Lepton flavor mixing from $\Delta(6n^2)$ series and generalized CP

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In collaboration with Stephen F. King, Thomas Neder

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Outline:

- Motivation for generalized CP symmetry
- Generalized CP transformation consistent with $\Delta(6n^2)$
- Lepton flavor mixing from $\Delta(6n^2)$ and generalized CP
- Summary

μτ reflection symmetry: a simple CP symmetry example

μτ exchange

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e \\ \nu_\tau \\ \nu_\mu \end{pmatrix}$$

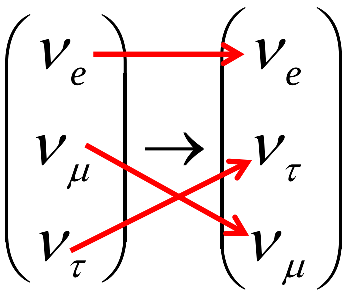
μτ reflection=μτ exchange +canonical CP

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e^c \\ \nu_\tau^c \\ \nu_\mu^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}$$

This CP transformation is **not** a unit matrix.

μτ reflection symmetry: a simple CP symmetry example

μτ exchange



μτ reflection=μτ exchange +canonical CP

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}\to\begin{pmatrix}\nu_e^c\\ \nu_\tau^c\\ \nu_\mu^c\end{pmatrix}=\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}\nu_e^c\\ \nu_\mu^c\\ \nu_\tau^c\end{pmatrix}$$

This CP transformation is **not** a unit matrix.

If the neutrino mass matrix is **invariant** under the μτ reflection,

$$P_{23}^T m_\nu P_{23} = m_\nu^*$$

neutrino mass terms: $\frac{1}{2}\nu_a^T C^{-1} m_{\nu,ab}\nu_b$

It leads to

$$\theta_{23}=\frac{\pi}{4},\quad \delta_{CP}=\pm\frac{\pi}{2}$$

[P. Harrison, W.Scott, Phys.Lett. B547(2002) 219;
W. Grimus, L. Lavoura,Phys.Lett. B579 (2004)113]

Our motivation: combining flavor symmetry and CP symmetry to predict both mixing angles and CP phases.

Consistency conditions

- Definition of generalized CP(GCP) transformation

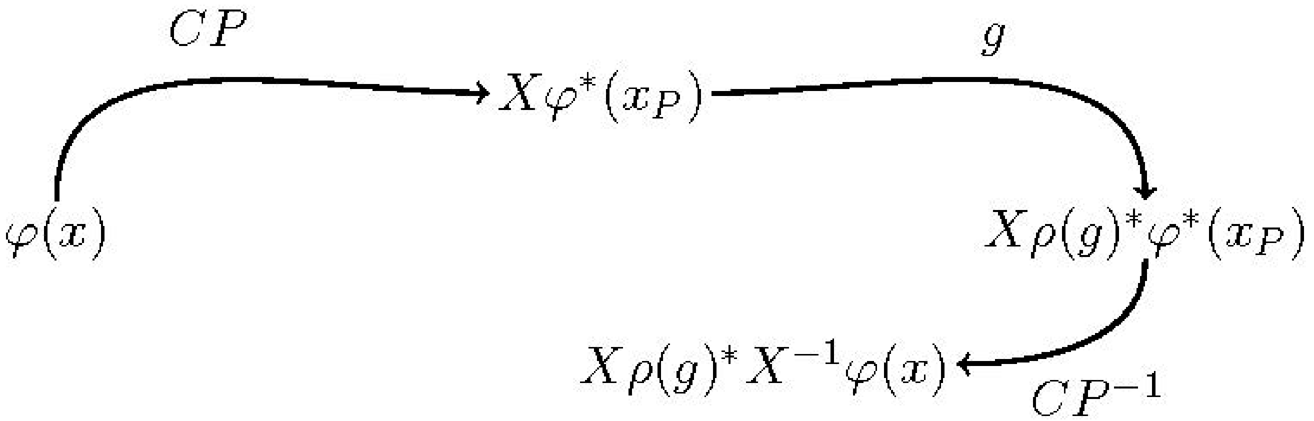
$$\varphi_i(x) \xmapsto{CP} X_{ij}\varphi_j^*(x_P)$$

[W. Grimus et al.,J. Phys. A 20 (1987) L807;
G.Branco et al., Rev .Mod .Phys. 84,515(2012).]

X is a unitary matrix.

- Combining GCP with flavor symmetry

"closure" relations have to hold!



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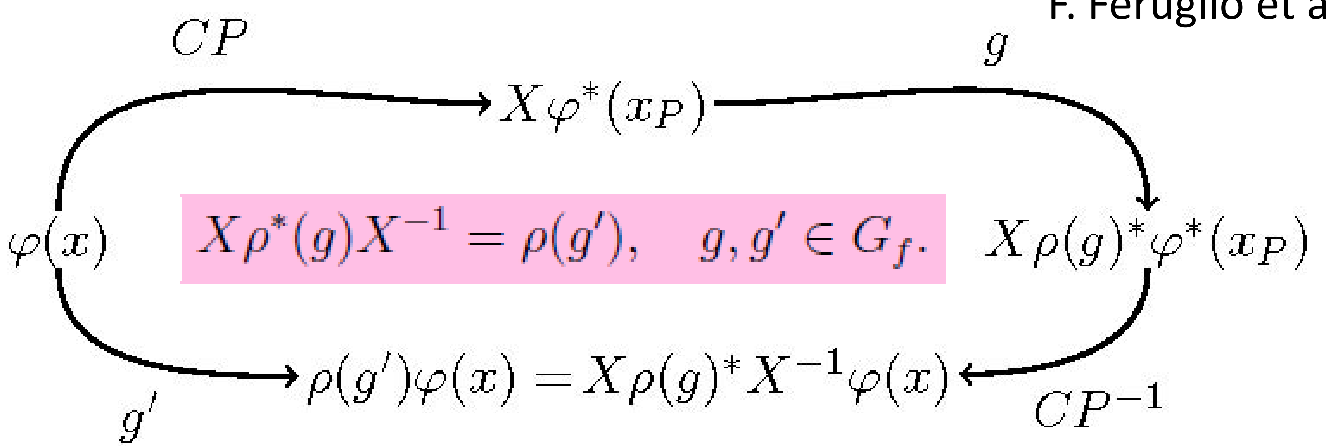

$$X\rho^*(g)X^{-1} = \rho(g'), \quad g, g' \in G_f.$$

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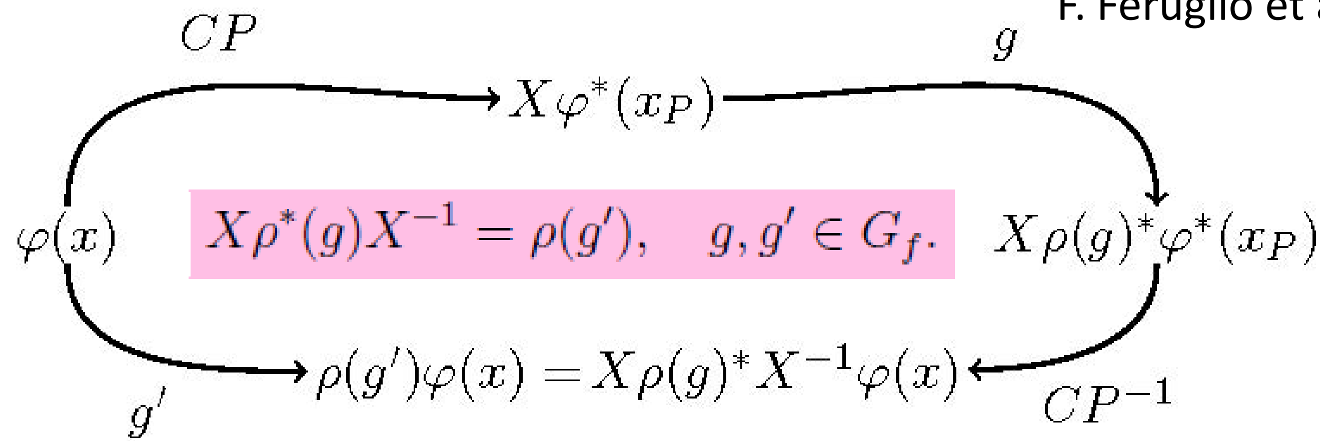

●Further requirement: **physical** GCP transformations have to map each representation into the complex conjugate. Mathematically, mapping induced by X should be class-inverting automorphism $g' \sim g^{-1}$.

[M. Ratz et al., Nucl.Phys.B 883, 267(2014).]

Group theory of Δ(6n²)

- $\Delta(6n^2)$ is a non-abelian finite subgroup of SU(3), it is isomorphic to $(Z_n \times Z_n) \rtimes S_3$. Its four generators satisfy:

$$a^3 = b^2 = (ab)^2 = 1,$$

$$c^n = d^n = 1, \qquad cd = dc,$$

$$aca^{-1} = c^{-1}d^{-1}, \quad ada^{-1} = c, \quad bcb^{-1} = d^{-1}, \quad bdb^{-1} = c^{-1}$$

- Familiar examples: $\Delta(6\times 1^2) \cong S_3, \quad \Delta(6\times 2^2) \cong S_4$

- Irreducible representations : 1-dim, 2-dim, 3-dim, 6-dim.

