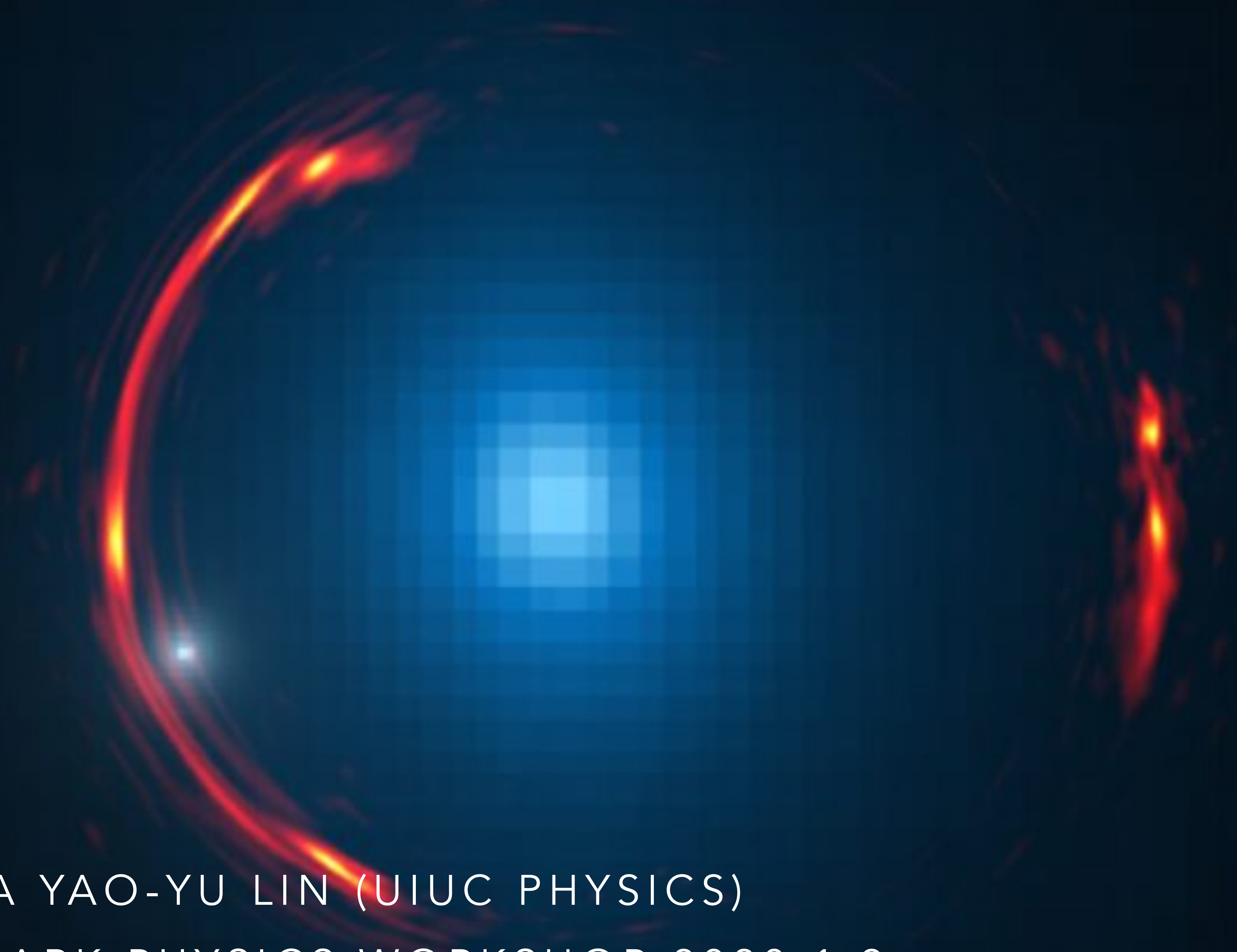
HUNTING FOR DARK MATTER SUBSTRUCTURES IN STRONG LENSING WITH NEURAL NETWORKS

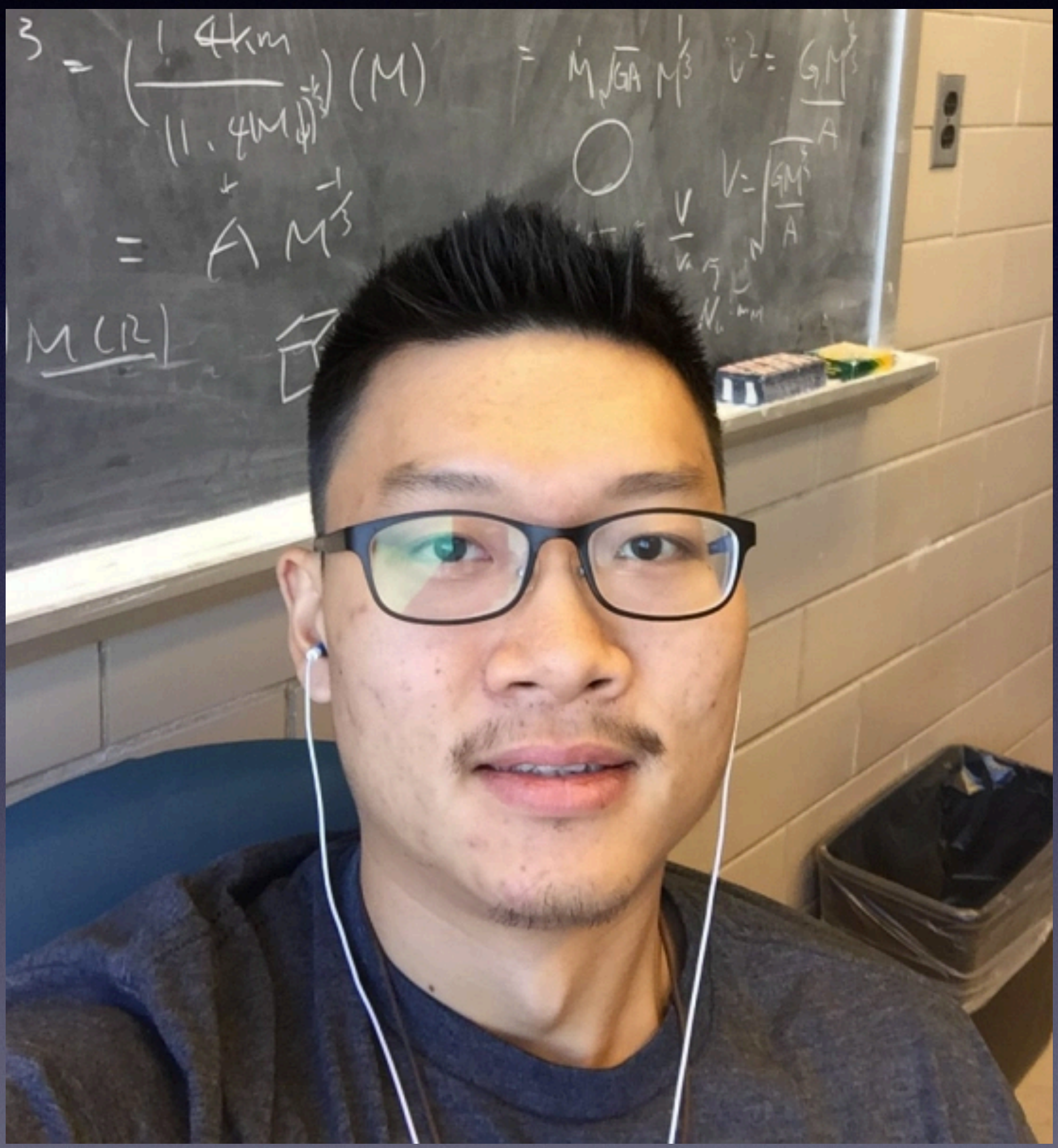


JOSHUA YAO-YU LIN (UIUC PHYSICS)

NCTS DARK PHYSICS WORKSHOP 2020.1.9

Lin et al. 2020 (In prep)

About myself



- NTHU Physics (Undergrad 2012)
- NTU Physics (MS 2015)
- UIUC Physics (Ph.D. candidate 2016-present)

Research Interest:

- Dark Matter Substructures in Strong Lensing

- Cosmic Neutrinos

- Black Holes images, Radio radio interferometry, Machine Learning

Gravitational Lensing of Cosmic Neutrino Background

Lin & Holder (arXiv:1910.03550)

Joshua Yao-Yu Lin (University of Illinois at Urbana-Champaign)

In collaboration with Gilbert Holder

2020.1.9 NCTS Dark Physics Workshop

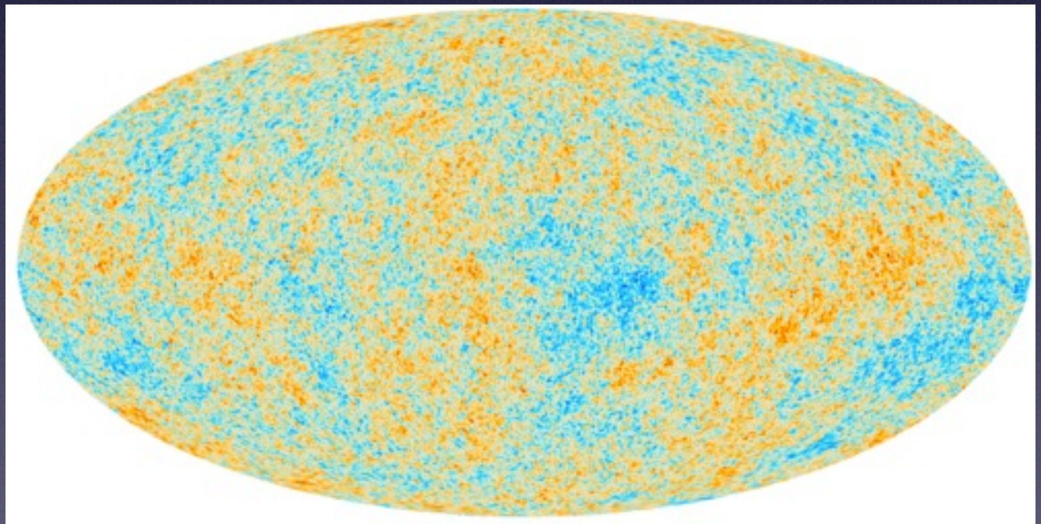
ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

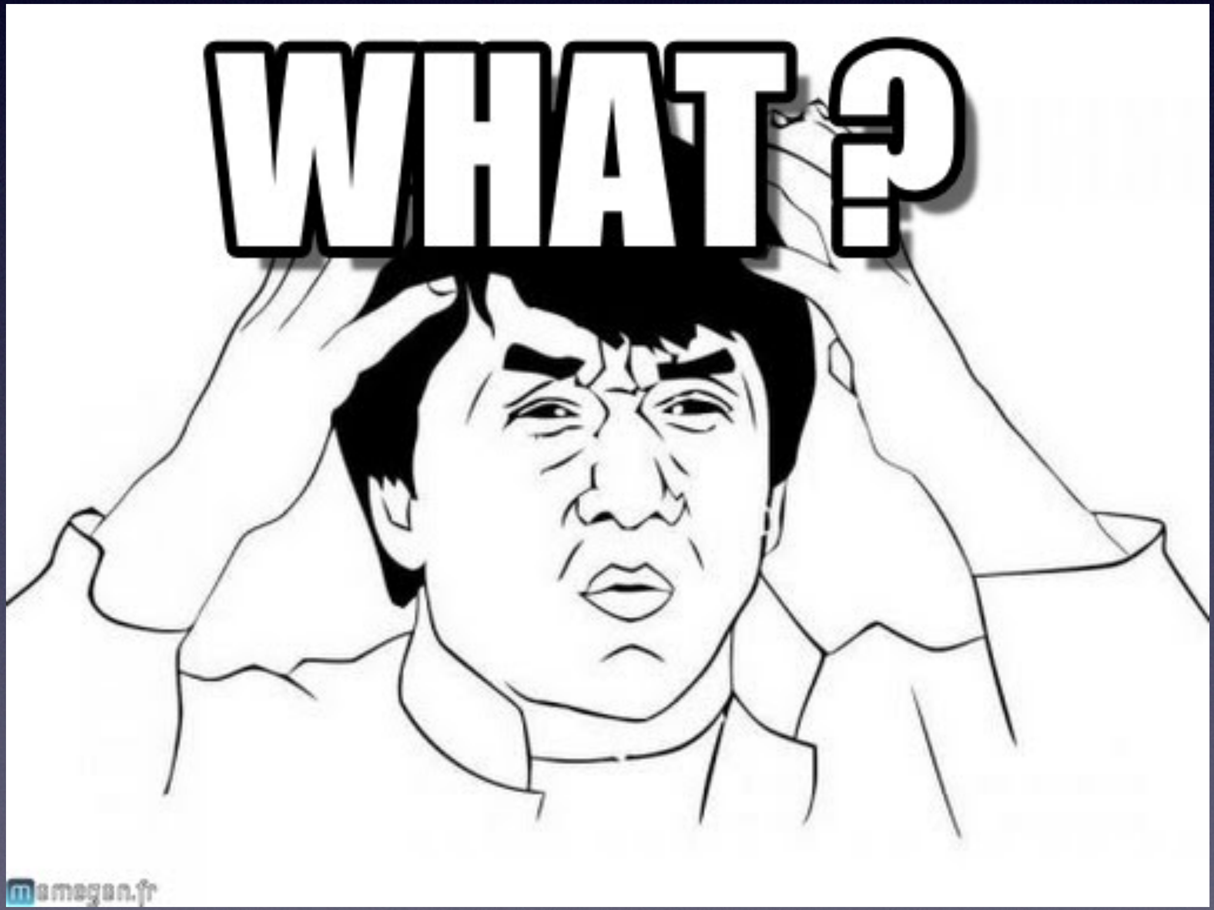
Lensing of CνB vs CMB

Similar:

Difference:

- Both (shall) have anisotropy and would be lensed by foreground gravitational potential





Thought Experiment!

Lensing of CνB vs CMB

Similar:

- Both (shall) have anisotropy and would be lensed by foreground gravitational potential

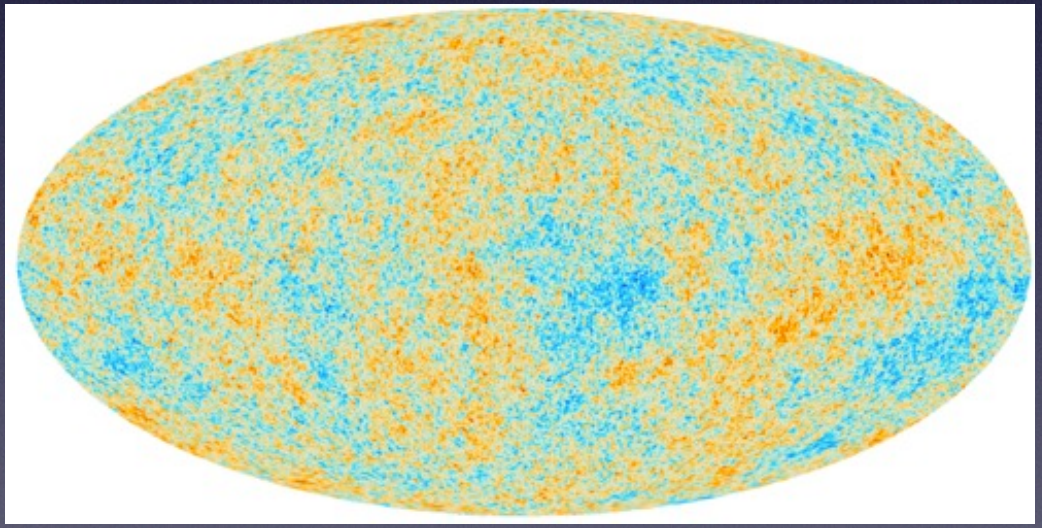
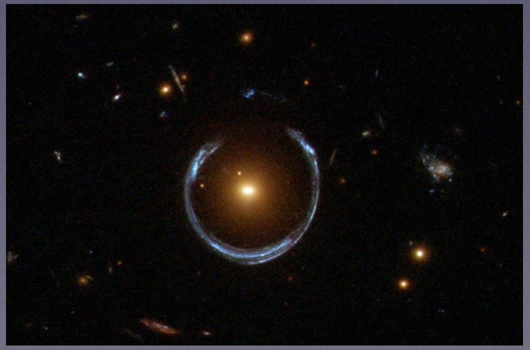


Image Credit: 1) Planck
2) ESA/NASA, Hubble



Difference:

- Neutrinos from the early universe will be non-relativistic today (massive neutrinos)
- Larger angles of deflection
- Closer surface of last scattering (compared to the cosmic microwave background)
- Could form multiple lensed images **[Strong gravitational lensing]**

Cosmic Neutrino Last Scattering Surface

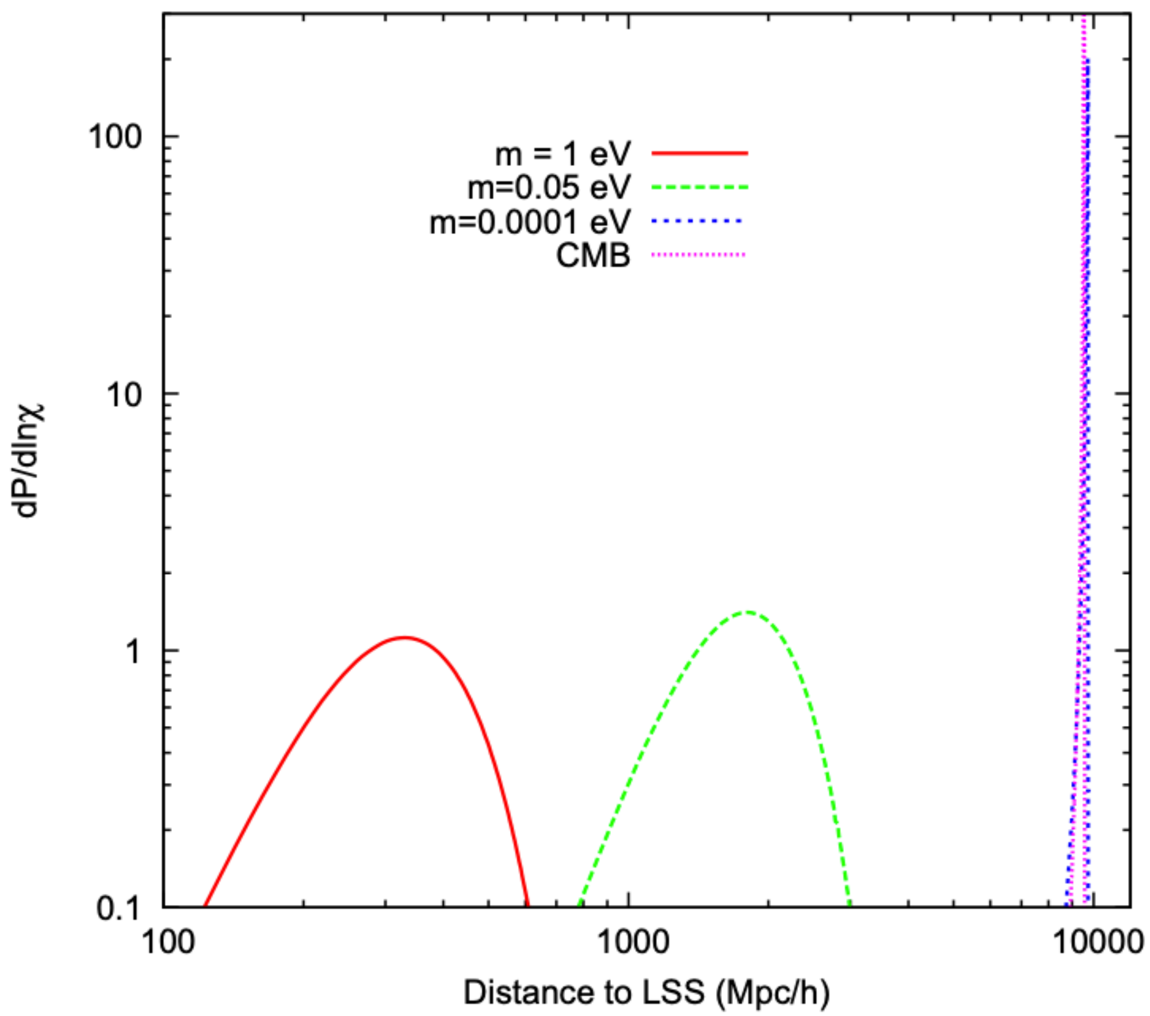


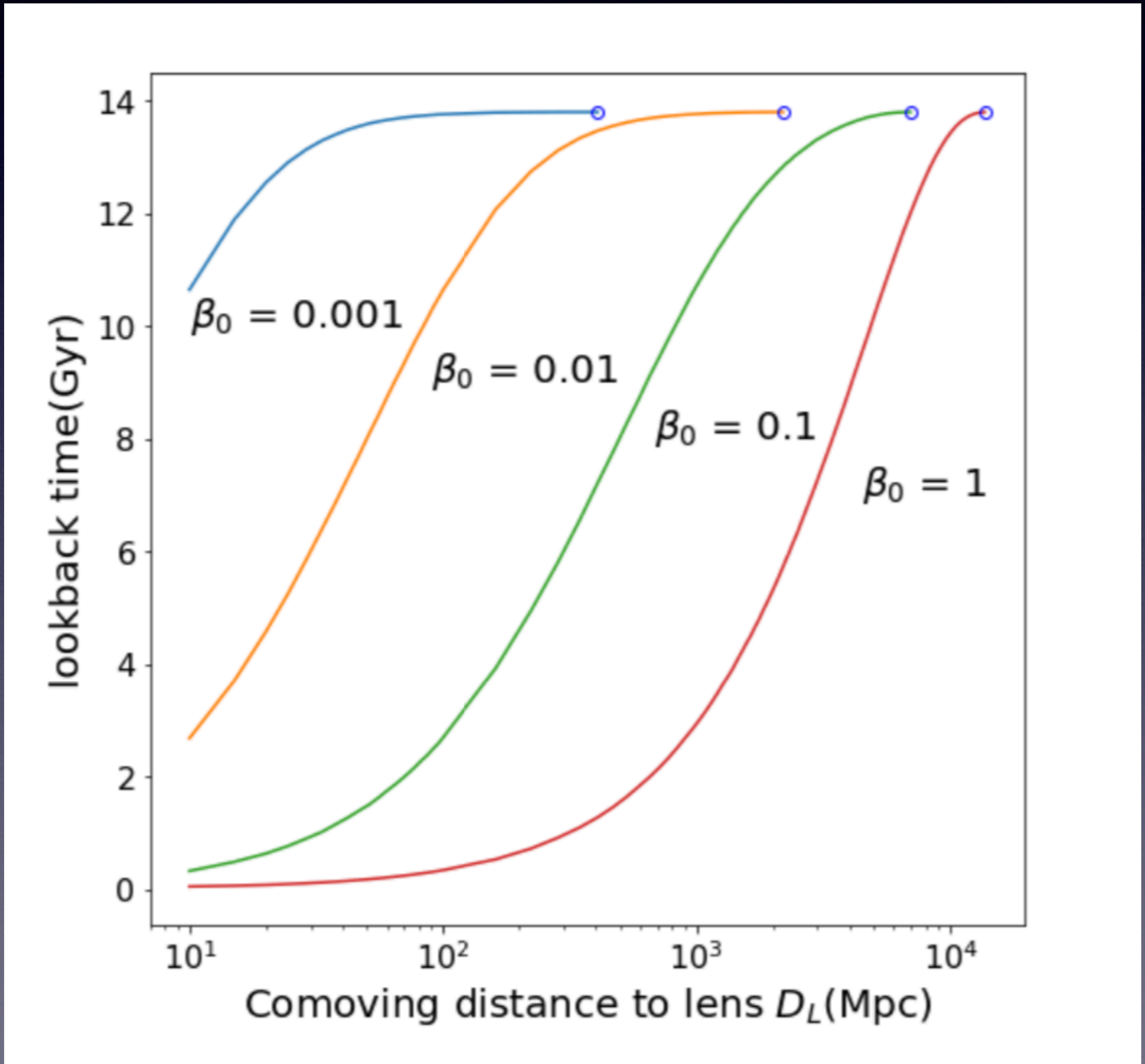
FIG. 2: The probability that a neutrino with mass m last scatters at a given comoving distance from us (the visibility function). Massive neutrinos travel more slowly than massless neutrinos so arrive here from much closer distances. Also shown is the last scattering surface of the cosmic microwave background, virtually indistinguishable from that of an $m_\nu = 10^{-4}$ eV neutrino.

$$v(a) = \frac{v_0}{\sqrt{a^2 + \frac{v_0^2}{c^2}(1-a^2)}}$$

$$D_S(v_0) = \int_{a_s}^{1} \frac{da}{a^2 H(a)} v(a)$$

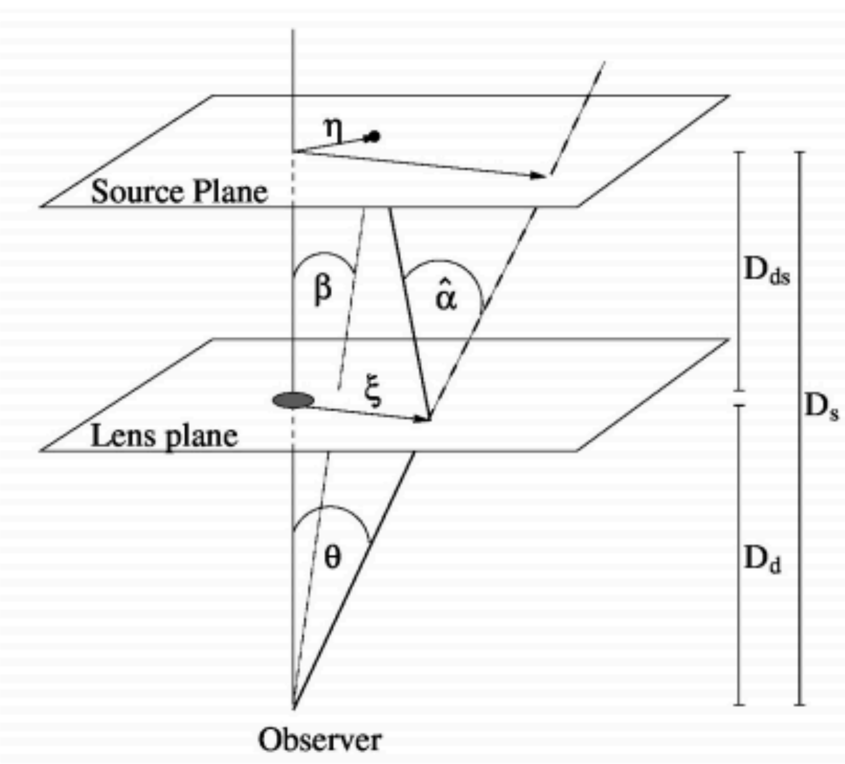
Dodelson & Vesterinen (PRL, 2009)

Cosmic Neutrino Last Scattering Surface



Strong lensing of cosmic neutrino

Lens equation



$$\boldsymbol{\eta} = \frac{D_s}{D_d}\boldsymbol{\xi} - D_{ds}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

In terms of angular coord.:

$$\boldsymbol{\eta} = D_s\boldsymbol{\beta}$$
$$\boldsymbol{\xi} = D_d\boldsymbol{\theta}$$

$$\boxed{\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})}$$

where

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_{ds}}{D_s}\hat{\boldsymbol{\alpha}}(D_d\boldsymbol{\theta})$$

[Schneider et al. 2006]

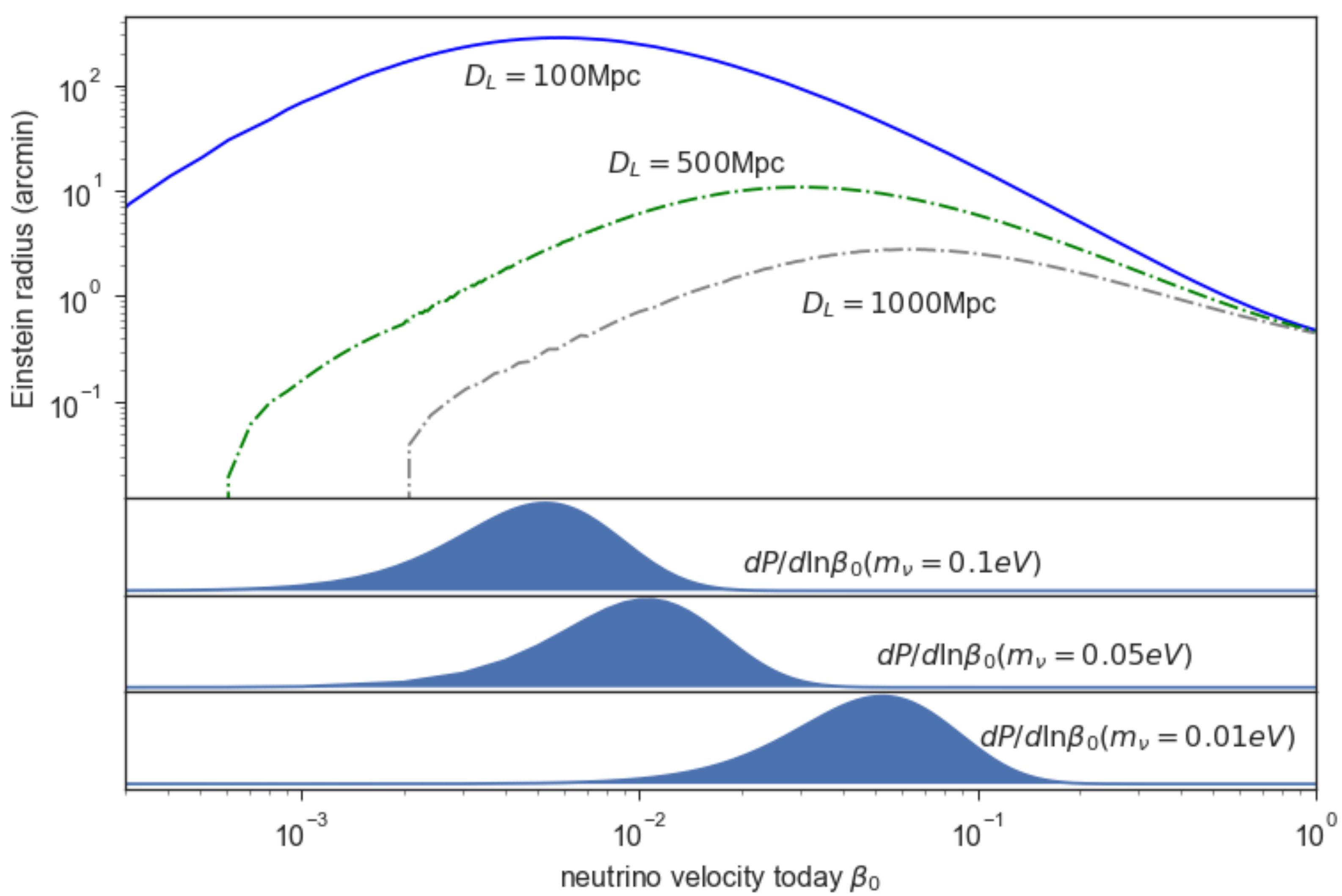
2

Slides credit: Sherry Suyu

- Angle of deflection for cosmic neutrino (post-Newtonian)

$$\alpha(R) = \frac{4GM(R)}{Rc^2}\frac{c^2 + v_{lens}^2}{2v_{lens}^2},$$

Einstein Radius: function of neutrino velocity/ Distance to Lens



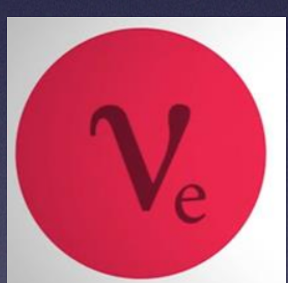
$$\frac{dP}{dp_0} = \frac{2}{3\zeta(3)k_B^3 T_\nu^3} \frac{p_0^2/c^3}{e^{p_0/k_B T_\nu} + 1}$$

Neutrinos momentum distribution

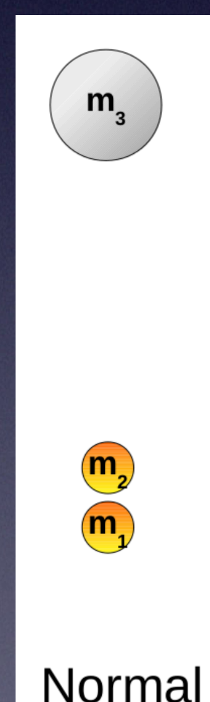
$$\theta_E^{\mathrm{SIS}} = \frac{4\pi\sigma_v^2}{c^2}\left(\frac{c^2 + v_{lens}^2}{2v_{lens}^2}\right)\frac{D_{LS}(v_0, D_L)}{D_S(v_0)}$$

SIS lens model: σv = 1000 km/s

Mass eigenstates splitting via gravitational potential



Neutrino source (flavor)

