metric

A connection determines
the covariant derivative $\nabla_\mu$ and parallel transport.

From a connection 2 tensor fields can be constructed,
curvature and torsion:

$$[\nabla_\mu, \nabla_\nu]V^\alpha = R^\alpha{}_{\beta\mu\nu}V^\beta - T^\gamma{}_{\mu\nu}\nabla_\gamma V^\alpha.$$

Physical spacetime has also a metric $g_{\mu\nu}$. It determines the causal structure, the magnitude of vectors, angles, path length and a relation between tangent vectors and covectors (one-forms).

Consider Riemann-Cartan geometry:
metric compatible connection $\nabla_\mu g_{\alpha\beta} = 0$.
i.e., lengths and angles are preserved under parallel transport.

# *local gauge theories*

- All the known physical interactions can be formulated in a common framework as *local gauge theories*:
- However the standard theory of gravity, Einstein's GR, based on the spacetime metric, is a rather unnatural gauge theory
- Physically (and geometrically) it is reasonable to consider gravity as a gauge theory of the local Poincaré symmetry of Minkowski spacetime
- There is a perfect correspondence between the natural geometric symmetries of Riemann-Cartan geometry and local Poincaré gauge symmetries.

# The Poincaré gauge theory of gravity (PG)

[Hehl '80, Hayashi & Shirafuji '80],
the local gauge potentials are, for translations, the orthonormal co-frame, (which determines the metric):

$$\vartheta^\alpha = e^\alpha{}_i \mathrm{d}x^i \to g_{ij} = e^\alpha{}_i e^\beta{}_j \eta_{\alpha\beta}, \quad \eta_{\alpha\beta} = \text{diag}(-1. + 1, +1, +1),$$

and, for Lorentz/rotations, the metric-compatible (Lorentz) connection (one-form):

$$\Gamma^{\alpha\beta}{}_i \mathrm{d}x^i = \Gamma^{[\alpha\beta]}{}_i \mathrm{d}x^i.$$

The associated field strengths are the *torsion* and *curvature* (2-forms):

$$T^\alpha \quad := D\vartheta^\alpha := \quad \mathrm{d}\vartheta^\alpha + \Gamma^\alpha{}_\beta \wedge \vartheta^\beta = \frac{1}{2} T^\alpha{}_{\mu\nu} \vartheta^\mu \wedge \vartheta^\nu,$$

$$R^{\alpha\beta} \quad := D\Gamma^{\alpha\beta} := \quad \mathrm{d}\Gamma^{\alpha\beta} + \Gamma^\alpha{}_\gamma \wedge \Gamma^{\gamma\beta} = \frac{1}{2} R^{\alpha\beta}{}_{\mu\nu} \vartheta^\mu \wedge \vartheta^\nu,$$

which satisfy the respective Bianchi identities:

$$DT^\alpha \equiv R^\alpha{}_\beta \wedge \vartheta^\beta, \qquad DR^\alpha{}_\beta \equiv 0.$$

# General PG Lagrangian

The general quadratic PG Lagrangian density has the form
(see [Baekler, Hehl & Nester PRD 2011, Baekler & Hehl CQG 2011])

$$\mathscr{L}[\vartheta, \Gamma] \sim \kappa^{-1}\left[\Lambda + \text{scalar curvature} + \text{pseudoscalar curvature} + \text{torsion}^2(3+2)\right] + \varrho^{-1}\left[\text{curvature}^2(6+4)\right].$$

where $\Lambda =$ cosmological constant, $\kappa = 8\pi G/c^4$, $\varrho^{-1}$ has the dimensions of action.

The general theory has 11 scalar parameters and 7 pseudoscalar parameters.

There are 3 total derivative "topological terms"
(one scalar and two pseudoscalar),
which effectively reduce the number of physical parameters to to 10 scalar and 5 pseudoscalar.

Note: There is no fundamental reason to expect gravity to be parity invariant so no fundamental reason to exclude odd parity coupling terms

# *field equations*

Gravitational field eqns are 2nd order eqns for the gauge potentials:

$$\delta\vartheta^{\alpha}{}_{i}: \qquad \Lambda + a_0 G_{\alpha}{}^{i} + DT + T^2 + R^2 = \text{energy-momentum density}$$

$$\delta\Gamma^{\alpha\beta}{}_{k}: \qquad T + DR = \text{source spin density}.$$

Bianchi identities $\Longrightarrow$
conservation of source energy-momentum & angular momentum.

# *good dynamic modes*

early studies did not seriously consider pseudoscalar parameters.

- Investigations of the linearized theory identified six possible dynamic connection modes carrying spin-$2^{\pm}$, $1^{\pm}$, $0^{\pm}$.
  [Hayashi & Shirafuji 1980, Sezgin & van Nuivenhuizen 1980, . . . ]
- A good dynamic mode transports positive energy at speed $\leq c$.
  At most three modes can be simultaneously dynamic;
  all the cases were tabulated;
  many combinations are satisfactory to linear order.
- A Hamiltonian analysis revealed the related constraints
  [Blagojević & Nicolić, 1983].
- Then detailed investigations
  [Hecht, N & Zhytnikov 1996, Chen, N & Yo 1998, Yo & N 1999, 2002]
  concluded that effects due to nonlinearities could be expected to render all of these cases physically unacceptable—
  except for the two "scalar modes": spin-$0^{+}$ and spin-$0^{-}$.

# *exploring the dynamics*

In order to explore the dynamics of these two scalar modes at NCU we considered cosmological models.

The $0^+$ mode was considered first:
[Yo & N, Mod Phys Lett A, 2007], [Shie, N & Yo PRD 2008]

The model was extended to also include the $0^-$ mode
[Chen et al JCAP 2009]

Those investigations did not consider any pseudoscalar parameter terms.

