Matter Power Spectra in Viable $f(R)$ Gravity Models with Massive Neutrinos

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Motivation

- Neutrino mass difference from oscillation experiments:

$$\Delta m_{21}^2 = (7.50 \pm 0.20) \times 10^{-5} eV^2 \,,$$
$$\Delta m_{32}^2 = \left(2.32^{+0.12}_{-0.08}\right) \times 10^{-3} eV^2 \,.$$

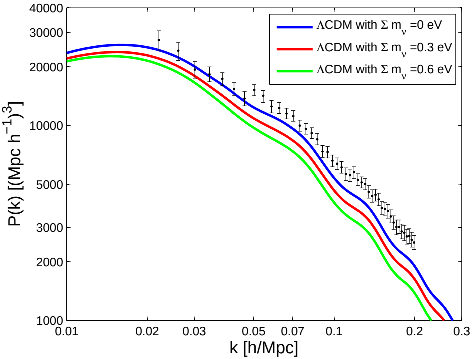
- Neutrono mass from cosmology:

$$\Sigma m_\nu < 0.23 eV \ \ (\text{best fit} : 0.001 eV) \,.$$
$$(95\%; \text{Planck} + \text{WMAP} + \text{highL} + \text{BAO})$$

Motivation

- Free streaming massive neutrinos suppress the Matter Power Spectrum $P(k)$ (data from SDSS DR7):

$$\langle \delta_m(\vec{k}) \delta_m^*(\vec{k'}) \rangle = (2\pi)^3 P(k) \delta^3(\vec{k} - \vec{k'}) \, .$$



Outline

$f(R)$ Gravity

- $f(R)$ gravity:
  One of the simplest modified gravity model, which extends Einstein-Hilbert action to higher order terms.
- The action of $f(R)$ gravity:

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m \,,$$

  where $f(R)$ is an arbitrary function.
- Under the metric formalism, modified Einstein equation:

$$f_R R_{\mu\nu} - \frac{f}{2} g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R = \kappa^2 T_{\mu\nu} \,,$$

  where the subscript R denotes d/dR and $T_{\mu\nu}$ is the energy-momentum tensor.

Viability Conditions

- $\frac{df(R)}{dR}$, $\frac{d^2 f(R)}{dR^2} > 0$ for $R > R_0$, where R_0 is the background curvature.
- Passing local gravity constraints.
- Having a stable late-time de-Sitter point
- $f(R) \to R - 2\Lambda$ in the large curvature regime ($R \gg R_0$).

$f(R)$ Gravity

- Hu-Sawicki model: (R_{ch} is a constant curvature for each model)

$$f(R) = R - R_{ch}^{(HS)} \frac{c_1 \left(R/R_{ch}^{(HS)}\right)^p}{c_2 \left(R/R_{ch}^{(HS)}\right)^p + 1}.$$

- Starobinsky model:

$$f(R) = R - \lambda R_{ch}^{(S)} \left[1 - \left(1 + \frac{R^2}{R_{ch}^{(S)\,2}}\right)^{-n}\right].$$

- Tsujikawa model:

$$f(R) = R - \mu R_{ch}^{(T)} \tanh\left(\frac{R}{R_{ch}^{(T)}}\right).$$

- Exponential gravity model:

$$f(R) = R - \beta R_{ch}^{(E)} \left(1 - e^{-R/R_{ch}^{(E)}}\right).$$

- Appleby-Battye model:

$$(1-g)R + g R_{ch}^{(AB)} \ln\left[\frac{\cosh\left(R/R_{ch}^{(AB)} - b\right)}{\cosh\, b}\right].$$

$f(R)$ Gravity

- $R_{ch} \sim \Lambda$: the same order of cosmological constant.
- In high redshift regime ($R \gg R_0$), these viable models reduce to:
  - Hu-Sawicki, Starobinsky model $\Rightarrow f(R) \simeq R - \lambda R_{ch} \left(1 - \left(\frac{R_{ch}}{R}\right)^{2n}\right)$.
  - Tsujikawa, Appleby-Battye model $\Rightarrow f(R) = R - \beta R_{ch} \left(1 - e^{-R/R_{ch}}\right)$.

Perturbation in $f(R)$ Gravity

- The perturbed FRW metric in Newtonian gauge:

$$ds^2 = a(\tau)^2 \left[-(1+2\Psi)d\tau^2 + (1-2\Phi)dx_i dx^i \right] ,$$

where τ is the conformal time.

