New theory of massive spin-two field

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Preliminary

Recently there have been much progress in the study of massive gravity, with which motivated, We propose a new kind of model for massive spin-two field and investigate the properties.

Motivation

- New mechanism for the supersymmetry.
- Accelerating expansion of the present universe.
- Dark matter $\cdots$ massive spin-two particle?

Fierz-Pauli action (linearized or free theory), 3/4 century ago

M. Fierz and W. Pauli, “On relativistic wave equations for particles of arbitrary spin in an electromagnetic field,” Proc. Roy. Soc. Lond. A **173** (1939) 211.

The Lagrangian of the massless spin-two field (graviton) $h_{\mu\nu}$

$$\mathcal{L}_0 = -\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\lambda h^\lambda{}_\mu \partial_\nu h^{\mu\nu} - \partial^\mu h_{\mu\nu}\partial^\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h\,, \quad \left(h \equiv h^\mu{}_\mu\right)\,.$$

The Lagrangian of the massive graviton with mass m

$$\mathcal{L}_m = \mathcal{L}_0 - \frac{m^2}{2}\left(h_{\mu\nu}h^{\mu\nu} - h^2\right) \quad \text{(Fierz-Pauli action)}\,.$$

Massless graviton: 2 degrees of freedom (helicity),
Massive graviton: 5 degrees of freedom ($2s+1$, spin $s=2$).

When $m=0$, gauge symmetry (linearized general covariance)

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \,,$$

$\xi_\mu(x)$: space-time dependent gauge parameter.

The combination $h_{\mu\nu}h^{\mu\nu} - h^2$:

Fierz-Pauli tuning (not related with any symmetry)

For the combination $h_{\mu\nu}h^{\mu\nu} - (1-a)h^2$,

if $a \neq 0$, there appears ghost scalar field with mass

$$m_g^2 = \frac{3-4a}{2a}m^2 \quad \left(m_g^2 \to \infty \text{ when } a \to 0\right)$$

Hamiltonian and counting of degrees of freedom:

five propagating degrees of freedom in four dimensions

Legendre transformation only with respect to the spatial components h_{ij}.

$$\pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ij}} = \dot{h}_{ij} - \dot{h}_{kk}\delta_{ij} - 2\partial_{(i}h_{j)0} + 2\partial_k h_{0k}\delta_{ij} \,,$$

$$\Rightarrow \; S = \int d^D x \left\{ \pi_{ij}\dot{h}_{ij} - \mathcal{H} + 2h_{0i}\left(\partial_j \pi_{ij}\right) + m^2 h_{0i}^2 \right.$$

$$\left. + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right)\right\} \,,$$

$$\mathcal{H} = \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik}$$

$$+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2(h_{ij}h_{ij} - h_{ii}^2) \,.$$

$m = 0$ case: h_{0i}, h_{00}: Lagrange multipliers $\rightarrow$ constraints

$$\partial_j \pi_{ij} = 0 \,, \quad \vec{\nabla}^2 h_{ii} - \partial_i \partial_j h_{ij} = 0 \,.$$

First class constraints $\rightarrow$ gauge symmetry ($\Leftarrow$ general covariance)

For $D = 4$, h_{ij} and π_{ij} each have 6 components, respectively.
$\rightarrow$ 12 dimensional phase space.
4 constraints $+$ 4 gauge invariances
$\rightarrow$ 4 dimensional phase space
(two polarizations (helicities) of massless graviton)

