Neutrino and dark matter in a modular symmetry

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1 Introduction

There are many flavor puzzles ; quarks and leptons masses and mixings, muon g-2, b->sμμ*, b->cτν*, photon anomalies (DM), etc. => Flavor dependent symmetries e.g..U(1)μ-τ provides good suggestions to resolve these anomalies.

- **Current situations for quarks and leptons masses and mixings**

The CKM mixing angles and CP violating phase of quarks have been precisely measured.

Next task is precisely to measure the neutrino oscillation observations and CP violation of lepton sector that will reach

at T2K and Nova experiments T2HK, DUNE.

What is the principle to control flavors of leptons ? => We expect a symmetry.

Abelian: Zn, U(1)μ-τ, etc. => Two-zero texture

Non-Abelian: SU(3), S3(61),
A4(>100), S4(54), etc, (since 2012).
=> Non-trivial matrix form

Why it is applied so many times?

Why A4???

Tri-bimaximal Mixing of Neutrino flavors.

$\sin^2\theta_{12} = 1/3,\ \sin^2\theta_{23} = 1/2,\ \sin^2\theta_{13} = 0,$

$$U_{\text{tri-bimaximal}} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

Harrison, Perkins, Scott (2002) proposed

Tri-bimaximal Mixing (TBM) is realized by the mass matrix

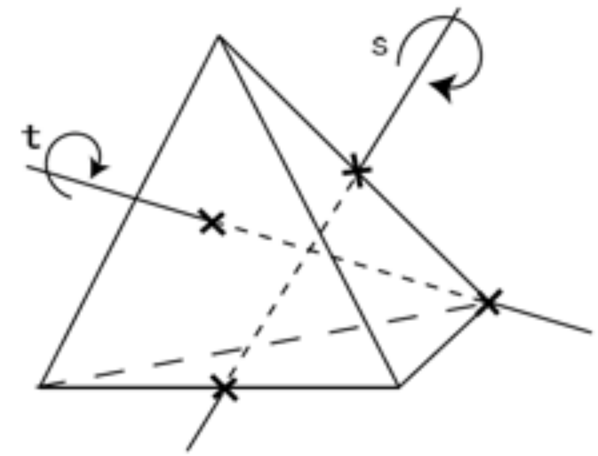
$$m_{TBM} = \frac{m_1+m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{m_2-m_1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1-m_3}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

in the diagonal basis of charged leptons.

A_4 symmetric

E. Ma, G. Rajasekaran 2001

A_4 group



Symmetry of tetrahedron

Even permutation group of four objects (1234)
12 elements (order 12) are generated by
S and T: $S^2=T^3=(ST)^3=1$: S=(14)(23), T=(123)

4 conjugacy classes

C1: 1 h=1
C3: S, T^2ST, TST^2 h=2
C4: T, ST, TS, STS h=3
C4': T^2, ST^2, T^2S, ST^2S h=3

	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0

Irreducible representations: 1, 1', 1", 3
The minimum group containing triplet that is identified as 3 flavor.

Multiplication rule of A_4 group

Irreducible representations: 1, 1', 1", 3

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}; \quad \omega = e^{2\pi i/3}$$ **for triplet**

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1 b_1 + a_2 b_3 + a_3 b_2)_{\mathbf{1}} \oplus (a_3 b_3 + a_1 b_2 + a_2 b_1)_{\mathbf{1'}}$$

$$\oplus (a_2 b_2 + a_1 b_3 + a_3 b_1)_{\mathbf{1''}}$$

$$\oplus \frac{1}{3}\begin{pmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{pmatrix}_{\mathbf{3}}$$

A_4 invariant Majorana neutrino mass term

$$(LL)_1 = L_1 L_1 + L_2 L_3 + L_3 L_2$$

3 x 3

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

A_4 invariant

In 2012, θ_{13} was measured by Daya Bay, RENO, Double Chooz, T2K, MINOS,

Tri-bimaximal mixing was ruled out !

$$\theta_{13} \simeq 9^\circ \simeq \theta_c/\sqrt{2}$$

Neutrino mixing matrix

$\nu_\alpha = (U_{PMNS})_{\alpha i} \nu_i$

$\alpha = e, \mu, \tau$ $i = 1, 2, 3$

flavor eigenstates mass eigenstates

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

c_{ij} and s_{ij} denote $\cos\theta_{ij}$ and $\sin\theta_{ij}$, respectively.

