# Rates and Asymmetries of $B \rightarrow \pi l^+ l^-$ Decays

$l = e$ or $\mu$

**Masaya Kohda (Chung Yuan Christian Univ.)**

甲田 昌也

Based on:

**Wei-Shu Hou, MK and Fanrong Xu, Phys.Rev.D90, 013002 (2014)**

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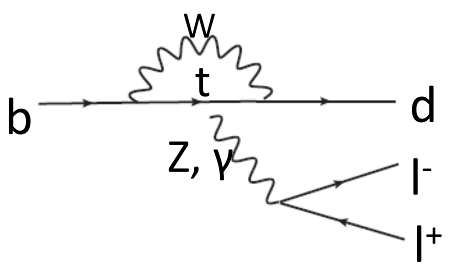
**2nd International Workshop on**
**Particle Physics and Cosmology after Higgs and Planck**

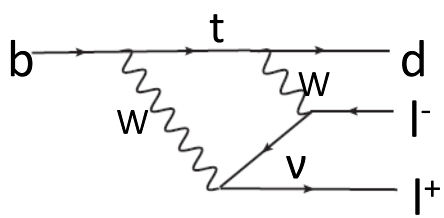
# Introduction

- Dominant quark-level process for $B \to \pi\, l^+ l^-$ is $b \to d\, l^+ l^-$

- This is Flavor Changing Neutral Current (FCNC) process




$$\sim \frac{g^4}{16\pi^2} V_{tb} V_{td}^*$$



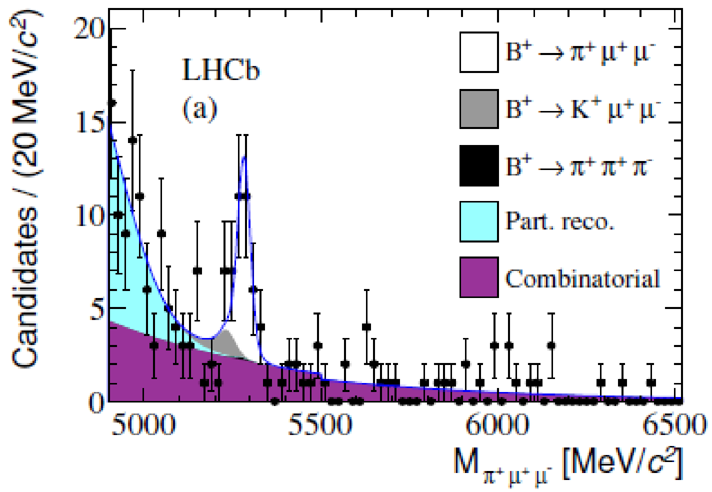

- ➢ Loop suppressed
- ➢ CKM suppressed

- Tiny SM rate, thus, good place to check New Physics effects

# First observation of $B^+ \to \pi^+\mu^+\mu^-$ by LHCb

LHCb, JHEP12(2012)125

- In 2012, LHCb reported the first observation of $b \to d\, l^+ l^-$ transition
- Using 1.0 $fb^{-1}$ of data, LHCb observed $25.3^{+6.7}_{-6.4}$ signal events (5.2$\sigma$ excess)




$$\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6\ (\text{stat.}) \pm 0.1\ (\text{syst.})) \times 10^{-8}$$

- They announced consistency with SM prediction: $B_{SM} = (2.0 \pm 0.2)\times 10^{-8}$

Wang, Wang, Xu and Yang, PRD77(2008)014017

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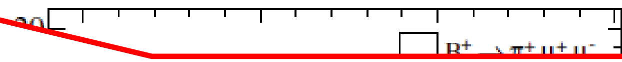


> - Can be still improved by current data (2 $fb^{-1}$ from LHC8)
> - Rich information is expected from future data (LHC14):
>   m($\mu\mu$)–distribution, AFB, ACP, …
> - $B^+ \to \pi^+ \mu\mu$ is a promising mode at LHC

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# Only upper bounds on $B^0 \to \pi^0 l^+ l^-$ by Belle & BaBar

- Experimental summary for measurement of $10^8$ x BR (90% CL)

| Mode | LHCb [1] | Belle [2] | BaBar [3] |
|---|---|---|---|
| $B^+ \to \pi^+\mu^+\mu^-$ | $2.3 \pm 0.6 \pm 0.1$ | $< 6.9$ | $< 5.5$ |
| $B^+ \to \pi^+ e^+ e^-$ | – | $< 8.0$ | $< 12.5$ |
| $B^+ \to \pi^+\ell^+\ell^-$ | – | $< 4.9$ | $< 6.6$ |
| $B^0 \to \pi^0\mu^+\mu^-$ | – | $< 18.4$ | $< 6.9$ |
| $B^0 \to \pi^0 e^+ e^-$ | – | $< 22.7$ | $< 8.4$ |
| $B^0 \to \pi^0\ell^+\ell^-$ | – | $< 15.4$ | $< 5.3$ |

**Belle, PRD78, 011101 (2008) [657M BBbar pairs]**

**BaBar, PRD88, 032012 (2013) [471M BBbar pairs]**

- Isospin symmetry tells, $B(B^0 \to \pi^0 l^+ l^-) \sim 1\times10^{-8}$
- Case of $B^0$ decays will be improved by Belle II, but how much?

# Theoretical status of $B \to \pi\, l^+ l^-$

- So far, most of TH works are based on "Naïve Factorization"
  - Naïve Factorization

    **Aliev, Savci (1998); Song-Lu-Lu (2008); Wang-Wang-Xu-Yang (2007)**
    **Ali, Parkomenko, Rusov (2013)**

  - pQCD **Wang, Xiao (2012)**

- Given that precise measurements are expected for $B^+ \to \pi^+ \mu^+\mu^-$ at LHCb, accurate theoretical studies are desirable

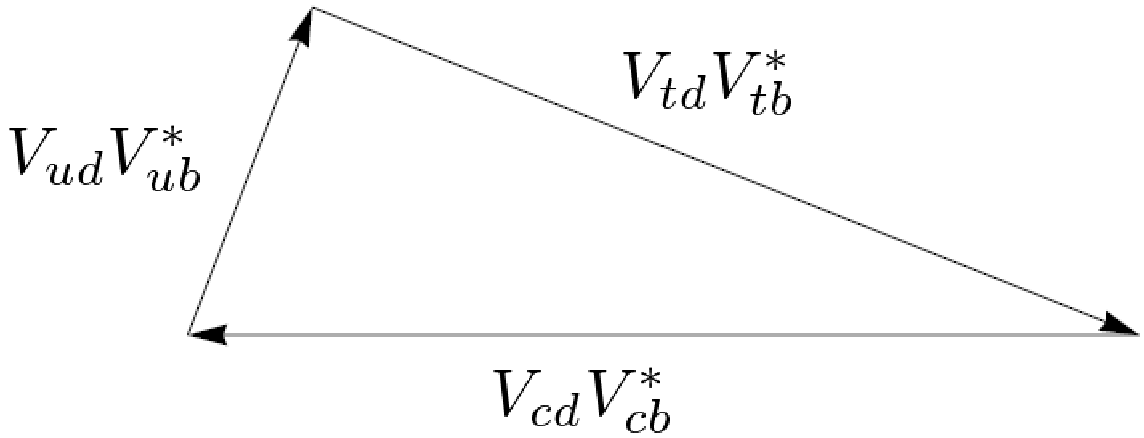
- We study $B^+ \to \pi^+ l^+l^-$ based on QCD factorization
  **Beneke, Feldmann, Seidel, NPB612(2001); EPJC41(2005)**
  - Theoretically well studied for exclusive $b \to s\, l^+l^-$ decays ($B \to K\, l^+l^-$, $B \to K^* l^+l^-$)
  - Provides a good description for experimental data of $B \to K^{(*)} l^+l^-$
  - Can include Weak Annihilation effects, missed in previous studies

# Boring repetition of $b \rightarrow s\, l^+ l^-$ ?

# Boring repetition of $b \rightarrow s\, l^+ l^-$? No!

# Boring repetition of b →s $l^+ l^-$ ? No!

Unitarity Triangle for b → d



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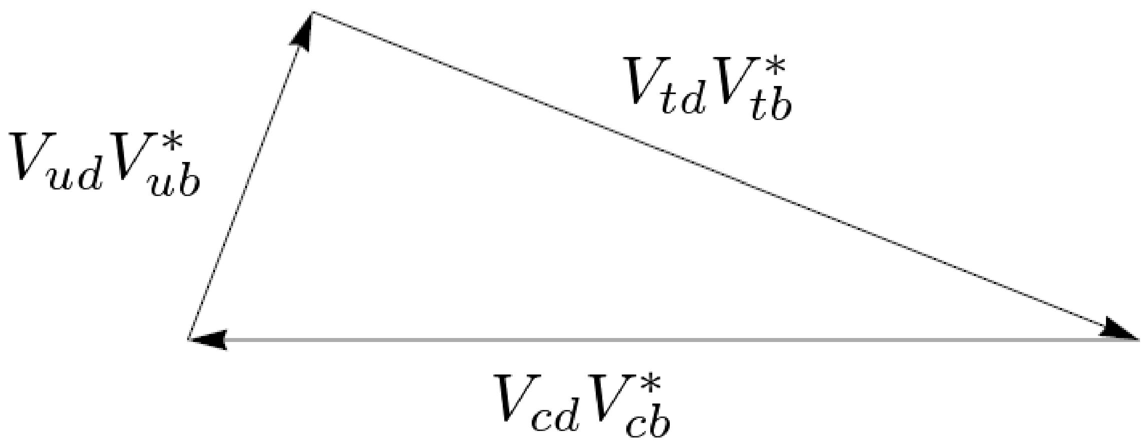
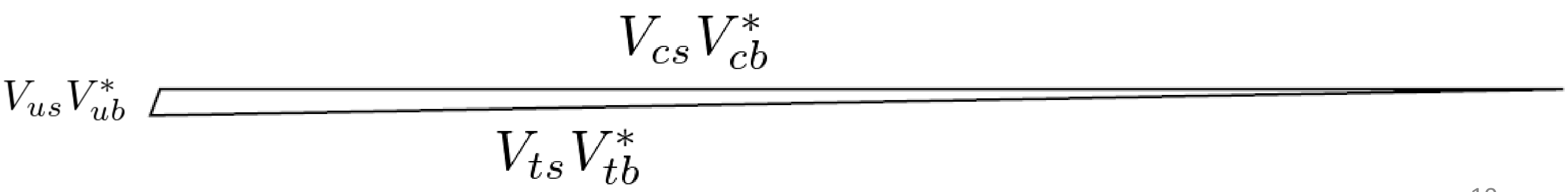



Unitarity Triangle for b → s



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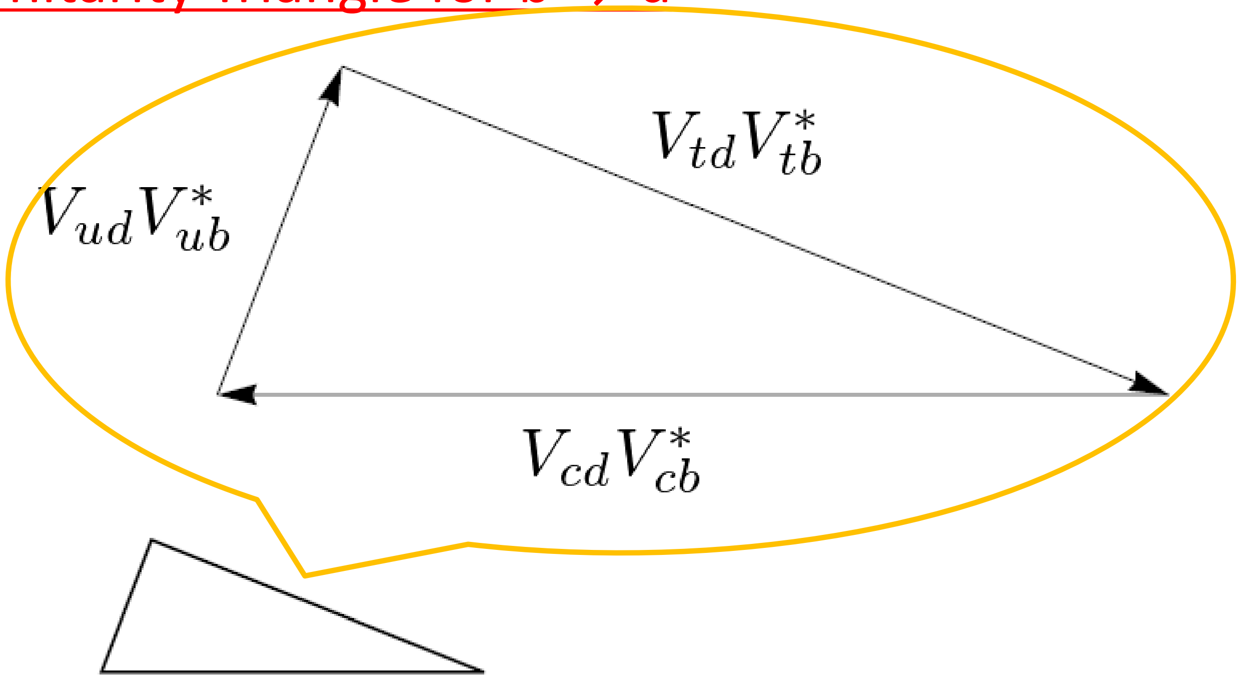
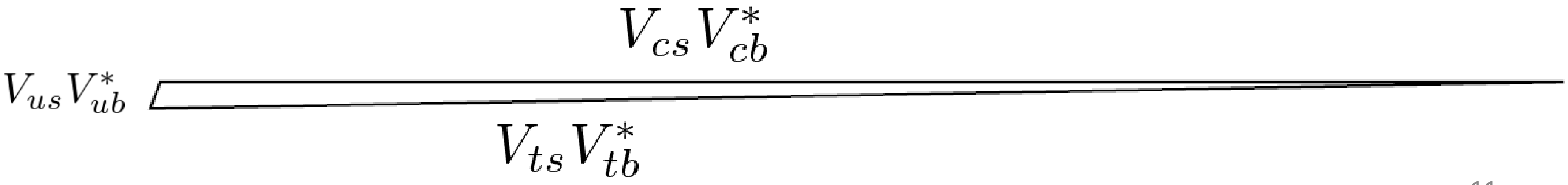