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix} \qquad \eta = e^{2\pi i/n}$$

➢Automorphism group of Δ($6n^2$)

n	G_f	GAP-Id	Out(G_f)	Num.
1	$\Delta(6) \equiv S_3$	[6,1]	Z_1	1
2	$\Delta(24) \equiv S_4$	[24,12]	Z_1	1
3	$\Delta(54)$	[54,8]	S_4	0
4	$\Delta(96)$	[96,64]	Z_2	1
5	$\Delta(150)$	[150,5]	Z_4	1
6	$\Delta(216)$	[216,95]	S_3	0
7	$\Delta(294)$	[294,7]	Z_6	1
8	$\Delta(384)$	[384,568]	K_4	1
9	$\Delta(486)$	[486,61]	$Z_3 \times S_3$	0
10	$\Delta(600)$	[600,179]	Z_4	1
11	$\Delta(726)$	[726,5]	Z_{10}	1
12	$\Delta(864)$	[864,701]	D_{12}	0
13	$\Delta(1014)$	[1014,7]	Z_{12}	1
14	$\Delta(1176)$	[1176,243]	Z_6	1
15	$\Delta(1350)$	[1350,46]	$Z_4 \times S_3$	0
16	$\Delta(1536)$	[1536,408544632]	$Z_4 \times Z_2$	1
17	$\Delta(1734)$	[1734,5]	Z_{16}	1
18	$\Delta(1944)$	[1944,849]	$Z_3 \times S_3$	0
19	$\Delta(2166)$	[2166,15]	Z_{18}	1

Num. ≡ Number of class-inverting outer automorphism.

Physical CP transformations are class-inverting outer automorphisms of the flavor symmetry group.

[M. Lindner et al., JHEP 1304, 122;
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↔ Num. ≡ Number of class-inverting outer automorphism.

The Δ (6n²) group with $n \neq 3\mathbb{Z}$ admits a **unique** class-inverting outer automorphism (for another proof: C. Hagedorn et al, arXiv:1408.7118).

The unique automorphism u is:

$$a \xmapsto{u} a^2, \quad b \xmapsto{u} b, \quad c \xmapsto{u} d, \quad d \xmapsto{u} c\,.$$

Physical CP transformations are class-inverting outer automorphisms of the flavor symmetry group.

[M. Lindner et al., JHEP 1304, 122;
M. Ratz et al., Nucl.Phys.B 883, 267(2014).]

➢Defining CP transformations compatible with $\Delta(6n^2)$

GCP transformation $X_r(u)$ for the unique automorphism u:

$$X_{\mathbf{r}}(u)\,\rho^*_{\mathbf{r}}(a)X^\dagger_{\mathbf{r}}(u)=\rho_{\mathbf{r}}(u(a))=\rho_{\mathbf{r}}(a^2)$$
$$X_{\mathbf{r}}(u)\,\rho^*_{\mathbf{r}}(b)X^\dagger_{\mathbf{r}}(u)=\rho_{\mathbf{r}}(u(b))=\rho_{\mathbf{r}}(b)$$
$$X_{\mathbf{r}}(u)\,\rho^*_{\mathbf{r}}(c)X^\dagger_{\mathbf{r}}(u)=\rho_{\mathbf{r}}(u(c))=\rho_{\mathbf{r}}(d)$$
$$X_{\mathbf{r}}(u)\,\rho^*_{\mathbf{r}}(d)X^\dagger_{\mathbf{r}}(u)=\rho_{\mathbf{r}}(u(d))=\rho_{\mathbf{r}}(c)$$

$$\Rightarrow X_{\mathbf{r}}(u)=\rho_{\mathbf{r}}(b)$$

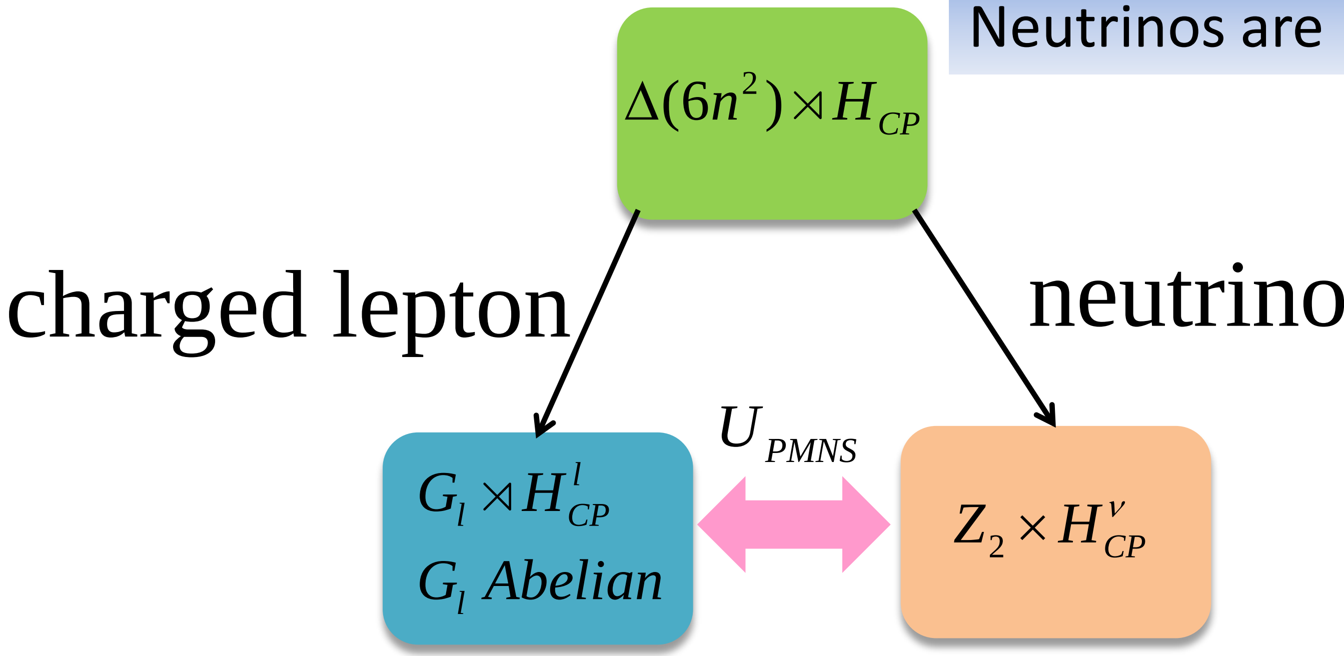
Including inner automorphism, the full GCP transformation is

$$X_{\mathbf{r}}=\rho_{\mathbf{r}}(g),\quad g\in\Delta(6n^2)$$

CP transformations and flavor symmetry transformations are of the same form in the chosen basis.

"Semi-direct" approach of flavor symmetry with GCP

Neutrinos are Majorana particles.



- The LH lepton doublets are assigned to be a $\Delta(6n^2)$ **triplet 3**;
- The lepton mixing arises from the mismatch between two remnant symmetries, and it is **independent** of how the remnant symmetries is dynamically realized(model independent).

Sketch of the Steps

Step 1: Constraints from the remnant flavor symmetries

Charged lepton

Invariant under $l \to \rho_3(g_l) l$

$$\rho_3^\dagger(g_l) m_l^\dagger m_l \rho_3(g_l) = m_l^\dagger m_l, \quad g_l \in G_l$$

$$\left[\rho_3(g_l),\ m_l^\dagger m_l\right] = 0$$

Neutrino

Invariant under $\nu \to \rho_3(g_\nu)\nu$

$$\rho_3^T(g_\nu) m_\nu \rho_3(g_\nu) = m_\nu, \quad g_\nu \in Z_2$$

$$\left[\rho_3(g_\nu),\ m_\nu^\dagger m_\nu\right] = 0$$

Mixing matrices diagonalizing m_ν, $m_l^\dagger m_l$ also diagonalize $\rho_3(g_\nu)$ and $\rho_3(g_l)$, respectively !

- The remnant flavor symmetry in the neutrino sector is relaxed from K_4 to Z_2, **only one column** can be fixed.

[D.Hernandez, A.Smirnov,Phys.Rev.D86 (2012).]

Column vector determined by remnant flavor symmetry

	$G_\nu = Z_2^{bc^x d^x}$	$G_\nu = Z_2^{c^{n/2}}$
$G_l = \langle c^s d^t \rangle$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ ✗	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ✗
$G_l = \langle bc^s d^t \rangle$	$\begin{pmatrix} 0 \\ \cos\left(\frac{s+t-2x}{2n}\pi\right) \\ \sin\left(\frac{s+t-2x}{2n}\pi\right) \end{pmatrix}$ ✗	$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ ✗
$G_l = \langle ac^s d^t \rangle$	$\sqrt{\frac{2}{3}}\begin{pmatrix} \sin\left(\frac{s-x}{n}\pi\right) \\ \cos\left(\frac{\pi}{6} - \frac{s-x}{n}\pi\right) \\ \cos\left(\frac{\pi}{6} + \frac{s-x}{n}\pi\right) \end{pmatrix}$ ✓	$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ ✓
$G_l = \langle abc^s d^t \rangle$	$\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$?	$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ ✗
$G_l = \langle a^2bc^s d^t \rangle$	$\frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$?	$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ ✗

Two cases are viable

Step 2: including remnant CP symmetry

remnant CP symmetry has to be compatible with remnant flavor symmetry

$$X_{\nu\mathbf{r}}\rho_{\mathbf{r}}^{*}(g_{\nu})X_{\nu\mathbf{r}}^{-1}=\rho_{\mathbf{r}}(g_{\nu}),\quad g_{\nu}\in Z_{2}$$

$g_\nu = bc^x d^x$	$X_{\nu\mathbf{r}} = \rho_{\mathbf{r}}(c^\gamma d^{-2x-\gamma}),\quad X_{\nu\mathbf{r}} = \rho_{\mathbf{r}}(bc^\gamma d^{-\gamma}),\quad \gamma = 0, 1, 2 \ldots n-1$
$g_\nu = c^{n/2}$	$X_{\nu\mathbf{r}} = \rho_{\mathbf{r}}(c^\gamma d^\delta),\quad X_{\nu\mathbf{r}} = \rho_{\mathbf{r}}(abc^\gamma d^\delta),\quad \gamma, \delta = 0, 1, 2 \ldots n-1$

