On The Inflationary Magnetogenesis

Wolung Lee (National Taiwan Normal University)
with Shu-Lin Cheng (NTNU) & Kin-Wang Ng (AS)
2nd Particle Physics & Cosmology after Higgs & Planck, Oct. 8, 2014

The origin of large-scale magnetic fields

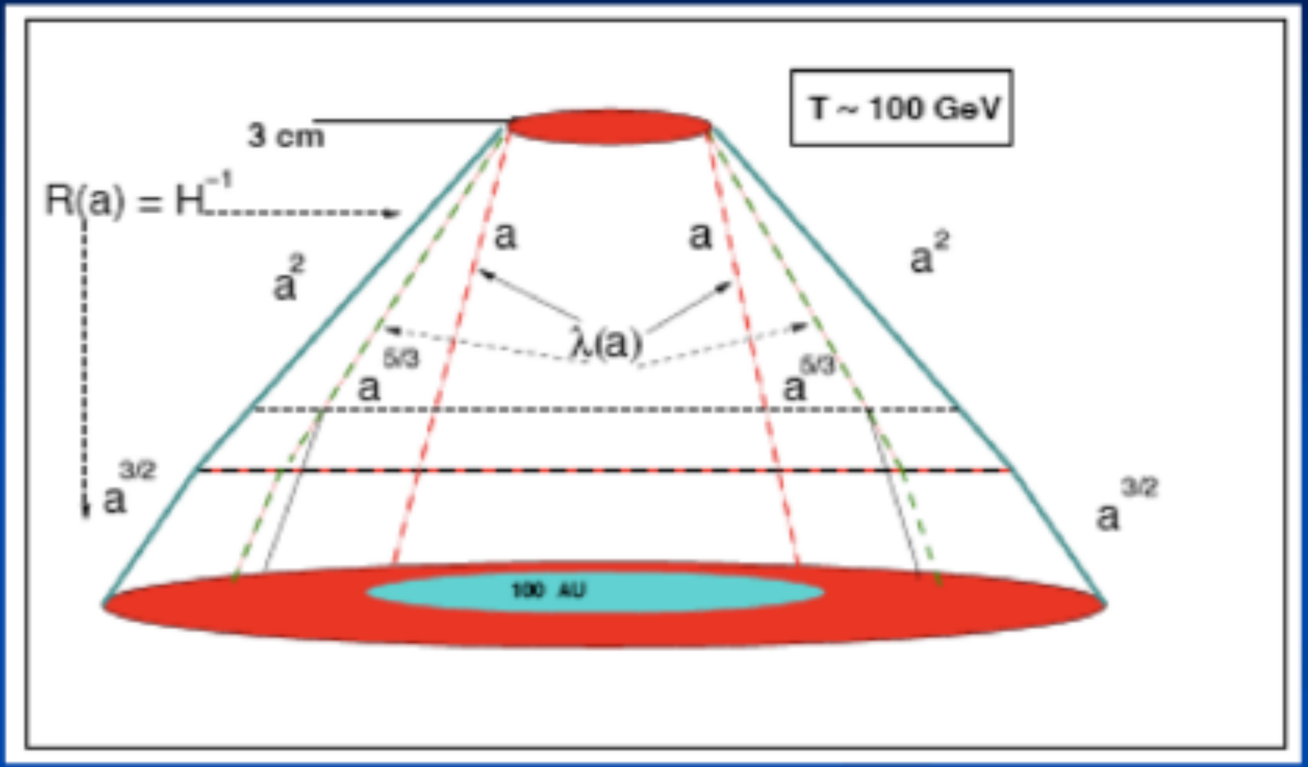
- The observed cluster and galactic magnetic fields of about a few μG may be resulted from the amplification of a seed field of $B_* \sim 10^{-23}$ G via the so-called galactic dynamo effect.

- Through most of its history, the universe has been a good conductor which preserves the magnetic flux:

$$r \equiv \frac{\rho_B}{\rho_\gamma} = \text{constant} \qquad \Rightarrow \qquad \rho_B \simeq 10^{-34} \rho_\gamma .$$

- The seed fields may be generated inside or outside the Hubble horizon.

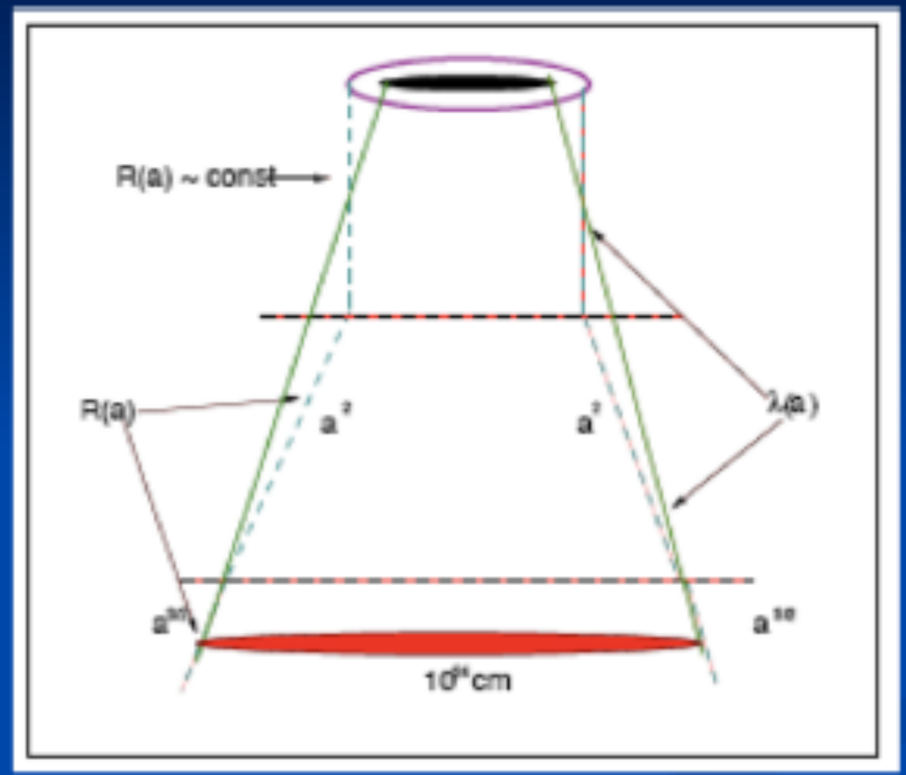
Magnetogensis Inside the Hubble Radius



- Inverse cascade: many small-scale magnetic domains coalesce giving rise to a magnetic domain of larger size but of smaller energy
- If inverse cascade are invoked, the correlation scale may grow up to 100 AU
- But cosmic magnetic fields are coherent over much larger scales (i.e. Mpc and even larger)

M. Giovannini: astro-ph/0612378

Magnetogensis Outside the Hubble Radius



- In inflationary models, the vacuum fluctuations of fields of various spin are amplified, typically fluctuations of spin 0 and spin 2 fields.
- Spin 1 fields ca be amplified during inflation only when the conformal invariance is broken.