$m_1 < m_2 < m_3$ $m_3 < m_1 < m_2$

observable	3σ range for NH	3σ range for IH
$\Delta m^2_{\rm atm}$	$(2.399 \sim 2.593) \times 10^{-3}{\rm eV}^2$	$(-2.562 \sim -2.369) \times 10^{-3}{\rm eV}^2$
$\Delta m^2_{\rm sol}$	$(6.80 \sim 8.02) \times 10^{-5}{\rm eV}^2$	$(6.80 \sim 8.02) \times 10^{-5}{\rm eV}^2$
$\sin^2\theta_{23}$	$0.418 \sim 0.613$	$0.435 \sim 0.616$
$\sin^2\theta_{12}$	$0.272 \sim 0.346$	$0.272 \sim 0.346$
$\sin^2\theta_{13}$	$0.01981 \sim 0.02436$	$0.02006 \sim 0.02452$

NuFIT3.2 (2018) $\Delta m^2_{\rm atm} = m_3^2 - m_1^2$, $\Delta m^2_{\rm sol} = m_2^2 - m_1^2$

- **Traditional application of A_4 to realize the current neutrino oscillation data**

A_4 is broken by flavon (SU_2 singlet scalors) VEV's.

A_4 controls Yukaw couplings among leptons and flavons with special vacuum alignments.

Minimal fields contents and their assignments under A_4

		Leptons	flavons	
A_4 triplets	L	(L_e, L_μ, L_τ)	$\phi_\nu(\phi_{\nu 1}, \phi_{\nu 2}, \phi_{\nu 3})$	couples to neutrino sector
			$\phi_E(\phi_{E1}, \phi_{E2}, \phi_{E3})$	couples to charged lepton sector

A_4 singlets $e_R : \mathbf{1}$ $\mu_R : \mathbf{1}''$ $\tau_R : \mathbf{1}'$

ξ: 1 and ξ': 1'(1'') flavors are also introduced.

Charged-Lepton is given by 5 dim., while the neutrino sector is given by 6 dim..(Weinberg operator.)

After spontaneously symmetry breaking **by Flavons:**

Neutrino sector

$$m_{\nu LL} \sim y_1 \begin{pmatrix} 2\langle\phi_{\nu 1}\rangle & -\langle\phi_{\nu 3}\rangle & -\langle\phi_{\nu 2}\rangle \\ -\langle\phi_{\nu 3}\rangle & 2\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle \\ -\langle\phi_{\nu 2}\rangle & -\langle\phi_{\nu 1}\rangle & 2\langle\phi_{\nu 3}\rangle \end{pmatrix} + y_2\langle\xi\rangle \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + y3\langle\xi'\rangle \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$3_L \times 3_L \times 3_{flavon} \rightarrow 1$

$$\langle\phi_{\nu 1}\rangle = \langle\phi_{\nu 2}\rangle = \langle\phi_{\nu 3}\rangle$$

Z_2 (1,S) residual symmetry in neutrinos

Charged-lepton sector

$$m_E \sim \begin{pmatrix} y_e\langle\phi_{E1}\rangle & y_e\langle\phi_{E3}\rangle & y_e\langle\phi_{E2}\rangle \\ y_\mu\langle\phi_{E2}\rangle & y_\mu\langle\phi_{E1}\rangle & y_\mu\langle\phi_{E3}\rangle \\ y_\tau\langle\phi_{E3}\rangle & y_\tau\langle\phi_{E2}\rangle & y_\tau\langle\phi_{E1}\rangle \end{pmatrix}$$

