Decays of doubly charmed baryon

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Outline

• Introduction: recent progress
• Theoretical framework
• Results and discussion
Introduction
Why charm?

Life moves on!
Face up to reality: “C”.

(SSS)

(uds)
Charm meson sector

- Direct CP violation: first CPV in charm has been observed.

\[ \Delta A_{CP} \equiv A_{CP}(D^0 \rightarrow K^- K^+) - A_{CP}(D^0 \rightarrow \pi^- \pi^+) \]

\[ \Delta A_{CP} = (-15.4 \pm 2.9) \times 10^{-4} \]

- Indirect CPV in D-Dbar mixing: the first successful quantitative attempt to understand x and y simultaneously.

\[ \begin{array}{|c|c|}
\hline
\text{Parameter} & \text{Avg value (HFLAV 2018) [\%]} \\
\hline
x & 0.36^{+0.21}_{-0.16} \\
\hline
y & 0.67^{+0.06}_{-0.13} \\
\hline
\end{array} \]

Charmed baryons

- Classification of Charmed Baryons

\[
\begin{align*}
4 \times 4 \times 4 &= 20_s + 20_{MA} + 20_{MS} + \bar{4}_A \\
3 \times 3 \times 3 &= 10_s + 8_{MA} + 8_{MS} + 1_A
\end{align*}
\]

- Singly charmed baryons

\[
\begin{align*}
3 \times 3 &= \bar{3}_A + 6_s
\end{align*}
\]

- Observations
- (Weak) Decays
- Lifetimes
- CPV
Progress in $\Lambda_c^+$: BESIII

- Based on 35 days’ data collecting at BESIII, 16 papers including 7 PRLs on $\Lambda_c^+$ have been published so far.
- About 20 channels have been measured, relevant PDG values has been revised.

- Absolute branching fraction of Golden mode: $\Lambda_c^+ \rightarrow pK\pi$.
- New technique to measure neutron final state.
- Single Cabibbo suppressed processes have been touched.
- Reach the sensitivity of decay asymmetry.
Progress in $\Lambda_c^+$: BESIII

- Based on 35 days’ data collecting at BESIII, 16 papers including 7 PRLs on $\Lambda_c^+$ have been published so far.
- Now new data collecting is being operated...
Progress in $\Xi_c$

- First measurement of $\Xi_c^0 \to \Xi^- \pi^+$

$$\mathcal{B}(B^- \to \bar{\Lambda}_c^- \Xi^0_c) = [9.51 \pm 2.10(\text{stat.}) \pm 0.88(\text{syst.})] \times 10^{-4}$$

$$\mathcal{B}(\Xi^0_c \to \Xi^- \pi^+) = [1.80 \pm 0.50(\text{stat.}) \pm 0.14(\text{syst.})]\%$$

Belle, PRL 122 (2019) 082001

- The branching fraction of $\Xi_c^+ \to \Xi^0 \pi^+$

$$\mathcal{B}(\bar{B}^0 \to \bar{\Lambda}_c^- \Xi^+_c) = [1.16 \pm 0.42(\text{stat.}) \pm 0.15(\text{syst.})] \times 10^{-3}$$

$$\mathcal{B}(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (2.86 \pm 1.21 \pm 0.38) \times 10^{-2}$$

Belle, 1904.12093

$$\Gamma(\Xi^+_c \to \Xi^0 \pi^+)/\Gamma(\Xi^+_c \to \Xi^- \pi^+ \pi^+) = (0.55 \pm 0.13 \pm 0.09)$$

CLEO, PLB373(1996)261

$$\mathcal{B}(\Xi^+_c \to \Xi^0 \pi^+) = (1.57 \pm 0.83)\%$$
Doubly charmed baryon

- First doubly charmed baryon is observed via
  \[ \Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+ \]
  LHCb, PRL 119, 112001 (2017)

- First observation of two body decay
  \[ \Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+ \]
  LHCb, PRL 121, 162002 (2018)

- Lifetime and precise mass of \( \Xi_{cc}^{++} \) have been measured.

  \[ \tau_{\Xi_{cc}^{++}} = 0.256^{+0.024}_{-0.022} \text{(stat)} \pm 0.014 \text{(syst)} \text{ ps} \]
  \[ m_{\Xi_{cc}^{++}} = 3621.55 \pm 0.23 \pm 0.30 \text{ MeV} \]
  LHCb, PRL 121, 052002 (2018)
  LHCb, 1911.08594 [hep-ex]
Recent study in theory

- **SU(3) approach**
  
  
  
  
  
  
  
  
  
  
  
  - H.J. Zhao, Yan-Li Wang, Y.-K. Hsiao, Y. Yu, “A diagrammatic analysis of two-body charmed baryon decays with flavor symmetry”, 1811.07265
Recent study in theory

- Calculation concerning factorizable contribution:
  - a series of work by C.D. Lu, W. Wang, F.S. Yu, Z.X. Zhao...