Step 3: Constructing neutrino mass matrix and extract PMNS matrix

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Step 3: Constructing neutrino mass matrix and extract PMNS matrix

●Case I: $G_l=\langle ac^s d^t\rangle,\quad G_\nu=Z_2^{bc^x d^x},\quad X_{\nu\mathbf{r}}=\{\rho_{\mathbf{r}}(c^\gamma d^{-2x-\gamma}),\rho_{\mathbf{r}}(bc^{x+\gamma}d^{-x-\gamma})\}$

$$U_{PMNS}^{I,1st}=\frac{1}{\sqrt{3}}\begin{pmatrix}\sqrt{2}\sin\varphi_1 & e^{i\varphi_2}\cos\theta-\sqrt{2}\sin\theta\cos\varphi_1 & e^{i\varphi_2}\sin\theta+\sqrt{2}\cos\theta\cos\varphi_1\\ \sqrt{2}\cos\left(\frac{\pi}{6}-\varphi_1\right) & -e^{i\varphi_2}\cos\theta-\sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}-\varphi_1\right) & -e^{i\varphi_2}\sin\theta+\sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}-\varphi_1\right)\\ \sqrt{2}\cos\left(\frac{\pi}{6}+\varphi_1\right) & e^{i\varphi_2}\cos\theta+\sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}+\varphi_1\right) & e^{i\varphi_2}\sin\theta-\sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}+\varphi_1\right)\end{pmatrix}$$

$$\varphi_1=\frac{\pi}{2}:\quad \frac{1}{\sqrt{6}}\begin{pmatrix}2\\1\\-1\end{pmatrix}$$

3 parameters involved

$$\varphi_1=\frac{s-x}{n}\pi=0,\pm\frac{1}{n}\pi,\pm\frac{2}{n}\pi,\ldots\pm\frac{n-1}{n}\pi$$

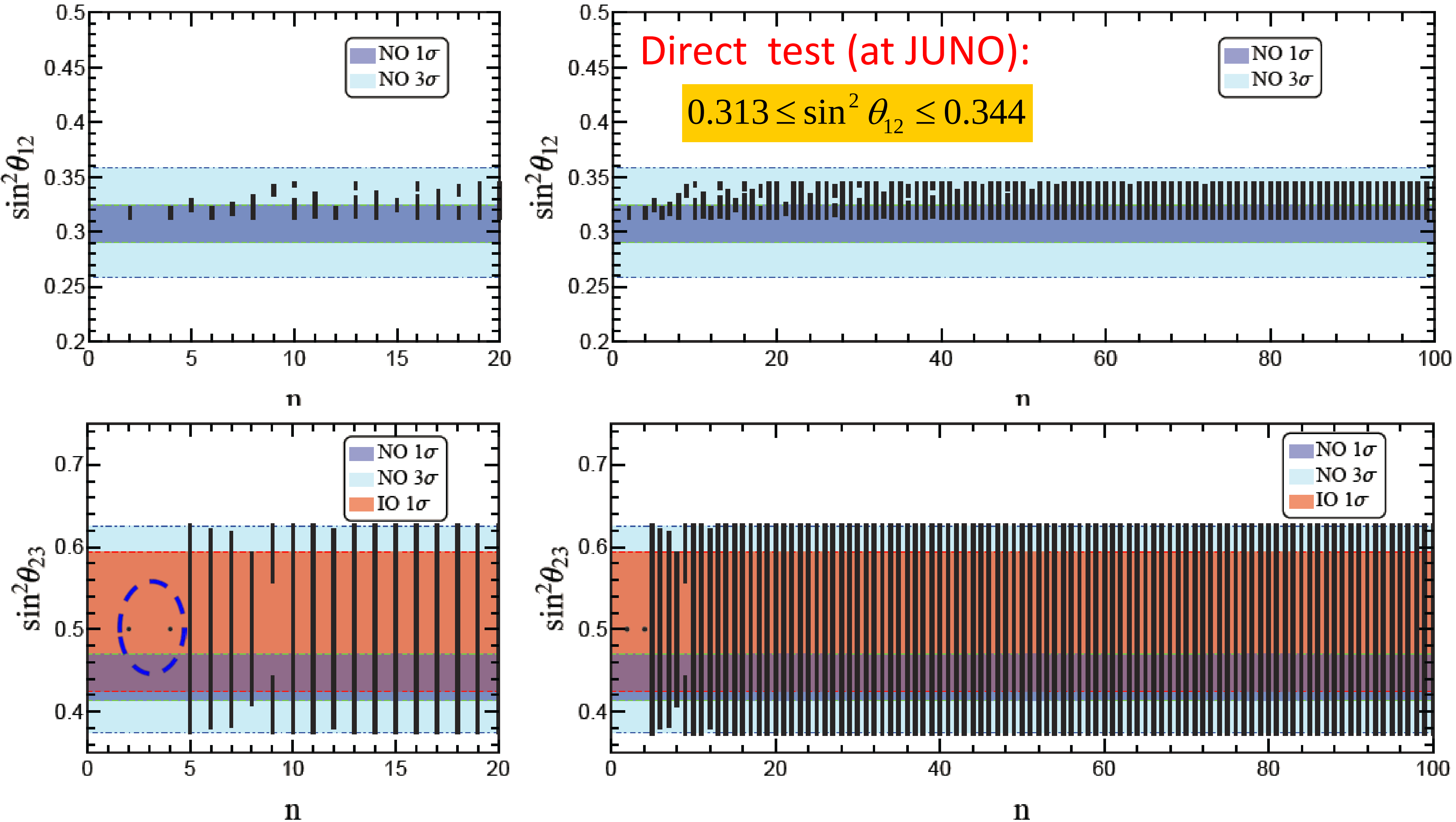
$$\varphi_2=\frac{2t-s-3(\gamma+x)}{n}\pi=0,\frac{1}{n}\pi,\frac{2}{n}\pi,\ldots\frac{2n-1}{n}\pi \bmod 2\pi$$

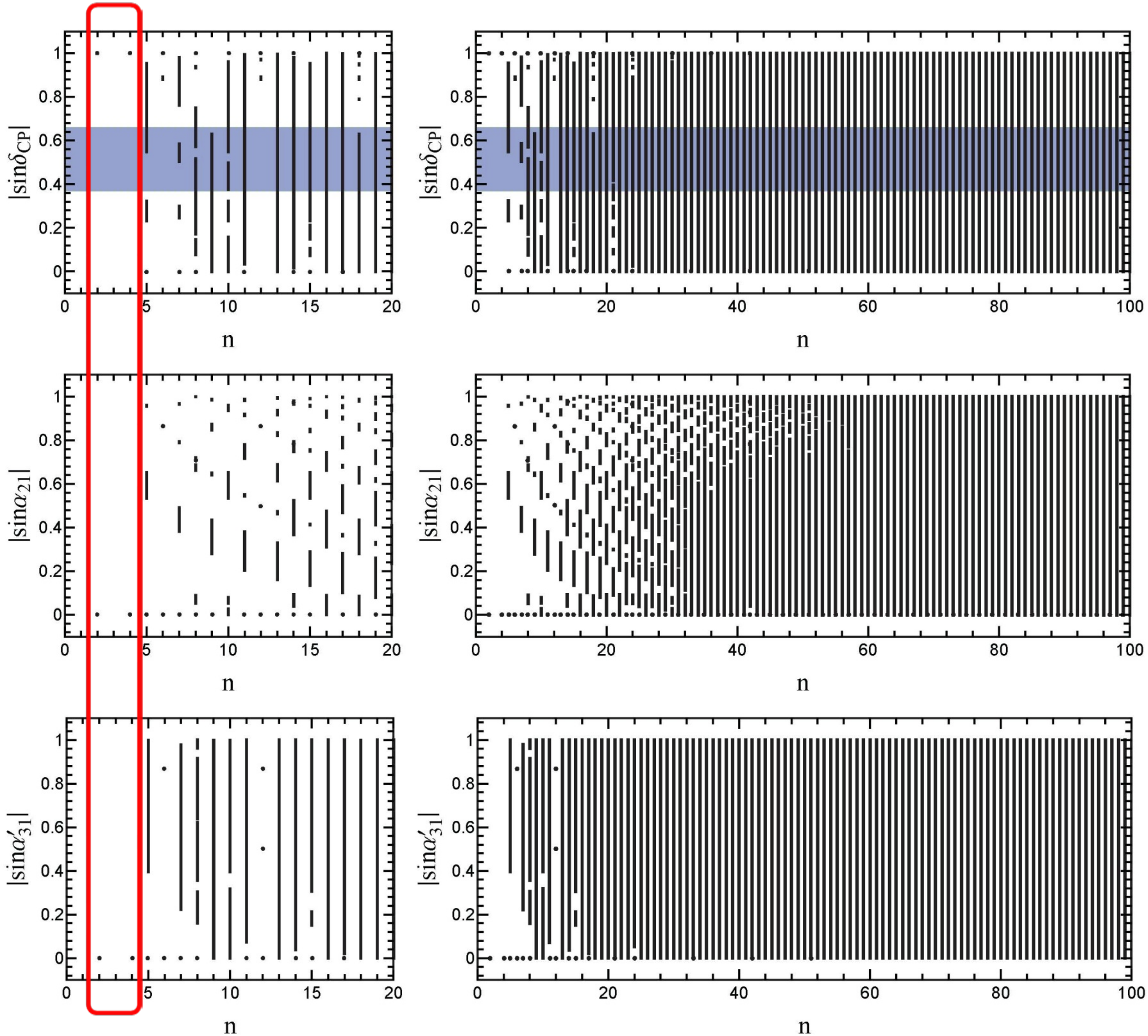
θ is a free parameter, it is fixed by the experimental data.