- The perturbed energy-momentum tensor is given by

$$\begin{aligned} T_0^0 &= -(\rho + \delta\rho) , \\ T_i^0 &= (1+w)\,\rho v_i , \\ T_j^i &= (P + \delta P)\,\delta_j^i . \end{aligned}$$

Perturbation in $f(R)$ Gravity

- Inside the subhorizon limit ($k^2 \gg \mathcal{H}^2$), the perturbation equations reduce to

$$\frac{k^2}{a^2}\Psi = -4\pi G\ \mu(k,a)\ \rho\Delta\,, \qquad \frac{\Phi}{\Psi} = \gamma(k,a)\,.$$

where

$$\mu(k,a) = \frac{1}{f_R}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,, \qquad \gamma(k,a) = \frac{1+2\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}\,,$$

and $\Delta = \frac{\delta\rho}{\rho} + 3\frac{\mathcal{H}}{k}(1+w)$ is the gauge-invariant matter density perturbation.

- In ΛCDM limit ($\mu = \gamma = 1$):

$$\frac{k^2}{a^2}\Psi = -4\pi G\rho\Delta\,, \qquad \Psi = \Phi\,.$$

Perturbation in $f(R)$ Gravity

- Evolution of matter density perturbation:
  $\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\,\mu(k,a)\,\rho_m\delta_m = 0$, where $\delta_m \equiv \delta\rho_m/\rho_m$.
- Matter power spectrum $P(k) \sim \langle\delta_m^2\rangle$.
- The growth of $P(k)$ is k-dependent in $f(R)$ gravity, and is scale independent in ΛCDM model.

CAMB and MGCAMB

- CAMB: Synchronous gauge.
  MGCAMB[1]: Newtonian gauge.
- Newtonian and synchronous gauge are related under the coordinate transformation $\hat{x}^\mu \to x^\mu + d^\mu$. Thus,

$$\Psi = \dot{\alpha} + \mathcal{H}\alpha\,,$$
$$\Phi = \eta - \mathcal{H}\alpha\,,$$

  where $\alpha = \left(\dot{h} + 6\dot{\eta}\right)/2k^2$.
- The gauge invariant equations:

$$\frac{k^2}{a^2}\Psi = -4\pi G\ \mu(k,a)\ \rho\Delta\,, \qquad \frac{\Phi}{\Psi} = \gamma(k,a)\,.$$

[1] A. Hojjati, G.B. Zhao, L. Pogosian and A. Silvestri, JCAP **1108** 005, http://www.sfu.ca/ aha25/MGCAMB.html

CAMB and MGCAMB

- Models:
  - Starobinsky model:

$$f(R) = R - \lambda R_{\mathrm{S}} \left[1 - \left(1 + \frac{R^2}{R_{\mathrm{S}}^2} \right)^{-n} \right] .$$

  - Exponential model:

$$f(R) = R - \beta R_s \left(1 - e^{-R/R_s} \right) .$$

- The total mass, Σm_ν, is given by

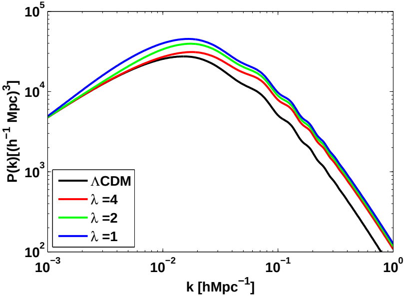
$$\Omega_\nu \simeq \frac{\Sigma m_\nu}{94 h^2 \mathrm{eV}} .$$

- Assumption:
  - The ΛCDM background evolution.
  - Only one massive neutrino.

Outline

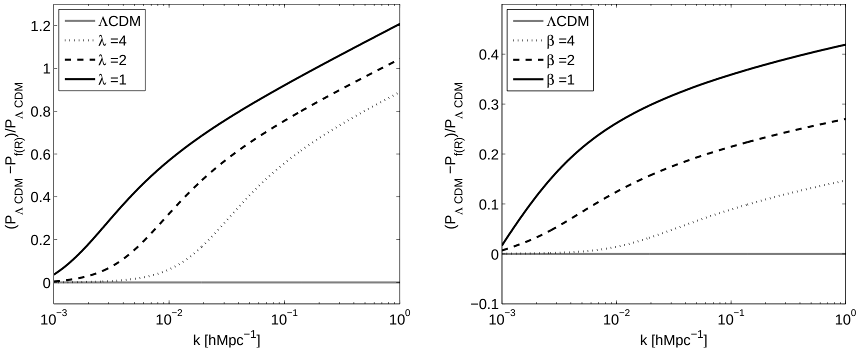
Numerical Results

- Matter power spectrum $P(k)$ in Starobinsky (left) and Exponential gravity (right) model:



Numerical Results

- Differences of the matter power spectra between $f(R)$ and ΛCDM models with Starobinsky (n=2) and exponential models
- The magnification of $P(k)$ in the Starobinsky model is more greater than that in the exponential one within the allowed viable model parameters.



