Can a holography be inferred simply from thermodynamics?

Jian-Xin Lu
(Work done with Shibaji, Wei, Xu and Ouyan)

The Interdisciplinary Center for Theoretical Study (ICTS)
University of Science & Technology of China

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BH thermal nature

Bekenstein (72) and Hawking (74) established:

- a stationary black hole with $M\,, Q\,\&\, J$ is a thermodynamical system,
- obeying the usual thermodynamical laws such as the first law

$$dM = TdS + \Omega dJ + \Phi dQ, \tag{1.1}$$

where $k = \hbar = c = G = 1$.

BH thermal nature

In particular, a black hole has a temperature given by

$$T_{\rm BH} = \frac{\kappa}{2\pi} \quad \left(= \frac{\hbar c \kappa}{2\pi k} \right), \tag{1.2}$$

with κ the so-called surface gravity of horizon, and an entropy given by

$$S = \frac{A}{4} \quad \left(= \frac{k c^3 A}{4 G \hbar} \right), \tag{1.3}$$

with A the area of the horizon.

$\Rightarrow$ Quantum Thermodynamics?

$\Rightarrow$ Part of Quantum Gravity?

Holography

't Hooft and Susskind went a step further to propose in the quantum theory of gravity:

In a region of space of volume V, the maximum entropy is proportional to the area and not the volume of the region.

A holography!

AdS/CFT

Maldacena (97) gave the first concrete realization of such a holography in string theory, namely,

IIB string theory on $(AdS_5 \times S^5)_N$

is equivalent to

$\mathcal{N} = 4, D = 3 + 1, U(N)$ SYM

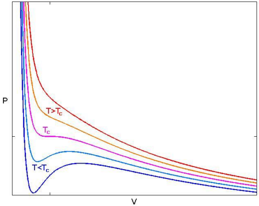
AdS BH

Chamblin et al (99)were first to study the thermodynamics of AdS black holes and in the charged case, the revealed phase structure is nothing but the familiar van der Waals-Maxwell gas-liquid type.

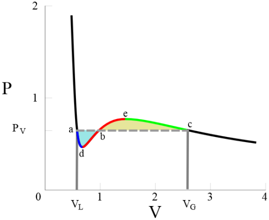
In particular, this phase structure was claimed to be the special property of AdS case only.

Chamblin, Emparan, Johnson & Myers, PRD60, 064018(99); 104026 (99)

Van der Waals isotherm



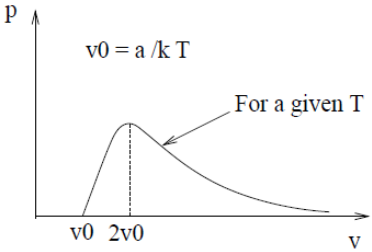
$$\left(p+\frac{a}{v^2}\right)(v-b)=kT,$$



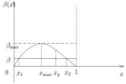
Van der Waals isotherm

When $b = 0$,

$$p = \frac{kTv - a}{v^2}, \qquad (1.4)$$



Analogous to the chargeless case:



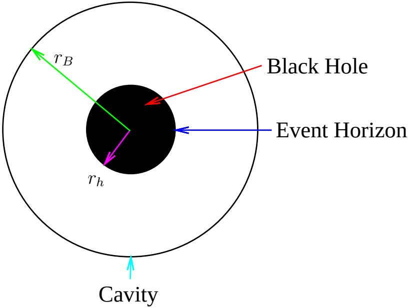
Black holes/branes

Carlip&Vaidya and Lundgren later found that black hole systems, when properly stabilized, appear to give the corresponding Universal Phase structure whether charged or not, independent of their asymptotical geometry, either AdS, dS, or flat.

Given the AdS/CFT, they went further to indicate that this universal phase structure may hint a holography in general.

Carlip&Vaidya CQG20,3827(03); Lundgren PRD77,044014(08)

Asymptotically flat BH



Black hole (r_h) placed in a cavity (r_B) with fixed T and V.