M. Giovannini: astro-ph/0612378

The Weyl (Conformal) Invariance

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = a^2(\tau)[d\tau^2 - dx^2]$$

$$S_{\rm em} = -\frac{1}{4} \int d^4x \sqrt{-g} F_{\mu\nu} F^{\mu\nu}$$

$$\Rightarrow \quad \partial_\mu (\sqrt{-g} F^{\mu\nu}) = 0$$

$$\sqrt{-g} F^{\mu\nu} = a^4(\tau) \frac{\eta^{\mu\alpha}}{a^2(\tau)} \frac{\eta^{\nu\beta}}{a^2(\tau)} F_{\alpha\beta} = F^{\mu\nu}$$

$\Rightarrow$ the evolution equations of Abelian gauge fields are the same in flat space-time and in a conformally flat FRW space-time

Breaking the Conformal Invariance

Turner & Widrow PRD (1988):

$$\frac{1}{m^2}F_{\mu\nu}F_{\alpha\beta}R^{\mu\nu\alpha\beta}, \qquad \frac{1}{m^2}R_{\mu\nu}F^{\mu\beta}F^{\nu\alpha}g_{\alpha\beta}, \qquad \frac{1}{m^2}F_{\alpha\beta}F^{\alpha\beta}R$$

Ratra ApJ (1992):

$$RA_\mu A^\mu, \qquad R_{\mu\nu}A^\mu A^\nu.$$

Carroll & Field (1990); Garretson et. al. (1992):

$$\sqrt{-g}c_{\psi\gamma}\alpha_{\rm em}\frac{\psi}{8\pi M}F_{\alpha\beta}\tilde{F}^{\alpha\beta}$$

Failing of an inflationary magnetogenesis

The coupled system of evolution equations to be solved in order to get the amplified field is

$$\begin{aligned}
\mathbf{B}'' - \nabla^2\mathbf{B} - \frac{\alpha_{\rm em}}{2\pi M}\nabla\times\mathbf{B} &= 0, \\
\psi'' + 2\mathcal{H}\psi' + m^2a^2\psi &= 0,
\end{aligned}$$

where $\mathbf{B} = a^2 B$. From the first equation, there is a maximally amplified physical frequency

$$\omega_{\rm max} \simeq \frac{\alpha_{\rm em}}{2\pi}\cdot m$$

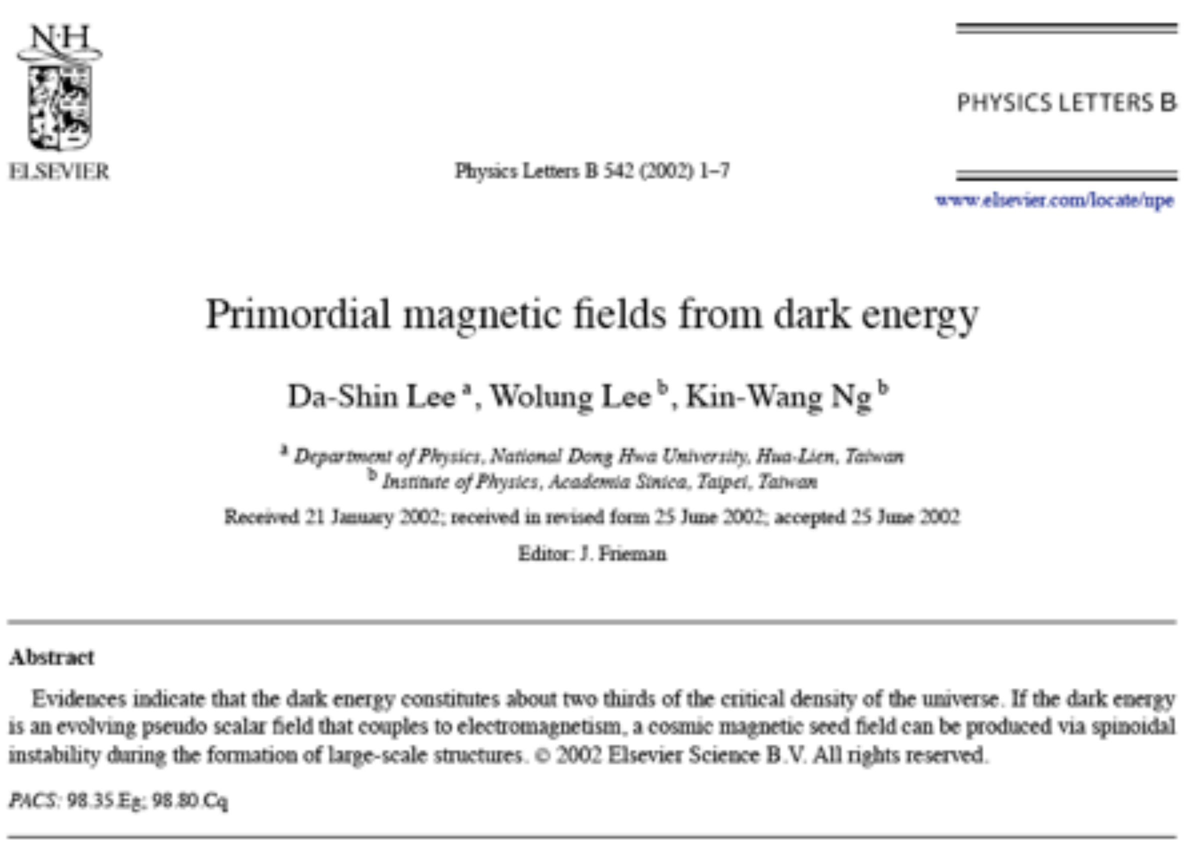
However, the amplification of ω from the inflaton dynamics is at the order of

$$\exp\left\{\left(\frac{\alpha_{\rm em}}{2\pi}\right)\frac{m}{H}\right\}$$

The modes which are substantially amplified are the ones for which $\omega_{\rm max} \gg H$, i.e. they are sub-horizon modes which could not lead to the large-scale magnetic fields we are interested in.

$$\exp\left[c\left(\frac{m}{H}\right)\right]$$

- As long as an ultralight ϕ field couples to photon where for a slow-roll condition, the mass m_ϕ is comparable to H_0, it is conceivable to have very long-wavelength electromagnetic fields generated via spinodal instabilities from the dynamics of ϕ as a possible source of seed magnetic fields for the galactic dynamo.


ELSEVIER

Physics Letters B 542 (2002) 1–7

PHYSICS LETTERS B

www.elsevier.com/locate/npe

Primordial magnetic fields from dark energy

Da-Shin Lee [a], Wolung Lee [b], Kin-Wang Ng [b]

[a] Department of Physics, National Dong Hwa University, Hua-Lien, Taiwan
[b] Institute of Physics, Academia Sinica, Taipei, Taiwan

