Identifying Inelastic WIMPs from Direct Dark Matter Detection Experiments

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Dark Matter, Dark Energy, and Matter-Antimatter Asymmetry
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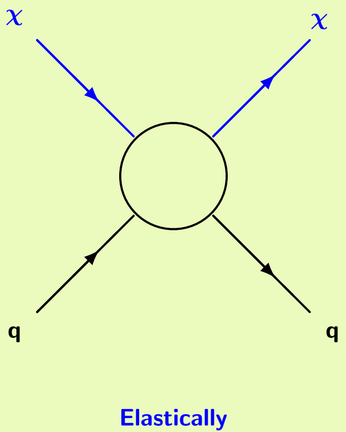
Motivation

- WIMP-nucleus scattering



- DAMA: 11-year annual modulation observation
- CoGeNT: a modulated component of unknown origin (modified somehow recently)
- CRESST-II: more observed nuclear recoil events than expected backgrounds

⇕

- CDMS-II, XENON100, ZEPLIN-II: non-observations (a few candidate events)

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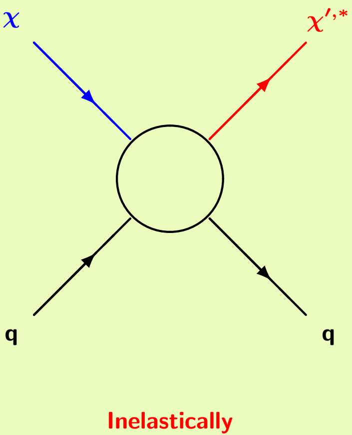

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- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}(Q)}^{v_{\rm max}} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min}(Q) = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector.

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r,N}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r,N}^2}} \qquad m_{\rm r,N} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth

σ_0: total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor

$f_1(v)$: one-dimensional velocity distribution of halo WIMPs

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Reconstruction of the WIMP velocity distribution - inelastic scattering

- Normalized one-dimensional WIMP velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ \left(\alpha\sqrt{Q} + \frac{\alpha_\delta}{\sqrt{Q}} \right) \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right)^{-1} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\} \right\}$$

with

$$Q(v) = \frac{v^2 - 2\alpha\alpha_\delta \pm v\sqrt{v^2 - 4\alpha\alpha_\delta}}{2\alpha^2}$$

and

$$\mathcal{N} = 2 \left\{ \int_0^\infty \left[\frac{1}{Q} \left(\alpha\sqrt{Q} - \frac{\alpha_\delta}{\sqrt{Q}} \right) \right] \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

Determinations of the WIMP mass and the mass splitting

- Threshold (minimal required) velocity of incident inelastic WIMPs
  - $v_{\rm min}(Q) = \alpha\sqrt{Q} + \frac{\alpha_\delta}{\sqrt{Q}}$

    $Q(v) = \frac{v^2 - 2\alpha\alpha_\delta \pm v\sqrt{v^2 - 4\alpha\alpha_\delta}}{2\alpha^2}$
  - $v_{\rm thre} = 2\sqrt{\alpha\alpha_\delta}$

    $Q_{v_{\rm thre}} \equiv Q(v = v_{\rm thre}) = \frac{\alpha_\delta}{\alpha} = \left(\frac{m_\chi}{m_\chi + m_{\rm N}}\right)\delta$
- Determining m_χ and δ (by combining two targets)

$$m_\chi = \frac{Q_{v_{\rm thre},Y}\, m_Y - Q_{v_{\rm thre},X}\, m_X}{Q_{v_{\rm thre},X} - Q_{v_{\rm thre},Y}}$$

$$\delta = \frac{Q_{v_{\rm thre},X}\, Q_{v_{\rm thre},Y}\, (m_Y - m_X)}{Q_{v_{\rm thre},Y}\, m_Y - Q_{v_{\rm thre},X}\, m_X}$$

Determinations of the WIMP mass and the mass splitting

- Differential event rate for inelastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}(Q)}^{v_{\rm max}} \left[\frac{f_1(v)}{v}\right] dv$$

Here

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- Differential event rate for inelastic WIMP-nucleus scattering

$$\left(\frac{dR}{dQ}\right)_{Q=Q_{v_{\rm thre}}} = \mathcal{A} F^2(Q_{v_{\rm thre}}) \left(\int_{v_{\rm thre}}^{v_{\rm max}} \left[\frac{f_1(v)}{v}\right] dv\right) \quad \text{maximal, indep't. of } f_1(v)$$