Asymptotically flat BH

Local stability condition:

define:

$$\begin{aligned} x &= \frac{r_h}{r_B} < 1, \quad q = \frac{Q}{r_B} < x, \quad \beta_B = \frac{1}{T_B}, \quad \beta(x) = \frac{1}{T(x)}, \\ \bar{b} &= \frac{\beta_B}{4\pi r_B}, \quad b_q(x) = \frac{\beta(x)}{4\pi r_B}. \end{aligned} \tag{2.1}$$

$$\frac{dF}{dx} \sim (\bar{b} - b_q(x)) = 0 \Rightarrow \bar{b} = b_q(\bar{x}) \quad (\mathrm{EOS}) \tag{2.2}$$

$$\left.\frac{d^2F}{dx^2}\right|_{x=\bar{x}} \sim -\frac{db_q(\bar{x})}{d\bar{x}} > 0 \Rightarrow \tag{2.3}$$

local minimal free energy $\Leftrightarrow$ the negative slope of $b_q(\bar{x})$

Uncharged Schwarzschild BH

$$b_0(x) = x(1-x)^{1/2} > 0. \tag{3.1}$$

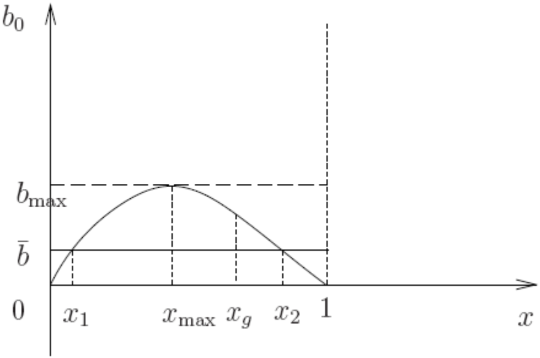
$$(\ 0 < x < 1)$$

$$\bar{b} = b_0(\bar{x}). \qquad (\mathrm{EOS}) \tag{3.2}$$

Note

$$b_0(x \to 0) \to 0, \qquad b_0(x \to 1) \to 0. \tag{3.3}$$

Uncharged Schwarzschild BH



Charged Reissner-Nordström BH

$$b_q(x) = \frac{x(1-x)^{1/2}\left(1-\frac{q^2}{x}\right)^{1/2}}{1-\frac{q^2}{x^2}}. \quad (3.4)$$

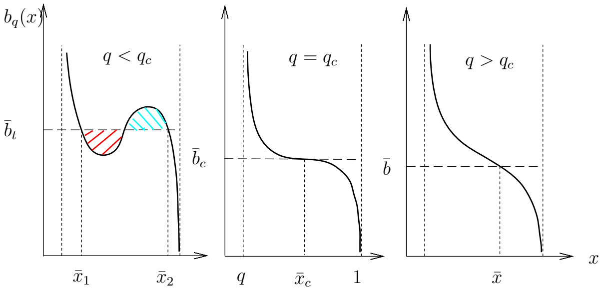
$$(q < x < 1)$$

$$\bar{b} = b_q(\bar{x}). \qquad \text{(EOS)} \quad (3.5)$$

Note

$$b_q(x \to q) \to \infty, \quad b_q(x \to 1) \to 0. \quad (3.6)$$

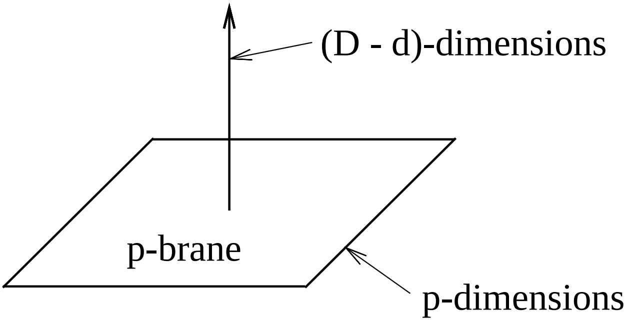
Charged Reissner-Nordström BH



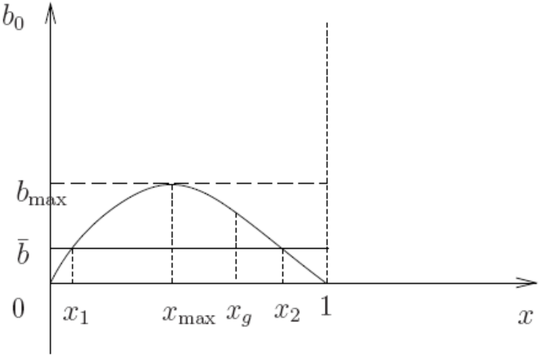
The typical behaviors of $b(x)$ vs x for $q < q_c, q = q_c, q > q_c$.

p-brane

The spatial dimensions transverse to the p-brane is $D - d = \tilde{d} + 2$ $(d = p + 1)$ and note $1 \leq \tilde{d} \leq 7$ (If D = 10, $0 \leq p \leq 6$).

