3rd International workshop on “DM, DE & M-AA”, NTHU & NTU

Gravitational Wave Background: Astrophysical Origin

Zhu, Zong-Hong

朱, 宗 宏 (zhuzh@bnu.edu.cn)

Beijing Normal University, Beijing 100875, China

北京 師範 大學, 北京 100875, 中國

&

Blair, Fan, Howell, Regimbau , Zhu

- ***Stochastic Gravitational Wave Background*** from Neutron Star r-mode Instability Revisited
  X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

- ***Stochastic Gravitational Wave Background*** from Coalescing Binary Black Holes
  X.-J. Zhu, E. Howell, T. Regimbau, D. Blair, Z-HZ 2011 ApJ, 739 , 86

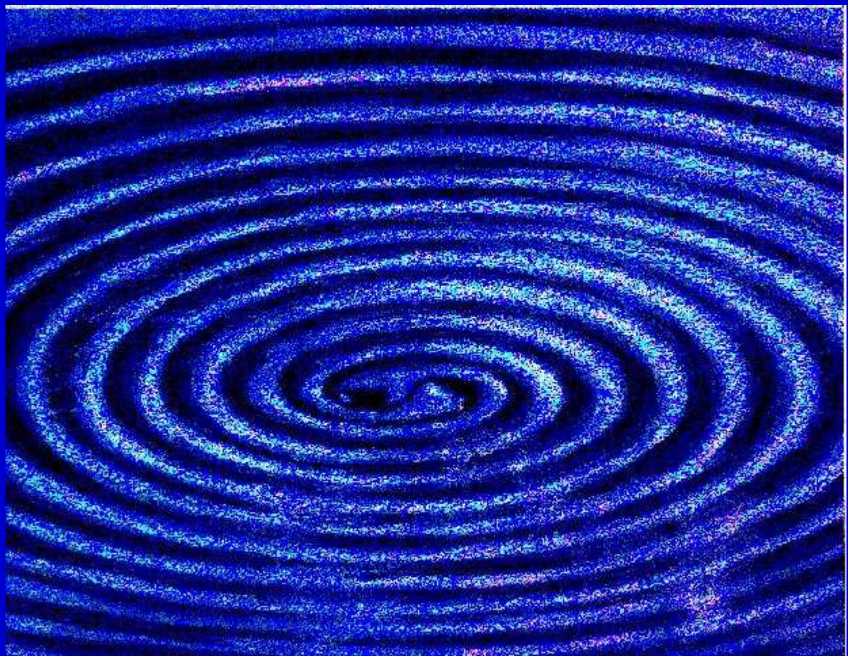
Contents

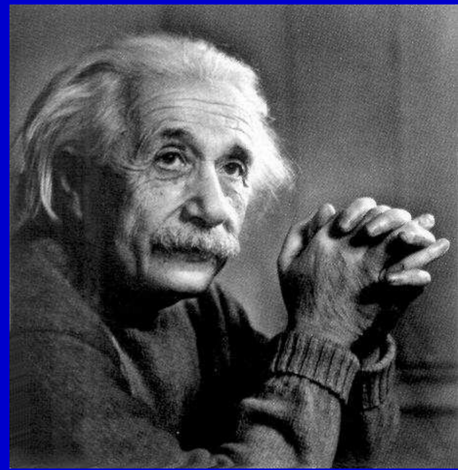
- Introduction
- SGWB from NS r-mode Instability Revisited
- SGWB from Coalescing Binary Black Holes
- Summary

Gravitation Theory: Einstein vs. Newton

Newton's Theory

"instantaneous action at a distance"



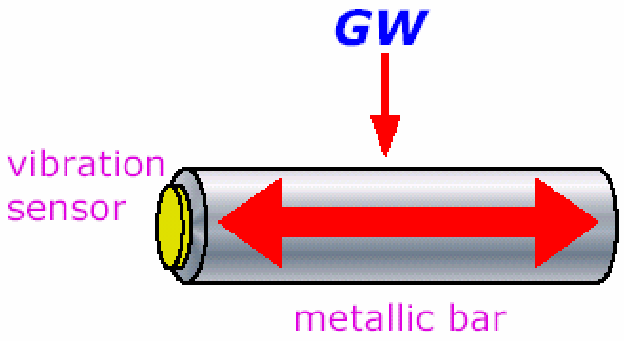


Einstein's Theory

information carried by gravitational radiation at the speed of light

Detection of Gravitational Wave(GW): Methods

- ***Resonant mass detector***

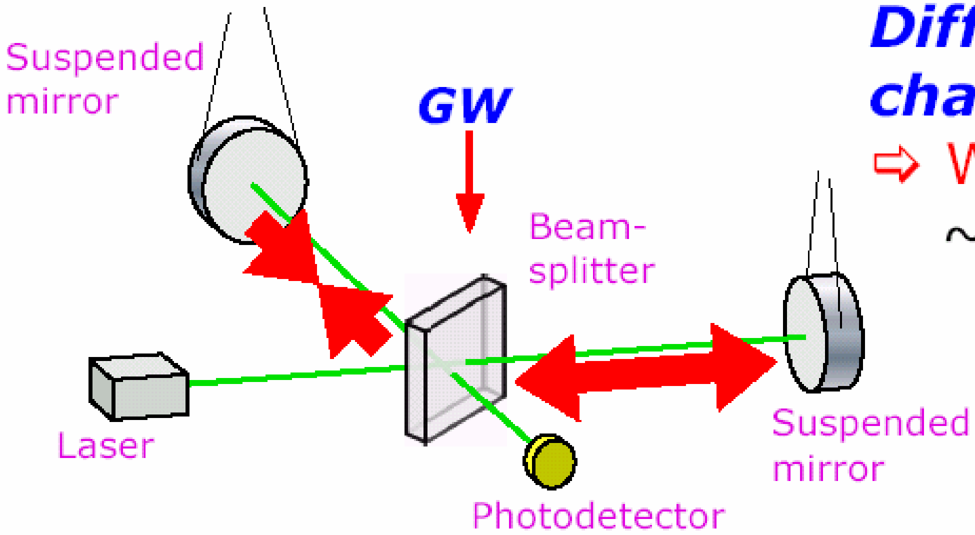


Excitation of quadrupole modes of the bar

⇨ Narrow-band detection

~ only at the resonant frequency

- ***Interferometric detector***



Differential pathlength change of two arms

⇨ Wide-band detection

~ waveform is preserved

Detection of GWs: Resonant mass detectors

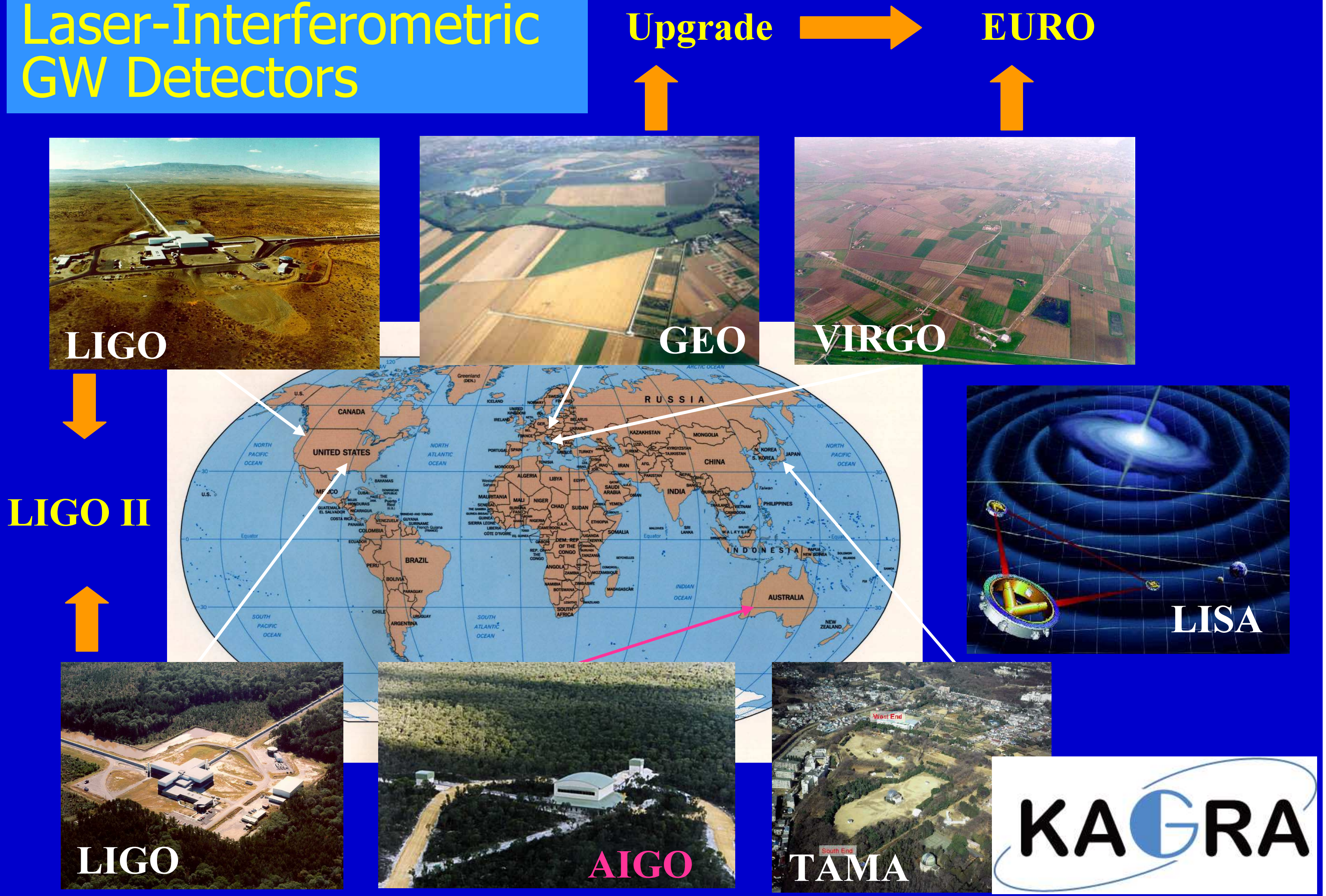
- Joseph Weber constructed the first practical instruments for detecting GWs in 1960.