$m \neq 0$: h_{0i} are no longer Lagrange multipliers $\delta h_{0i} \;\Rightarrow\; h_{0i} = -\frac{1}{m^2}\partial_j \pi_{ij}$,

$$S = \int d^D x \left\{ \pi_{ij}\dot{h}_{ij} - \mathcal{H} + h_{00}\left(\vec{\nabla}^2 h_{ii} - \partial_i\partial_j h_{ij} - m^2 h_{ii}\right)\right\} ,$$

$$\mathcal{H} = \frac{1}{2}\pi_{ij}^2 - \frac{1}{2}\frac{1}{D-2}\pi_{ii}^2 + \frac{1}{2}\partial_k h_{ij}\partial_k h_{ij} - \partial_i h_{jk}\partial_j h_{ik}$$
$$+ \partial_i h_{ij}\partial_j h_{kk} - \frac{1}{2}\partial_i h_{jj}\partial_i h_{kk} + \frac{1}{2}m^2\left(h_{ij}h_{ij} - h_{ii}^2\right) + \frac{1}{m^2}\left(\partial_j \pi_{ij}\right)^2 .$$

h_{00}: Lagrange multiplier $\rightarrow$ single constraint

$$\mathcal{C} = -\vec{\nabla}^2 h_{ii} + \partial_i\partial_j h_{ij} + m^2 h_{ii} = 0 ,$$

Secondary constraint:

$$\{H, \mathcal{C}\}_{\mathrm{PB}} = \frac{1}{D-2} m^2 \pi_{ii} + \partial_i\partial_j \pi_{ij} = 0 , \quad H = \int d^d x \, \mathcal{H} ,$$

Two second class constraints.

For $D = 4$,
12 dimensional phase space $-$ 2 constraints $= 10$ degrees of freedom
(5 polarizations of the massive graviton and their conjugate momenta).

Boulware-Deser ghost

D. G. Boulware and S. Deser, "Classical General Relativity Derived from Quantum Gravity," Annals Phys. **89** (1975) 193.

In non-linear (interacting) theory, 6th degree of freedom appears as a ghost.

Non-linear massive gravity action with flat metric $\eta_{\mu\nu}$, $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$

$$S = \frac{1}{2\kappa^2} \int d^D x \left[\sqrt{-g} R - \frac{1}{4} m^2 \eta^{\mu\alpha} \eta^{\nu\beta} \left(h_{\mu\nu} h_{\alpha\beta} - h_{\mu\alpha} h_{\nu\beta} \right) \right] .$$

δh_{00} does not give a constraint.

$\Rightarrow$ 12 phase space degrees of freedom, or 6 real degrees of freedom.

$\Rightarrow$ One extra degree of freedom, compared with linearized theory

$\Rightarrow$ ghost scalar **Boulware-Deser ghost**

Massive gravity without ghost

C. de Rham and G. Gabadadze, "Generalization of the Fierz-Pauli Action," Phys. Rev. D **82**, 044020 (2010) [arXiv:1007.0443 [hep-th]],

C. de Rham, G. Gabadadze and A. J. Tolley, "Resummation of Massive Gravity," Phys. Rev. Lett. **106** (2011) 231101 [arXiv:1011.1232 [hep-th]].

S. F. Hassan and R. A. Rosen, "Resolving the Ghost Problem in non-Linear Massive Gravity," Phys. Rev. Lett. **108** (2012) 041101 [arXiv:1106.3344 [hep-th]].

Non-dynamical metric $f_{\mu\nu}\,(\sim \eta_{\mu\nu})$, $\sqrt{g^{-1}f}$: $\sqrt{g^{-1}f}\sqrt{g^{-1}f} = g^{\mu\lambda}f_{\lambda\nu}$

Minimal extension of Fierz-Pauli action:

$$S = M_p^2 \int d^4x \sqrt{-g}\, \left[R - 2m^2 \left(\mathrm{tr}\, \sqrt{g^{-1}f} - 3\right)\right] .$$

$$\Rightarrow \qquad S = M_p^2 \int d^4x \sqrt{-g} \left[R + 2m^2 \sum_{n=0}^{3} \beta_n \, e_n(\sqrt{g^{-1}f}) \right] ,$$

$$e_0(\mathbb{X}) = 1 \,, \quad e_1(\mathbb{X}) = [\mathbb{X}] \,, \quad e_2(\mathbb{X}) = \tfrac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]) \,,$$