Correlation between θ_{12} and θ_{13}

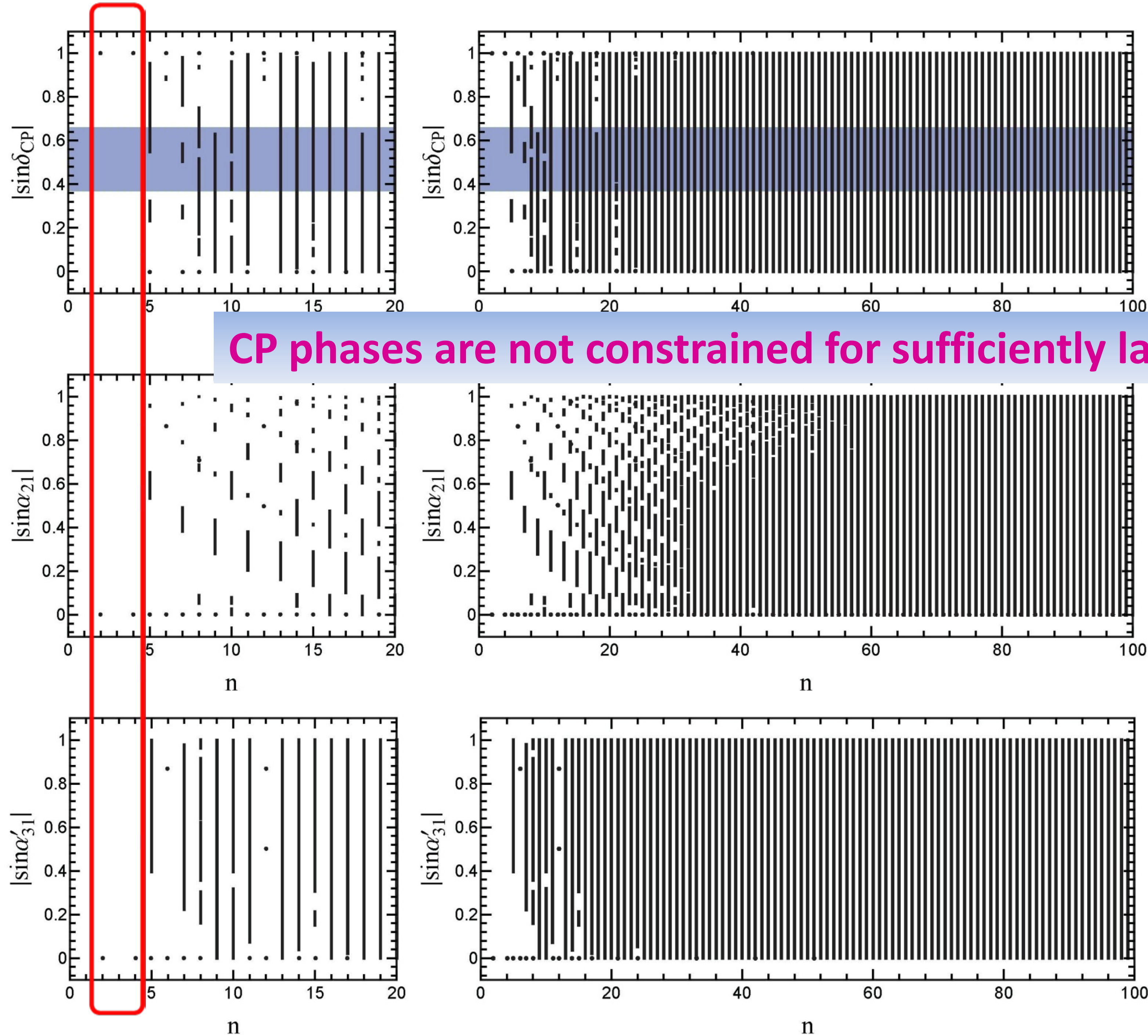
$$3\cos^2\theta_{12}\cos^2\theta_{13} = 2\sin^2\varphi_1$$

Independent of θ





|sinδ_CP|
|sinα_21|
|sinα'_31|
n
0
5
10
15
20
40
60
80
100
0.2
0.4
0.6
0.8
1



CP phases are not constrained for sufficiently large *n*.

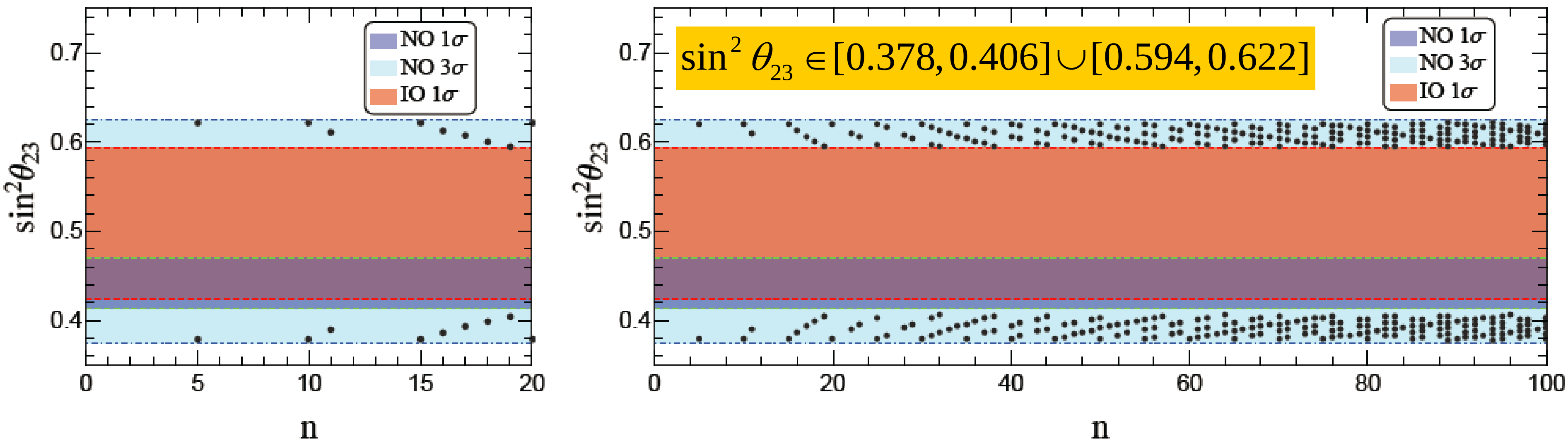
Permutation the columns

$$U_{PMNS}^{I,2nd} = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\varphi_2}\sin\theta + \sqrt{2}\cos\theta\cos\varphi_1 & e^{i\varphi_2}\cos\theta - \sqrt{2}\sin\theta\cos\varphi_1 & \sqrt{2}\sin\varphi_1 \\ -e^{i\varphi_2}\sin\theta + \sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}-\varphi_1\right) & -e^{i\varphi_2}\cos\theta - \sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}-\varphi_1\right) & \sqrt{2}\cos\left(\frac{\pi}{6}-\varphi_1\right) \\ e^{i\varphi_2}\sin\theta - \sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}+\varphi_1\right) & e^{i\varphi_2}\cos\theta + \sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}+\varphi_1\right) & \sqrt{2}\cos\left(\frac{\pi}{6}+\varphi_1\right) \end{pmatrix}$$

Both θ_{13} and θ_{23} only depend on the discrete parameter φ_1

$$\sin^2\theta_{23} = \frac{1}{2} \pm \frac{1}{2}\tan\theta_{13}\sqrt{2-\tan^2\theta_{13}} \quad \Rightarrow \quad \theta_{23} \approx \frac{\pi}{4} \pm \frac{\theta_{13}}{\sqrt{2}}$$

Direct test:



CP phases are also not constrained for very large *n*.

●Case II: $G_l = \langle a c^s d^t \rangle, \quad G_\nu = Z_2^{c^{n/2}}, \quad X_{\nu\mathrm{r}} = \rho_\mathrm{r}(c^\gamma d^\delta)$

$$U^{II}_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\varphi_4}\cos\theta - e^{i\varphi_5}\sin\theta & 1 & e^{i\varphi_4}\sin\theta + e^{i\varphi_5}\cos\theta \\ \omega e^{i\varphi_4}\cos\theta - \omega^2 e^{i\varphi_5}\sin\theta & 1 & \omega e^{i\varphi_4}\sin\theta + \omega^2 e^{i\varphi_5}\cos\theta \\ \omega^2 e^{i\varphi_4}\cos\theta - \omega e^{i\varphi_5}\sin\theta & 1 & \omega^2 e^{i\varphi_4}\sin\theta + \omega e^{i\varphi_5}\cos\theta \end{pmatrix}$$

with

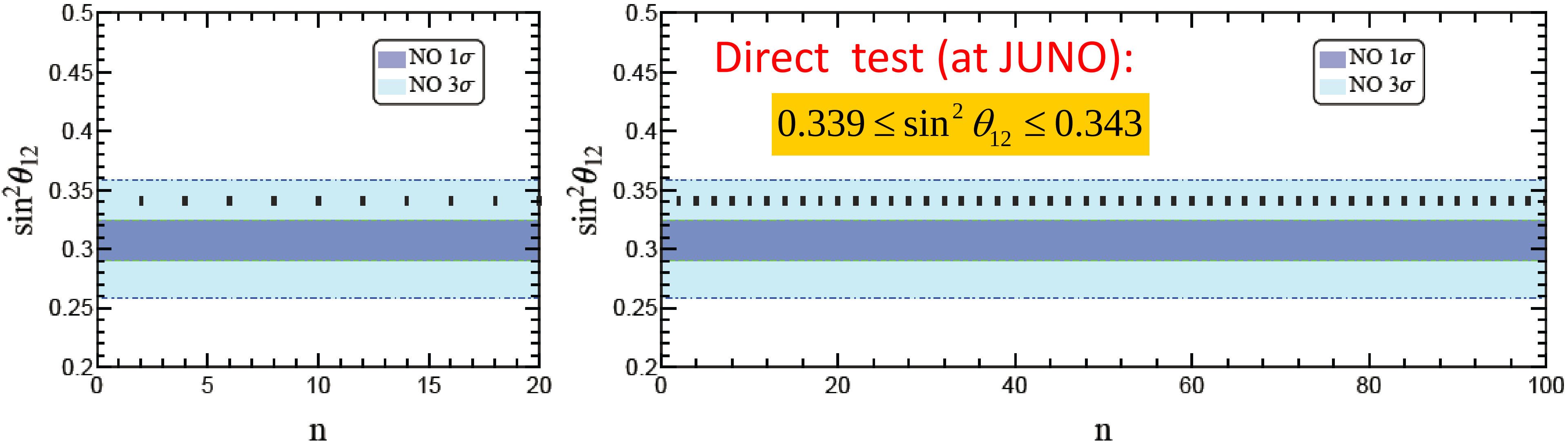
$$\varphi_4 = \frac{\gamma+\delta+2s}{n}\pi, \qquad \varphi_5 = \frac{2\delta-\gamma+2t}{n}\pi$$

$$\varphi_4, \varphi_5 \ \mathrm{mod}\ 2\pi = 0, \frac{1}{n}\pi, \frac{2}{n}\pi, \ldots, \frac{2n-1}{n}\pi$$