# BHN Lagrangian

- Now, the model has been extended to include parity violating terms [Baekler Hehl & N PRD 2011].
- The Lagrangian of the BHN model is

$$\mathcal{L}[\vartheta,\Gamma] = \frac{1}{2\kappa}\left[-2\Lambda + a_0 R - \frac{1}{2}\sum_{\mathsf{n}=1}^{3} a_{\mathsf{n}} \overset{(\mathsf{n})}{T}{}^2 + b_0 X + 3\sigma_2 T_\mu P^\mu\right] + \frac{1}{2\varrho}\left[\frac{w_6}{12}R^2 + \frac{w_3}{12}X^2 + \frac{\mu_3}{12}RX\right],$$

where $R$ & $X = 6R_{[0123]}$ are the scalar & pseudoscalar curvatures, $T_\mu \equiv T^\alpha{}_{\alpha\mu}$, $P_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}T^\nu{}_{\alpha\beta}$ are the torsion trace & axial vectors and $b_0$ & $\sigma_2$ & $\mu_3$ are the odd parity coupling constants.

There is one odd parity topological identity (Nieh-Yan)

$$d(\vartheta^\alpha \wedge T_\alpha) = T^\alpha \wedge T_\alpha - \vartheta^\alpha \wedge R_{\alpha\beta} \wedge \vartheta^\beta$$

# Cosmological models

- Earlier PGT cosmology: Minkevich [e.g., 1980, 1983, 1995, 2007] and Goenner & Müller-Hoissen [1984];
  recent: Shie, N & Yo [2008], Wang & Wu [2009], Chen et al [2009], Li, Sun & Xi [2009ab], Ao, Li & Xi [2010], Baekler, Hehl & N [2011], Ao & Li [2012], Ho & N [2011, 2012], Tseng, Lee & Geng [2012].
- **manifestly** homogeneous & isotropic models: Bianchi I & IX
  **manifestly** isotropic orthonormal coframe:

$$\vartheta^0 := dt, \qquad \vartheta^a := a\sigma^a,$$

where $a = a(t)$ is the scale factor and $\sigma^j$ depends on the (never needed) spatial coordinates in such a way that

$$d\sigma^i = \zeta \epsilon^i{}_{jk} \sigma^j \wedge \sigma^k,$$

where $\zeta = 0$ for Bianchi I (equivalent to FLRW $k = 0$, which appears to describe our physical universe) and $\zeta = 1$ for Bianchi IX (FLRW $k = +1$), thus $\zeta^2 = k$, the curvature parameter.

- isotropy $\Longrightarrow$ non-vanishing connection one-form coefficients

$$\Gamma^a{}_0 = \psi(t)\,\sigma^a, \quad \Gamma^a{}_b = \chi(t)\epsilon^a{}_{bc}\,\sigma^c,$$

$\Longrightarrow$ nonvanishing curvature components:

$$R^{a0}{}_{b0} = a^{-1}\dot{\psi}\delta^a_b, \quad R^{ab}{}_{0c} = a^{-1}\dot{\chi}\epsilon^{ab}{}_c,$$

$$R^{a0}{}_{bc} = 2a^{-2}\psi(\chi - \zeta)\epsilon^a{}_{bc}, \quad R^{ab}{}_{cd} = a^{-2}(\psi^2 - \chi^2 + 2\chi\zeta)\delta^{ab}_{cd}.$$

$\Longrightarrow$ scalar and pseudoscalar curvatures:

$$\begin{aligned} R &= 6[a^{-1}\dot{\psi} + a^{-2}(\psi^2 - [\chi - \zeta]^2 + \zeta^2)], \\ X &= 6[a^{-1}\dot{\chi} + 2a^{-2}\psi(\chi - \zeta)]. \end{aligned}$$

- isotropy $\Longrightarrow$ nonvanishing torsion tensor components

$$T^a{}_{b0} = u(t)\delta^a_b, \qquad T^a{}_{bc} = -2x(t)\epsilon^a{}_{bc}.$$

they depend on the gauge variables:

$$u = a^{-1}(\dot{a} - \psi), \qquad x = a^{-1}(\chi - \zeta).$$

- isotropy $\Longrightarrow$ energy-momentum tensor has the perfect fluid form with an energy density and pressure: $\rho, p$.

  - When $p = 0$, the gravitating material behaves like dust with

$$\rho a^3 = \text{constant.}$$

- isotropy $\Longrightarrow$ most of the source spin density components vanish. We assume they all vanish (reasonable except in the very early universe).

# *effective Lagrangian, eqns*

- The dynamical equations for the homogeneous cosmology can be obtained by imposing the Bianchi symmetry on the field equations found by BHN from the BHN *Lagrangian density*
- *These same dynamical equations* can be obtained directly from a classical mechanics type effective *Lagrangian*, which in this case can be simply obtained by restricting the BHN Lagrangian density to the Bianchi symmetry.
- This procedure is known to be successful for all Bianchi class A models in GR, and it is conjectured to also be true for the PG theory.

- The effective Lagrangian $L_{\text{eff}} = L_{\text{G}} + L_{\text{int}}$ includes the *interaction* Lagrangian:

$$L_{\text{int}} = pa^3, \qquad p = p(t) \qquad \text{pressure},$$

and the *gravitational* Lagrangian:

$$L_{\text{G}} = \frac{a^3}{\kappa}\left[-\Lambda + \frac{a_0}{2}R + \frac{b_0}{2}X - \frac{3}{2}a_2u^2 + 6a_3x^2 + 6\sigma_2 ux\right] + \frac{a^3}{\varrho}\left[-\frac{w_6}{24}R^2 + \frac{w_3}{24}X^2 - \frac{\mu_3}{24}RX\right]$$

with $a_2 < 0, \quad w_6 < 0, \quad w_3 > 0, \quad -4\alpha := 4w_3w_6 + \mu^2 < 0$
signs necessary for least action/positive kinetic energy

- In the following we often take for simplicity units such that $\kappa = 1 = \varrho$.
- For convenience we introduce the *modified* parameters $\tilde{a}_2, \tilde{a}_3, \tilde{\sigma}_2$ with the definitions

$$\tilde{a}_2 := a_2 - 2a_0, \qquad \tilde{a}_3 := a_3 - \frac{1}{2}a_0, \qquad \tilde{\sigma}_2 := \sigma_2 + b_0.$$