$$e_3(\mathbb{X}) = \tfrac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]) \,,$$

$$e_4(\mathbb{X}) = \tfrac{1}{24}([\mathbb{X}]^4 - 6[\mathbb{X}]^2[\mathbb{X}^2] + 3[\mathbb{X}^2]^2 + 8[\mathbb{X}][\mathbb{X}^3] - 6[\mathbb{X}^4]) \,,$$

$$e_k(\mathbb{X}) = 0 \quad \text{for } k > 4 \,,$$

$$\mathbb{X} = (X^\mu{}_\nu) \,, \quad [\mathbb{X}] \equiv X^\mu{}_\mu \,,$$

δg_{00} gives a constraint, $+$ secondary constraint $= 2$ constraints.

12 components in phase space $-\ 2$ constraints
$= 10$ components (massive spin 2)

Bimetric gravity (bigravity)

S. F. Hassan and R. A. Rosen, "Bimetric Gravity from Ghost-free Massive Gravity," JHEP **1202** (2012) 126 [arXiv:1109.3515 [hep-th]].

Dynamical $f_{\mu\nu}$ (background independent).

$$
\begin{aligned}
S =& M_g^2 \int d^4x \sqrt{-\det g}\, R^{(g)} + M_f^2 \int d^4x \sqrt{-\det f}\, R^{(f)} \\
& + 2m^2 M_{\text{eff}}^2 \int d^4x \sqrt{-\det g} \sum_{n=0}^{4} \beta_n\, e_n(\sqrt{g^{-1}f})\,, \\
& 1/M_{\text{eff}}^2 \equiv 1/M_g^2 + 1/M_f^2\,.
\end{aligned}
$$

$R^{(g)}$: scalar curvature for $g_{\mu\nu}$, $R^{(f)}$: scalar curvature for $f_{\mu\nu}$.

Massless graviton + Massive graviton

$F(R)$ gravity extension

$\rightarrow$ arbitrary evolution of the expansion of the universe can be reproduced.

S. Nojiri, S. D. Odintsov and N. Shirai, "Variety of cosmic acceleration models from massive $F(R)$ bigravity," JCAP **1305** (2013) 020 [arXiv:1212.2079 [hep-th]].

J. Klusoň, S. Nojiri and S. D. Odintsov, "New proposal for non-linear ghost-free massive $F(R)$ gravity: Cosmic acceleration and Hamiltonian analysis," Phys. Lett. B **726** (2013) 918 [arXiv:1309.2185 [hep-th]].

K. Bamba, A. N. Makarenko, A. N. Myagky, S. Nojiri and S. D. Odintsov, "Bounce cosmology from $F(R)$ gravity and $F(R)$ bigravity," JCAP01(2014)008 [arXiv:1309.3748 [hep-th]].

K. Bamba, Y. Kokusho, S. Nojiri and N. Shirai, "Cosmology and stability in scalar?tensor bigravity," Class. Quant. Grav. **31** (2014) 075016 [arXiv:1310.1460 [hep-th]].

etc.

New theory of massive spin-two field

Y. Ohara, S. Akagi and S. Nojiri, "Renormalizable toy model of massive spin two field and new bigravity" Phys. Rev. D **90** (2014) 043006 [arXiv:1402.5737 [hep-th]],

ibid., "Black hole entropy of new bigravity," arXiv:1407.5765 [hep-th] + paper in preparation.

New ghost free interactions $\cdots$ "pseudo" linear terms

K. Hinterbichler, "Ghost-Free Derivative Interactions for a Massive Graviton," JHEP **1310** (2013) 102 [arXiv:1305.7227 [hep-th]].