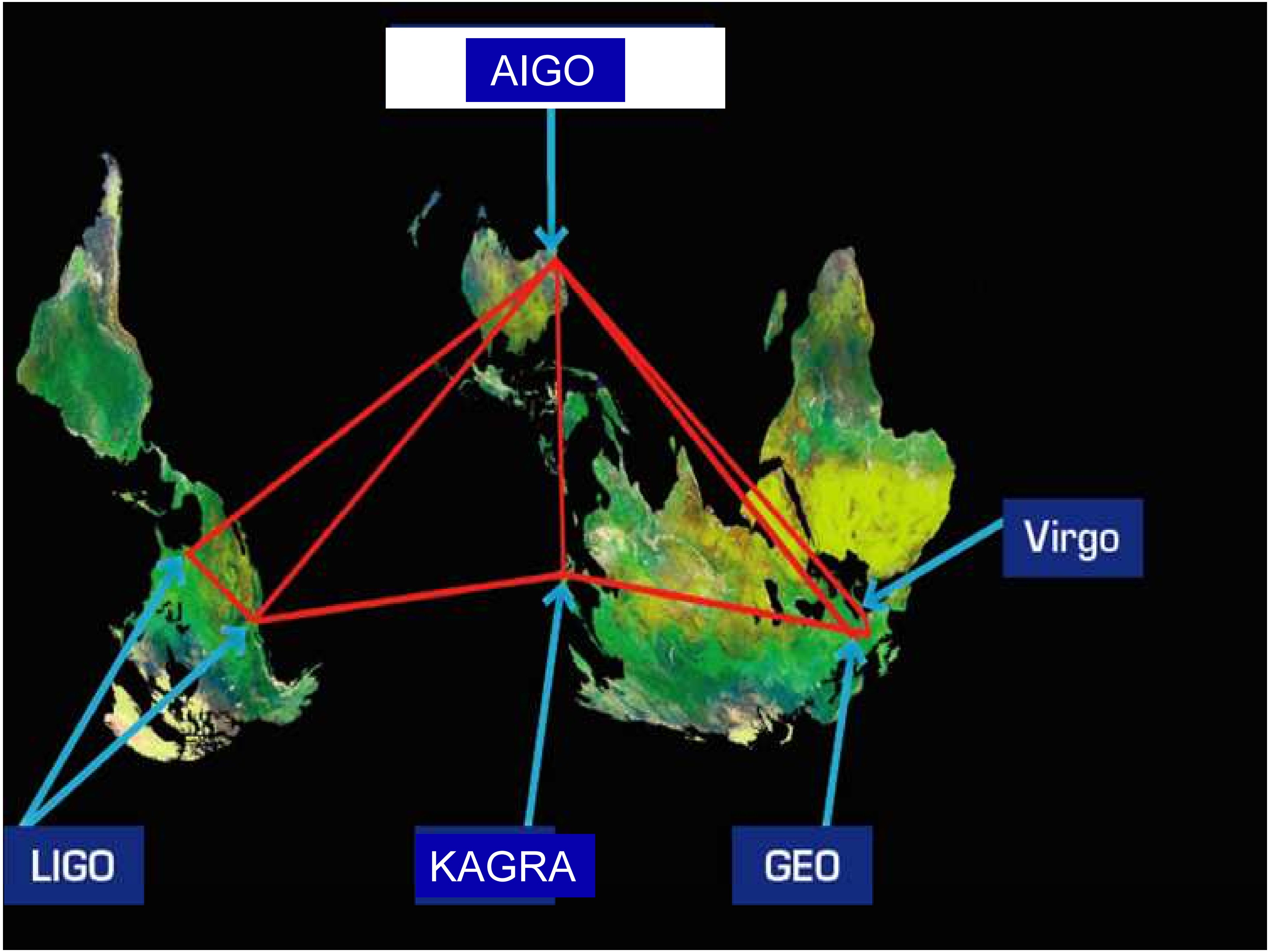
- 1990s, cryogenic bar detectors have been operating (sensitivity ~ 10^{-20}).
  - NAUTILUS (Rome, Italy,907 & 922Hz),
  - AURIGA (Padua, Italy，911 & 939Hz),
  - EXPLORER (CERN, 905 & 921Hz),
  - ALLEGRO (Louisiana, USA, 897 & 920Hz),
  - NIOBE (Perth, Australia, 694.6 & 713Hz).
- 2000s, cryogenic sphere detectors will operate (sensitivity ~ 10^{-21}).

Detection of GWs: Interferometric detectors

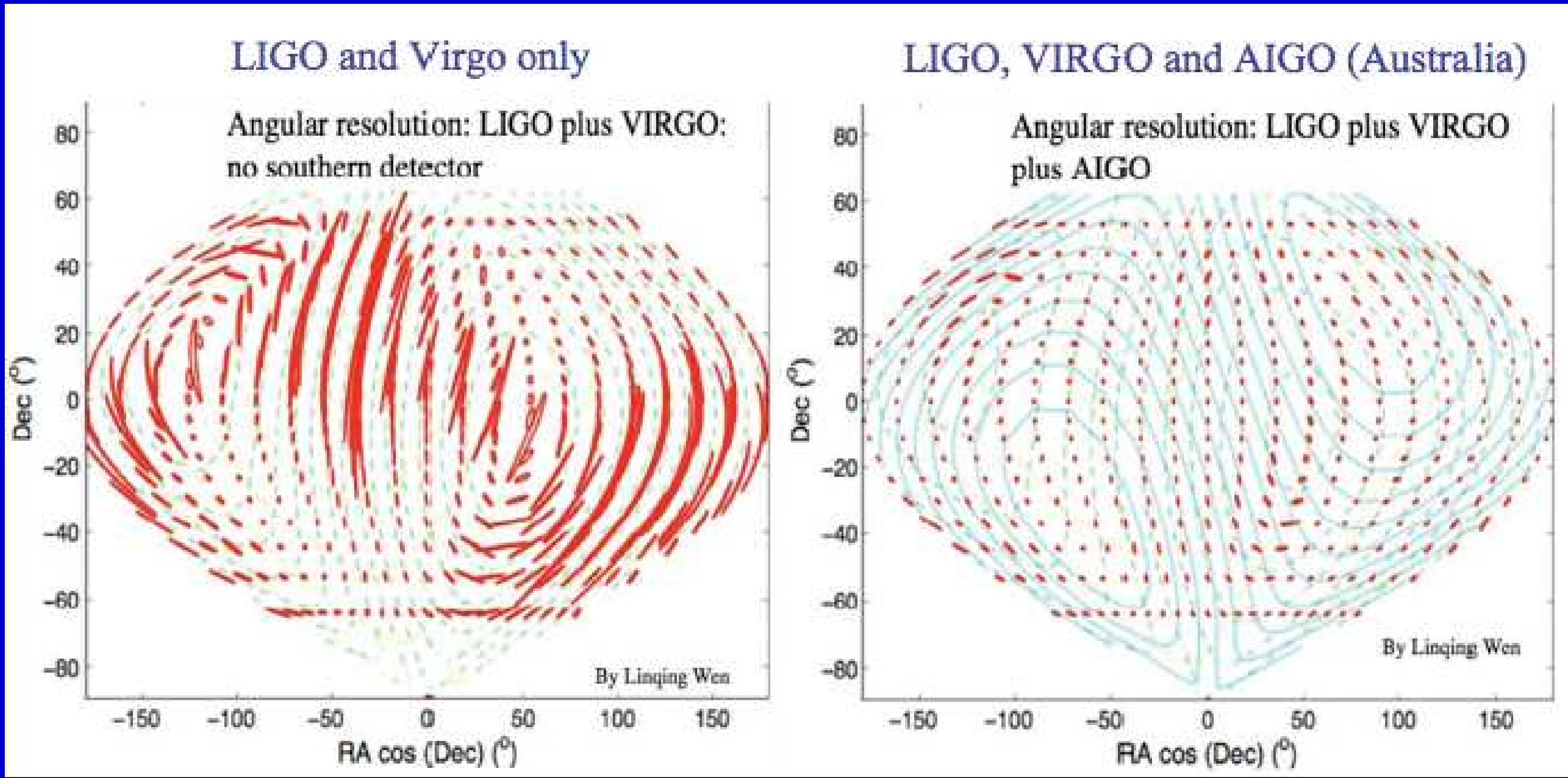
- Laser interferometry started in 1970s.
- Three prototype interferometers appeared by early 1980s in Glasgow, Garching near Munich, & at MIT.
- A 40m interferometer was constructed at Caltech by the end of 1980s.
- LIGO, VIRGO, GEO600 & TAMA300 started in 1990s.
- LIGOII, EURO, KAGRA（LCGT）; LISA; DECIGO

Laser-Interferometric GW Detectors




AIGO
Virgo
LIGO
KAGRA
GEO

Detectors at southern hemisphere are important!



GW bands, detection methods & sources

北京師範大學 BEIJING NORMAL UNIVERSITY

Frequency (Hz)	**Detection**	**Examples**
HF $(1 \sim 10^4)$	Ground based: LIGO, Virgo, KAGRA	BH or NS binaries Supernova Pulsars
LF $(10^{-4} \sim 1)$	Space: proposed LISA	binary stars in the Galaxy Massive binary BHs
VLF $(10^{-9} \sim 10^{-7})$	PTA	Super massive binary BHs
ELF $(10^{-18} \sim 10^{-15})$	CMB	Early Universe processes e.g., inflation & phase transitions

Why astrophysical background ?

- Primordial GWs from the big bang
  - the holy grail (rich information about the early universe)
- Astrophysical background (AB)
  - mask the primordial background
  - star formation history, source population
- The strength of the AB depends on source rates and individual energy emissions
  - high rate & strong energy emission → strong background

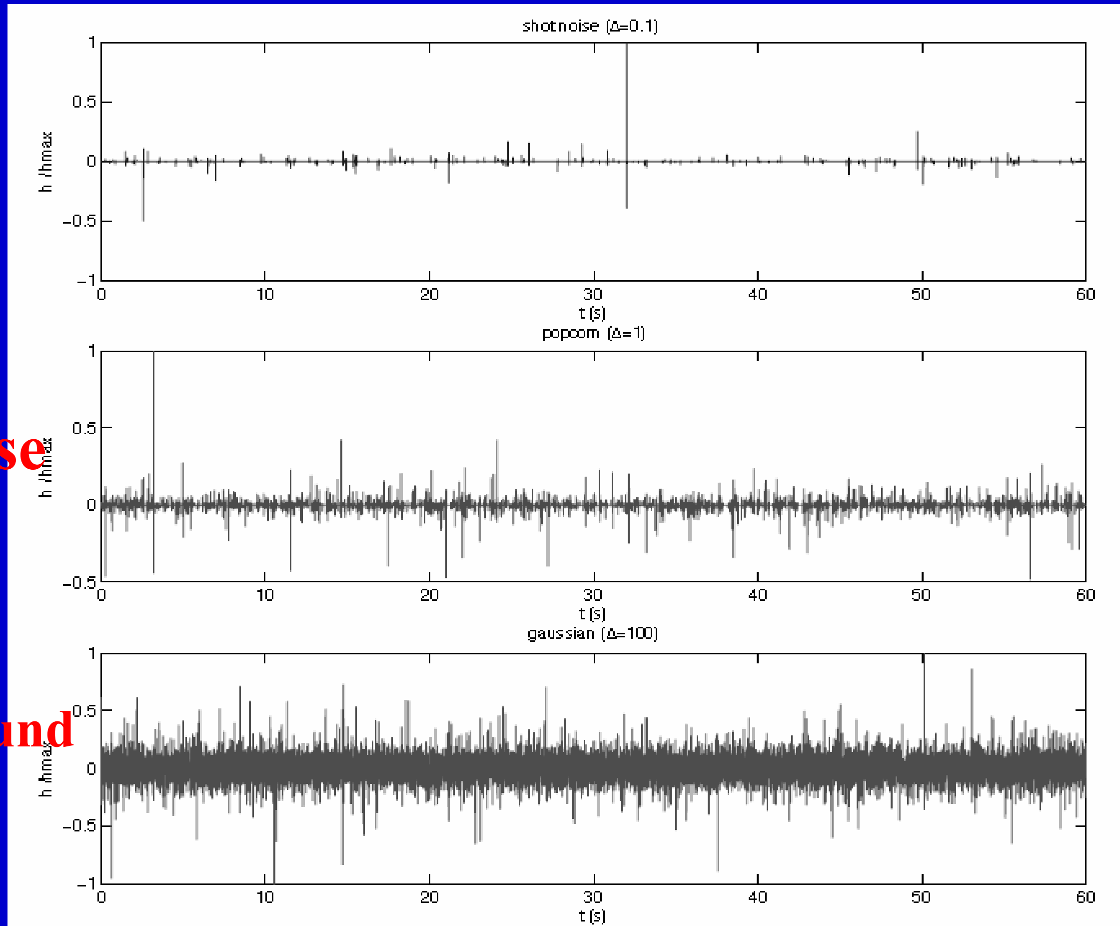
Behaviours of AB in time

- **Duty Cycle:** the ratio of the **average event duration** to the **time interval between successive events.**