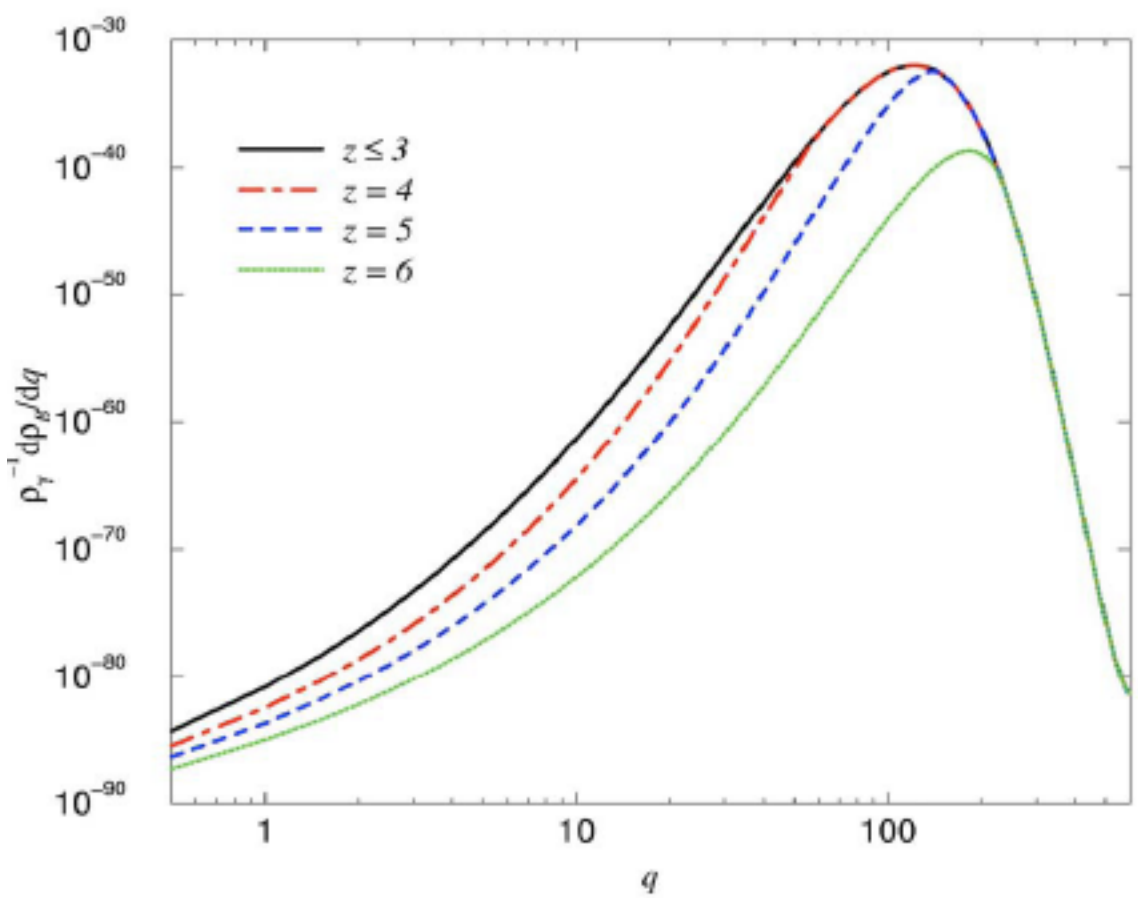
Received 21 January 2002; received in revised form 25 June 2002; accepted 25 June 2002

Editor: J. Frieman

Abstract

Evidences indicate that the dark energy constitutes about two thirds of the critical density of the universe. If the dark energy is an evolving pseudo scalar field that couples to electromagnetism, a cosmic magnetic seed field can be produced via spinoidal instability during the formation of large-scale structures.

PACS: 98.35.Eg; 98.80.Cq



Axial coupling magnetogenesis

- In a spatially flat universe, we consider an evolving pseudo-scalar field ϕ characterized by the action

$$S = \int d^4x \sqrt{g} \left\{ \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} g^{\alpha\mu} g^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} + \frac{1}{\sqrt{g}} L_{\phi\gamma} \right\},$$

where $L_{\phi\gamma}$ is the ϕ-photon coupling given by

$$L_{\phi\gamma} = \frac{c}{M_{\mathrm{Pl}}} \phi \, \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

- The comoving magnetic field can be expressed in terms of the comoving coordinates $(\mathbf{x}, \eta)$ as

$$\left(\nabla^2 - \frac{\partial^2}{\partial \eta^2} \right) \mathbf{B} = 4c \frac{d\phi}{d\eta} \nabla \times \mathbf{B}$$

Magnetic mode equation

- The comoving magnetic field can be expressed in terms of the comoving coordinates $(\mathbf{x}, \eta)$ as

$$\left(\nabla^2 - \frac{\partial^2}{\partial \eta^2}\right) \mathbf{B} = 4c \frac{d\phi}{d\eta} \nabla \times \mathbf{B}$$

- The magnetic field can be recast as $\mathbf{B} = \nabla \times \mathbf{A}_T$, in which the transverse field $\mathbf{A}_T(\eta, \mathbf{x})$ can be further decomposed into Fourier modes such that

$$\mathbf{A}_T = \int \frac{d^3\mathbf{k}}{\sqrt{2(2\pi)^3 k}} \left[e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{\lambda=\pm} b_{\lambda\mathbf{k}} V_{\lambda\mathbf{k}}(\eta) \epsilon_{\lambda\mathbf{k}} + \text{h.c.} \right],$$

where $b_{\pm\mathbf{k}}$ are destruction operators, and $\epsilon_{\pm\mathbf{k}}$ are circular polarization unit vectors. Then, it is straightforward to deriving the mode equations from Eqs. (4) and (5) as

$$\frac{d^2}{d\eta^2} V_{\pm q} + \left(q^2 \mp 4cq \frac{d\phi}{d\eta} \right) V_{\pm q} = 0,$$

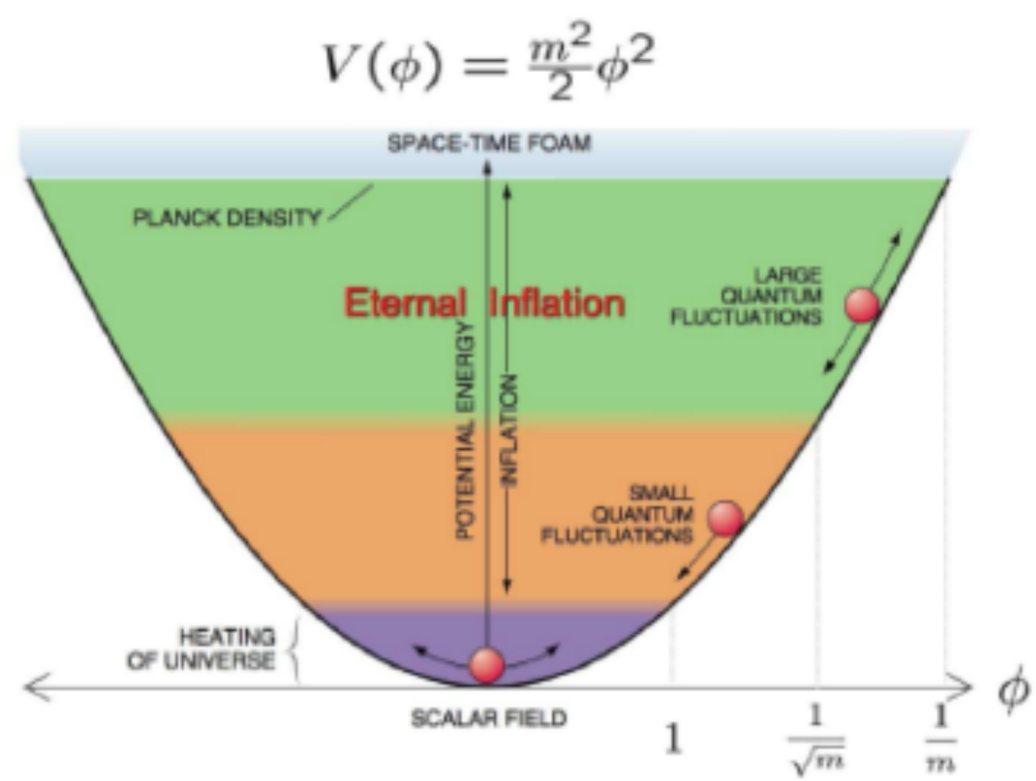
where the dimensionless comoving wavenumber $q = k/H$.