- Rescattering approach

- Dynamic calculation based on pole model
Theoretical working frame
New platform                   Traditional observables

(a) The angular distribution of the decay pion from a completely polarized hyperon at rest. It has been pointed out before\(^1\) that the distribution is proportional to

\[
[1 + \alpha \cos \chi] \, d\Omega,
\]

(b) The longitudinal polarization of the nucleon emitted in the decay of unpolarized hyperons at rest.

Lee-Yang, 1957
Topological diagram approach

\[ M(B_i \rightarrow B_f P) = i \bar{u}_f (A - B \gamma_5) u_i \]

\[ A = A^{\text{fac}} + A^{\text{nf}} \]

\[ B = B^{\text{fac}} + B^{\text{nf}} \]

Factorizable contribution

- Naïve factorization

\[ M = \langle PB_c | \mathcal{H}_{\text{eff}} | B_{cc} \rangle = \begin{cases} \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_1 \langle P | (\bar{u}d) | 0 \rangle \langle B_c | (\bar{s}c) | B_{cc} \rangle, & P = \pi^+, \\ \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* a_2 \langle P | (\bar{s}d) | 0 \rangle \langle B_c | (\bar{u}c) | B_{cc} \rangle, & P = \bar{K}^0, \end{cases} \]

\[ A^{\text{fac}} = \frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} - m_{B_c}) f_1 (q^2), \]

\[ B^{\text{fac}} = -\frac{G_F}{\sqrt{2}} a_{1,2} V_{ud}^* V_{cs} f_P (m_{B_{cc}} + m_{B_c}) g_1 (q^2). \]
Factorizable contribution

- Wilson coefficient

\[ a_1 = c_1 + \frac{c_2}{N_c} , \quad a_2 = c_2 + \frac{c_1}{N_c} \]

- Baryon transition form factor
  - light-front quark model; light-cone sum rules; non-relativistic quark model
  - MIT bag model

\[
f_1^{B_f B_i} (q_{\text{max}}^2) = \langle B_f \uparrow | b_{q_1}^{\dagger} b_{q_2} | B_i \uparrow \rangle \int d^3 r (u_{q_1} (r) u_{q_2} (r) + v_{q_1} (r) v_{q_2} (r)) ,
\]

\[
g_1^{B_f B_i} (q_{\text{max}}^2) = \langle B_f \uparrow | b_{q_1}^{\dagger} b_{q_2} \sigma_z | B_i \uparrow \rangle \int d^3 r (u_{q_1} (r) u_{q_2} (r) - \frac{1}{3} v_{q_1} (r) v_{q_2} (r))
\]
Factorizable contribution: form factor

• Baryon transition form factor: comparison

<table>
<thead>
<tr>
<th>$B_{cc} \rightarrow B_c$</th>
<th>$f_1(m^2_\pi)$</th>
<th>$g_1(m^2_\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MIT</td>
<td>LFQM(I)</td>
</tr>
<tr>
<td>$\Xi^{++}_{cc} \rightarrow \Xi^+_c$</td>
<td>0.577</td>
<td>0.920</td>
</tr>
<tr>
<td>$\Xi^{++}_{cc} \rightarrow \Xi^{'+}_c$</td>
<td>0.386</td>
<td>0.541</td>
</tr>
<tr>
<td>$\Xi^+_{cc} \rightarrow \Xi^0_c$</td>
<td>0.606</td>
<td>0.920</td>
</tr>
<tr>
<td>$\Xi^+_c \rightarrow \Xi^0_c$</td>
<td>0.435</td>
<td>0.541</td>
</tr>
<tr>
<td>$\Omega^+_c \rightarrow \Omega^0_c$</td>
<td>0.505</td>
<td>0.758</td>
</tr>
</tbody>
</table>
Nonfactorizable: pole model

• **Pole model**: initial baryon decays through a pole,
  ➢ *s*-wave amplitude: excited state $\frac{1}{2}^-$ should be taken into account.
  ➢ *p*-wave amplitude: ground state $\frac{1}{2}^+$ contributes.