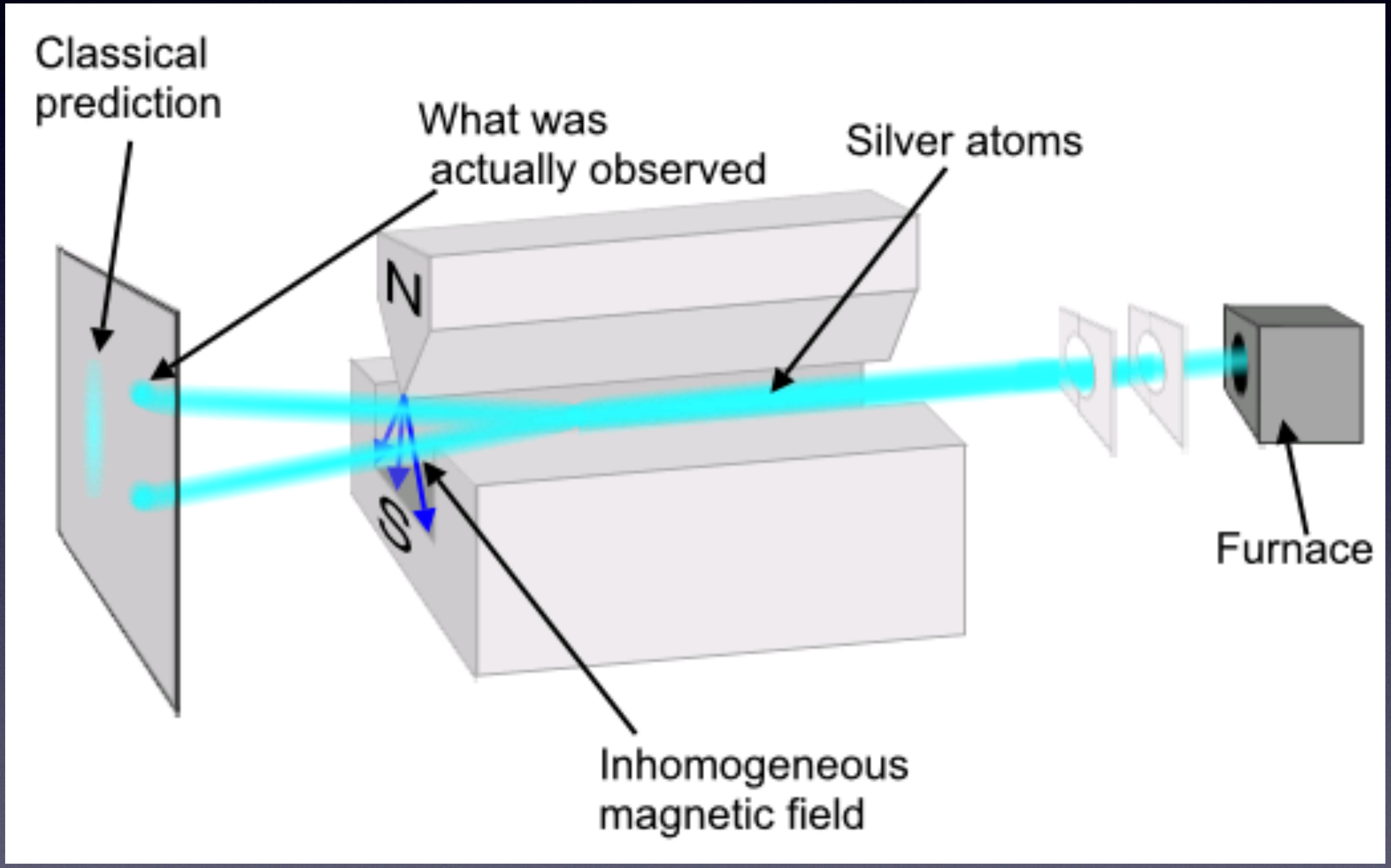


Mass eigenstates

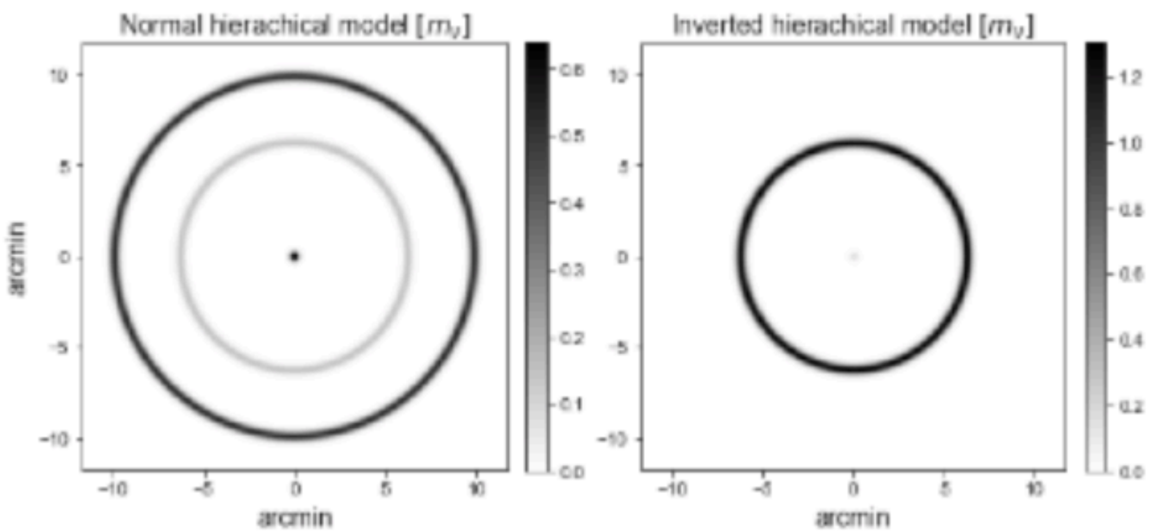
Gravitational Lens:
Spectrometer

Mass eigenstates
splitting

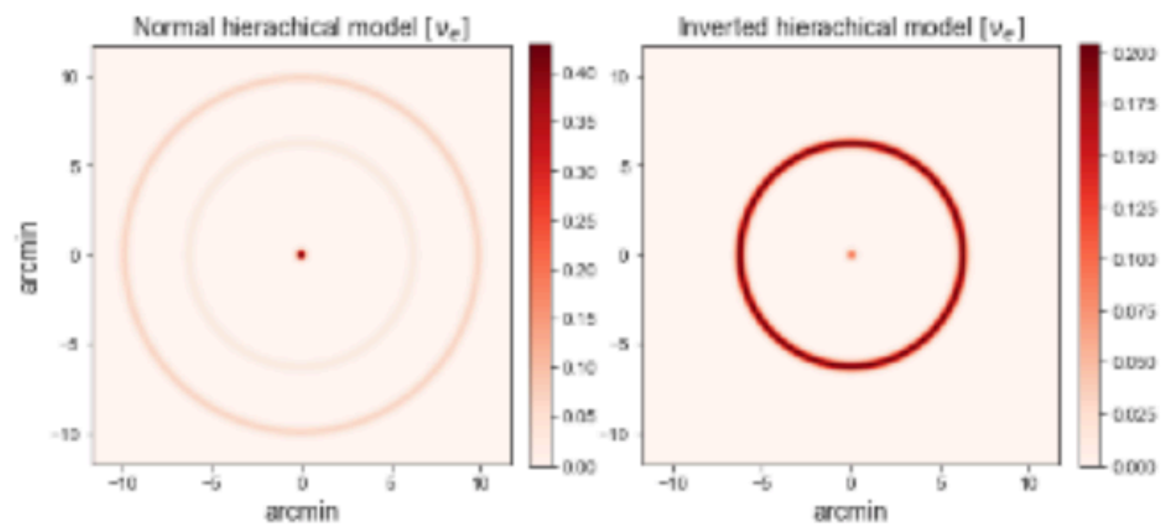
Spectrometer: Stern–Gerlach experiment



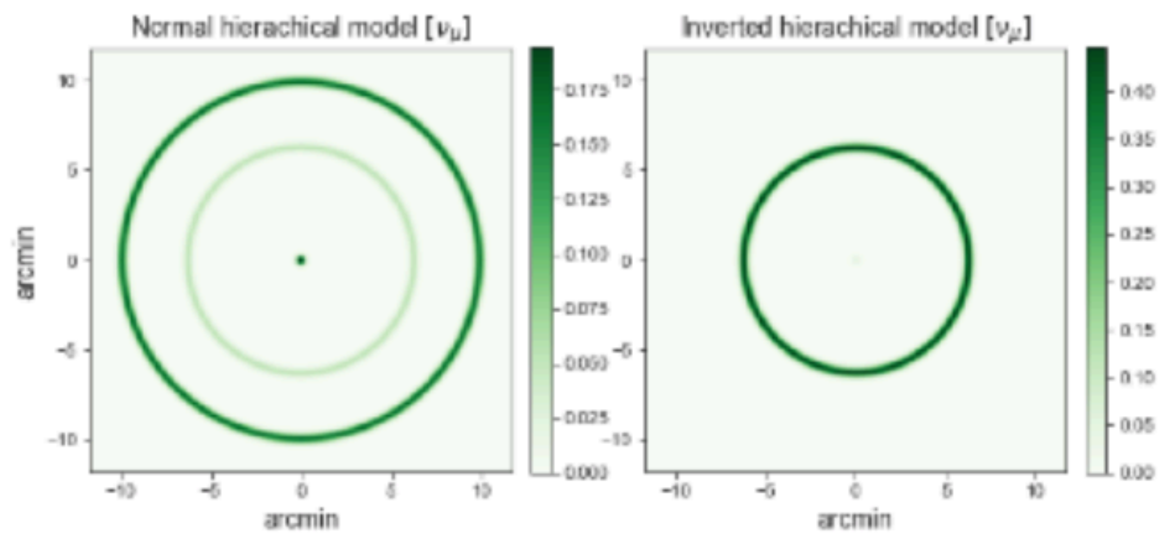
Mass eigenstates splitting via gravitational potential



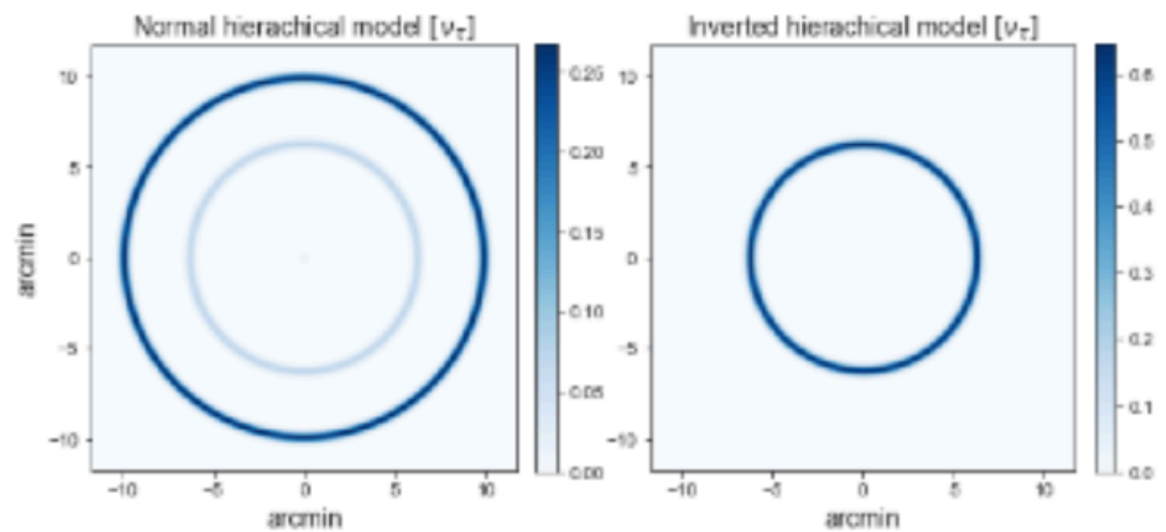
(a) Neutrino flux for mass eigenstates



(b) Neutrino flux for electron neutrinos



(c) Neutrino flux for muon neutrinos



(d) Neutrino flux for tau neutrinos

Discussion & Summary

- Strong lensing of CνB could be the largest Stern–Gerlach experiment in our universe
- Neutrino oscillations are not relevant in this case, as the mass eigenstates get dispersed in angular space
- Time evolution of the halo could also be probed
- Interesting, with extremely rich source of information: strong lensing, neutrino mass, quantum properties

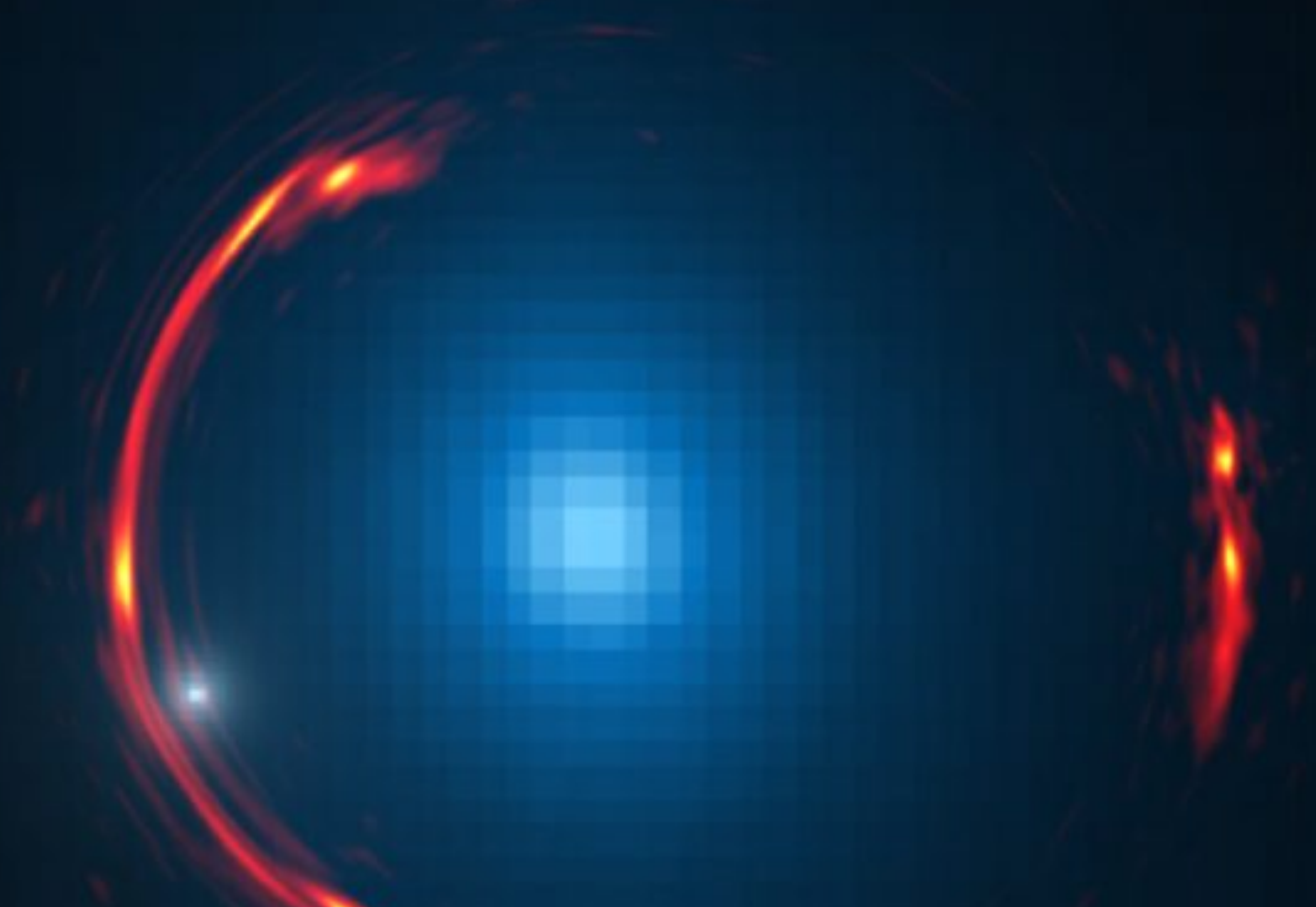
Lin & Holder (arXiv:1910.03550)

Lin & Holder (arXiv:1910.03550)

University of Illinois

Image credit: Aram Grigoryan/Getty Images

HUNTING FOR DARK MATTER SUBSTRUCTURES IN STRONG LENSING WITH NEURAL NETWORKS



JOSHUA YAO-YU LIN (UIUC PHYSICS)

NCTS DARK PHYSICS WORKSHOP 2020.1.9

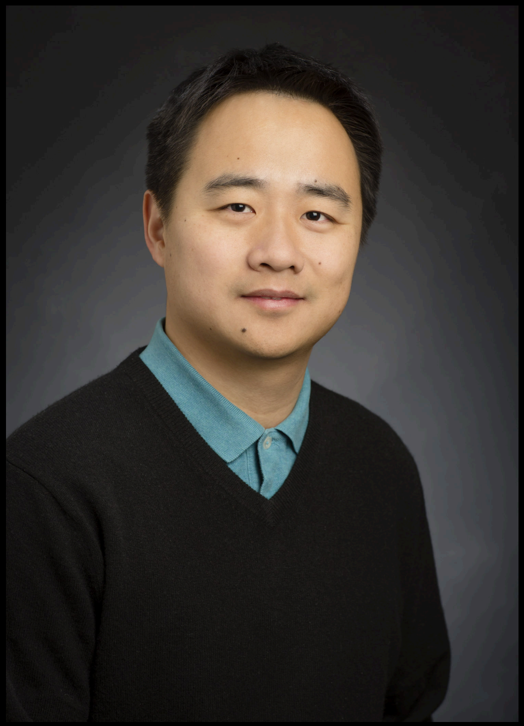
Lin et al. 2020 (In prep)

PEOPLE I WORK WITH

LOCAL GROUP

Hang Yu
[UIUC]

Warren
Morningstar
[Stanford]

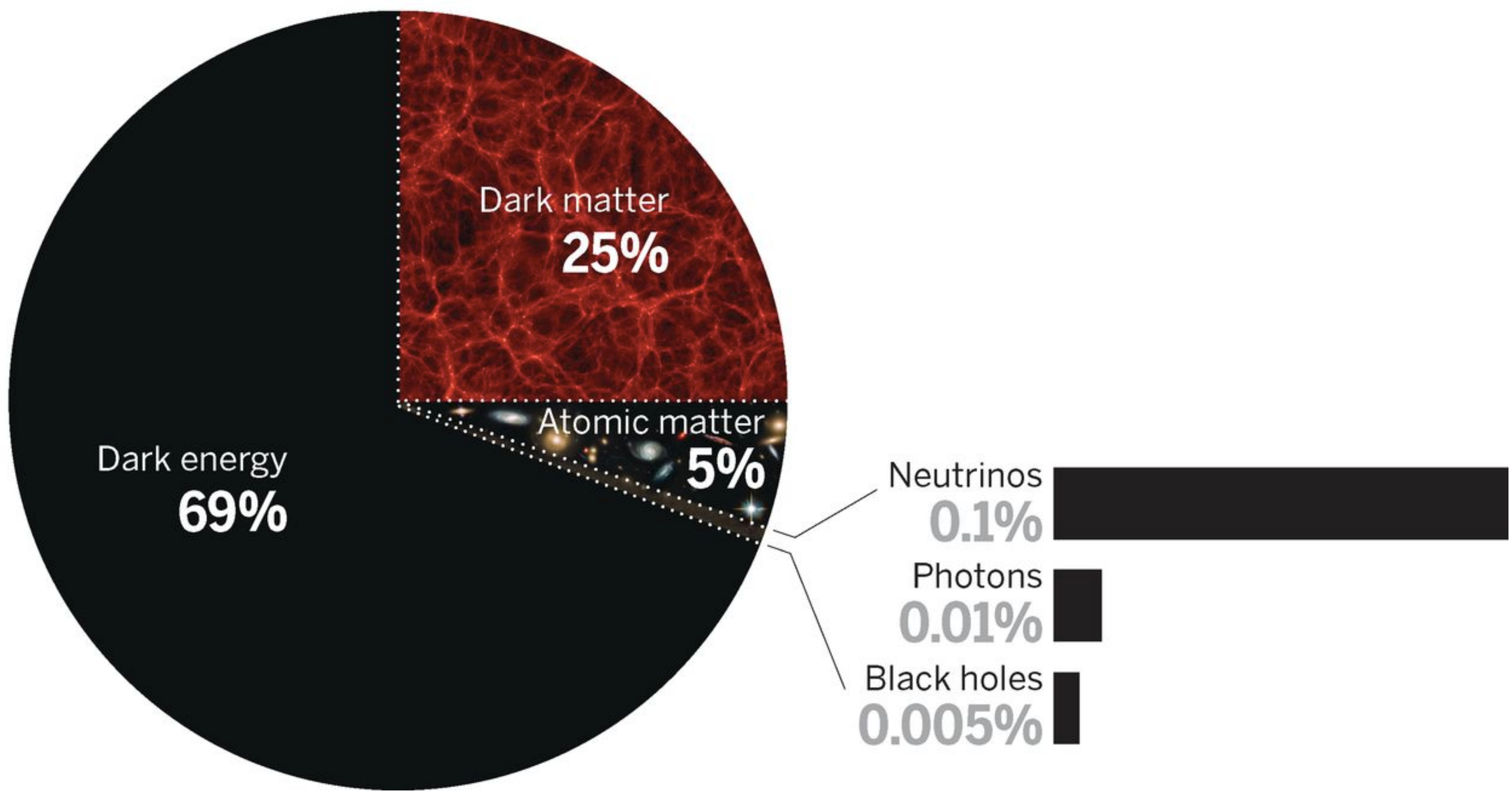


Jian Peng
[CS@UIUC]

Gil Holder
[UIUC]

The multiple components that compose our universe

Current composition (as the fractions evolve with time)



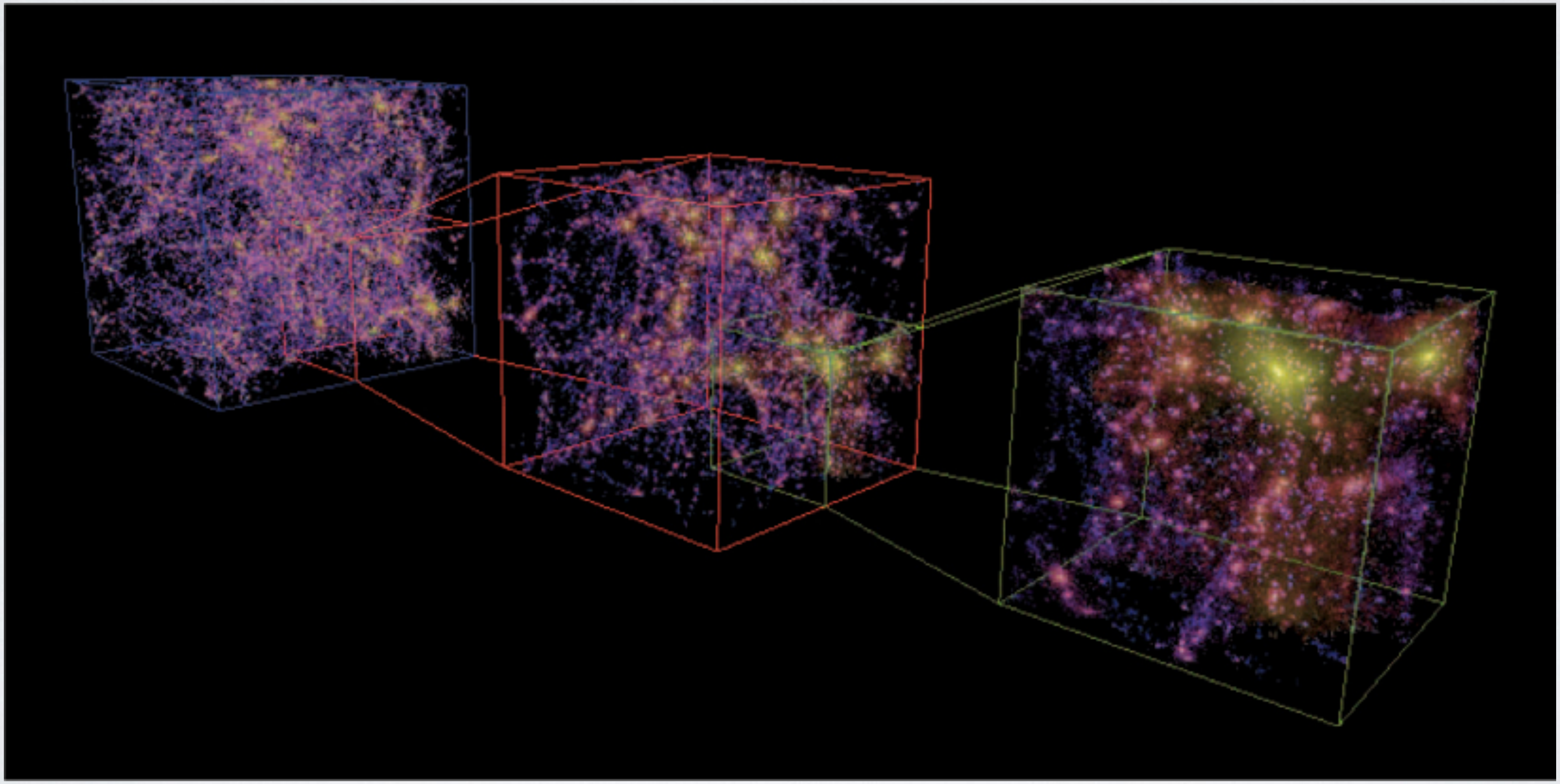
Three different types of neutrinos comprise at least 0.1%, the cosmic background radiation makes up 0.01%, and black holes comprise at least 0.005%.

David Spergel (Science, 2015)

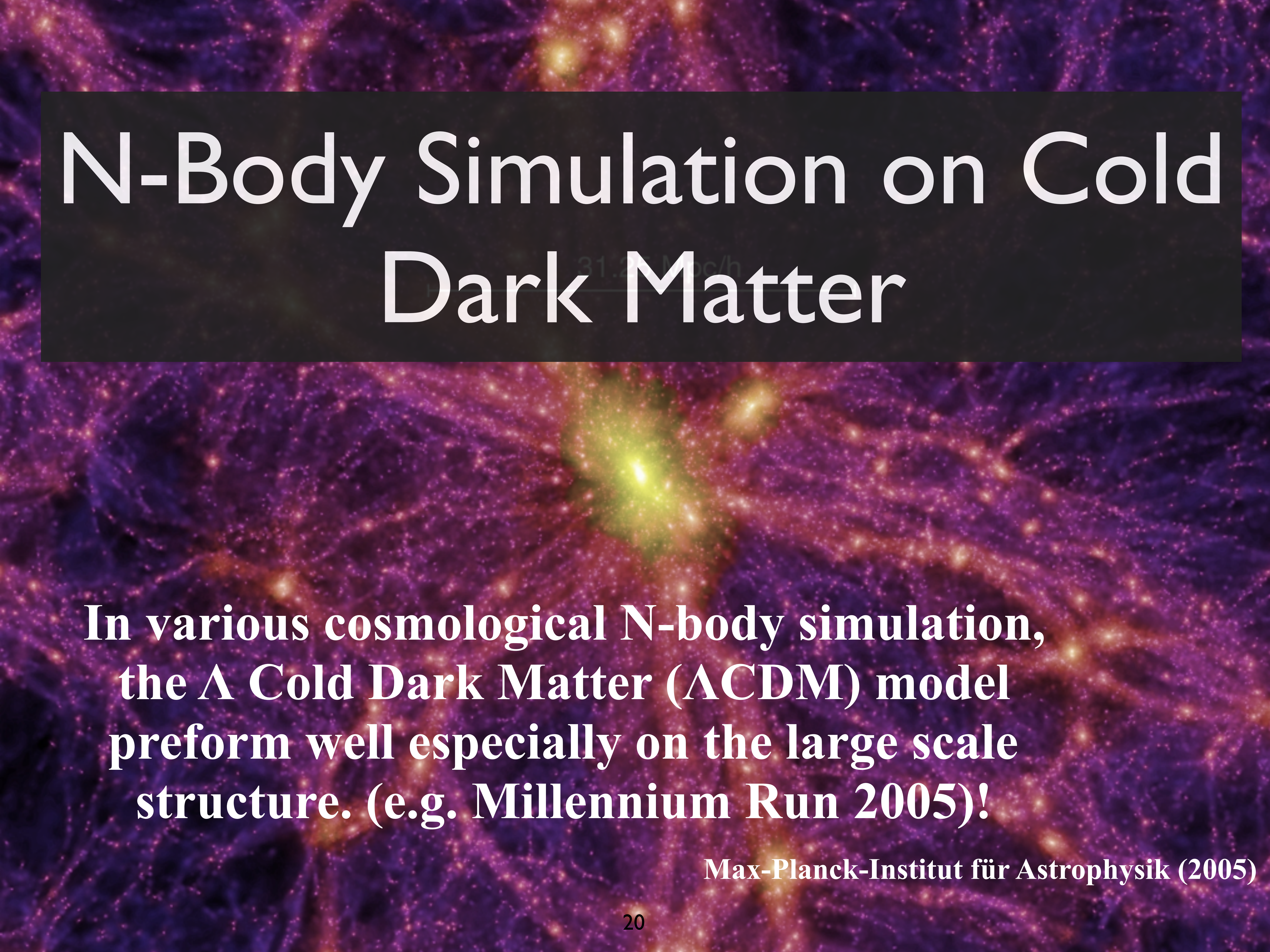
Dark Matter

- 84% of the matter is Dark(DM)
- DM interacts through gravity.
- Further DM interactions unobserved so far. Such couplings must be very weak, much weaker than weak interactions.

N-BODY SIMULATION: STUDY THE PROPERTY OF DARK MATTER



Max-Planck-Institut für Astrophysik (2005)



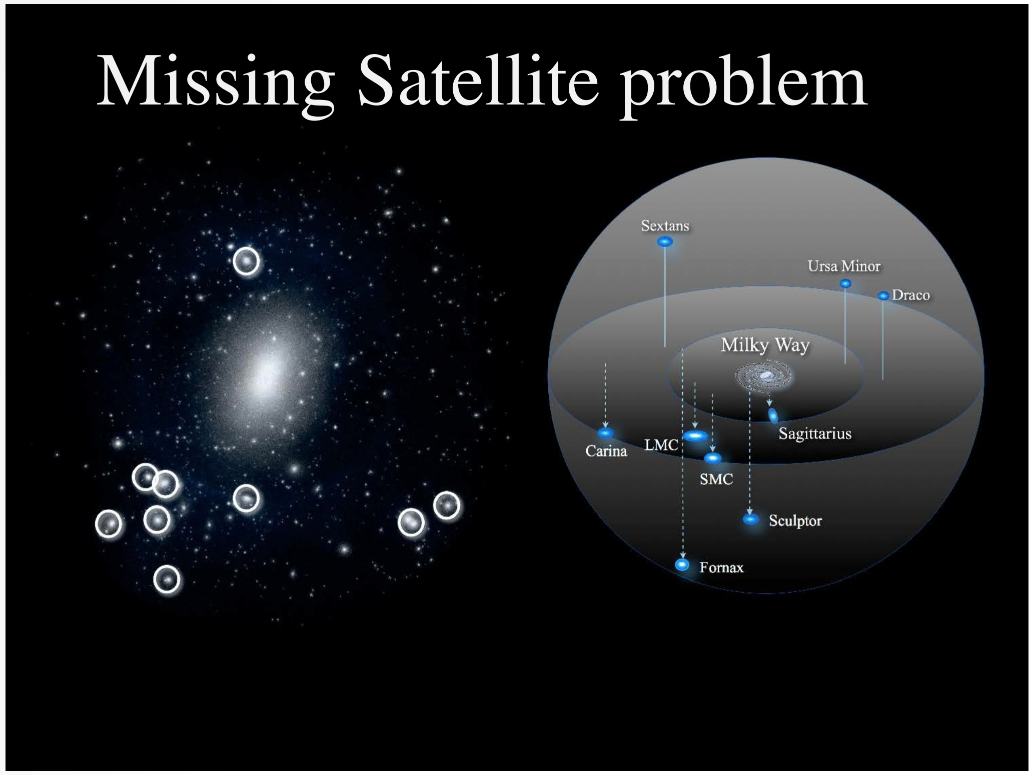
N-Body Simulation on Cold Dark Matter

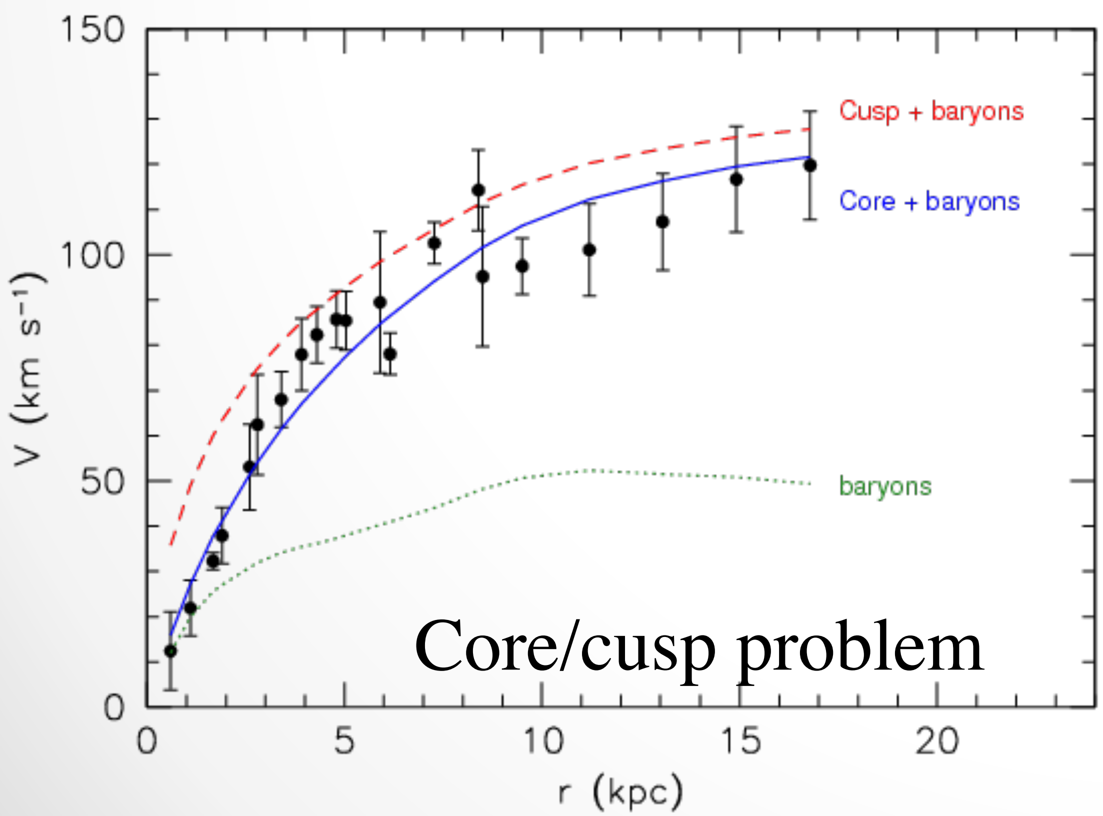
In various cosmological N-body simulation, the Λ Cold Dark Matter (ΛCDM) model preform well especially on the large scale structure. (e.g. Millennium Run 2005)!

