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Localization of fermions on brane

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Based on: [PRD 89(2014)086001], [PRD 86(2012)127502].

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- 3. Localization of fermions on scalar-tensor brane
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1983: Domain Wall Scenario [Akama, Rubakov, Shaposhnikov]

- Our 4D world is a brane embedded in 5D flat space-time
- **Infinite** extra dimension
- Generated by a scalar field: $\phi(y) = v_0 \tanh(ky)$
- Fermions can be localized on DW by **Yukawa coupling** $\eta\bar{\Psi}\phi\Psi$

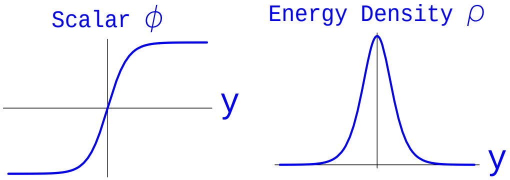


Figure: **Domain wall**

- **Newton's law can not be recovered on the DW**

1999: Warped Extra Dimension (RS Brane Scenario) [Randall and Sundrum]

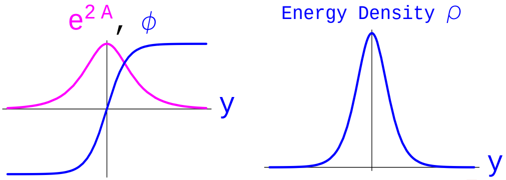
- $$ds^2 = e^{-2k|y|}\,\eta_{\mu\nu}(x)\,dx^\mu dx^\nu + dy^2,\quad \text{ED: } S^1/Z_2$$
- Our 4D world is a brane embedded in a 5D space-time
- SM fields are assumed to be confined on brane, and gravity propagates in the whole space-time
- To solve the gauge hierarchy and cosmological problems
- Fermions can be localized on brane by "mass term":
$$\eta\bar{\Psi}M\epsilon(y)\Psi$$
- Newton's law can be recovered on brane
- Energy density: $\rho(y) \propto \sigma_1\delta(y) + \sigma_2\delta(y-y_b)$

1999—Now: Thick braneworlds (Domain Walls) [Bazeia, Csaki, DeWolfe, Freedman, Hollowood, Giovannini, Goldberger, Gremm, Gubser, Kodama, Rubakov, Volkas, Schnabl, ...]

- $ds^2 = e^{2A(y)}\, \hat{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + dy^2$
- Infinite but warped extra dimension
- Braneworlds are generated by scalar fields, e.g.

$$\phi(y) = v_0 \tanh(ky)$$

- Newton's law can be recovered on the brane
- Fermions can be localized on the brane by **Yukawa coupling** $\eta\bar{\Psi}\phi\Psi$.



- **In braneworld model, (3+1)-dimensional gravitons, fermions, and gauge fields are zero modes of bulk gravity, spinor and vector fields, respectively.**
- **So, an important question is how to localize gravitons and matters (scalars, vectors, and fermions) on a brane.**

Localization of fermions on domain wall/ brane

- **Domain wall/brane is generated by a scalar field.**
- **If the scalar $\phi(y)$ is an odd function of extra dimension y (Z_2 odd), fermions can be localized on the brane by Yukawa coupling $\eta\bar{\Psi}\phi\Psi$.**

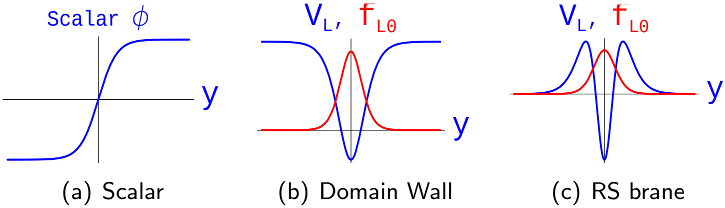


Figure: Effective potential V_L and zero mode f_{L0} for left-handed fermion

- **There are a lot of papers on this scenario.**

Localization of fermions on domain wall/ brane

- **If the scalar is Z_2 even, the effective potential $V_L(y)$ is not Z_2 even anymore, how to localize fermions on the brane?**

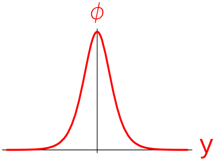


Figure: Z_2 even scalar $\phi(y)$

- **We will introduce new localization mechanism.**

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2. New localization mechanism

Suppose the brane is generated by a Z_2 even scalar field ϕ, namely, ϕ is an even function of extra dimension.

