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# GAUGE HIERARCHY PROBLEM: SCALE INVARIANCE AND POINCARE PROTECTION

**Ray Volkas**
**School of Physics**
**The University of Melbourne**

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1. SM without gravity
2. Radiative symmetry breaking
3. SM extensions without gravity
4. Gravity
5. Final remarks

**WARNING!**

**Almost everything in this talk will be stuff everybody knows!**

# 1. SM without gravity

By SM, I mean "Plain SM": no new physics at any higher mass scale, and not SM+GR.

`t Hooft criterion of technical naturalness:

A special approximate relationship amongst a priori independent parameters is "technically natural" (stable under radiative corrections) if the exact relation increases the symmetry of the theory.

The gauge hierarchy problem is a concern over the radiative stability of the electroweak Higgs mass.

$$V = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

Taking $\mu \longrightarrow 0$ makes the Plain SM Lagrangian scale invariant. (Assuming Dirac neutrinos, to begin with, for simplicity.)

So, it seems an arbitrarily small Higgs mass meets the `t Hooft criterion. But, *let's think a bit more* …

… $\mu$ is the only *dimensionful* parameter in the Plain SM Lagrangian (with Dirac neutrinos).

What does it mean for it to be small?
Small relative to what?
Why are you even worried? How can a one-scale theory possibly have a hierarchy problem?

OK … but we should talk about the quantum theory not the classical theory, and a QFT is not just its Lagrangian: there is also the quantisation procedure … and we know that new dimensionful parameters appear through quantisation.

Two types of quantal scales to worry about (in principle):

- dimensional transmutation scales e.g. $\Lambda_{QCD}$
- UV cut-off $\Lambda$

Indeed, the usual objection to classical scale invariance as an approach to the hierarchy problem is that it is *anomalous*: not a symmetry of the quantum theory even if it is of the classical theory.

## The scale anomaly:

But, the scale anomaly leads to logarithmic violations of scale invariance, not power-law as would be needed for it to destabilise the Higgs mass.

E.g. running couplings
Coleman-Weinberg potential

# Easy to see this in dimensional regularisation:

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow (\tilde{\mu})^{2\epsilon} \int \frac{d^{4-2\epsilon} k}{(2\pi)^{4-2\epsilon}}$$

W.A. Bardeen FERMILAB-CONF-95-391-T
K.A. Meissner and H. Nicolai, PLB648, 312 (2007); PLB660, 260 (2008)

$$(\tilde{\mu})^{2\epsilon} = 1 + \epsilon \ln \tilde{\mu}^2 + \ldots \quad \text{as } \epsilon \longrightarrow 0$$

**Any divergent integral goes as 1/ $\varepsilon$ .**
**The two epsilon's cancel out, and one has a physical, logarithmic violation of scale invariance.**

Quadratically-divergent integral:

$$\int \frac{d^{4-2\epsilon} k_E}{(2\pi)^{4-2\epsilon}} \frac{1}{k_E^2 + \Delta} = \frac{1}{(4\pi)^{2-\epsilon}} \Gamma(-1+\epsilon) \Delta^{1-\epsilon}$$

With $\Delta$ =(SM mass)$^2$, there are no large radiative corrections proportional to any physical mass.

For $\Delta = 0$ (classical scale invariance), the integral vanishes.

## UV cut-off:

Suppose you use some form of momentum cut-off regularisation rather than DR.

Then, famously, one consequence is:

$$m^2 = m_0^2 + a\Lambda^2 + \ldots$$

There is, in a sense, a large radiative correction.

But the bare mass $m_0$ is unphysical and $\Lambda$ is an unphysical regularisation parameter (remember, we are assuming no new high scale physics, and gravity is switched off).

You choose $m_0$ so that m is the observed physical mass (in some scheme). Indeed, all renormalised quantities are $\Lambda$-independent.

Of course they are; we have a renormalisable theory after all.

# SUMMARY

For the Plain SM (no new physics at any high scale, no gravity) there is no hierarchy problem in any meaningful sense.

- Only scales are $\mu$ and $\Lambda_{QCD}$, and the latter, one consequence of the scale anomaly, does not destabilise the former (or vice-versa).
- How do you even define a hierarchy problem for what is essentially a one-scale theory?
- UV cut-off dependence disappears from all renormalised and physical quantities.

YOU HEAR ALL THE TIME: “THE SM HAS A HIERARCHY PROBLEM”.

THIS TAKES POETIC LICENCE: NOT PLAIN SM, BUT SM + NEW PHYSICS AND/OR SM + GRAVITY.