(See also, S. Folkerts, A. Pritzel and N. Wintergerst, "On ghosts in theories of self-interacting massive spin-2 particles," arXiv:1107.3157 [hep-th].)

$$
\begin{aligned}
\mathcal{L}_{d,n} \sim & \eta^{\mu_1\nu_1\cdots\mu_n\nu_n}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\cdots\partial_{\mu_{d-1}}\partial_{\nu_{d-1}}h_{\mu_d\nu_d}h_{\mu_{d+1}\nu_{d+1}}\,,\ h_{\mu\nu}=g_{\mu\nu}-\eta_{\mu\nu}\,,\\
\eta^{\mu_1\nu_1\mu_2\nu_2} \equiv & \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}-\eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\,,\\
\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3} \equiv & \eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}-\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_2}+\eta^{\mu_1\nu_2}\eta^{\mu_2\nu_3}\eta^{\mu_3\nu_1}\\
& -\eta^{\mu_1\nu_2}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_3}+\eta^{\mu_1\nu_3}\eta^{\mu_2\nu_1}\eta^{\mu_3\nu_2}-\eta^{\mu_1\nu_3}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_1}\,.
\end{aligned}
$$

- Linear with respect to h_{00} in the Hamiltonian.

$$\eta^{\mu_1\nu_1\mu_2\nu_2} h_{\mu_1\nu_1} h_{\mu_2\nu_2} \sim h_{00}\left(h_{11}+h_{22}+h_{33}\right) + \text{terms not including } h_{00}\,,$$

$$\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1} h_{\mu_2\nu_2}\right) h_{\mu_3\nu_3} \sim \left(\partial_1^2 h_{00}\right)\left(h_{22}+h_{33}-2h_{23}h_{32}\right) + \cdots\,.$$

- Do not appear the terms which include both of h_{00} and h_{0i}.

Variation of h_{00}

$\Rightarrow$ a constraint for h_{ij} and their conjugate momenta π_{ij}

$+$ secondary constraint

$\Rightarrow$ eliminate the ghost.

Power-counting renormalizable model of the massive spin two particle

$$
\begin{aligned}
\mathcal{L}_{h0} =& \frac{1}{2}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\left(\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2}\right)h_{\mu_3\nu_3} - \frac{m^2}{2}\eta^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} \\
& - \frac{\mu}{3!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} \\
& - \frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4} \\
=& \frac{1}{2}\left(h\Box h - h^{\mu\nu}\Box h_{\mu\nu} - h\partial^\mu\partial^\nu h_{\mu\nu} - h_{\mu\nu}\partial^\mu\partial^\nu h + 2h_\nu{}^\rho\partial^\mu\partial^\nu h_{\mu\rho}\right) \\
& - \frac{m^2}{2}\left(h^2 - h_{\mu\nu}h^{\mu\nu}\right) - \frac{\mu}{3!}\left(h^3 - 3hh_{\mu\nu}h^{\mu\nu} + 2h_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\mu\right) \\
& - \frac{\lambda}{4!}\left(h^4 - 6h^2h_{\mu\nu}h^{\mu\nu} + 8hh_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\mu - 6h_\mu{}^\nu h_\nu{}^\rho h_\rho{}^\sigma h_\sigma{}^\mu + 3\left(h_{\mu\nu}h^{\mu\nu}\right)^2\right) .
\end{aligned}
$$

m, μ: parameters with the dimension of mass
λ: dimensionless parameters.
$\Rightarrow$ power-counting renormalizable (free from ghost)

Propagator

$$
\begin{aligned}
D^m_{\alpha\beta,\rho\sigma} =& \frac{1}{2\left(p^2+m^2\right)}\left\{P^m_{\alpha\rho}P^m_{\beta\sigma} + P^m_{\alpha\sigma}P^m_{\beta\rho} - \frac{2}{D-1}P^m_{\alpha\beta}P^m_{\rho\sigma}\right\} , \\
P^m_{\mu\nu} \equiv& \eta_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} .
\end{aligned}
$$

$p^2 \to \infty \Rightarrow D^m_{\alpha\beta,\rho\sigma} \sim \mathcal{O}\left(p^2\right) \cdots$ Not renormalizable

Classical solution

Assume $h_{\mu\nu} = C\eta_{\mu\nu}$, C: constant

$$S = -\int d^4x V(C)\,, \quad V(C) \equiv -6m^2C^2 + 4\mu C^3 + \lambda C^4\,,$$

$C \Leftarrow V'(C) = 0$.