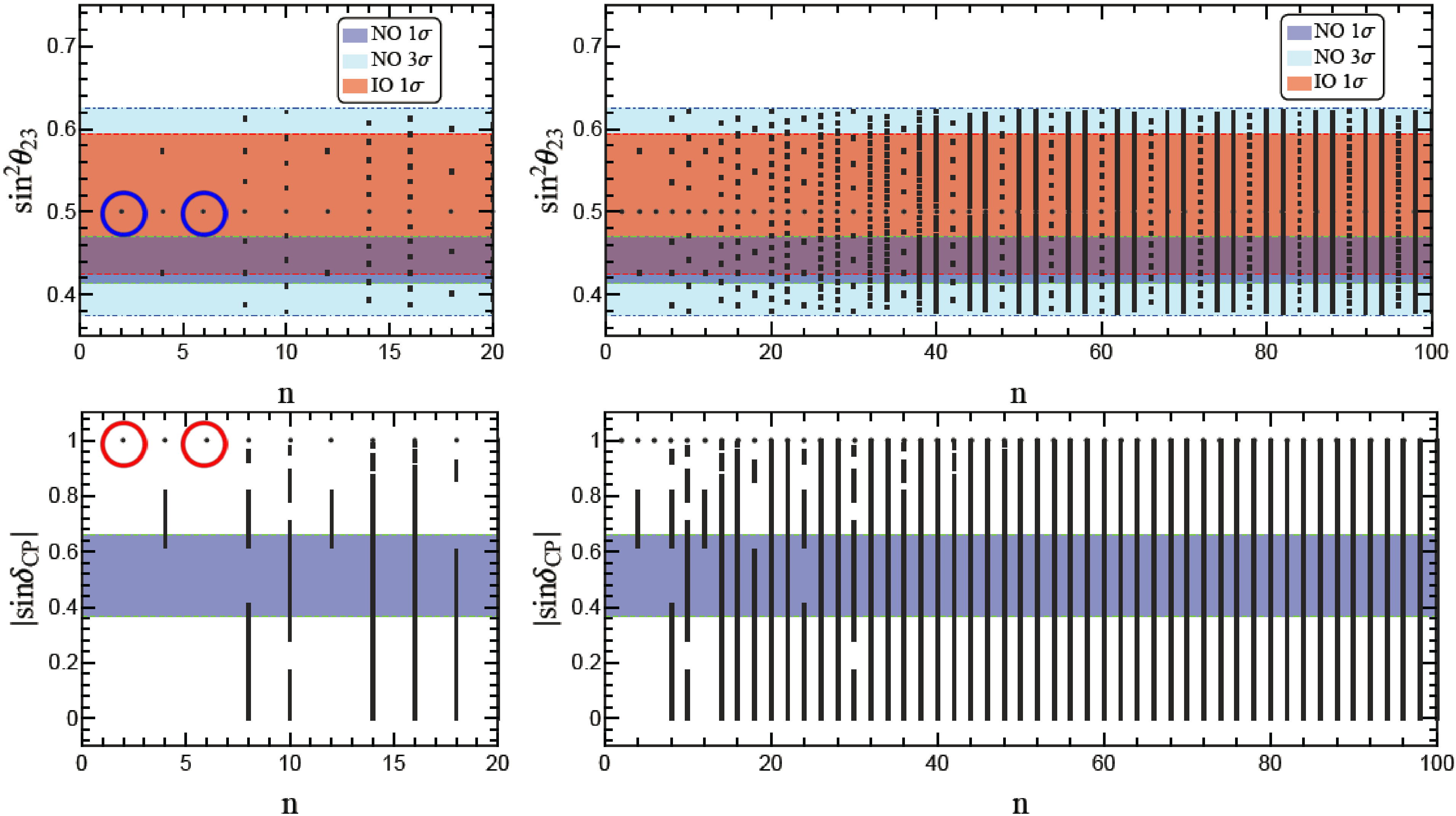
Trimaximal mixing

[X.G.He and A.Zee, Phys.Lett.B645(2007)427; Phys.Rev. D84 (2011) 053004]

Correlation between θ_{12} and θ_{13} : $\cos^2\theta_{13}\sin^2\theta_{12} = \frac{1}{3}$



$0.339 \le \sin^2\theta_{12} \le 0.343$

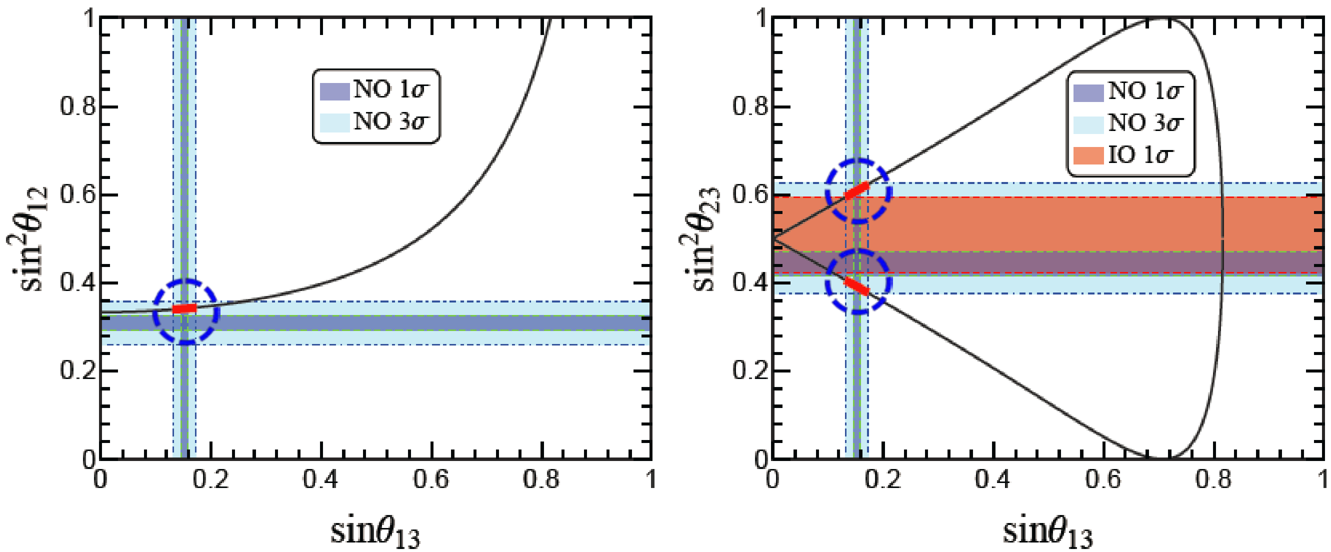

NO 1σ
NO 3σ
IO 1σ
$\sin^2\theta_{23}$
$|\sin\delta_{CP}|$
n

● **Case III:** $G_l = \langle ac^s d^t \rangle, \quad G_\nu = Z_2^{c^{n/2}}, \quad X_{\nu r} = \rho_r(abc^\gamma d^\delta)$

$$U_{PMNS}^{III} = \frac{1}{\sqrt{3}} \begin{pmatrix} -i\sqrt{2}e^{i\varphi_6}\sin\theta & 1 & \sqrt{2}e^{i\varphi_6}\cos\theta \\ i\sqrt{2}e^{i\varphi_6}\cos\left(\theta-\frac{\pi}{6}\right) & 1 & \sqrt{2}e^{i\varphi_6}\sin\left(\theta-\frac{\pi}{6}\right) \\ -i\sqrt{2}e^{i\varphi_6}\cos\left(\theta+\frac{\pi}{6}\right) & 1 & -\sqrt{2}e^{i\varphi_6}\sin\left(\theta+\frac{\pi}{6}\right) \end{pmatrix}$$

Mixing angles**(only depend on θ)**:

$$\sin^2\theta_{13} = \frac{1}{3}[1+\cos(2\theta)], \quad \sin^2\theta_{12} = \frac{1}{2-\cos(2\theta)}, \quad \sin^2\theta_{23} = \frac{1-\sin(2\theta+\pi/6)}{2-\cos(2\theta)}$$