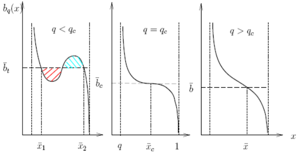


Chargeless p-brane Phase structure



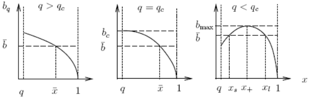
Charged Black p-brane Phase structure

$\tilde{d} > 2,$



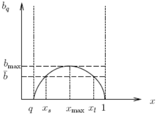
The typical behaviors of $b(x)$ vs x for $q < q_c, q = q_c, q > q_c$.

$\tilde{d} = 2$



The typical behaviors of $b_q(x)$ vs x for $\tilde{d} = 2$.

$\tilde{d} = 1$

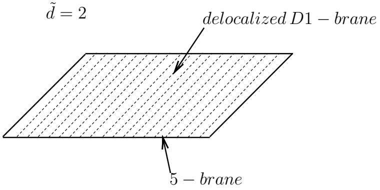


The typical behavior of $b_q(x)$ vs x for $\tilde{d} = 1$.

Lu et al JHEP1101,133,JHEP1105, 091(11), NPB854,913(12)

D1/D5 (F/NS5)

The D1/D5 (F/NS5) system:



D1/D5 (F/NS5)

The EOS

$$\bar{b} = b_{q1,q5}(\bar{x}), \qquad q_5 < x < 1 \tag{3.7}$$

$$b_{q1,q5}(x) = \frac{1}{2}x^{1/2}\left(\frac{1-x}{1-\frac{q_5^2}{x}}\right)^{1/2}\left[\frac{1+\sqrt{1+4q_1^2\frac{\frac{1-x}{1-q_5^2/x}}{\left(1-\frac{1-x}{1-q_5^2/x}\right)^2}}}{2}\right]^{1/2}. \tag{3.8}$$

Note now

$$b_{q_1,q_5}(x \to q_5) \to \infty, \quad b_{q_1,q_5}(x \to 1) \to 0, \tag{3.9}$$

therefore similar to $p < 5$, having a van der Waals-Maxwell liquid-gas type.

Lu et al JHEP1212,012(12)

D0/D6 Phase structure

D0/D6 with both the delocalized charged D0 branes and the charged black D6 branes having the same amount of charge:

$$b_{q,q}(x) = \frac{x(1-x)^{1/2}\left(1-\frac{q^2}{x}\right)^{1/2}}{1-\frac{q^2}{x^2}}. \quad (3.10)$$

$$(q < x < 1)$$

Note again

$$b_{q,q}(x \to q) \to \infty, \quad b_{q,q}(x \to 1) \to 0, \quad (3.11)$$

therefore having also a van der Waals-Maxwell liquid-gas type.

Lu & Wei JHEP04(2013)100, Lu et al PRD90,066003 (2014)

Discussion

In summary, just like the AdS case, the asymptotically flat black holes/branes, when properly stabilized, all share the same phase structure of van der Waals-Maxwell gas-liquid type.

Given the AdS/CFT, does this imply a holography in general?

If so, what is the corresponding field theory?

Discussion

If we assume the existence of such a holography,

the corresponding field theory is hard to guess in the black hole case

but for a Dp-brane system in string theory, it should be related to its worldvolume theory, i.e., the $(p+1)$-dimensional SYM, with relevant fields satisfying certain boundary conditions as set by the cavity under consideration.

Discussion

However, the D6/D0 system provides a counter example to the above assertion.

The low-energy worldvolume theory of this system is a $(6+1)$-SYM with background flux $\int F^n \neq 0$ for $n = 3$ only but this theory doesn't exist in the UV (there is actually no $(6+1)$-dimensional field theory without gravity in the UV) and the corresponding D6 brane theory is as complicated as the M-theory itself and for any given D6 brane number N, it is described in the UV by M-theory in a flat background with a A_{N-1} singularity (an ALE space).

Seiberg PRL79,3577(97); Sen ATMP2,51(98); Itzhaki et al PRD58,046004(98)

Summary

The important lessons learned in this study,

- A universal thermal property and a general holography may not be necessarily related to each other.
- On the other hand, if there is a holography, one should expect to see the same feature on both sides.

Lu et al PRD90,066003 (2014)

THANK YOU!