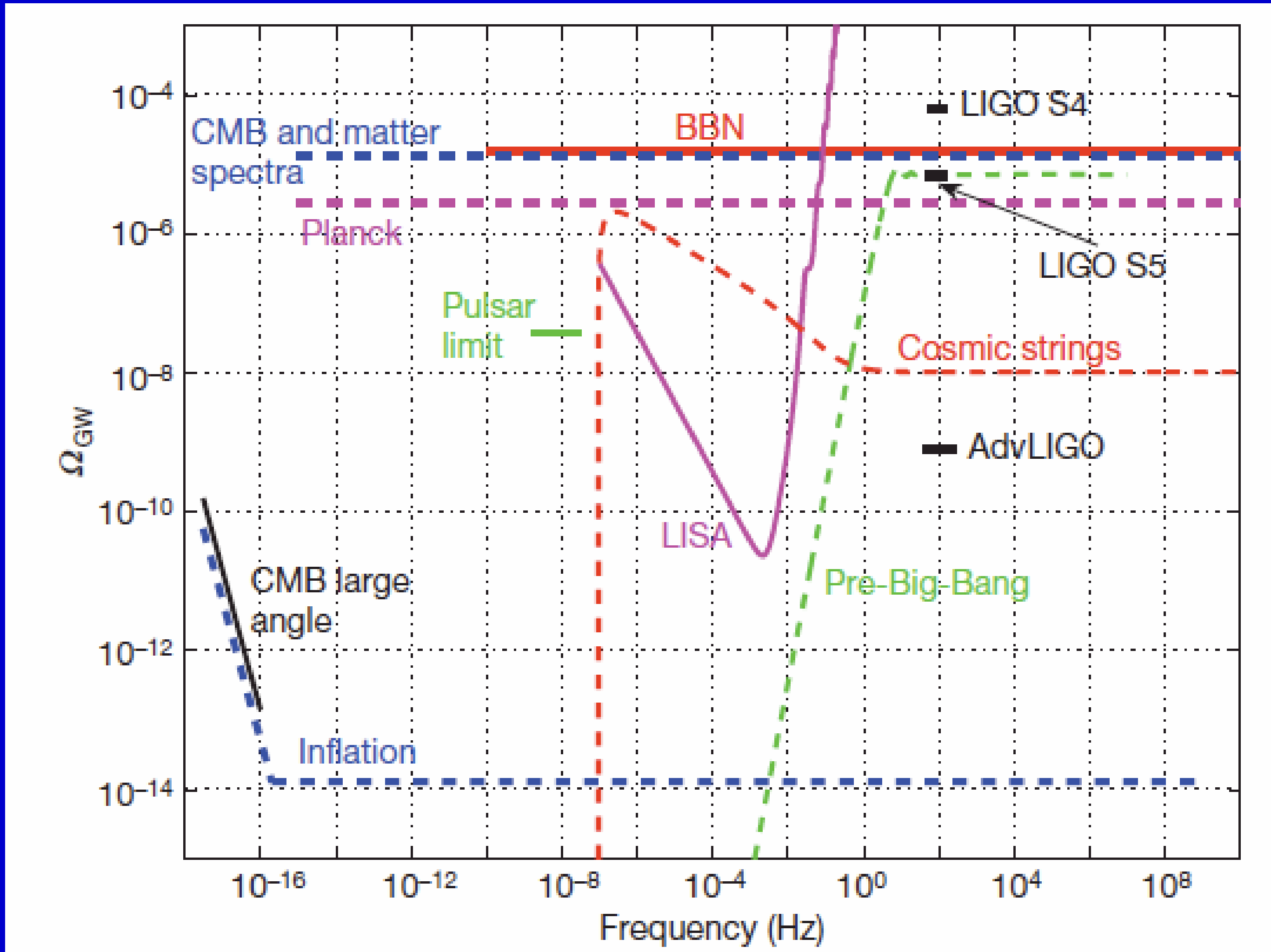
DC=0.1, Shot noise

DC=1, Popcorn noise

DC=100, Gaussian background



Primordial SGWB: measurements vs models

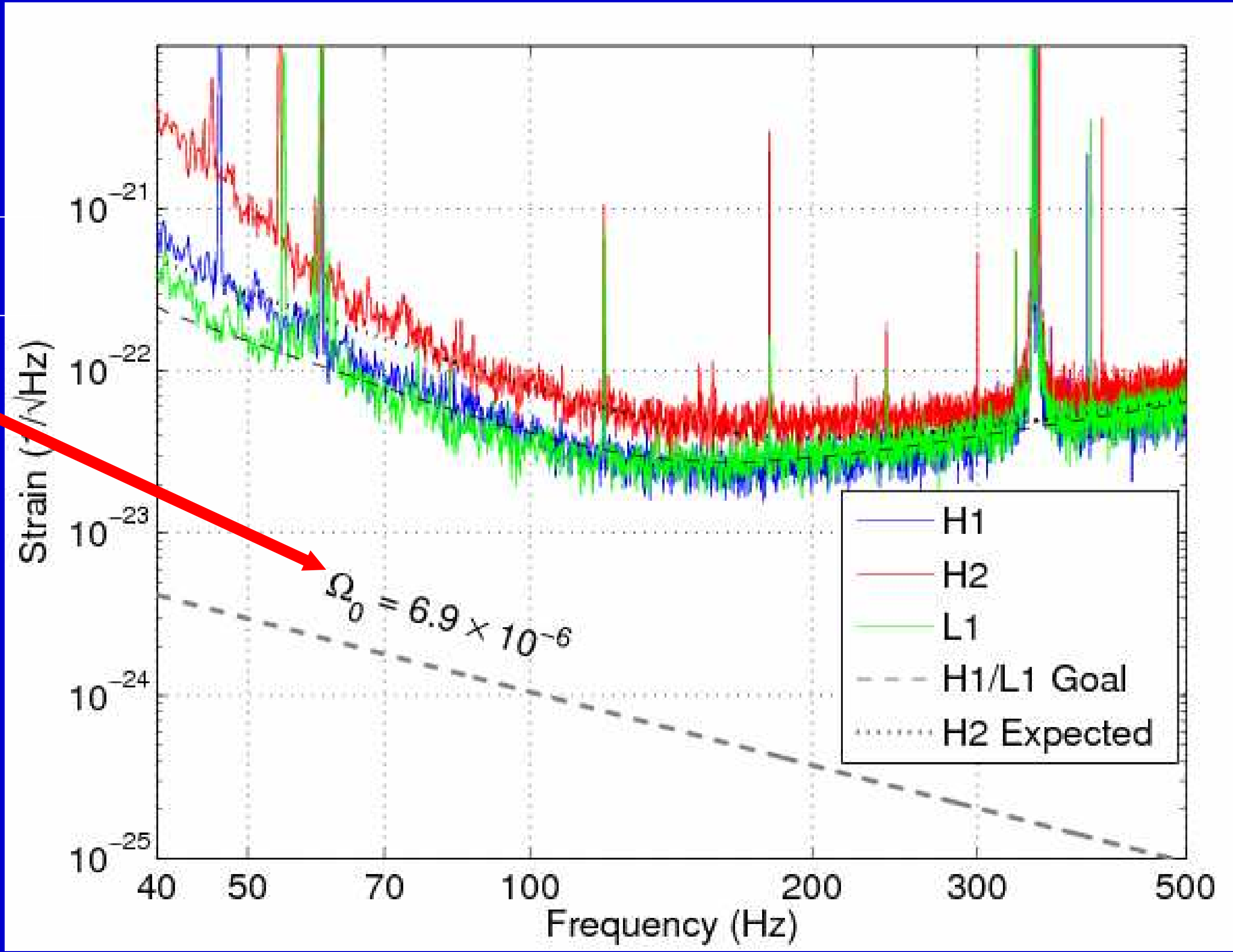


From the LIGO & Virgo Collaboration, 2009, Nature 460, 990

LIGO achieved its design sensitivity during the fifth science run

at 41.5-169.25 Hz

Upper limit



Computation of Astrophysical Background (AB)

- Depends on cosmological parameters, source event rate, and emission strength of individual sources

$$\Omega_{GW}(\nu_{obs}) = \frac{8\pi G}{3c^3 H_0^2}\nu_{obs} F_\nu(\nu_{obs})$$

$$F_\nu(\nu_{obs}) = \int_{z_{\min}}^{z_{\max}} f_\nu(\nu_{obs}, z)\frac{dR}{dz}(z)dz$$

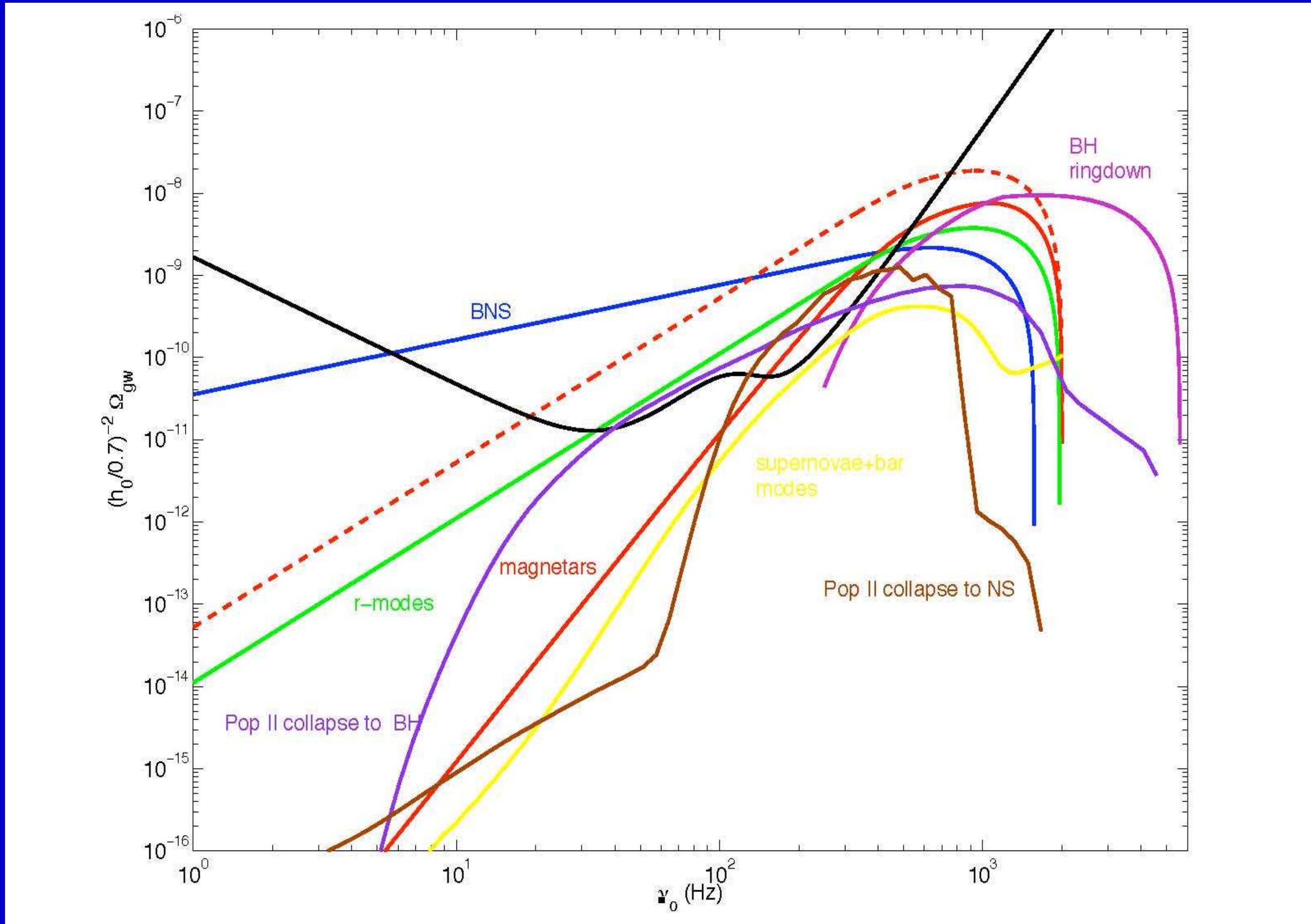
$$z_{\min} = \max(0, \nu_{\min}/\nu_{obs} - 1) \qquad z_{\max} = \min(z_*, \nu_{\max}/\nu_{obs} - 1)$$