Unitarity Triangle for b → s



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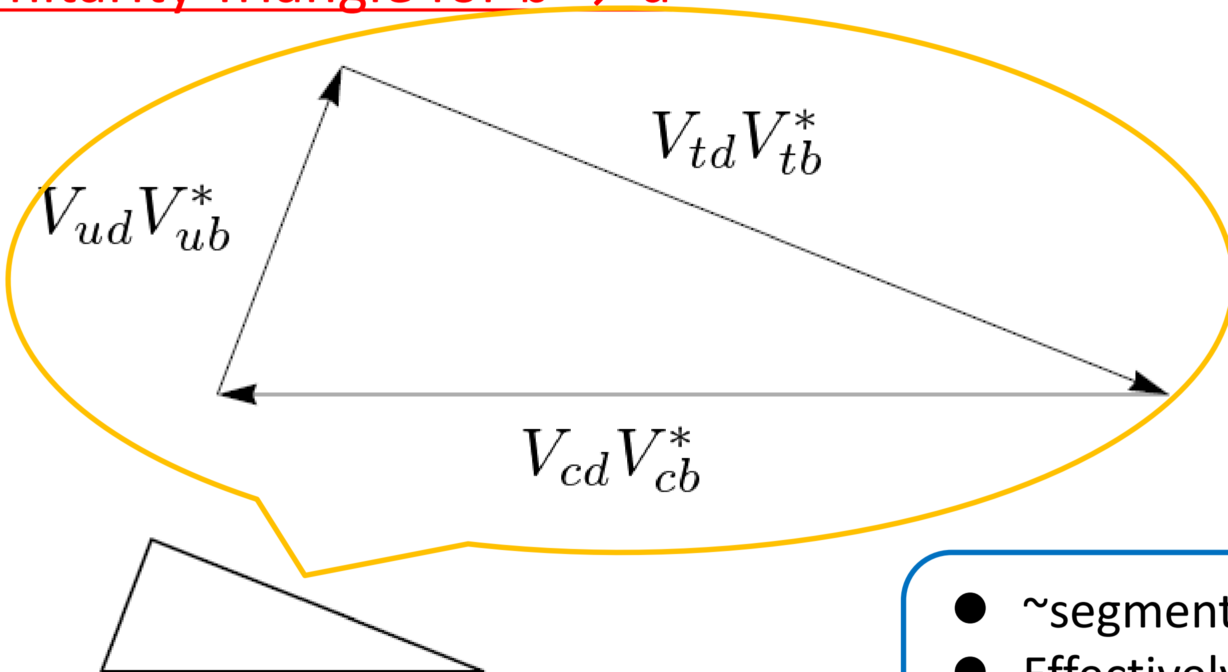
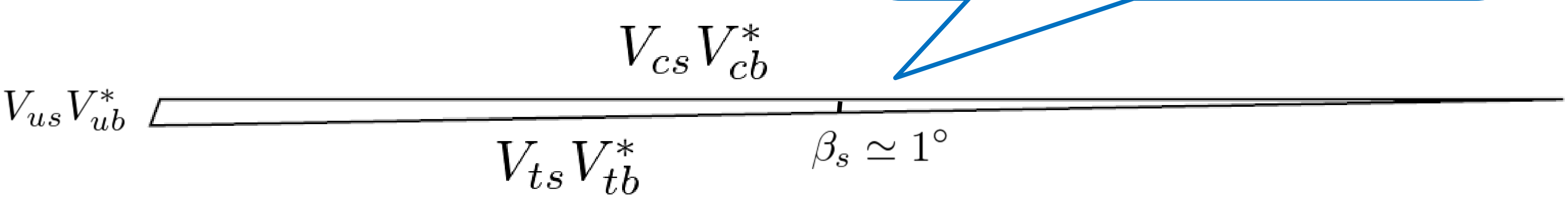



- ~segment
- Effectively single parameter

$$|V_{ts}V_{tb}^*|$$

Unitarity Triangle for b → s



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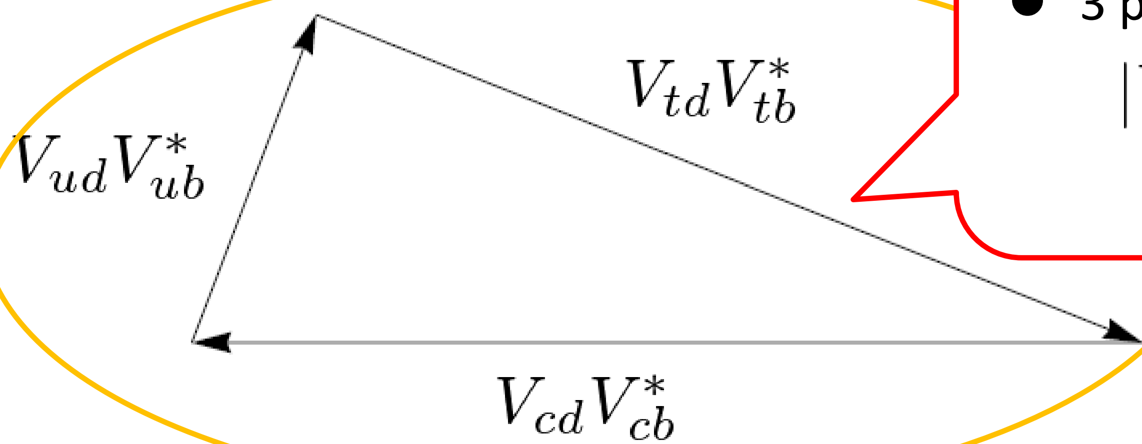


- Genuine triangle
- 3 parameters

$$|V_{td}V_{tb}^*| \quad |V_{ud}V_{ub}^*|$$

$$\phi_2 \equiv \alpha \simeq 89^\circ$$

Unitarity Triangle for b → s

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- Effectively single parameter

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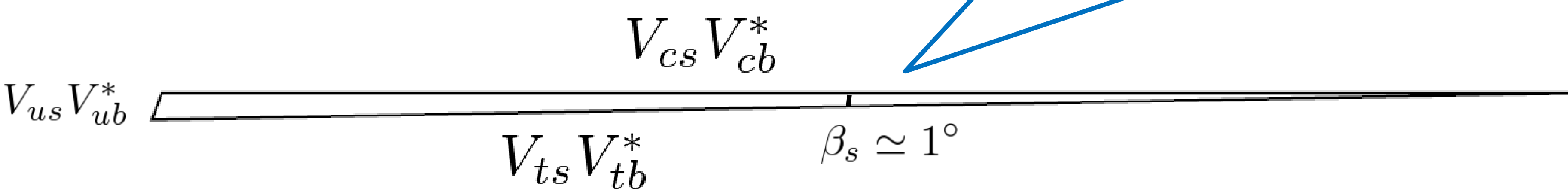

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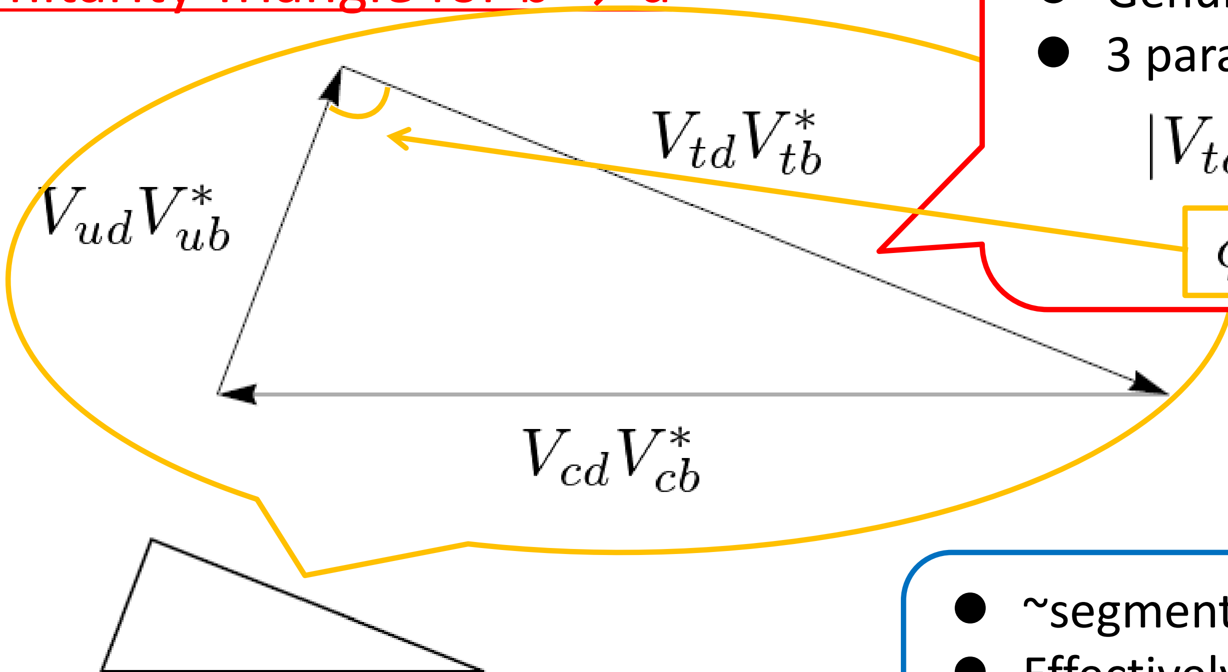


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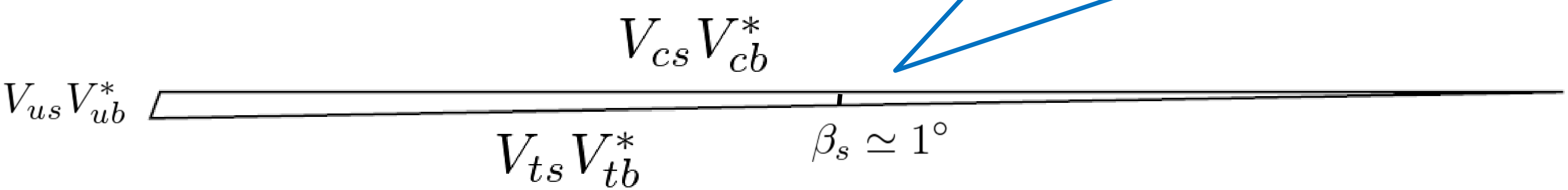


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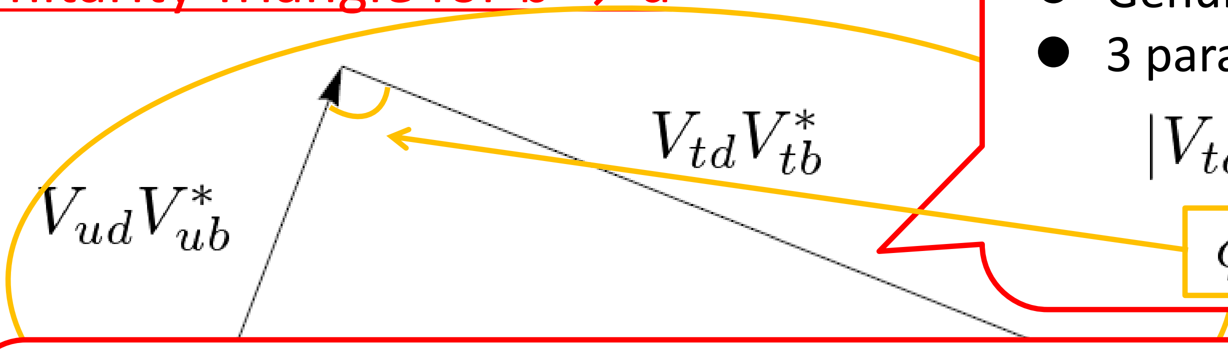


- Genuine triangle
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$|V_{td}V_{tb}^*|$ $|V_{ud}V_{ub}^*|$

$\phi_2 \equiv \alpha \simeq 89^\circ$

- CKM structure of b → d $l^+l^-$ is numerically more complicated
- So, much richer phenomenology is expected
- Especially, large CP violation in decay may happen

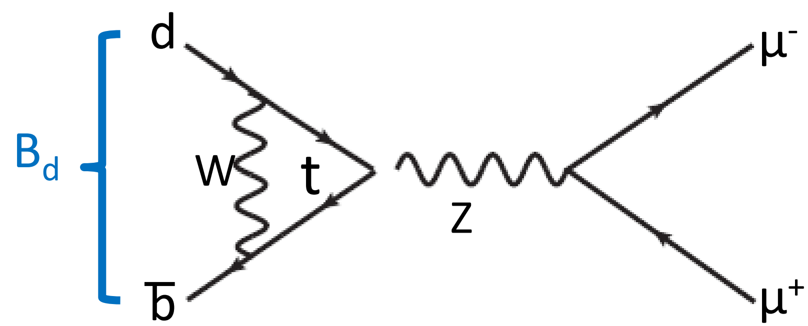
$|V_{ts}V_{tb}^*|$

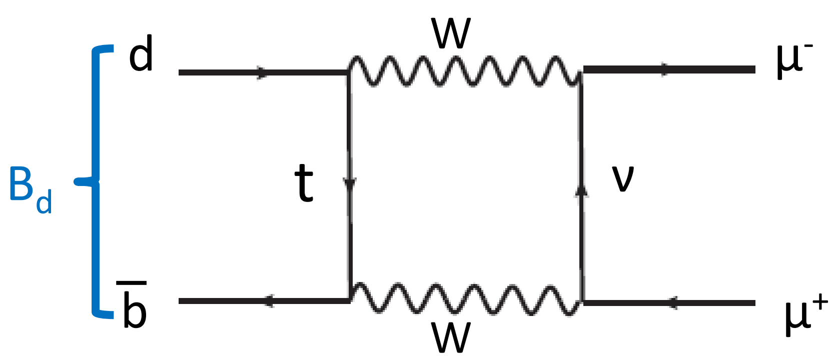
Unitarity Triangle for b → s

# A mild hint for New Physics from $B_d \to \mu^+\mu^-$

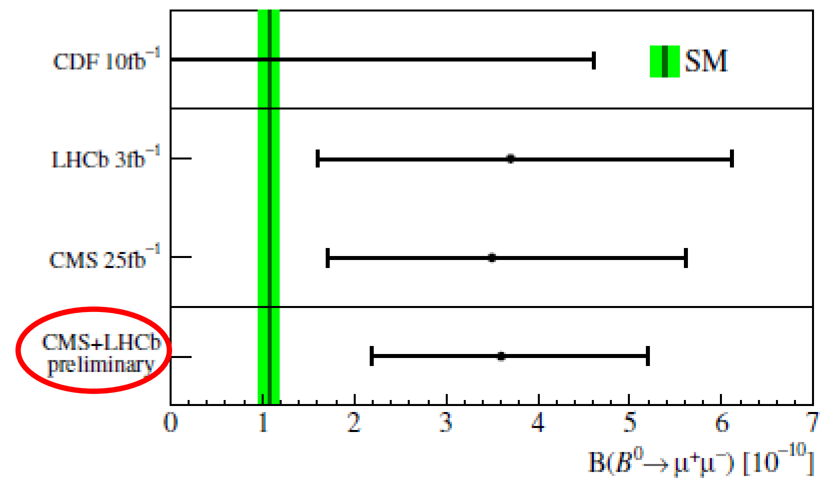
LHCb-CONF-2013-012, CMS-PAS-BPH-13-007

- Also mediated by $b \to d$ current




$B(B_d \to \mu^+\mu^-)$




- Both LHCb and CMS found excess over SM

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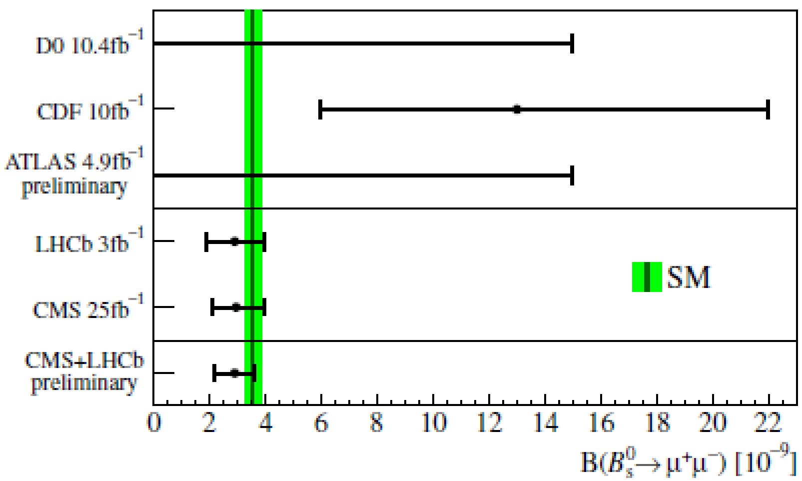