Here

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Reconstruction of the recoil spectrum - elastic scattering

- Integrals over two simplest theoretical velocity distributions

$$\left(\frac{dR}{dQ}\right)_{\rm Gau} \propto F^2(Q)\, e^{-\alpha^2 Q/v_0^2}$$

$$\left(\frac{dR}{dQ}\right)_{\rm sh} \propto F^2(Q)\left[\mathrm{erf}\left(\frac{\alpha\sqrt{Q}+v_{\rm e}}{v_0}\right) - \mathrm{erf}\left(\frac{\alpha\sqrt{Q}-v_{\rm e}}{v_0}\right)\right]$$

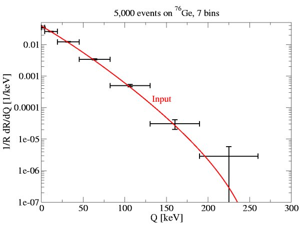
- Exponential ansatz for reconstructing the measured recoil spectrum (in the nth Q-bin)

$$\left(\frac{dR}{dQ}\right)_{{\rm expt},\, Q\simeq Q_n} \equiv r_n\, e^{k_n(Q-Q_{s,n})} \qquad r_n \equiv \frac{N_n}{b_n}$$

- Logarithmic slope and shifted point (in the nth Q-bin)

$$\overline{Q-Q_n}|_n \equiv \frac{1}{N_n}\sum_{i=1}^{N_n}(Q_{n,i}-Q_n) = \left(\frac{b_n}{2}\right)\coth\left(\frac{k_n b_n}{2}\right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n}\ln\left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2}\right]$$



[M. Drees and CLS, JCAP 0706, 011]

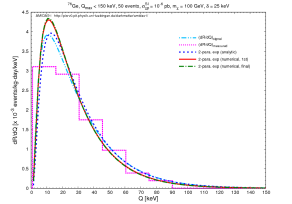
Reconstruction of the recoil spectrum - inelastic scattering

- Integrals over two simplest theoretical velocity distributions

$$\left(\frac{dR}{dQ}\right)_{\mathrm{in,\ Gau}} \propto F^2(Q)\, e^{-\left(\alpha^2 Q + \alpha_\delta^2 / Q\right) / v_0^2}$$

$$\left(\frac{dR}{dQ}\right)_{\mathrm{in,\ sh}} \propto F^2(Q)\left[\mathrm{erf}\left(\frac{\alpha\sqrt{Q} + \alpha_\delta / \sqrt{Q} + v_e}{v_0}\right) - \mathrm{erf}\left(\frac{\alpha\sqrt{Q} + \alpha_\delta / \sqrt{Q} + v_e}{v_0}\right)\right]$$

- Typical recoil spectrum



- 2-para. exp. ansatz for reconstructing the measured recoil spectrum

$$\left(\frac{dR}{dQ}\right)_{\mathrm{in,\ expt}} = r_0\, e^{-kQ - k' / Q} \qquad\qquad r_0 = \frac{N_{\mathrm{tot}}}{\mathcal{E}\int_{Q_{\mathrm{min}}}^{Q_{\mathrm{max}}} e^{-kQ-k'/Q}\, dQ}$$

Reconstruction of the recoil spectrum - inelastic scattering

- Moments of the 2-parameter exponential recoil spectrum

$$\int \frac{1}{\sqrt{x}}\, e^{-ax-b/x}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\left[e^{2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}+\frac{\sqrt{b}}{\sqrt{x}}\right)+e^{-2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}-\frac{\sqrt{b}}{\sqrt{x}}\right)\right]$$

$$\begin{aligned}\int \sqrt{x}\, e^{-ax-b/x}\, dx = \; & \frac{1}{2}\sqrt{\frac{\pi}{a}}\left\{\frac{1}{2a}\left[e^{2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}+\frac{\sqrt{b}}{\sqrt{x}}\right)+e^{-2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}-\frac{\sqrt{b}}{\sqrt{x}}\right)\right]\right. \\ & \left. -\sqrt{\frac{b}{a}}\left[e^{2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}+\frac{\sqrt{b}}{\sqrt{x}}\right)-e^{-2\sqrt{a}\sqrt{b}}\,\mathrm{erf}\left(\sqrt{a}\sqrt{x}-\frac{\sqrt{b}}{\sqrt{x}}\right)\right]\right\} \\ & -\frac{1}{a}\left(\sqrt{x}\, e^{-ax-b/x}\right)\end{aligned}$$