In order to localize fermion on the brane, we introduce the following **new coupling** between fermion Ψ and scalar ϕ:

$$\lambda\bar{\Psi}\Gamma^M\partial_M F(\phi)\gamma^5\Psi. \tag{1}$$

So, the Dirac action is given by

$$S_{\frac{1}{2}} = \int d^5x\sqrt{-g}\Big[\bar{\Psi}\Gamma^M(\partial_M+\omega_M)\Psi+\lambda\bar{\Psi}\Gamma^M\partial_M F(\phi)\gamma^5\Psi\Big], \tag{2}$$

The metric of the background space-time is

$$\begin{aligned} ds^2 &= e^{2A(y)}\,\hat{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + dy^2 \\ &= e^{2A(y(z))}\,\big[\hat{g}_{\mu\nu}(x)\,dx^\mu dx^\nu + dz^2\big]. \end{aligned} \tag{3}$$

With the non-vanishing components of ω_M:
$\omega_\mu = \frac{1}{2}\partial_z A\gamma_\mu\gamma_5 + \hat{\omega}_\mu$, the Dirac equation reads as

$$\Big[\gamma^\mu(\partial_\mu + \hat{\omega}_\mu) + \gamma^5\big(\partial_z + 2\partial_z A(z)\big) + \lambda\partial_z F(\phi)\Big]\Psi = 0. \tag{4}$$

2. New localization mechanism

We make the general KK decomposition in terms of 4D effective Dirac fields $\psi_{L,R}(x)$:

$$\Psi(x,z) = e^{-2A}\left[\sum_n \psi_{Ln}(x) f_{Ln}(z) + \sum_n \psi_{Rn}(x) f_{Rn}(z)\right]. \qquad (5)$$

Then the left- and right-handed KK modes $f_n^{L,R}(z)$ satisfy the following equations:

$$\left[-\partial_z^2 + V_{L,R}(z)\right] f_n^{L,R} = m_n^2 f_n^{L,R}, \qquad (6)$$

$$V_{L,R}(z) = \lambda^2 \left[\partial_z F(\phi)\right]^2 \pm \lambda \partial_z^2 F(\phi). \qquad (7)$$

- In brane models, the extra dimension has Z_2 symmetry.
- So the effective potentials $V_{L,R}(z)$ should be Z_2 even.
- This can be ensured for even scalar $\phi(z)$.

Note that if we consider Yukawa coupling $\eta\bar{\Psi}F(\phi)\Psi$, then the effective potentials are

$$V_{L,R}(z) = \eta\left[\mathbf{e}^{A}F(\phi)\right]^2 \mp \eta\partial_z\left[\mathbf{e}^{A}F(\phi)\right]. \tag{8}$$

They are even only for odd scalar.

2. New localization mechanism

The above Schrödinger–like equations (6) for the left– and right–handed KK modes of fermions can be recast into

$$Q^{\dagger}Qf_n^{L,R}(z) \;=\; m_n^2 f_n^{L,R}, \quad \Rightarrow \quad m_n^2 \geq 0, \tag{9}$$

where $Q = \partial_z - \lambda\partial_z F(\phi)$. So, there are no tachyon fermion KK modes with negative mass square m_n^2.

On the other hand, by introducing the following orthonormalization conditions for f_{Ln} and f_{Rn}

$$\int f_{Lm,Rn}^2(z)dz = \delta_{LR}\delta_{mn}. \tag{10}$$

the 5D action (2) reduces to the 4D effective actions of a massless chiral fermion and a series of massive fermions:

$$S_{\frac{1}{2}} = \sum_n \int d^4x\sqrt{-\hat{g}}\;\bar{\psi}_n\left[\gamma^{\mu}(\partial_{\mu}+\hat{\omega}_{\mu}) - m_n\right]\psi_n. \tag{11}$$

- The zero modes are

$$f_{L0,R0}(z) \propto \exp\Big[\pm\lambda F(\phi)\Big]. \tag{12}$$

- In order to localize fermions on the brane, the zero modes need to satisfy the normalization conditions:

$$\int f^2_{L0,R0}(z)dz < \infty. \tag{13}$$

Content

3. Localization of fermions on scalar-tensor brane

As an example, we consider the scalar-tensor brane presented in [K Yang, YX Liu, et al, PRD 86(2012)127502].

- In the RS-1 brane model [PRL83(1999)3370], there are two 3-branes.
- In order to solve gauge hierarchy problem, our universe is located on the negative tension brane.
- However, this would give a "wrong-signed" Friedmann-like equation, which leads to a severe cosmological problem [PLB 462(1999)34;PRL83(1999)4245].
- In the RS-2 brane model, the cosmological problem has been solved, but the gauge hierarchy problem is left.
- Recently, in order to solve this problem, we gave a simple generalization of the RS1 model in the scalar-tensor gravity [Yang, Liu, et al, PRD 86(2012)127502].

Scalar-tensor brane

The action for the scalar-tensor gravity is given by

$$S_5 = \frac{1}{2}\int d^5x\sqrt{-g}e^{\gamma\phi}\left[R-(3+4\gamma)(\partial\phi)^2\right]. \tag{14}$$

The braneworld is generated by the scalar ϕ.