Magnetogenesis during inflation

$$\frac{d^2V_\pm}{d\eta^2} = \left(-q^2 \pm 4cq \cdot \frac{d\theta}{d\eta}\right) \cdot V_\pm \Rightarrow V_\pm \propto \exp\left\{\left(\sqrt{\pm 4cq\frac{d\theta}{d\eta} - q^2}\right) \cdot \eta\right\}$$

Hence the growing mode can be identified with $V_{+q} \propto \exp(\omega\eta)$ with

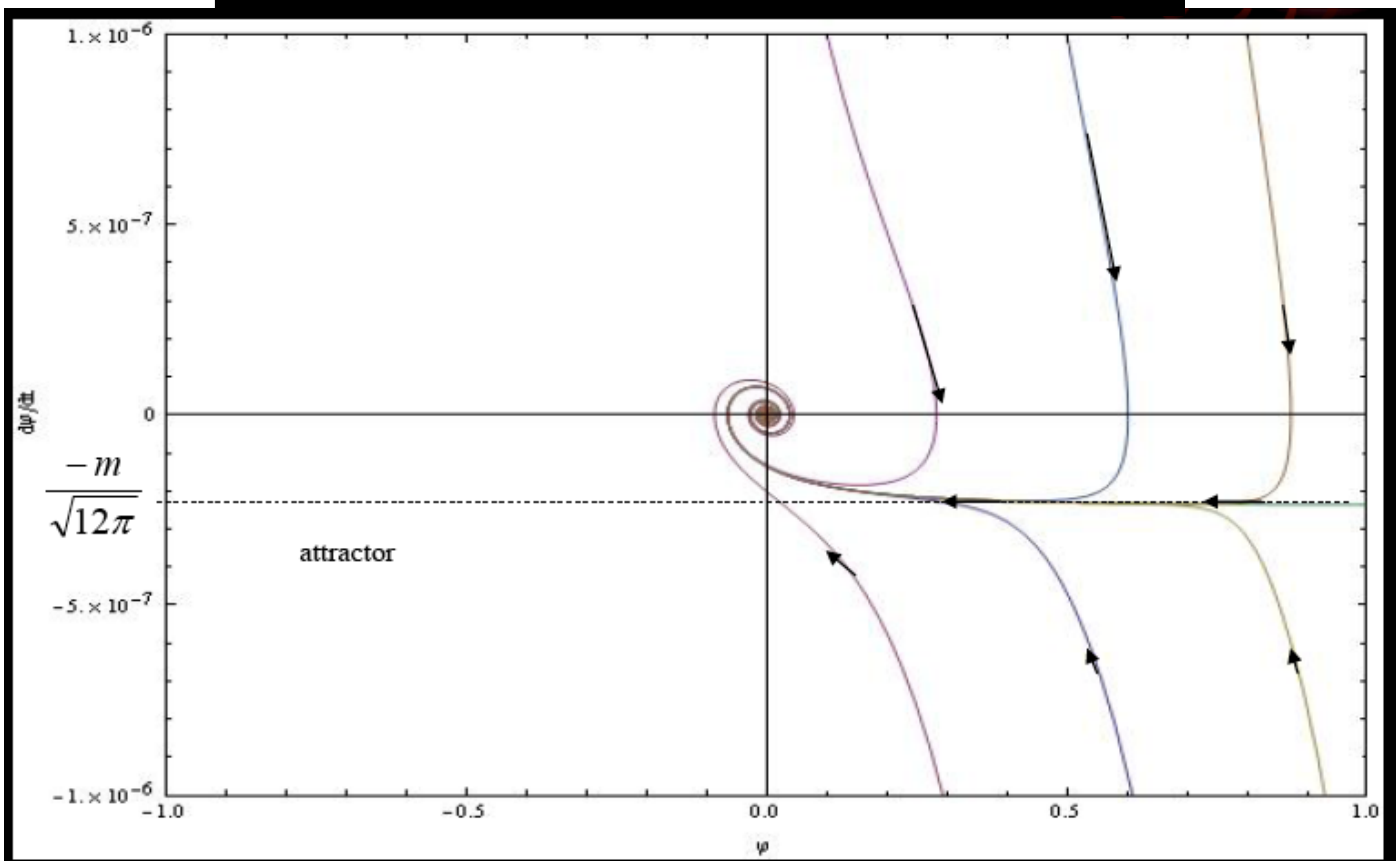
$$\omega \equiv \sqrt{4cq\left|\frac{d\theta}{d\eta}\right| - q^2}$$



$$\ddot{\phi} + (3H + \gamma)\dot{\phi} + V'(\phi) = 0$$

$$\dot{\rho} + 4H\rho = \gamma\dot{\phi}^2$$

$$H^2 = \frac{8\pi}{3M_{Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + \frac{m^2}{2}\phi^2 + \rho\right)$$