$3_L \times 1_R (1_R', 1_R'') \times 3_{flavon} \rightarrow 1$

$$\langle\phi_{E2}\rangle = \langle\phi_{E3}\rangle = 0$$

Assumptions

Z_3 $(1, T, T^2)$ residual symmetry in charged leptons

Additional Matrix

$$M_\nu = a\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + b\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix} + d\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$$

$$a = \frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{\Lambda}, \quad b = -\frac{y^\nu_{\phi_\nu}\alpha_\nu v_u^2}{3\Lambda}, \quad c = \frac{y^\nu_{\xi}\alpha_\xi v_u^2}{\Lambda}, \quad d = \frac{y^\nu_{\xi'}\alpha_{\xi'} v_u^2}{\Lambda} \qquad a = -3b$$

$$M_\nu = V_{\text{tri-bi}}\begin{pmatrix}a+c-\frac{d}{2} & 0 & \frac{\sqrt{3}}{2}d\\ 0 & a+3b+c+d & 0\\ \frac{\sqrt{3}}{2}d & 0 & a-c+\frac{d}{2}\end{pmatrix}V^T_{\text{tri-bi}} \qquad V_{\text{tri-bi}} = \begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

Non zero θ13 can be given by the 1-3(3-1) component of the above matrix.

Unsatisfactory points for traditional flavor models

1.Here, one might think;

"Can we reduce the number of scalar?"

even though it is phenomenologically okay…

2.Vacuum alignment would not be a unique solution but one of the solutions; it is like an assumption…

Modular inspired flavor symmetry can resolve these issues!!!

2 Modular Group and its application

It is well known that the superstring theory on certain compactifications lead to non-Abelian finite groups.

Indeed, torus compactification leads to Modular symmetry, which includes S_3, A_4, S_4, A_5 as its congruence subgroup.

Advantages:

1. Additional **scalar bosons contributing to mass matrix are not needed, because Yukawa coupling has its structure originated from a modular group; More predictions without assumptions!**

2. DM stability can be assured by modular number that is required by a modular group; Additional symmetry (Z_2) is not needed!

Recent papers on modular symmetries

R.Toorop, F.Feruglio, C.Hagedorn:1112.1340,
F.Feruglio:1706.08749; J.C.Criado, F.Feruglio:1807.01125,
T.Kobayashi,N.Omoto,Y.Shimizu,K.Takagi,M.T,T.H.Tatsuishi:1808.03012,
H.O., M. Tanimoto: 1812.09677 ; T. Nomura, H. O.: 1904.03937,
H.O., M. Tanimoto :1905.13421, F. J. de Anda, S. F. King and E. Perdomo:1812.05620, P. P. Novichkov, S. T. Petcov and M. Tanimoto:1812.11289, T. Nomura and H. O.:1906.03927, G. J. Ding, S. F. King and X. G. Liu:1907.11714, H. O. and Y. Orikasa:1907.13520, T. Nomura, H. Okada and O. Popov, arXiv:1908.07457, T. Kobayashi, Y. Shimizu, K. Takagi, M. Tanimoto and T. H. Tatsuishi:1909.05139,T. Asaka, Y. Heo, T. H. Tatsuishi and T. Yoshida:1909.06520, G. J. Ding, S. F. King, X. G. Liu and J. N. Lu: 1910.03460p-ph], D. Zhang:1910.07869(Texture); (17) A_4

J.T.Penedo, S.T.Petcov, arXiv:1806.11040, P. P. Novichkov, J. T. Penedo, S. T. Petcov and A. V. Titov: 1811.04933, T. Kobayashi, Y. Shimizu, K. Takagi, M. Tanimoto and T. H. Tatsuishi:1907.09141,S. F. King and Y. L. Zhou, arXiv:1908.02770, H. O and Y. Orikasa:1908.08409, J. C. Criado,Feruglio and S.