The solution is given by [Yang, Liu, et al, PRD 86(2012)127502]

$$e^{A(z)} = (1+k|z|)^{\frac{1}{3+2\gamma}}, \tag{15}$$

$$\phi(z) = \frac{2}{3+2\gamma}\ln(1+k|z|), \tag{16}$$

where the parameters $k>0$ and $\gamma<-\frac{3}{2}$. The scalar $\phi(z)$ is an even function of z.

Scalar-tensor brane

The property and advantage of the scalar-tensor brane [Yang, Liu, et al, PRD 86(2012)127502]:

- There are two branes in this model: a positive tension brane at the origin $z=0$ and a negative one at the boundary z_b.
- The massless graviton is localized on the negative tension brane, which is opposite to the case of the RS1 model.
- Then, if we suppose that the Standard Model fields are trapped on the positive tension brane, the gauge hierarchy problem and cosmological problem can be solved in this brane model.

Localization of fermions

Can fermions be localized on the positive tension brane?

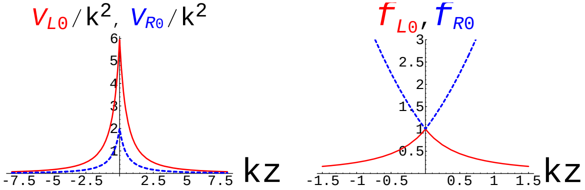
- **Since the scalar field $\phi(z)$ is even, we can not use the Yukawa coupling $\eta\bar{\Psi}F(\phi)\Psi$ anymore.**
- **We consider the new coupling $\lambda\bar{\Psi}\Gamma^M\partial_M F(\phi)\gamma^5\Psi$ with $F(\phi)=\phi^q$ ($q=1,2,3,\cdots$).**
- **The potentials (7) are given by** [Liu, et al, PRD 89(2014)086001]

$$V_{L,R}(z) = \frac{4qk^2\phi^{q-2}}{(3+2\gamma)^2(1+k|z|)^2}\Big(q\phi^q\lambda^2 \pm \big(q-1-\ln(1+k|z|)\big)\lambda\Big) + \frac{4kq}{(3+2\gamma)}\delta_{q,1}\delta(z). \quad (17)$$

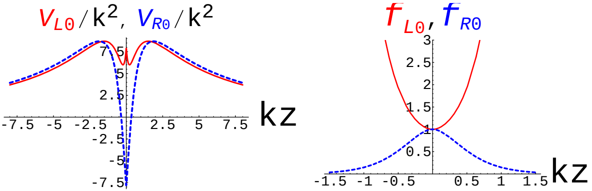
- **The fermion zero modes are**

$$f_{L0,R0}(z) \propto \exp\left[\pm\lambda\left(\frac{2}{3+2\gamma}\ln(1+k|z|)\right)^q\right]. \quad (18)$$

Localization of fermions



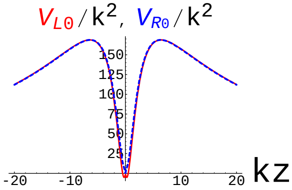
(a) $F(\phi) = \phi$ $(q = 1)$



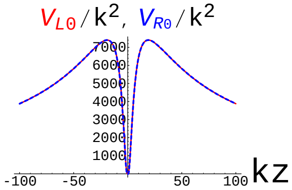
(b) $F(\phi) = \phi^2$ $(q = 2)$

Figure: The potentials and fermion zero modes with $\gamma = -2$, $\lambda = 1$.

Localization of fermions



(a) $F(\phi) = \phi^3$ $(q = 3)$



(b) $F(\phi) = \phi^4$ $(q = 4)$

Figure: The potentials and fermion zero modes with $\gamma = -2$, $\lambda = 1$.

Localization of fermions

- For $q = 1$, if $\lambda > \lambda_0 \equiv -\frac{3+2\gamma}{4} (> 0)$, $f_{L0}(z)$ is normalizable, so the left-handed fermion zero mode can be localized on the positive tension brane.
- For odd $q = 3, 5, \cdots$, $V_L(z)$ around the positive tension brane is negative, which leads to a bound left-handed fermion zero mode localized on the brane if $\lambda > 0$.
- For even $q = 2, 4, \cdots$, $V_R(z)$ around the positive tension brane is negative, which results in a bound right-handed fermion zero mode localized on the brane if $\lambda > 0$.

4. Conclusion

- In order to localize fermions on branes generated by **odd** scalar field $\phi_O(z)$ (such as kink scalar), we need to introduce the Yukawa coupling

$$\eta\bar{\Psi}F(\phi_O)\Psi.$$

- In order to localize fermions on branes generated by **even** scalar field $\phi_E(z)$ (such as dilaton), we need to introduce new fermion-scalar coupling

$$\lambda\bar{\Psi}\Gamma^M\partial_M F(\phi_E)\gamma^5\Psi.$$

Thanks for your listening!