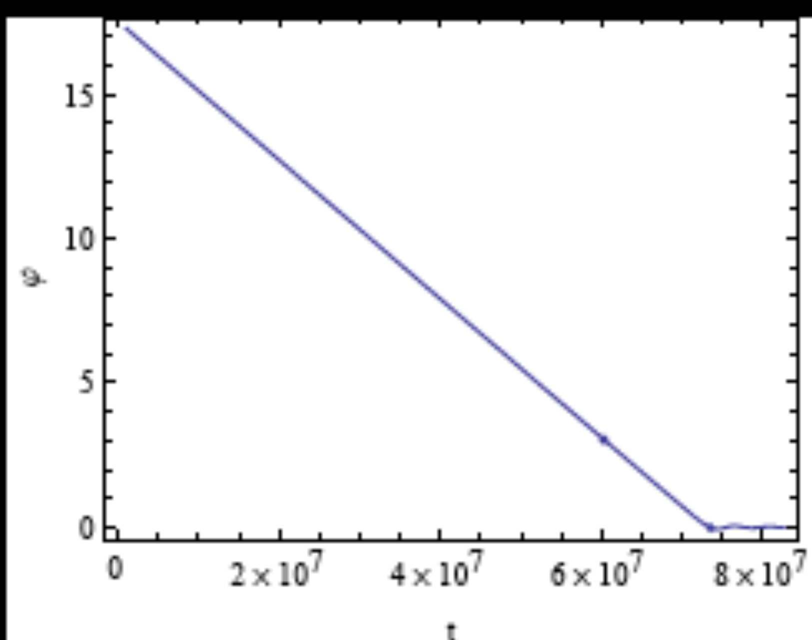
Magnetogenesis during inflation

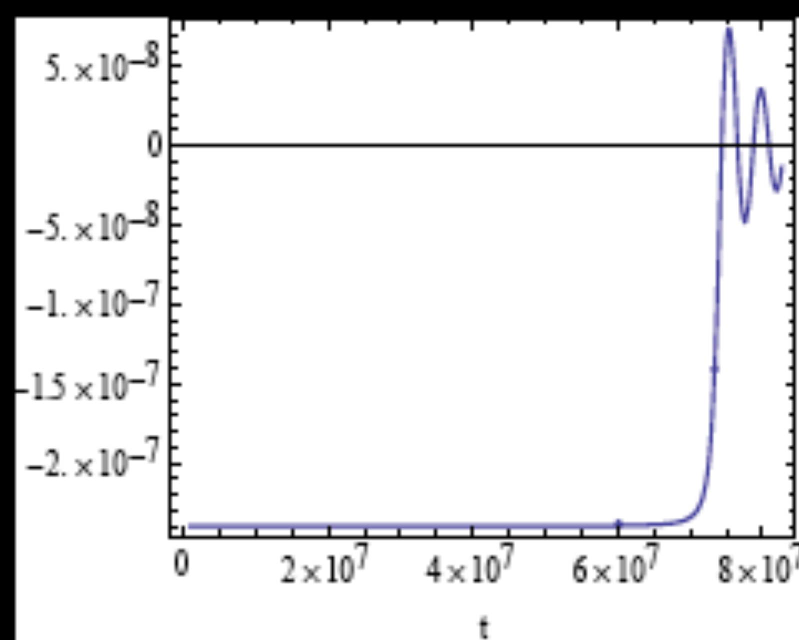
- 慢滚階段（slow-roll stage）

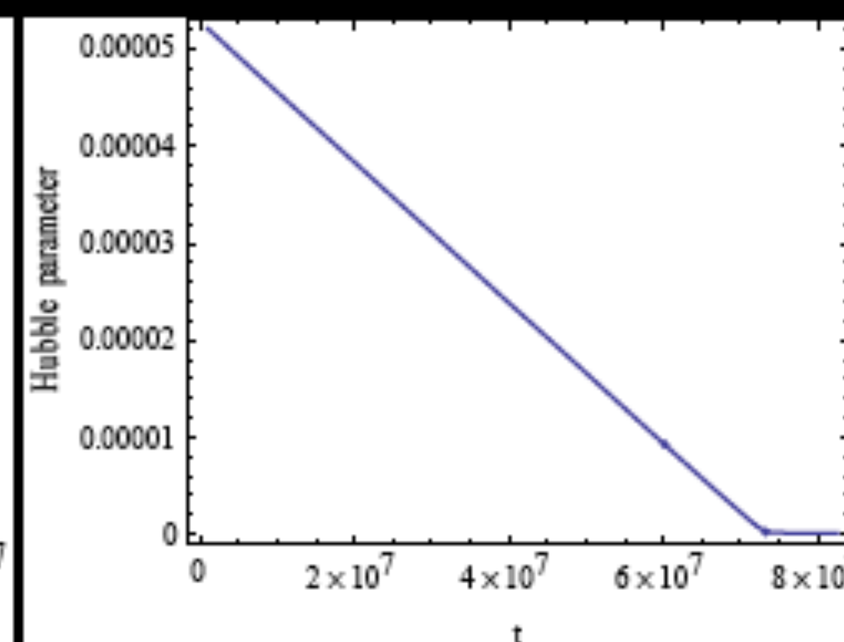
$$\frac{d^2}{d\eta^2}V_\pm + (q^2 \mp 4\cdot C\cdot q\cdot \frac{d\tilde{\phi}}{d\eta})V_\pm = 0 \longrightarrow V_\pm \propto Exp\left[\left(\sqrt{\pm 4\cdot C\cdot q\cdot\frac{d\tilde{\phi}}{d\eta} - q^2}\right)\cdot\eta\right]$$

$$\frac{a}{H}\cdot d\eta = dt$$

$$\frac{a^2}{H^2}\left(\frac{\partial^2}{\partial\tilde{t}^2}V_\pm + \frac{1}{a}\frac{\partial a}{\partial\tilde{t}}\frac{\partial}{\partial \tilde{t}}V_\pm\right) + \left(q^2 \pm 4\cdot C\cdot q\cdot\frac{a}{H}\frac{d\tilde{\phi}}{d\tilde{t}}\right)V_\pm = 0$$