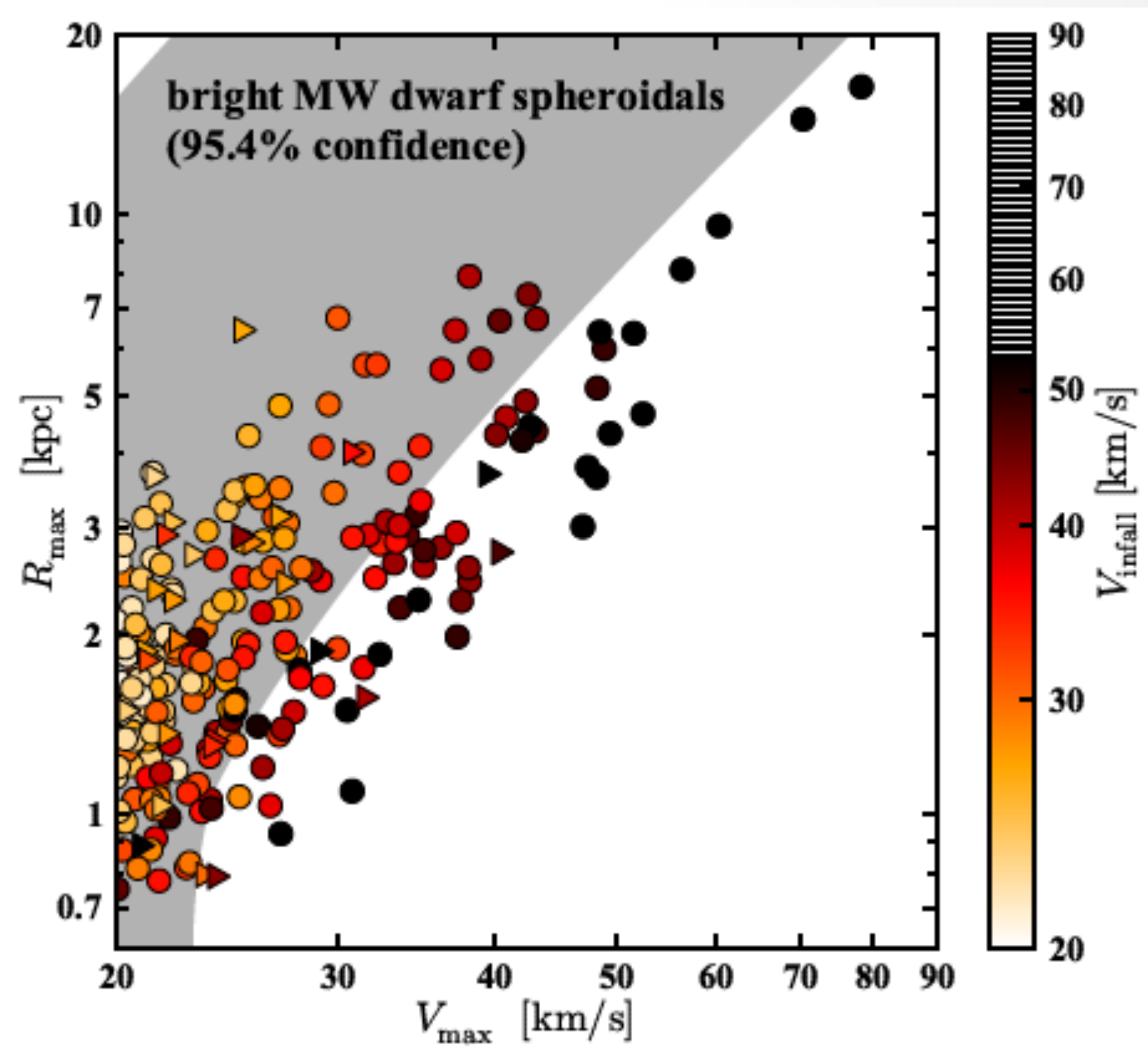
Max-Planck-Institut für Astrophysik (2005)

Controversies on Galactic Scale

-- Comparison of Numerical Simulation & Observational Data



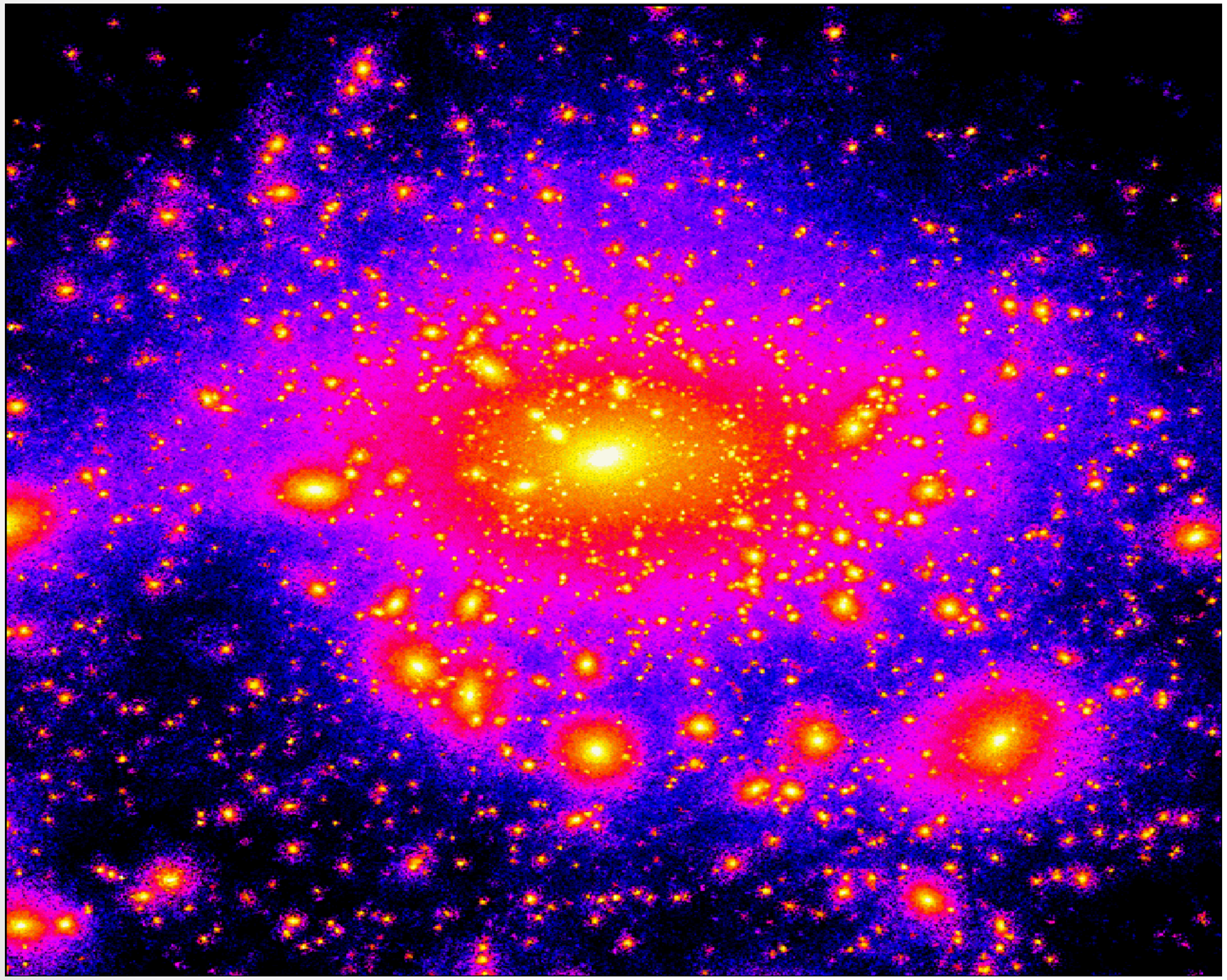




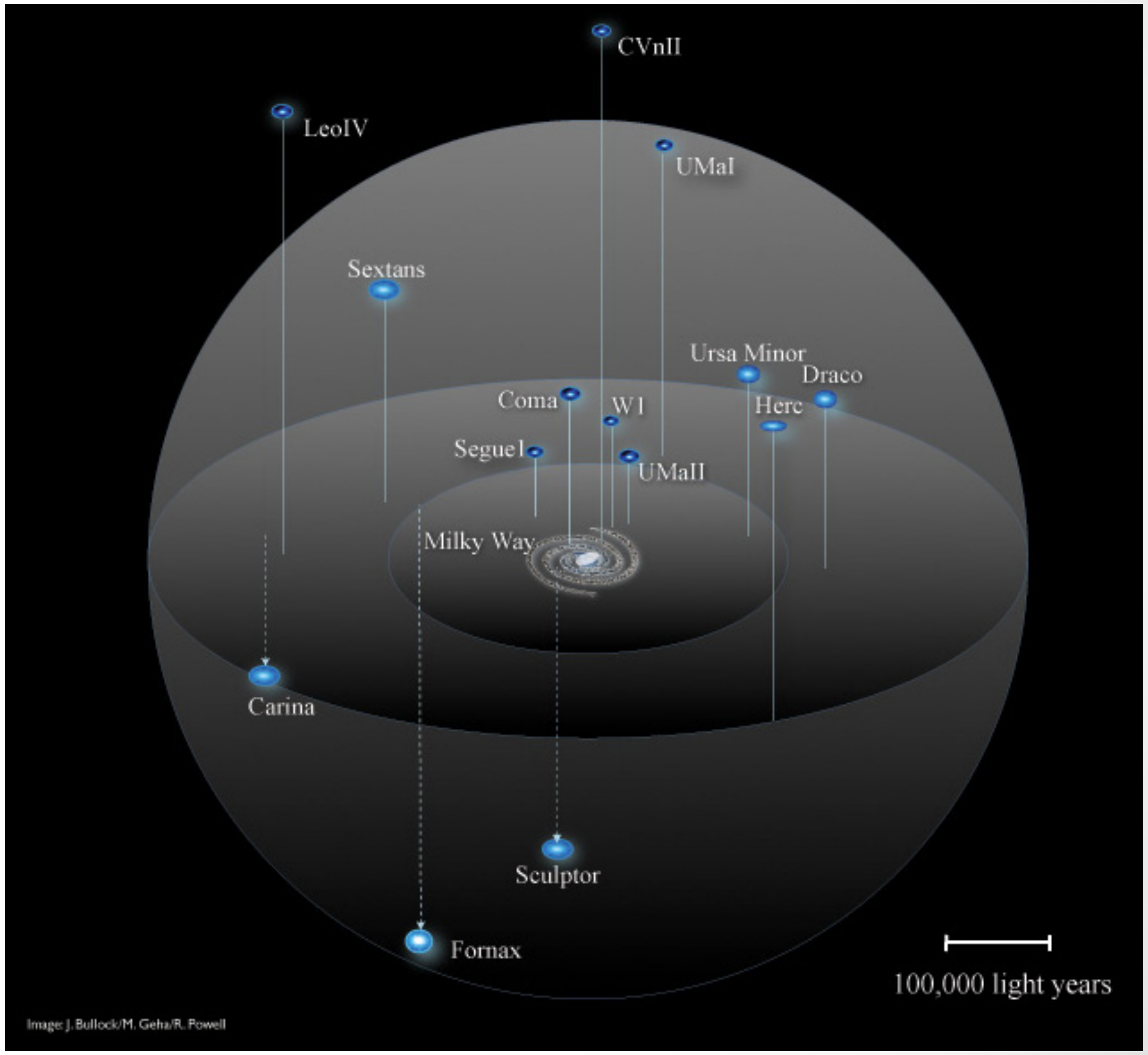
Too Big to Fail problem

D. H. Weinberg et al. (2013) (arXiv:1306.0913)

Missing Satellites Problem



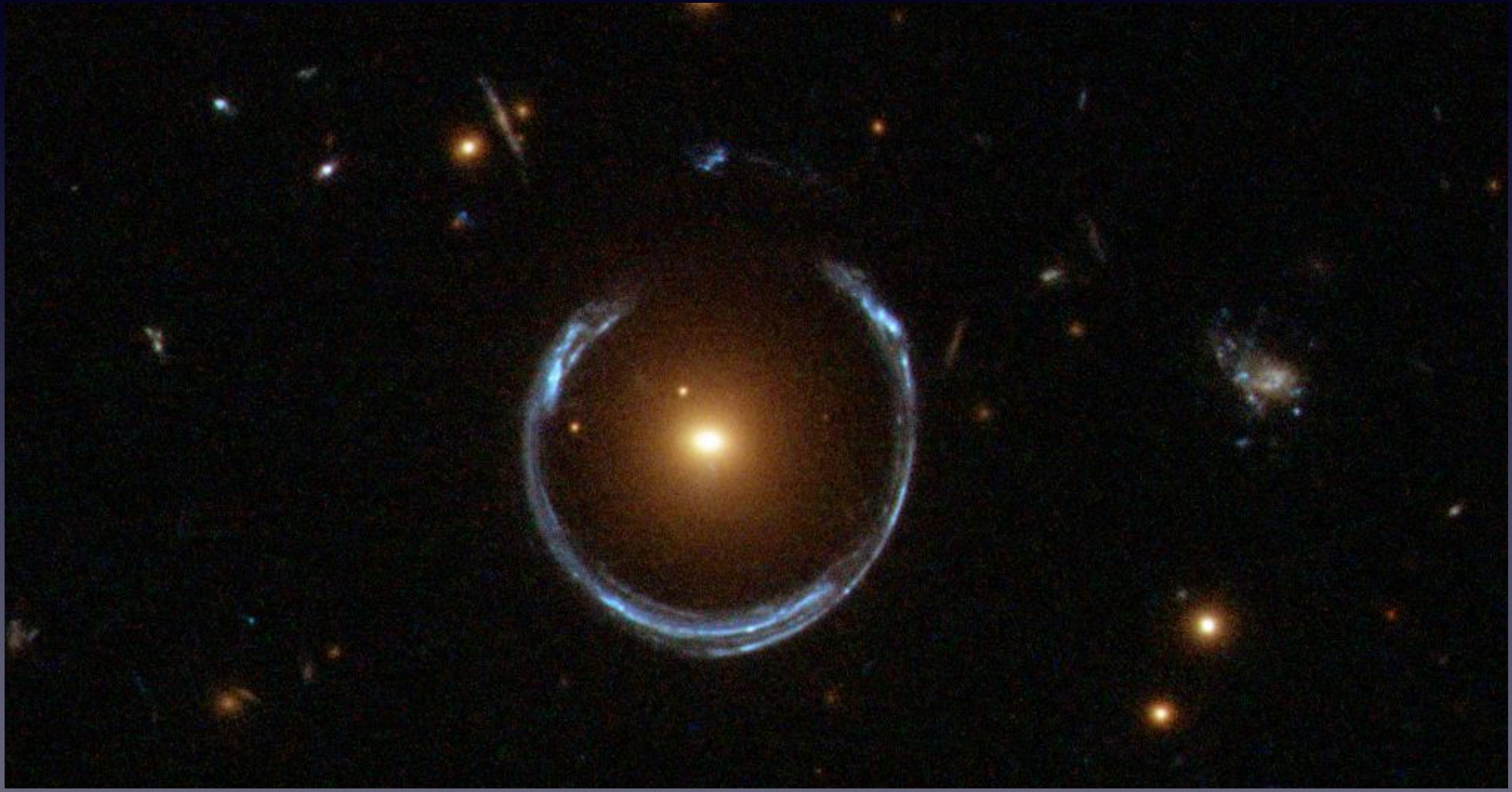
CDM Simulation (Mayer and Kazantzidis)



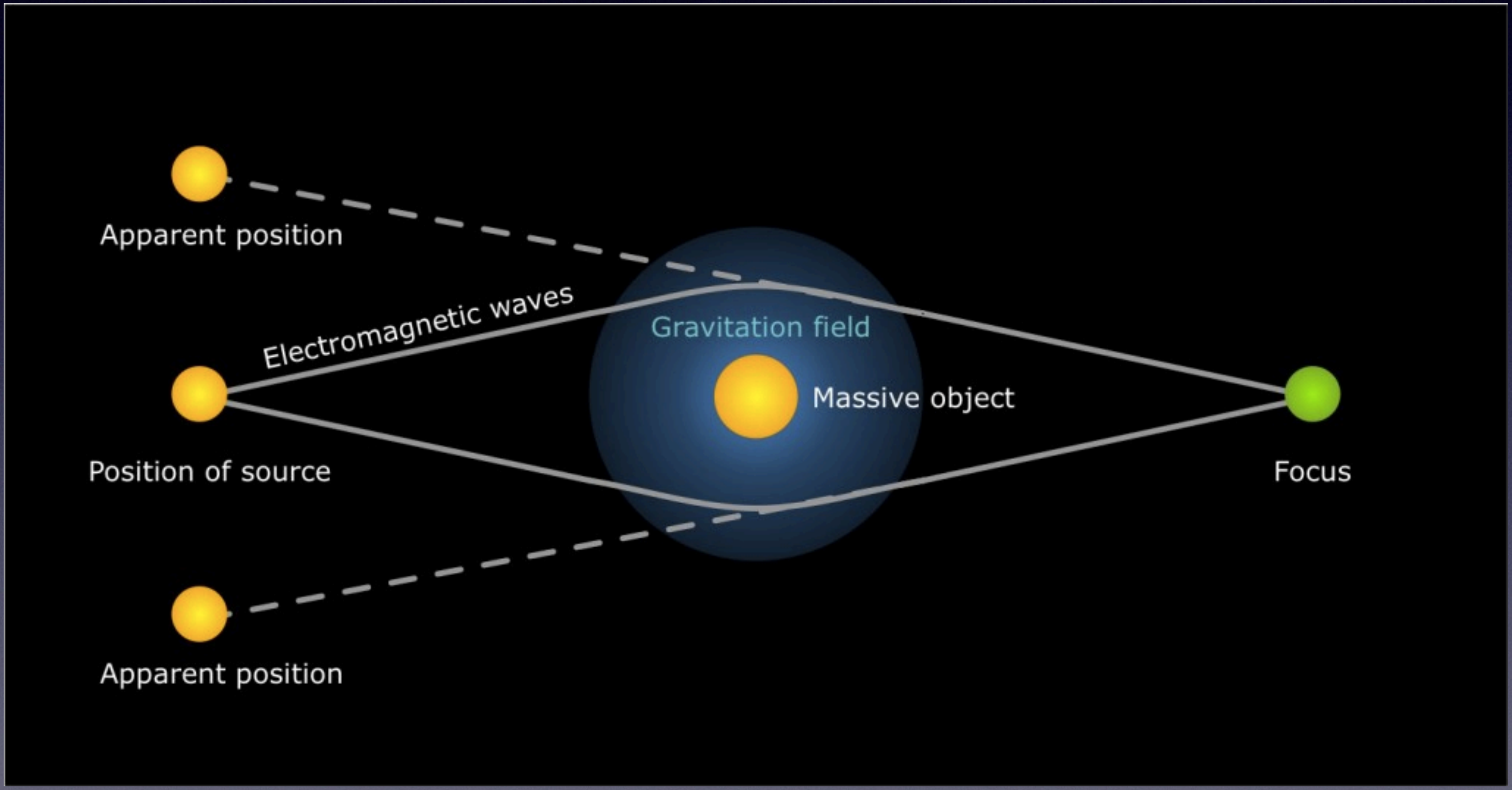
Satellite galaxies of the Milky Way (Observation)

Credit: J. Bullock, M. Geha, R. Powell

Strong lensing: Natural place to probe dark matter substructures

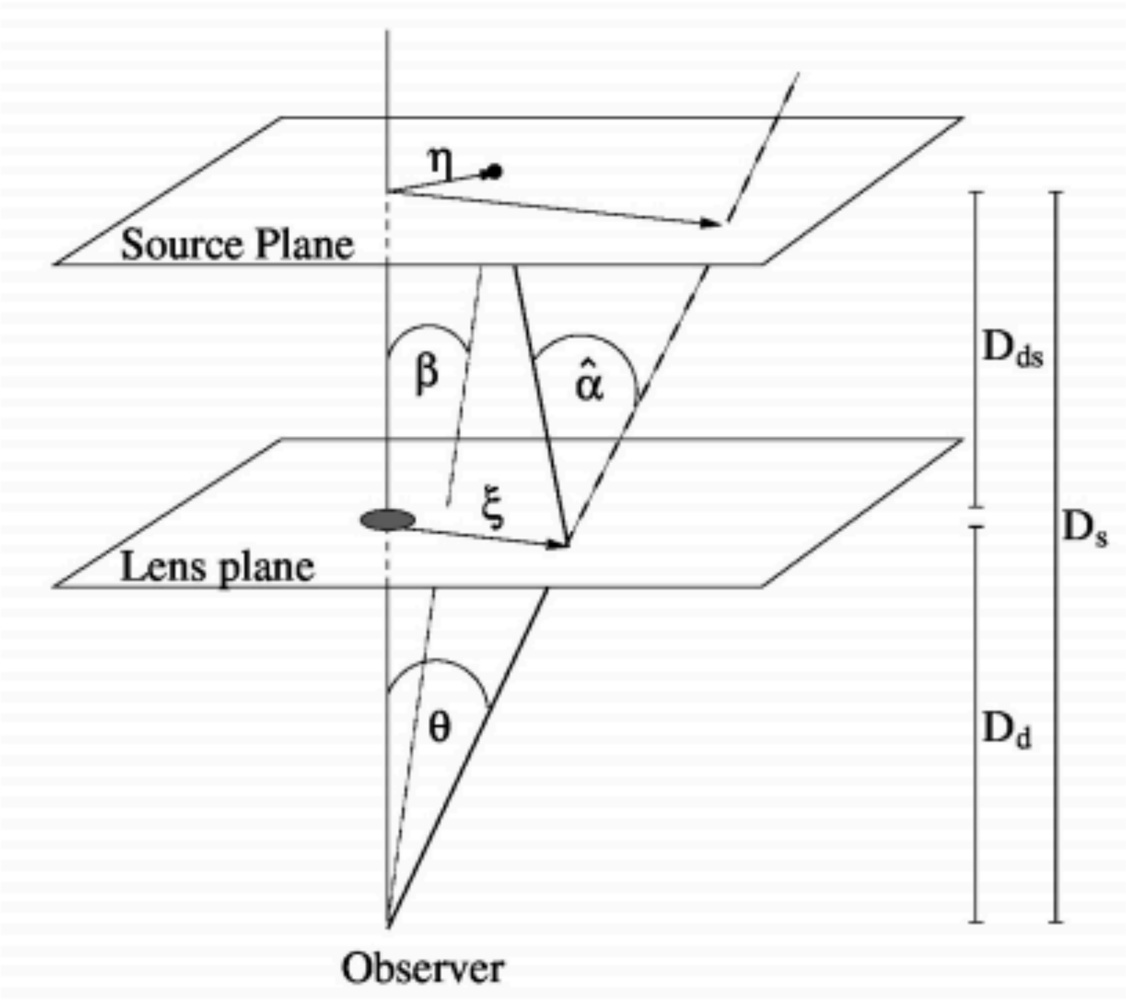


Strong lensing



Lensing basic

Lens equation



[Schneider et al. 2006]

$$\boldsymbol{\eta} = \frac{D_\mathrm{s}}{D_\mathrm{d}}\boldsymbol{\xi} - D_\mathrm{ds}\hat{\boldsymbol{\alpha}}(\boldsymbol{\xi})$$

In terms of angular coord.:

$$\boldsymbol{\eta} = D_\mathrm{s}\boldsymbol{\beta}$$

$$\boldsymbol{\xi} = D_\mathrm{d}\boldsymbol{\theta}$$

$$\boxed{\boldsymbol{\beta} = \boldsymbol{\theta} - \boldsymbol{\alpha}(\boldsymbol{\theta})}$$

where

$$\boldsymbol{\alpha}(\boldsymbol{\theta}) = \frac{D_\mathrm{ds}}{D_\mathrm{s}}\hat{\boldsymbol{\alpha}}(D_\mathrm{d}\boldsymbol{\theta})$$

2

Slides credit: Sherry Suyu

Lensing basic

Source

Lens Image

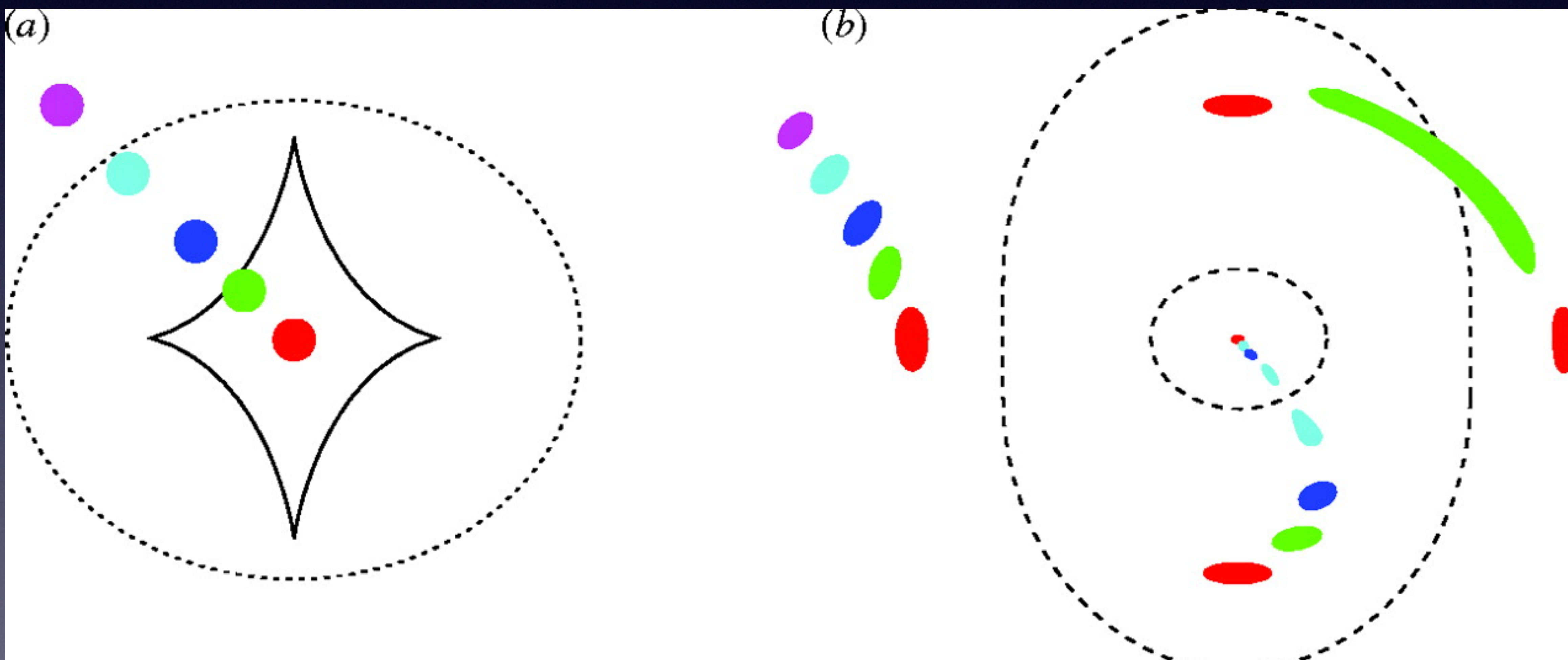


Figure from Narayan & Bartelmann (1995)

Subhalo detection

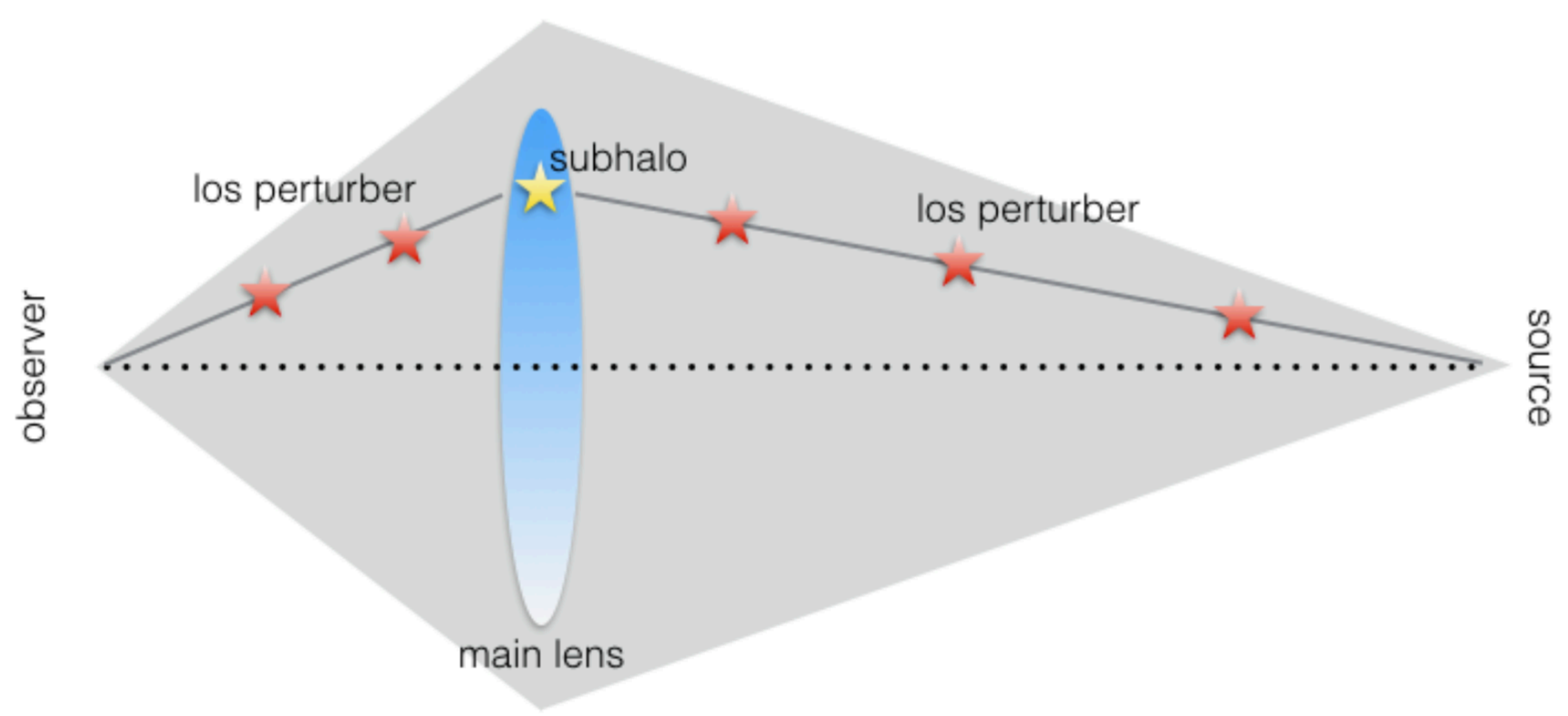
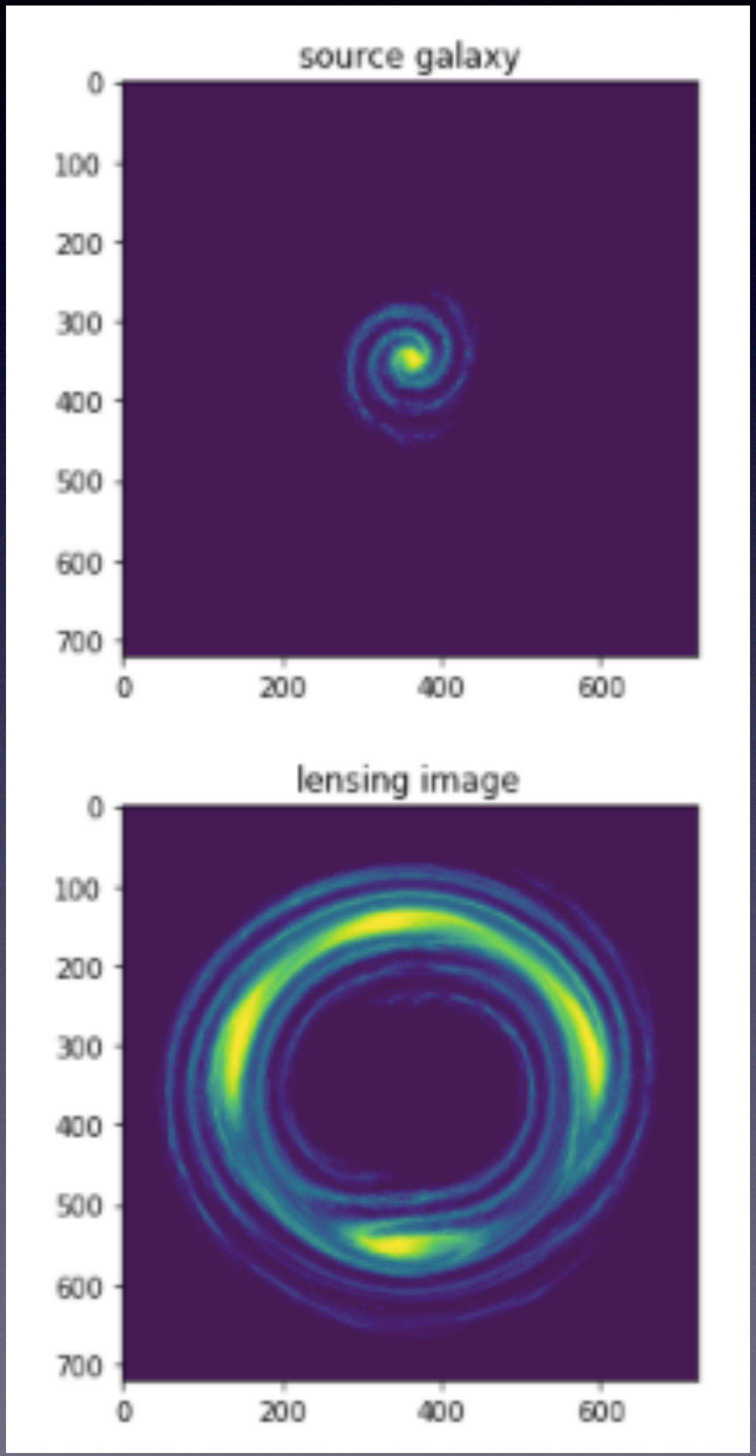


Figure 2. A simple sketch of the method we used to create our mock data; subhaloes and line-of-sight haloes are placed so that their lensing effect lies in the same projected position on the plane of the main lens; the grey region gives an example of the line-of-sight volume that is taken into account.