- Infinite moments of the 2-parameter exponential recoil spectrum
$\left(Q_{\rm min} \to 0,\ Q_{\rm max} \to \infty,\ v_{\rm max} \to \infty\right)$

$$\langle 1/\sqrt{x}\rangle_{\rm inf} \equiv \int_0^\infty \frac{1}{\sqrt{x}}\, e^{-ax-b/x}\, dx = \sqrt{\frac{\pi}{a}}\, e^{-2\sqrt{a}\sqrt{b}}$$

$$\langle \sqrt{x}\rangle_{\rm inf} \equiv \int_0^\infty \sqrt{x}\, e^{-ax-b/x}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\, e^{-2\sqrt{a}\sqrt{b}}\left(\sqrt{\frac{b}{a}}\cdot 2+\frac{1}{a}\right)$$

Reconstruction of the recoil spectrum - inelastic scattering

- Estimating k and k' analytically

$$k_{\rm ana} = \frac{1}{2}\left(\frac{\left\langle Q^{-1/2}\right\rangle_{\rm inf}\left\langle Q^{-3/2}\right\rangle_{\rm inf}}{\left\langle Q^{1/2}\right\rangle_{\rm inf}\left\langle Q^{-3/2}\right\rangle_{\rm inf} - \left\langle Q^{-1/2}\right\rangle_{\rm inf}^2}\right) \qquad k'_{\rm ana} = \frac{1}{2}\left(\frac{\left\langle Q^{-1/2}\right\rangle_{\rm inf}\left\langle Q^{-3/2}\right\rangle_{\rm inf}}{\left\langle Q^{-1/2}\right\rangle_{\rm inf}\left\langle Q^{-5/2}\right\rangle_{\rm inf} - \left\langle Q^{-3/2}\right\rangle_{\rm inf}^2}\right)$$

$$\left\langle Q^{\lambda}\right\rangle_{\rm inf} \equiv \frac{\int_0^\infty Q^\lambda \left(dR/dQ\right)_{\rm in,\ expt} dQ}{\int_0^\infty \left(dR/dQ\right)_{\rm in,\ expt} dQ} \to \frac{1}{N_{\rm tot}} \sum_a Q_a^\lambda$$

- Solving k and k' numerically (with $k_{\rm ana}$ and $k'_{\rm ana}$ as the starting point)

$$\left\langle Q^{\lambda}\right\rangle (k_{\rm num}, k'_{\rm num}) = \frac{\left\langle x^{\lambda}\right\rangle (k_{\rm num}, k'_{\rm num}; x = Q_{\rm max}) - \left\langle x^{\lambda}\right\rangle (k_{\rm num}, k'_{\rm num}; x = Q_{\rm min})}{\int_{Q_{\rm min}}^{Q_{\rm max}} e^{-k^* x - k'^* / x}\, dx}$$

- Equation for solving $Q_{v_{\rm thre}}$

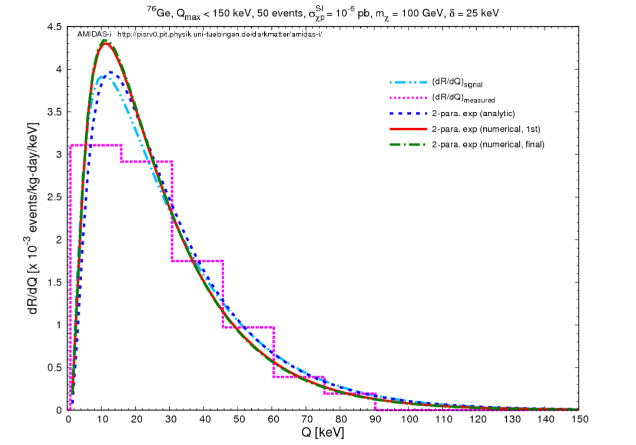
$$\left(k - \frac{k'}{Q_{v_{\rm thre}}^2}\right) + \frac{2}{F\left(Q_{v_{\rm thre}}\right)}\left(\frac{dF}{dQ}\right)_{Q = Q_{v_{\rm thre}}} = 0$$