J. D. King:1908.11867, X. Wang and S. Zhou:1910.09473; **(7)** S_4

T.Kobayashi, K.Tanaka, T.H.Tatsuishi, arXiv:1803.10391, T. Kobayashi, Y. Shimizu, K. Takagi, M. Tanimoto, T. H. Tatsuishi and H. Uchida,:1812.11072, T. Kobayashi, Y. Shimizu, K. Takagi, M. Tanimoto and T. H. Tatsuishi, arXiv:1906.10341; H.O. and Y. Orikasa, 1907.04716; (4) **S_3**

J. C. Criado, Feruglio and S. J. D. King:1908.11867, P. P. Novichkov, J. T. Penedo, S. T. Petcov and A. V. Titov:1812.02158, G. J. Ding, S. F. King and X. G. Liu:1903.12588; (3) **A5**

A. Baur, H. P. Nilles, A. Trautner and P. K. S. Vaudrevange:1901.03251, I. de Medeiros Varzielas, S. F. King and Y. L. Zhou, arXiv:1906.02208 [hep-ph]; (2) **Others**

A concrete model

A modular A4 symmetric scotogenic model
Takaaki Nomura (Korea Inst. Advanced Study, Seoul), Hiroshi Okada (APCTP, Pohang), Oleg Popov (Seoul Natl. U.). Aug 19, 2019. 12 pp.
KIAS-P19048, APCTP Pre2019 - 022
e-Print: arXiv:1908.07457 [hep-ph] | PDF

Field contents and their assignments

Fermions:

A_4 singlets L_e, e_R 1 ; L_μ, μ_R 1" ; L_τ τ_R 1'...with zero modular weight.

A_4 triplet 3 (N_{eR}, $N_{\mu R}$, $N_{\tau R}$)...with -1 modular weight.

Bosons:

A_4 singlets H(0)... Higgs boson, η*(-3) ...inert boson

all the bosons are trivial singlets with different modular weights!

	Couplings		
	$Y_{\mathbf{1}}^{(6)}$	$Y_{\mathbf{3}}^{(2)}$	$Y_{\mathbf{3}}^{(4)}$
A_4	**1**	**3**	**3**
$-k$	6	2	4

...Needed Yukawa couplings

(Only even modular weight can be allowed!)

$$-\mathcal{L}_{Lepton} = \sum_{\ell=e,\mu,\tau} y_\ell \bar{L}_{L_\ell} H e_{R_\ell}$$
$$+ \alpha_\nu \bar{L}_{L_e} (Y_{\mathbf{3}}^{(4)} \otimes N_R)_{\mathbf{1}} \tilde{\eta} + \beta_\nu \bar{L}_{L_\mu} (Y_{\mathbf{3}}^{(4)} \otimes N_R)_{\mathbf{1}''} \tilde{\eta} + \gamma_\nu \bar{L}_{L_\tau} (Y_{\mathbf{3}}^{(4)} \otimes N_R)_{\mathbf{1}'} \tilde{\eta}$$
$$+ M_1 (Y_{\mathbf{3}}^{(2)} \otimes \bar{N}_R^C \otimes N_R) + \text{h.c.},$$

Sum of weights vanish.

$Y_{\mathbf{1}}^{(6)} (\eta^\dagger H)^2$. **...Non-trivial quartic term to get nonzero neutrino mass at one-loop level**

$(\eta^\dagger H)$

$\bar{L}_{L_a} \tilde{H} N_{R_b}$

→ These terms are forbidden by modular weight!