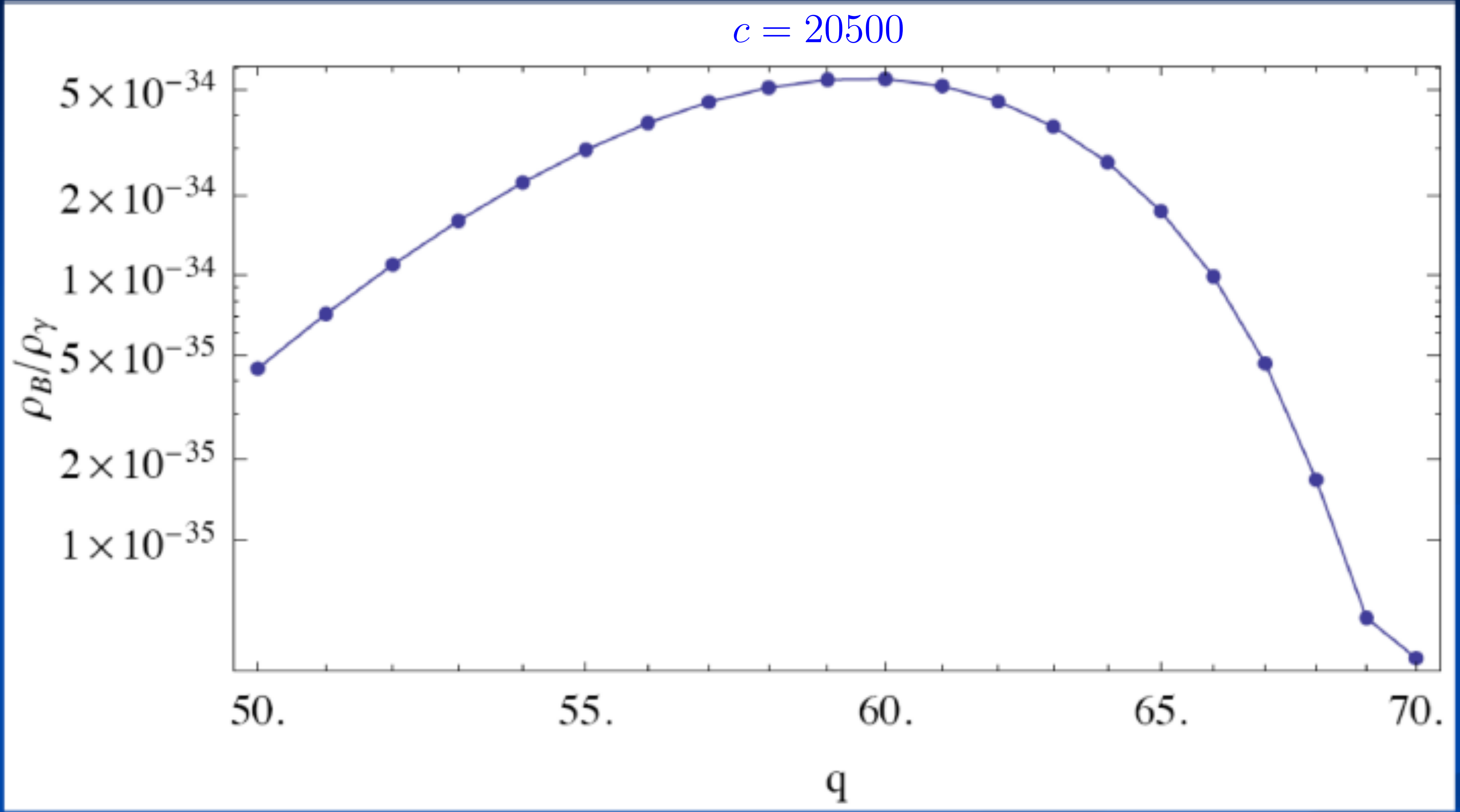




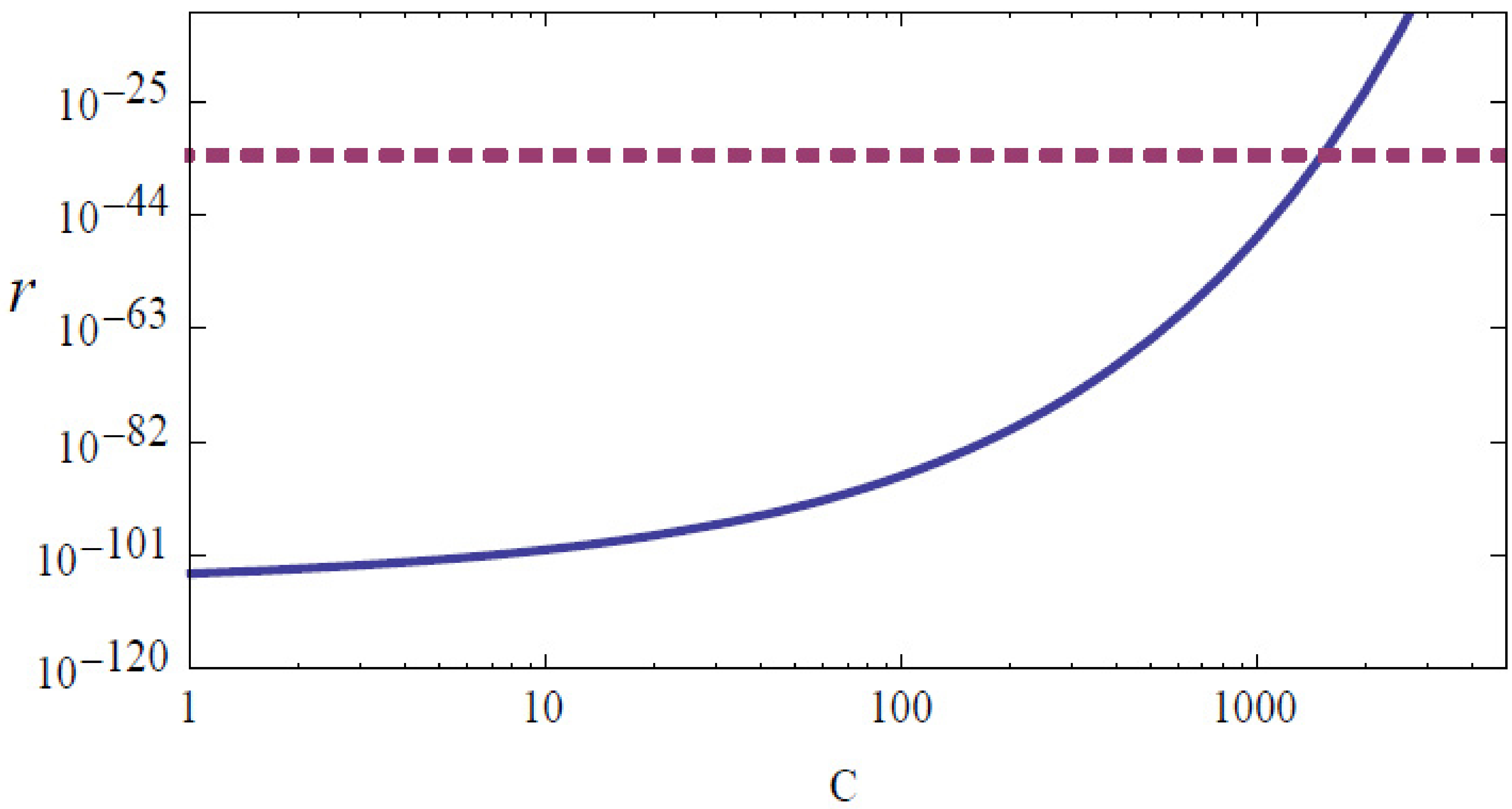


Inflationary PMF

$$\frac{d\rho_B}{dq} = \frac{q^3 H^4}{32\pi^3 a_{\rm end}^4} \sum_{i=\pm} |V_{iq}|^2$$