- Energy flux of signal source

$$f_\nu(\nu_{obs}, z) = \frac{1}{4\pi d_L^2(z)}\frac{dE_{GW}}{d\nu}(\nu_{obs})$$

- Event rate $\frac{dR}{dz}$, derived from star formation rate

SGWB from various astrophysical sources



From T. Regimbau, 2011, RAA, 11, 369

Contents

- Introduction
- SGWB from NS r-mode Instability Revisited
- SGWB from Coalescing Binary Black Holes
- Summary

NS r-mode instability

- Proposed in the end of 1990s, GW production as high as 0.01 $M_{\odot}$

- Owen et al. (1998), Ferrari et al. (1999): the background Ω_{GW} peaking at several hundred Hz at~10^{-8}

- Within the detection range of advanced LIGO

However…

- **GW emission efficiency depends on the saturation amplitude α**

- **Owen et al. and Ferrari et al. assumed α takes a value of 1，later detailed studies indicated it is at most 10^{-3}-10^{-2}**

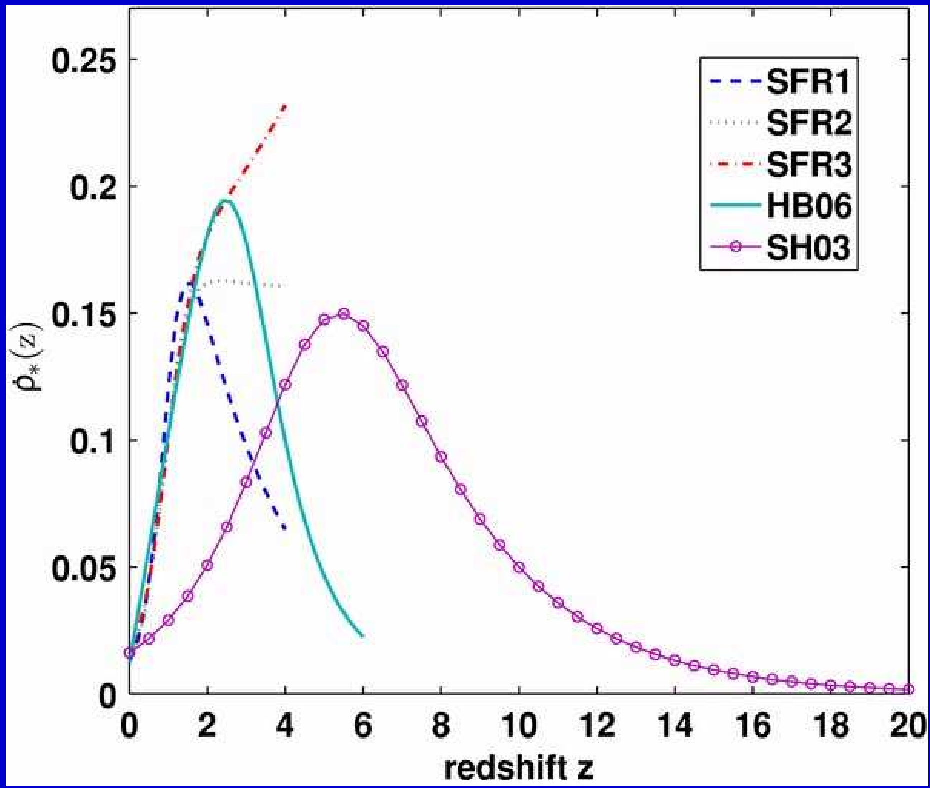
- **The background was overestimated by 4-6 orders of magnitude**

Our work

- Considered different SFR models, calculated the NS formation rate, and investigated their effects on GW background
- Used the single source emission model based on new numerical simulation results where differential rotation was taken into account in the non-linear evolution of r-mode instability
- Detection prospects for network of GW detectors

Cosmic Star Formation Rate (CSFR)

- Rest-frame ultraviolet light is considered to be an indicator of SF because it is mainly radiated by short-lived massive stars.
- $\rho_*(z)$: $M_\odot yr^{-1} Mpc^{-3}$
- Cosmology: 737



GW event rate

- NS formation

$$R(z) = \int_0^z \dot{\rho}_*(z') \frac{dV}{dz'} dz' \int_{m_{min}}^{m_u} \Phi(m) dm,$$

SFR density function — comoving volume element — initial mass

GW event rate

- NS formation

$$R(z) = \int_0^z \dot{\rho}_*(z') \frac{\mathrm{d}V}{\mathrm{d}z'} \mathrm{d}z' \int_{m_{\min}}^{m_{\mathrm{u}}} \Phi(m) \mathrm{d}m,$$

SFR density function — comoving volume element — initial mass

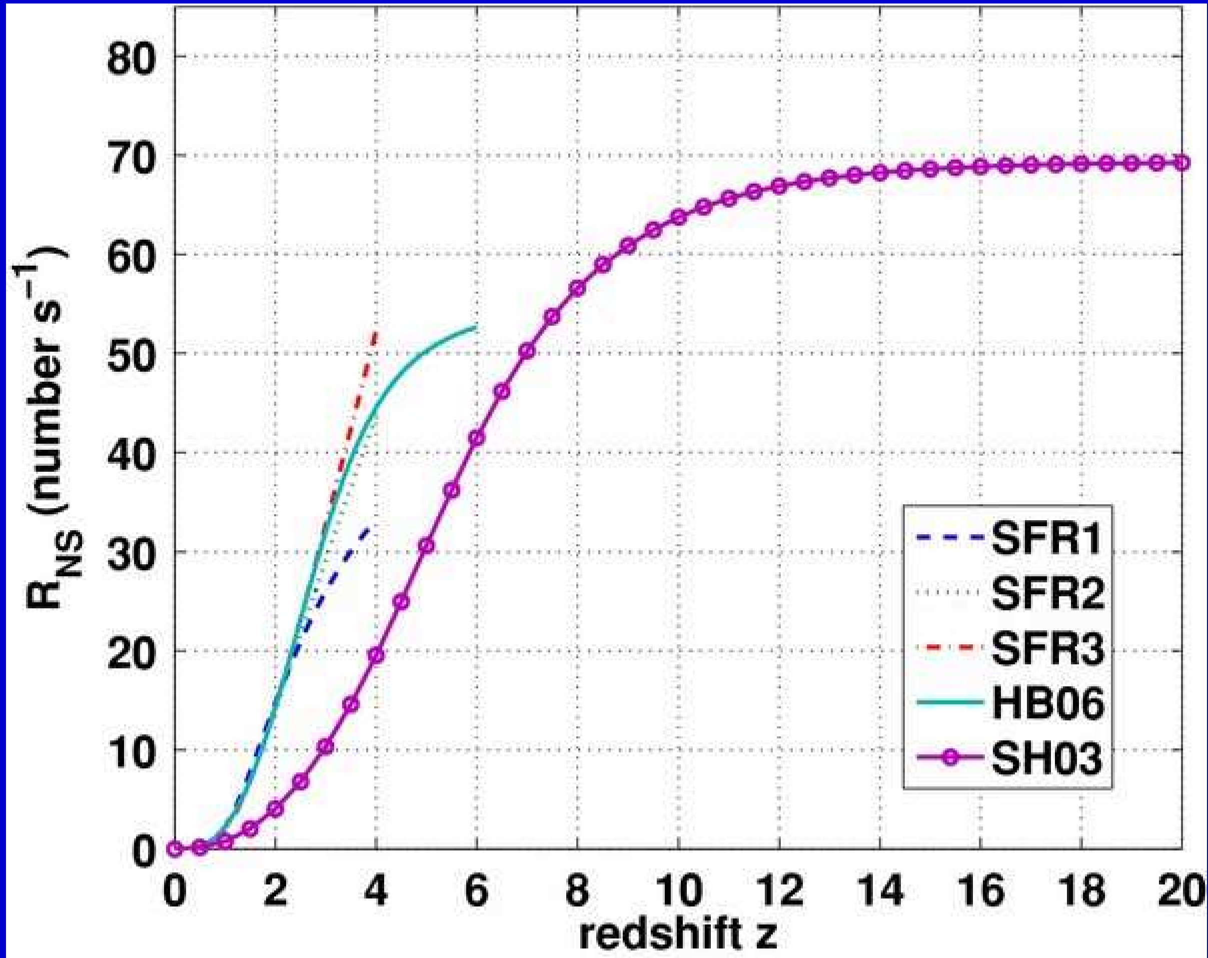
- CBC:

$$dR = r_0 e(z) \frac{dV}{dz} dz, \quad e(z) = \dot{\rho}_{*,c}(z) / \dot{\rho}_{*,c}(0)$$

$$\dot{\rho}_{*,c}(z) = \int \frac{\dot{\rho}_*(z_f)}{(1+z_f)} P(t_d) dt_d, \quad t_d = \int_z^{z_f} \frac{dz'}{(1+z')H(z')}.$$

Distribution — delay time

Neutron Star Formation Rate



X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

GW from NS r-mode instability

- GW emitted by a single NS

$$h_c(\nu) = \frac{5.5\times 10^{-22}}{\sqrt{K+2}}\sqrt{\frac{\nu}{\nu_{\max}}}\left(\frac{20\ \mathrm{Mpc}}{d_L}\right),$$

Sá, P. M., & Tomé, B. 2006, Phys. Rev. D, 74, 044011

- SGWB from all NSs

$$\Omega_{\mathrm{GW}}(\nu_{\mathrm{obs}}) = \frac{4\pi^2(1.1\times 10^{-20})^2}{3H_0^2(K+2)}\frac{\nu_{\mathrm{obs}}^2}{\nu_{\max}} \times\left[\int_{z_{\min}}^{z_{\max}}\int_{m_{\min}}^{m_{\max}}\dot{\rho}_*(z)(1+z)\left(\frac{1\ \mathrm{Mpc}}{d_L}\right)^2\frac{dV}{dz}\Phi(m)dm\,dz\right].$$