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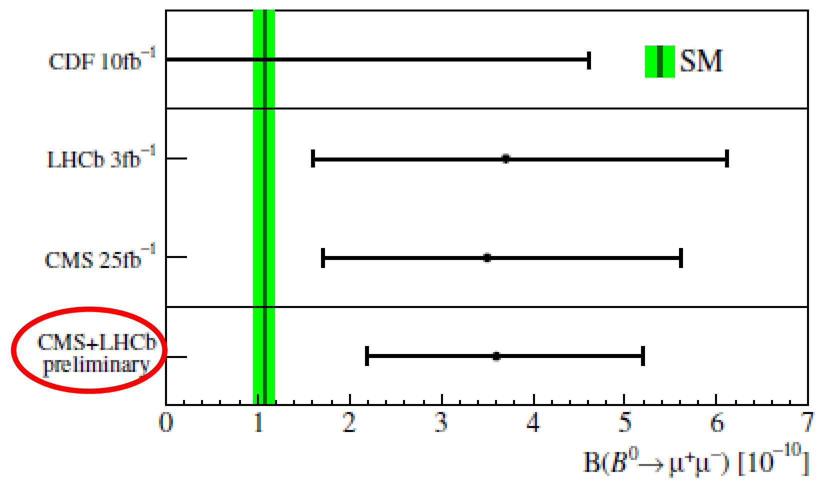



◆ Good agreement with SM

◆ Both LHCb and CMS found excess over SM

# CMS + LHCb (full likelihood combo)

## Fit result

Talk by F. Archilli at CKM2014

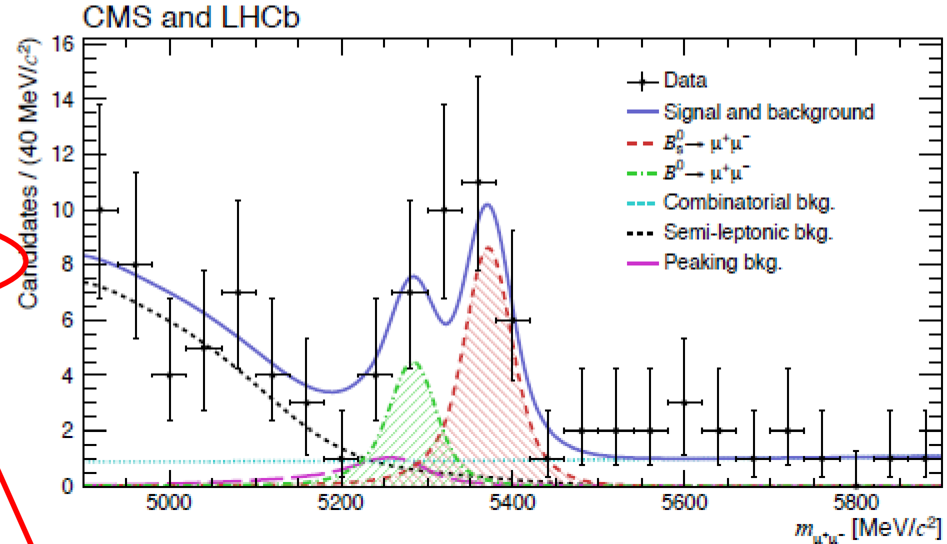
from the simultaneous fit we get:

$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = 2.8^{+0.7}_{-0.6} \times 10^{-9}$$

$$\mathcal{B}(B^0 \to \mu^+\mu^-) = 3.9^{+1.6}_{-1.4} \times 10^{-10}$$

Using the Wilks' theorem the statistical significance from the likelihood is:

- 6.2 $\sigma$ for the $B^0_s \to \mu^+\mu^-$ (Expected SM 7.6 $\sigma$)
  - ✦ First **observation**
- 3.2 $\sigma$ for the $B^0 \to \mu^+\mu^-$ (Expected SM 0.8 $\sigma$)




$$\mathcal{B}(B^0 \to \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}$$ (SM)

Bobeth et al. [PRL 112 (2014) 101801]

◆ Compatibility with SM is 2.2$\sigma$

Wilks' theorem assumes asymptotic behaviour, Feldman-Cousin approach is used for $B^0 \to \mu^+\mu^-$

# Our study on B → $\pi$ $l^+ l^-$

- Theoretical framework
  - Effective Hamiltonian + QCD factorization
- Numerical results for several B → $\pi$ $l^+l^-$ observables
  - Branching Ratio, Direct CP asymmetry, Isospin asymmetry
- Implication for determination of CKM parameters by making projections for near future experimental data

# Effective Hamiltonian for b → d l⁺ l⁻

$$\mathcal{H}_{\rm eff} = -\frac{G_F}{\sqrt{2}}\left[\lambda_t \mathcal{H}_{\rm eff}^{(t)} + \lambda_u \mathcal{H}_{\rm eff}^{(u)}\right] + {\rm h.c.}$$

- 2 parts (top and up parts) after utilizing unitarity of CKM

$$\lambda_t \equiv V_{td}^* V_{tb} \qquad \lambda_u \equiv V_{ud}^* V_{ub}$$

$$\mathcal{H}_{\rm eff}^{(t)} \equiv C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{10} C_i \mathcal{O}_i \qquad \mathcal{H}_{\rm eff}^{(u)} \equiv C_1 \left(\mathcal{O}_1^c - \mathcal{O}_1^u\right) + C_2 \left(\mathcal{O}_2^c - \mathcal{O}_2^u\right)$$

- Important operators

$$\mathcal{O}_7 = -\frac{e\hat{m}_b}{8\pi^2}\bar{d}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$$

$$\mathcal{O}_9 = \frac{\alpha}{2\pi}\left[\bar{d}\gamma^\mu(1-\gamma_5)b\right]\left[\bar{\ell}\gamma_\mu \ell\right]$$

$$\mathcal{O}_{10} = \frac{\alpha}{2\pi}\left[\bar{d}\gamma^\mu(1-\gamma_5)b\right]\left[\bar{\ell}\gamma_\mu \gamma_5 \ell\right]$$

$$\mathcal{O}_2^q = [\bar{d}\gamma^\mu(1-\gamma_5)q][\bar{q}\gamma_\mu(1-\gamma_5)b] \ (q = u, c)$$

- Wilson coefficients at NNLO ($\mu = m_b$)

**Bobeth et al., NPB(2000), Chetyrkin et al., PLB(1997)**

**Gambino et al., NPB(2003); Gorbahn et al., NPB(2005)**

$C_7^{eff} \sim -0.30$

$C_9 \sim 4.3$

$C_{10} \sim -4.2$

$C_2 \sim 1.0$

# Naïve factorization

- B → P $l^+l^-$ amplitude (P = $\pi$) is simply obtained from the case of free quark decay

$$
\begin{aligned}
&i\mathcal{M}(\bar{B} \to P\ell^+\ell^-) \\
&= i\frac{G_F\alpha}{2\sqrt{2}\pi}\Big\{\lambda_t^{(d)\mathrm{SM}}\Big[C_9^{\mathrm{eff}}\langle P|\bar{d}\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle\bar{\ell}\gamma_\mu\ell + C_{10}\langle P|\bar{d}\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle\bar{\ell}\gamma_\mu\gamma_5\ell \\
&\quad - 2\frac{\hat{m}_b}{q^2}C_7^{\mathrm{eff}}\langle P|\bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\bar{B}\rangle\bar{\ell}\gamma_\mu\ell\Big] + \lambda_u^{(d)}Y^{(u)}(q^2)\langle P|\bar{d}\gamma^\mu(1-\gamma_5)b|\bar{B}\rangle\bar{\ell}\gamma_\mu\ell\Big\}
\end{aligned}
$$

- 3 Form Factors:

$$
c_P\langle P(p')|\bar{q}\gamma^\mu b|\bar{B}(p)\rangle = f_+(q^2)\left[p^\mu + p'^\mu - \frac{M_B^2 - m_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2 - m_P^2}{q^2}q^\mu
$$

$$
c_P\langle P(p')|\bar{q}\sigma^{\mu\nu}q_\nu b|\bar{B}(p)\rangle = \frac{if_T(q^2)}{M_B + m_P}[q^2(p^\mu + p'^\mu) - (M_B^2 - m_P^2)q^\mu]
$$

$$
c_P = \begin{cases} 1, & \text{for } \pi^-, \bar{K}^0, K^- \\ -\sqrt{2}, & \text{for } \pi^0. \end{cases}
$$

  - ➢ Needs input from nonperturbative calculations/experimental data

- Misses to include Weak Annihilation term

# QCD factorization

Beneke, Feldmann, NPB592(2001)

- Based on Heavy Quark Effective Theory + Large Energy Effective Theory

  $m_b >> \Lambda_{QCD}$ $m_b \sim E_P >> \Lambda_{QCD}$ (at rest frame of B)

- The 3 FFs ($f_{+,0,T}$) are not independent and can be described by a single FF with known corrections via hard gluon exchange

$$f_+(q^2) \equiv \xi_P(q^2)$$

- Factorization formula (schematic)

$$\langle \ell^+ \ell^- P | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = C_P \xi_P + \Phi_B \otimes T_P \otimes \Phi_P + \mathcal{O}(1/m_b)$$

  - $\Phi_{B,P}$: Light-cone Distribution Amplitude (LDA)
  - 2nd term describes hard-spectator-scattering (weak annihilation, …)

# Valid kinematic range

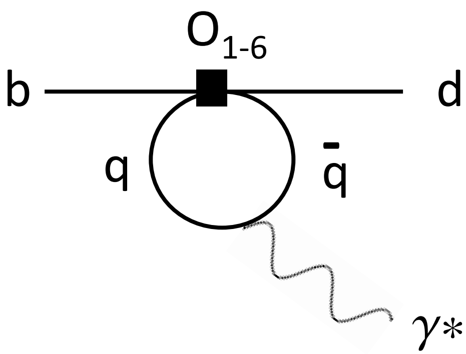
1. Only applicable when dilepton invariant mass ($q^2$) is small

$M_B \sim E_\pi >> \Lambda_{QCD} \rightarrow q^2 << M_B^2$

$\pi$–energy at B-rest frame

$$E_\pi = \frac{M_B}{2}\left(1 - \frac{q^2}{M_B^2} + \frac{m_\pi^2}{M_B^2}\right)$$

2. Nonperturbative effects enter when q qbar in loop is near threshold

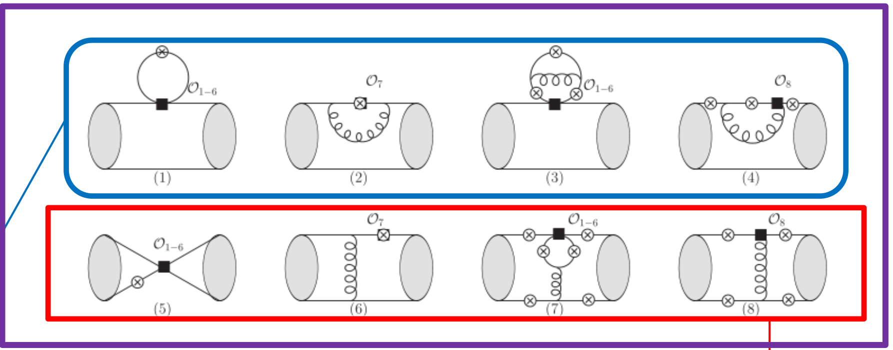



- $q^2 < (2m_c)^2$ (c cbar loop)
- $m_{\rho,\omega}^2 < q^2$ (u ubar loop)

- Practically, we set the valid kinematic range as

$$2\ \text{GeV}^2 < q^2 < 6\ \text{GeV}^2$$

# Formulas




* crossed circles represent photon emission

$$c_P\langle\gamma^*(q,\mu)P(p')|\mathcal{H}_{\text{eff}}^{(i)}|\bar{B}(p)\rangle = -\frac{em_b}{4\pi^2}\frac{\mathcal{T}_P^{(i)}(q^2)}{M_B}[q^2(p^\mu+p'^\mu)-(M_B^2-m_P^2)q^\mu],$$

$$\mathcal{T}_P^{(i)} = \xi_P C_P^{(i)} + \frac{\pi^2}{N_c}\frac{f_B f_P}{M_B}\sum_{\pm}\int_0^\infty \frac{d\omega}{\omega}\Phi_{B,\pm}(\omega)\int_0^1 du\ \phi_P(u)T_{P,\pm}^{(i)}(u,\omega)$$

$$\mathcal{M}(\bar{B}\to P\ell^+\ell^-) = \frac{G_F\alpha}{2\sqrt{2}\pi}c_P^{-1}\xi_P\left[\left(\lambda_t\mathcal{C}_{9,P}^{(t)}+\lambda_u\mathcal{C}_{9,P}^{(u)}\right)(p^\mu+p'^\mu)(\bar{\ell}\gamma_\mu\ell) + \lambda_t C_{10}(p^\mu+p'^\mu)(\bar{\ell}\gamma_\mu\gamma_5\ell)\right]$$

$$\mathcal{C}_{9,P}^{(t)}(q^2) = C_9 + \frac{2m_b}{M_B}\frac{\mathcal{T}_P^{(t)}(q^2)}{\xi_P(q^2)},$$