Numerical results

Reconstruction of the recoil spectrum

- Measured recoil spectrum

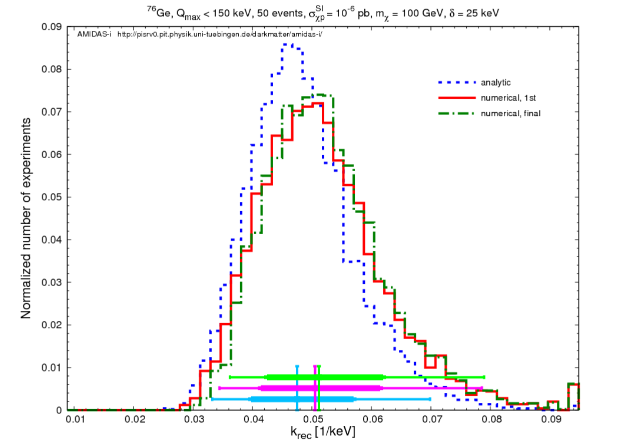
(^{76}Ge, 0 – 150 keV, 50 events, $m_{\chi,\rm in} = 100$ GeV, $\delta_{\rm in} = 25$ keV)



Reconstruction of the recoil spectrum

- Distribution of the reconstructed $k_{\rm rec}$

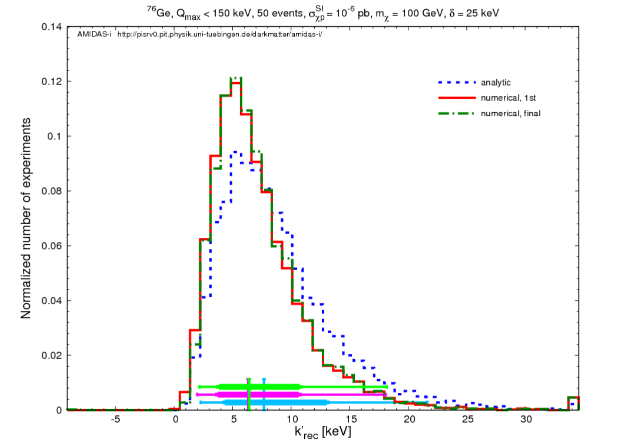
  (^{76}Ge, 0 – 150 keV, 50 events, $m_{\chi,\rm in} = 100$ GeV, $\delta_{\rm in} = 25$ keV)



Reconstruction of the recoil spectrum

- Distribution of the reconstructed $k'_{\rm rec}$

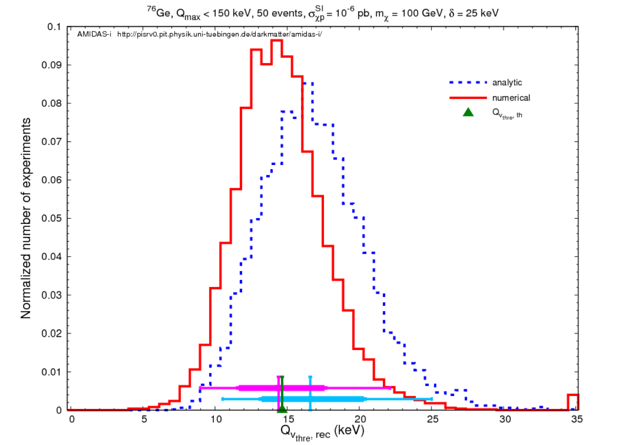
(^{76}Ge, 0 − 150 keV, 50 events, $m_{\chi,\rm in} = 100$ GeV, $\delta_{\rm in} = 25$ keV)



Reconstruction of the recoil spectrum

- Distribution of the reconstructed $Q_{v_{\rm thre},{\rm rec}}$

  (^{76}Ge, $0 - 150$ keV, 50 events, $m_{\chi,{\rm in}} = 100$ GeV, $\delta_{\rm in} = 25$ keV)