When $\mu = \lambda = 0$, $V(C)$ (Fierz-Pauli model) is unbounded below

$\cdots$ no inconsistency.

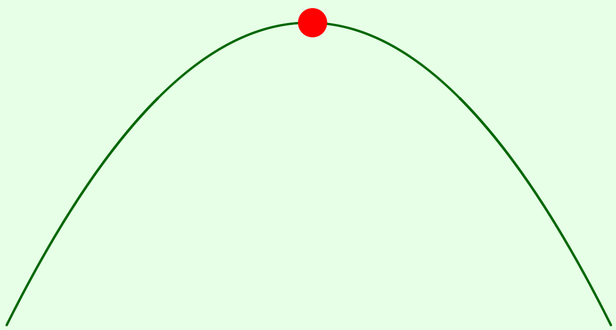
C does not propagate and does not roll down the potential.

(See soon.)

On the other hand,

on the local minimum of the potential ($m^2 < 0$), $h_{\mu\nu}$ becomes tachyon.

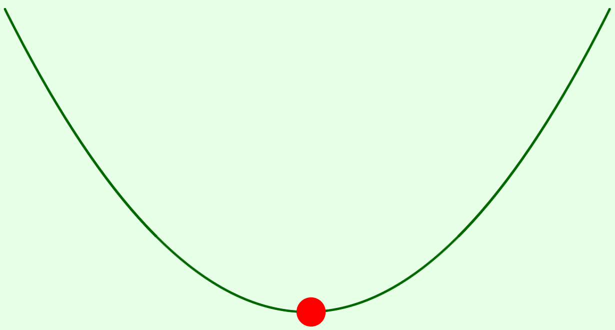
local maximum



no tahyon for $h_{\mu\nu}$ $(m^2 > 0)$

stable

C does not roll down



local minimum

tahyon for $h_{\mu\nu}$ $(m^2 < 0)$!

unstable

We now show C is always a constant.

Eq. of motion:

$$
\begin{aligned}
0 =& \eta^{\mu\nu\mu_1\nu_1\mu_2\nu_2}\partial_{\mu_1}\partial_{\nu_1}h_{\mu_2\nu_2} - m^2\eta^{\mu\nu\mu_1\nu_1}h_{\mu_1\nu_1} - \frac{\mu}{2}\eta^{\mu\nu\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} \\
&- \frac{\lambda}{3!}\eta^{\mu\nu\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} \,.
\end{aligned}
$$

Assume $h_{\mu\nu} = C\eta_{\mu\nu}$ but C is not a constant,

$$
0 = \eta^{\mu\nu}\left(2\Box C - 3m^2C - 3\mu C^2 + 3\lambda C^3\right) - 2\partial^\mu\partial^\nu C \,.
$$

$\Rightarrow$ C should be a constant.
Even if C is on the local maximum of the potential, C does not roll down.

Parametrize m^2 and μ by

$$m^2 = -\frac{\lambda}{3} C_1 C_2 \, , \quad \mu = -\frac{\lambda}{3} \left(C_1 + C_2 \right) \, .$$