# Energy function

- The energy function obtained from $L_G$ is an effective energy; $G_{00}$, the "Hamiltonian constraint" with magnitude $-a^3\rho$:

$$
\begin{aligned}
\mathcal{E} \;=\; & a^3\Bigg\{\frac{3}{2}\tilde{a}_2 u^2 - 3a_0 H^2 - 6\tilde{a}_3 x^2 - 3\tilde{a}_2 u H + \Lambda \\
& +6\tilde{\sigma}_2 x(H-u) - 3a_0\frac{\zeta^2}{a^2} \\
& -\frac{w_6}{24}\left[R^2 - 12R\left\{(H-u)^2 - x^2 + \frac{\zeta^2}{a^2}\right\}\right] \\
& +\frac{w_3}{24}\left[X^2 + 24Xx(H-u)\right] \\
& -\frac{\mu_3}{24}\left[RX - 6X\left\{(H-u)^2 - x^2 + \frac{\zeta^2}{a^2}\right\} + 12Rx(H-u)\right]\Bigg\},
\end{aligned}
$$

time independent Lagrangian $\Longrightarrow$ work-energy relation:

$$\frac{d(\rho a^3)}{dt} = -p\frac{da^3}{dt}, \quad \text{if } p = 0 = \Lambda, \text{late-time field fall-off } \; a^{-3/2}$$

# The Dynamical Equations

- 2nd order Lagrange eqns for $\psi$, $\chi$ and a:

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{\psi}} &= \frac{d}{dt}\left(a^2\left[3a_0 - \frac{w_6}{2}R - \frac{\mu_3}{4}X\right]\right) = \frac{\partial L_G}{\partial \psi} \\
&= 3(a_2 u - 2\sigma_2 x)a^2 + \left[6a_0 - w_6 R - \frac{\mu_3}{2}X\right] a\psi \\
&\quad + \left[6b_0 - \frac{\mu_3}{2}R + w_3 X\right] a(\chi - \zeta), \qquad \Longrightarrow \dot{R}, \dot{X}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{\chi}} &= \frac{d}{dt}\left(a^2\left[3b_0 - \frac{\mu_3}{4}R + \frac{w_3}{2}X\right]\right) = \frac{\partial L_G}{\partial \chi} \\
&= -6(2a_3 x + \sigma_2 u)a^2 - \left[6a_0 - w_6 R - \frac{\mu_3}{2}X\right] a(\chi - \zeta) \\
&\quad + \left[6b_0 - \frac{\mu_3}{2}R + w_3 X\right] a\psi, \qquad \Longrightarrow \dot{R}, \dot{X}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\frac{\partial L_G}{\partial \dot{a}} &= \frac{d}{dt}\left(-a^2 3[a_2 u - 2\sigma_2 x]\right) = \frac{\partial L_G}{\partial a} + \frac{\partial L_{int}}{\partial a} \\
&= 3a^{-1}L - \left(\frac{a_0}{2} - \frac{w_6}{12}R - \frac{\mu_3}{24}X\right)[a^2 R + 6(\psi^2 - [\chi - \zeta]^2 + \zeta^2)] \\
&\quad - \left(\frac{b_0}{2} + \frac{w_3}{12}X - \frac{\mu_3}{24}R\right)[a^2 X + 12\psi(\chi - \zeta)] \\
&\quad + 3a^2(a_2 u - 2\sigma_2 x)u - 6a^2[2a_3 x + \sigma_2 u]x + 3pa^2, \quad \Longrightarrow \dot{u}, \dot{x}.
\end{aligned}
$$

- First order eqns from:

$$
\begin{aligned}
\dot{a} &= aH \qquad \text{Hubble relation} \\
\dot{x} &= -Hx - \frac{X}{6} - 2x(H - u), \\
\dot{H} - \dot{u} &= \frac{R}{6} - H(H - u) - (H - u)^2 + x^2 - \frac{\zeta^2}{a^2}.
\end{aligned}
$$

# First order equations with parity coupling

$$
\begin{aligned}
\dot{a} &= aH, \\
\dot{H} &= \frac{1}{6a_2}(\tilde{a}_2 R - 2\tilde{\sigma}_2 X) - 2H^2 + \frac{\tilde{a}_2 - 4\tilde{a}_3}{a_2}x^2 - \frac{\zeta^2}{a^2} \\
&\quad + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2}, \\
\dot{u} &= -\frac{1}{3a_2}(a_0 R + \tilde{\sigma}_2 X) - 3Hu + u^2 - \frac{4a_3}{a_2}x^2 \\
&\quad + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2}, \\
\dot{x} &= -\frac{X}{6} - (3H - 2u)x, \\
-\frac{w_6}{2}\dot{R} - \frac{\mu_3}{4}\dot{X} &= \left[3\tilde{a}_2 + w_6 R + \frac{\mu_3}{2}X\right]u + \left[-6\tilde{\sigma}_2 + \frac{\mu_3}{2}R - w_3 X\right]x \\
-\frac{\mu_3}{4}\dot{R} + \frac{w_3}{2}\dot{X} &= \left[-6\tilde{\sigma}_2 + \frac{\mu_3}{2}R - w_3 X\right]u - \left[12\tilde{a}_3 + w_6 R + \frac{\mu_3}{2}X\right]x
\end{aligned}
$$

For our numerical evolution we consider only $p = 0$, dust.

Note obvious special constant curvature solutions.

# Isotropic Bianchi V

Isotropic Bianchi V is equivalent to the FLRW $k = -1$ model.

It is a class B model; the effective Lagrangian method is not expected to succeed. Let us try it and see what happens.

Following the above approach with suitably modifications:

**Type V coframe**

$$\vartheta^0 := dt, \qquad \vartheta^a := a\sigma^a,$$

where $a = a(t)$ is the scale factor and $\sigma^j$ depends on the (never needed) spatial coordinates in such a way that

$$d\sigma^i = \sigma^1 \wedge \sigma^i.$$

**connection** one-form components

$$\Gamma^{[ab]} := (\chi\epsilon^{ab}{}_c + \delta^{ab}_{c1})\sigma^c, \qquad \Gamma^a{}_0 := \psi\sigma^a,$$

where $\chi = \chi(t),\ \psi = \psi(t)$.