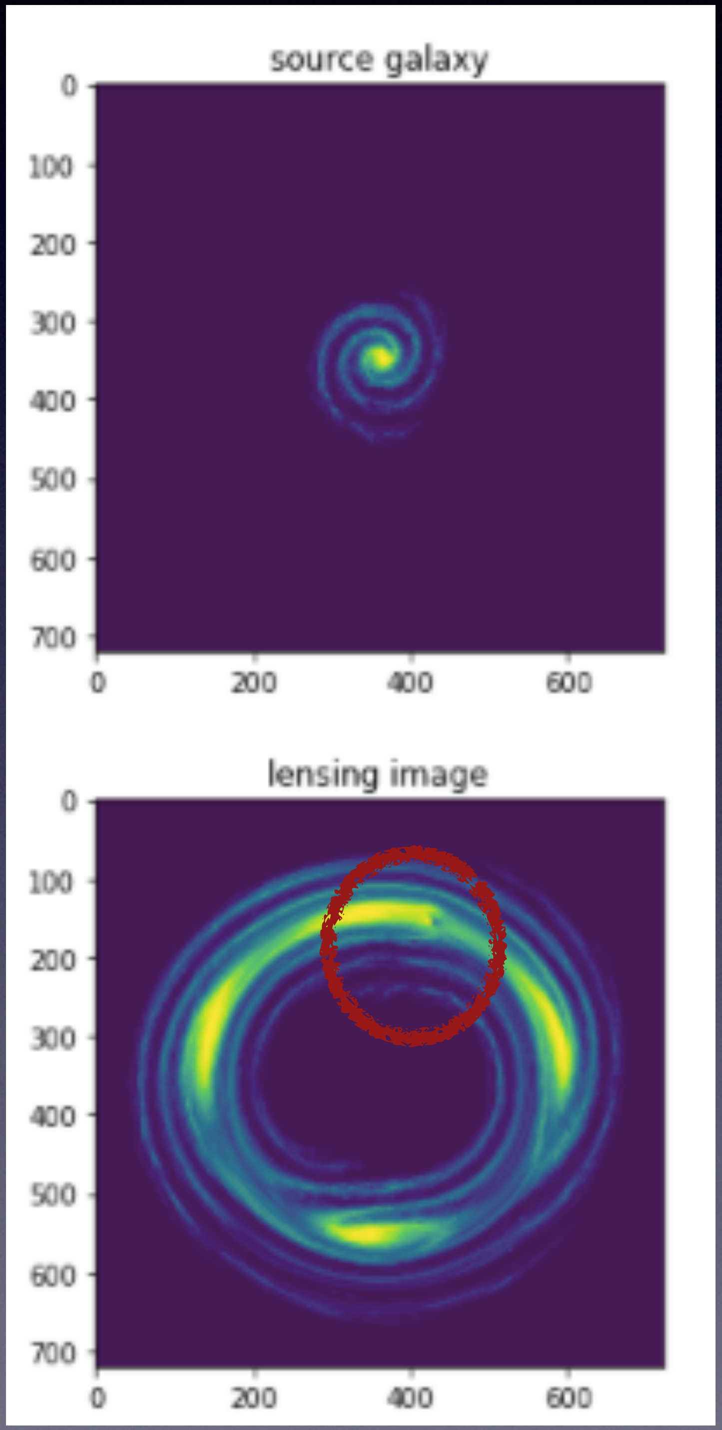
$$\vec{\alpha}_{lens} = \vec{\alpha}_{smooth} + \sum_{i=0}^{N} \vec{\alpha}_{i,subhalo}$$

Despali et al. (1710.05029)

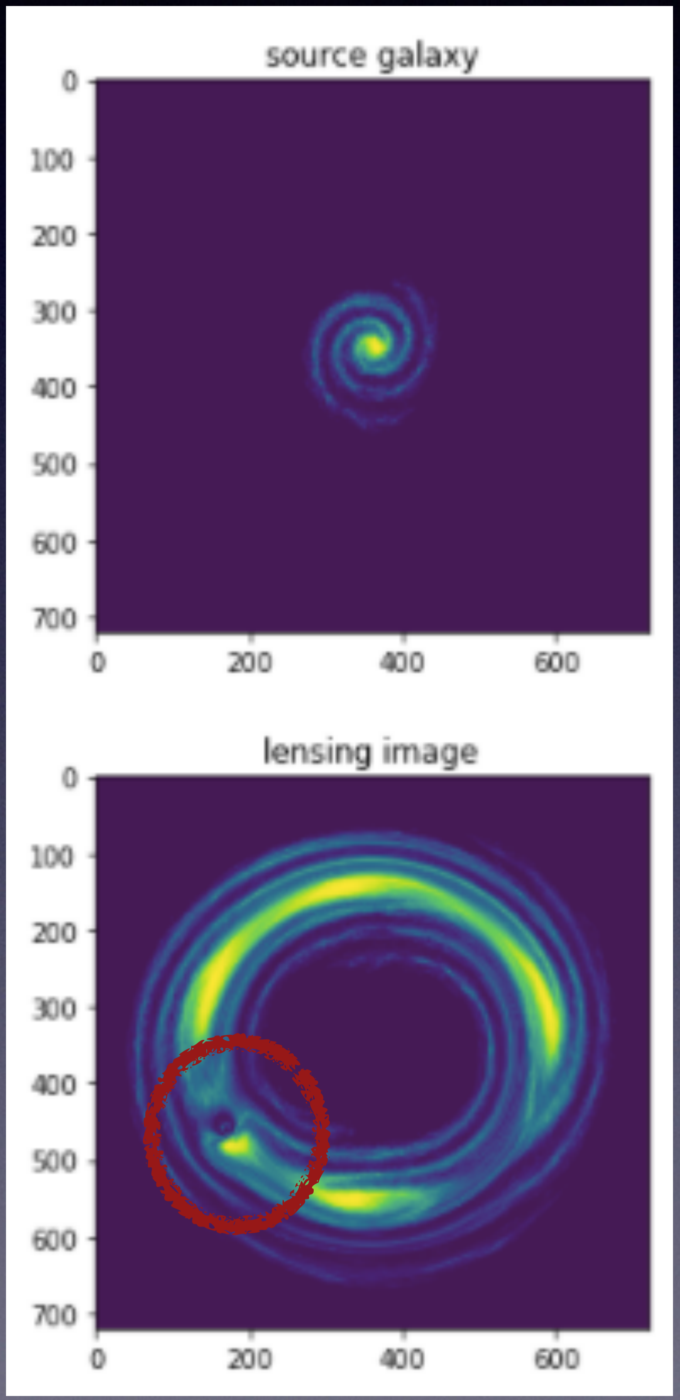
Strong lensing with substructure as perturber



Lensing without perturber

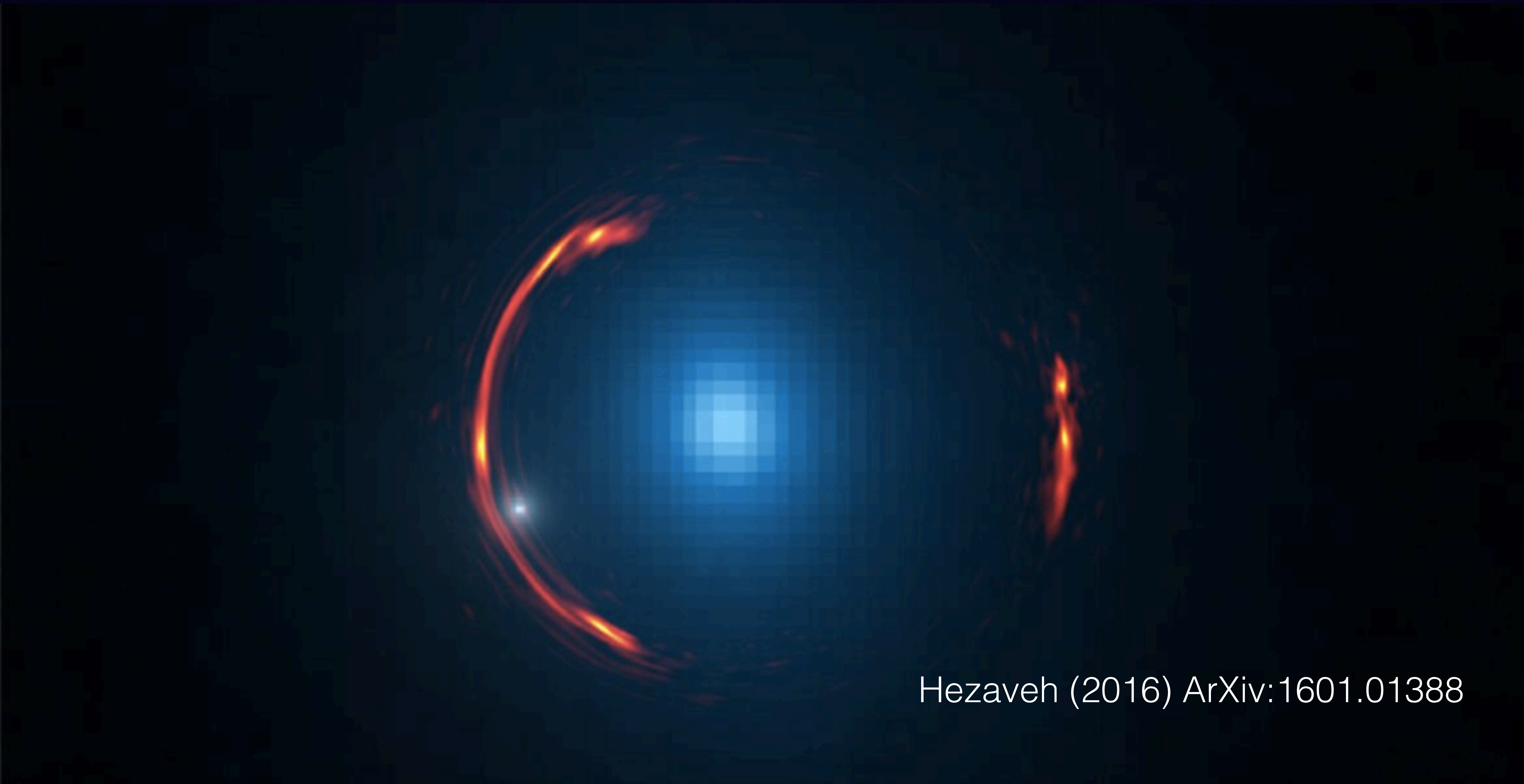


Lensing with perturber



Lensing with (large)
massive perturber

Subhalo Hidden in ALMA Gravitational Lens Image



Hezaveh (2016) ArXiv:1601.01388

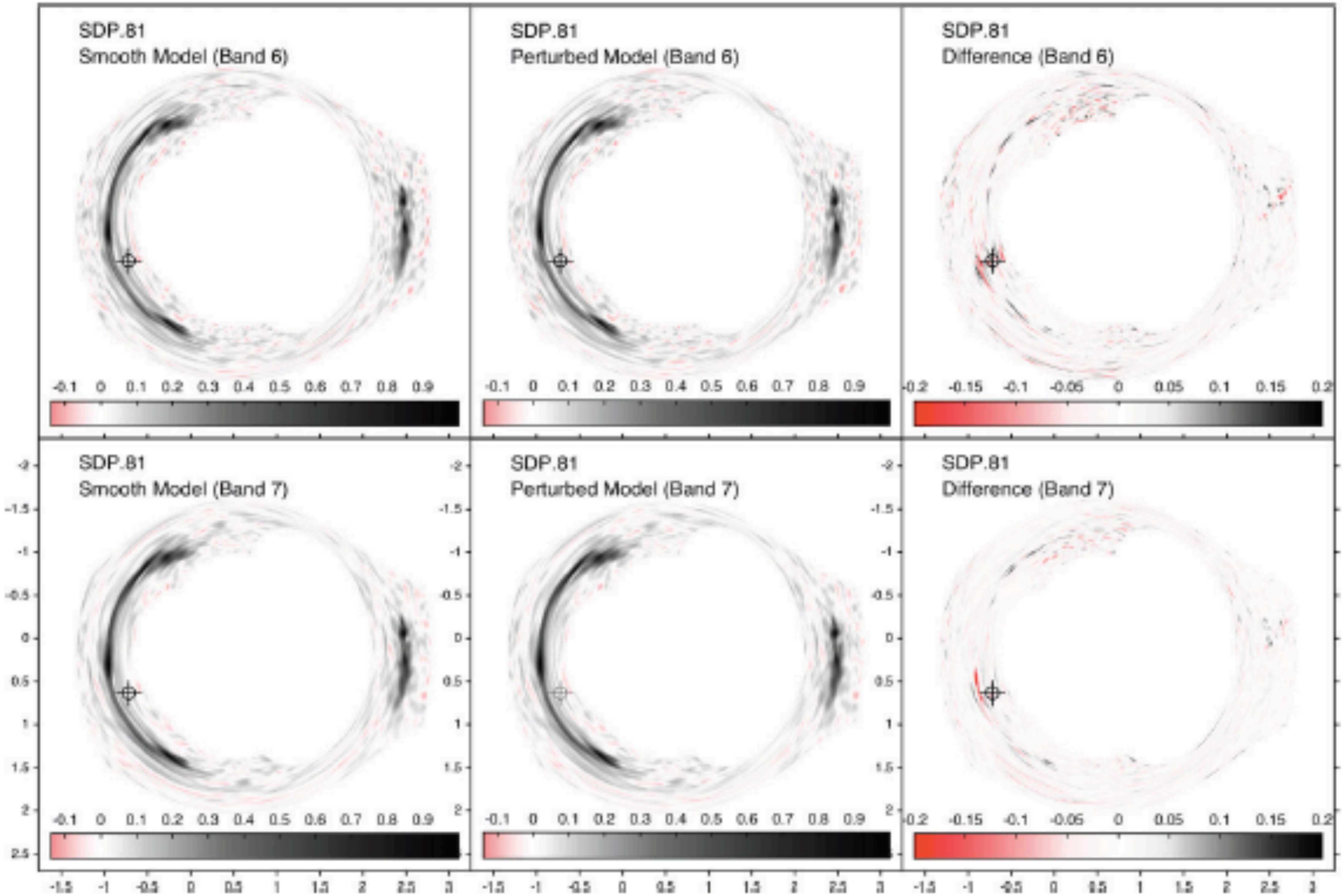


Figure 6. Top left: the sky emission model in band 6 for the best-fit smooth lens parameters for the SDP.81 data. Top middle: the same for the perturbed model. Top right: the difference between the two models. The bottom panels show the same for band 7. The bright feature in the difference plots is mainly caused by the astrometric anomaly of the arc. In each row, the images have been scaled to the peak flux of the smooth model.

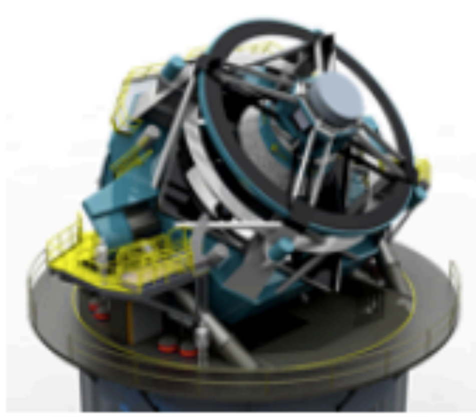
Hezaveh et al., (2016)

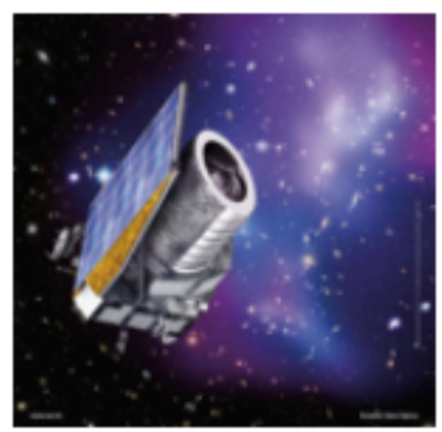
LOOKING INTO THE FUTURE:

New **Lenses**

For future surveys we find that, assuming Poisson limited lens galaxy subtraction, searches of the DES, LSST, and Euclid data sets should discover **2400**, **120000**, and **170000** galaxy–galaxy strong lenses, respectively

Collett, ApJ. 2015





Slides from Laurence Perreault Levasseur

Can AI (deep learning) help?

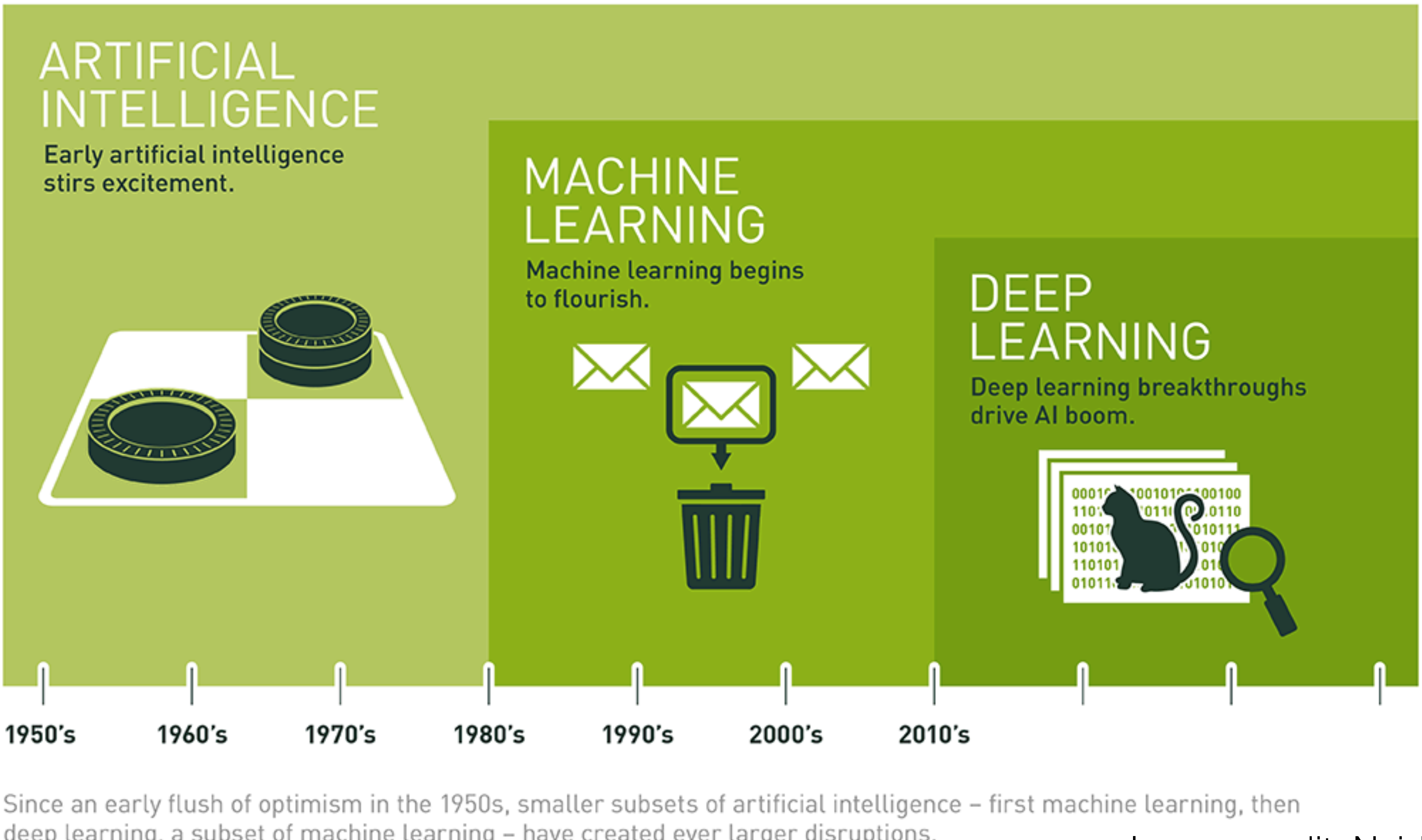
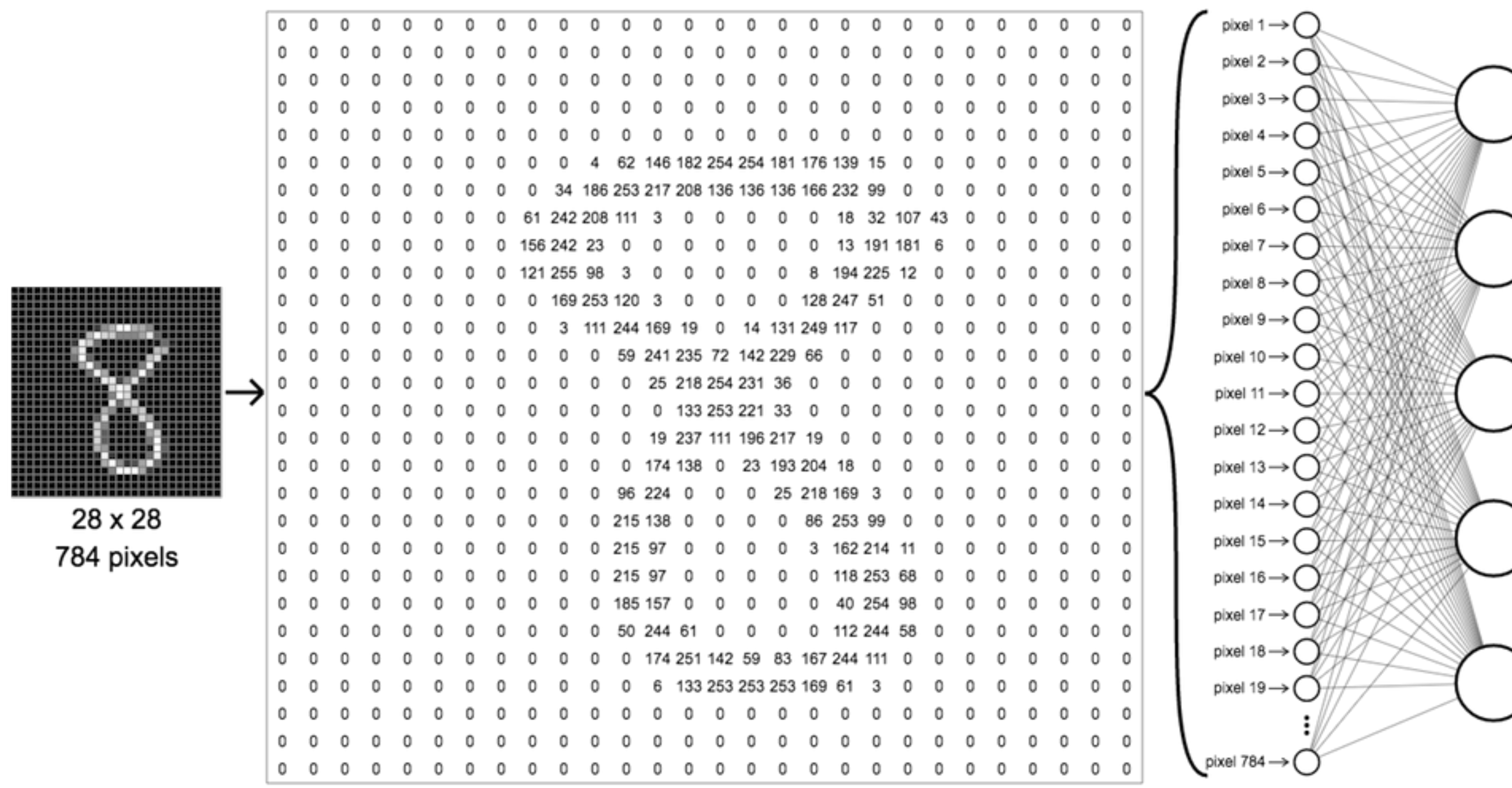


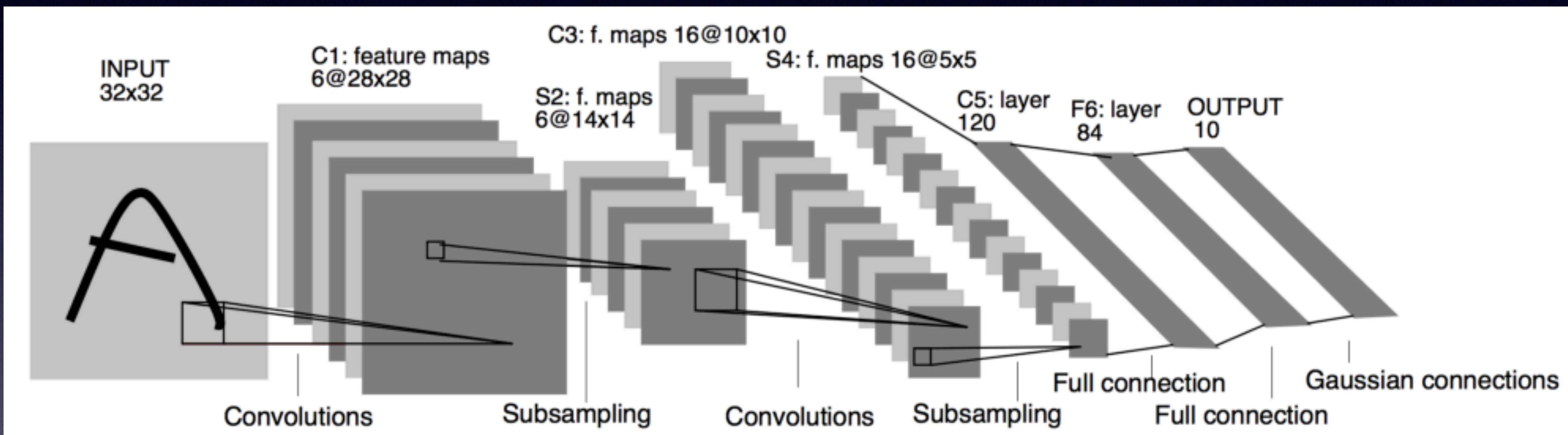
Image credit: Nvidia

Brief Intro to deep learning: MNIST dataset



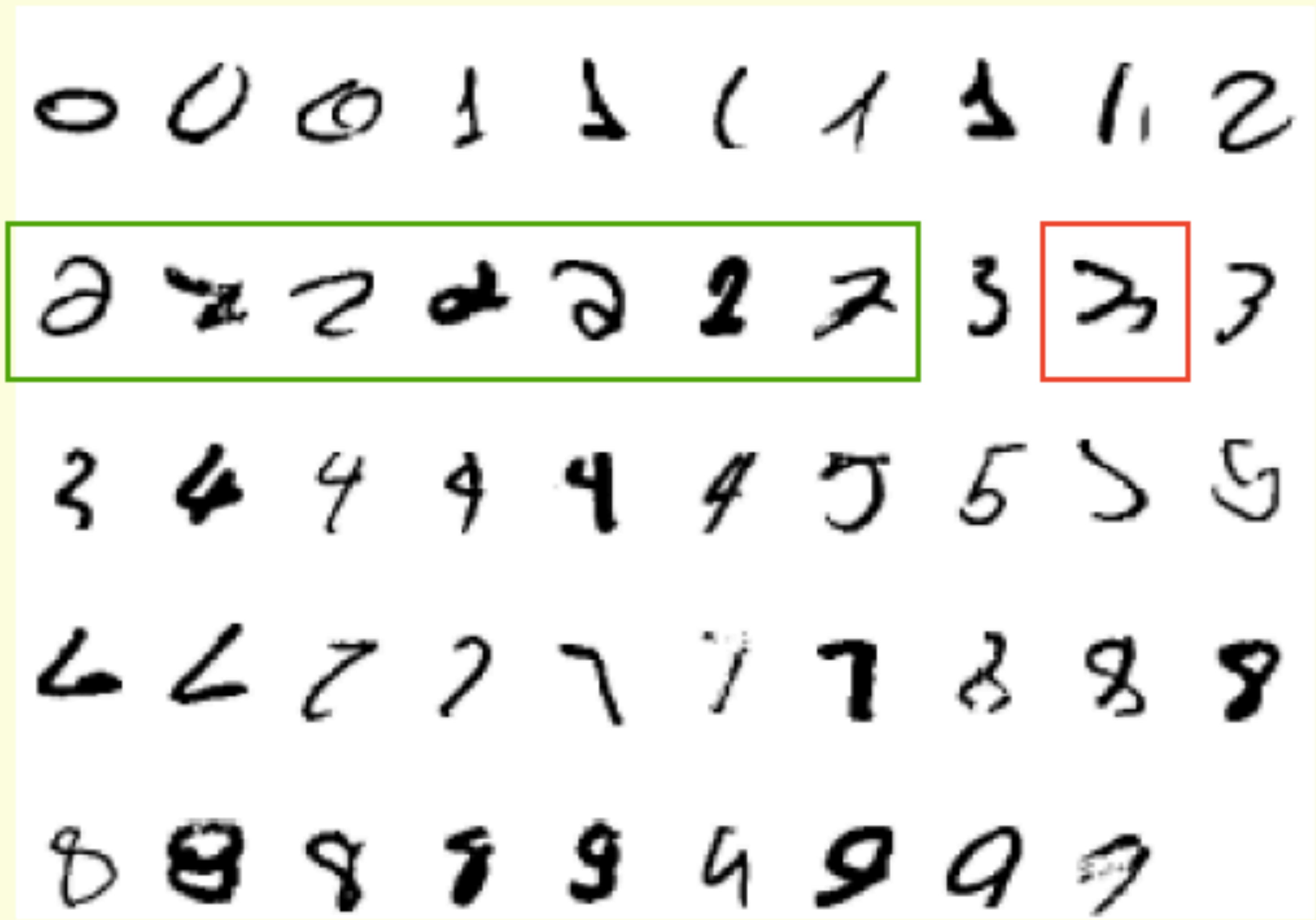
Credit: http://yann.lecun.com/exdb/mnist/

Brief Intro to deep learning: Deep Neural Networks



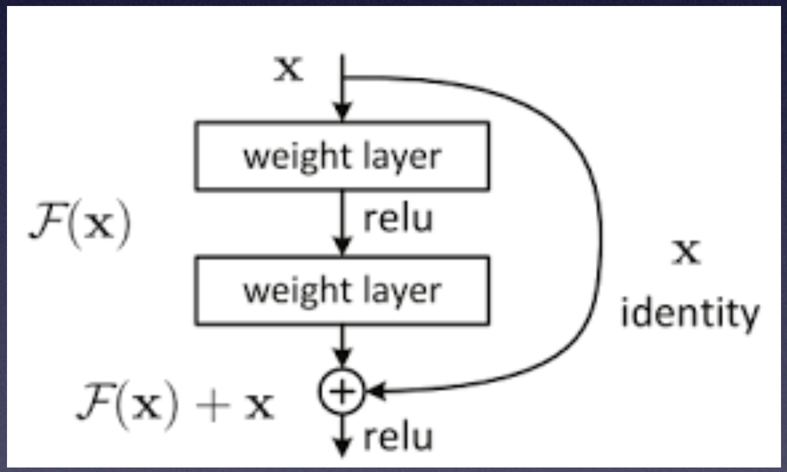
Credit:LeNet by Yann LeCun (1998)

Examples of handwritten digits that can be recognized correctly the first time they are seen



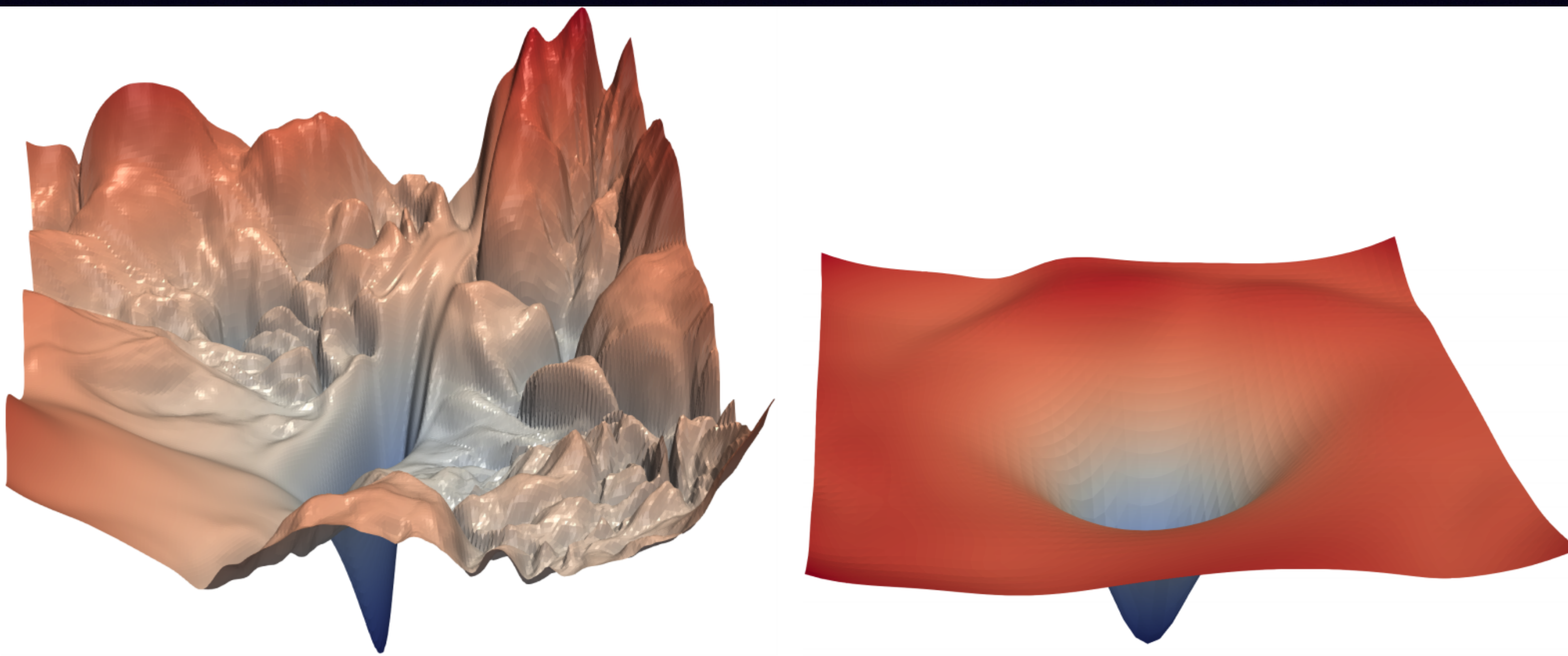
Slides from Geoff Hinton (Coursera, 2012)

DenseNet and ResNet



"Deep Residual Learning for Image Recognition" Kaiming He et al (arXiv:1512.03385)

Loss Surface w Skip Connection

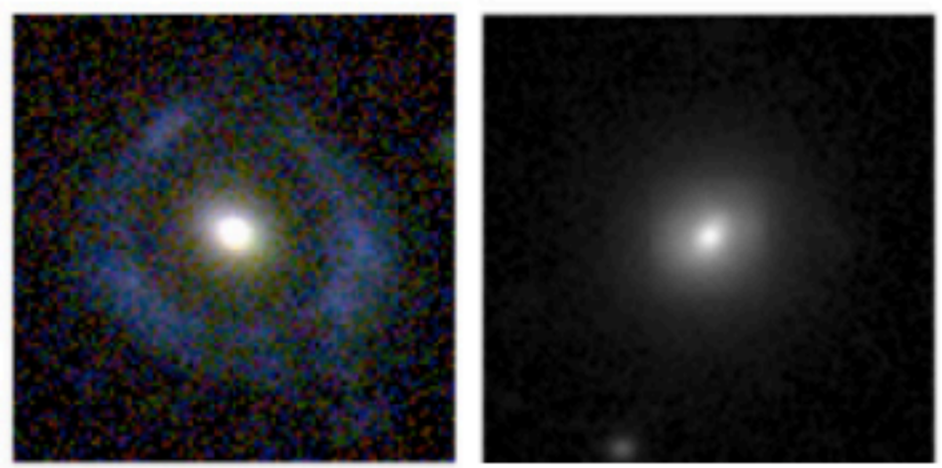


(a) without skip connections (b) with skip connections

Li et al. NIPS 2017 (ArXiv: 1712.09913)

Convolutional Neural Networks: Previously used to find lenses (Classification)

They can be trained to classify images:
Two classes: lenses vs. non-lenses



CMU DeepLens: Deep Learning For Automatic Image-based Galaxy-Galaxy Strong Lens Finding

François Lanusse,[1*] Quanbin Ma,[2] Nan Li,[3,4] Thomas E. Collett,[5] Chun-Liang Li,[2] Siamak Ravanbakhsh,[2] Rachel Mandelbaum[1] and Barnabás Póczos[2]

[1] McWilliams Center for Cosmology, Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[2] School of Computer Science, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[3] High Energy Physics Division, Argonne National Laboratory, Lemont, IL 60439, USA
[4] Department of Astronomy & Astrophysics, The University of Chicago, 5640 South Ellis Avenue, Chicago, IL 60637, USA
[5] Institute of Cosmology and Gravitation, University of Portsmouth, Burnaby Rd, Portsmouth, PO1 3FX, UK

Finding Strong Gravitational Lenses in the Kilo Degree Survey with Convolutional Neural Networks

C. E. Petrillo[1*], C. Tortora[1], S. Chatterjee[1], G. Vernardos[1], L. V. E. Koopmans[1], G. Verdoes Kleijn[1], N. R. Napolitano[2], G. Covone[3], P. Schneider[4], A. Grado[2], J. McFarland[1]

[1] Kapteyn Astronomical Institute, University of Groningen, Postbus 800, 9700 AV, Groningen, The Netherlands
[2] INAF - Osservatorio Astronomico di Capodimonte, Salita Moiariello, 16, 80131 Napoli, Italy
[3] Dipartimento di Scienze Fisiche, Università di Napoli Federico II, Compl. Univ. Monte S. Angelo, 80126 Napoli, Italy
[4] Argelander-Institut für Astronomie, Auf dem Hügel 71, D-53121 Bonn, Germany

LETTER

doi:10.1038/nature23463

Fast automated analysis of strong gravitational lenses with convolutional neural networks

Yashar D. Hezaveh[1,2]*, Laurence Perreault Levasseur[1,2]* & Philip J. Marshall[1,2]

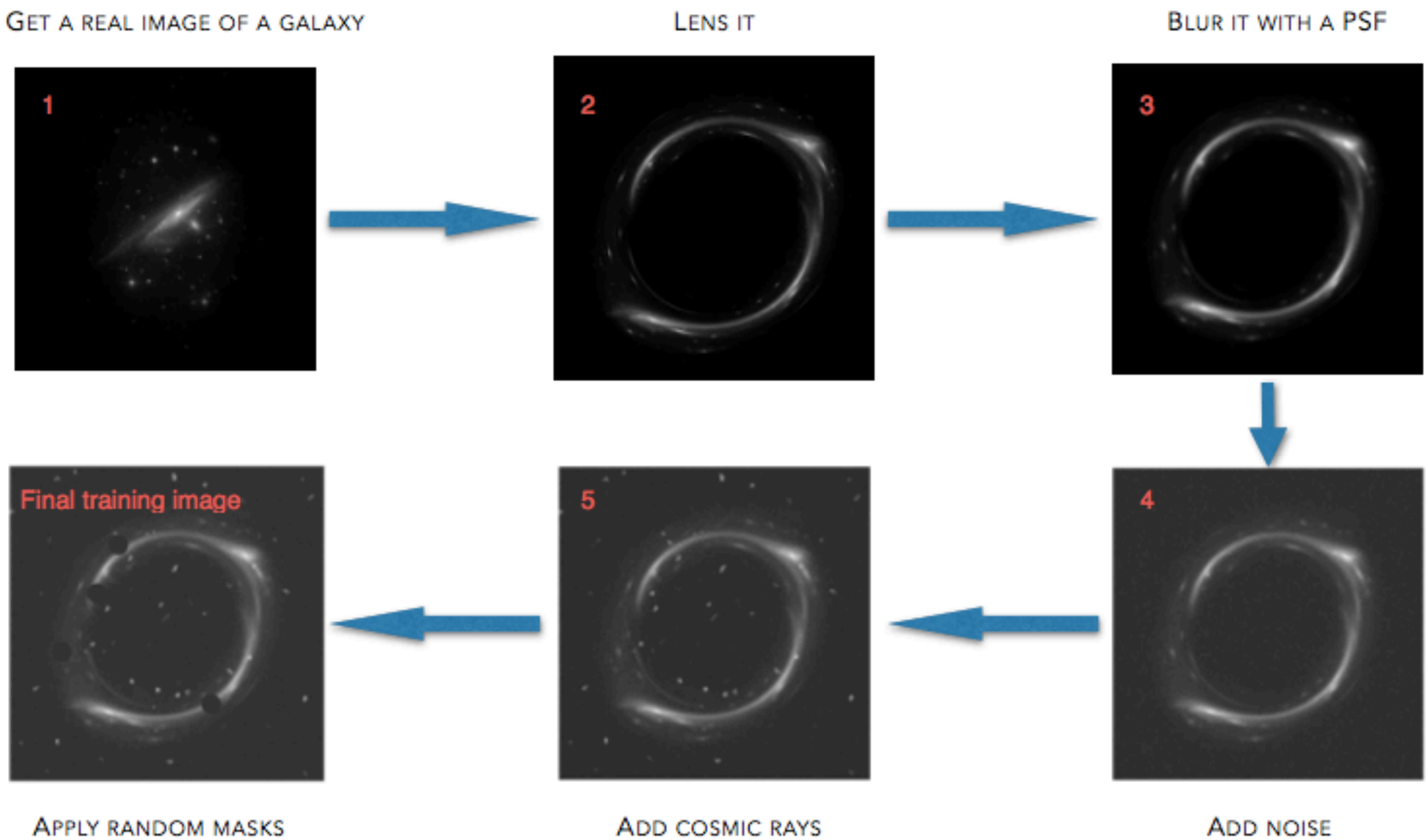
Quantifying image distortions caused by strong gravitational lensing—the formation of multiple images of distant sources due to the deflection of their light by the gravity of intervening structures—and estimating the corresponding matter distribution of these structures (the 'gravitational lens') has primarily been performed using maximum likelihood modelling of observations. This procedure is typically time- and resource-consuming, requiring sophisticated lensing codes, several data preparation steps, and finding the maximum likelihood model parameters in a computationally expensive process with downhill optimizers[1]. Accurate analysis of a single gravitational lens can take up to a few weeks and requires expert knowledge of the physical processes and methods involved. Tens of thousands of new lenses are expected to be discovered with the upcoming generation of ground and space surveys[2,3]. Here we report the use of deep convolutional neural networks to estimate lensing parameters in an extremely fast and automated way, circumventing the difficulties that are faced by maximum likelihood methods. We also show that the removal of lens light can be made fast and automated using independent component analysis[4] of multi-filter imaging data. Our networks can recover the parameters of the 'singular isothermal ellipsoid' density profile[5], which is commonly used to model strong lensing systems, with an accuracy comparable to the uncertainties of sophisticated models but about ten million times faster: 100 systems in approximately one second on a single graphics processing unit. These networks can provide a way for non-experts to obtain estimates of lensing parameters for large samples of data.

deep learning, convolutional neural networks (Methods) have been shown to excel at many image recognition and classification tasks[6]. This makes them a particularly promising tool for the analysis of gravitational lenses. Recently, these networks have been used to search for gravitational lenses in large volumes of telescope data[7–9] and to simulate weakly lensed galaxy images[10]. Here we show that these networks can also be used for data analysis and parameter estimation.