The Plain SM may still have a problem: the hypercharge Landau pole.

# 2. Radiative symmetry breaking

Suppose you want to explore the classically scale-invariant SM, with $\mu$ =0.

It is well known that, with only SM particle content, radiative electroweak symmetry breaking requires $m_t < 40$ GeV and predicts $m_h < 10$ GeV.

But simple extensions, with new bosonic degrees of freedom do work.

Gildener & S. Weinberg – PRD13, 3333 (1976) – explained how to analyse CW symmetry breaking for weakly-coupled massless scalar field theories:

- 1-loop CW potential dominates along flat direction of tree-level potential $f_{ijkl}S_iS_jS_kS_l$
- Quartic couplings are running parameters $f_{ijkl}(\mu)$
- Get flat direction by suitable relation amongst $f_{ijkl}$ at certain scale $\mu = \Lambda$.

The relation replaces one $f_{ijkl}$ with quantally-generated scale $\Lambda$: dimensional transmutation (not fine-tuning).

$\Lambda$ is free parameter; all masses related to it.

# There are many possible models:

Some examples:

R. Hempfling, PLB379, 153 (1996)
R. Foot, A. Kobakhidze and RV, PLB655, 156 (2007); PRD82, 035005 (2010); PRD84, 075101 (2011)
R. Foot, A. Kobakhidze, K. McDonald and RV, PRD76, 075014 (2007); PRD77, 035006 (2008)
K.A. Meissner and H. Nicolai, PLB648, 312 (2007); Eur. Phys. J C57, 493 (2008); PRD80, 086005 (2009)
W.F. Chang, J.N. Ng and J.M.S. Wu, PRD75, 115016 (2007)
J.R. Espinosa and M. Quiros, Phys.Rev.D76:076004 (2007)
S. Iso, N. Okada and Y. Orikasa, PLB676, 81 (2009)
M. Holthausen, M. Lindner and M.A. Schmidt, PRD82, 055002 (2010)
L. Alexander-Nunneley and A. Pilaftsis, JHEP 1009, 021 (2010)
J.S. Lee and A. Pilaftsis, PRD86, 035004 (2012)
C. Englert, J. Jaeckel, V. Khoze and M. Spannowsky, JHEP 1304, 060 (2013)
V. Khoze, JHEP 1311, 215 (2013)
M. Holthausen, J. Kubo, K.S. Lim and M. Lindner, JHEP 1312, 076 (2013)

I won't attempt a survey. I'll briefly comment on a scheme that is now named after the Higgs portal.

$\Phi$ = EW Higgs doublet    $S_1$, $S_2$, ... gauge singlets

In limit where S-sector decouples from SM-sector:

$$V(\phi, S_1, S_2, \ldots) = V(\phi) + V(S_1, S_2, \ldots)$$

V($\Phi$) is the scale-invariant SM effective potential: it fails to radiatively induce an appropriate nonzero VEV for $\Phi$.

But V(S1, S2, ...) can give radiatively-induced nonzero VEVs to the S fields.

Now switch on small coupling between sectors:

$$\sum_i \lambda_x^i \phi^\dagger \phi S_i^2$$

Negative $\lambda_x$ induce negative squared-mass for $\Phi$, hence nonzero VEV for $\Phi$.

But as $\lambda_x \to 0$, we get $m_\Phi \to 0$, so

$$\frac{m_\phi}{\langle S \rangle} \ll 1$$

is a technically-natural hierarchy.

# 3. SM extensions without gravity

RV, A. Davies, G. Joshi, PLB215, 133 (1988)
R. Foot, A. Kobakhidze, K. McDonald, RV, PRD89, 115018 (2014)

There are many excellent reasons for new physics:

- Dark matter (incontrovertible)
- Neutrino masses (incontrovertible)
- Baryogenesis
- Strong CP problem
- Compelling gauge extensions, e.g. LR sym, Pati-Salam, GUTs

Some of these require new high physical mass scales, and all may involve them.

Indeed, the GUT context is where the gauge hierarchy problem was first identified – E. Gildener, PRD14, 1667 (1976).

With a physical scale >> EW scale
(i.e. cannot be treated as a convenient but unphysical regularisation tool)
destabilisation of the weak scale is a real concern.