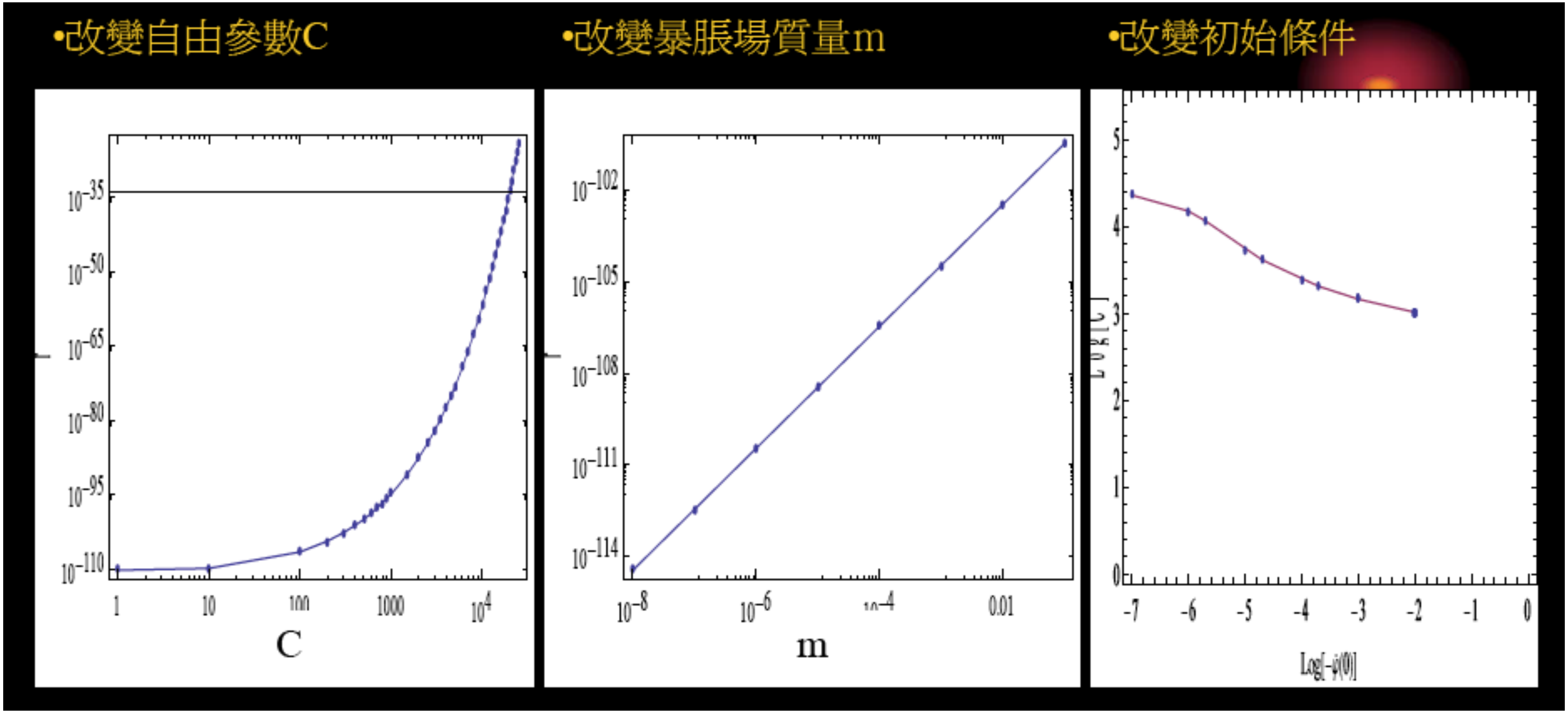


A slow-roll is not enough



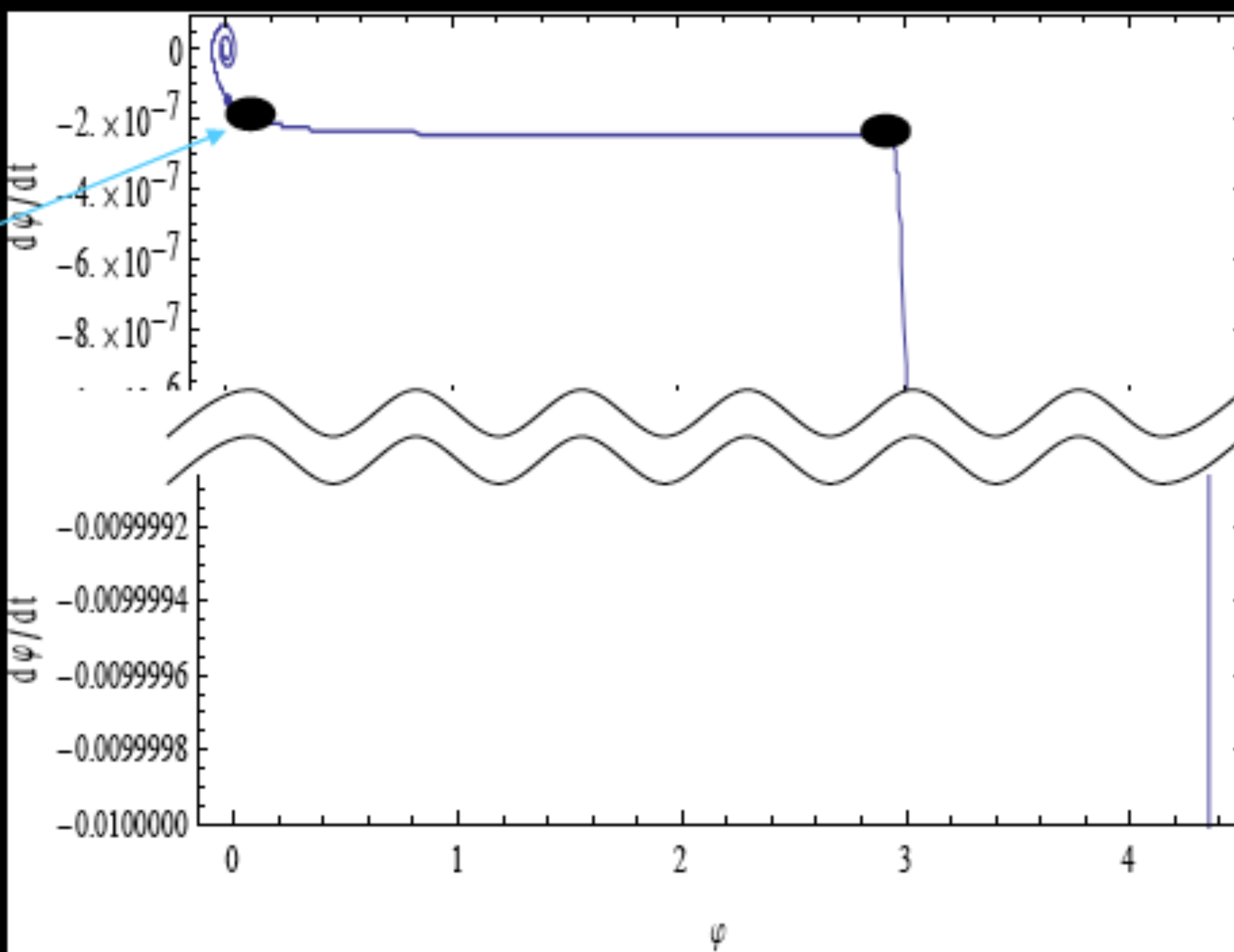
The r vs c diagram suggests that the megnetogenesis induced by the spinodal instability in the slow roll regime is not efficient enough. The dashed line marks the line of $r = 10^{-34}$.

Slow-roll is not enough!