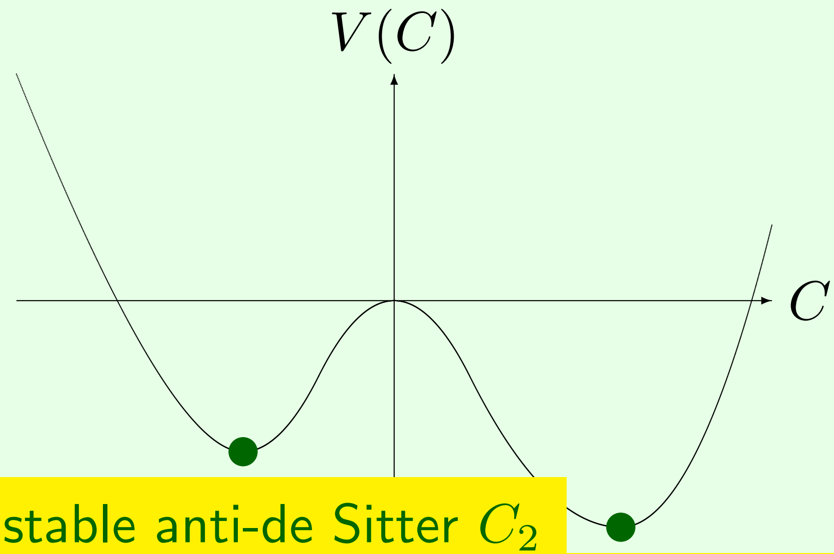
$V'(C) = 0 \Rightarrow C = 0,\, C_1,\, C_2$

$$V\left(C_1\right) = \frac{\lambda}{3} C_1^3 \left(-C_1 + 2C_2\right) \, , \quad V\left(C_2\right) = \frac{\lambda}{3} C_2^3 \left(-C_2 + 2C_1\right) \, .$$

If we couple the model with gravity, $V(C) \sim$ cosmological constant.

$\Rightarrow$ accelerating expansion of the universe if $V > 0$.

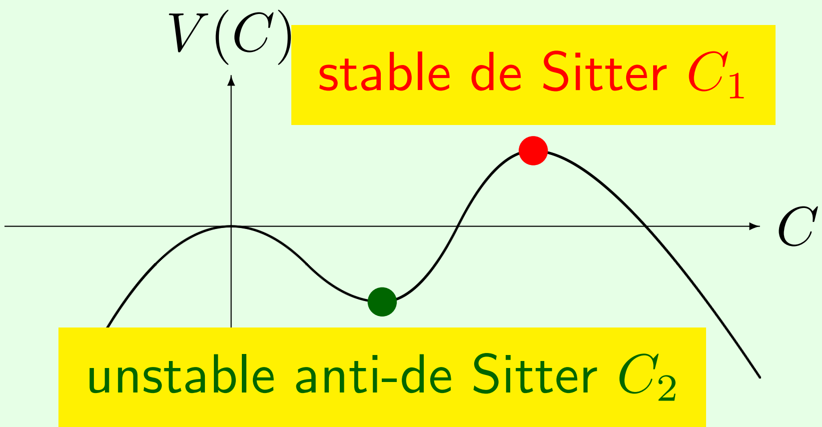
$\lambda > 0$

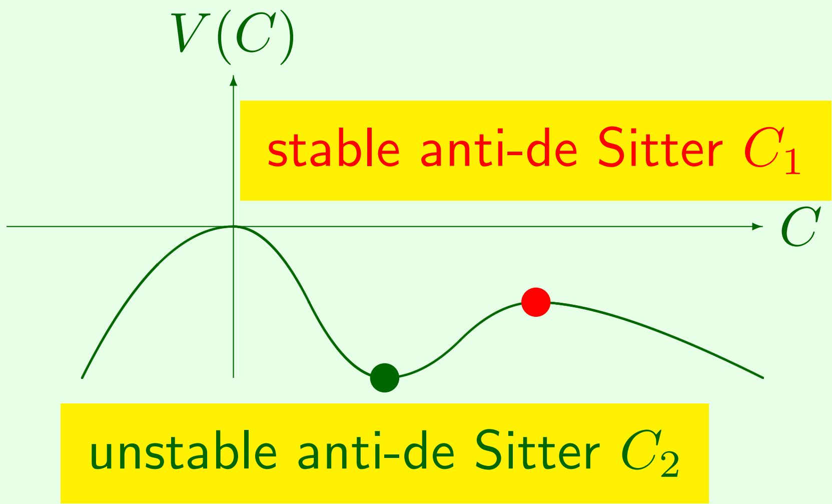


unstable anti-de Sitter C_2

unstable anti-de Sitter C_1

$\lambda < 0$





When we consider the supersymmetric model, if $E > 0$, the breaking of supersymmetry?