The famous example of (non-susy) GUTs:

need $m_h, v_{\rm EW} \ll v_{\rm GUT}$

$$V = -m^2\phi^\dagger\phi + \lambda\phi^\dagger\phi\Phi^2 - M^2\Phi^2 + \ldots$$

But large VEV for $\Phi$ induces large mass term for $\phi$
Need to make $\lambda$ very small. But then common gauge interactions spoil this at 1-loop, etc.
( $\Phi$ does not decouple)

**This is where the famous equation**

$$m^2 = m_0^2 + a\Lambda^2 + \ldots$$

**does reveal a problem. Although $\Lambda$ can always be absorbed by bare parameters, the dependence of m on $\Lambda$ correctly traces the dependence of m on a real high physical scale M:**

$$m^2 = m_0^2 + a\Lambda^2 + bM^2 \ln(M^2/\Lambda^2) + \ldots$$

**$\Lambda$ serves as a proxy for the physical scale M. Note, though, that this calculation is done in the full theory, with the new physics fully dynamical.**

This is what happens in the GUT case, of course. But this analysis also reveals another type of scenario:

The tree-level tuning will suffice provided that $\Phi$ is in a hidden sector from $\phi$.

Not possible for the GUT and similar cases, but actually quite interesting in others.

We just saw an example of this:

“ Now switch on small coupling between sectors:

$$\sum_i \lambda_x^i \phi^\dagger \phi S_i^2$$

Negative $\lambda_x$ induce negative squared-mass for $\Phi$, hence nonzero VEV for $\Phi$.

But as $\lambda_x \to 0$, we get $m_\Phi \to 0$, so

$$\frac{m_\phi}{\langle S \rangle} \ll 1$$

is a technically-natural hierarchy. ”

The $\lambda_x \to 0$ limit makes the S-sector hidden.

All (non-gravitational) physical effects between the SM and S-sector are proportional to powers of $\lambda_x$ and thus automatically small.

Note: the classical scale invariance feature of this example has the $\lambda_x$ play a dual role: control inter-sector effects AND control the magnitude of the ratio.

There is a symmetry reason for this.

We have called it “Poincaré protection”.

R. Foot, A. Kobakhidze, K. McDonald, RV, PRD89, 115018 (2014)

$$S = \int d^4x \mathcal{L}(\phi, \partial\phi) + \int d^4x \mathcal{L}(\Phi, \partial\Phi) + \int d^4x \mathcal{L}_{\rm int}(\phi, \Phi, \partial\phi, \partial\Phi)$$

If interaction terms are zero, then independent Poincaré transformations can be done in the two sectors.

H. Georgi pointed this out to me while refereeing RV, A. Davies, G. Joshi, PLB215, 133 (1988)

# The invisible axion example:

J. Kim, PRL43, 103 (1979)
A. Zhitnitski, SJNP31, 260 (1980)
M. Dine, W. Fischler, M. Srednicki PLB104, 199 (1981)

Two EW Higgs doublets $\Phi_1, \Phi_2$ and singlet N.
Peccei-Quinn U(1) imposed.

Need <N> >> $<\Phi_{1,2}>$.

Scalar potential has the cross terms:

$$\lambda_{1N}\, \phi_1^\dagger \phi_1 N^\dagger N + \lambda_{2N}\, \phi_2^\dagger \phi_2 N^\dagger N + (n\, \phi_1^\dagger \phi_2 N^2 + H.c.)$$

$\boxed{\lambda_{1N},\ \lambda_{2N},\ n \to 0}$ protects the hierarchy to all orders; N is hidden sector