## torsion

$$T^0 = 0, \qquad T^a = [\dot{a} - \psi] dt \wedge \sigma^a - a\chi \epsilon^a{}_{bc} \sigma^b \wedge \sigma^c, \quad \text{(2-form)}$$

$$T^a{}_{0b} = u\delta^a_b = a^{-1}[\dot{a} - \psi]\delta^a_b, \qquad T^a{}_{bc} = -2x\epsilon^a{}_{bc} = -2a^{-1}\chi\epsilon^a{}_{bc},$$

(frame components)

$$u = a^{-1}[\dot{a} - \psi], \qquad x = a^{-1}\chi. \quad \text{(torsion scalar and pseudoscalar)}$$

Note that $a^{-1}\psi = H - u$, where $H = a^{-1}\dot{a}$ is the *Hubble function*.

**curvature** components

$$
\begin{aligned}
R^{ab}{}_{0c} &= a^{-1}\dot{\chi}\epsilon^{ab}{}_{c}, \\
R^{ab}{}_{cd} &= a^{-2}[\psi^2 - \chi^2 - 1], \\
R^{a0}{}_{b0} &= a^{-1}\dot{\psi}\delta^a_b, \\
R^{a0}{}_{bc} &= 2a^{-1}\psi\chi\epsilon^{a}{}_{bc}.
\end{aligned}
$$

**scalar curvature**

$$R = 6[a^{-1}\dot{\psi} + a^{-2}(\psi^2 - \chi^2 - 1)].$$

**pseudoscalar curvature**

$$X = 6[a^{-1}\dot{\chi} + 2a^{-2}\psi\chi].$$

# *Isotropic Bianchi V energy function & dynamical equations*

The results obtained from the detailed calculations using the above effective Lagrangian are similar to those found for the Bianchi I,IX expressions:

One need merely make the simple replacements $\chi - \zeta \to \chi$, $\zeta^2 = k \to -1$.

The final equations completely agree with the BHN FLRW $k = -1$ case.

# Hamiltonian formulation

From the effective Lagrangian we have also found a Hamiltonian formulation for the manifestly homogeneous-isotropic Bianchi I,V,IX models.

This is the most powerful formulation for analytical investigations.

# Constant Curvature: a special subclass

From the 6 linear equations, for $\dot{R} = \dot{X} = 0$, $\Rightarrow \tilde{a}_2 = 4\tilde{a}_3$:

$$R = \frac{3}{\alpha}(w_3\tilde{a}_2 - \mu_3\tilde{\sigma}_2) \quad \text{and} \quad X = \frac{3}{\alpha}(\frac{\mu_3}{2}\tilde{a}_2 + 2w_6\tilde{\sigma}_2).$$

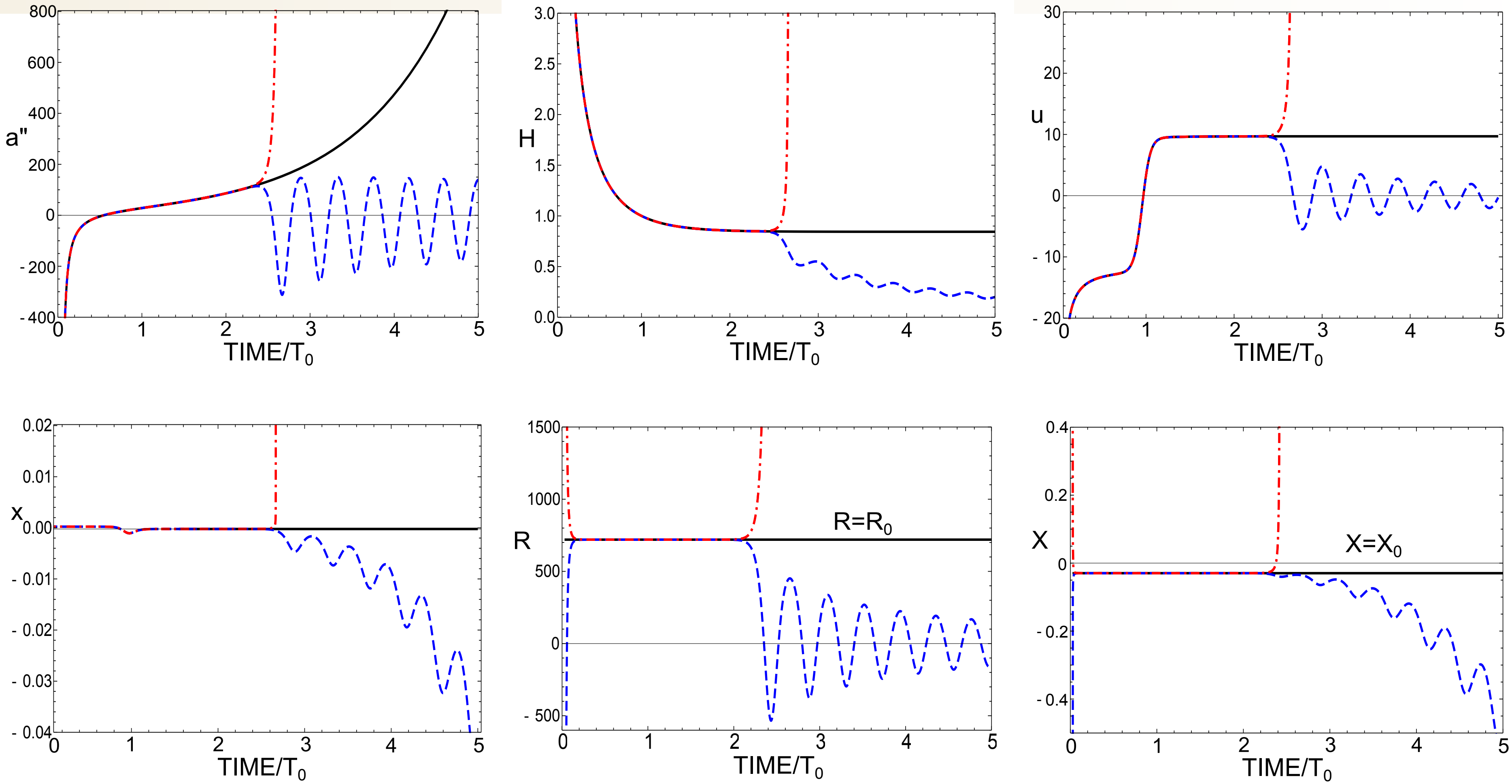
In this case the 6 equations reduce to the 4 dynamical equations:

$$\begin{aligned}
\dot{a} &= aH, \\
\dot{H} &= -2H^2 - \frac{\zeta^2}{a^2} + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2} + \frac{1}{6a_2}(\tilde{a}_2 R_0 - 2\tilde{\sigma}_2 X_0), \\
\dot{u} &= -3Hu + u^2 - x^2 + \frac{(\rho - 3p)}{3a_2} + \frac{4\Lambda}{3a_2} - \frac{1}{3a_2}(a_0 R_0 + \tilde{\sigma}_2 X_0), \\
\dot{x} &= -\frac{X}{6} - (3H - 2u)x,
\end{aligned}$$

along with the energy with constant curvatures $R_0$ and $X_0$:

$$\mathcal{E} = -a^3\rho = a^3\left[\frac{3w_3}{8\alpha}(\tilde{a}_2 - \frac{\mu_3\tilde{\sigma}_2}{w_3})^2 + \frac{3\tilde{\sigma}_2^2}{8w_3} - \frac{3}{2}a_2(H^2 + \frac{k}{a^2}) + \Lambda\right],$$

To have a positive $\rho$ (with vanishing $\Lambda, \zeta$) we tried evolution with some "unphysical" parameter values; it is unstable.




The (black) solid lines represent the result of $R(t=0) = R_0$, $X(t=0) = X_0$ the (blue) dashed lines represent the result of $R(t=1) = R_0 - 10^{-8}$, and the (red) dot-dashed lines represent the result of $R(t=1) = R_0 + 10^{-8}$ while all the other initial choices are fixed.

# Linearized first order equations

$$
\begin{aligned}
\dot{a} &= aH,\\
3a_2\dot{H} &= \frac{1}{2}\tilde{a}_2 R - \tilde{\sigma}_2 X,\\
3a_2\dot{u} &= -a_0 R - \tilde{\sigma}_2 X,\\
\dot{x} &= -\frac{X}{6},\\
-\frac{w_6}{2}\dot{R} - \frac{\mu_3}{4}\dot{X} &= 3\tilde{a}_2 u - 6\tilde{\sigma}_2 x,\\
-\frac{\mu_3}{4}\dot{R} + \frac{w_3}{2}\dot{X} &= -6\tilde{\sigma}_2 u - 12\tilde{a}_3 x,
\end{aligned}
$$

the associated "energy"

$$
\begin{aligned}
\mathcal{E} \;=\; a^3 \Bigg\{ & \frac{3}{2}\tilde{a}_2 u^2 - 3a_0 H^2 - 6\tilde{a}_3 x^2 - 3uH\tilde{a}_2 \\
& + 6\tilde{\sigma}_2 x(H-u) - \frac{w_6}{24}R^2 + \frac{w_3}{24}X^2 - \frac{\mu_3}{24}RX \Bigg\}.
\end{aligned}
$$

# normal modes

The variable combination

$$z := a_0 H + \frac{\tilde{a}_2}{2} u - \tilde{\sigma}_2 x,$$

to linear order is *constant*. It describes a *zero frequency normal mode*.

Two pairs of equations have a neat matrix form:

$$\mathbb{T} \begin{pmatrix} \dot{R} \\ \dot{X} \end{pmatrix} = -6\mathbb{M} \begin{pmatrix} u \\ x \end{pmatrix}, \qquad \begin{pmatrix} \dot{u} \\ \dot{x} \end{pmatrix} = -\mathbb{N} \begin{pmatrix} R \\ X \end{pmatrix},$$

which combine to give a 2nd order system:

$$\mathbb{T} \begin{pmatrix} \ddot{R} \\ \ddot{X} \end{pmatrix} = -\mathbb{V} \begin{pmatrix} R \\ X \end{pmatrix},$$

The matrices $\mathbb{T}, \mathbb{V}$ turn out to be *symmetric*, so a standard technique gives 2 orthogonal normal modes and their eigenfrequencies.

# Late time asymptotical expansion

- At late times the scale factor $a$ is large. For $\Lambda = 0$ the quadratic terms will dominate, then $H$, $u$, $x$, $R$, and $X$ should have a $a^{-3/2}$ fall off. Let

$$H = \underline{H} a^{-3/2},\ u = \underline{u} a^{-3/2},\ x = \underline{x} a^{-3/2},\ R = \underline{R} a^{-3/2},\ X = \underline{X} a^{-3/2},$$

dropping higher order terms, gives 6 linear equations with odd parity coupling:

$$\dot{a} = a^{-1/2} \underline{H}, \qquad \dot{\underline{H}} = \frac{1}{6a_2}[\tilde{a}_2 \underline{R} - 2\tilde{\sigma}_2 \underline{X}],$$

$$\dot{\underline{x}} = -\frac{\underline{X}}{6}, \qquad \dot{\underline{u}} = -\frac{1}{3a_2}[a_0 \underline{R} + \tilde{\sigma}_2 \underline{X}],$$