We train four networks, Inception-v4[11], AlexNet[12], OverFeat[13] and a network of our own design, to analyse strongly lensed systems, by removing their final classification layer and interpreting the outputs of the last fully connected layer as a prediction for lensing parameters, with all weights initialized at random. We train the networks to predict the five parameters of the singular isothermal ellipsoid profile: the Einstein radius, the complex ellipticity and the coordinates of the centre of the lens. We use a squared-difference cost function, averaged over the five parameters. Although in many situations in machine learning collecting sufficiently large training sets is one of the main challenges, here it is possible to simulate the training data extremely fast. We train the networks on half a million simulated strong lensing systems. The lensed background sources are composed of three equal sets of images: the first and second comprise real galaxy images from the Galaxy Zoo[14] machine learning challenge and high-quality images from the GREAT3 training data[15], and the third set is composed of simulated clumpy galaxies with Sérsic and Gaussian clump profiles. The position of the background galaxy in the source plane is chosen randomly for each sample, but limited to regions where strong lensing occurs, that is, inside or on the caustics.

Hezaveh et al. (Nature, 2017)

Producing the Training Data



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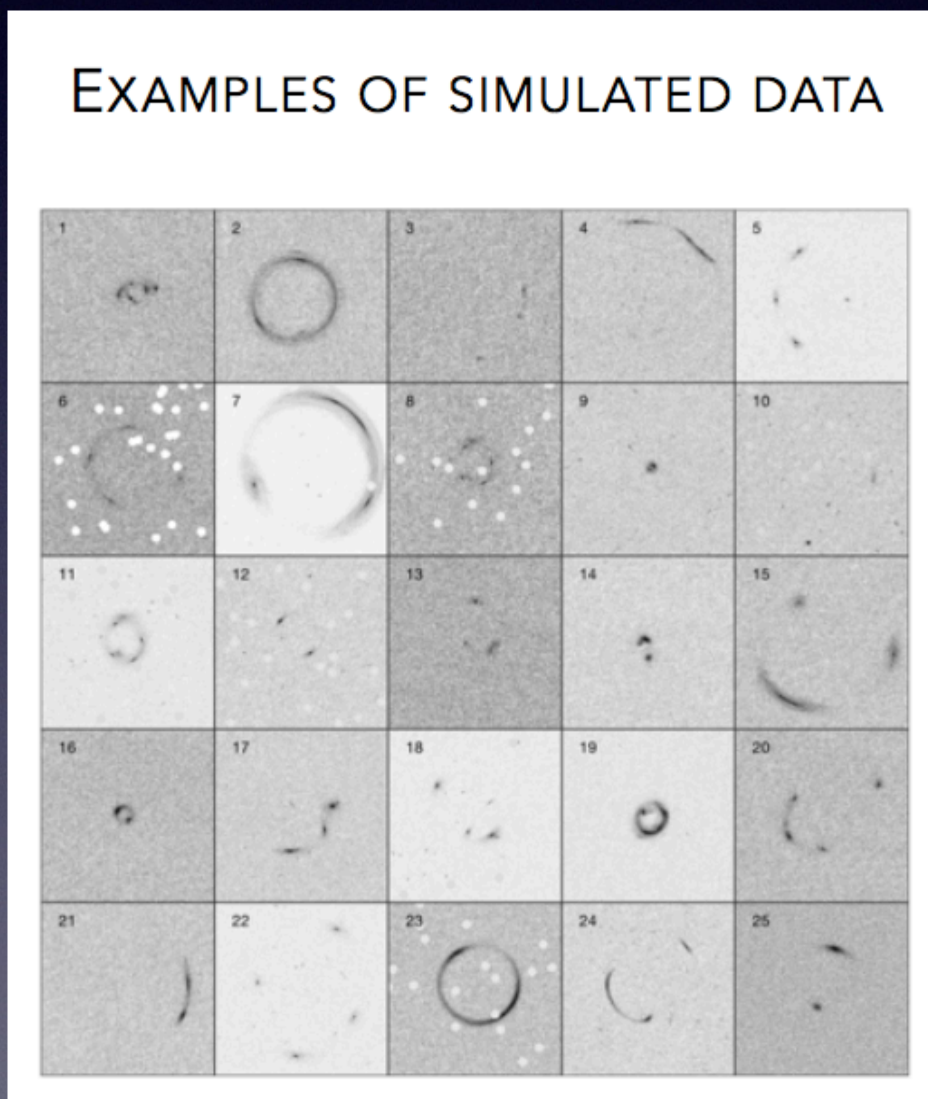


500,000 simulated data

- “label” SIE model with 5 parameters: Einstein radius[θ_E (arcsec)], x, y, e_x, e_y

Table 1 | Errors of the individual and combined networks

Network	θ_E (arcsec)	ε_x	ε_y	x (arcsec)	y (arcsec)
Inception-v4[11]	0.03	0.04	0.05	0.06	0.06
AlexNet[12]	0.03	0.04	0.04	0.05	0.06
OverFeat[13]	0.04	0.05	0.05	0.06	0.06
Our network	0.03	0.05	0.06	0.05	0.05
Combined network	0.02	0.04	0.04	0.04	0.04

The columns present the 68% errors for the Einstein radius (θ_E), the x and y components of complex ellipticity (ε_x and ε_y), and the coordinates of the lensing galaxy (x and y) for each individual network and the combined network. The angular parameters (θ_E, x and y) are given in units of arcseconds.

4 CNNs gives pretty good results!

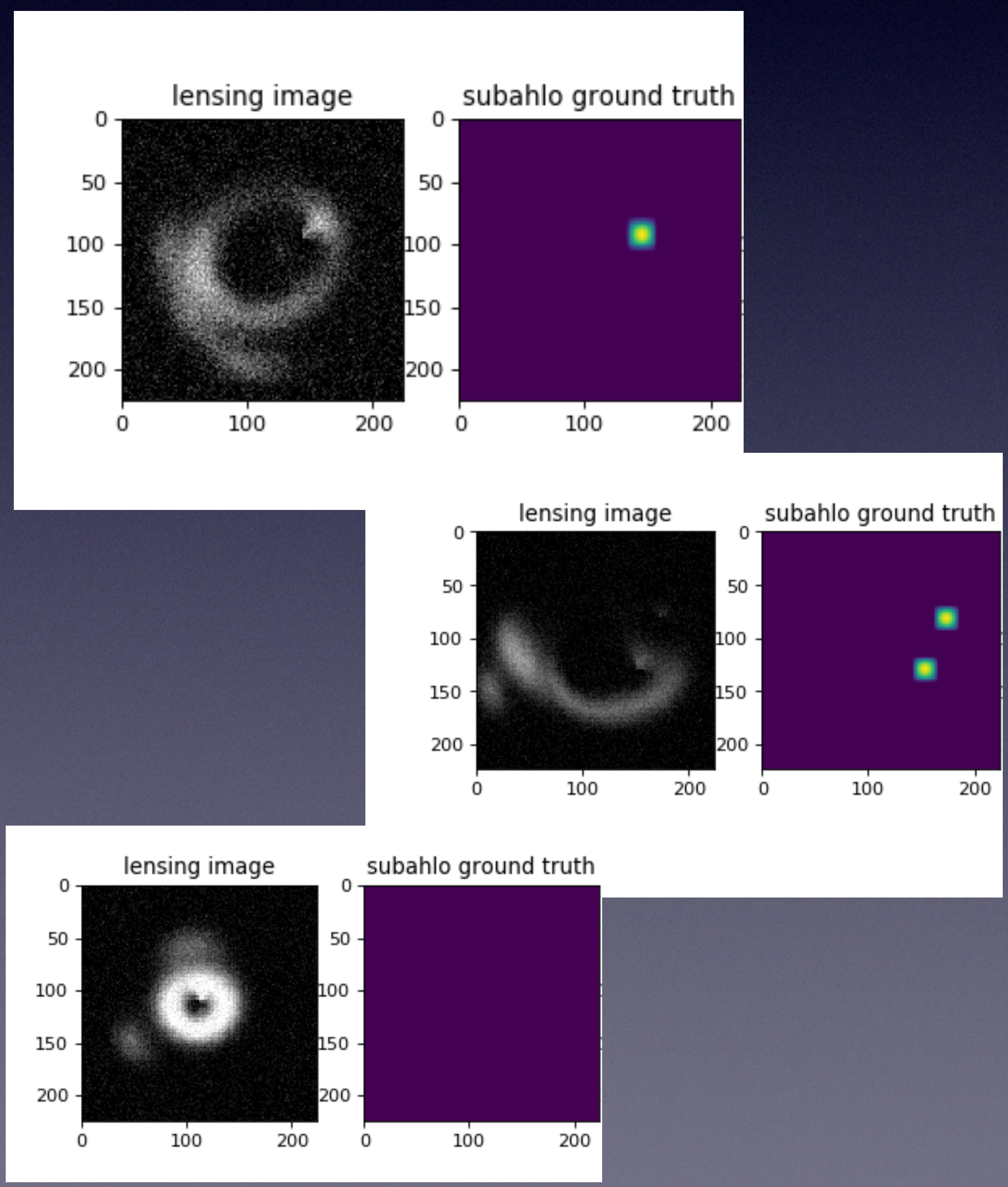
Hezaveh et al. (Nature, 2017)

Could we detect dark matter subhalos in Strong lensing with Neural networks?

- Simulated data could help us understand the problem
- - strong gravitational lensing are insufficient ($\leq$ O(10)) for typical size of the training set for deep learning
- - Simulated data would be good for supervised learning since we know the ground truth

Deep learning setup

20000 simulated data as training set

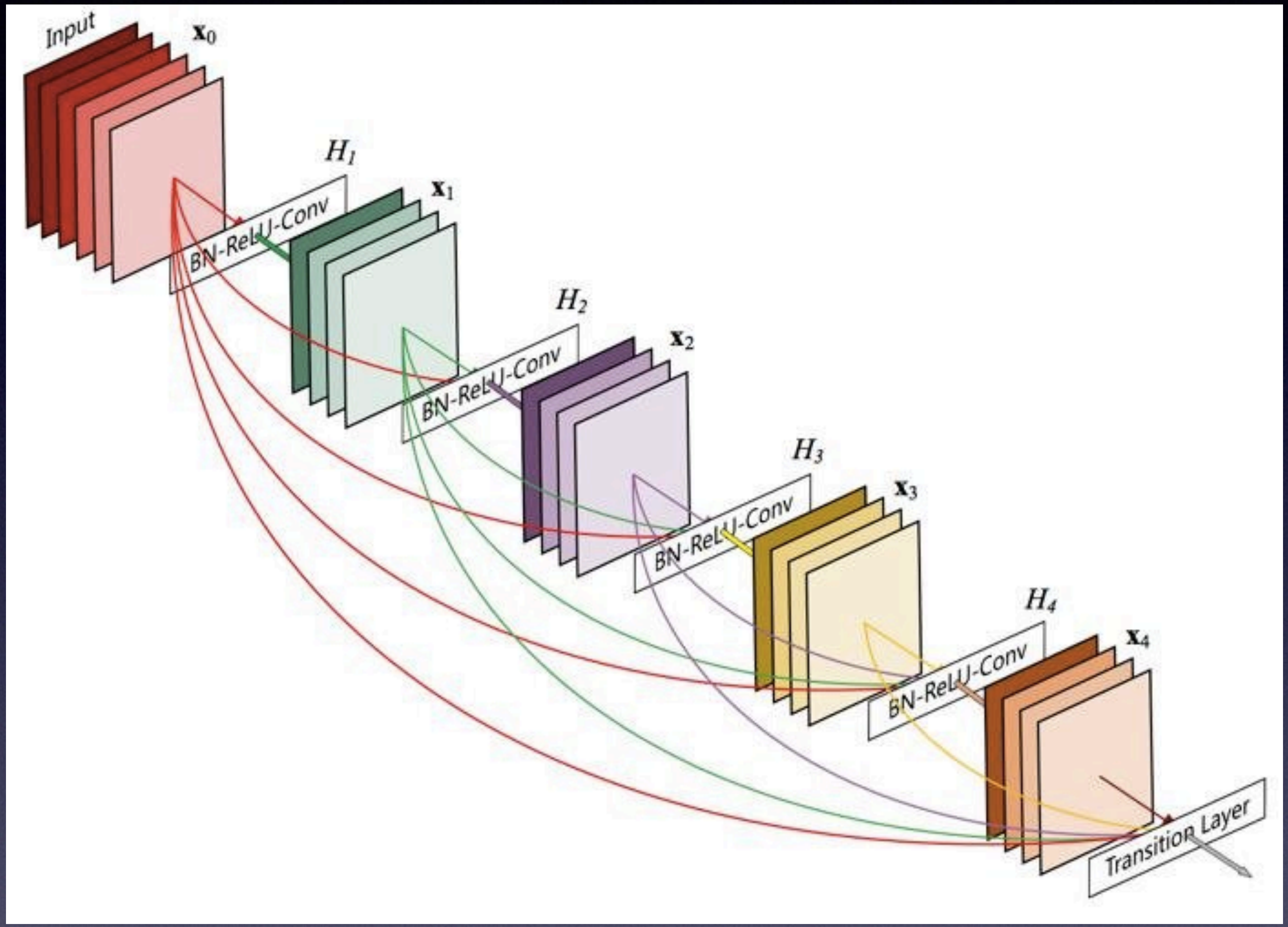


- 20000 simulated data [images and subhalo ground truth]as training set
- 2000 “DIFFERENT” data [images] as test set
- Simulation with SIE (marco lens) contains 0-5 subhalos(perturbers)
- Loss function: Binary Cross Entropy Loss (of subhalo probability map)

$$\mathrm{BCE} = t_i \log(p_i) + (1 - t_i)\log(1 - p_i)$$

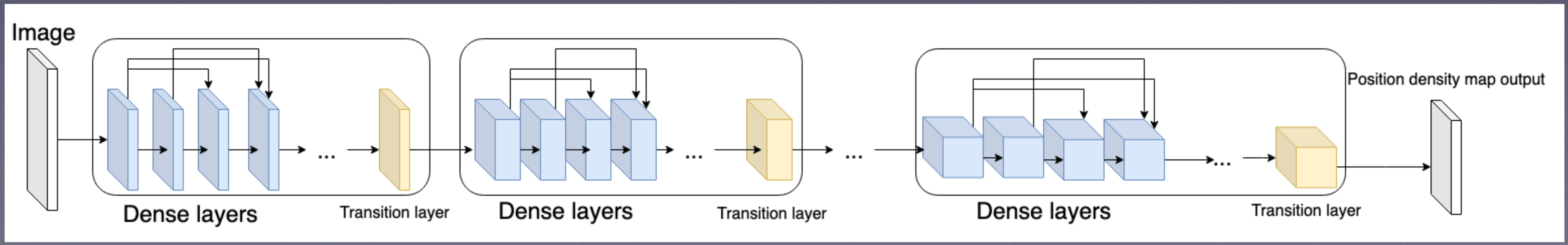
- Adam Optimizer, learning rate = 1e-4
- NN model: DenseNet (~53 layers)
- Nvidia GPU: 1080Ti

DenseNet



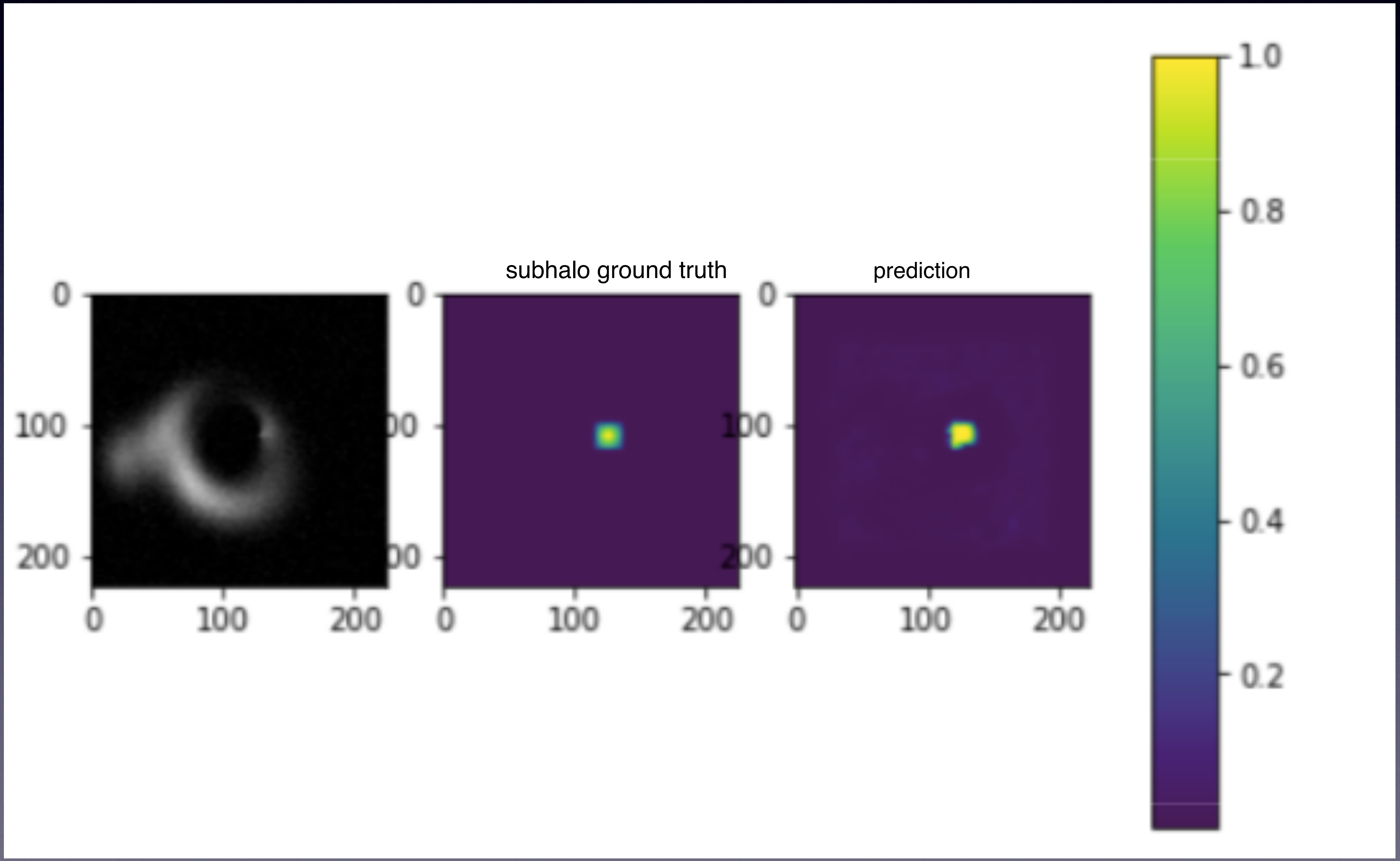
DenseNet architecture (121 layers)

Gao Huang et al., ArXiv:1608.06993

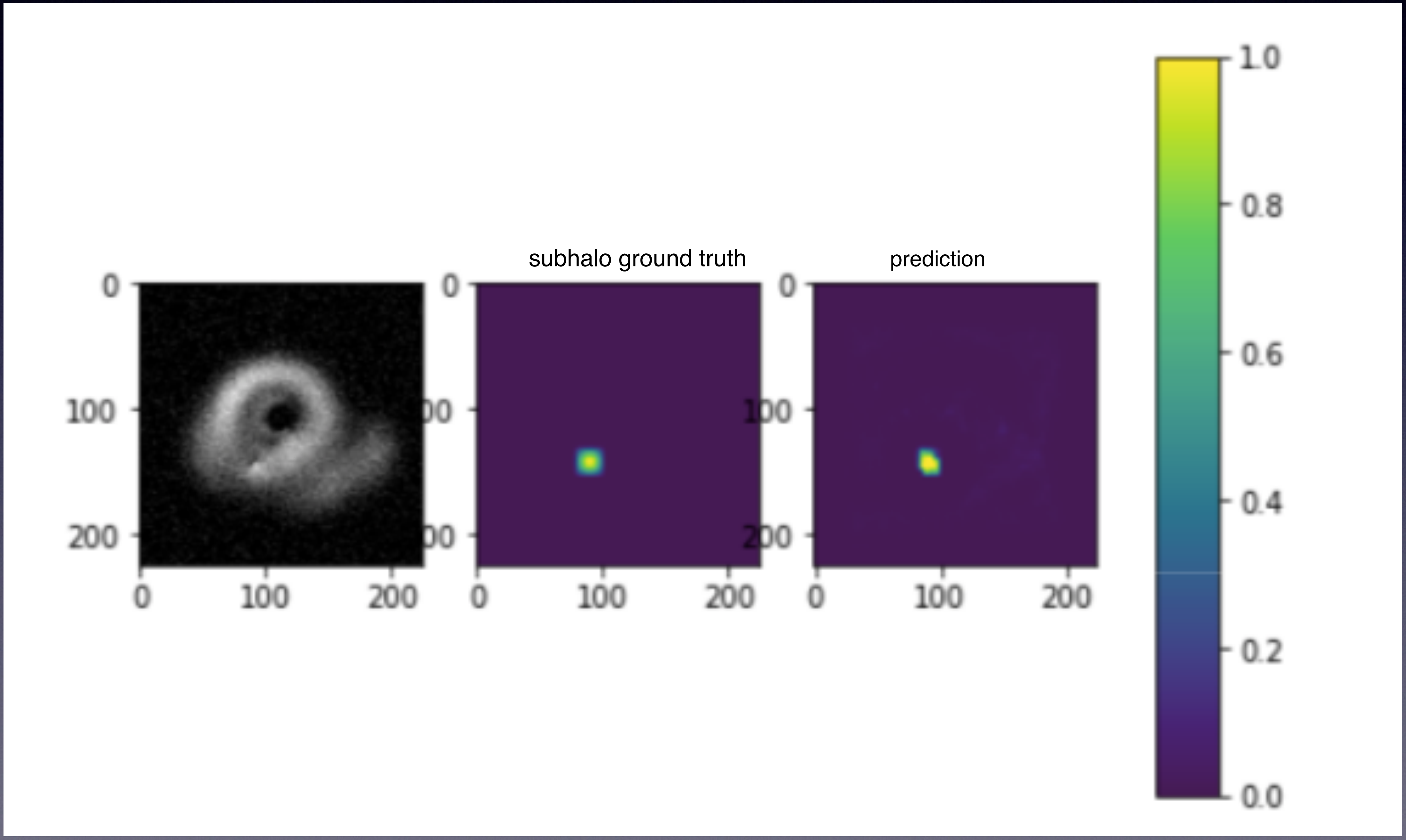


Let’s check how NN is doing

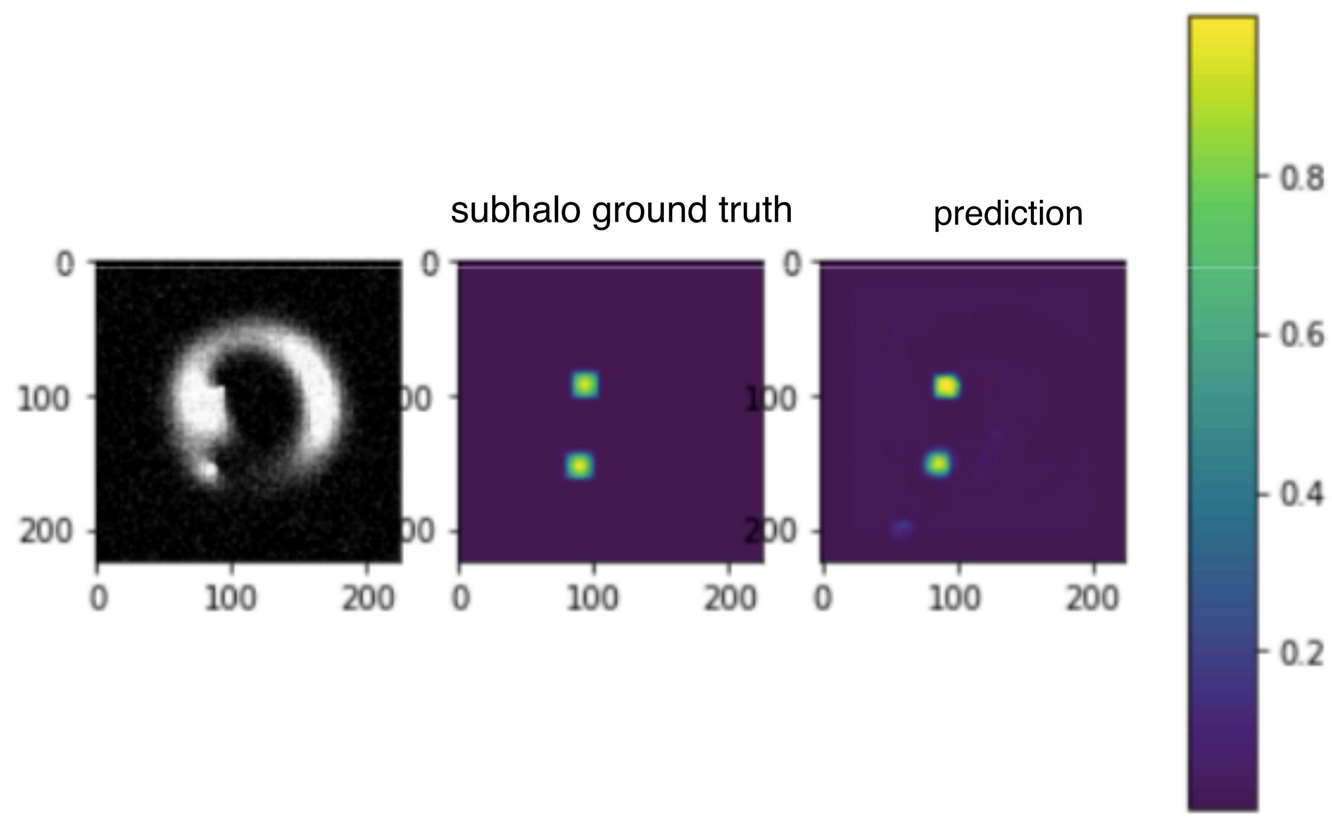
Prediction: subhalo detected!



Prediction: subhalo detected!

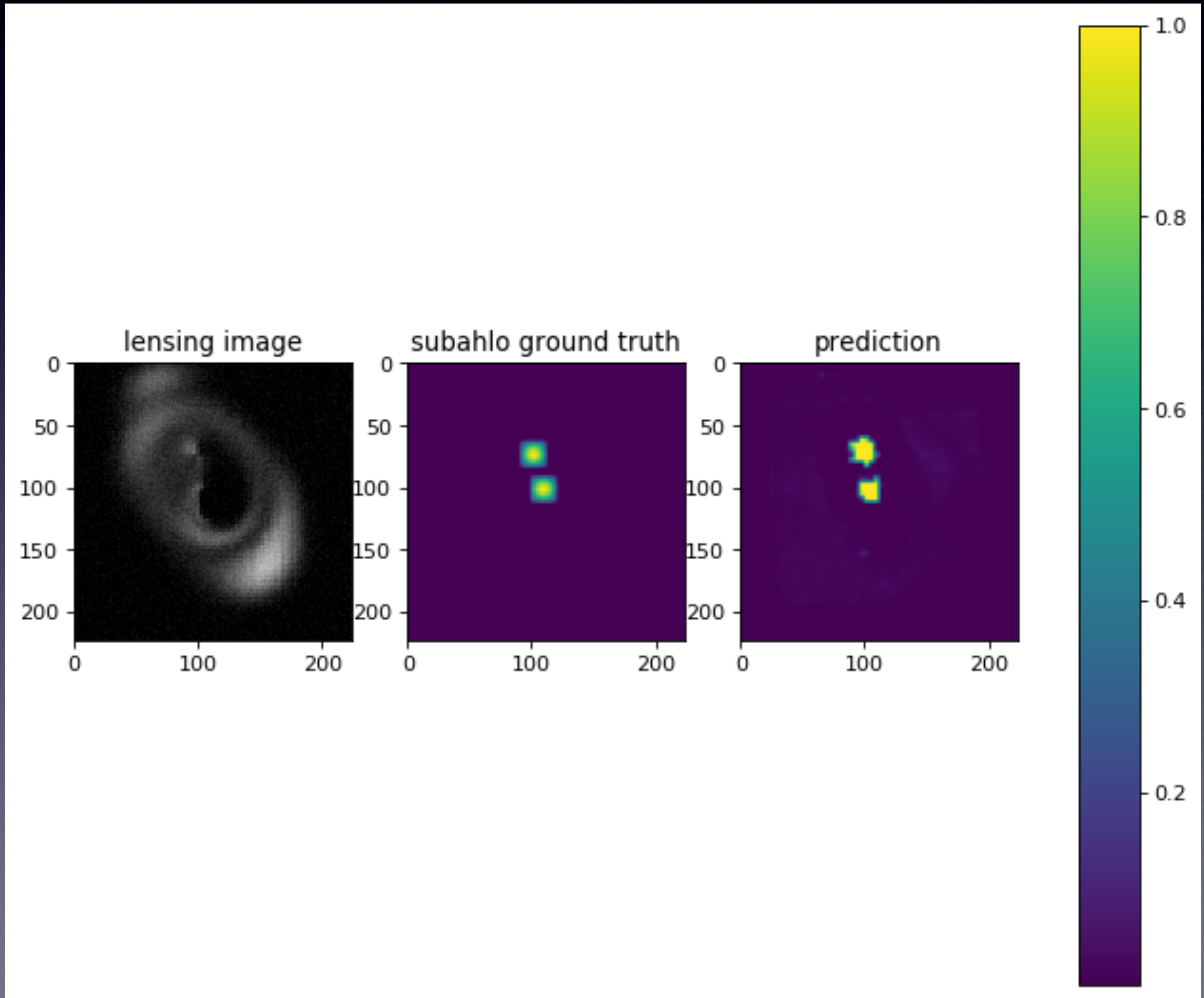


Prediction: multiple subhalos!