↓

Leading to assure the stability of DM!

$$Y_{\mathbf{3}}^{(2)} = \begin{aligned} y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\ y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\ y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right). \end{aligned}$$

$$Y_{\mathbf{3}}^{(4)} = \begin{bmatrix} y_1^2 - y_2y_3 \\ y_3^2 - y_1y_2 \\ y_2^2 - y_1y_3 \end{bmatrix}. \qquad Y_{\mathbf{1}}^{(6)} = y_1^3 + y_2^3 + y_3^3 - 3y_1y_2y_3,$$

η: Dedekind eta-function

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad q = e^{2\pi i\tau}$$

$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau), \qquad \eta(\tau+1) = e^{i\pi/12}\eta(\tau)$$

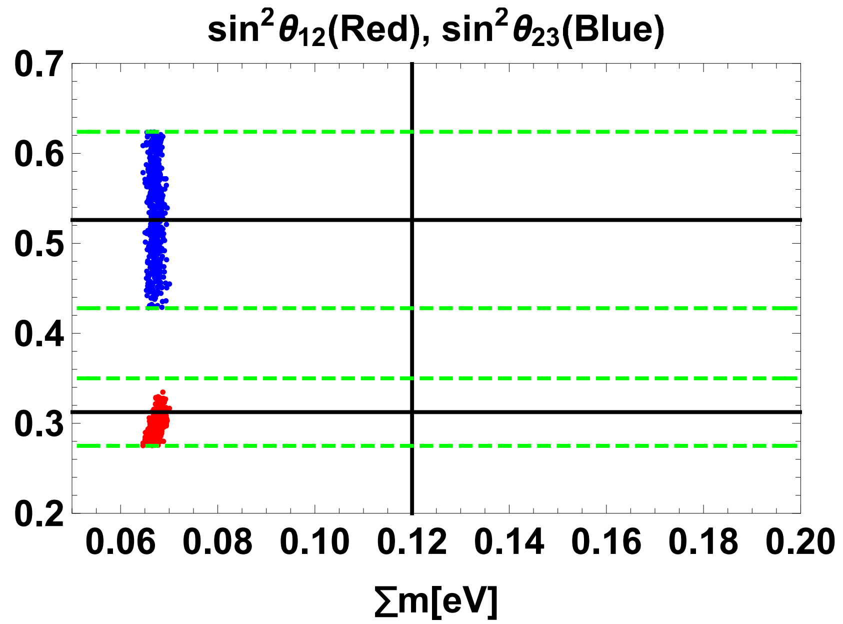
$$y_\eta = \begin{bmatrix} \alpha_\nu & 0 & 0 \\ 0 & \beta_\nu & 0 \\ 0 & 0 & \gamma_\nu \end{bmatrix} \begin{bmatrix} y'_1 & y'_3 & y'_2 \\ y'_3 & y'_2 & y'_1 \\ y'_2 & y'_1 & y'_3 \end{bmatrix}, \qquad M_N = \frac{M_1}{3} \begin{bmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_{3,1} & 2y_3 \end{bmatrix}.$$

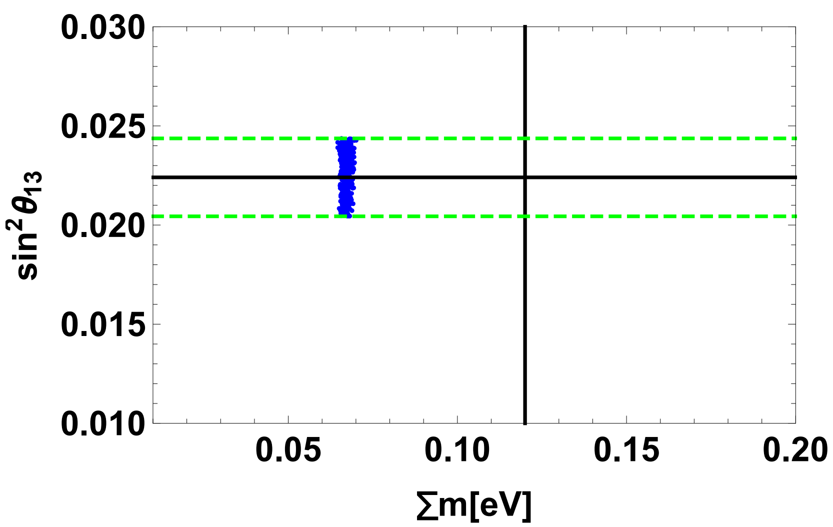
m_e, m_μ, m_τ fix ylе, ylμ, ylτ.