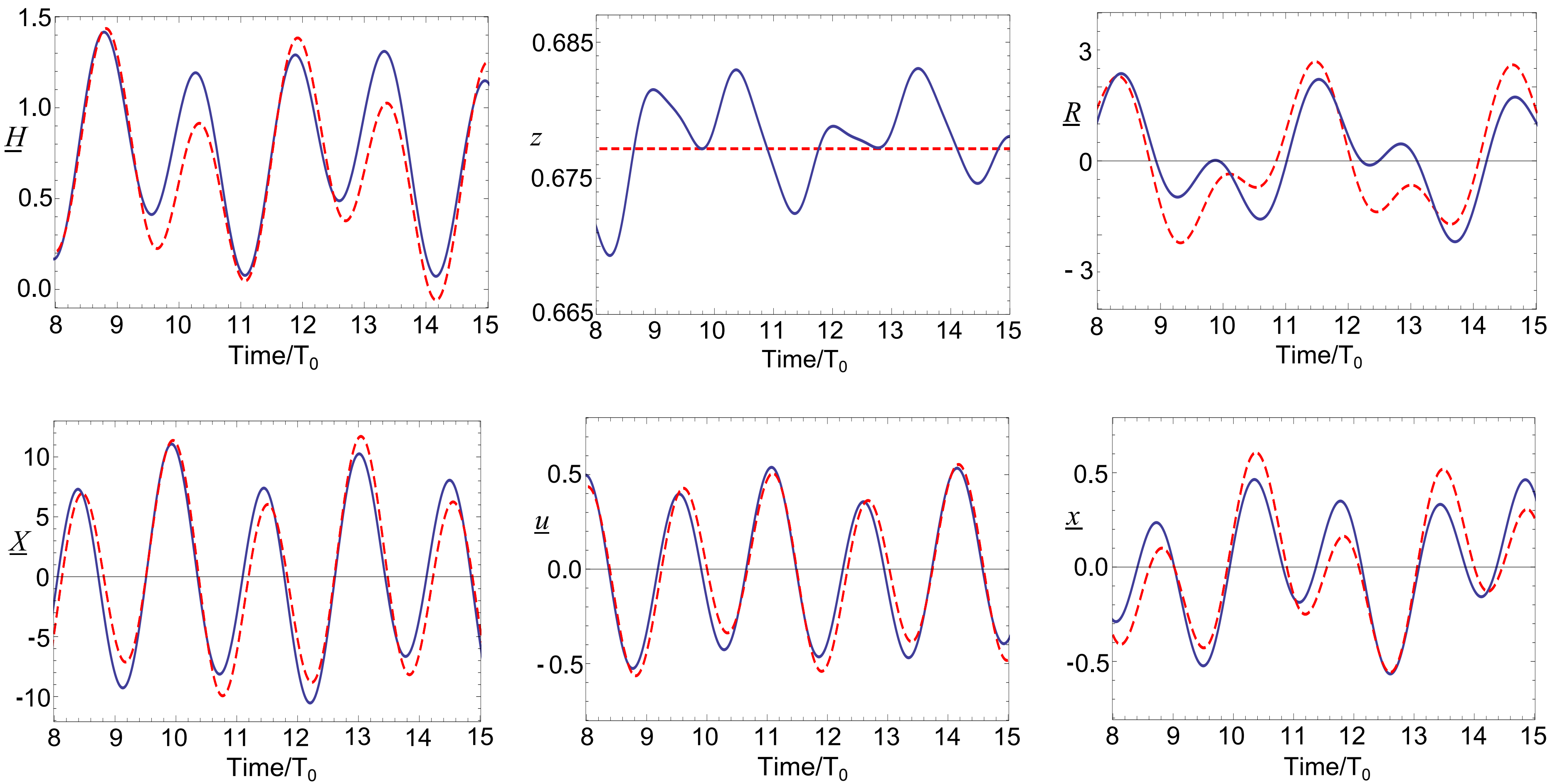
$$\dot{\underline{R}} = \frac{6}{\alpha}\left[(w_3 \tilde{a}_2 - \mu_3 \tilde{\sigma}_2)\underline{u} - 2(w_3 \tilde{\sigma}_2 + \mu_3 \tilde{a}_3)\underline{x}\right],$$

$$\dot{\underline{X}} = \frac{6}{\alpha}[(2w_6 \tilde{\sigma}_2 + \frac{1}{2}\mu_3 \tilde{a}_2)\underline{u} + (4w_6 \tilde{a}_3 - \mu_3 \tilde{\sigma}_2)\underline{x}],$$

plus the energy constraint

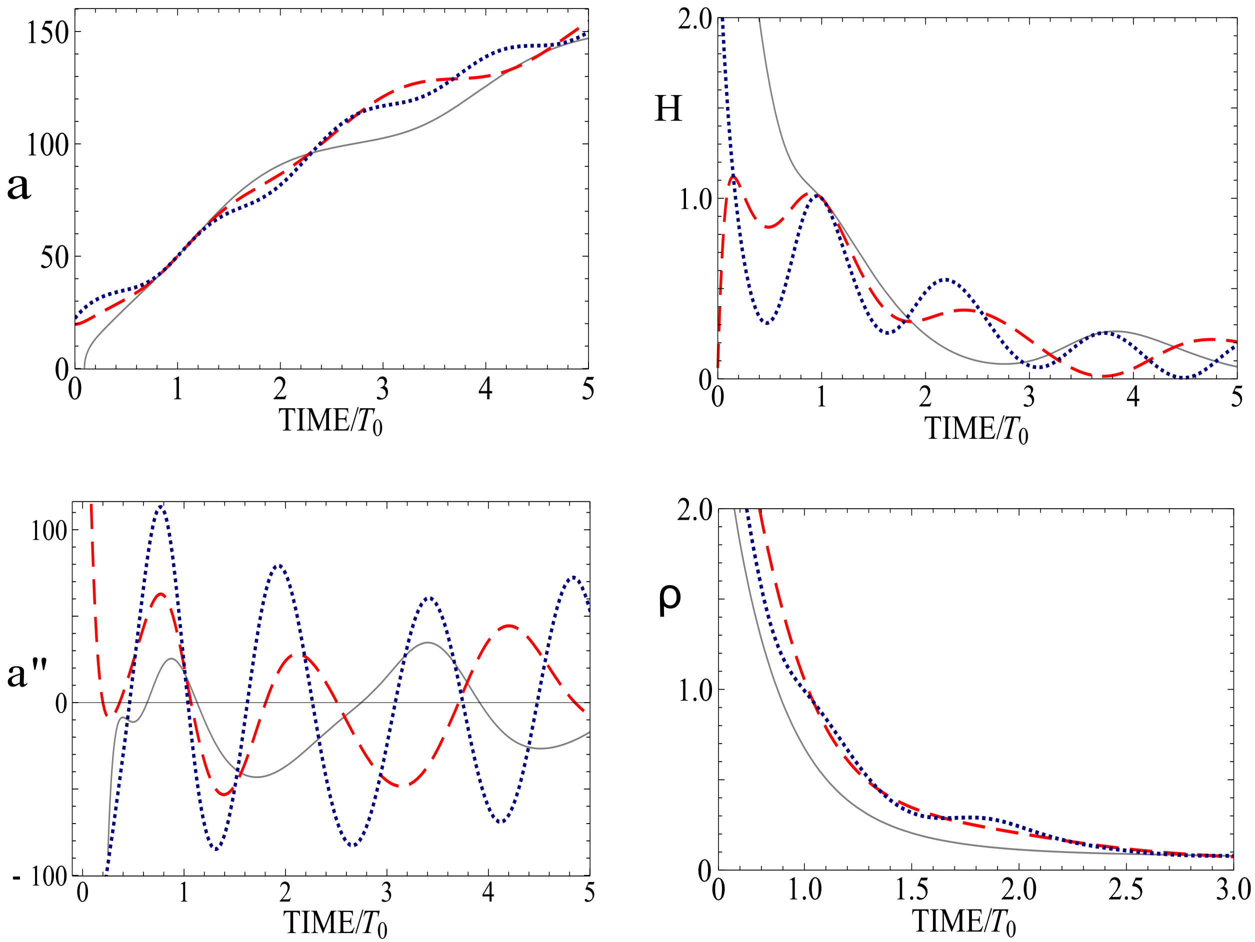
$$-a^3 \kappa \rho = \frac{3\tilde{a}_2}{2}(\underline{H} - \underline{u})^2 - \frac{3}{2} a_2 \underline{H}^2 + 6\tilde{\sigma}_2 \underline{x}(\underline{H} - \underline{u}) - 6\tilde{a}_3 \underline{x}^2 + \frac{w_3}{24}\underline{X}^2 - \frac{w_6}{24}\underline{R}^2 - \frac{\mu_3}{24} RX.$$