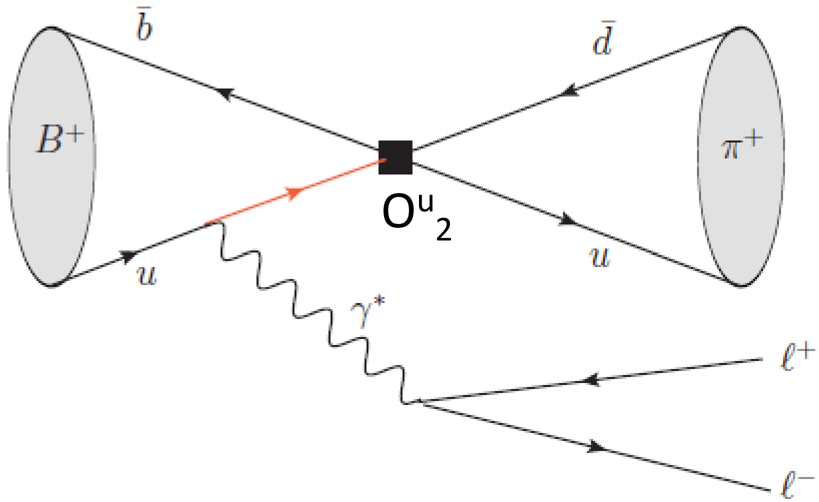
$$\mathcal{C}_{9,P}^{(u)}(q^2) = \frac{2m_b}{M_B}\frac{\mathcal{T}_P^{(u)}(q^2)}{\xi_P(q^2)}.$$

$$\frac{d\Gamma}{dq^2}(\bar{B}\to P\ell^+\ell^-) = S_P\frac{G_F^2M_B^3}{96\pi^3}\left(\frac{\alpha}{4\pi}\right)^2\lambda(q^2,m_P^2)^3\xi_P(q^2)^2\left(|\lambda_t\mathcal{C}_{9,P}^{(t)}(q^2)+\lambda_u\mathcal{C}_{9,P}^{(u)}(q^2)|^2+|\lambda_t|^2C_{10}^2\right)$$

$$S_P = \frac{1}{c_P^2} \qquad c_P = \begin{cases}1, & \text{for } \pi^-, \bar{K}^0, K^- \\ -\sqrt{2}, & \text{for } \pi^0.\end{cases}$$

$$\phi_2 \equiv \alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

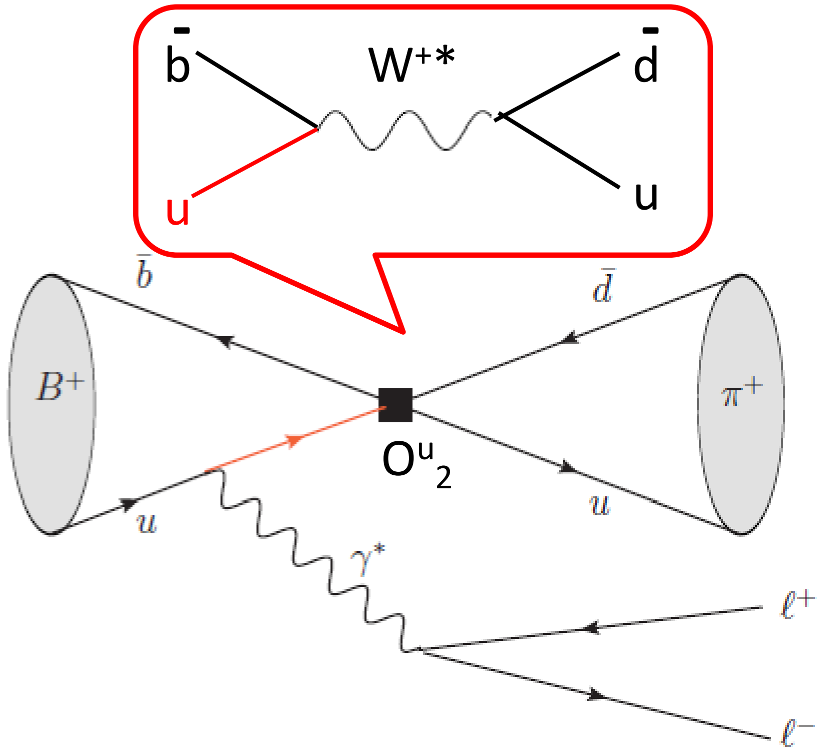
# Weak annihilation




- Microscopically, the dominant process is bbar u → W* → dbar u with photon emitted by u inside B meson --> specific to charged B decay

- Onshell spectator quark provides a strong phase

# Weak annihilation

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# Hadronic inputs

- For B→$\pi$ form factor and $\pi$–LDA, we use QCD sum rule results

**Duplancic, Khodjamirian, Mannel, Melic and Offen, JHEP0804.014**

$$\xi_\pi(q^2) = \frac{\xi_\pi(0)}{(1-q^2/m_{B^*}^2)(1-\alpha_{\rm BK} q^2/m_B^2)}$$

$$\xi_\pi(0) = 0.26^{+0.04}_{-0.03} \qquad \alpha_{\rm BK} = 0.53 \pm 0.06$$

$$\phi_\pi(u) = 6u(1-u)\Big[1 + a_2^\pi\, C_2^{(3/2)}(2u-1) + a_4^\pi\, C_4^{(3/2)}(2u-1) + \cdots\Big]$$

$$a_2^\pi = 0.25 \pm 0.15 \qquad a_4^\pi = -a_2^\pi + (0.1 \pm 0.1)$$

- We adopt model-functions for B-LDAs **Grozin and Neubert, PRD55(1997)**

$$\Phi_{B,+}(\omega) = \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} \qquad \Phi_{B,-}(\omega) = \frac{1}{\omega_0} e^{-\omega/\omega_0}$$

$$\lambda_{B,+}(1.5\ \mathrm{GeV}) = 0.485 \pm 0.115\ \mathrm{GeV}$$

QCDSR [Braun et al., PRD69(2004)]

  - enter amplitude via two moments

$$\lambda_{B,+}^{-1} = \int_0^\infty d\omega \frac{\Phi_{B,+}(\omega)}{\omega} = \omega_0^{-1} \qquad \lambda_{B,-}^{-1}(q^2) = \int_0^\infty d\omega \frac{\Phi_{B,-}(\omega)}{\omega - q^2/M_B - i\epsilon}$$

$$= \frac{e^{-q^2/(M_B\omega_0)}}{\omega_0}\left[-\mathrm{Ei}\left(q^2/M_B\omega_0\right) + \underline{i\pi}\right]$$

# SM predictions

# Branching Ratio for $B^+ \to \pi^+ ll$

- Integrated BR in well-controlled $q^2$ region

$$\int_{2\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} dq^2 \frac{d\mathcal{B}(B^+ \to \pi^+ \ell^+ \ell^-)}{dq^2} = \left(0.44^{+0.03}_{-0.02}\big|_{\mathrm{CKM}} {}^{+0.13}_{-0.10}\big|_{\mathrm{had.}} {}^{+0.02}_{-0.01}\big|_{\mu}\right) \times 10^{-8}$$
$$= \left(0.44^{+0.13}_{-0.10}\right) \times 10^{-8}$$

- Error can be reduced by taking ratio with $B \to \pi l \nu$ rate

$$\frac{d\hat{\mathcal{B}}(B \to \pi \ell^+ \ell^-)}{dq^2} \equiv \frac{d\mathcal{B}(B \to \pi \ell^+ \ell^-)/dq^2}{\mathcal{B}_{\pi\ell\nu}} \mathcal{B}^{\mathrm{exp}}_{\pi\ell\nu}$$

$$\mathcal{B}_{\pi\ell\nu} \equiv \mathcal{B}(B^0 \to \pi^- \ell^+ \nu_\ell)$$
$$= \frac{\tau_{B^0} G_F^2 |V_{ub}|^2 M_B^3}{192\pi^3} \int_{q_i^2}^{q_f^2} dq^2 \lambda(q^2, m_{\pi^-}^2)^3 \xi_\pi(q^2)^2$$

$$\mathcal{B}^{\mathrm{exp}}_{\pi\ell\nu} \equiv \mathcal{B}(B^0 \to \pi^- \ell^+ \nu_\ell)^{\mathrm{exp}}_{q^2 < 12\ \mathrm{GeV}^2}$$
$$= (0.81 \pm 0.02|_{\mathrm{stat.}} \pm 0.03|_{\mathrm{syst.}}) \times 10^{-4}$$

- Improved prediction for Integrated BR

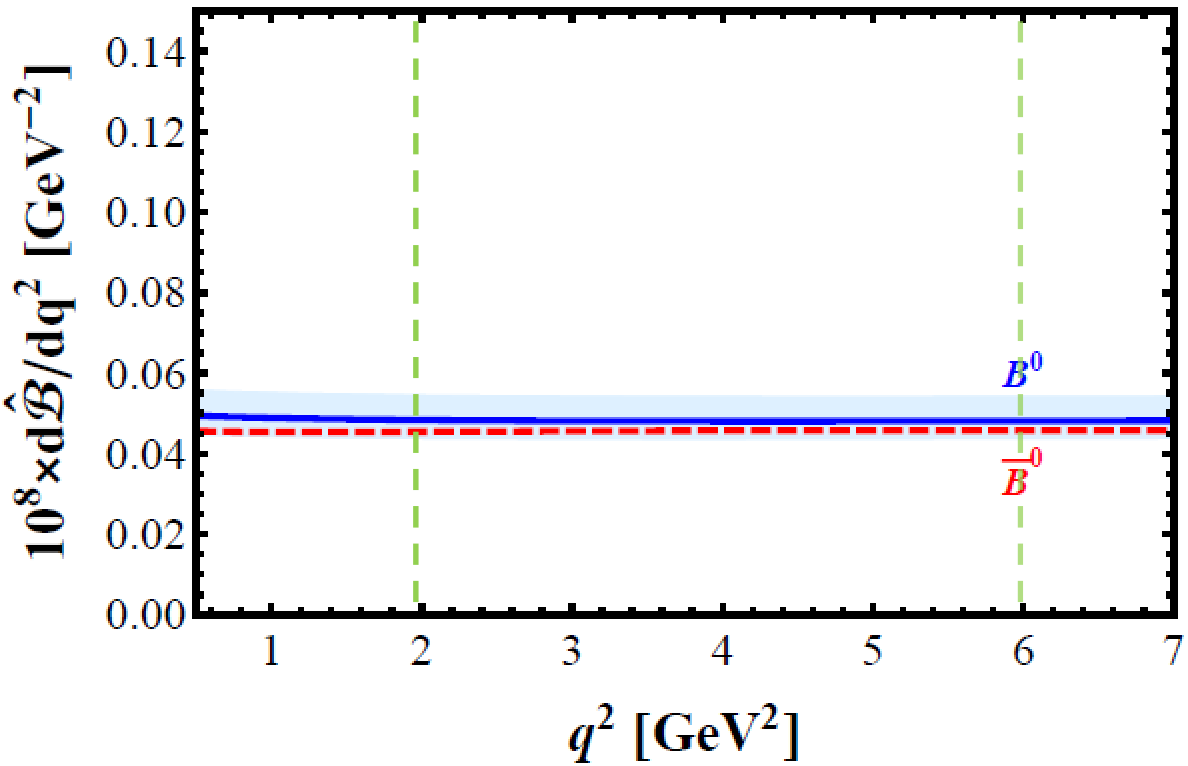
$$\int_{2\ \mathrm{GeV}^2}^{6\ \mathrm{GeV}^2} dq^2 \frac{d\hat{\mathcal{B}}(B^+ \to \pi^+ \ell^+ \ell^-)}{dq^2} = \left(0.47^{+0.05}_{-0.03}\big|_{\mathrm{CKM}} {}^{+0.01}_{-0.01}\big|_{\mathrm{had.}} {}^{+0.02}_{-0.01}\big|_{\mu} {}^{+0.02}_{-0.02}\big|_{\pi\ell\nu}\right) \times 10^{-8}$$
$$= \left(0.47^{+0.06}_{-0.04}\right) \times 10^{-8}$$

# Prediction for BR of all four modes

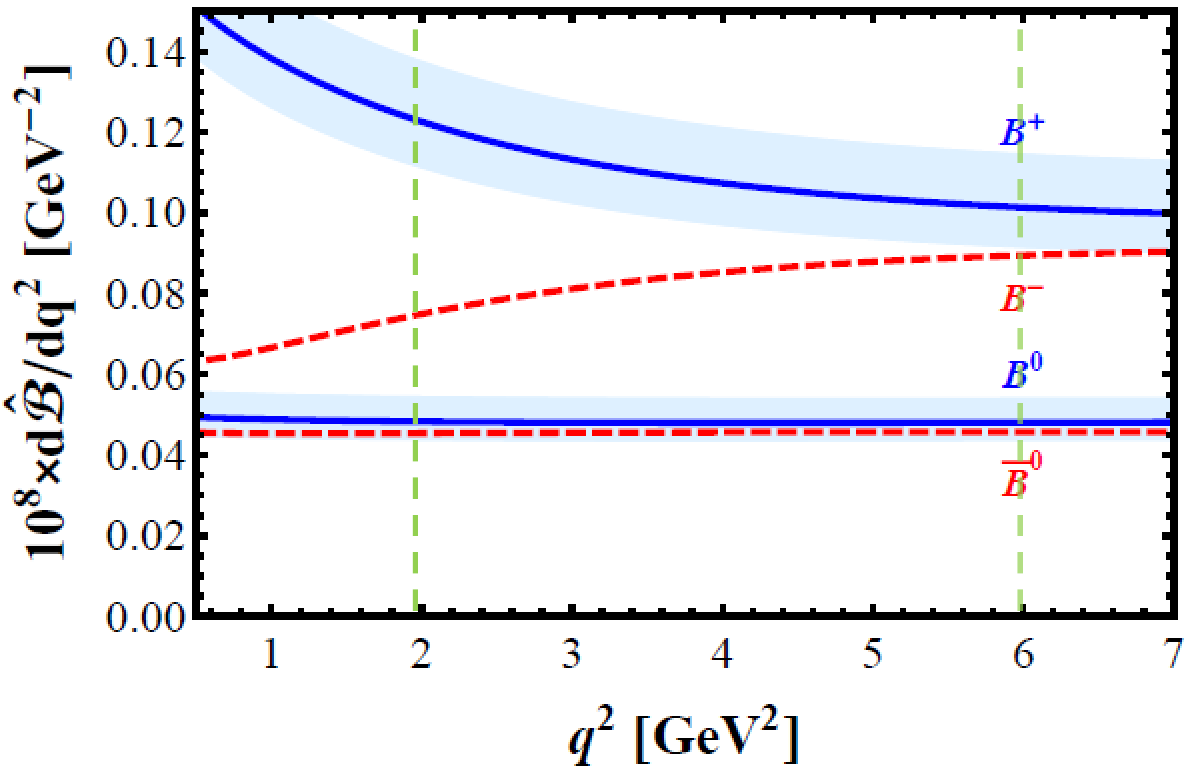
| | Original | Improved |
|---|---|---|
| $B^+ \to \pi^+\ell^+\ell^-$ | $0.44^{+0.03}_{-0.02}\big\vert_{\rm CKM}{}^{+0.13}_{-0.10}\big\vert_{\rm had.}{}^{+0.02}_{-0.01}\big\vert_{\mu}$ | $0.47^{+0.05}_{-0.03}\big\vert_{\rm CKM}{}^{+0.01}_{-0.01}\big\vert_{\rm had.}{}^{+0.02}_{-0.01}\big\vert_{\mu}{}^{+0.02}_{-0.02}\big\vert_{\pi\ell\nu}$ |
| $B^- \to \pi^-\ell^+\ell^-$ | $0.34^{+0.03}_{-0.02}\big\vert_{\rm CKM}{}^{+0.11}_{-0.08}\big\vert_{\rm had.}{}^{+0.02}_{-0.02}\big\vert_{\mu}$ | $0.36^{+0.04}_{-0.03}\big\vert_{\rm CKM}{}^{+0.01}_{-0.01}\big\vert_{\rm had.}{}^{+0.02}_{-0.02}\big\vert_{\mu}{}^{+0.02}_{-0.02}\big\vert_{\pi\ell\nu}$ |
| $B^0 \to \pi^0\ell^+\ell^-$ | $0.18^{+0.01}_{-0.01}\big\vert_{\rm CKM}{}^{+0.06}_{-0.04}\big\vert_{\rm had.}{}^{+0.01}_{-0.01}\big\vert_{\mu}$ | $0.19^{+0.02}_{-0.01}\big\vert_{\rm CKM}{}^{+0.00}_{-0.00}\big\vert_{\rm had.}{}^{+0.01}_{-0.01}\big\vert_{\mu}{}^{+0.01}_{-0.01}\big\vert_{\pi\ell\nu}$ |
| $\bar{B}^0 \to \pi^0\ell^+\ell^-$ | $0.17^{+0.01}_{-0.01}\big\vert_{\rm CKM}{}^{+0.05}_{-0.04}\big\vert_{\rm had.}{}^{+0.01}_{-0.01}\big\vert_{\mu}$ | $0.18^{+0.02}_{-0.01}\big\vert_{\rm CKM}{}^{+0.00}_{-0.00}\big\vert_{\rm had.}{}^{+0.01}_{-0.01}\big\vert_{\mu}{}^{+0.01}_{-0.01}\big\vert_{\pi\ell\nu}$ |

TABLE V. Integrated branching ratios of $B \to \pi\ell\ell$ in unit of $10^{-8}$ for 2 GeV$^2$ $< q^2 <$ 6 GeV$^2$, obtained in two different ways: "original" definition using Eq. (10); "improved" formula of Eq. (24), by taking the ratio with the $B \to \pi\ell\nu$ rate. The scale uncertainty (denoted with subscript $\mu$) is estimated by varying the scale $\mu \in [m_b/2, 2m_b]$.