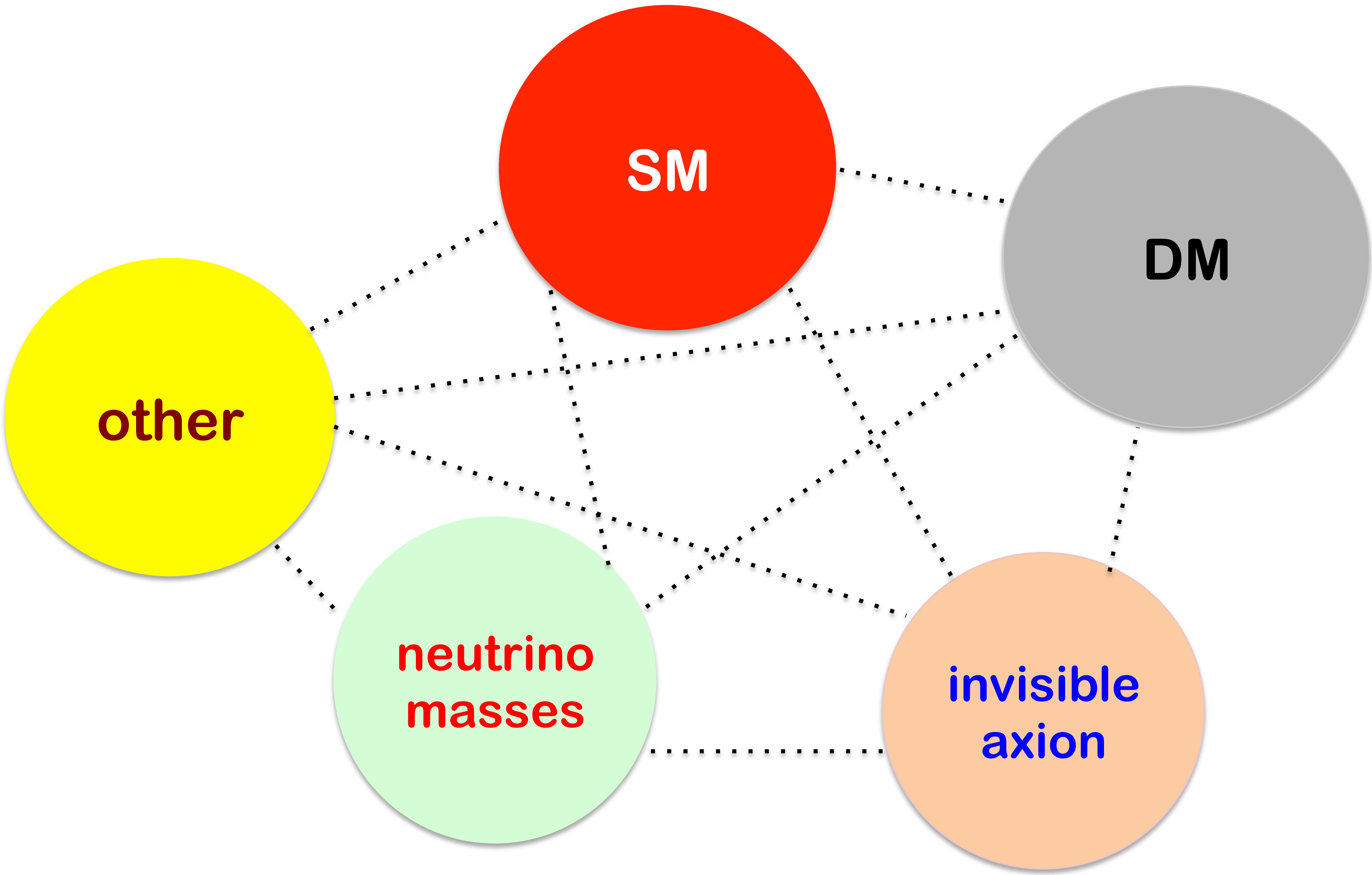
# The type-1 see-saw example:

F. Vissani PRD57, 7027 (1998)

No hierarchy problem if neutrino Yukawas are small enough – the RH neutrinos become the hidden sector.

Need $M_R < 10^7$ GeV.

# Non-susy-GUT scenario for BSM physics:



This approach cannot accommodate generic new physics, e.g. grand unification.

By contrast, susy is more generic, apart from the requirement of susy itself.

The hidden-sector approach is there to be falsified. But maybe it is even true.

Alternative: no new high scales, exemplified by $\nu$ SM of Shaposhnikov et al. Does $\nu$ mass, DM and baryogenesis with sub-EW scale sterile neutrinos (caveat: invisible axion).

# 4. Gravity

Is $\Lambda$-dependence a correct tracer for effect of Planck scale on weak scale? E.g. are there particle-like states at the Planck scale?

It tends not to be backed up by a full gravity calculation, unlike the m,M, $\Lambda$ story told earlier.

Poincaré protection is broken.
There certainly could be a problem.
Is there definitely a problem?

I find it interesting that a lot of non-gravity physics can be framed with no susy & no hierarchy problem.

An aside: in scale-invariant theories one can generate Planck scale from $\sqrt{-g}\, S^2 R$ term.

# 5. Final Remarks

1. Plain SM (no new physics, no gravity) has no hierarchy problem.
2. Scale anomaly is logarithmic, and Plain SM has rad. corrs. ~ (SM mass)$^2$ ln( $\mu$ ).
3. For Plain SM + hidden sectors, hierarchy problem does not arise (Poincaré protection).

> Hidden sectors can do dark matter, $\nu$ mass, invisible axion, … so they solve real problems, and this vision is there to be falsified.

4. This BSM paradigm is incompatible with GUTs.
5. Effect of gravity is the only concern.