# Linear late time evolution




Hubble function $\underline{H}$, “constant mode” $z$, scalar curvature $\underline{R}$, pseudoscalar curvature $\underline{X}$, scalar torsion $\underline{u}$ and pseudoscalar torsion, $\underline{x}$. The blue (solid) lines represent the rescaled late time evolution and the red (dashed) lines represent the linear approximation evolution.

# The effect of odd coupling parameters:




The effect of the cross coupling odd parity parameters $\sigma_2$ and $\mu_3$ (i). The red (dashed) line represents the evolution with the parameter $\sigma_2$ activated. The blue (doted) line represents the evolution including both pseudoscalar parameters $\sigma_2$ and $\mu_3$.

# Typical time evolution:

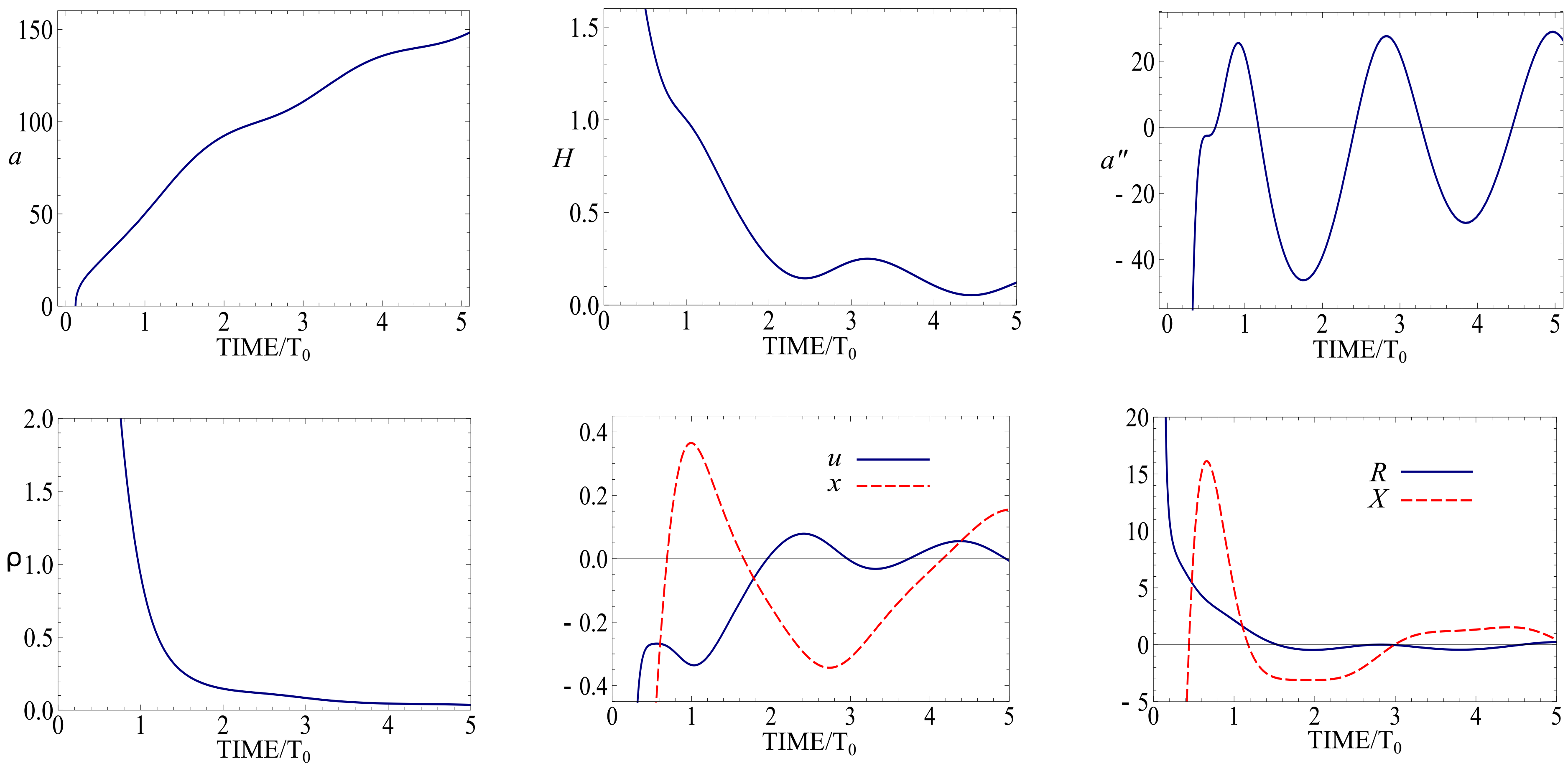



Figure 1: Full evolution of $a$, $H$, $\ddot{a}$, $\rho$, $u$ and $x$, $R$, $X$ for Case I.