$\Delta m^2_{sol} / \Delta m^2_{atm}$ and θ_{23}, θ_{12}, θ_{13} fix αν, βν, γν and τ .

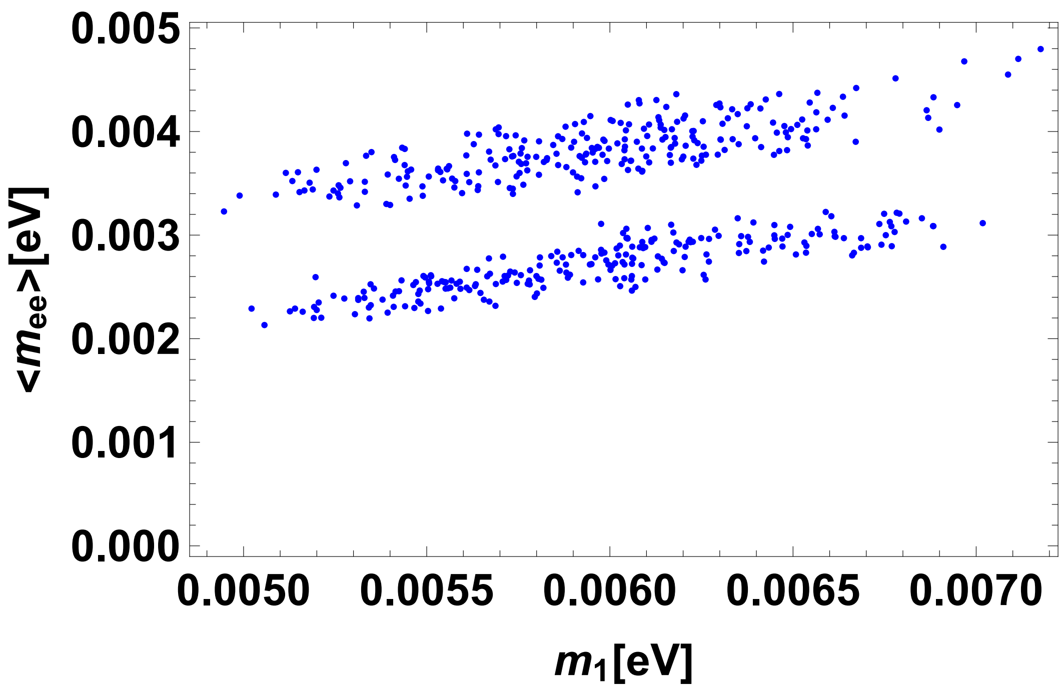
Free parameters for neturino: αν, βν, γν and τ