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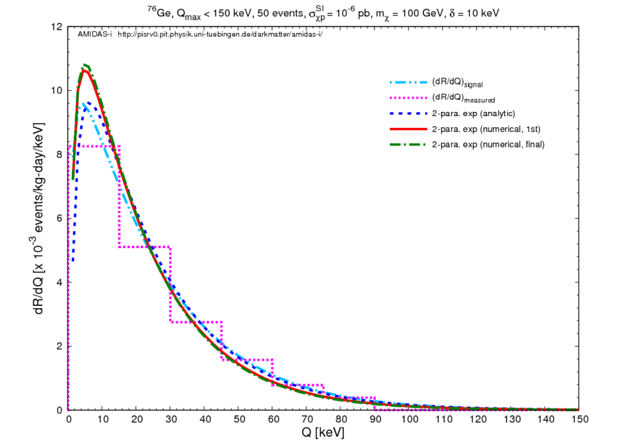
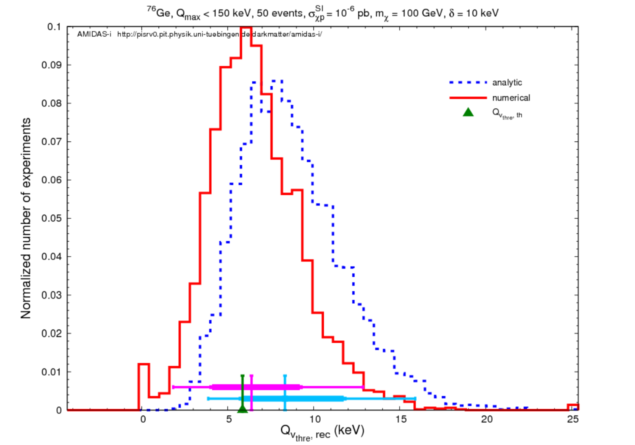


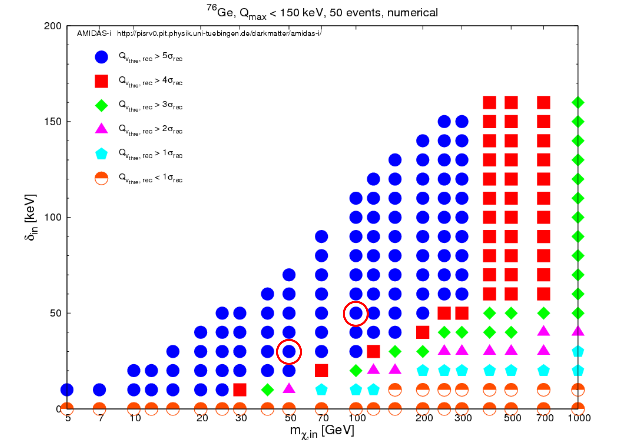
Reconstruction of the recoil spectrum

- Distribution of the reconstructed $Q_{v_{\rm thre},\rm rec}$
  (^{76}Ge, 0 – 150 keV, 50 events, $m_{\chi,\rm in} = 100$ GeV, $\delta_{\rm in} = 10$ keV)



Identification of the inelastic scattering

- $Q_{v_{\rm thre},{\rm rec}}$ in unit of $\sigma(Q_{v_{\rm thre},{\rm rec}})$
  (^{76}Ge, 0 – 150 keV, 50 events, numerical)



Identification of the inelastic scattering

- $Q_{v_{\rm thre},{\rm th}} - Q_{v_{\rm thre},{\rm rec}}$ in unit of $\sigma(Q_{v_{\rm thre},{\rm rec}})$
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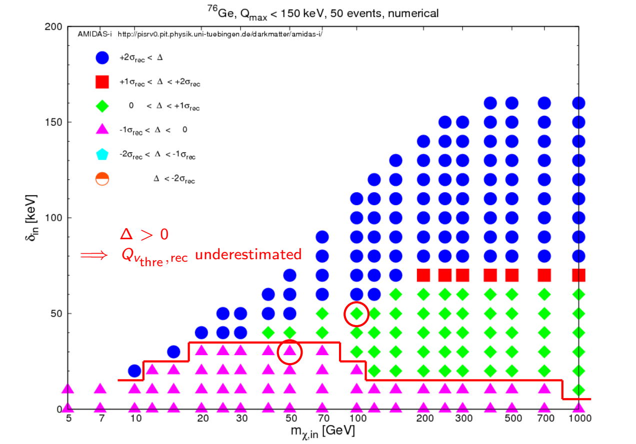
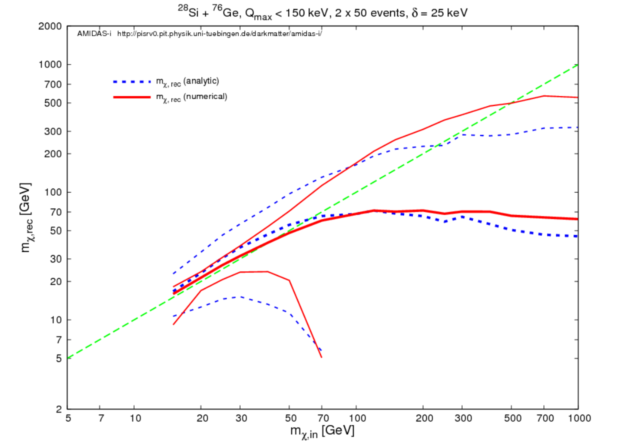