The relation between $C_{1,2}$ and the corresponding space-time and the stability of the solutions

	$0 < \lambda$	$-\frac{2\mu^2}{3m^2} < \lambda < 0$	$-\frac{3\mu^2}{4m^2} < \lambda < -\frac{2\mu^2}{3m^2}$
de Sitter	no solution	C_2 (stable)	no solution
Anti-de Sitter	C_1 (unstable) C_2 (unstable)	C_1 (unstable)	C_1 (unstable) C_2 (stable)

$$C_1 = \frac{-3\mu + \sqrt{9\mu^2 + 12m^2\lambda}}{2\lambda}, \quad C_2 = \frac{-3\mu - \sqrt{9\mu^2 + 12m^2\lambda}}{2\lambda}.$$

New bigravity

Couples with gravity $\sim$ new bigravity (gravity is not renormalizable)

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}\nabla_{\mu_1}\nabla_{\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3} - \frac{1}{2}m^2 g^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2} \right.$$
$$-\frac{\mu}{3!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}$$
$$\left. -\frac{\lambda}{4!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}\right\},$$

$h_{\mu\nu}$ is not the perturbation in $g_{\mu\nu}$ but $h_{\mu\nu}$ is a field independent of $g_{\mu\nu}$.
Cosmology with the Einstein-Hilbert action:

$$S_{\rm EH} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}R\,.$$

Naively if we work in the local Lorentz frame, no ghost?

I. L. Buchbinder, D. M. Gitman, V. A. Krykhtin and V. D. Pershin, "Equations of motion for massive spin-2 field coupled to gravity," Nucl. Phys. B **584** (2000) 615 [hep-th/9910188],

I. L. Buchbinder, V. A. Krykhtin and V. D. Pershin, "On consistent equations for massive spin two field coupled to gravity in string theory," Phys. Lett. B **466** (1999) 216 [hep-th/9908028].

Even in case of the Fierz-Pauli model, consistent theory should be

$$
\begin{aligned}
S = \int d^D x \sqrt{-g} \left\{ \frac{1}{4} \nabla_\mu h \nabla^\mu h - \frac{1}{4} \nabla_\mu h_{\nu\rho} \nabla^\mu h^{\nu\rho} - \frac{1}{2} \nabla^\mu h_{\mu\nu} \nabla^\nu h \right. \\
\left. + \frac{1}{2} \nabla_\mu h_{\nu\rho} \nabla^\rho h^{\nu\mu} + \frac{\xi}{2D} R h_{\mu\nu} h^{\mu\nu} + \frac{1-2\xi}{4D} R h^2 - \frac{m^2}{4} h_{\mu\nu} h^{\mu\nu} + \frac{m^2}{4} h^2 \right\} .
\end{aligned}
$$

Furthermore $R_{\mu\nu} = \frac{1}{D} g_{\mu\nu} R$ or $k^\mu k^\nu R_{\mu\nu} = \frac{1}{D} k^2 R$.

k^μ: time-like Killing vector of the background.