$z_{\min} = \max(0, \nu_{\min}/\nu_{\mathrm{obs}} - 1)$ $z_{\max} = \min(z_*, \nu_{\max}/\nu_{\mathrm{obs}} - 1)$

SGWB from NS r-mode Instability Revisited

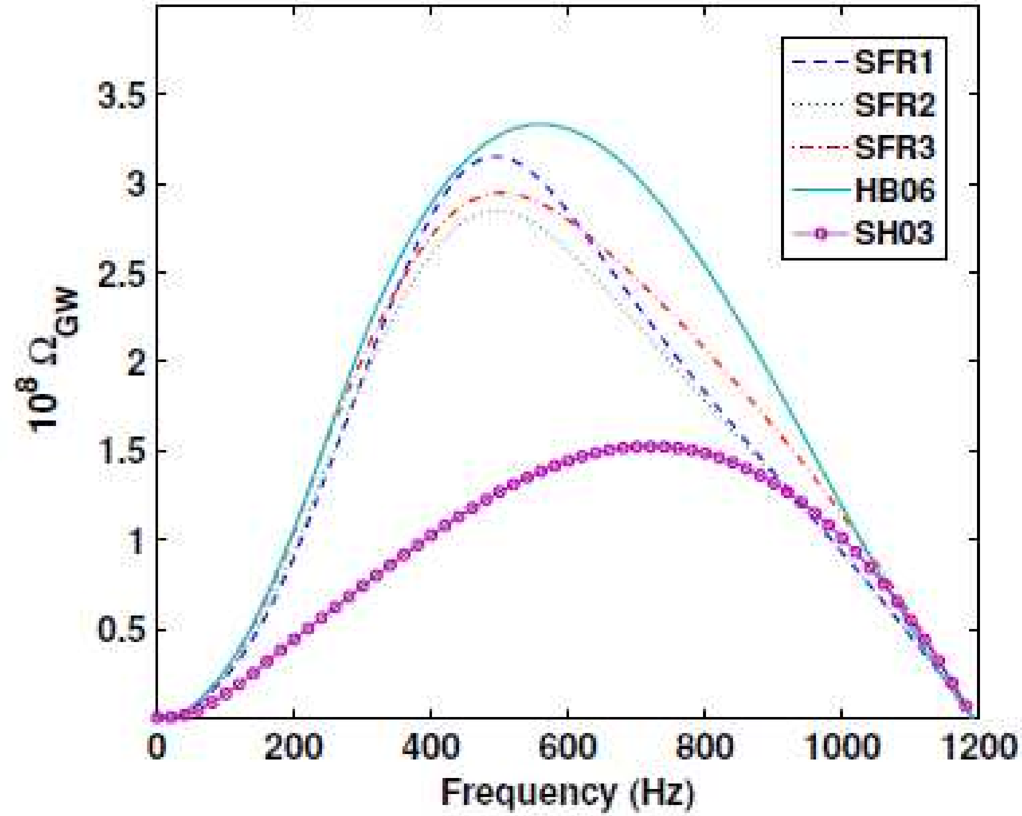


Figure 3. Dimensionless energy density Ω_{GW} as a function of observed frequency ν_{obs}, calculated for five CSFR models and by setting $K = -5/4$, $\nu_{max} = 1191$ Hz.

X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

SGWB from NS r-mode Instability Revisited

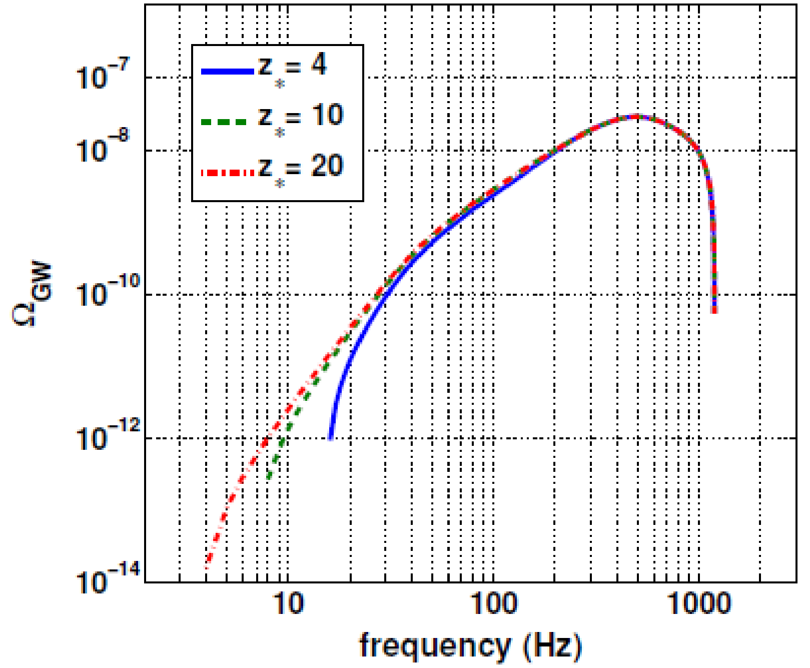


Figure 4. Ω_{GW} as a function of observed frequency ν_{obs}, calculated for the SFR2 model with three different values of maximum redshift z_*; for other parameters, the same as Figure 3.

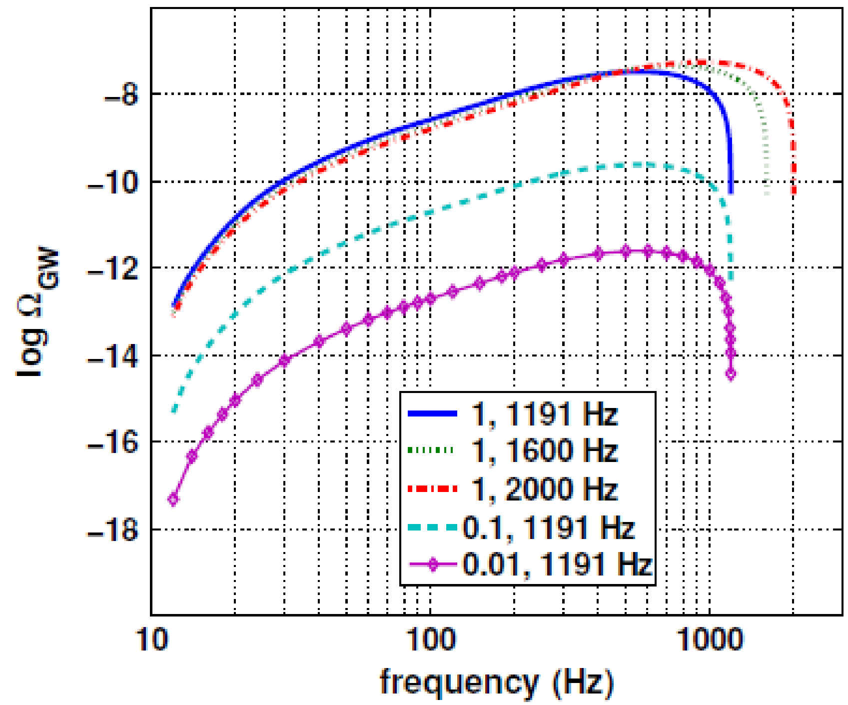
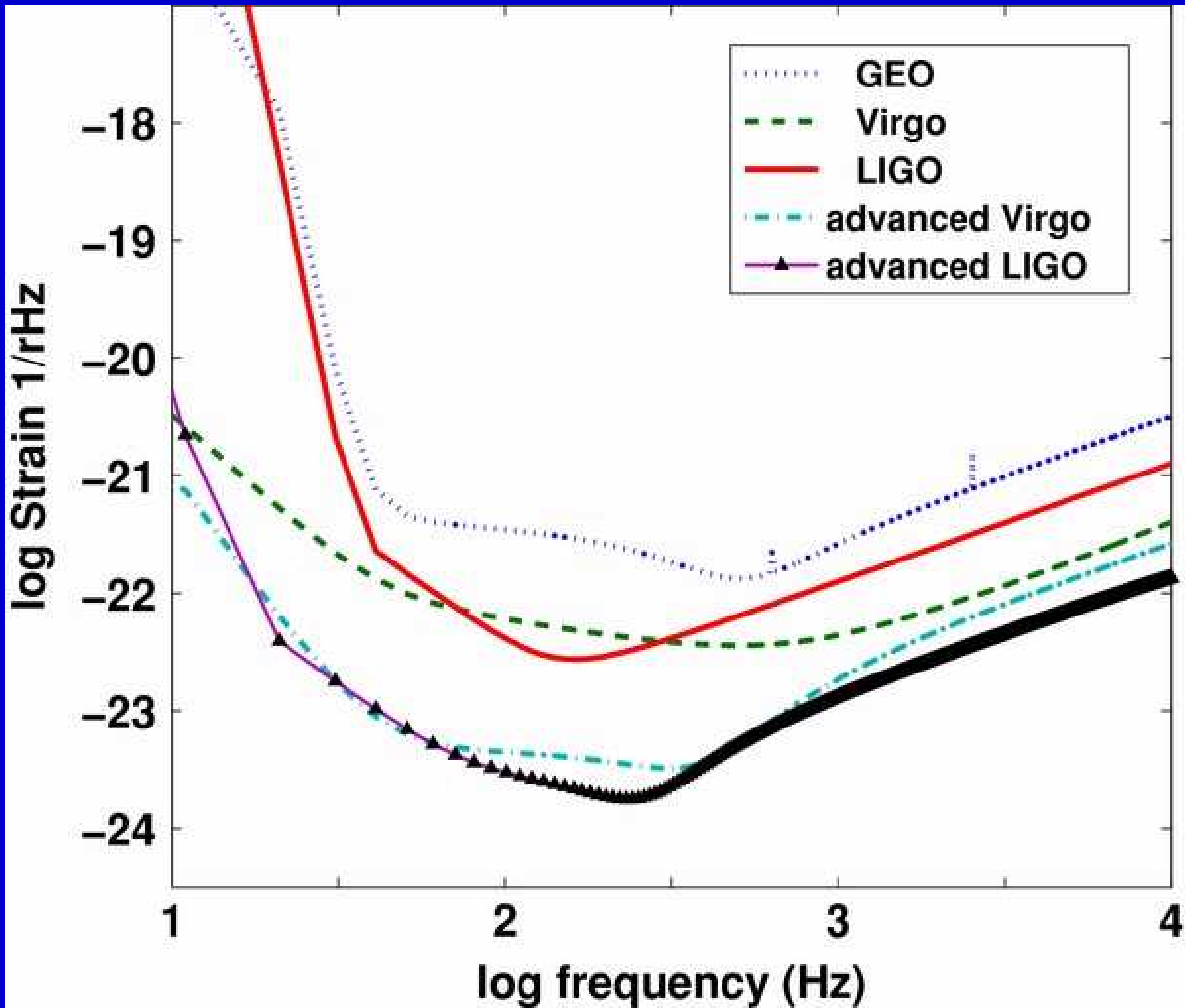


Figure 5. Ω_{GW} as a function of ν_{obs}, calculated for different values of (α, ν_{max}).