CAMB and MGCAMB

- Matter density perturbation:
  $\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\,\mu(k,a)\,\rho_m\delta_m = 0,$
  where $\mu(k,a) = \frac{1}{f_R}\frac{1+4\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}{1+3\frac{k^2}{a^2}\frac{f_{RR}}{f_R}}$.
- In viable model, $1 > f_R > 0$ and $f_{RR} > 0$.
- The growth of δ_m is enhanced in subhorizon scale, especially in large k, but the free streaming massive neutrino suppresses $P(k)$.

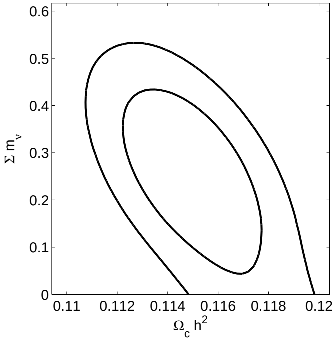
Numerical Results

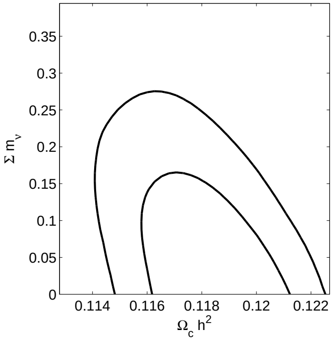
- CosmoMC program with MGCAMB.
- Dataset:
  - CMB: Planck ($l < 50$ and $50 < l < 2500$) and WMAP (low-l).
  - BAO: BOSS (Baryon Oscillation Spectroscopic Survey).
  - SNIa: SNLS (Supernova Legacy Survey).
  - Matter power spectrum: SDSS DR4 (Sloan Digital Sky Survey) and WiggleZ Dark Energy Survey.

Numerical Results

- Σm_ν and $\Omega_c h^2$ with 95% confidence level:

$f(R)$ model	Σm_ν	$\Omega_c h^2$
ΛCDM	< 0.200 eV	$0.117^{+0.004}_{-0.002}$
Starobinsky	$0.248^{+0.203}_{-0.232}$ eV	$0.114^{+0.004}_{-0.002}$
Exponential	< 0.214 eV	0.118 ± 0.03

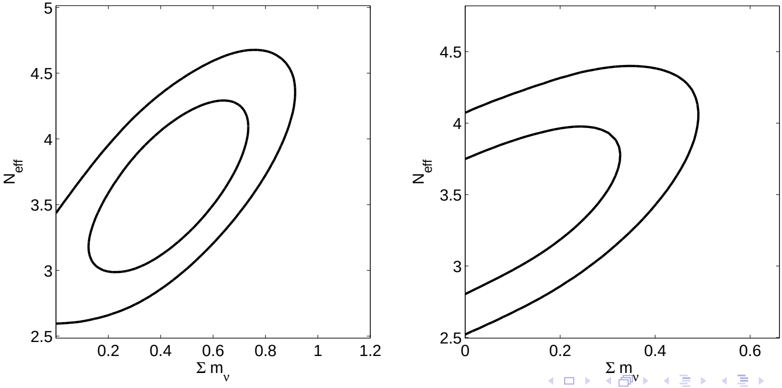




Numerical Results

- N_{eff}, Σm_ν and $\Omega_c h^2$ with 95% confidence level:

	N_{eff}	Σm_ν	$\Omega_c h^2$
ΛCDM	$3.47^{+0.82}_{-0.47}$	< 0.351 eV	$0.125^{+0.012}_{-0.008}$
Starobinsky	$3.78^{+0.64}_{-0.84}$	$0.533^{+0.254}_{-0.411}$ eV	$0.124^{+0.010}_{-0.011}$
Exponential	$3.47^{+0.74}_{-0.60}$	< 0.386 eV	$0.124^{+0.011}_{-0.009}$



Outline

Conclusion

- The $f(R)$ gravity enhances the matter power spectrum, especially in small scale, and the massive neutrino suppress the matter power spectrum in sub-horizon scale.
- 3 active neutrino case:
  - In Starobinsky model, the neutrino mass best-fit locates at $\Sigma m_\nu = 0.248$ eV, which is consistent with the result from neutrino oscillation experiments.
  - The Exponential gravity model might not be distinguishable from ΛCDM model.
- Examine $N_{\rm eff}$ case:
  - The best-fit in Starobinsky model, $N_{\rm eff} = 3.78$, leaves more room for a dark sector.
  - The Exponential gravity model is still indistinguishable from ΛCDM model.

Summary

Thank you for your attention!!