# *supernova distance modulus $\mu$ vs redshift $z$*

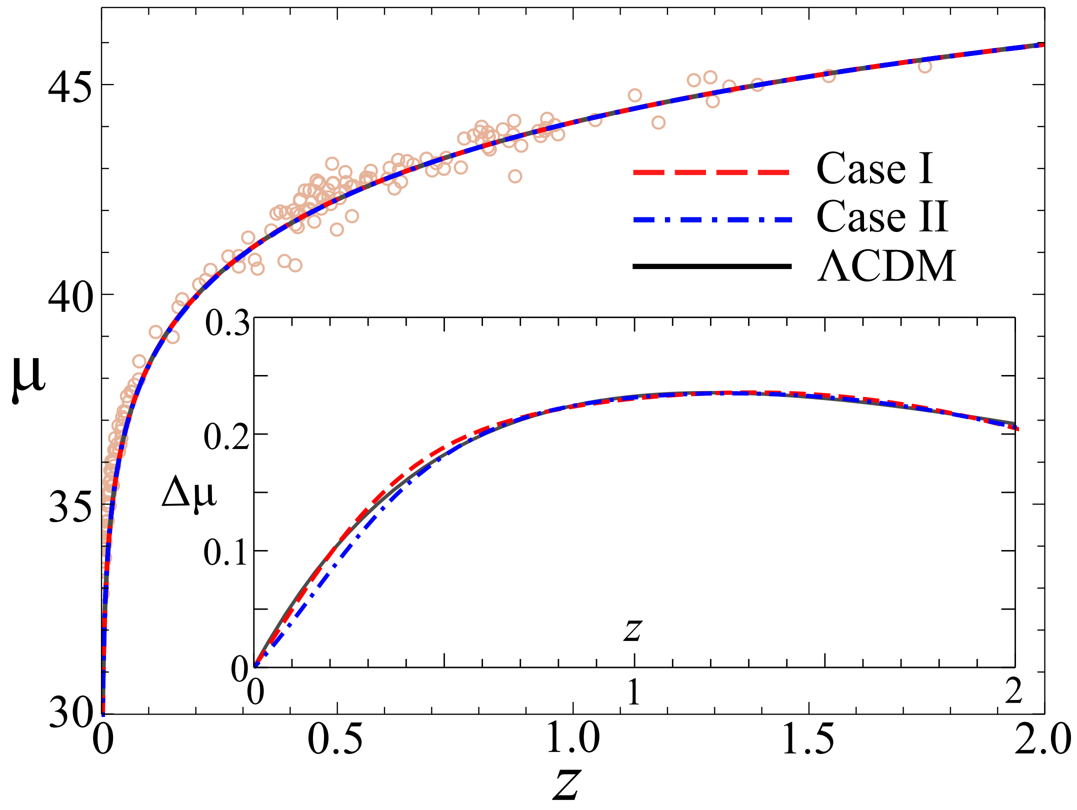



Figure 2:
Supernovae data (brown) circles. Standard $\Lambda$CDM model ($\Omega_{\rm m} = 0.3$, $\Omega_\Lambda = 0.7$) bold solid line. Scalar models I, II, and III: **red** dashed line, the **green** dot-dashed line, and the **blue** dotted line. Inset, the models and data relative to an empty universe model ($\Omega = 0$).

# Summary and concluding remarks

- We considered the dynamics of the BHN model in the context of manifestly homogeneous and isotropic Bianchi I, V & IX cosmologies.
- The BHN cosmological system of ODEs resemble those of a particle with 3 degrees of freedom. Putting the homogeneous & isotropic Bianchi I, V and IX symmetry into the BHN PG theory Lagrangian density leads to an effective Lagrangian which directly gives the evolution equations. We also obtained the associated Hamilton equations.
- Imposing symmetries and variations do not commute in general. For GR they commute for all Bianchi class A models. Here, for the BHN PG model we found that they commute for the class A isotropic Bianchi I and IX, ($\equiv$ FLRW $k = 0$ and $k = +1$) models and, surprisingly, also for the class B isotropic Bianchi V Model (FLRW $k = -1$).

- The system of first order equations obtained from an effective Lagrangian was linearized, the normal modes were identified, and it was shown analytically how they control the late time asymptotics.
- numerical evolution examples show that the late time linear mode approximation is good,
- In these models, at late times the acceleration oscillates. It can be positive at the present time.
- The scalar and pseudoscalar torsion modes do not directly couple to any known form of matter,
- the scalar mode does couple directly to the Hubble expansion, and thus it can directly influence the acceleration of the universe.

# *References*


Hehl, von der Heyde, Kerlik & Nester, Rev Mod Phys **48** (1976) 393

Hecht, Nester & Zhytnikov, Phys Lett A **222** (1996) 37 (1996)

Chen, Nester & Yo, Acta Phys Pol B **29** (1998) 961

Yo & Nester, Int J Mod Phys D **8** (1999) 459; **11** (2002) 747

Yo & Nester, Mod Phys Lett A **22** (2007) 2057

Shie, Nester, & Yo, Phys Rev D **78** (2008) 023522

Chen, Ho, Nester, Wang & Yo, JCAP10 (2009) 027

Baekler, Hehl & Nester, Phys Rev D **83** (2011) 024001

Ho & Nester, J Phys: Conf Ser **330** (2011) 012005 [arXiv:1105.5001];
Int J Mod Phys D **20** (2011) 2125 [arXiv:1106.0711];
Ann Phys (Berlin) **524** (2012) 97

H. Chen, PhD Thesis NCU (2012)