\[
A^{\text{pole}} = - \sum_{B^*_n(1/2^-)} \left[ \frac{g_{B_fB_n^*P} b_{n^*i}}{m_i - m_{n^*}} + \frac{b_{fn^*} g_{B_n^*B_iP}}{m_f - m_{n^*}} \right]
\]

\[
B^{\text{pole}} = \sum_{B_n} \left[ \frac{g_{B_fB_n^*P} a_{ni}}{m_i - m_n} + \frac{a_{fn} g_{B_nB_iP}}{m_f - m_n} \right],
\]

requires more effort
Current algebra: soft pseudoscalar final state

- Soft-pseudoscalar limit
- Goldberger-Treiman relation

\[
A^{\text{com}} = -\frac{\sqrt{2}}{f_{Pa}} \langle B_f | [Q^a, H^{PV}_{\text{eff}}] | B_i \rangle = \frac{\sqrt{2}}{f_{Pa}} \langle B_f | [Q^a, H^{PC}_{\text{eff}}] | B_i \rangle
\]

\[
B^{ca} = \frac{\sqrt{2}}{f_{Pa}} \sum_{B_n} \left[ g_{B_f B_n}^A \frac{m_f + m_n}{m_i - m_n} a_{ni} + a_{fn} \frac{m_i + m_n}{m_f - m_n} g_{B_n B_i}^A \right]
\]

limitation: only applicable in the decays to soft pseudoscalar modes
Baryon matrix elements & strong coupling

• The two nonperturbative quantities is model dependent.

• In MIT bag model

\[ a_{B'B} \equiv \langle B' | \mathcal{H}_{\text{eff}}^{PC} | B \rangle = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud}^* c_{-} \langle B' | O_- | B \rangle, \]

\[ \langle \Xi_c^+ | O_- | \Xi_{cc}^+ \rangle = 4\sqrt{6} X_2(4\pi), \quad \langle \Xi_c'{}^+ | O_- | \Xi_{cc}^+ \rangle = -\frac{4\sqrt{2}}{3} X_1(4\pi), \]

\[ g_{B'B}^{A(P)} = \langle B' \uparrow | b_{q_1}^+ b_{q_2} \sigma_z | B \uparrow \rangle \int d^3r \left( u_{q_1} u_{q_2} - \frac{1}{3} v_{q_1} v_{q_2} \right) \]

\[ g_{\Xi_{cc}^+ \Xi_{cc}^+}^{A(\pi^+)} = -\frac{1}{3} (4\pi Z_1), \quad g_{\Xi_{cc}^+ \Xi_{cc}^+}^{A(\pi^0)} = \frac{1}{6} (4\pi Z_1), \quad g_{\Xi_{cc}^+ \Xi_{cc}^+}^{A(\eta_s)} = -\frac{1}{6\sqrt{3}} (4\pi Z_1) \]
Results of doubly charmed baryon and discussion
Predictions to Cabibbo-allowed process