Determinations of the WIMP mass and the mass splitting

- Reconstructed $m_{\chi,\mathrm{rec}}$

  ($^{28}\mathrm{Si} + ^{76}\mathrm{Ge}$, $0 - 150$ keV, 2×50 events, $\delta_{\mathrm{in}} = 25$ keV)



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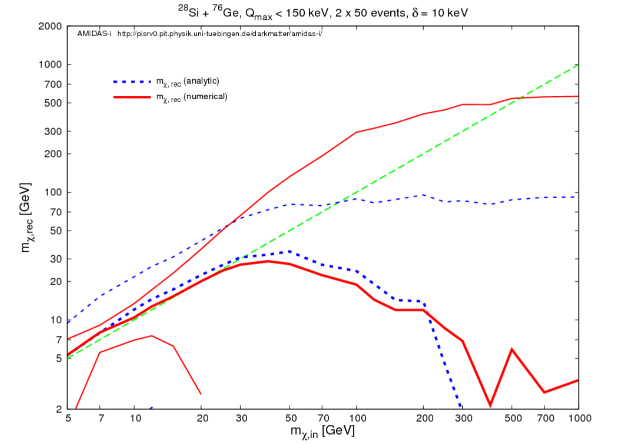

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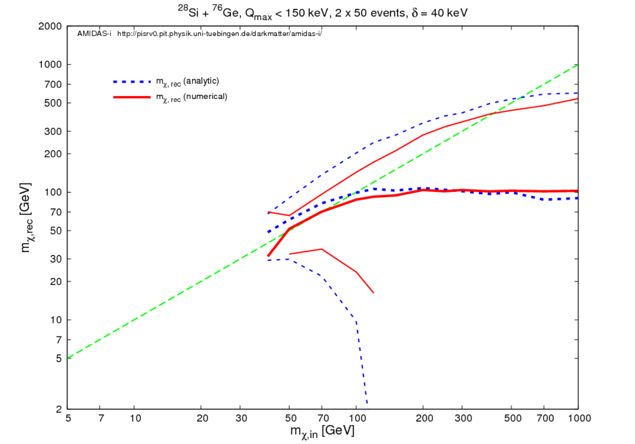
- Reconstructed $m_{\chi,\rm rec}$
  ($^{28}\rm Si + ^{76}\rm Ge$, $0 - 150$ keV, 2×50 events, $\delta_{\rm in} = 40$ keV)



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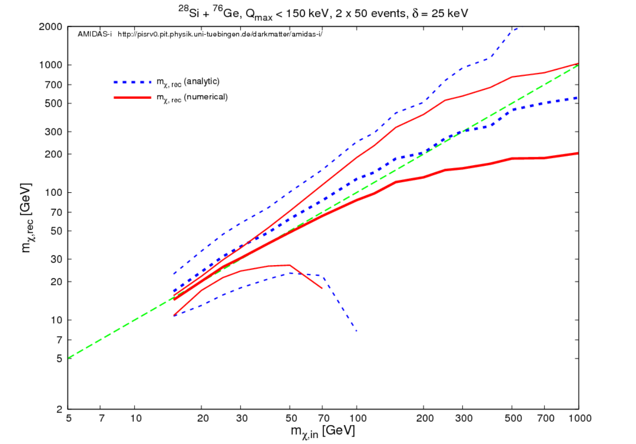