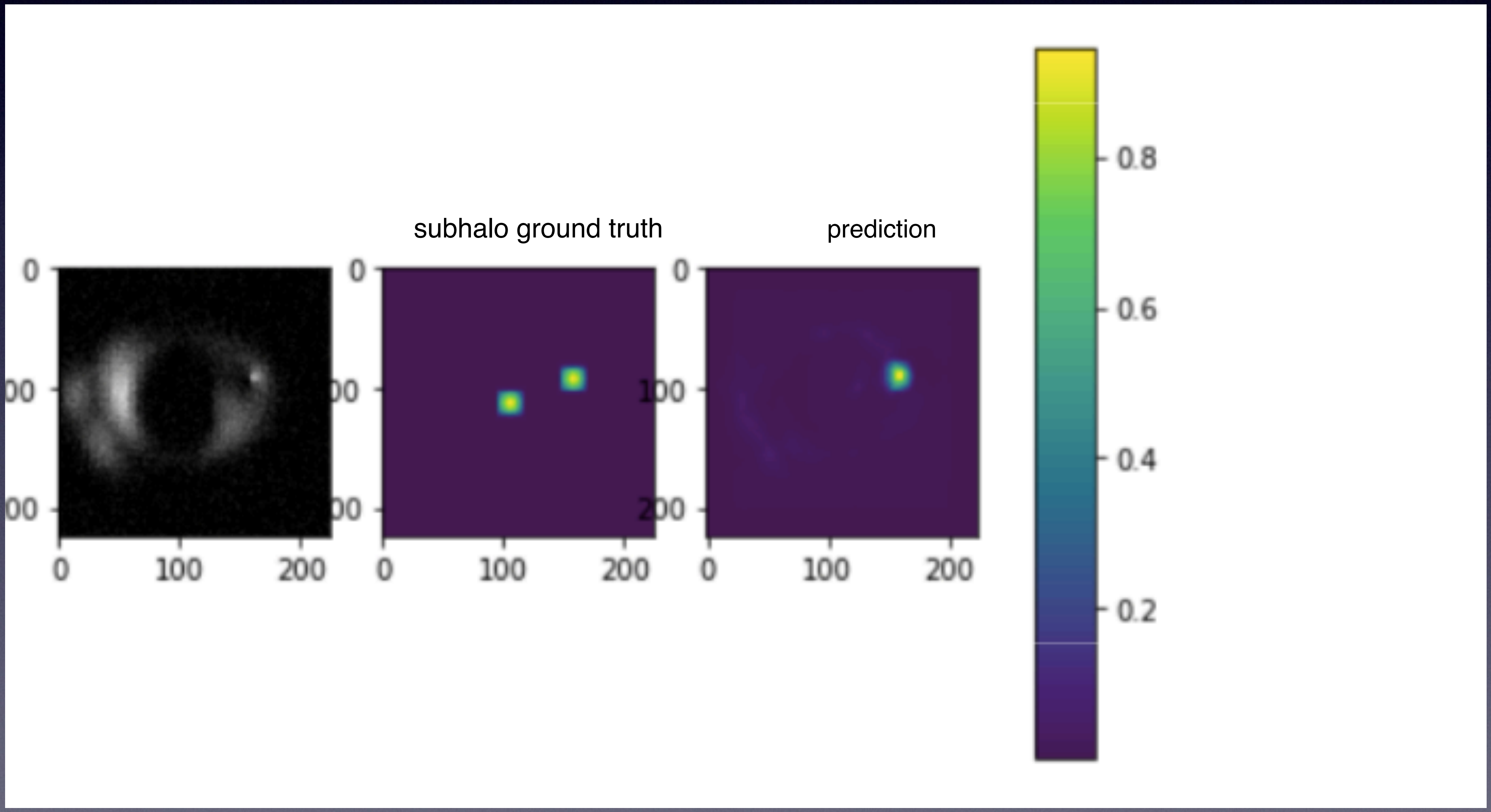



```
Macro:
pred [0.2658, 0.0345, 0.0166, 0.1423, 0.9035, 0.0023, -0.0046, 1.0]
target [ 0.2831 -0.0776  0.1189  0.0781  0.7854  0.       0.       1.      ]
#-----------------------------------------------------------------------
```

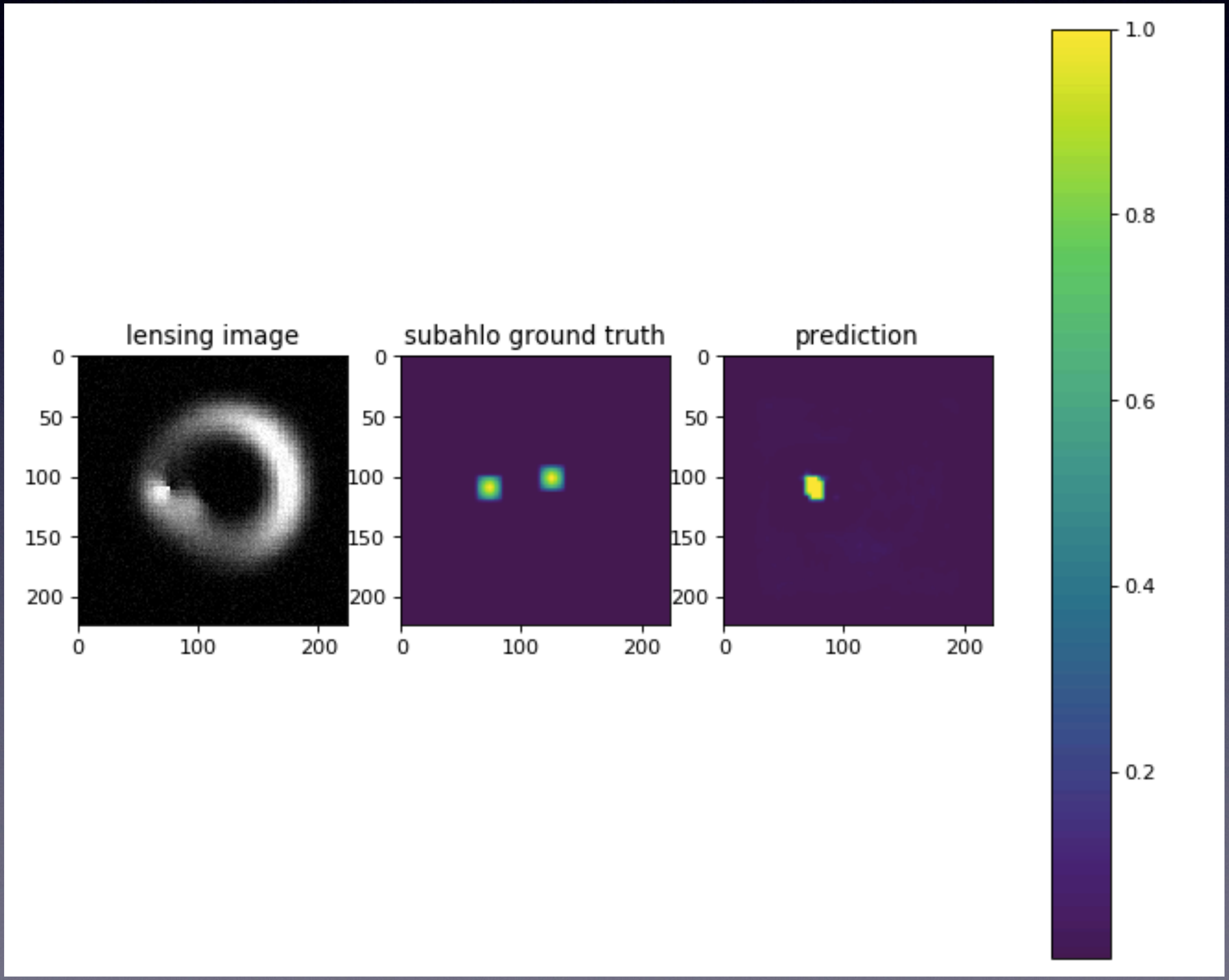
Prediction: multiple subhalos!



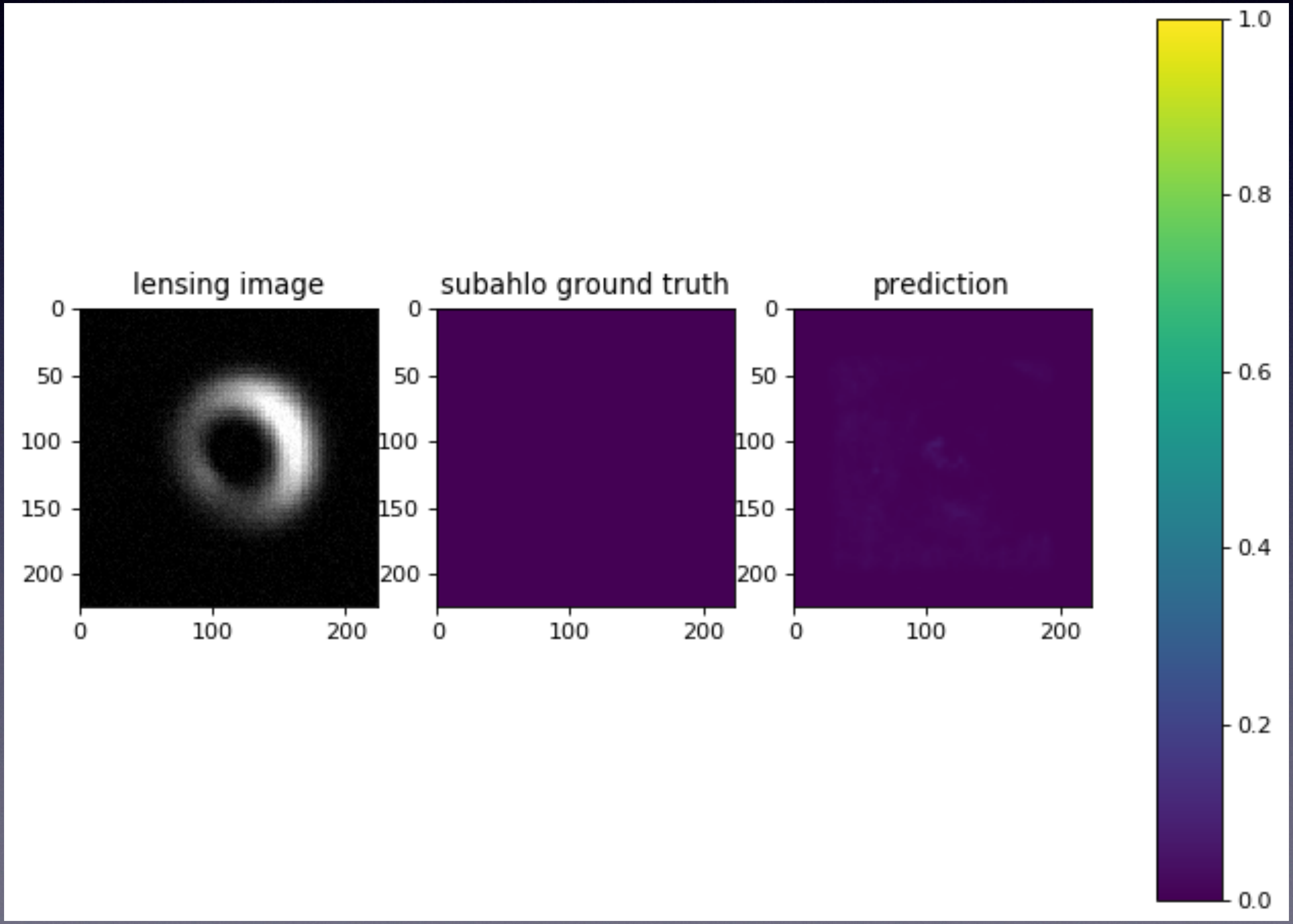
“Failed” good examples: Can’t see in the dark



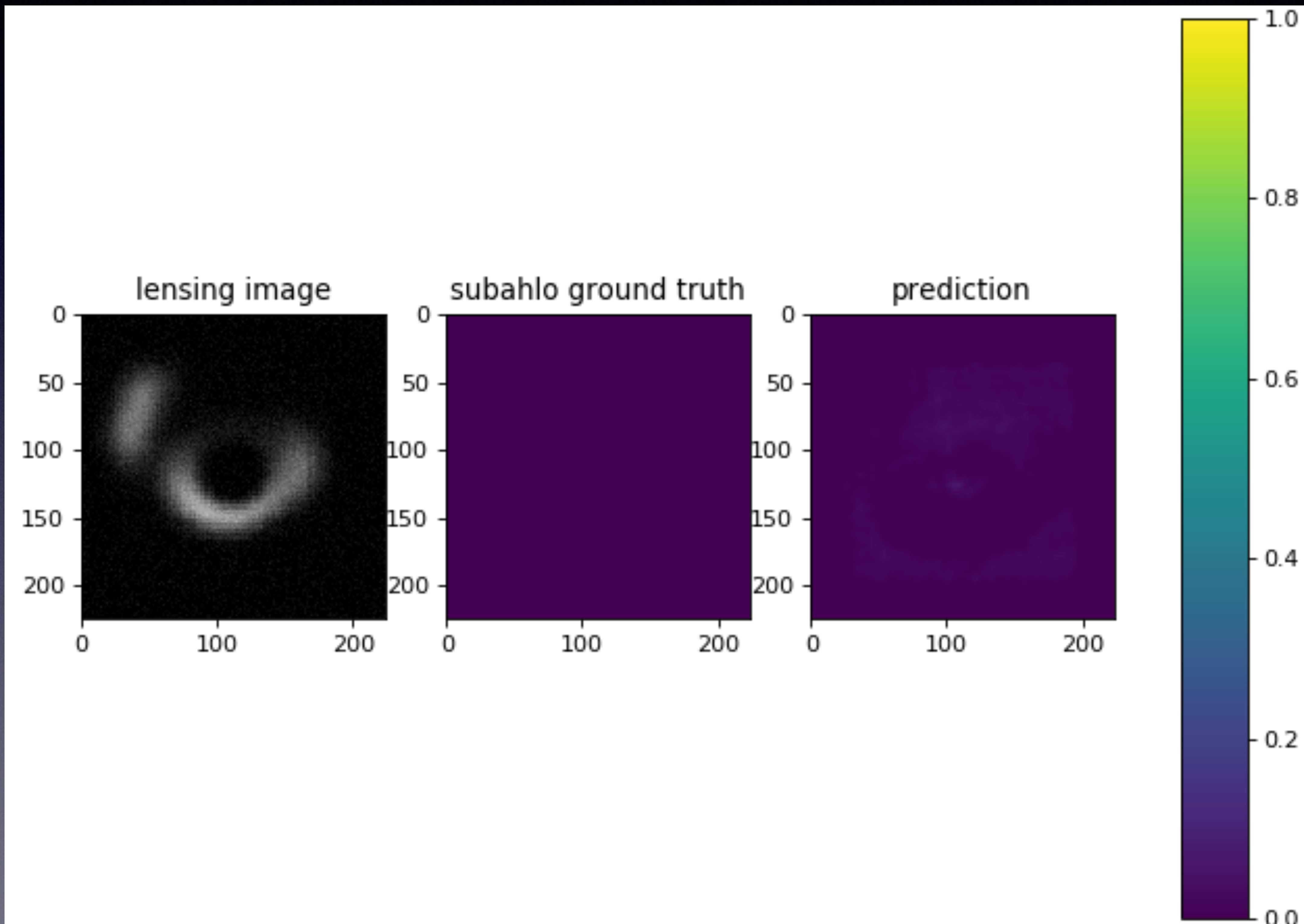
"Failed" good examples: Can't see in the dark



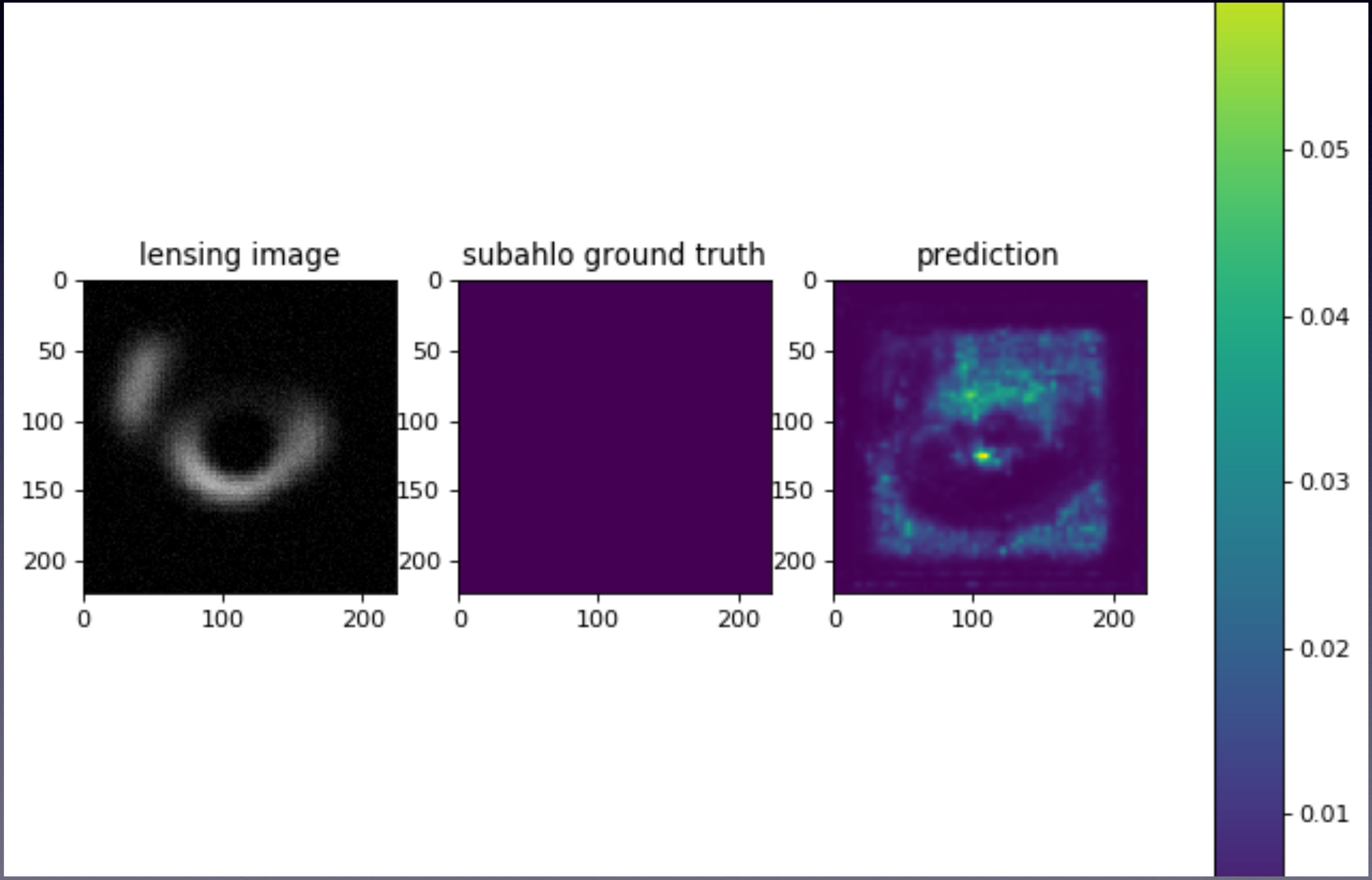
Prediction: No subhalo



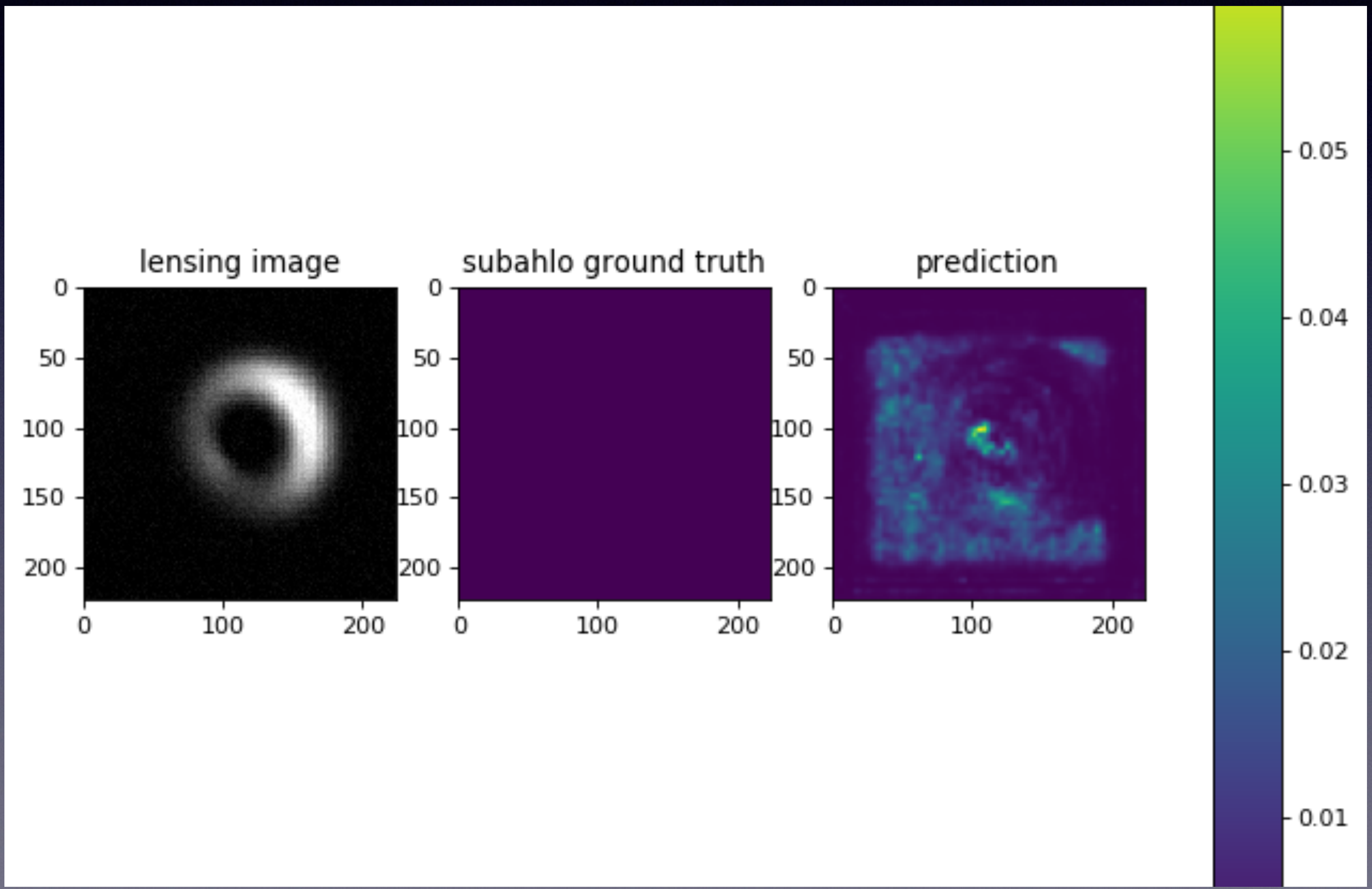
Prediction: No subhalo



"Rejection" of subhalo(s) around the arc



"Rejection" of subhalo(s) around the arc



Summary for Sim I

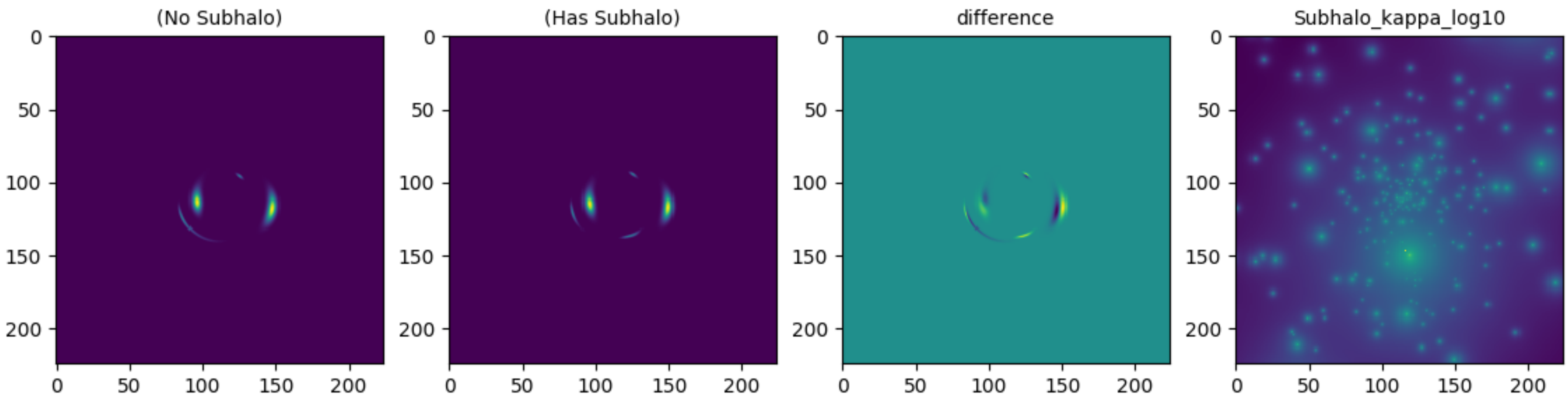
- Deep learning shows some promising result in dark matter substructures detection in lensing.
- “rejections” for no subhalos around the strong lensing arc.
- “detections” and “regression” for subhalos around the strong lensing arc.
- More realistic simulation needed

Simulation II

- Simulation II: More realistic, Evillens Based
- Sources
- Power law elliptical model
- Subhalos - Pseudo-Jaffe Profile, Cumulative mass function, and radial distribution function from Aquarius Simulation
- Angular resolution: 0.02”
- 500, 000/ 100,000 images as train/test set

https://github.com/wmorning/EvilLens

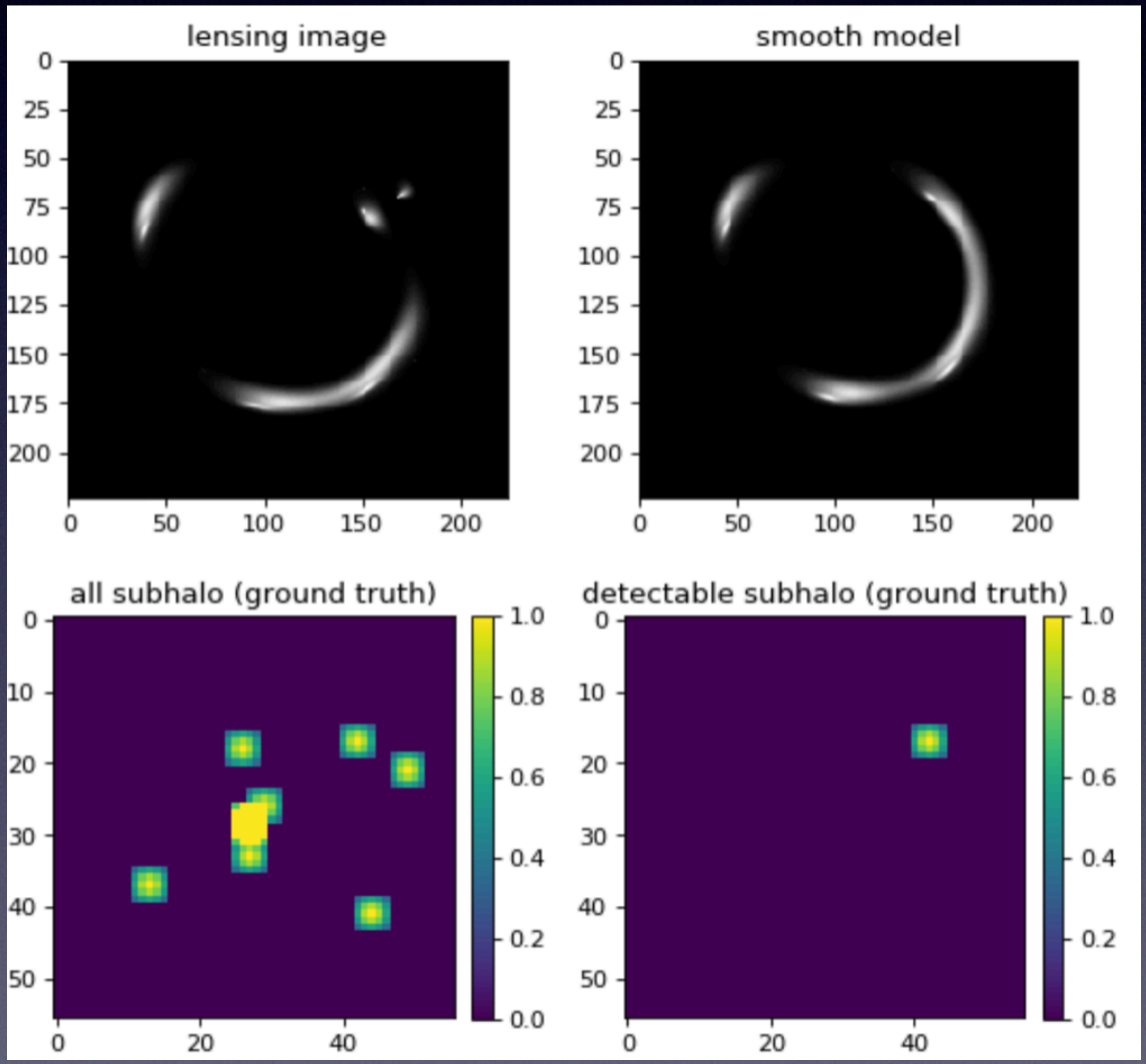
Simulation II



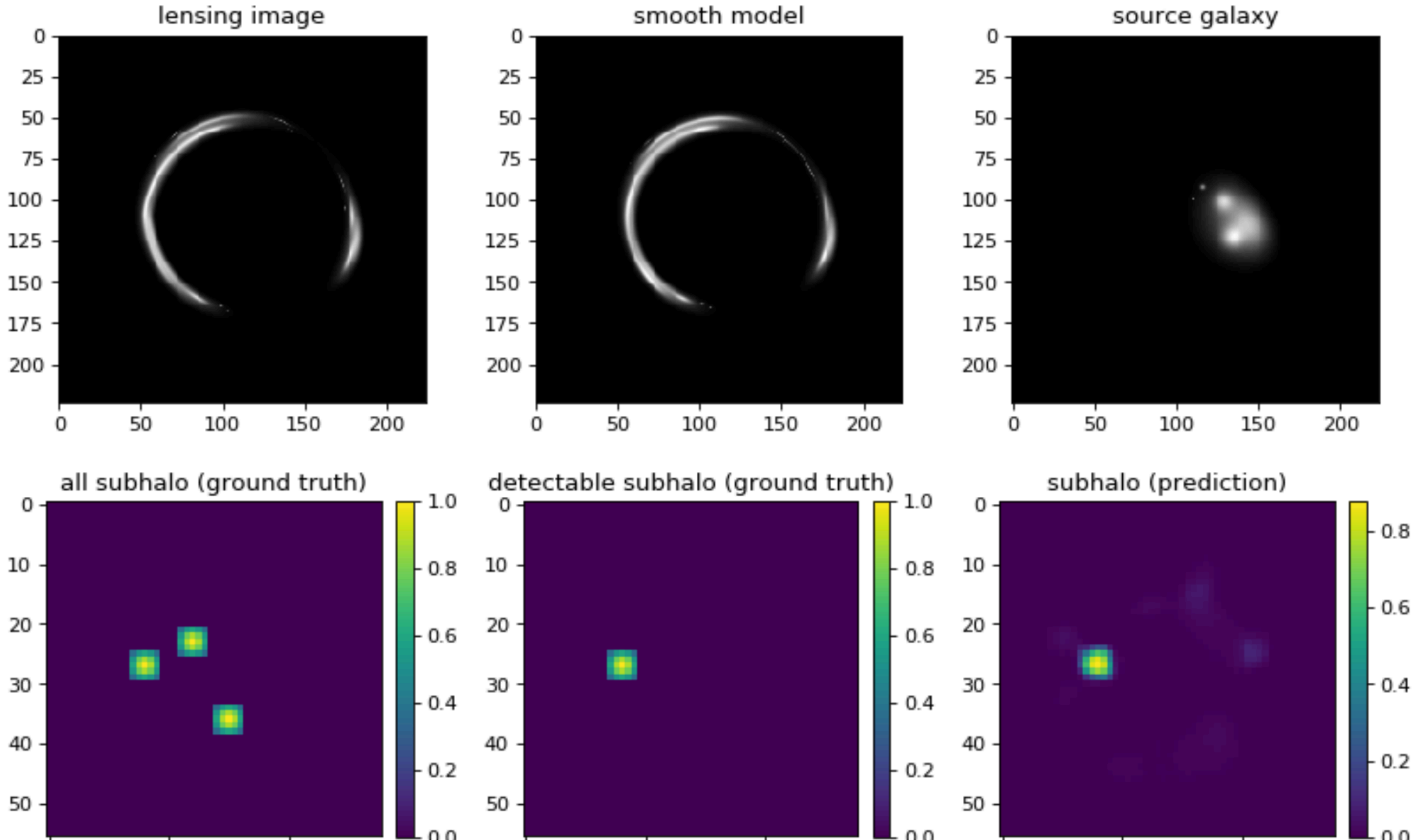
- Perturbations are small
- Most of the subhalos are not detectable (far away from lensed source)

Detectable subhalos

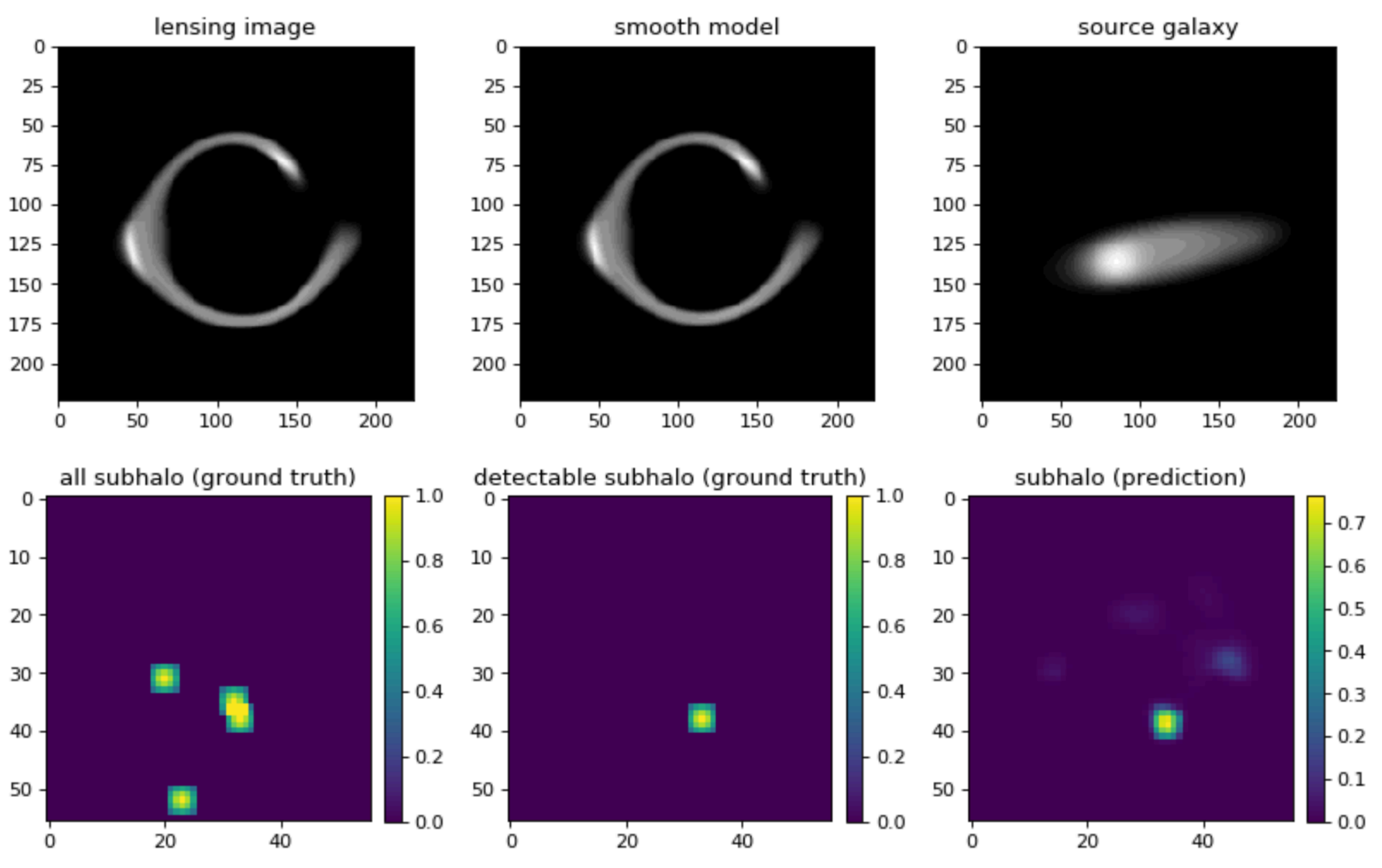
- Subhalos near lensing arc
- Einstein radius of subhalo overlap with lensed source (>max pixel/15)
- We treat all the subhalos (with different masses) with the same target probability density



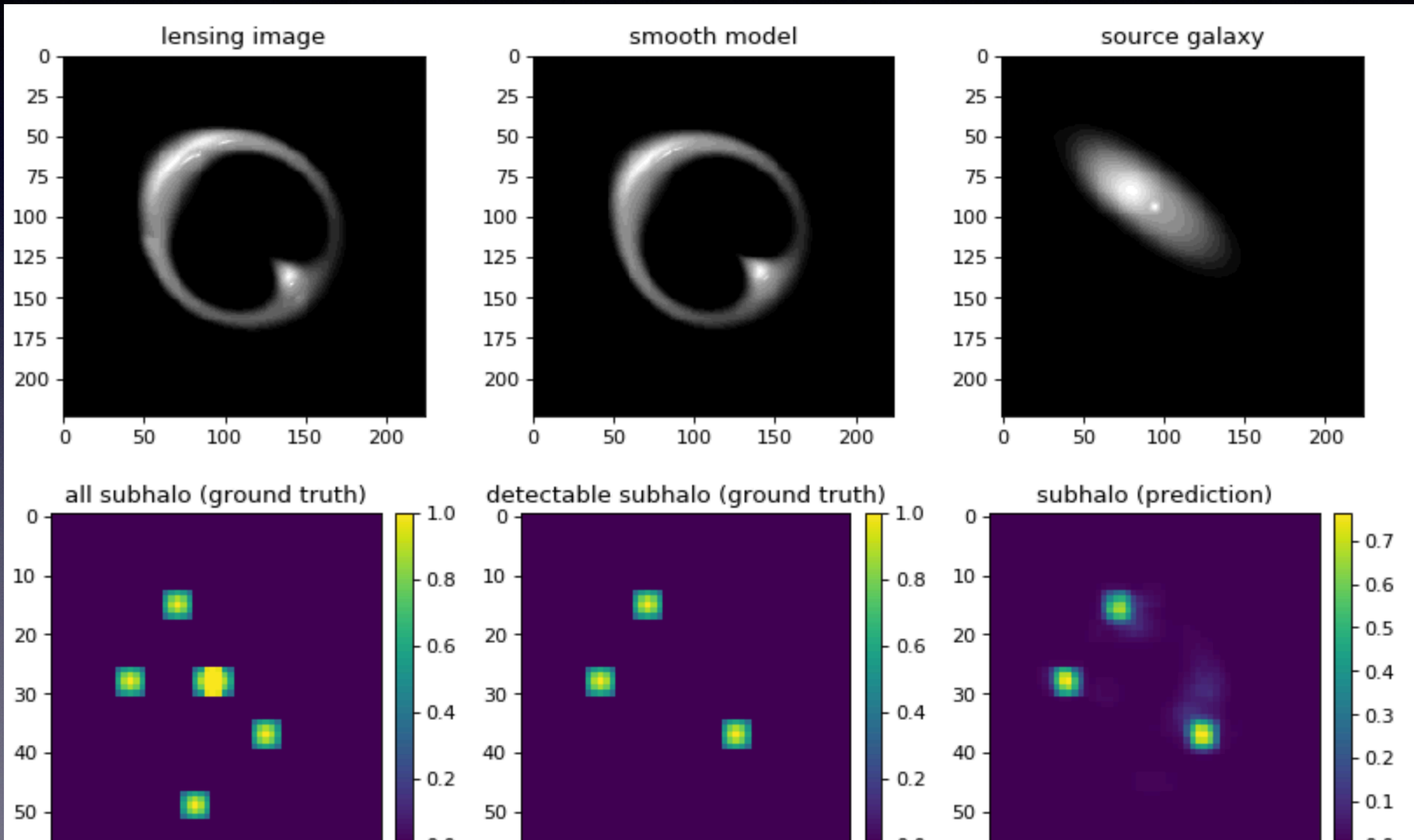
Prediction: subhalo detected!