<table>
<thead>
<tr>
<th>Channel</th>
<th>$A^{\text{fac}}$</th>
<th>$A^{\text{com}}$</th>
<th>$A^{\text{tot}}$</th>
<th>$B^{\text{fac}}$</th>
<th>$B^{\text{ca}}$</th>
<th>$B^{\text{tot}}$</th>
<th>$B_{\text{theo}}$</th>
<th>$\alpha_{\text{theo}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi^{++} \to \Xi^{+}_c\pi^+$</td>
<td>7.40</td>
<td>-10.79</td>
<td>-3.38</td>
<td>-15.06</td>
<td>18.91</td>
<td>3.85</td>
<td>0.69</td>
<td>-0.41</td>
</tr>
<tr>
<td>$\Xi^{++}_c \to \Xi^{+}_c\pi^+$</td>
<td>4.49</td>
<td>-0.04</td>
<td>4.45</td>
<td>-48.50</td>
<td>0.06</td>
<td>-48.44</td>
<td>4.65</td>
<td>-0.84</td>
</tr>
<tr>
<td>$\Xi^{++} \to \Sigma^{++}_c\bar{K}^0$</td>
<td>-2.67</td>
<td>0</td>
<td>-2.67</td>
<td>25.11</td>
<td>0</td>
<td>25.11</td>
<td>1.36</td>
<td>-0.89</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{0}_c\pi^+$</td>
<td>8.52</td>
<td>10.79</td>
<td>19.31</td>
<td>-16.46</td>
<td>-0.08</td>
<td>-16.54</td>
<td>3.84</td>
<td>-0.31</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{0}_c\pi^+$</td>
<td>5.05</td>
<td>0.04</td>
<td>5.09</td>
<td>-52.31</td>
<td>-17.63</td>
<td>-69.94</td>
<td>1.55</td>
<td>-0.73</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{+}_c\pi^0$</td>
<td>0</td>
<td>15.26</td>
<td>15.26</td>
<td>0</td>
<td>-10.49</td>
<td>-10.49</td>
<td>2.38</td>
<td>-0.25</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{+}_c\eta$</td>
<td>0</td>
<td>0.06</td>
<td>0.06</td>
<td>0</td>
<td>-24.97</td>
<td>-24.97</td>
<td>0.17</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{+}_c\eta$</td>
<td>0</td>
<td>21.75</td>
<td>21.75</td>
<td>0</td>
<td>4.86</td>
<td>4.86</td>
<td>4.18</td>
<td>0.07</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Xi^{+}_c\pi^0$</td>
<td>0</td>
<td>0.09</td>
<td>0.09</td>
<td>0</td>
<td>-17.87</td>
<td>-17.87</td>
<td>0.05</td>
<td>-0.07</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Sigma^{++}_c\pi^-$</td>
<td>0</td>
<td>0.07</td>
<td>0.07</td>
<td>0</td>
<td>22.14</td>
<td>22.14</td>
<td>0.13</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Lambda^{+}_c\bar{K}^0$</td>
<td>-3.37</td>
<td>8.90</td>
<td>5.53</td>
<td>5.62</td>
<td>-0.07</td>
<td>5.55</td>
<td>0.31</td>
<td>0.40</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Sigma^{+}_c\bar{K}^0$</td>
<td>-2.17</td>
<td>0.04</td>
<td>-2.14</td>
<td>19.37</td>
<td>15.64</td>
<td>35.02</td>
<td>0.38</td>
<td>-0.62</td>
</tr>
<tr>
<td>$\Xi^{+}_c \to \Omega^{+}_c\bar{K}^0$</td>
<td>0</td>
<td>0.05</td>
<td>0.05</td>
<td>0</td>
<td>-22.98</td>
<td>-22.98</td>
<td>0.06</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\Omega^{+}_c \to \Omega^{0}_c\pi^+$</td>
<td>5.71</td>
<td>0</td>
<td>5.71</td>
<td>-67.48</td>
<td>0</td>
<td>-67.48</td>
<td>3.96</td>
<td>-0.83</td>
</tr>
<tr>
<td>$\Omega^{+}_c \to \Xi^{+}_c\bar{K}^0$</td>
<td>2.62</td>
<td>-8.90</td>
<td>-6.28</td>
<td>-5.29</td>
<td>13.40</td>
<td>8.11</td>
<td>1.15</td>
<td>-0.45</td>
</tr>
<tr>
<td>$\Omega^{+}_c \to \Xi^{+}_c\bar{K}^0$</td>
<td>-1.68</td>
<td>-0.04</td>
<td>-1.72</td>
<td>17.44</td>
<td>0.06</td>
<td>17.50</td>
<td>0.29</td>
<td>-0.88</td>
</tr>
</tbody>
</table>

- **Lifetime**
  \[ \tau(\Xi^{+}_c) = 0.45 \times 10^{-13} \text{s} \]
  \[ \tau(\Omega^{+}_c) \sim (0.75 \sim 1.80) \times 10^{-13} \text{s} \]

- **Essential role of nonfactorizable contribution**

H.-Y. Cheng, Y.L. Shi,
PRD98(2018) 113005
$\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+}\pi^{+}$: the examining channel

\[
\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+}\pi^{+}) \times \mathcal{B}(\Xi_{c}^{+} \rightarrow pK^{-}\pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_{c}^{+}K^{-}\pi^{+}\pi^{+}) \times \mathcal{B}(\Lambda_{c}^{+} \rightarrow pK^{-}\pi^{+})} = 0.035 \pm 0.009(\text{stat.}) \pm 0.003(\text{syst.})
\]

LHCb, PRL 121 (2018) 162002

\[
\mathcal{B}(\Lambda_{c}^{+} \rightarrow pK^{-}\pi^{+}) = (6.28 \pm 0.32)\% \\
\mathcal{B}(\Xi_{c}^{+} \rightarrow pK^{-}\pi^{+}) = (0.45 \pm 0.21 \pm 0.07)\%
\]