# Differential Branching Ratios

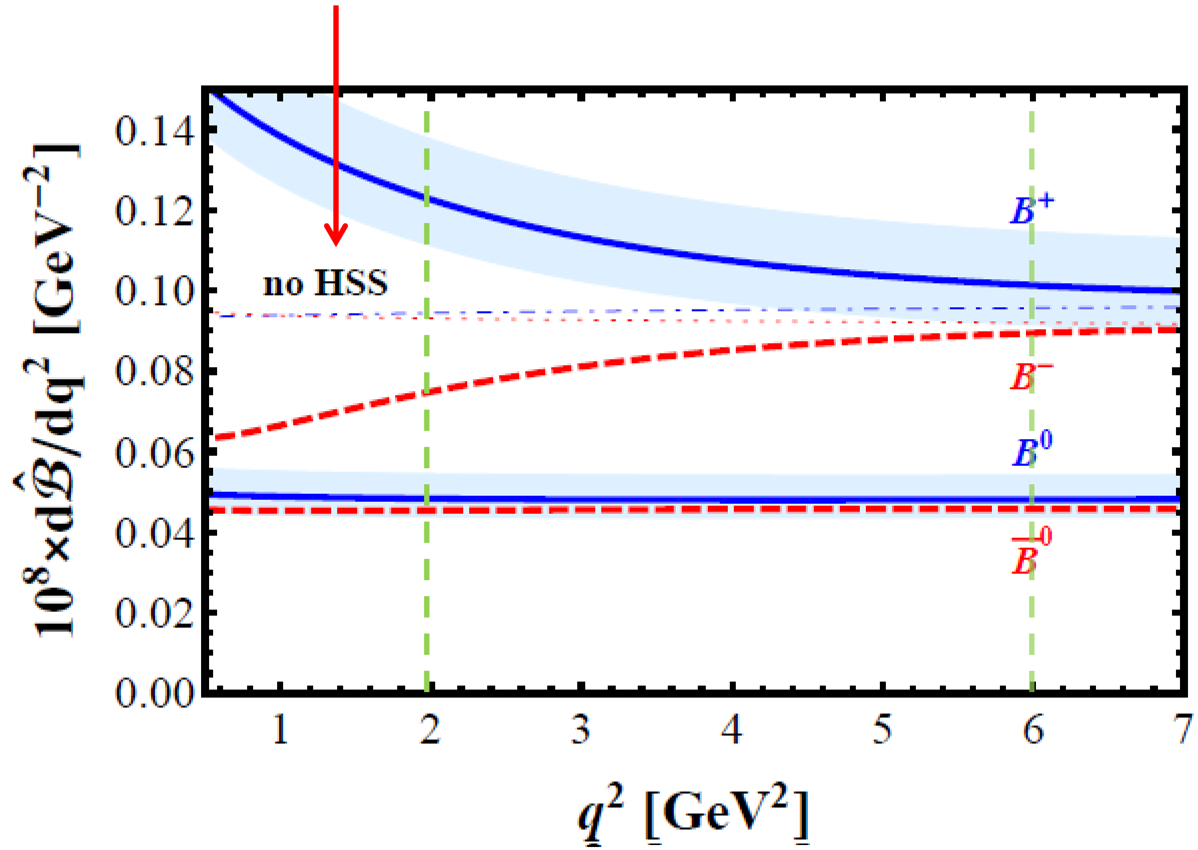


# Differential Branching Ratios



# Differential Branching Ratios

Result without Hard-Spectator-Scattering ~ Naïve fact.

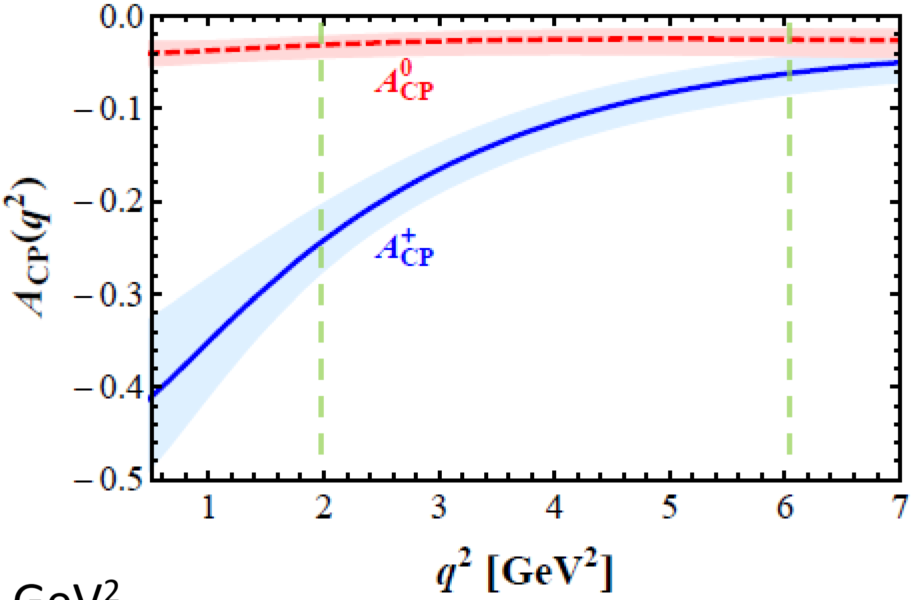


# Direct CP asymmetries

- $q^2$-dependent CP asymmetries

$$A^+_{\rm CP}(q^2) \equiv \frac{d\mathcal{B}(B^- \to \pi^- \ell\ell)/dq^2 - d\mathcal{B}(B^+ \to \pi^+ \ell\ell)/dq^2}{d\mathcal{B}(B^- \to \pi^- \ell\ell)/dq^2 + d\mathcal{B}(B^+ \to \pi^+ \ell\ell)/dq^2},$$

$$A^0_{\rm CP}(q^2) \equiv \frac{d\mathcal{B}(\bar{B}^0 \to \pi^0 \ell\ell)/dq^2 - d\mathcal{B}(B^0 \to \pi^0 \ell\ell)/dq^2}{d\mathcal{B}(\bar{B}^0 \to \pi^0 \ell\ell)/dq^2 + d\mathcal{B}(B^0 \to \pi^0 \ell\ell)/dq^2}.$$




- $q^2$-averaged CP asymmetries in [2,6] GeV$^2$

$$\langle A^+_{\rm CP}\rangle \equiv \frac{\mathcal{B}(B^- \to \pi^- \ell\ell) - \mathcal{B}(B^+ \to \pi^+ \ell\ell)}{\mathcal{B}(B^- \to \pi^- \ell\ell) + \mathcal{B}(B^+ \to \pi^+ \ell\ell)},$$

$$\langle A^0_{\rm CP}\rangle \equiv \frac{\mathcal{B}(\bar{B}^0 \to \pi^0 \ell\ell) - \mathcal{B}(B^0 \to \pi^0 \ell\ell)}{\mathcal{B}(\bar{B}^0 \to \pi^0 \ell\ell) + \mathcal{B}(B^0 \to \pi^0 \ell\ell)},$$

$$\langle A^+_{\rm CP}\rangle = -0.13^{+0.01}_{-0.01}\Big|_{\rm CKM}{}^{+0.02}_{-0.02}\Big|_{\rm had.}{}^{+0.01}_{-0.02}\Big|_{\mu} = -0.13^{+0.02}_{-0.03},$$

$$\langle A^0_{\rm CP}\rangle = -0.03^{+0.00}_{-0.00}\Big|_{\rm CKM}{}^{+0.00}_{-0.00}\Big|_{\rm had.}{}^{+0.01}_{-0.02}\Big|_{\mu} = -0.03^{+0.01}_{-0.02},$$

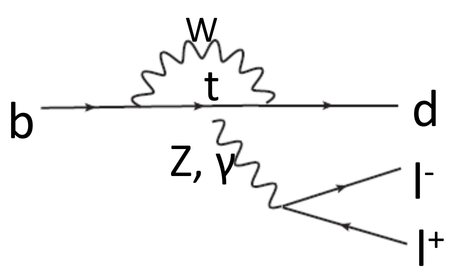
# Mechanism of large ACP$^+$ at low $q^2$

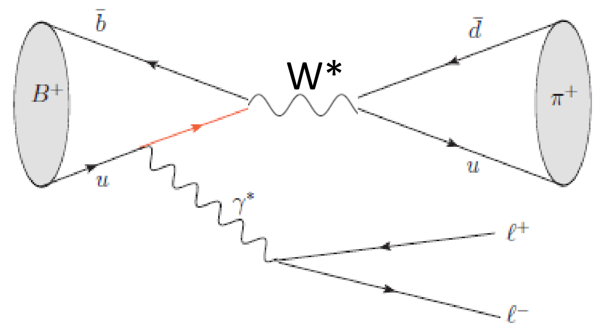
- In general, to have large direct ACP, amplitude should contain two terms with different CP-odd (weak) phase and CP-even (strong) phase

$$\mathcal{M} = A_1 + A_2 e^{i(\delta+\phi)} \qquad \overline{\mathcal{M}} = A_1 + A_2 e^{i(\delta-\phi)}$$

CP asymmetry: $$|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2 = -4 A_1 A_2 \sin\delta \sin\phi$$

- Satisfied by interference between EW penguin/box and Weak Annihilation




$$\lambda_t \equiv V_{td}^* V_{tb}$$

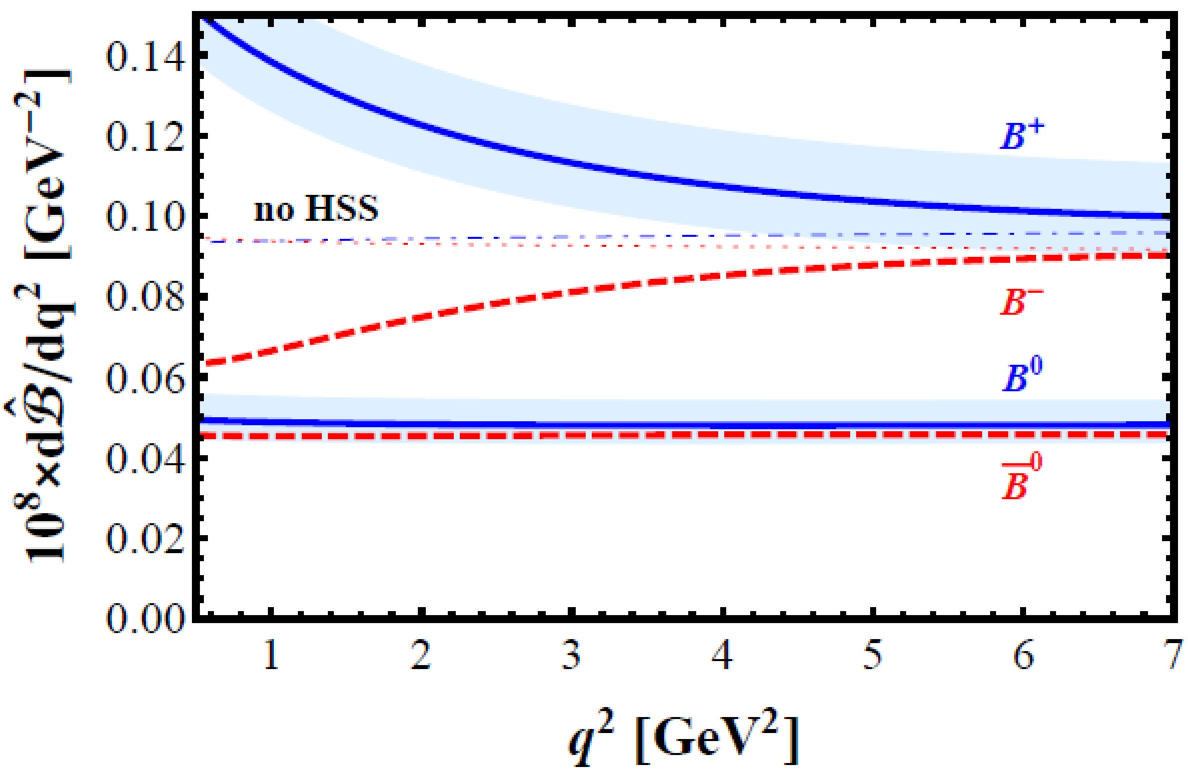
$\longleftrightarrow$ $\phi_2 = \alpha$

$$\lambda_u \equiv V_{ud}^* V_{ub}$$

N.B. negligible for b-->s

# Measuring large ACP$^+$ at low $q^2$

- Current LHCb result ($1 fb^{-1}$) only provides CP-averaged BR in full $q^2$ range
- Thus, hard to discriminate QCDF and Naïve fact. results
- LHCb with full 2011-2012 data or future data should be able to measure