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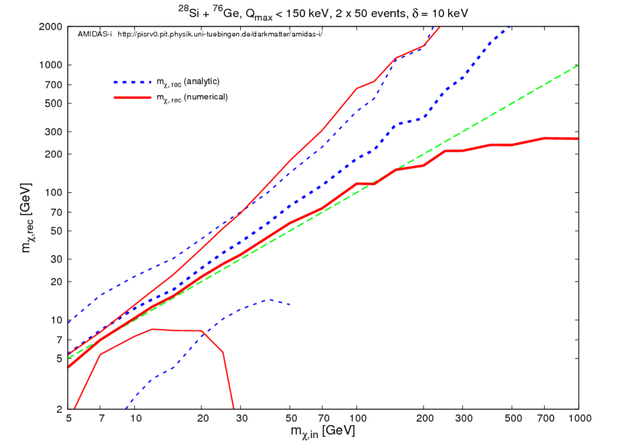

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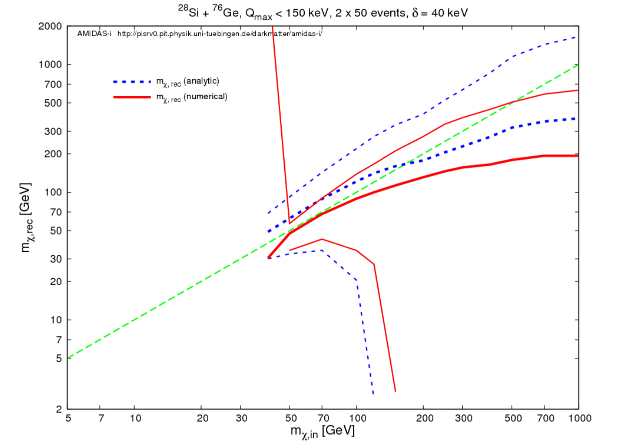
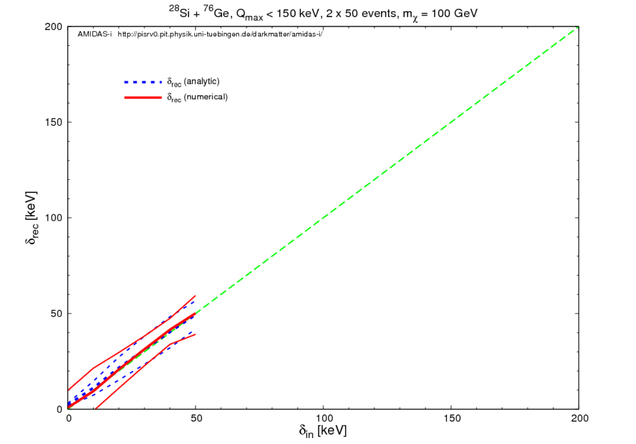


Determinations of the WIMP mass and the mass splitting

- Reconstructed δ_{rec}

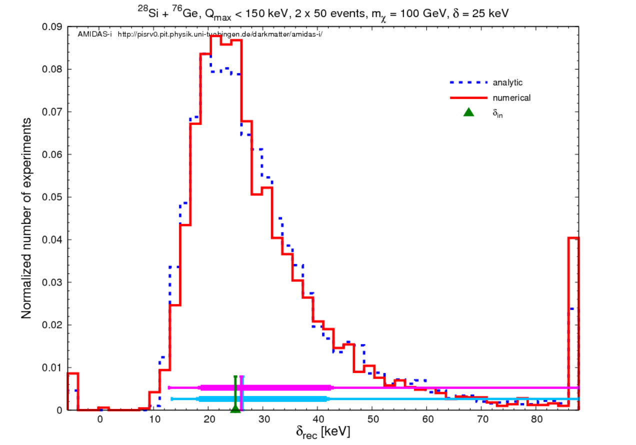
($^{28}Si +^{76} Ge$, $0 - 150$ keV, 2×50 events, $m_{\chi,in} = 100$ GeV)



Determinations of the WIMP mass and the mass splitting

- Distribution of the reconstructed $\delta_{\rm rec}$

($^{28}{\rm Si} + ^{76}{\rm Ge}$, $0 - 150$ keV, 2×50 events, $m_{\chi,{\rm in}} = 100$ GeV, $\delta_{\rm in} = 25$ keV)