BESIII

\[
\frac{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+}\pi^{+})}{\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_{c}^{+}K^{-}\pi^{+}\pi^{+})} = 0.49 \pm 0.27
\]

PDG2018

Belle, PRD100(2019) 031101

\[
\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Lambda_{c}^{+}K^{-}\pi^{+}\pi^{+}) \approx \frac{2}{3}\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_{c}^{++}\bar{K}^{*0}) \quad \text{assumption}
\]

T. Gutsche, et. al. PRD100(2019) 114037

\[
\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Sigma_{c}^{++}\bar{K}^{*0}) = 5.61\%
\]

\[
\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+}\pi^{+})_{\text{expt}} \approx (1.83 \pm 1.01)\% \\
\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{+}\pi^{+}) \approx 0.7\%
\]

more promising: $\mathcal{B}(\Xi_{cc}^{++} \rightarrow \Xi_{c}^{'+}\pi^{+}) = 4.65\%$
$\Xi_{cc}^+$ and $\Omega_{cc}^+$: the undiscovered ones

• The failure of searching $\Xi_{cc}^+$ through:
  $$\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^− π^+ \rightarrow p K^− π^+ K^− π^+$$


• The reason can be understood from our prediction:
  $$\Xi_{cc}^+ \rightarrow \Sigma_c^{++} K^- \rightarrow \Lambda_c^+ K^− π^+ \rightarrow p K^− π^+ K^− π^+$$
  $$B(\Xi_{cc}^+ \rightarrow \Sigma_c^{++} K^-) = 0.13\%$$

• A suggested discovery channel for $\Xi_{cc}^+$:
  $$\Xi_{cc}^+ \rightarrow \Xi_c^0 π^+ \rightarrow \Xi^- π^− π^+ π^+ \rightarrow \Lambda π^- π^+ π^+ \rightarrow p π^- π^- π^+ π^+$$
  $$B(\Xi_{cc}^+ \rightarrow \Xi_c^0 π^+)= 3.84\%$$ (large Br, this work)
  $$B(\Xi_c^0 \rightarrow \Xi^- π^+)= 6.47\%$$ (the largest channel)

(J. Zou, FX, G. Meng, H.-Y. Cheng, 1910.13626, accepted by PRD)

• A similar suggested discovery channel for $\Omega_{cc}^+$:
  $$B(\Omega_{cc}^+ \rightarrow \Omega_c^0 π^+)= 3.96\%$$ (this work)
## Comparison

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Xi^{++}<em>{cc} \rightarrow \Xi^{+}</em>{c} \pi^{+}$</td>
<td>0.69</td>
<td>6.64 (N)</td>
<td>0.70</td>
<td>6.18</td>
<td>7.01</td>
<td>3.48 ± 0.46</td>
<td>3.1 ± 0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.19 (H)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi^{++}<em>{cc} \rightarrow \Xi^{'}</em>{c} \pi^{+}$</td>
<td>4.65</td>
<td>5.39 (N)</td>
<td>3.03</td>
<td>4.33</td>
<td>5.85</td>
<td>1.96 ± 0.24</td>
<td>0.93 ± 0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.34 (H)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Xi^{++}<em>{cc} \rightarrow \Sigma^{++}</em>{c} \bar{K}^{0}$</td>
<td>1.36</td>
<td>2.39 (N)</td>
<td>1.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.69 (H)</td>
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</tbody>
</table>

- Our calculation agrees with Gutsche et al.
- For the mode of $\Xi^{++}_{cc} \rightarrow \Xi^{+}_{c} \pi^{+}$, Dhir-Shama’s result disagrees with ours due to the sign of baryon transition form factor.
- All the nonperturbative quantities are calculated in a consistent model, which guarantees the relative signs at least.
Summary

• All the CF weak decays of doubly charmed baryon has been calculated.
• Receiving large destructive contribution, $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ is predicted to be 0.7%, which is consistent with LHCb experiment in the lower end.
• A discovery mode for $\Xi_{cc}^+$ is suggested from $\Xi_{cc}^+ \rightarrow \Xi_c^0 \pi^+$, due to its large constructive interference in S-wave amplitude.
• The discovery mode for $\Omega_{cc}^+$ is also suggested from $\Omega_{cc}^+ \rightarrow \Omega_c^0 \pi^+$, with large pure factorization contribution.