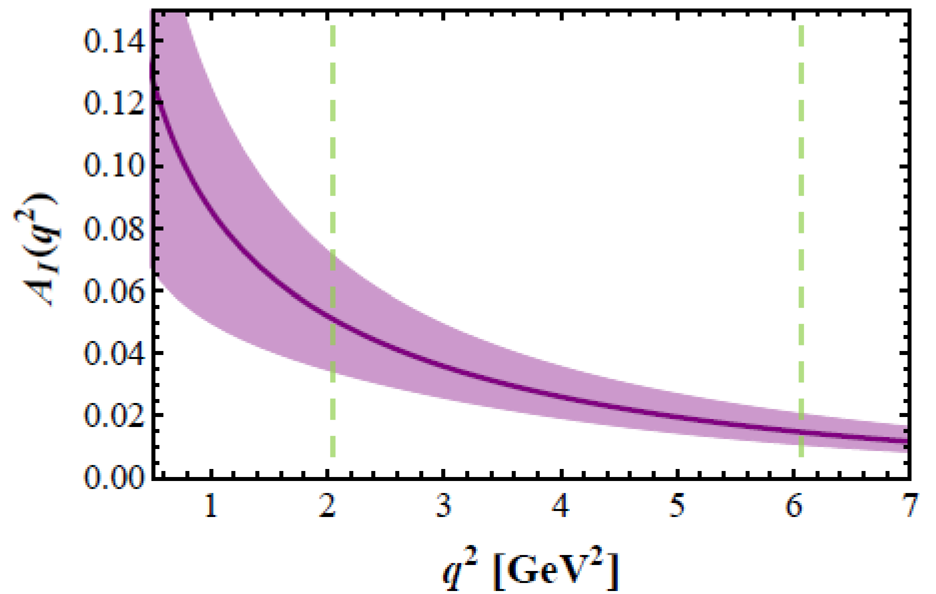


# Isospin asymmetry

- $q^2$-dependent isospin asymmetry

$$A_{\rm I}(q^2) \equiv \frac{\tau_{B^0}}{2\tau_{B^\pm}} \frac{d\overline{\mathcal{B}}(B^+ \to \pi^+ \ell\ell)/dq^2}{d\overline{\mathcal{B}}(B^0 \to \pi^0 \ell\ell)/dq^2} - 1$$

  - use CP-averaged rates



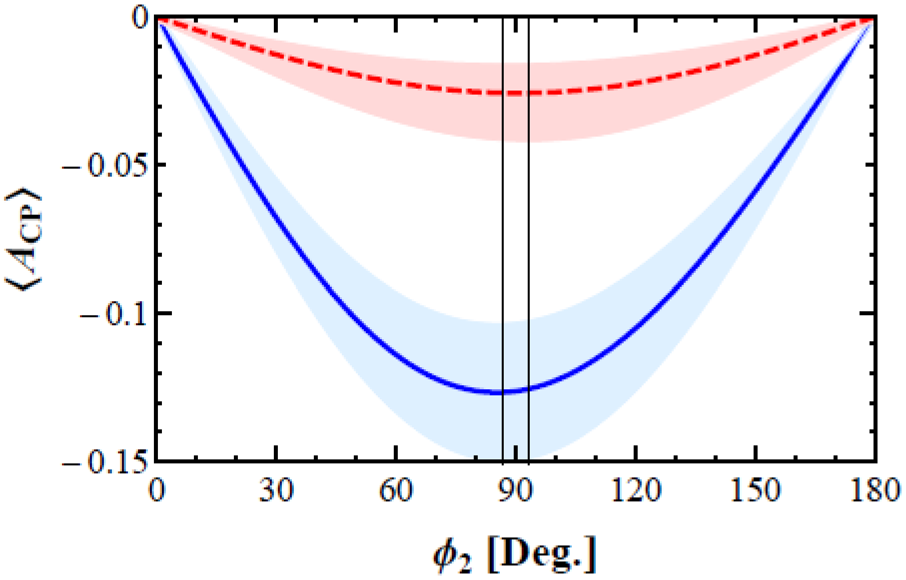

- $q^2$-averaged isospin asymmetry

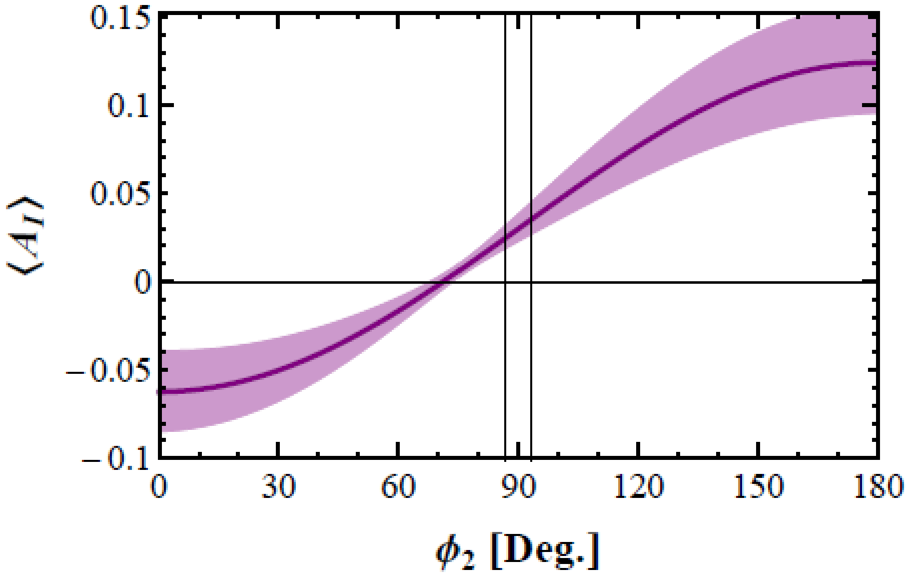
$$\langle A_{\rm I} \rangle \equiv \frac{\tau_{B^0}}{2\tau_{B^\pm}} \frac{\overline{\mathcal{B}}(B^+ \to \pi^+ \ell\ell)}{\overline{\mathcal{B}}(B^0 \to \pi^0 \ell\ell)} - 1$$

$$\langle A_{\rm I} \rangle = 0.03^{+0.01}_{-0.00}\Big|_{\rm CKM}{}^{+0.01}_{-0.01}\Big|_{\rm had.}{}^{+0.00}_{-0.00}\Big|_{\mu} = 0.03 \pm 0.01.$$

# Weak phase dependence

- $\phi_2 = \alpha$ dependence of asymmetries for $R_{ut} = |\lambda_u/\lambda_t| = 0.39$

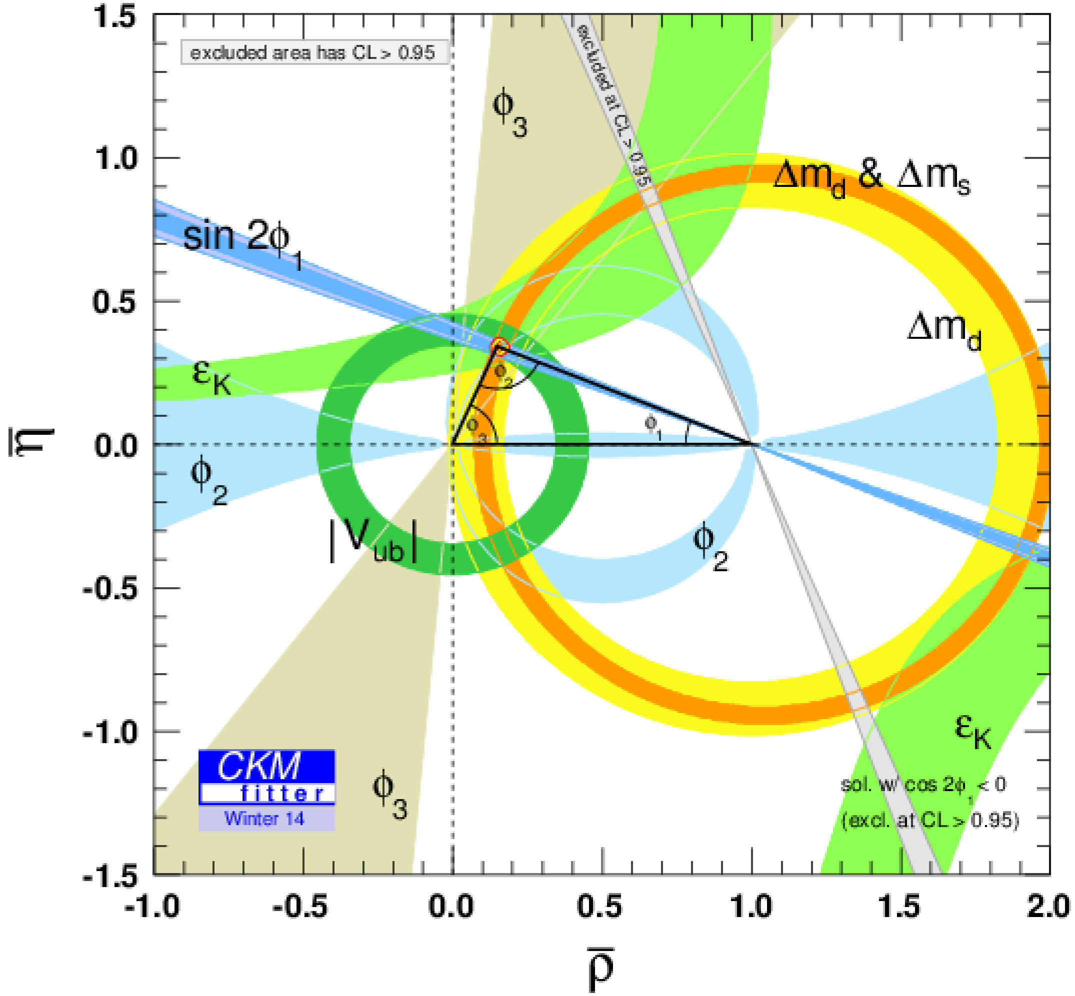



➢ Maximized nearby global fit value

➢ global fit value locates near vanishing point

# Future implication for CKM parameters



# Other useful observable

- Had. error can be reduced by taking ratio with B → K ll rate

$$R_+ \equiv \frac{\overline{\mathcal{B}}(B^+ \to \pi^+ \ell^+ \ell^-)}{\overline{\mathcal{B}}(B^+ \to K^+ \ell^+ \ell^-)}$$

$$\frac{\xi_K(0)}{\xi_\pi(0)} = 1.38^{+0.11}_{-0.10}$$

Duplancic and Melic, PRD78(2008)

- Our QCDF prediction in [2,6] $\mathrm{GeV}^2$ region

$$R_+ = \left|\frac{V_{td}}{V_{ts}}\right|^2 F_+^2 \left[1 - c_+ R_{ut} \cos\phi_2 + d_+ R_{ut}^2\right]$$

$$F_+^2 = 0.58^{+0.09}_{-0.08}, \quad c_+ = 0.25^{+0.07}_{-0.06}, \quad d_+ = 0.13^{+0.04}_{-0.03}.$$

- Can be used to measure $|Vtd/Vts|^2$

# A possible info on UT with Run 2 data

- Projection aiming for future LHCb result with Run 2 data (~ 5 $fb^{-1}$) assuming SM-like central values (due to naïve scaling of current statistical error)

$$\overline{\mathcal{B}}(B^+ \to \pi^+ \ell^+ \ell^-)^{\rm exp} = (0.42 \pm 0.04) \times 10^{-8}$$

$$R_+^{\rm exp} = 0.027 \pm 0.003 \quad \langle A_{\rm CP}^+ \rangle^{\rm exp} = -0.13 \pm 0.10$$

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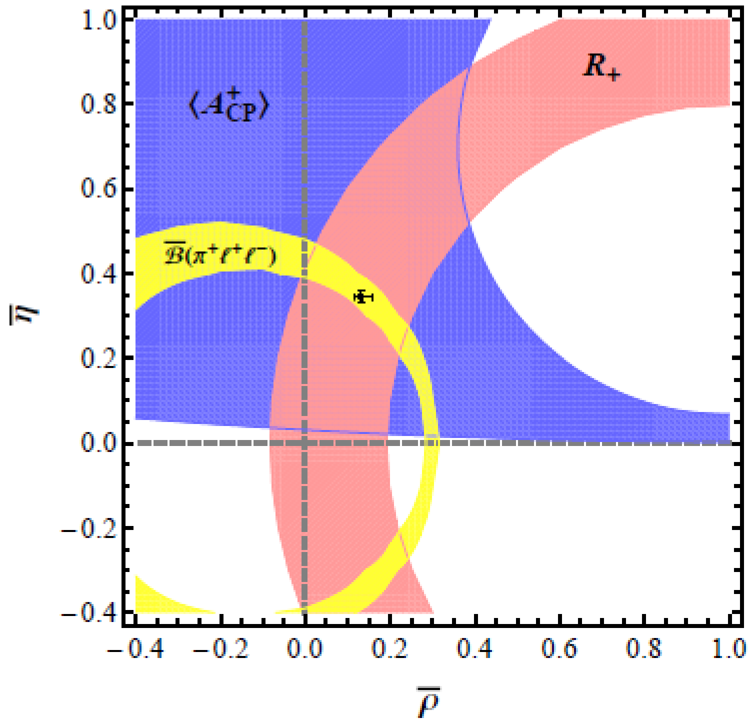



* Black dot with error bars corresponds to the current global fit result

* TH and EXP errors are added linearly

# Summary

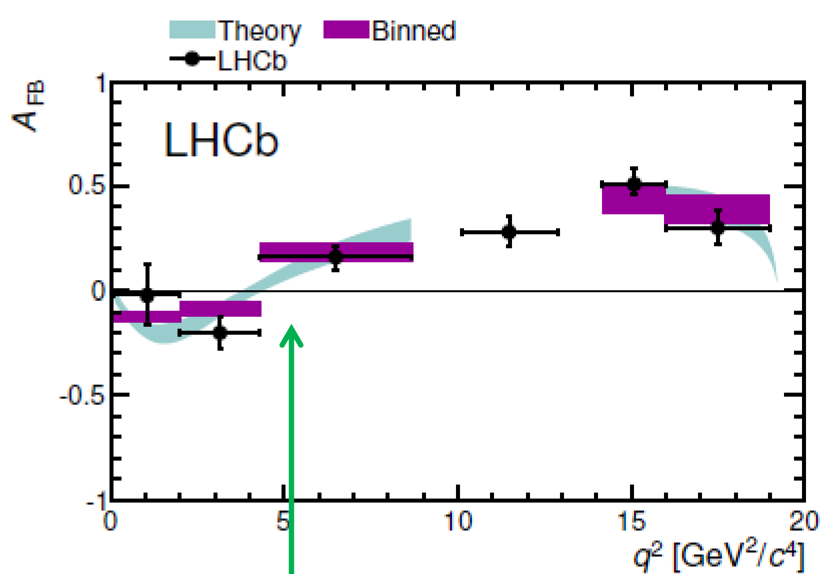
- FCNC decay $B^+ \to \pi^+ l^+l^-$ is a promising mode at LHCb
- We studied $B \to \pi ll$ in SM based on QCDF for low $q^2$
- We predicted BRs rather accurately (~10%), by taking ratio with $B \to \pi l \nu$
- We found large direct CP asymmetry (~-30%) in charged B decay at small $q^2$, which is generated by Weak-annihilation
- $B^+ \to \pi^+ l^+l^-$ observables alone can still provide meaningful information on Unitarity Triangle
- Near future LHCb data should be able to discriminate QCDF prediction from Naïve factorization predictions for charged B decays
- Extension to New Physics study is possible