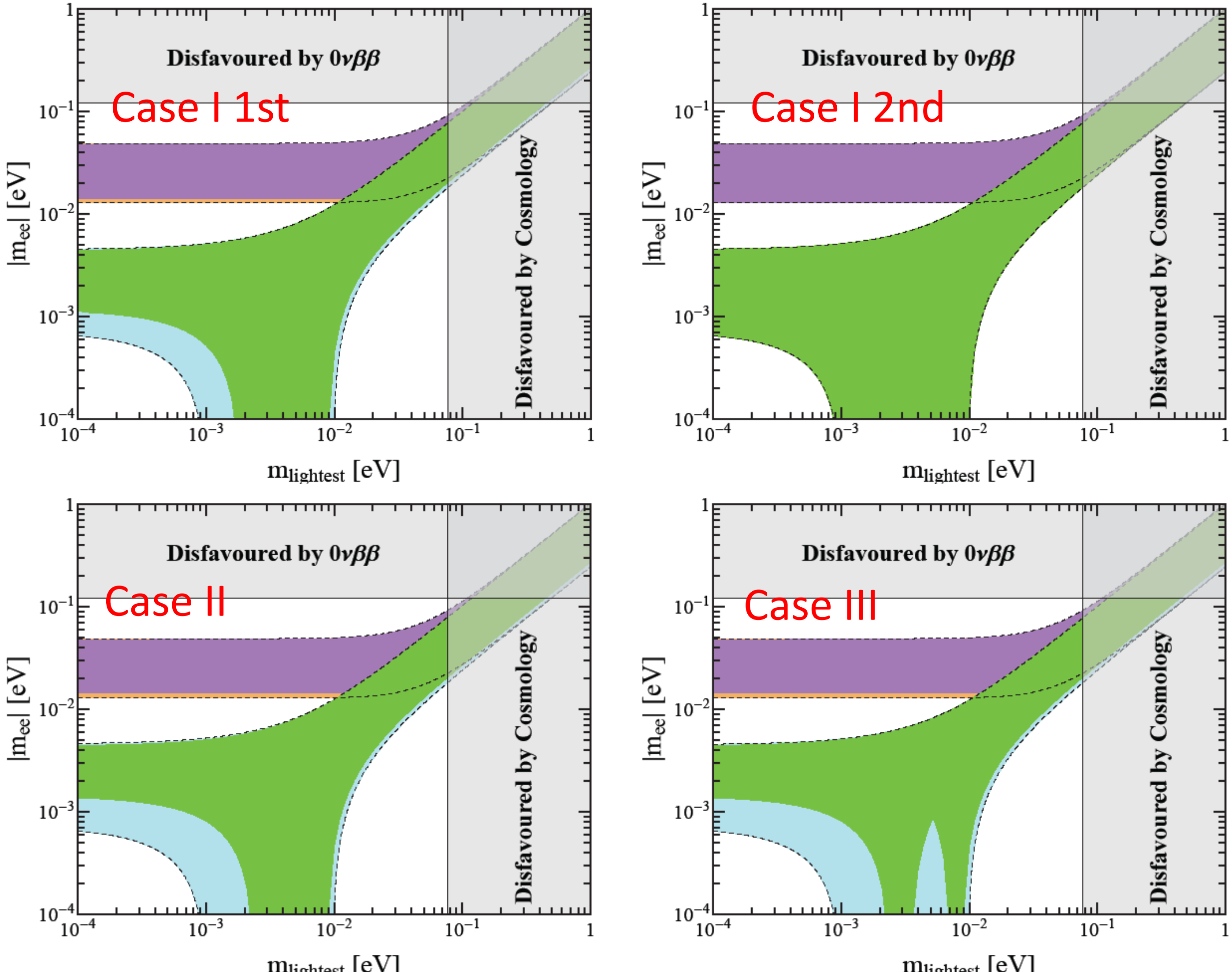
CP phases:

$$\tan\delta_{CP} = \tan\alpha_{31} = 0, \qquad |\tan\alpha_{21}| = |\tan(2\varphi_6)|$$

① Both δ_{CP} and α_{31} are conserved; ② $\alpha_{21} = 0, \frac{1}{n}\pi, \ldots, \frac{2n-1}{n}\pi$

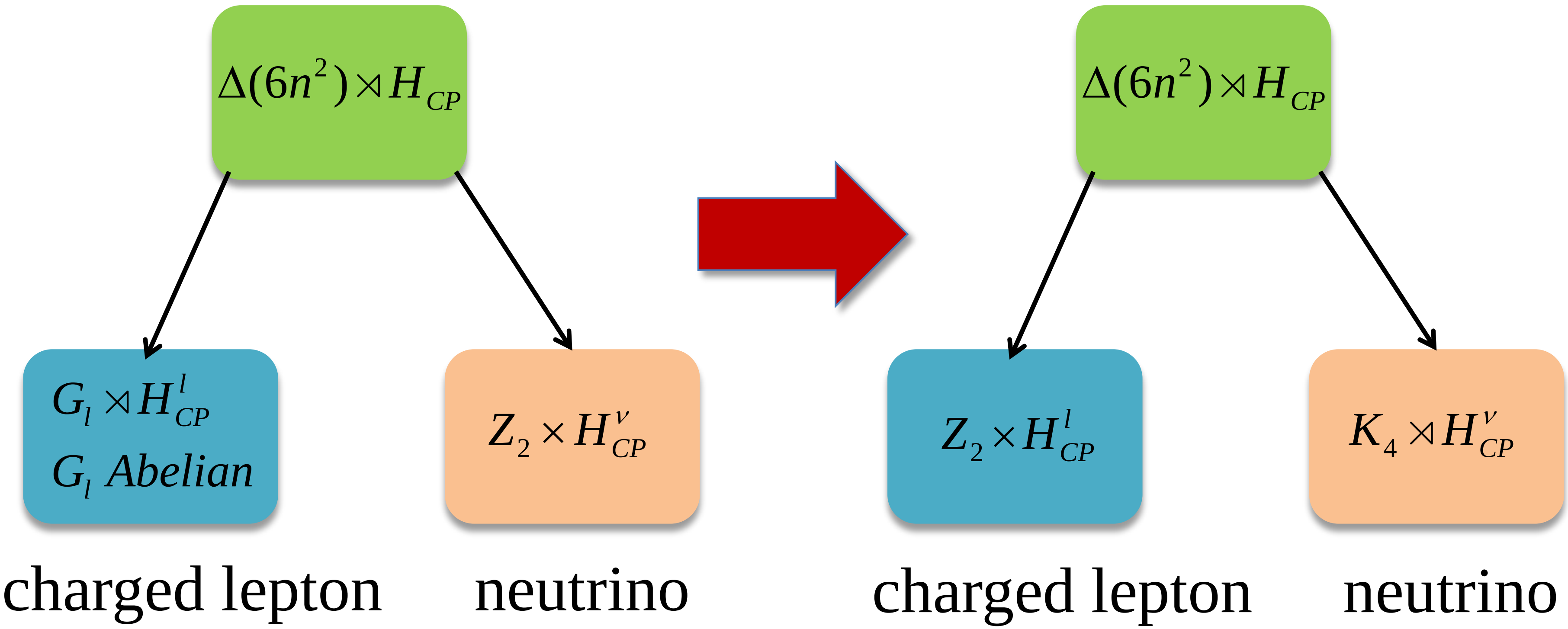
Predictions for neutrinoless double decay (in the limit n → ∞):

$$|m_{ee}| = \left|(m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\alpha_{21}})c_{13}^2 + m_3 s_{13}^2 e^{i(\alpha_{31}-2\delta_{CP})}\right|$$



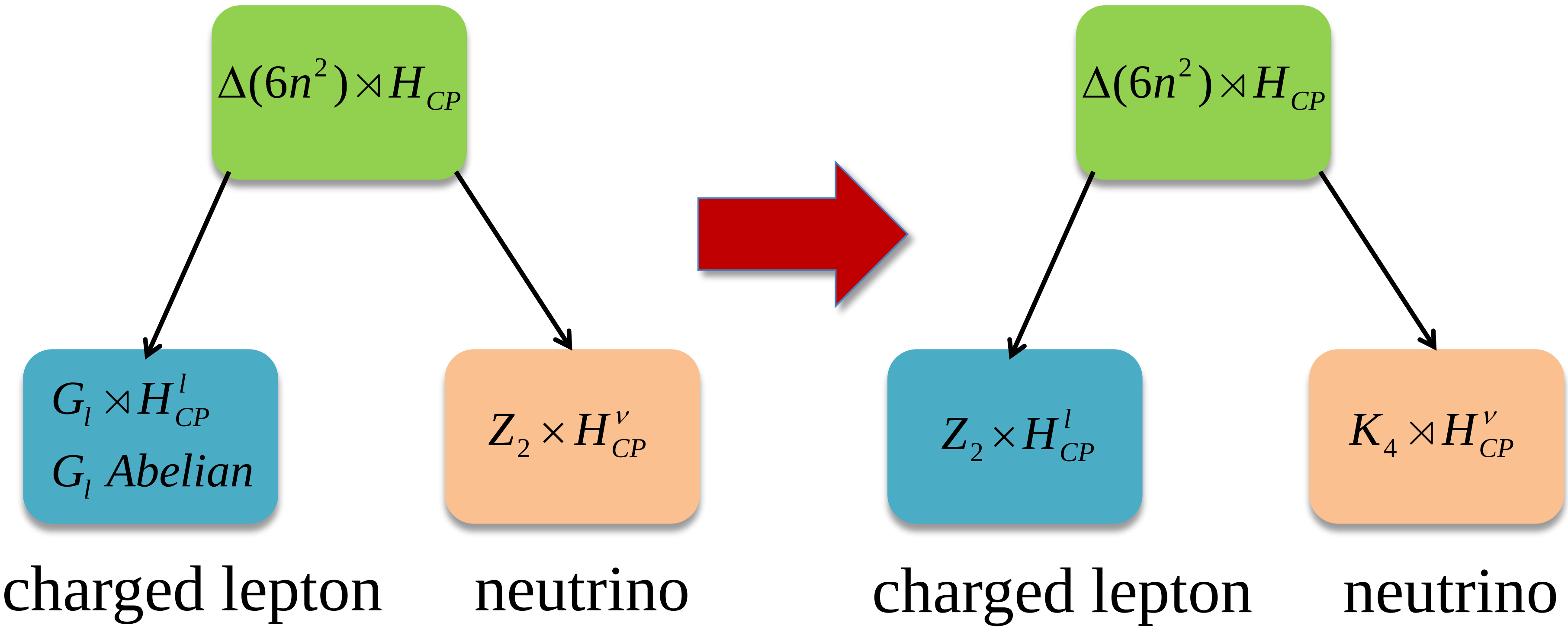
Difficult to test due to limitation of sensitivity

Extending the "Semi-direct" approach



Only one row can be fixed in the new scheme.