X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

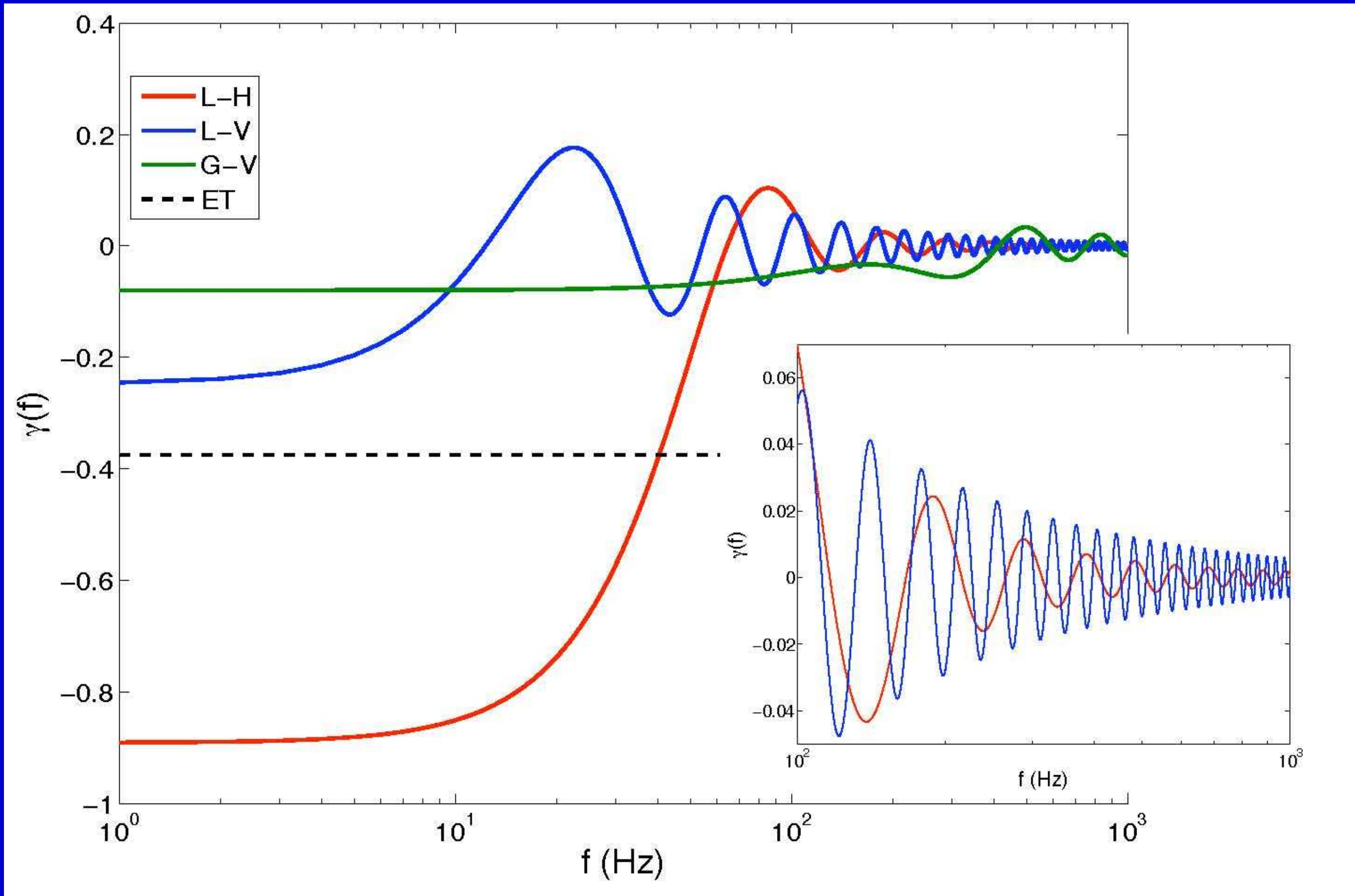
Noise Power Spectrum of Ground-based Detectors

北京師範大學 BEIJING NORMAL UNIVERSITY



Overlap Reduction Function

The sensitivity loss due to detector separation and orientation

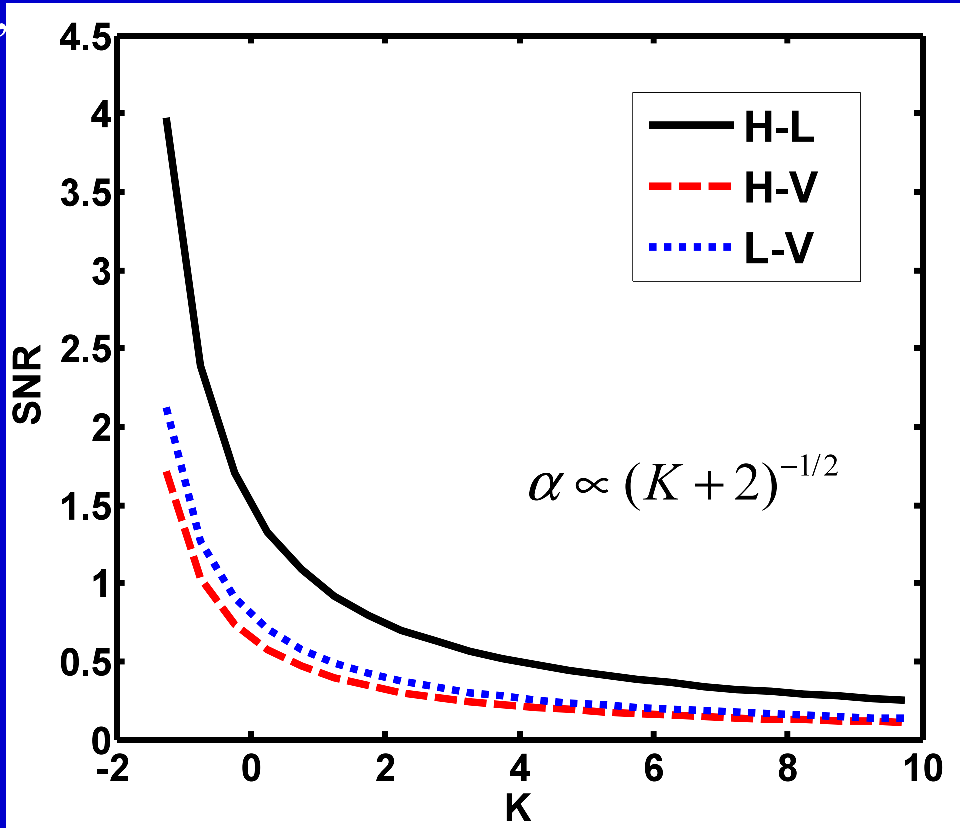


Detectability

For cross correlation of two detectors, the signal to noise ratio is：

$$\left(\frac{S}{N}\right)^2 = \frac{9H_0^4}{50\pi^4} T \int_0^\infty df \frac{\gamma^2(f)\Omega_{GW}^2(f)}{f^6 P_1(f) P_2(f)}$$

For three detector pairs H-L, H-V & L-V, T=3yr, SNR as a function of K, the differential rotation parameter. In order to let SNR>1, we used a sensitivity 10 times better than advanced LIGO and advanced Virgo, and put ν_{max}=1191 Hz。



X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

Combining multiple detectors

Method I : directly correlate the outputs of multiple detectors;

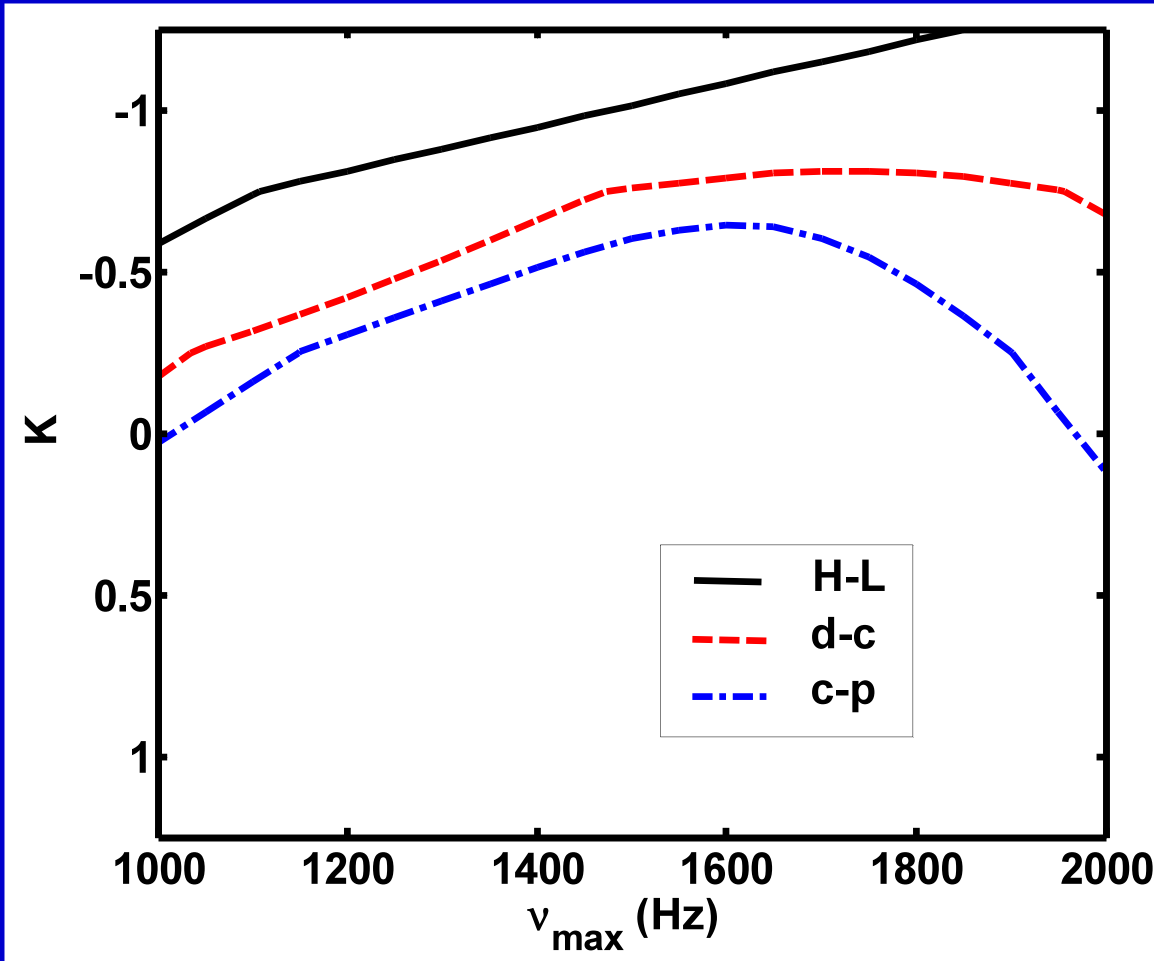
$$\left(\frac{S}{N}\right)^2_{\mathrm{opt\,II}} \approx {}^{(12)}\left(\frac{S}{N}\right)^{2\,(34)}\left(\frac{S}{N}\right)^2 \cdots {}^{(2N-1,2N)}\left(\frac{S}{N}\right)^2 + \text{all possible permutations.}$$

Method II: correlating the outputs of a pair of detectors, then combining measurements from multiple pairs

$$\left(\frac{S}{N}\right)^2_{\mathrm{opt\,I}} = \sum_{\mathrm{pair}}\left(\frac{S}{N}\right)^2_{\mathrm{pair}},$$

"Observable" region of parameter space

Region of parameter space for which the SGWB produced from an ensemble of NSs with *r*-mode instability is detectable by the H–L pair, the approach of directly combining (labeled "d–c") four third-generation detectors, and the approach of combining multiple pairs of detectors (labeled "c–p"). (an order of magnitude improvement in all detector sensitivities is assumed)



X.-J. Zhu, X.-L. Fan, Z-HZ 2011 ApJ 729, 59

Conclusions

BEIJING NORMAL UNIVERSITY

- To be detectable with 2nd generation detectors, one must have α ~1. But the upper limit on α is 10^{-3}-10^{-2}. The r-mode background is not going to be detected.