Invoking a fast-roll stage

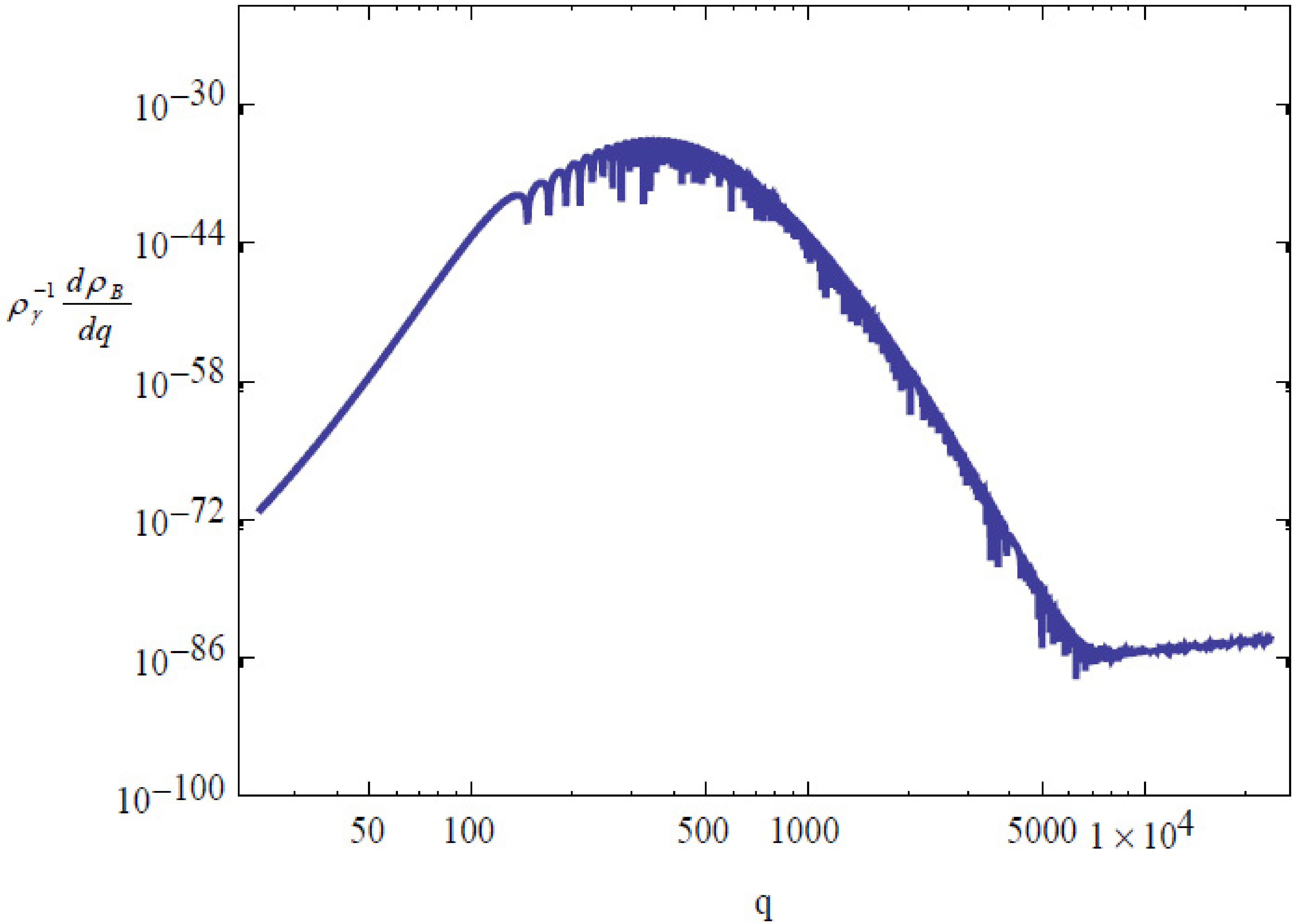
慢滚階段（slow-roll stage）



再熱化（reheating）

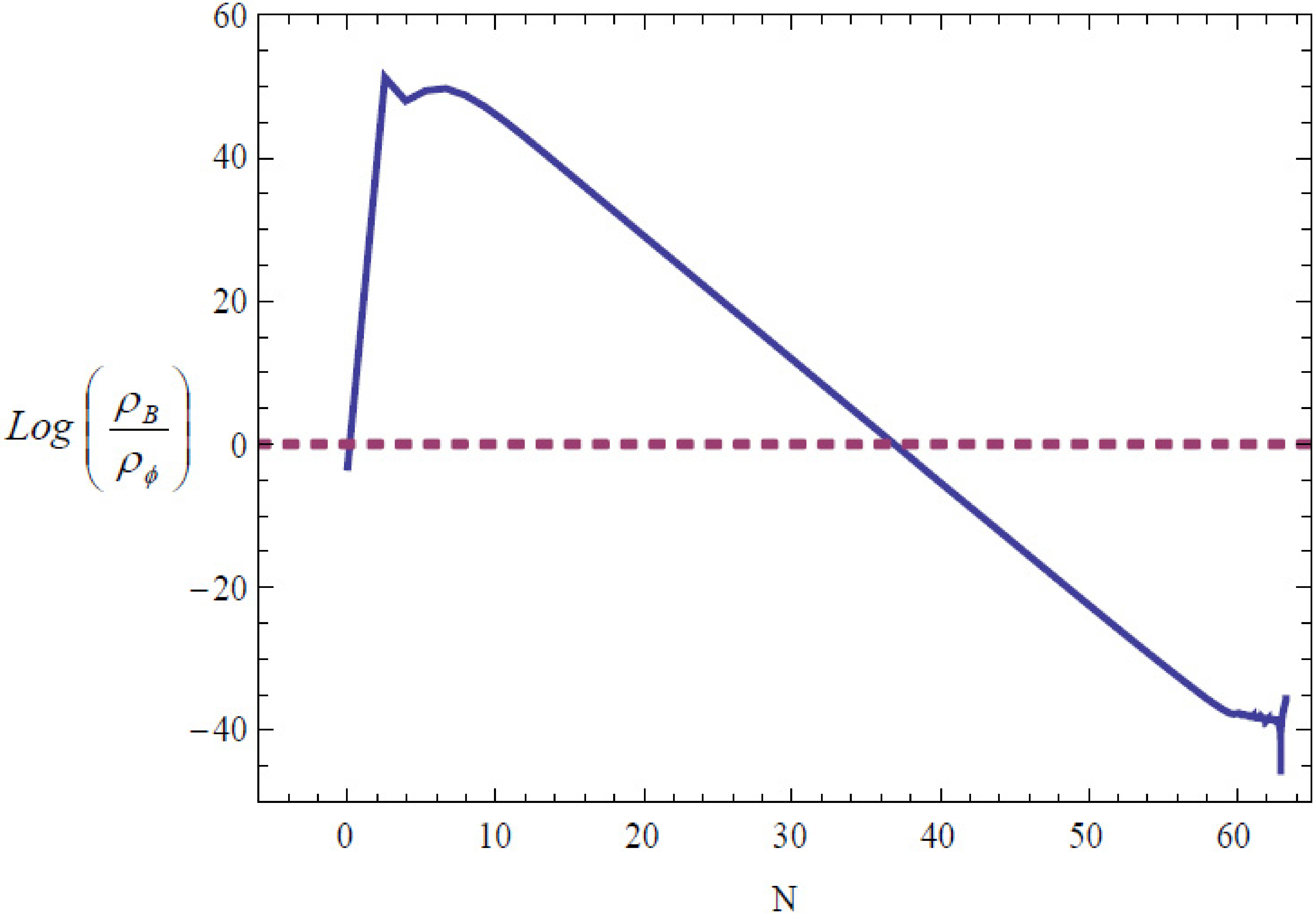
快滚階段（fast-roll stage）

A fast-rolling inflationary PMF



The differential spectrum of magnetic energy density normalized to the photon energy density during the inflation. The coupling constant is set to $c = 12.7$. The magnetic peak mode crossing out the Hubble horizon at $q = 425.5$ corresponds to a comoving distance of about 10 Mpc.

The energy constraint



The evolution of the energy density ratio $\rho_{\rm B}/\rho_\phi$ within the inflationary epoch. The peak mode crossing out the horizon at $q_{\rm peak} \simeq 425$ corresponds to a comoving distance of about 10 Mpc. Apparently, $\rho_{\rm B} \gg \rho_\phi$ for the most time during the inflation.

Conclusion: a no-go theorem

With certain modifications, nonetheless, such a magnetogenesis scenario of spinodal amplification may not be completely ruled out yet. Since the magnetic energy density $\rho_{\rm B}$ scales as a^{-4}, and ρ_γ is only important in the post inflationary reheating stage, the condition for a viable mechanism of magnetogenesis that $\rho_{\rm B} \lesssim \rho_\phi$ at all times then gives rise to

$$\left.\frac{\rho_{\rm B}}{\rho_\phi}\right|_{N=60} \approx \left.\frac{\rho_{\rm B}/\rho_\gamma}{\rho_\phi/\rho_\gamma}\right|_{N=60} \simeq 10^{-34}\left(\frac{a_{60}}{a_*}\right)^4 \lesssim 1,$$

where $a_{60} \propto \exp(60)$ represents the scale factor at the end of inflation, and $a_* \propto \exp(N_*)$ signifies the scale factor at the moment when the kinetic energy of the fast inflaton is thoroughly transformed into the magnetic energy. This criterion can be translated to $N_* \gtrsim 40$, *i.e.* the regime of spinodal instability should not be terminated prior to 20 e-folds after the onset of inflation.