We consider the case of Normal neutrino mass hierarchy $m_1 < m_2 < m_3$



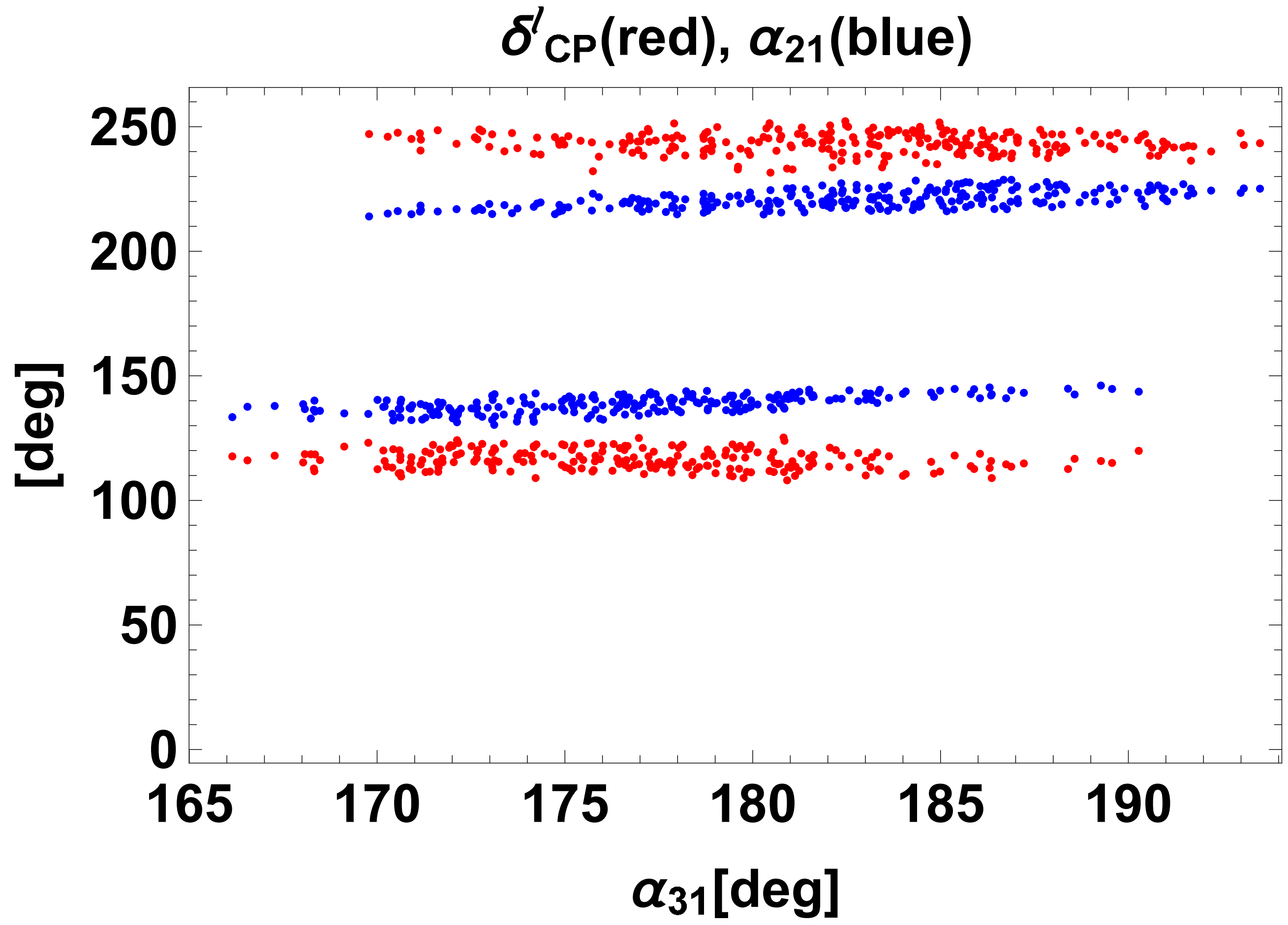


$\sum m$ = [0.065-0.070] eV <0.12 eV



$0.0049 \lesssim m_1 \lesssim 0.0072$ eV and $0.002 \lesssim \langle m_{ee} \rangle \lesssim 0.005$ eV

Predicted Phases



Predicted δcp…[100-120,230-250] deg,

Predicted α21…[130-150,210-230] deg,

Predicted α31…[165-190] deg.

3 Summary

- Modular inspired non-Abelian discrete flavor symmetries provides several predictions without introducing so many Higgs fields.

- Mass matrices of A_4 model are determined essentially by the modular parameter τ.

- Predictions are sharp and testable in the future without any assumptions unlikely to traditional model buildings.

- Further applications are expected to explain any flavor dependent anomalies; muon related anomalies, dark matter, etc…

More phenomenological discussions can be possible or needed !!!