Extending the "Semi-direct" approach



Only one row can be fixed in the new scheme.

- **Case IV:** $G_l = Z_2^{bc^{x'}d^{x'}}$, $X_{l\mathrm{r}} = \rho_{\mathrm{r}}(c^{\gamma'}d^{-2x'-\gamma'})$, $G_\nu = K_4^{(c^{n/2}, abc^y)}$ and $X_{\nu\mathrm{r}} = \rho_{\mathrm{r}}(c^{\gamma}d^{2y+2\gamma})$

$$U^{IV}_{PMNS} = \frac{1}{2}\begin{pmatrix} \sin\theta + \sqrt{2}e^{i\varphi_8}\cos\theta & \sin\theta - \sqrt{2}e^{i\varphi_8}\cos\theta & \sqrt{2}e^{i\varphi_9}\sin\theta \\ 1 & 1 & -\sqrt{2}e^{i\varphi_9} \\ \cos\theta - \sqrt{2}e^{i\varphi_8}\sin\theta & \cos\theta + \sqrt{2}e^{i\varphi_8}\sin\theta & \sqrt{2}e^{i\varphi_9}\cos\theta \end{pmatrix}$$

Relation between θ_{13} and θ_{23}

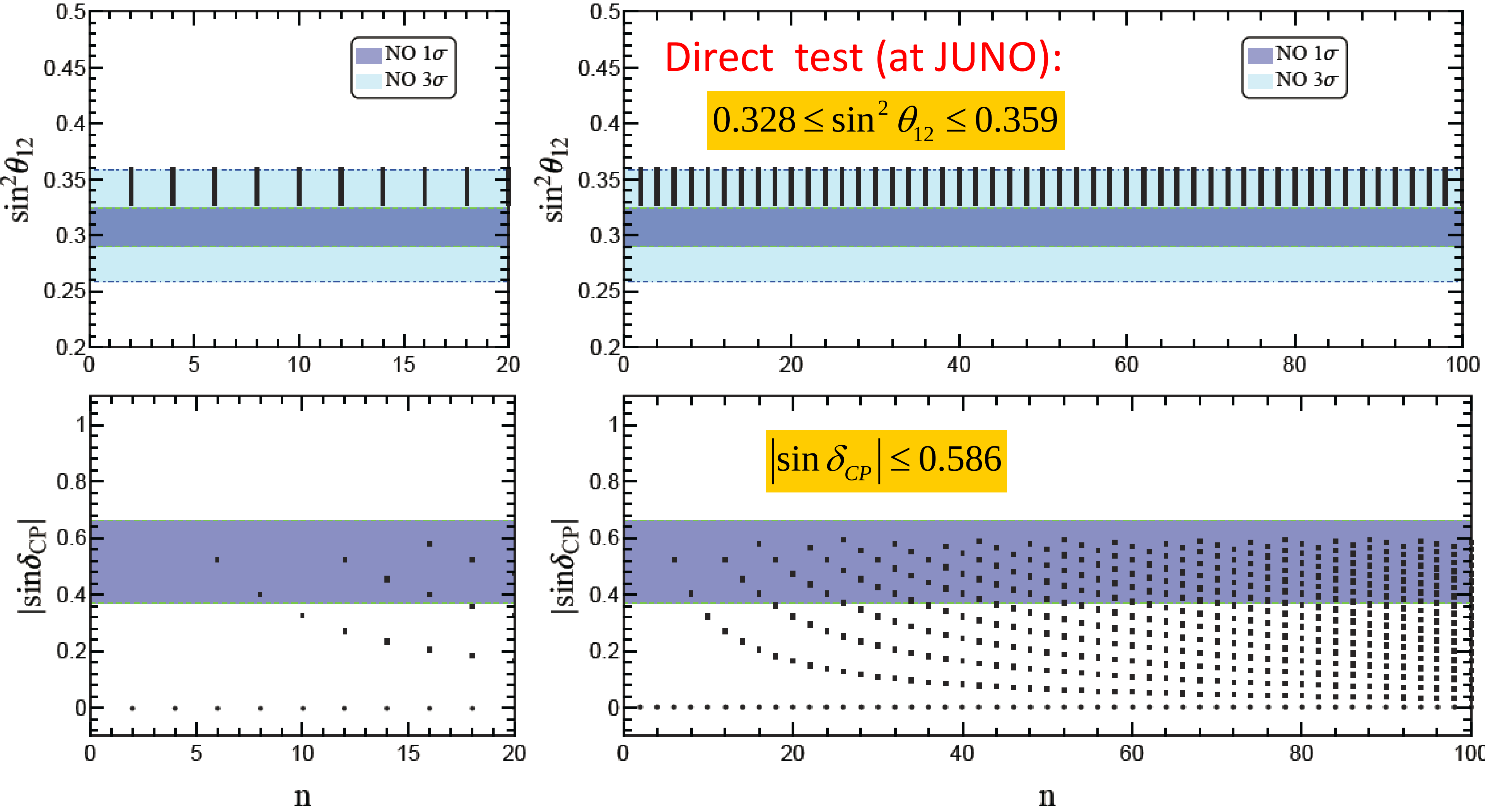
$$2\cos^2\theta_{13}\sin^2\theta_{23}=1 \qquad \text{or} \qquad 2\cos^2\theta_{13}\sin^2\theta_{23}=\cos 2\theta_{13}$$

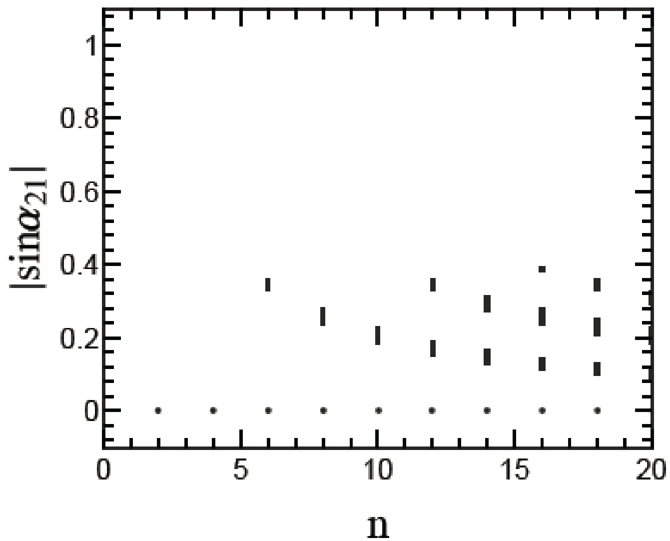
$$\therefore \quad \sin^2\theta_{23}\approx 0.488,\ \ 0.512$$ ← Precisely measure θ_{23}

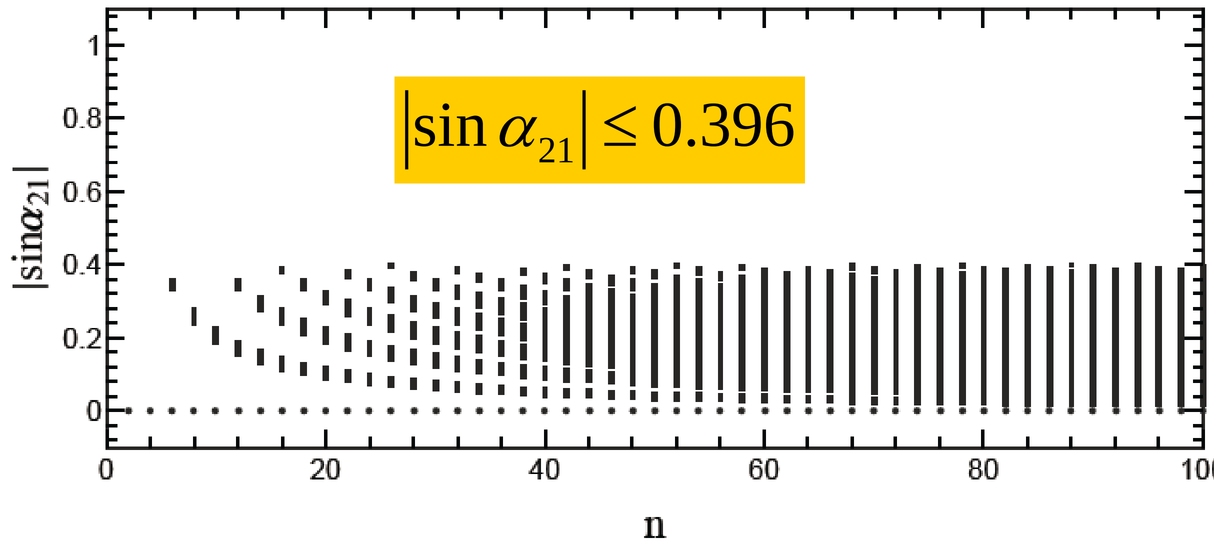
Direct test (at JUNO):