- Constraints on GW production can be obtained： i) two co-located advanced LIGO detectors (γ(f)=1): $E_{GW} < 10^{-3}\ M_{\odot}c^2$; ii) two ET type detectors: $E_{GW} < 2\times 10^{-5}\ M_{\odot}c^2$

- The main contribution to an astrophysical background comes from sources at z<2

- Combining measurements from multiple detector pairs is more efficient in terms of improving the detection ability.

Contents

Compact Binary Coalesce（CBC）

- CB,including NS-NS,NS-BH &BH-BH: the most promising GW sources because a huge amount GW energy will be produced during their final coalescence process.
- Coalescing BH-BH: the most energetic GW event in our Universe, ~1 $M_\odot c^2$, GW luminosity ~ 10^{22} $L_\odot$ > all stars. **However event rate r_0 is very small and uncertain!** **(**r_0~10^{-4} - 0.3 $Mpc^{-3}Myr^{-1}$;most likely 4×10^{-3}**)**
- NS-NS has been confirmed by pulsar binary observations, and it was regarded as the main CBC event for ground-based detectors (a few/yr).
- Coalescing rate of BH-BH (NS-BH) can only be estimated from simulation of binary evolution. **However recent theoretical calculations and observational break-through indicate that BH-BH might have a much higher event rate than expected before!**

Coalescence rate r_0 of BH-BH

北京師範大學 BEIJING NORMAL UNIVERSITY

- Recent astronomical observation is changing r_0
  - Two WR-BH were observed——>r_0 of BH-BH might be 0.36 (*arXiv: 0803.3516*)。WR is progenitor of SNIb/c forming BH.
  - Recent SDSS observations indicate that about half stars are formed at galaxies with low metallicities——>assuming half stars have $1Z_\odot$ and half $0.1Z_\odot$， r_0 of BH-BH could be 3.1×10^{-2} - 0.43 （*Belczynski et al. 2010*）。
- Therefore coalescence rates of BH-BH and NS-NS are probably the same order. Cobsidering the much higher GW luminosity, coalescing BH-BH could be 25 times higher than coalescing NS-NS for observation. （*Belczynski et al. 2010*）

GW backgrounds from CBC

- It was proposed long time ago that all NS-NS coalescence would form a strong GW background for ground-based observation band, e.g., *Regimbau & de Freitas Pacheco 2006; Regimbau & Chauvineau 2007.*

- No detail considering SGWB from coalescing BH-BH, why？
  - It was widely thought their coalescing rate was too low.
  - Many uncertainties on mass distribution, coalesce time and spin etc. and their effects on GW signal.
  - No body knows what kinds of GW signal will be produced during the final stage of BH-BH coalescence until 2005.

- Although optimistic estimates on r_0 of BH-BH as well as higher GW luminosity indicate a stronger GW background produced by BH-BH than NS-NS, and even dominating ground-based observation band, all above questions should be investigated carefully.

Evolution model for coalescence rate

In order to consider contributions from the events at z>0, one should model the evolution for event rate. Differential event rate happaned at redshift from z to z+dz:

$$dR = r_0 e(z) \frac{dV}{dz} dz,$$

where r_0 is local event rate, dV/dz comoving volume element, $e(z)$ dimensionless normalized factor, i.e., evolution of event rate:

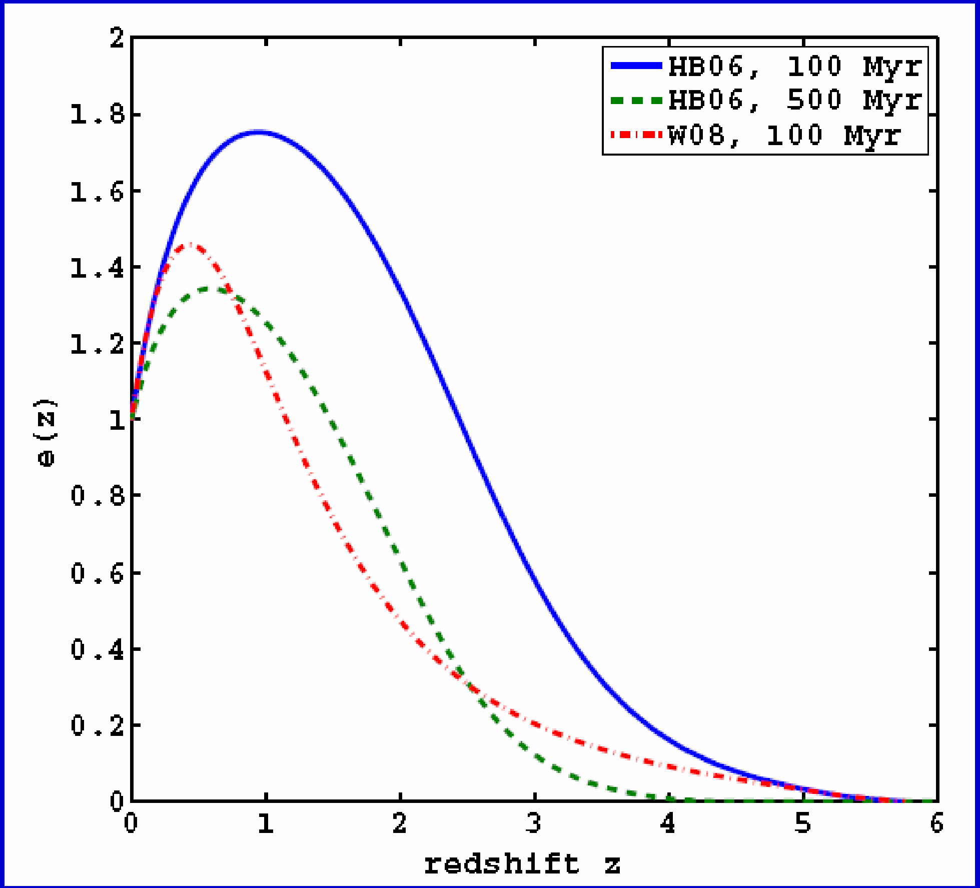
$$e(z) = \dot{\rho}_{*,c}(z)/\dot{\rho}_{*,c}(0)$$

$$\dot{\rho}_{*,c}(z) = \int \frac{\dot{\rho}_*(z_f)}{(1+z_f)} P(t_d) dt_d,$$

z is redshift where BH-BH coalesce while z_f is redshift where BH-BH was formed. Their time interval or coalesce time is t_d while $P(t_d)$ is its probability distribution. ρ_* is SFR, and $\rho_{*,c}$ relates event rate to SFR.

Evolution of event rate: our result

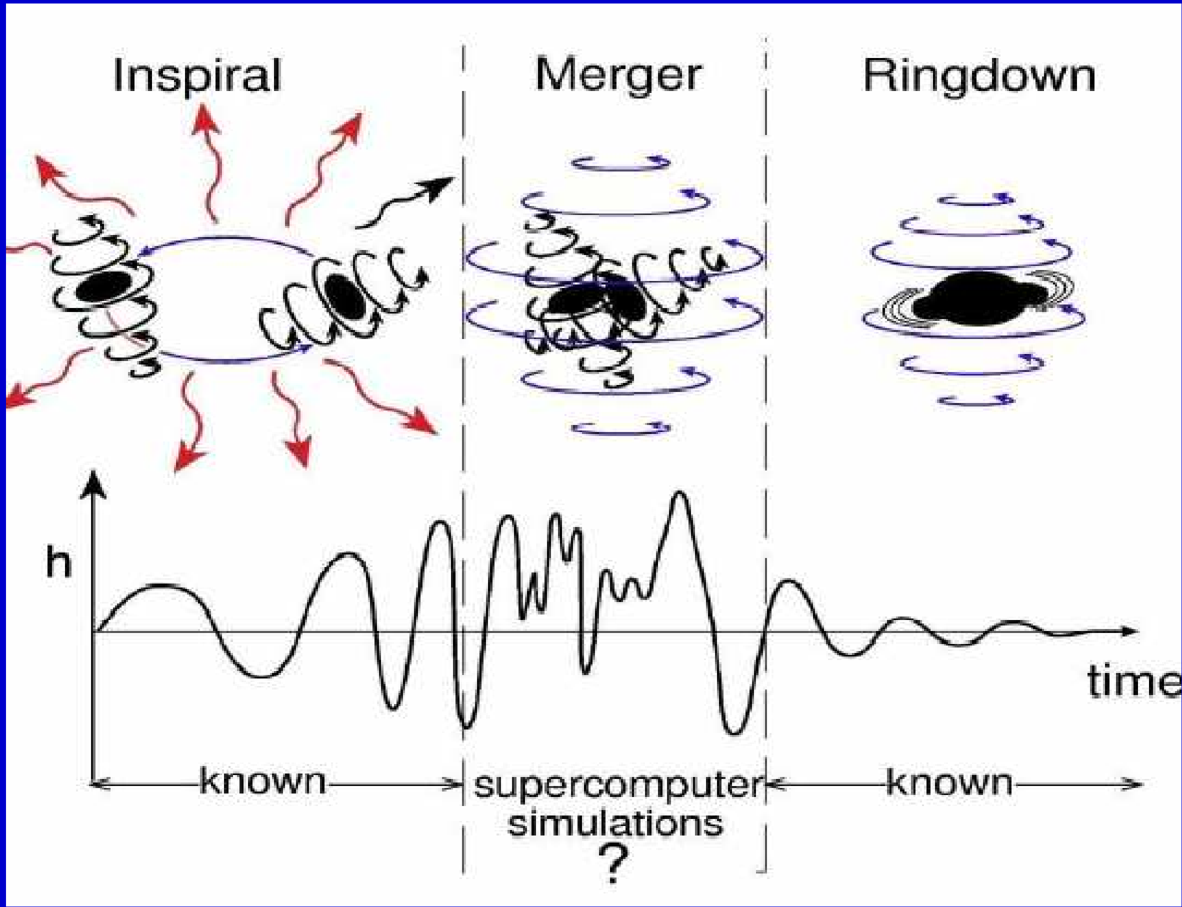
- Using two SFR model HB06 - *Hopkins & Beacom (2006)* and W08 - *Wilkins et al. (2008)*, the resulting e(z) is shown in the right figure. Assuming τ_0= 100 Myr, 500 Myr respectively.



X.-J. Zhu, …, Z-HZ 2011 ApJ, 739 , 86

GW spectrum of coalescing BH-BH

Three stages for BH-BH：inspiral, merger, and ringdown。The merger stage can only be investigated by numerical simulations, which made breakthrough in 2005.