Prediction: subhalo detected!



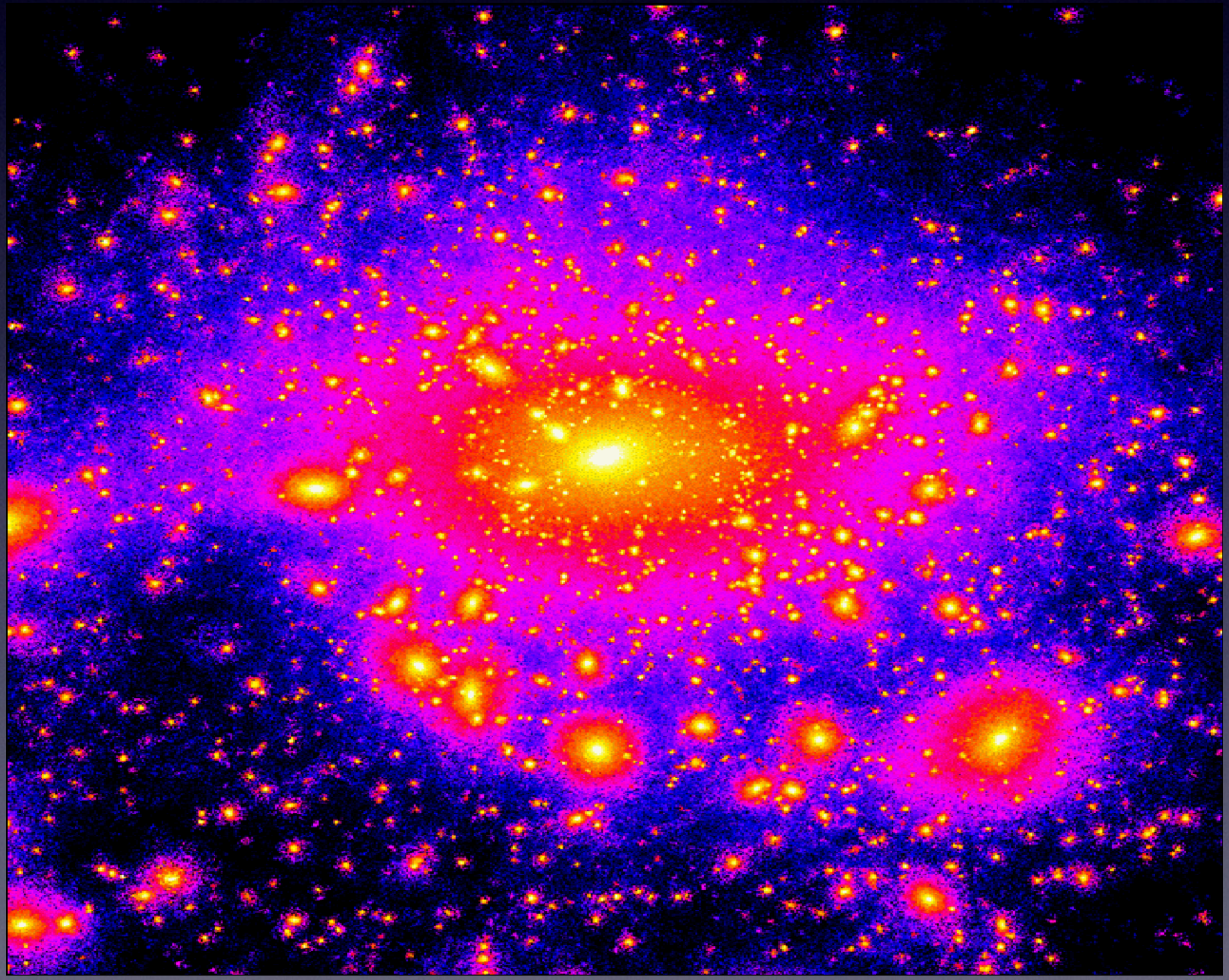
Multiple subhalos detected



Discussion

- Neural network are able to find subhalos in a more realistic simulation
- Unlabeled subhalos could be detected if close enough to the lens
- Some summary statistics would be needed

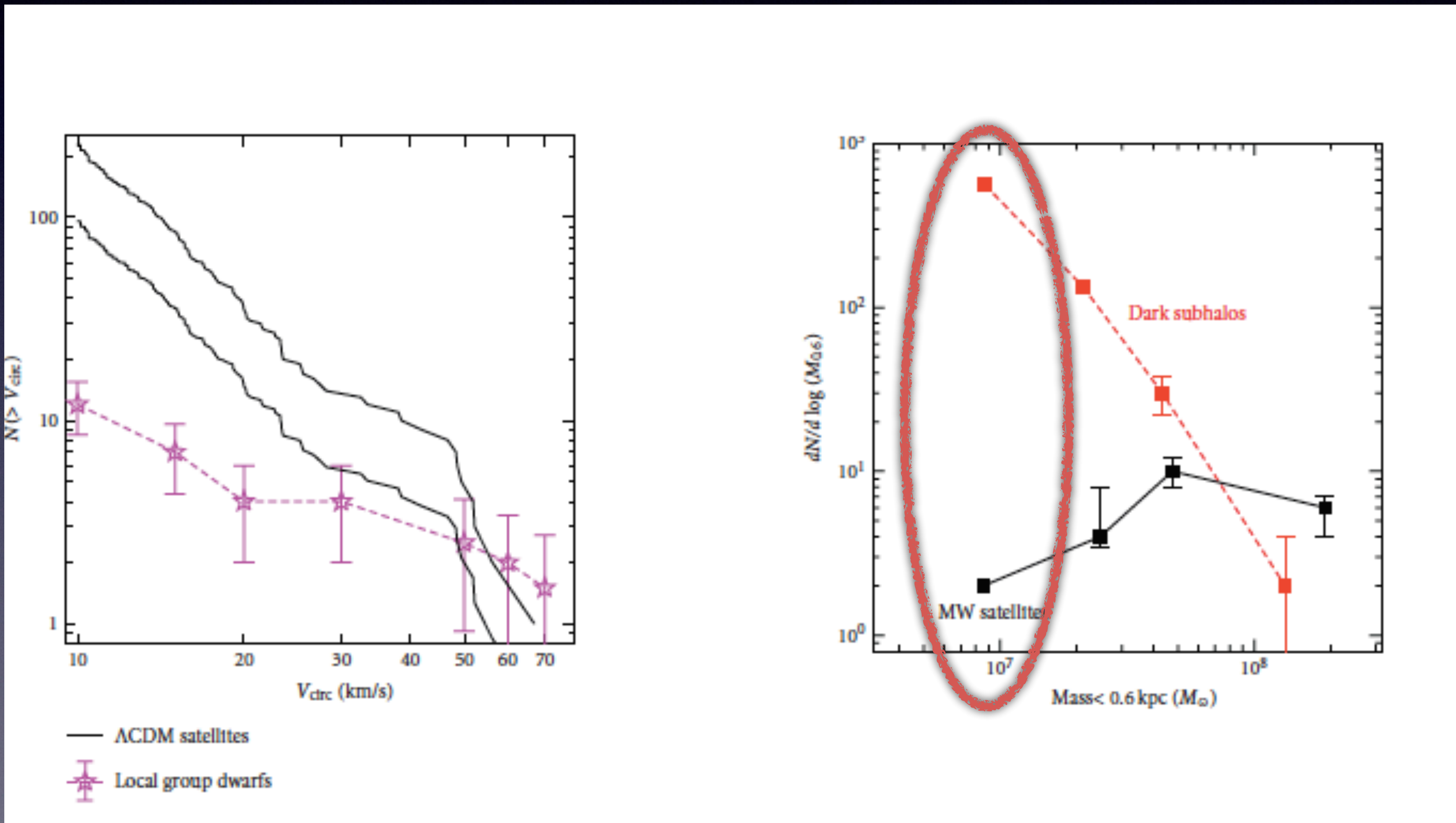

WAIT! WAIT! WAIT!
HOLD ON A SECOND!



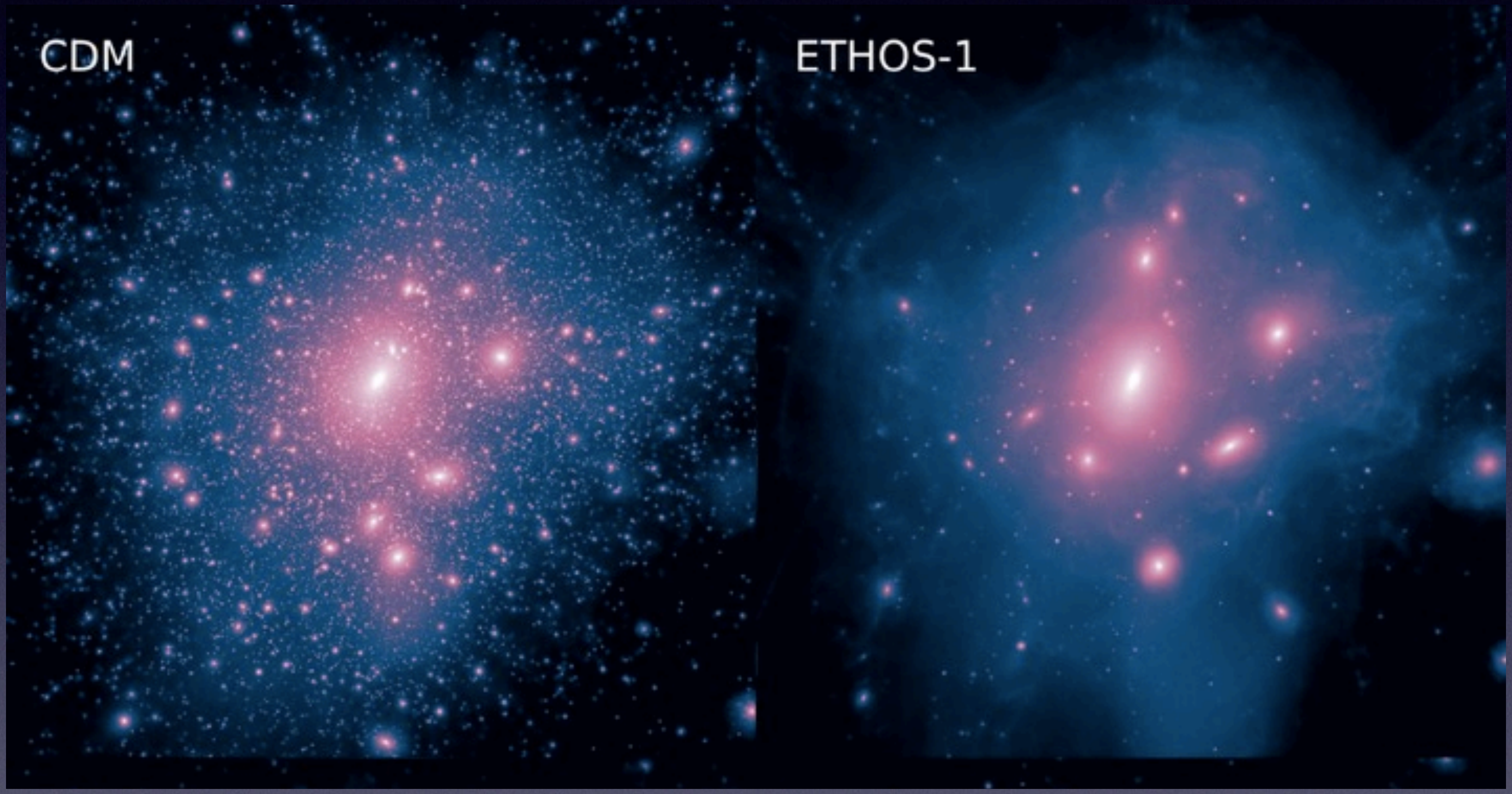
CDM Simulation (Mayer and Kazantzidis)

- What about Low mass subhalos?

Lots of low mass (< 10^8 M_sun) subhalos contributes!



CDM vs SIDM



Lots of low mass subhalos contributes in CDM!

Image credit: Mark Vogelsberger

Power spectrum of DM models

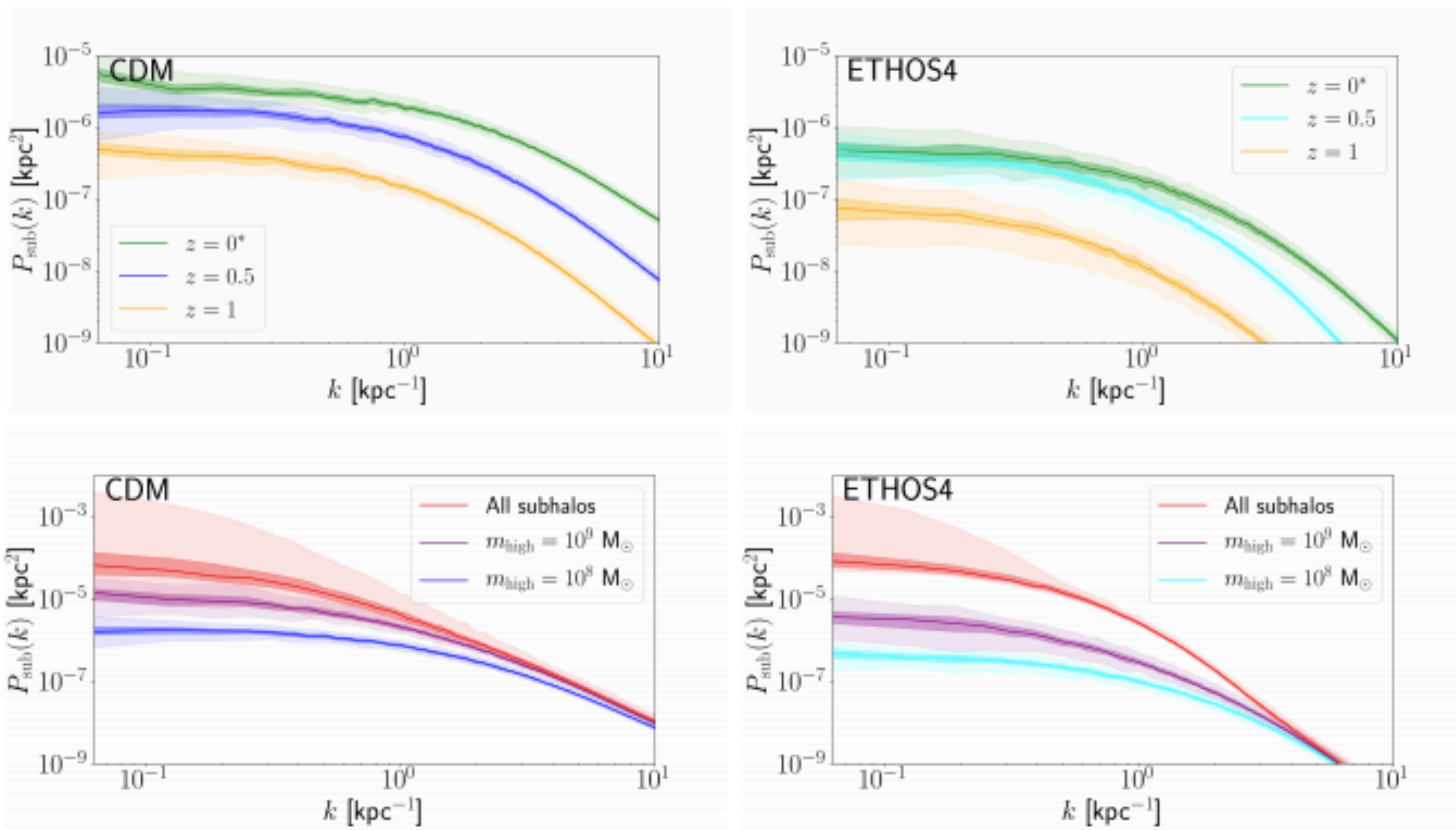
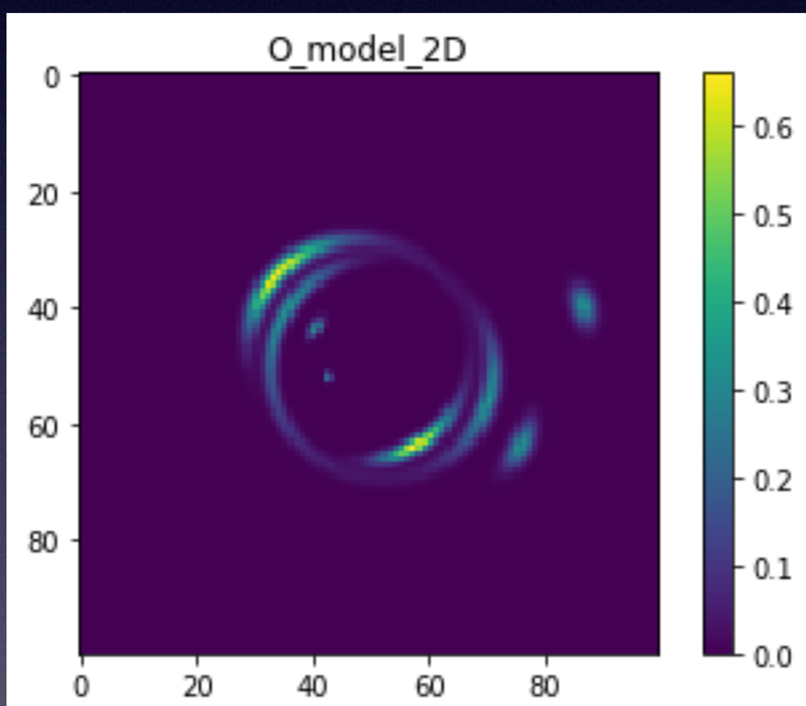


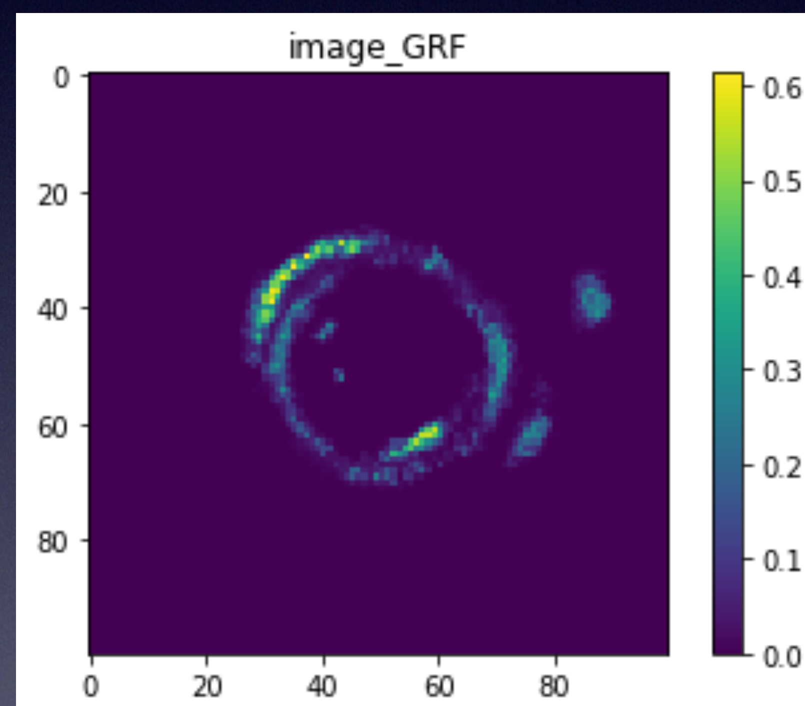
Figure 3. *Top Left*: redshift dependence of the convergence power spectrum for the CDM simulation. *Top Right*: redshift dependence of the convergence power spectrum for the ETHOS4 simulation. *Bottom Left*: mass dependence of the convergence power spectrum for the CDM simulation. *Bottom Right*: mass dependence of the convergence power spectrum for the ETHOS4 simulation. Note that the y-axis is the same for a given row but differs between rows. The wavenumbers k are in comoving coordinates. *As discussed in the text, the $z = 0$ power spectra are computed using the subhalo catalog at $z = 0$ but the distance between the observer and the lens $D_{\rm ol}$ is fixed to be the same as for a lens at $z = 0.5$ because $\Sigma_{\rm crit}$ diverges as $z \to 0$.

Díaz Rivero et al. (ArXiv:1809.00004)

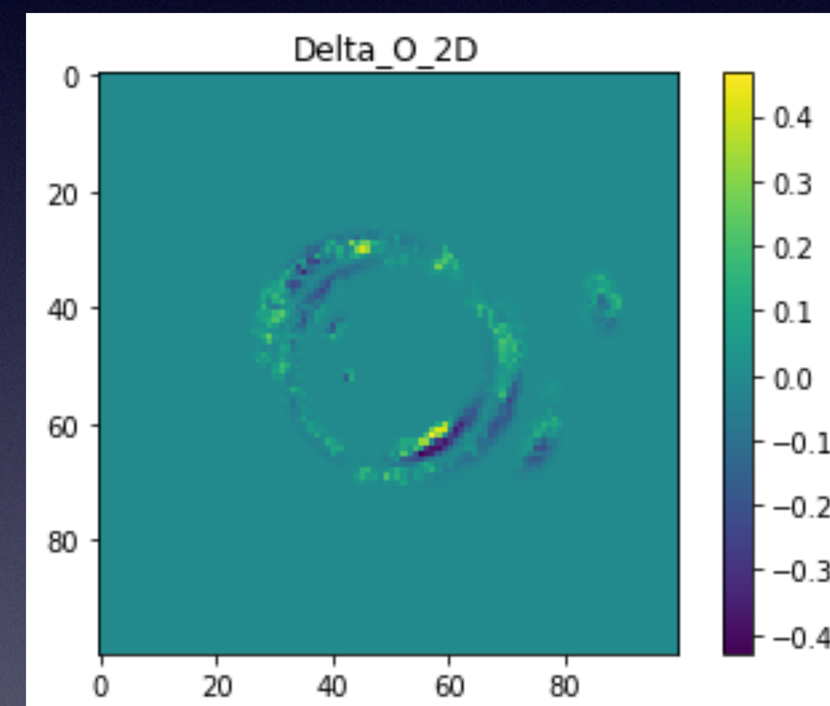
Statistical detection of dark matter power spectrum



Smooth Lens

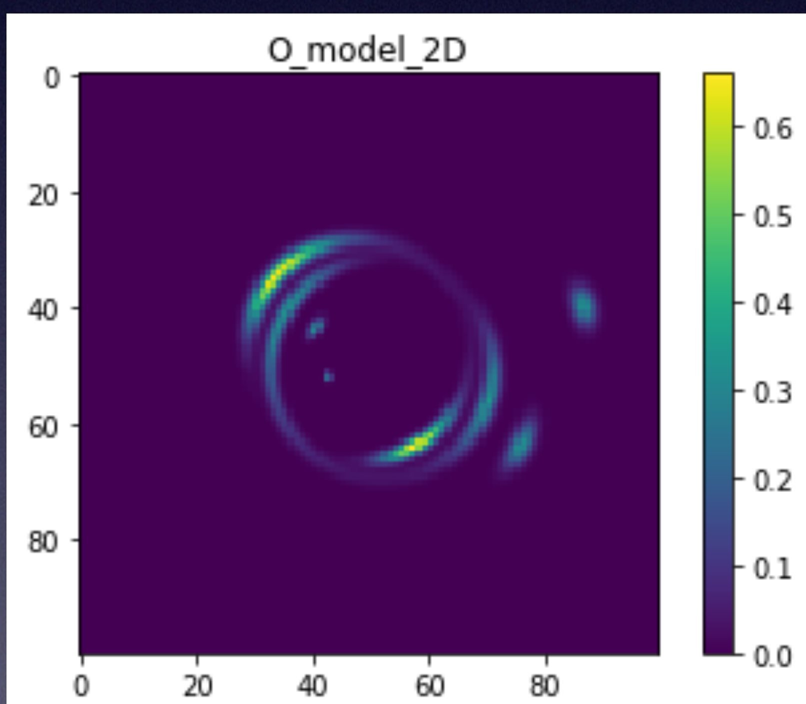


with Gaussian Random Field

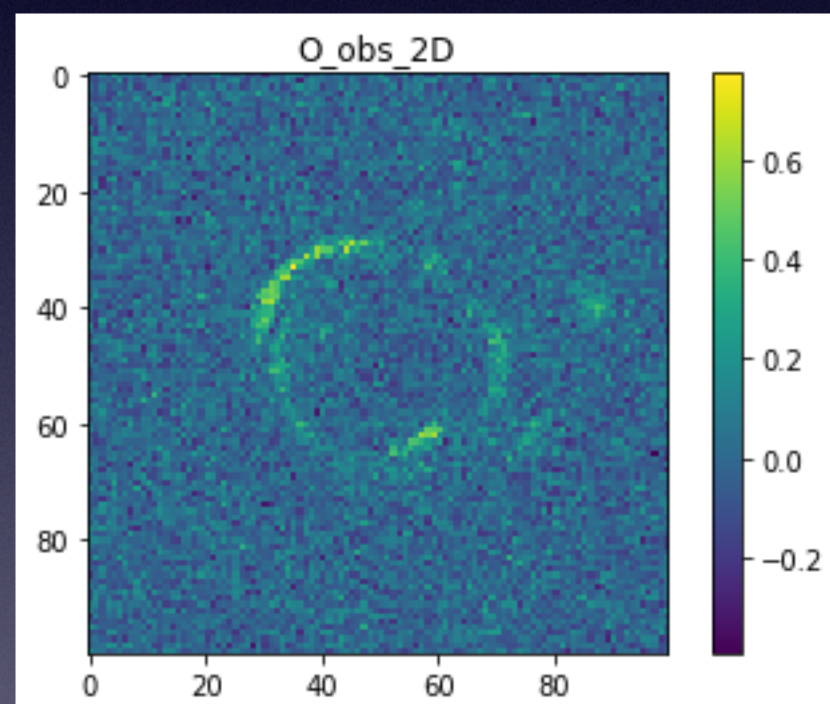


Difference

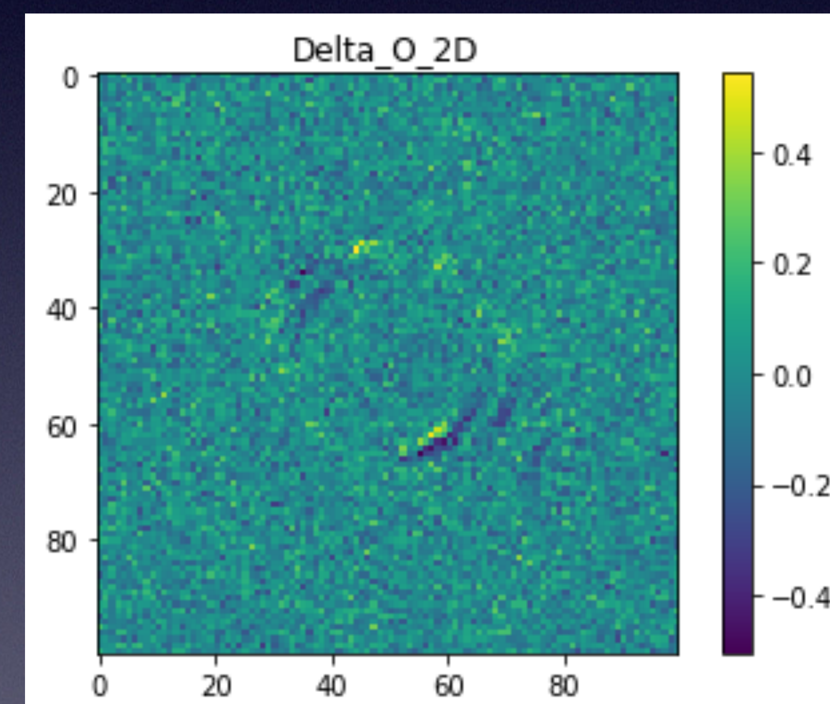
Noise



Smooth Lens



with Gaussian Random Field

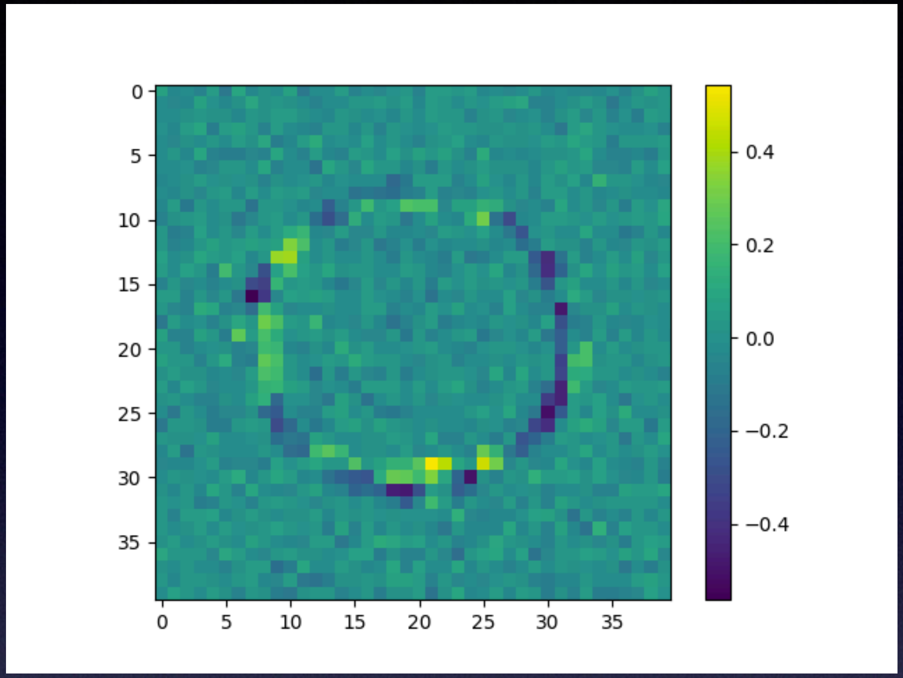


Difference

Likelihood function

$$\mathcal{L}(\boldsymbol{O}_{\rm obs},\boldsymbol{p}) = \int d^n N d^{2n}\alpha\, P(\boldsymbol{N})P(\boldsymbol{\alpha})$$
$$\delta\left[\boldsymbol{O}_m(\boldsymbol{p}) + \frac{\partial O}{\partial \alpha}\Delta\boldsymbol{\alpha} + \boldsymbol{N} - \boldsymbol{O}_{\rm obs}\right] P_{\rm p}(\boldsymbol{p}) \qquad (6)$$

$$\mathcal{L}(C_\alpha) = \left(|C_N|\,|C_\alpha|\,|C_p||M|\right)^{-1/2} e^{\frac{1}{2}B^{\mathsf{T}}\,M\,B}$$
$$e^{-\frac{1}{2}\left(\Delta\boldsymbol{O}^{\mathsf{T}} C_N^{-1}\Delta\boldsymbol{O} + \boldsymbol{p}_0 C_p^{-1}\boldsymbol{p}_0\right)} \qquad (9)$$