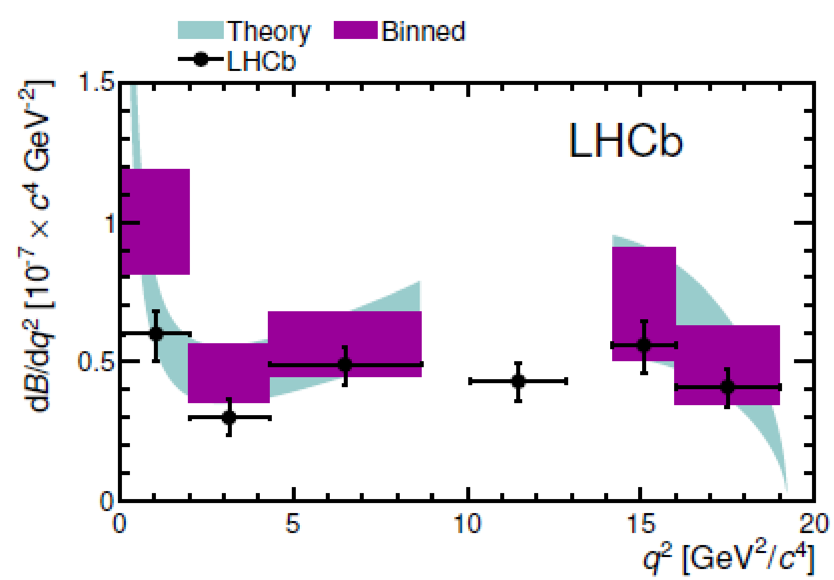
***Back Up Slides***

# $A_{FB}(B \to K^* \mu^+ \mu^-)$ at LHC

LHCb, JHEP1308.131 (7 TeV, 1 $fb^{-1}$)

- AFB is in good agreement with SM




Indicating zero-crossing $q_0^2 = 4.9 \pm 0.9\,\mathrm{GeV}^2$
(consistent with SM prediction)

# Input parameters

| | | | |
|---|---|---|---|
| $\alpha$ | $1/137$ | $\lambda$ | $0.22535 \pm 0.00065$ |
| $\sin^2\theta_W$ | $0.23$ | $A$ | $0.811^{+0.022}_{-0.012}$ |
| $G_F$ | $1.166 \times 10^{-5}$ GeV$^{-2}$ | $\bar{\rho}$ | $0.131^{+0.026}_{-0.013}$ |
| $\alpha_s(M_Z)$ | $0.1184 \pm 0.0007$ | $\bar{\eta}$ | $0.345^{+0.013}_{-0.014}$ |
| $M_W$ | 80.4 GeV | $\xi_\pi(0)$ | $0.26^{+0.04}_{-0.03}$ [31] |
| $M_Z$ | 91.2 GeV | $\alpha_{\rm BK}$ | $0.53 \pm 0.06$ [31] |
| $m_{t,\rm pole}$ | 173.5 GeV | $a_2^\pi$ | $0.25 \pm 0.15$ [32] |
| $m_{b,\rm PS}(2\text{ GeV})$ | $(4.6 \pm 0.1)$ GeV [29] | $a_4^\pi$ | $-a_2^\pi + (0.1 \pm 0.1)$ [33] |
| $m_{c,\rm pole}$ | 1.67 GeV | $\xi_K(0)$ | $0.36^{+0.05}_{-0.04}$ [34] |
| $f_\pi$ | $(130.41 \pm 0.03 \pm 0.20)$ MeV | $a_1^K$ | $0.10 \pm 0.04$ [35] |
| $f_K$ | $(156.1 \pm 0.2 \pm 0.8 \pm 0.2)$ MeV | $a_2^K$ | $0.25 \pm 0.15$ [32, 34] |
| $f_B$ | $(190.6 \pm 4.7)$ MeV [30] | $\xi_K(0)/\xi_\pi(0)$ | $1.38^{+0.11}_{-0.10}$ [34] |
| $\tau_{B^0}$ | $1.52 \times 10^{-12}$ s | $\lambda_{B,+}(1.5\text{ GeV})$ | $(0.485 \pm 0.115)$ GeV [15, 36] |
| $\tau_{B^\pm}$ | $1.64 \times 10^{-12}$ s | $\mathcal{B}^{\rm exp}_{\pi\ell\nu}$ | $(0.81 \pm 0.02 \pm 0.03) \times 10^{-4}$ [37] |

TABLE II. Summary of input parameters, taken from the Particle Data Group [17] unless otherwise stated. We use three-loop running for QCD coupling $\alpha_s$ with the listed initial value $\alpha_s(M_Z)$. Our treatment of $m_b$ follows Ref. [12, 15] by choosing the potential-subtracted (PS) mass [29] as input. We define $\mathcal{B}^{\rm exp}_{\pi\ell\nu} = \mathcal{B}(B^0 \to \pi^- \ell^+ \nu)^{\rm exp}_{q^2 < 12\text{ GeV}^2}$ as described in the text. The errors explicitly shown are taken into account in our error analysis.

# Wilson Coefficients

- We adopted Next-to-next-to-leading logarithmic (NNLL) formula for $C_9$
  - 2-loop matching condition **Bobeth, Misiak and Urban, NPB(2000)**
  - 3-loop anomalous dimension matrix **Chetyrkin, Misiak and Munz, PLB(1997)**
    **Gambino, Gorbahn and Haisch, NPB(2003); Gorbahn and Haisch, NPB(2005)**

- Wilson coefficients at $\mu$ = $m_b$

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ | $C_7^{\rm eff}$ | $C_8^{\rm eff}$ | $C_9$ | $C_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| LL | $-0.5093$ | 1.0256 | $-0.0050$ | $-0.0686$ | 0.0005 | 0.0010 | $-0.3189$ | $-0.1505$ | 2.0111 | 0 |
| NLL | $-0.3001$ | 1.0080 | $-0.0047$ | $-0.0827$ | 0.0003 | 0.0009 | $-0.2969$ | $-0.1642$ | 4.1869 | $-4.3973$ |
| NNLL | – | – | – | – | – | – | – | – | 4.2607 | $-4.2453$ |

N.B. These Wilson coefficients include $O(\alpha_s)$ terms

- We evaluate hadronic matrix elements based on Heff with these WC

# Breakdown of B→$\pi$ $l^+l^-$ amplitudes

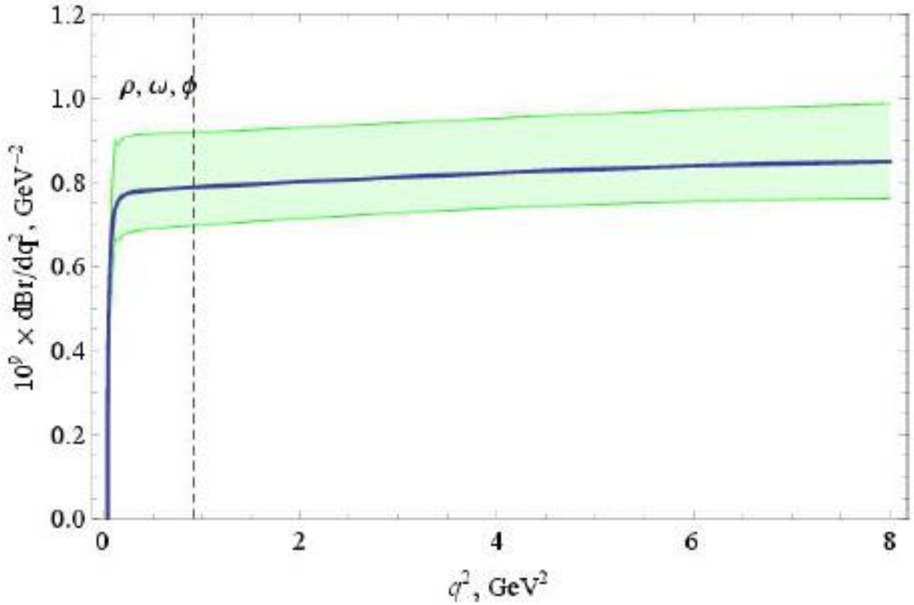
| | $q^2 = 2$ GeV$^2$ | | $q^2 = 5$ GeV$^2$ | |
|---|---|---|---|---|
| | $\mathcal{C}_{9,P}^{(t)}$ | $\mathcal{C}_{9,P}^{(u)}$ | $\mathcal{C}_{9,P}^{(t)}$ | $\mathcal{C}_{9,P}^{(u)}$ |
| $C_9$ | $4.26$ | $0$ | $4.26$ | $0$ |
| $Y(q^2)$ | $0.38 + 0.06i$ | $-0.50 - 0.85i$ | $0.44 + 0.06i$ | $-0.17 - 0.85i$ |
| $aC_7^{\rm eff}$ | $-0.52$ | $0$ | $-0.52$ | $0$ |
| $C^{(1)}$ | $-0.24 + 0.01i$ | $0.24 + 0.77i$ | $-0.27 - 0.01i$ | $0.03 + 0.66i$ |
| $T^{(0)}$ $(\pi^-)$ | $0.03 - 0.08i$ | $1.06 - \underline{2.58i}$ | $0.03 - 0.02i$ | $1.01 - 0.67i$ |
| $(\pi^0)$ | $-0.02 + 0.04i$ | $0.11 - 0.26i$ | $-0.02 + 0.01i$ | $0.10 - 0.07i$ |
| $T^{(1)}$ $(\pi^-)$ | $0.03 - 0.01i$ | $-0.08 - 0.03i$ | $0.02 - 0.01i$ | $-0.04 - 0.02i$ |
| $(\pi^0)$ | $0.01 - 0.01i$ | $-0.05 + 0.00i$ | $0.01 - 0.01i$ | $-0.04 - 0.01i$ |
| sum $(\pi^-)$ | $3.95 - 0.06i$ | $0.73 - 2.69i$ | $3.97 + 0.03i$ | $0.84 - 0.88i$ |
| $(\pi^0)$ | $3.87 + 0.10i$ | $-0.20 - 0.34i$ | $3.91 + 0.05i$ | $-0.08 - 0.26i$ |

TABLE IV. Numerical values and breakdowns for the amplitudes $\mathcal{C}_{9,P}^{(t,u)}$ at $q^2 = 2$ GeV$^2$ and 5 GeV$^2$, for $B \to \pi\ell\ell$ (Table 5 in Ref. [15] gives analogous values for $B \to \rho\ell\ell$ at $q^2 = 5$ GeV$^2$). Each term is classified into two categories: (1) form factor term, which includes $C_9$, $Y(q^2)$, $aC_7^{\rm eff} \equiv (2m_b/M_B)C_7^{\rm eff}$, and the $\mathcal{O}(\alpha_s)$ correction $C^{(1)}$; (2) hard-spectator-scattering term, which includes weak annihilation $T^{(0)}$ (with the main source of strong phase underlined) and $\mathcal{O}(\alpha_s)$ hard-gluon-exchange $T^{(1)}$ terms. The "sum" represents the numerical values of $\mathcal{C}_{9,P}^{(t,u)}$ themselves. Following the argument of Ref. [15] for $\mathcal{C}_{9,\parallel}^{(t,u)}$, we do not include $1/m_b$ corrections to the second category. See Appendix A for details.

# Comparison with literature

- NF prediction by Ali et al., arXiv:1312.2523 (CP averaged)

$$\mathcal{B}(B^+ \to \pi^+\mu^+\mu^-;\, 1\,\mathrm{GeV}^2 \le q^2 \le 8\,\mathrm{GeV}^2) = \left(0.57^{+0.07}_{-0.05}\right) \times 10^{-8}$$




- Our prediction in same range (CP non-averaged)

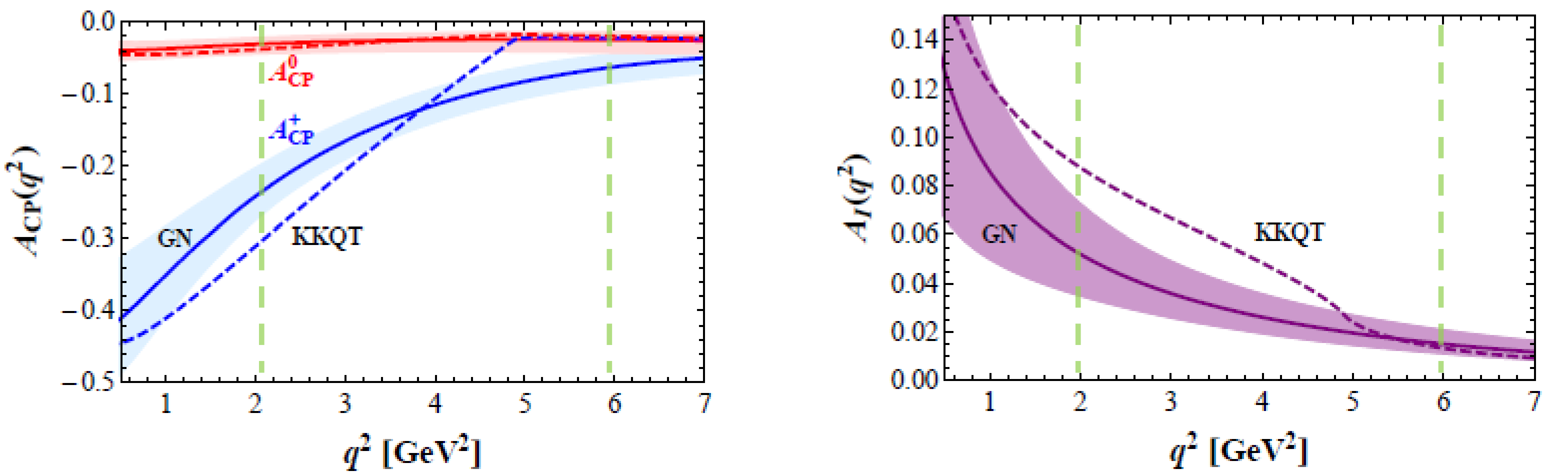
$$\hat{\mathcal{B}}(B^+ \to \pi^+\mu^+\mu^-) = (0.82^{+0.10}_{-0.07})10^{-8} \quad \hat{\mathcal{B}}(B^- \to \pi^-\mu^+\mu^-) = (0.63^{+0.09}_{-0.07})10^{-8}$$