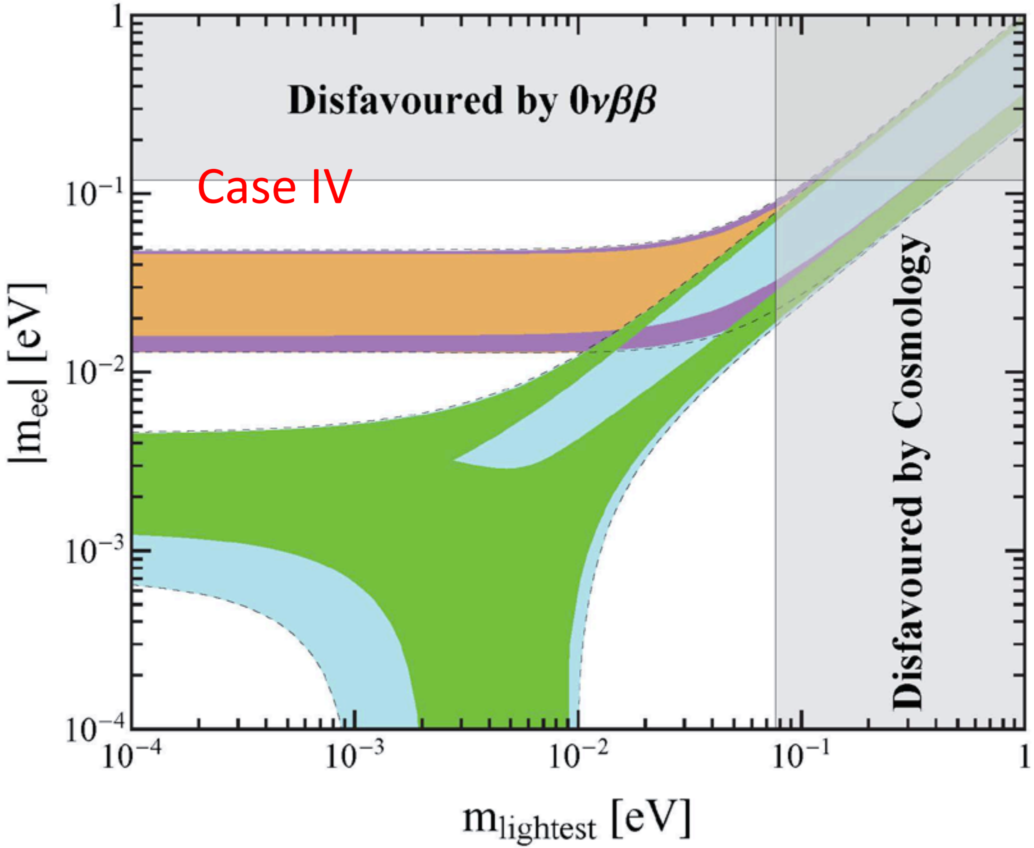
$$0.328\le\sin^2\theta_{12}\le 0.359$$

$$|\sin\delta_{CP}|\le 0.586$$









The effective mass is around the 3σ lower or upper bounds for Inverted hierarchy.

Summary:

- The predictions of $\Delta(6n^2)$ with generalized CP symmetry for lepton mixing are studied in a model independent way.
- There are only four viable cases. We find the mixing angles are constrained within certain ranges. Precise measurement of θ_{12} (at JUNO) and θ_{23} can directly test this scenario.
- CP phases are generally predicted to take regular values $0, \pi$ or $\pm\pi/2$ for small n, while they are usually not constrained for large value of n .
- Exploring the phenomenological predictions for leptogenesis, electric dipole moments and more phenomena related with CP.

Thank you for your attention!

Backup

Where do we stand?

Taken from NuFIT, arXiv:1409.5439

	Normal Ordering ($\Delta\chi^2 = 0.97$)		Inverted Ordering (best fit)		Any Ordering
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.304^{+0.013}_{-0.012}$	$0.270 \to 0.344$	$0.270 \to 0.344$
$\theta_{12}/^\circ$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$33.48^{+0.78}_{-0.75}$	$31.29 \to 35.91$	$31.29 \to 35.91$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.382 \to 0.643$	$0.579^{+0.025}_{-0.037}$	$0.389 \to 0.644$	$0.385 \to 0.644$
$\theta_{23}/^\circ$	$42.3^{+3.0}_{-1.6}$	$38.2 \to 53.3$	$49.5^{+1.5}_{-2.2}$	$38.6 \to 53.3$	$38.3 \to 53.3$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0186 \to 0.0250$	$0.0219^{+0.0011}_{-0.0010}$	$0.0188 \to 0.0251$	$0.0188 \to 0.0251$
$\theta_{13}/^\circ$	$8.50^{+0.20}_{-0.21}$	$7.85 \to 9.10$	$8.51^{+0.20}_{-0.21}$	$7.87 \to 9.11$	$7.87 \to 9.11$
$\delta_{CP}/^\circ$	306^{+39}_{-70}	$0 \to 360$	254^{+63}_{-62}	$0 \to 360$	$0 \to 360$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.50^{+0.19}_{-0.17}$	$7.02 \to 8.09$	$7.02 \to 8.09$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.457^{+0.047}_{-0.047}$	$+2.317 \to +2.607$	$-2.449^{+0.048}_{-0.047}$	$-2.590 \to -2.307$	$\begin{bmatrix} +2.325 \to +2.599 \\ -2.590 \to -2.307 \end{bmatrix}$

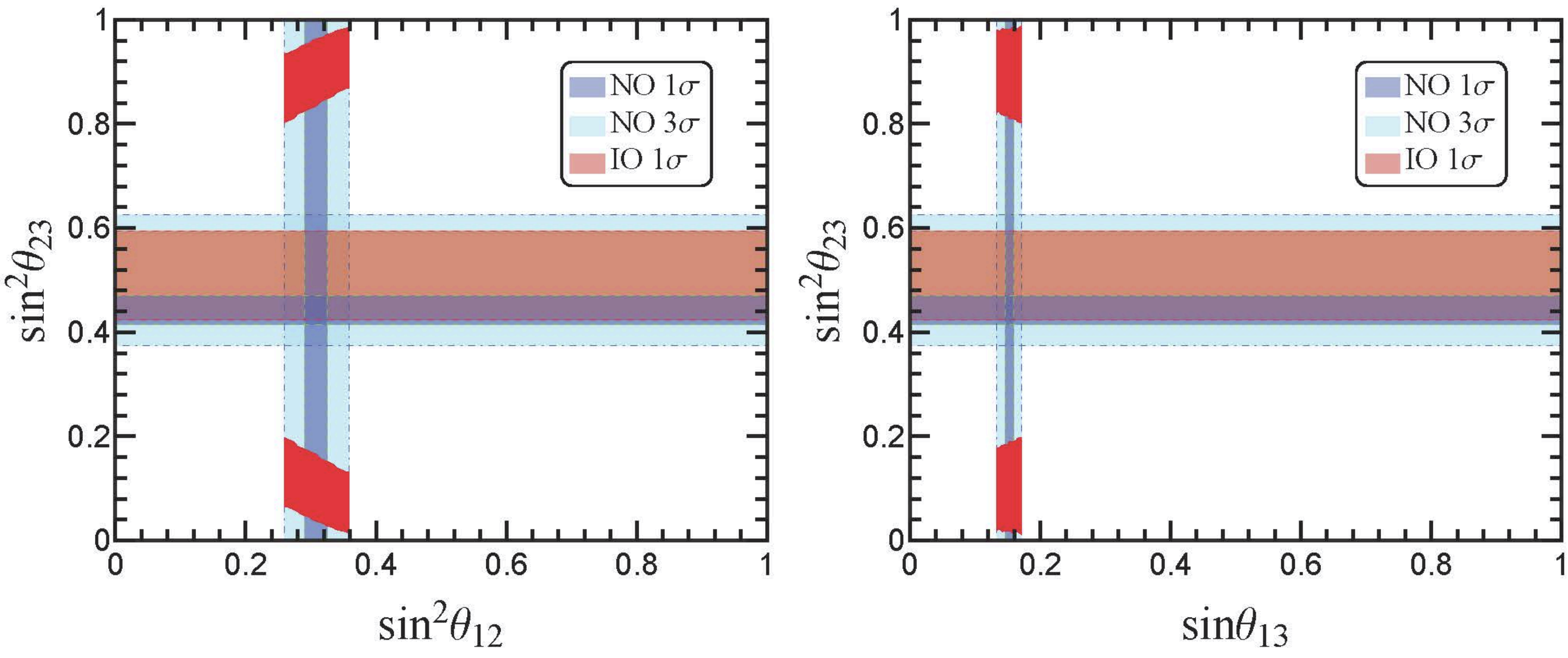
Unknown quantities: ①CP phases: δ_{CP} , α_{21} and α_{31}; ② neutrino mass order.

Inner automorphism σ_h is defined as

$$\sigma_h : g \rightarrow hgh^{-1}, \qquad h, g \in G_f$$

Fixed vector in the 2^{nd} column

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} e^{i\varphi_2}\cos\theta - \sqrt{2}\sin\theta\cos\varphi_1 & \sqrt{2}\sin\varphi_1 & e^{i\varphi_2}\sin\theta + \sqrt{2}\cos\theta\cos\varphi_1 \\ -e^{i\varphi_2}\cos\theta - \sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}-\varphi_1\right) & \sqrt{2}\cos\left(\frac{\pi}{6}-\varphi_1\right) & -e^{i\varphi_2}\sin\theta + \sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}-\varphi_1\right) \\ e^{i\varphi_2}\cos\theta + \sqrt{2}\sin\theta\sin\left(\frac{\pi}{6}+\varphi_1\right) & \sqrt{2}\cos\left(\frac{\pi}{6}+\varphi_1\right) & e^{i\varphi_2}\sin\theta - \sqrt{2}\cos\theta\sin\left(\frac{\pi}{6}+\varphi_1\right) \end{pmatrix}$$



The correct values of θ_{12}, θ_{13} and θ_{23} can not be achieved simultaneously.