GW spectrum of coalescing BH-BH

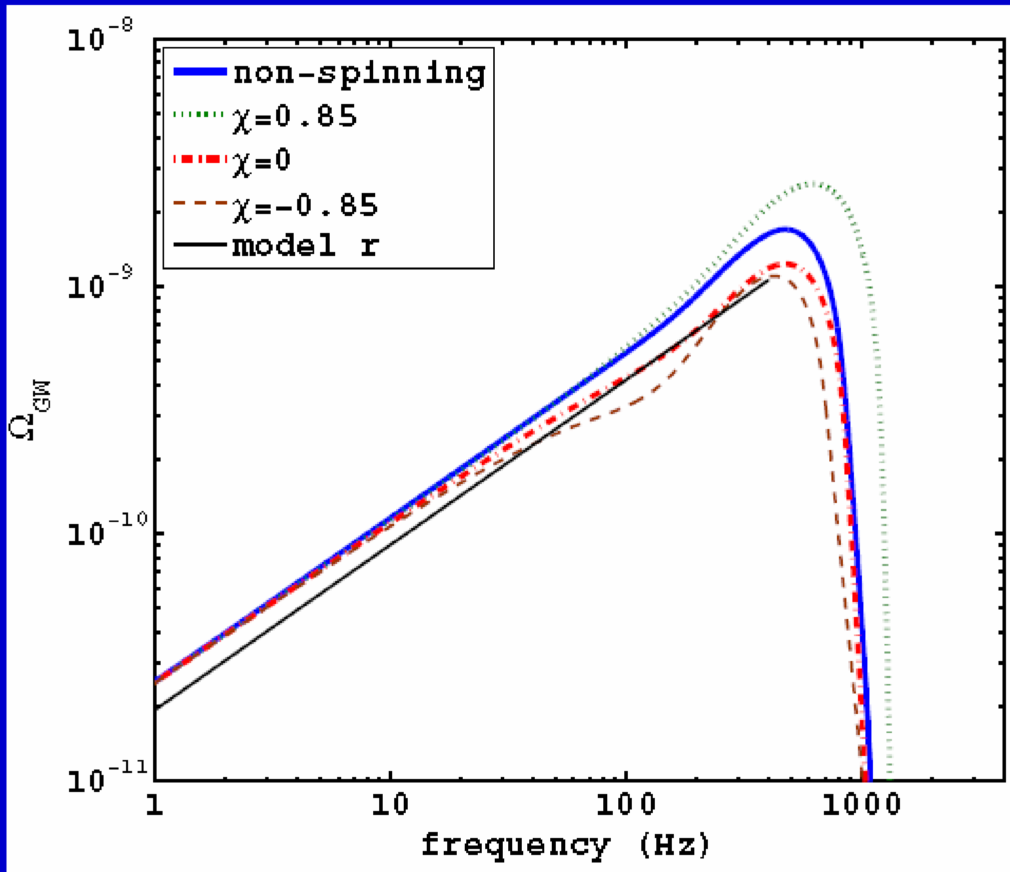
北京師範大學 BEIJING NORMAL UNIVERSITY

$$\frac{dE}{d\nu} \equiv \frac{(G\pi)^{2/3} M_c^{5/3}}{3} \begin{cases} \nu^{-1/3} & \text{if } \nu < \nu_1 \\ \omega_1 \nu^{2/3} & \text{if } \nu_1 \leqslant \nu < \nu_2 \\ \omega_2 \left[\dfrac{\nu}{1+(\frac{\nu-\nu_2}{\sigma/2})^2} \right]^2 & \text{if } \nu_2 \leqslant \nu < \nu_3 \end{cases}$$

Ajith et al 2011 PRL, 106, 241101
Ajith et al 2008 PRD, 77, 104017

SGWB from coalescing BH-BH: our result

- Using SFR model HB06, τ_0=100 Myr, and $r_0=3.1\times10^{-2}$.



X.-J. Zhu, …, Z-HZ 2011 ApJ, 739 , 86

Noise Power Spectrum of Ground-based Detectors

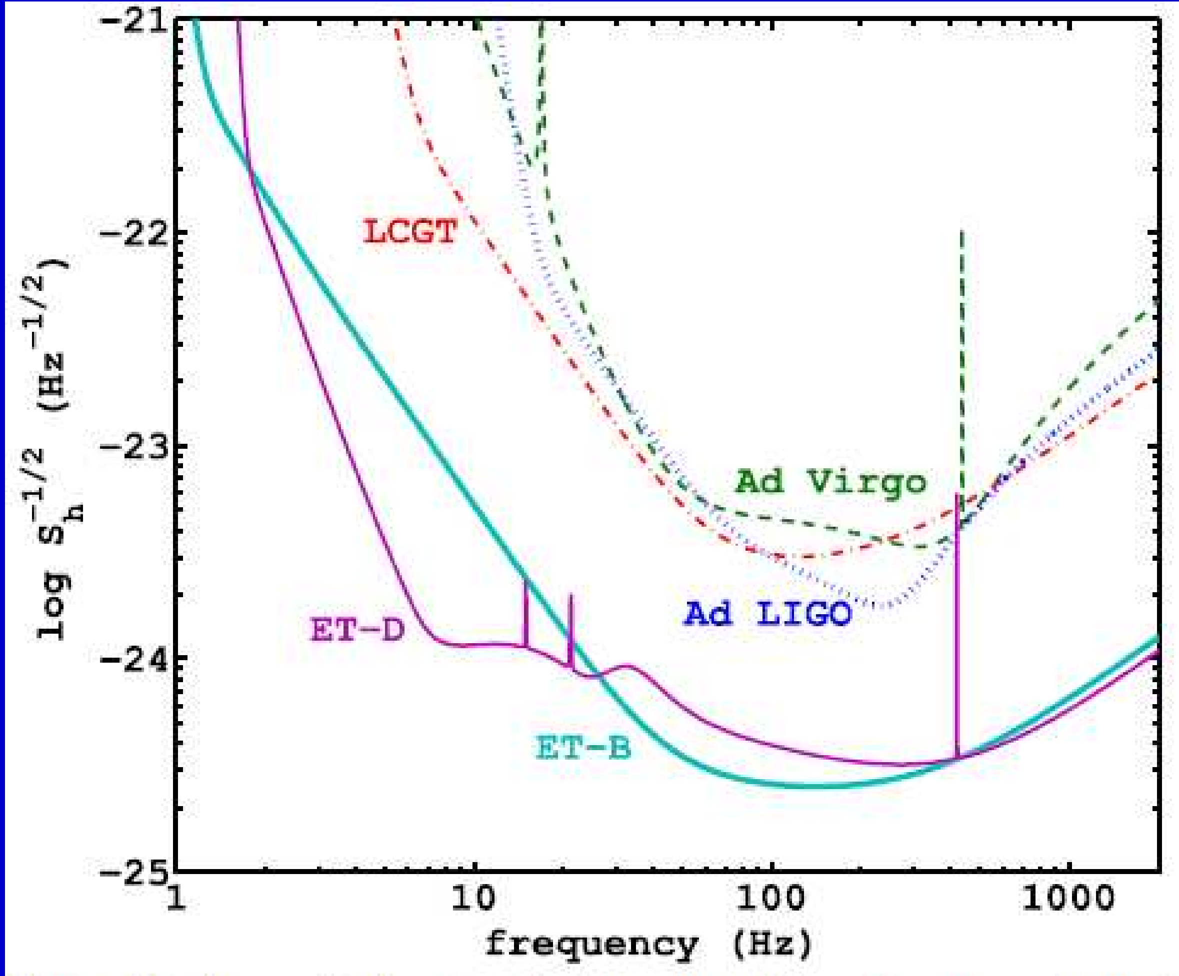


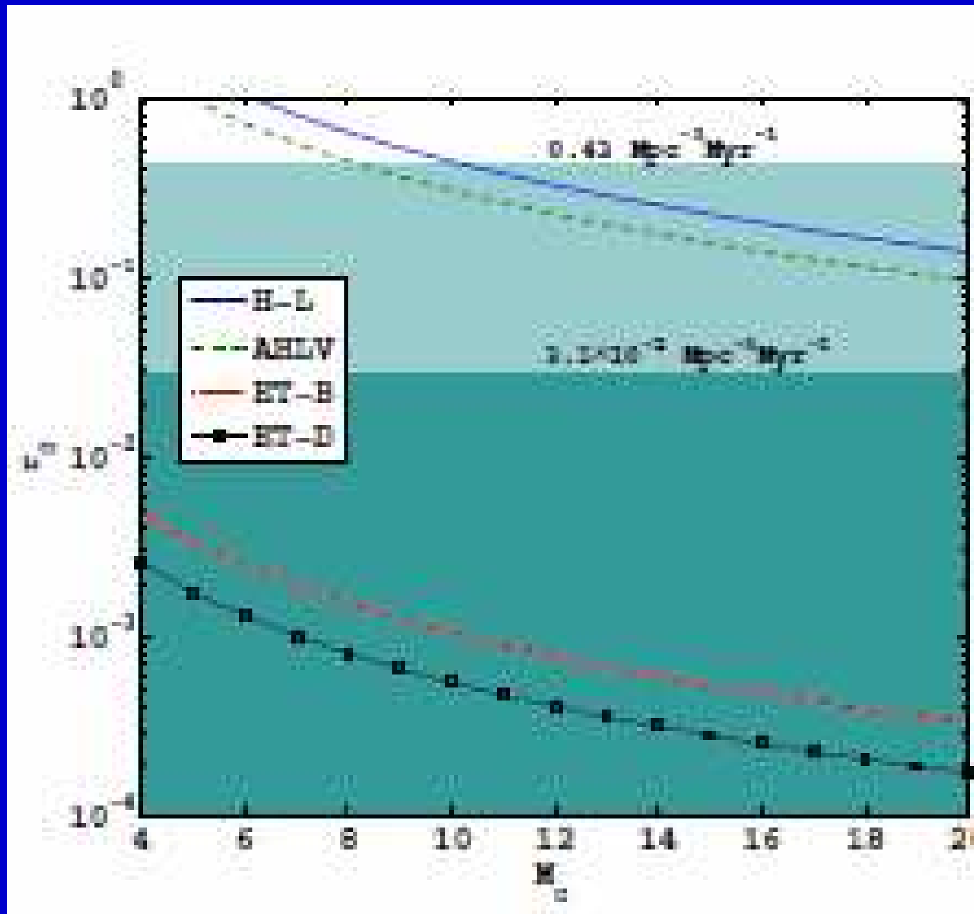
Figure 4. Design sensitivity curves for future ground-based detectors: Advanced Virgo (Ad Virgo), LCGT, Advanced LIGO (Ad LIGO), and two possible configurations of the Einstein Telescope, ET-B and ET-D.

Detectability: parameter space r_0–M_c

- $3.1\times10^{-2} \leqslant r_0 \leqslant 0.43$;
  $4 \leqslant M_c \leqslant 20$

$$M_c^{5/3} = m_1 m_2 (m_1 + m_2)^{-1/3},$$

- SNR=3
- H-L: Hanford and Livingston; AHLV: four 2nd detectors and CP; ET: the 3rd detector.



X.-J. Zhu, …, Z-HZ 2011 ApJ, 739 , 86

Conclusions

- Astrophysical background is an important detection target

  - information about the star formation history, the statistical properties of source populations

- Coalescing binary BHs could dominate the GW signal of ground-based interferometers. It could mask the primordial background at around 100 Hz.

- Detection depends highly on the event rate. 2^{nd} generation detectors require a rate at the high end of predictions. 3^{rd} generation detector ET could detect it easily.

Thanks for your attention !