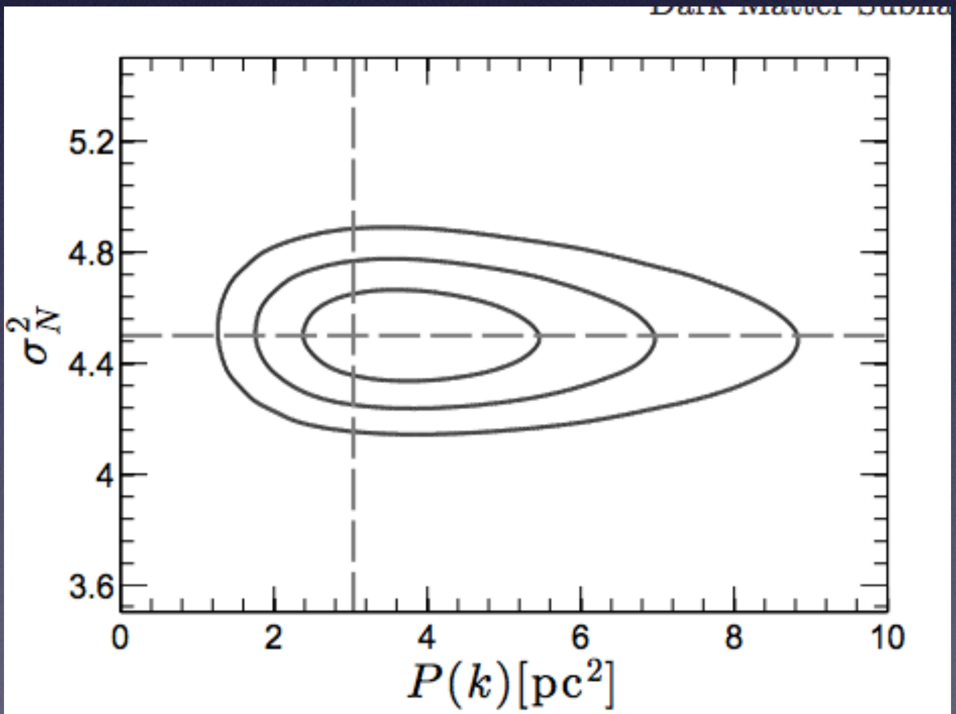
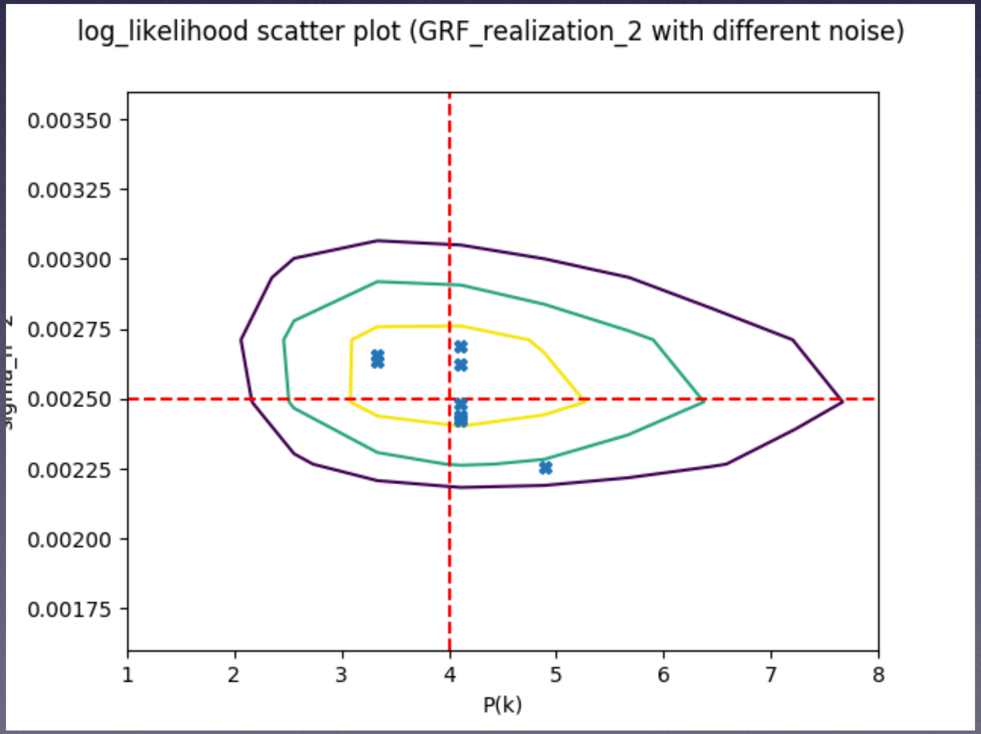
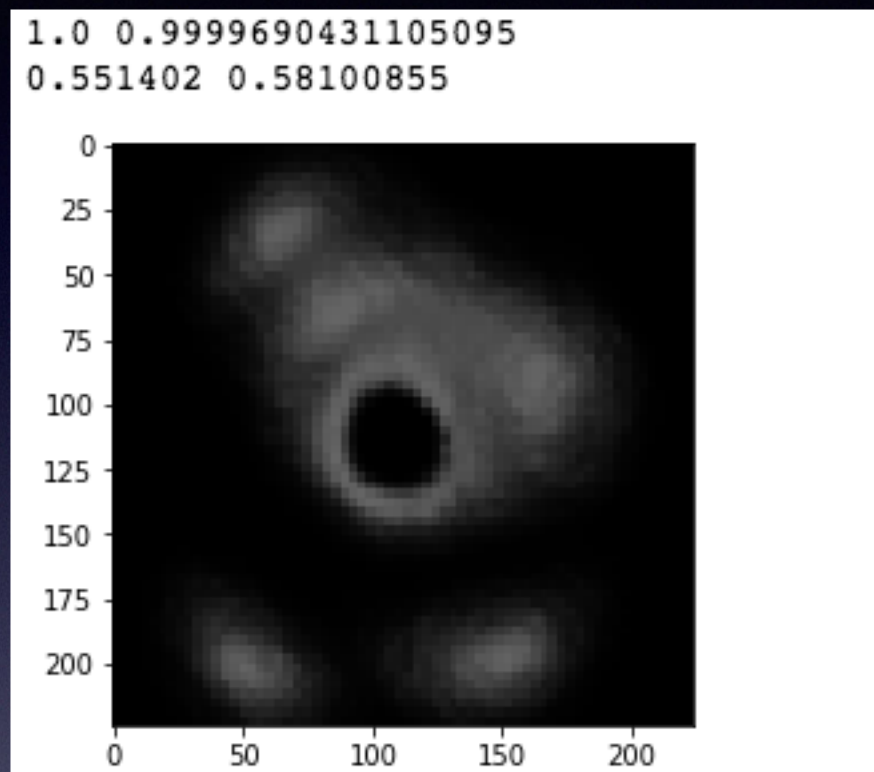


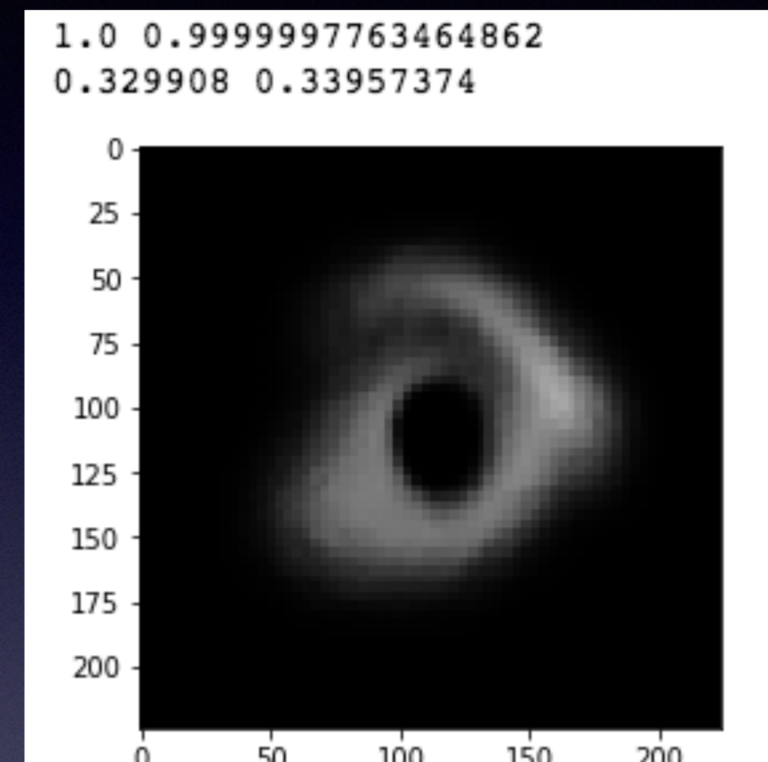
FIG. 2.— Joint-likelihood of noise and the amplitude of the power spectrum, mapped by evaluating Equation (9) using mock observations described in §4, lensed by a density field which includes substructure with a flat power spectrum. The dashed lines show the true values which were used in the mock observation. The input amplitude of the power spectrum is successfully recovered, with little if any degeneracy between instrumental noise and substructure fluctuations.

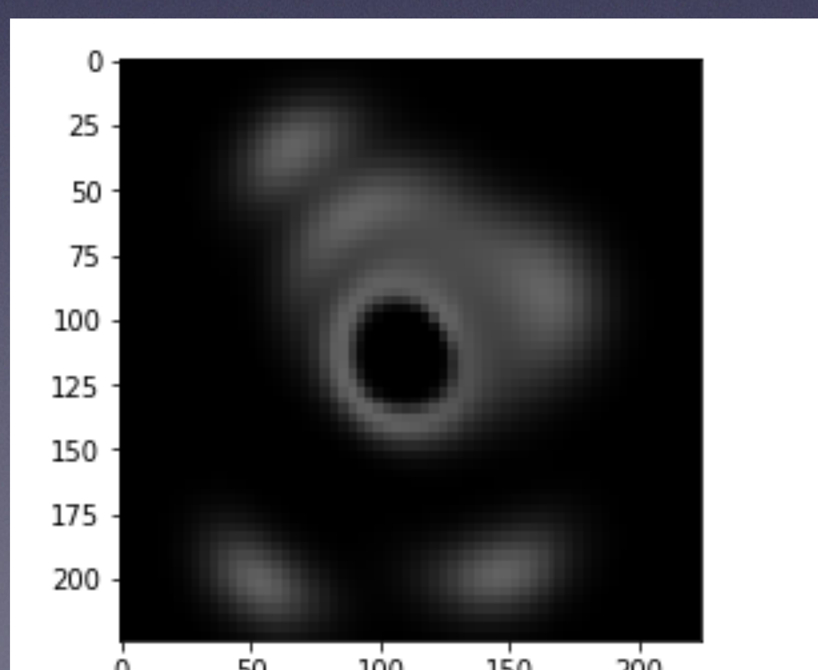


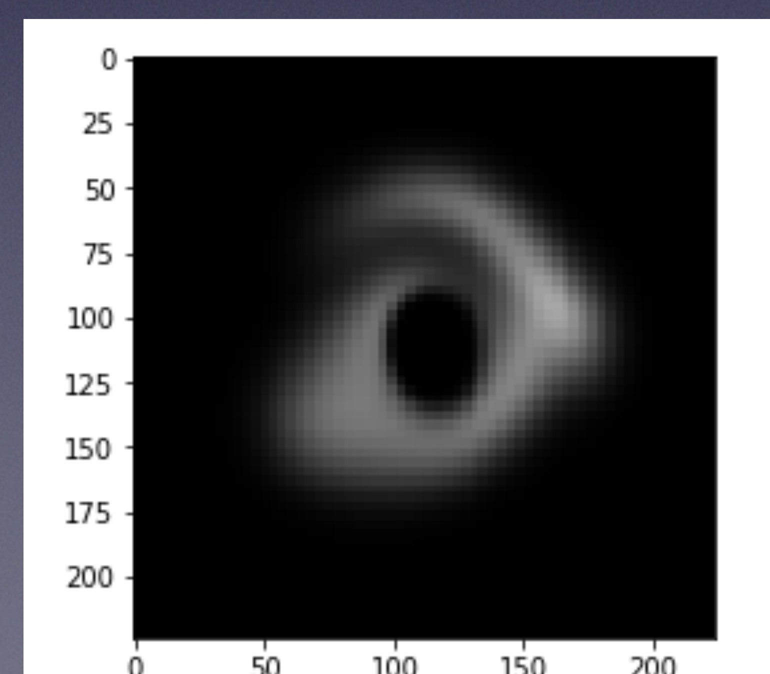
Hezaveh et al. (2016)

Preliminary result with NN









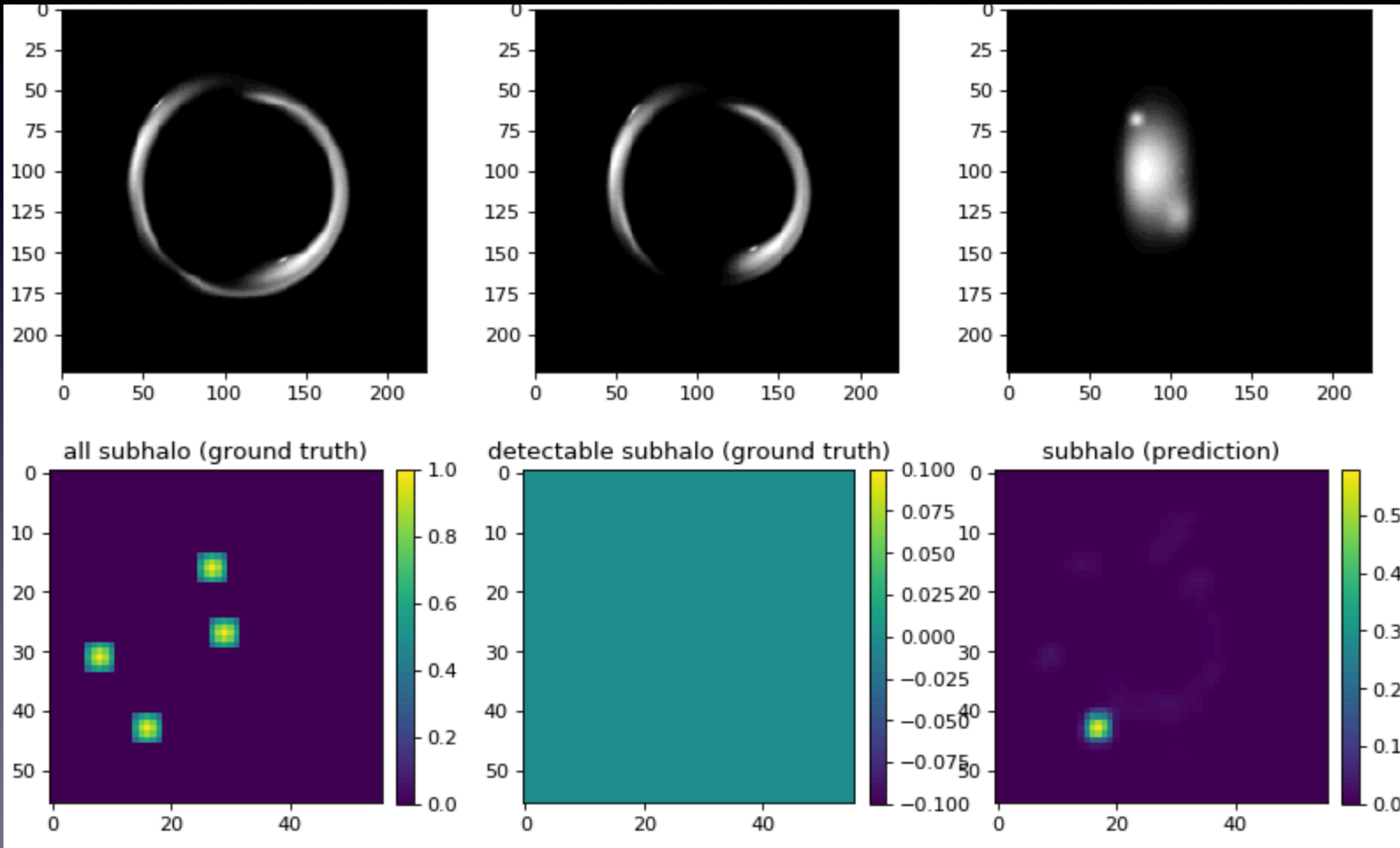
Discussion

- Maximum likelihood could serve as a tool to probe power spectrum of dark matter substructures in strong lensing system
- ML also could serve as a interesting tool for DMS power spectrum

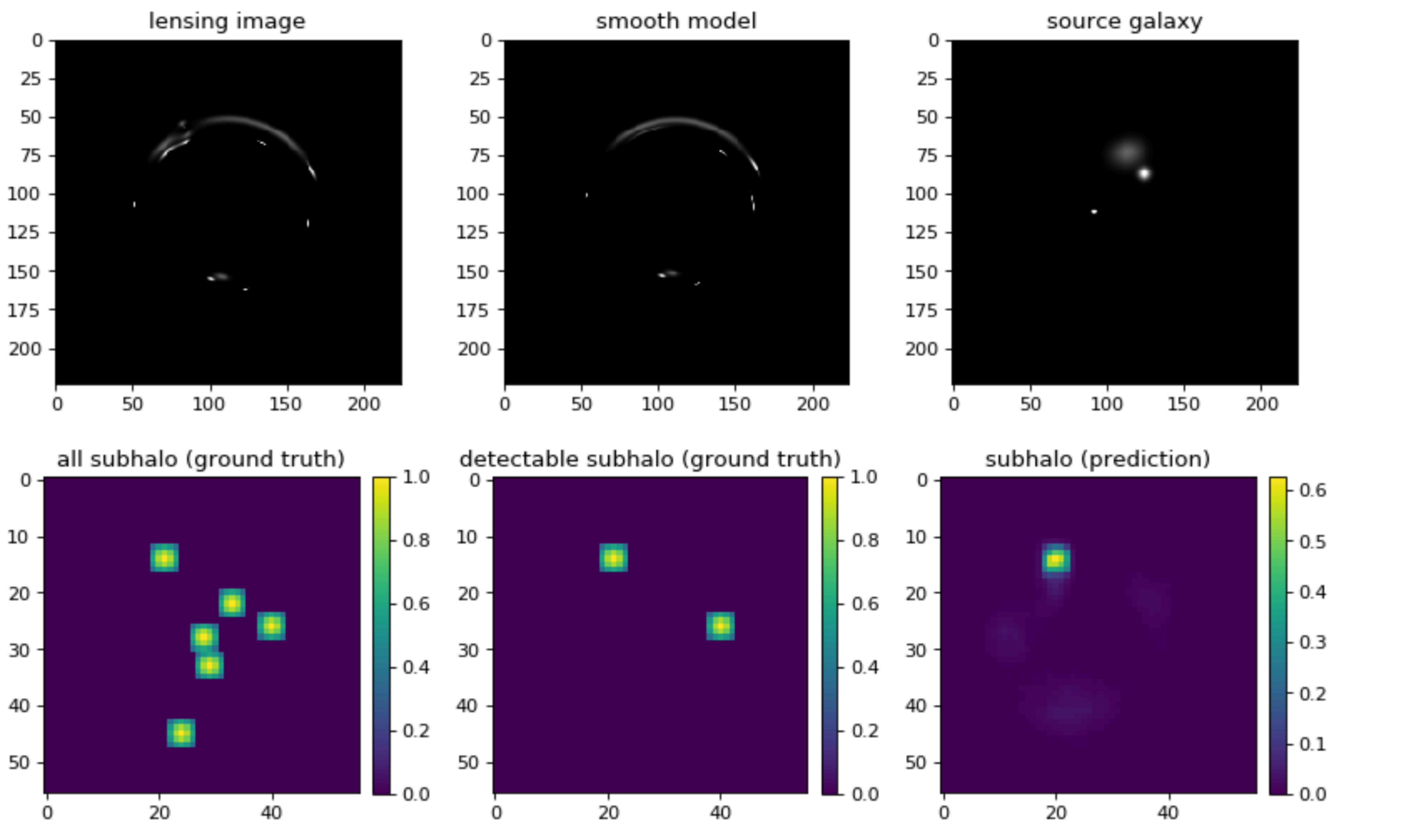
Thank you!

Image Credit: Hubble/STScI & NASA

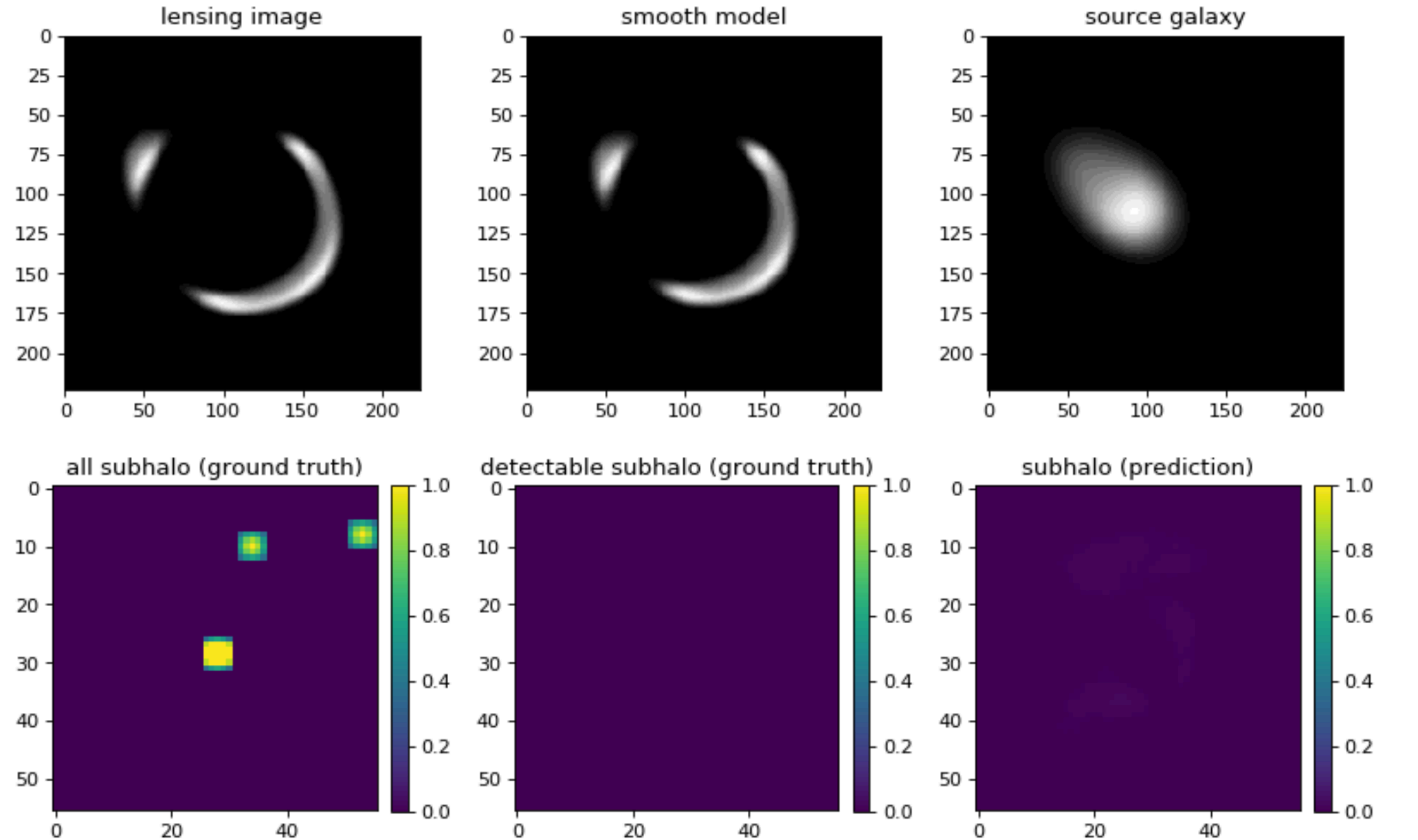
Prediction: unlabeled subhalo detected!



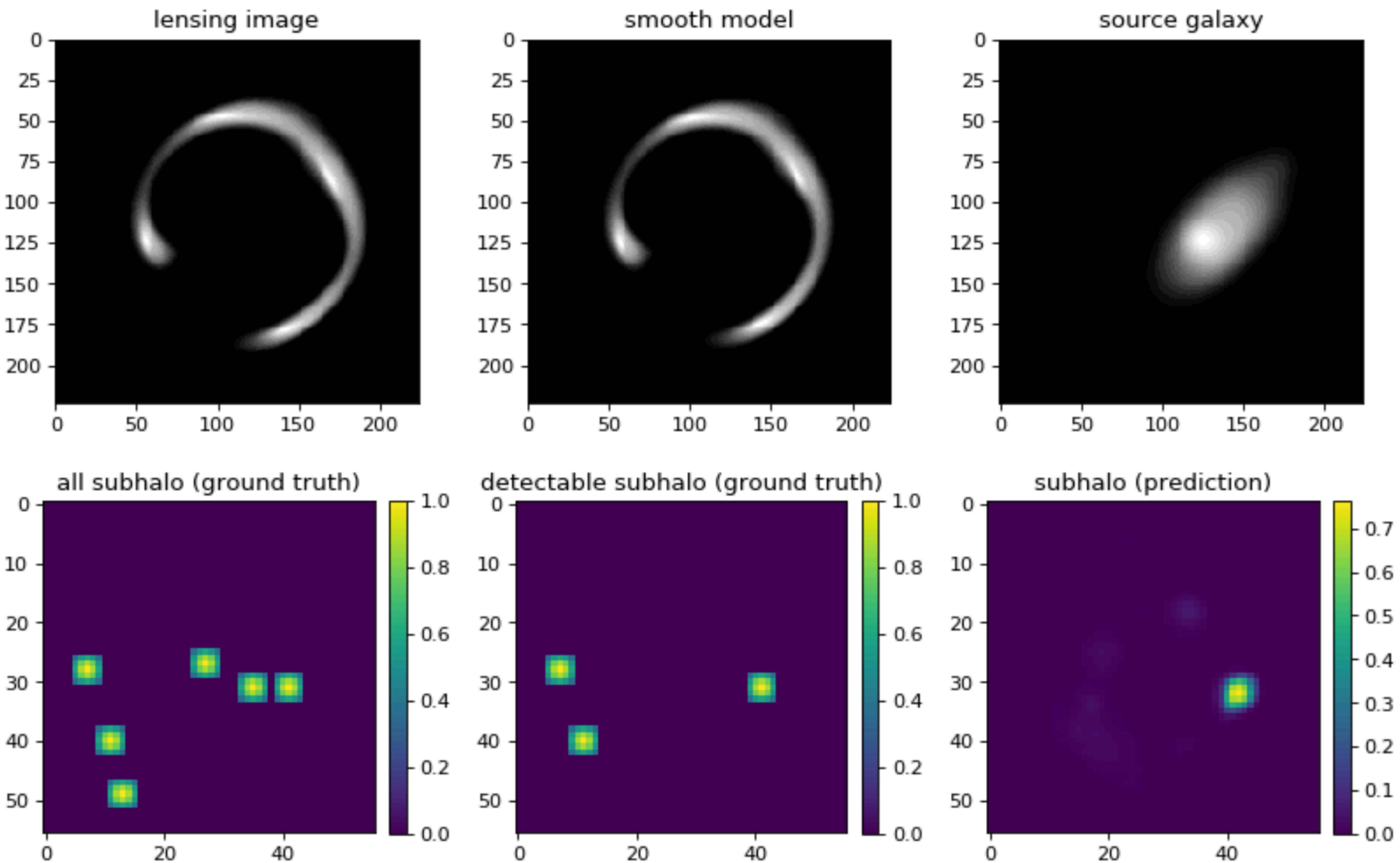
Detection: Complicated Lens



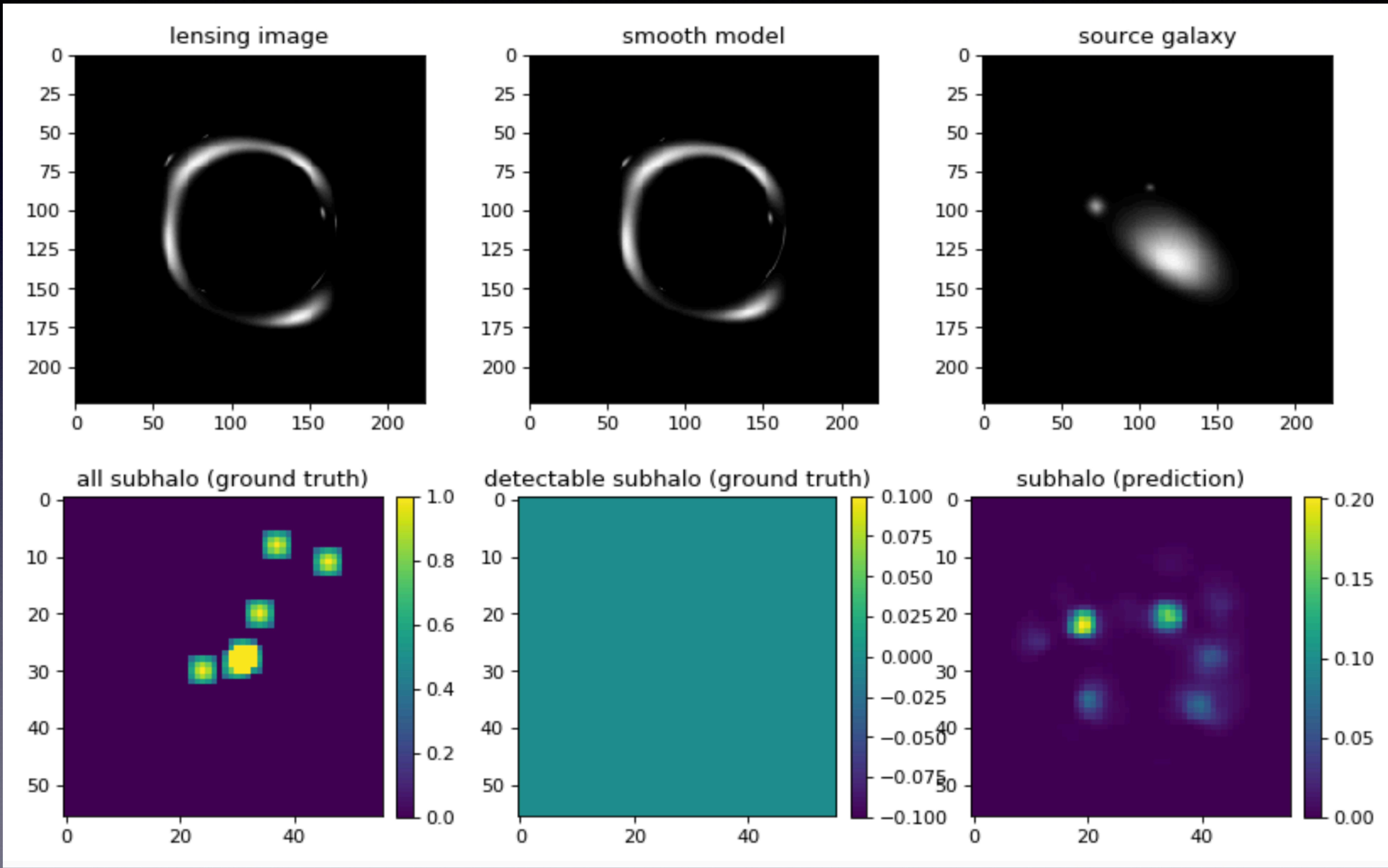
No detection



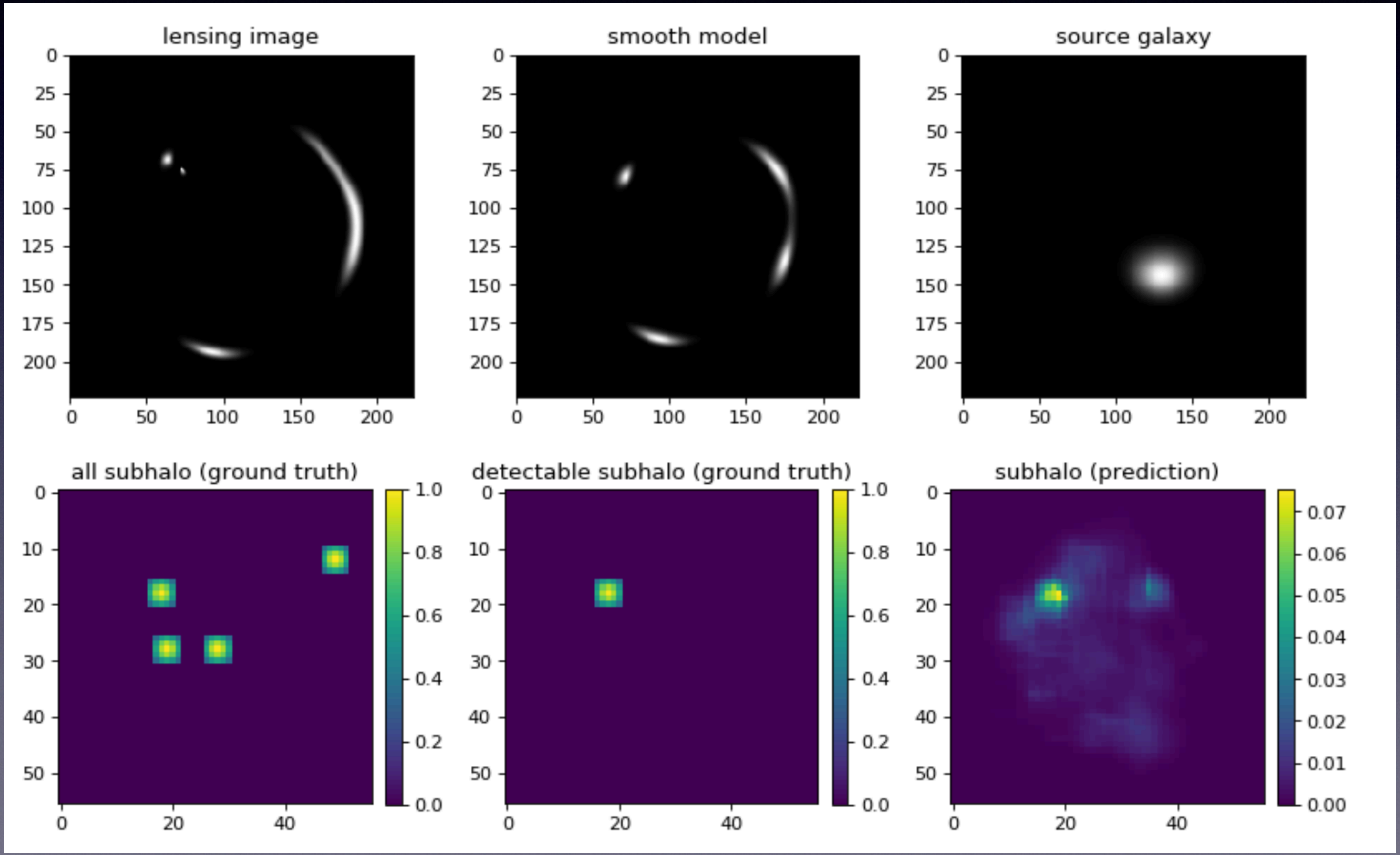
Fail to detect



False positive: Degeneracy



Degeneracy


lensing image
smooth model
source galaxy
all subhalo (ground truth)
detectable subhalo (ground truth)
subhalo (prediction)

True positive