Determinations of the WIMP mass and the mass splitting

- Distribution of the reconstructed $\delta_{\rm rec}$

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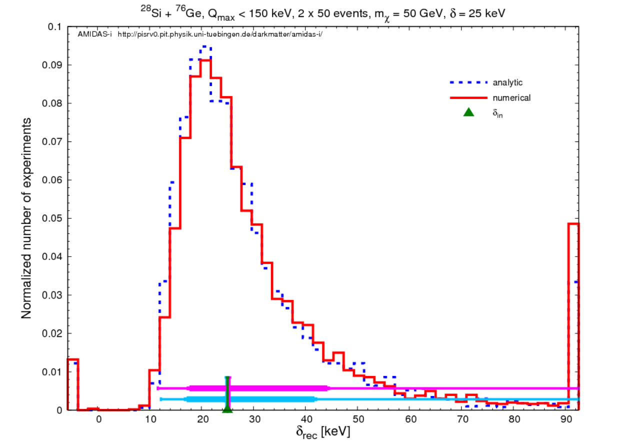
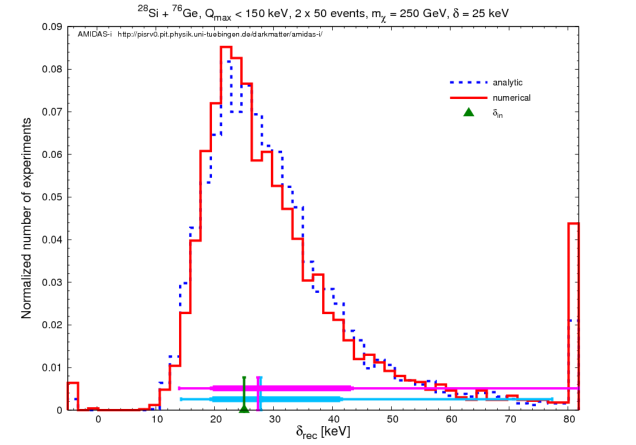


Determinations of the WIMP mass and the mass splitting

- Distribution of the reconstructed δ_{rec}

($^{28}Si + ^{76}Ge$, 0 – 150 keV, 2 × 50 events, $m_{\chi,in} = 250$ GeV, $\delta_{in} = 25$ keV)



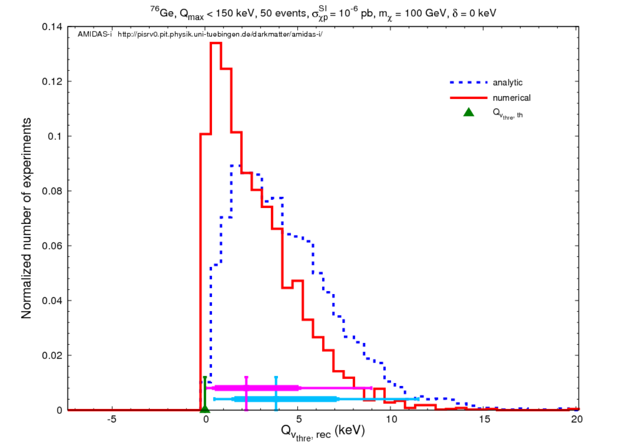
Determinations of the WIMP mass and the mass splitting

- The two-parameter exponential ansatz for reconstructing the recoil spectrum could be a good approximation.
- With $\mathcal{O}(50)$ events one could in principle identify inelastic WIMPs from the elastic one model-independently.
- For inelastic ($\delta > 0$) case, we could
  - observe positive $Q_{v_{\rm thre}}$ with a 2σ - 5σ confidence level.
  - observe positive δ (although this could be slightly underestimated).
  - give an upper bound of m_χ (underestimated, improvement required).
- For elastic ($\delta = 0$) case, we could observe
  - very small, but non-zero positive $Q_{v_{\rm thre}}$...
  - very small, but non-zero positive δ...
  - ☛ (unphysically) negative m_χ: the larger the input m_χ, the larger the absolute value of the reconstructed m_χ

Determinations of the WIMP mass and the mass splitting

- Distribution of the reconstructed $Q_{v_{\rm thre},\rm rec}$

(^{76}Ge, $0 - 150$ keV, 50 events, $m_{\chi,\rm in} = 100$ GeV, $\delta_{\rm in} = 0$ keV)



Summary and outlook

Summary and outlook

- With a single experiment one could identify inelastic WIMPs from the elastic one model-independently.
- By combining two experiments with different target nuclei, one could determine the mass splitting and (for small mass splitting) give a rough upper bound of the mass of inelastic WIMPs.
- Possible improvement of the determination of the (upper bound of the) WIMP mass is currently under investigation: e.g., the use of the pretty certainly determined δ.
- The reconstruction of the 1-D WIMP velocity distribution could then be achieved.

Thank you very much for your attention!