# B-LDA model dependence

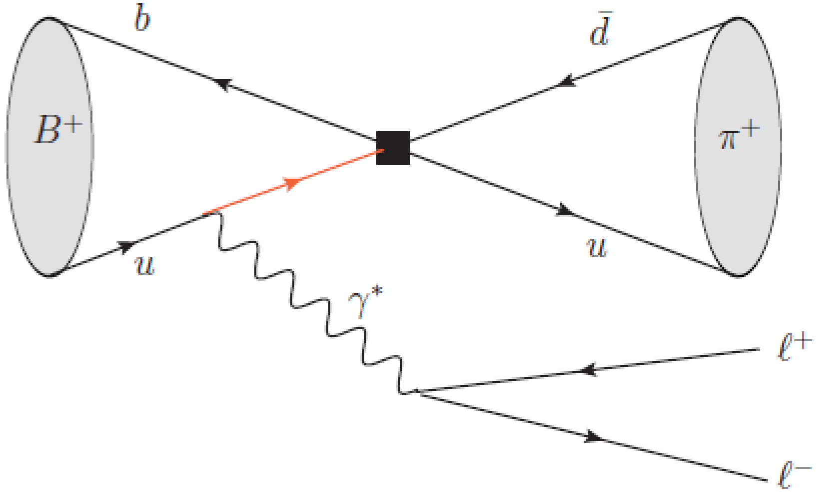
- Another B-LDA model **Kawamura-Kodaira-Qiao-Tanaka, PLB523(2001)**

$$\Phi_{B,+}^{\mathrm{KKQT}}(\omega) = \frac{\omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda}-\omega), \quad \Phi_{B,-}^{\mathrm{KKQT}}(\omega) = \frac{2\bar{\Lambda}-\omega}{2\bar{\Lambda}^2}\theta(2\bar{\Lambda}-\omega),$$

- KKQT vs. GN (used in main part of this talk)



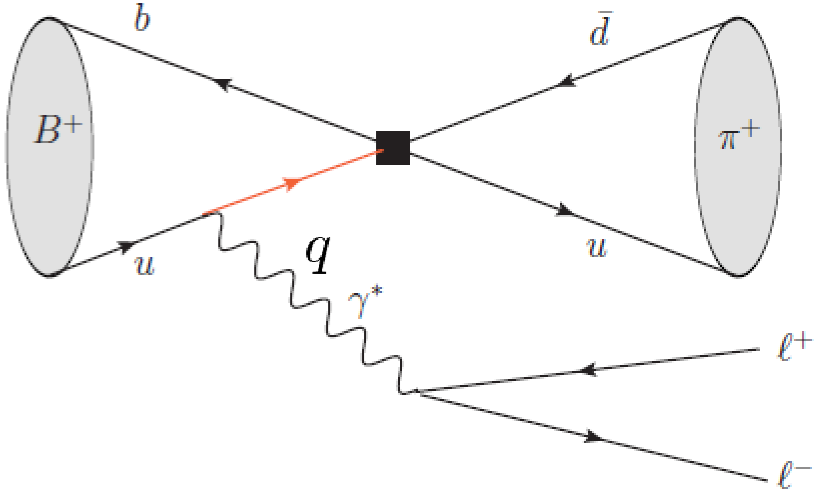
# Weak annihilation



# Weak annihilation

- Kinematics at B-meson rest frame

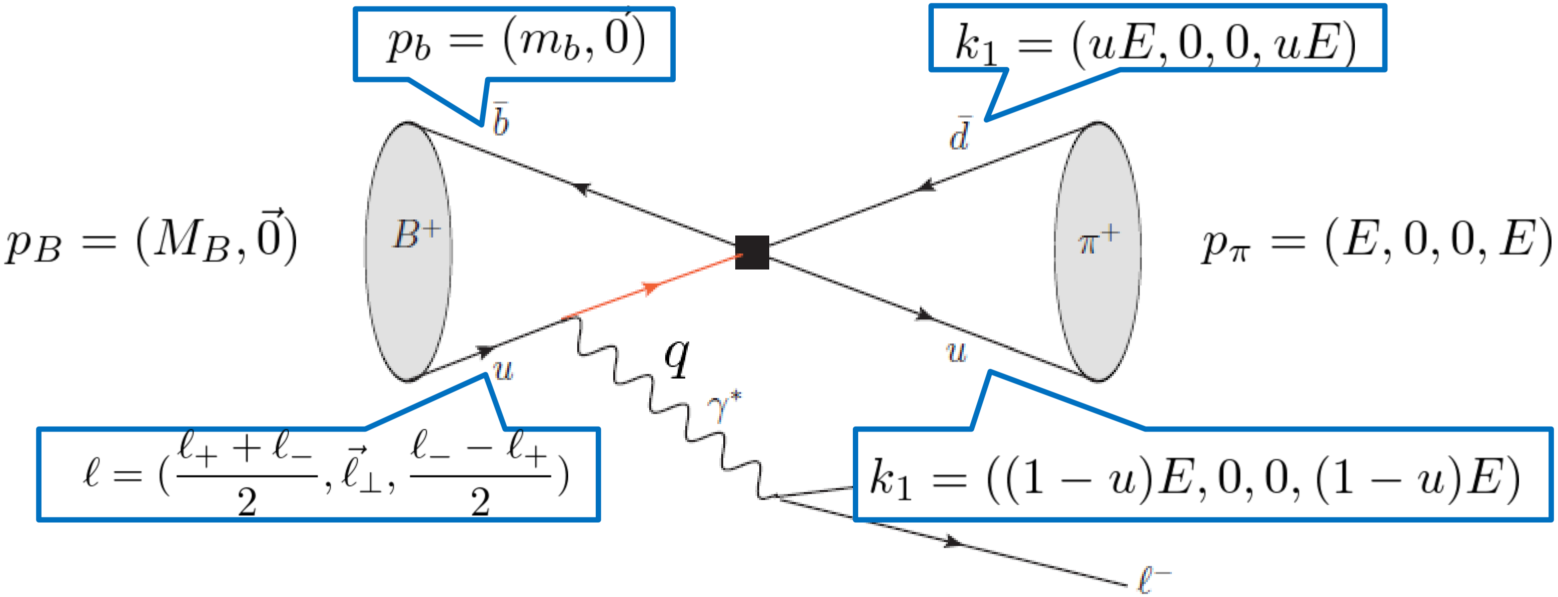
$p_B = (M_B, \vec{0})$




$p_\pi = (E, 0, 0, E)$

# Weak annihilation

- Kinematics at B-meson rest frame in heavy quark and large recoil limit




  - QCD scale terms are neglected except spectator quark momentum

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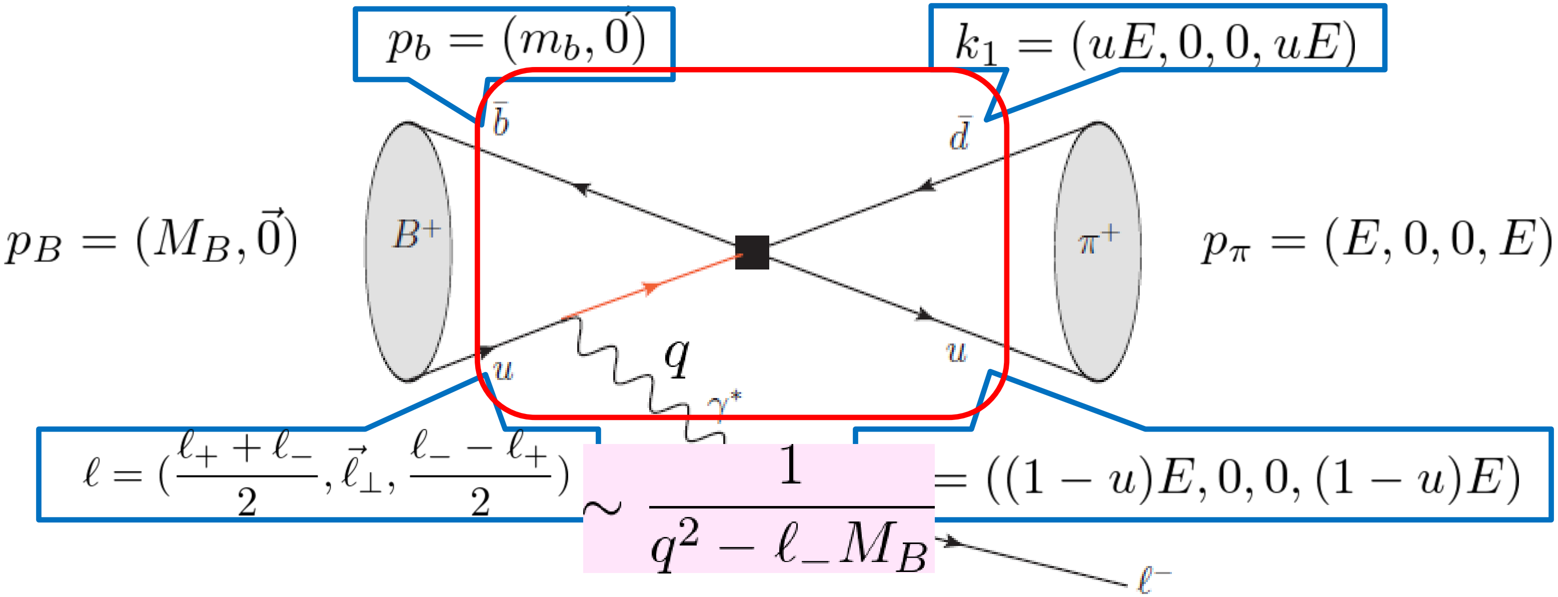

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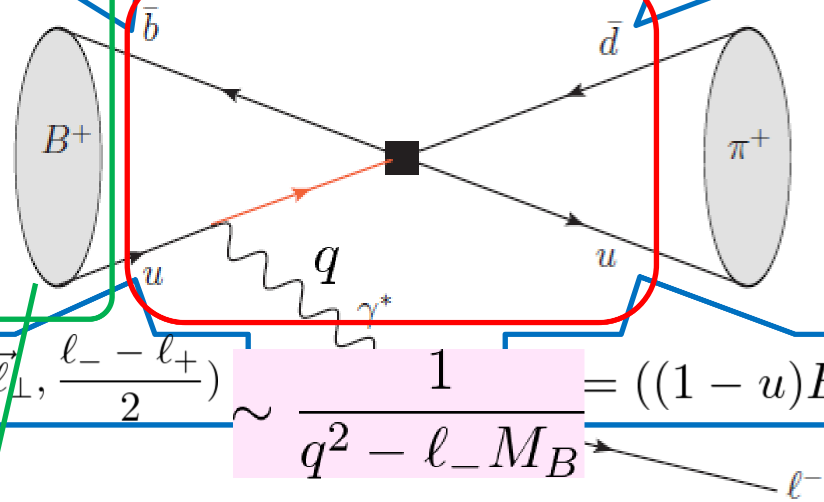
$p_b = (m_b, \vec{0})$

$k_1 = (uE, 0, 0, uE)$

$p_B = (M_B, \vec{0})$




$p_\pi = (E, 0, 0, E)$

$\ell = (\frac{\ell_+ + \ell_-}{2}, \vec{\ell}_\perp, \frac{\ell_- - \ell_+}{2})$

$= ((1-u)E, 0, 0, (1-u)E)$

$\sim \frac{1}{q^2 - \ell_- M_B}$

$$\sim \int_0^\infty d\ell_- \; [\Phi_{B,+}(\ell_-) \not{n}_+ + \Phi_{B,-}(\ell_-) \not{n}_-]$$

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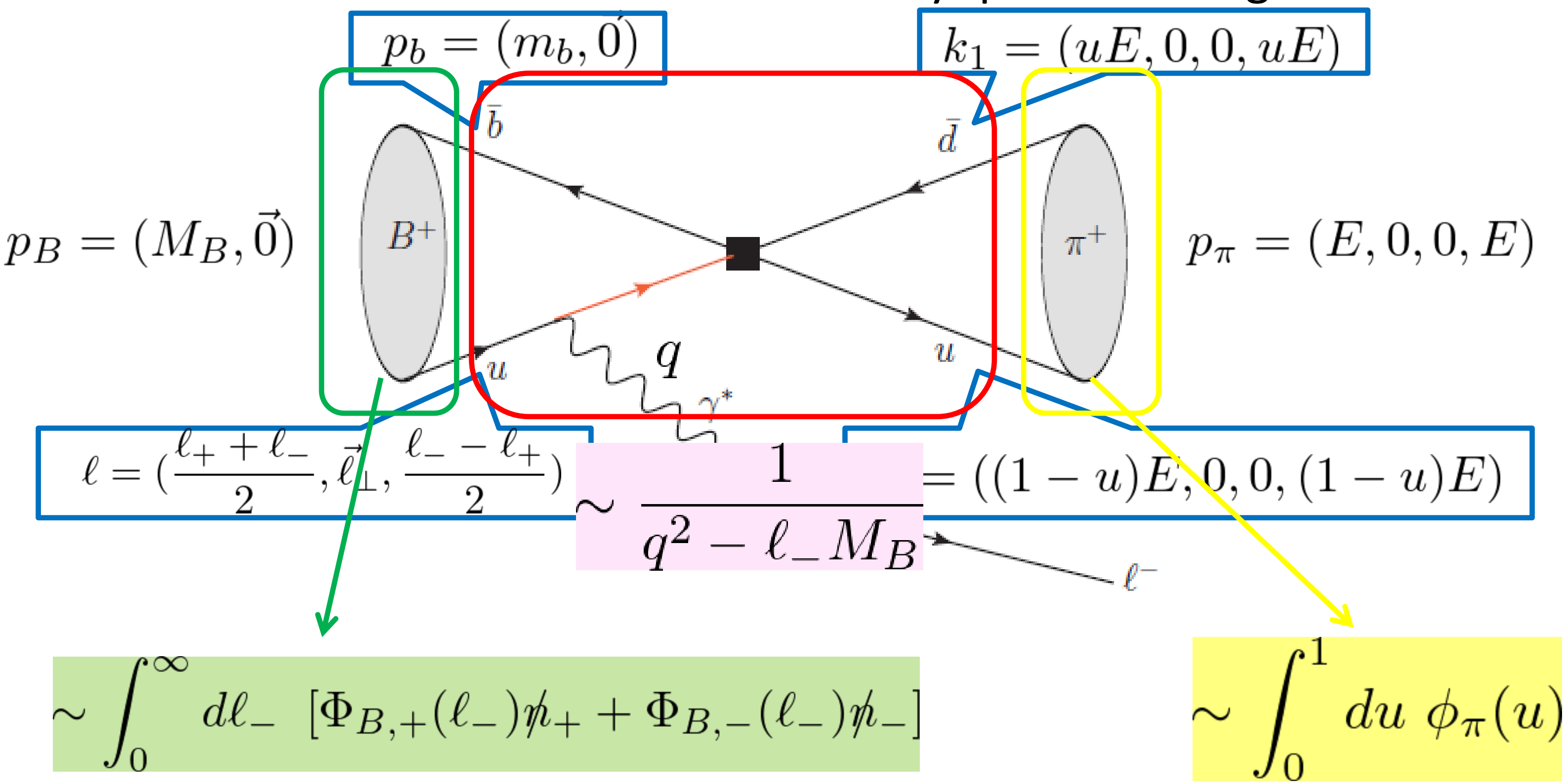


N.B. Most of Lorentz structures are neglected for sake of illustration