In case of interacting model,

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{2}\nabla_\mu h\nabla^\mu h - \frac{1}{2}\nabla_\mu h_{\nu\rho}\nabla^\mu h^{\nu\rho} - \nabla^\mu h_{\mu\nu}\nabla^\nu h \right.$$
$$+ \nabla_\mu h_{\nu\rho}\nabla^\rho h^{\nu\mu} + \frac{\xi}{4}Rh_{\alpha\beta}h^{\alpha\beta} + \frac{1-2\xi}{8}Rh^2 + \frac{m^2}{2}g^{\mu_1\nu_1\mu_2\nu_2}h_{\mu_1\nu_1}h_{\mu_2\nu_2}$$
$$- \frac{\mu}{3!}g^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}$$
$$\left. -\frac{\lambda}{4!}\eta^{\mu_1\nu_1\mu_2\nu_2\mu_3\nu_3\mu_4\nu_4}h_{\mu_1\nu_1}h_{\mu_2\nu_2}h_{\mu_3\nu_3}h_{\mu_4\nu_4}\right\},$$

Changes are only for quadratic terms as in the Fierz-Pauli model.
$\Rightarrow$ We may consider (anti-)de Sitter (-Schwarzschild or Kerr) space-time as exact solutions.

Y. Ohara, S. Akagi and S. Nojiri, in preparation.

Assume $h_{\mu\nu} = C g_{\mu\nu}$, C: constant

$$S = - \int d^4x \sqrt{-g} V(C) + \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} R \,,$$

$$V(C) = - \left\{ 6m^2 + (2 - 3\xi) R \right\} C^2 + 4\mu C^3 + \lambda C^4 \,,$$

$$S = \left\{ (2 - 3\xi) C^2 + \frac{1}{2\kappa^2} \right\} \int d^4x \sqrt{-g} \left[R - 2\Lambda_{\mathrm{eff}} \right] \,,$$

Effective mass M: $M^2 \equiv m^2 - 2\mu C - \lambda C^2 \,,$

Effective cosmological constant: $\Lambda_{\mathrm{eff}} \equiv \dfrac{\kappa^2 \left(-6m^2 C^2 + 4\mu C^3 + \lambda C^4 \right)}{2\kappa^2 C^2 (2 - 3\xi) + 1} \,.$

$\Rightarrow R = 4\Lambda_{\rm eff} \Rightarrow$

$$V_0{}'(C) = 4C\left\{-2\mu\zeta C^3 + \left(\lambda + 6\zeta m^2\right)C^2 + 3\mu C - 3m^2\right\} = 0\,.$$

$\zeta \equiv \kappa^2\,(2 - 3\xi)$.

Trivial solution $C = 0$.

$$-2\mu\zeta C^3 + \left(\lambda + 6\zeta m^2\right)C^2 + 3\mu C - 3m^2 = 0\,.$$

$\Rightarrow$

$$C = x + \frac{\lambda + 6\zeta m^2}{6\mu\zeta}\,,$$

$$p = -\frac{1}{3}\left\{\left(\frac{\lambda + 6\zeta m^2}{2\mu\zeta}\right)^2 + \frac{9}{2\zeta}\right\}\,,\quad q = \frac{2}{27}\left(\frac{\lambda + 6\zeta m^2}{2\mu\zeta}\right)^3 - \frac{\lambda}{4\mu\zeta^2}\,.$$

$\omega \equiv \mathrm{e}^{i2\pi/3} \Rightarrow$ Solution

$$x = \omega^k \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \omega^{3-k} \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}\,, \quad k = 1, 2, 3\,,$$

Determinant $D = -27q^2 - 4p^3 = -2^2 \cdot 3^3 \left\{ \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3 \right\}$.

Except the case $q = p = 0$,

1. $D > 0$ There are three different real solutions.

2. $D < 0$ There is only one real solution.

3. $D = 0$ There are three real solutions but two of them are degenerate with each other.

Stability $\Leftrightarrow$ Higuchi bound

A. Higuchi, “Forbidden Mass Range for Spin-2 Field Theory in De Sitter Space-time,” Nucl. Phys. B **282** (1987) 397.

Black hole solution

(ant-) de Sitter-Schwarzschild (Kerr) black hole space-time is an exact solution.

Entropy would not be changed from the case of the Einstein gravity.

c.f. Hassan-Rosen bigravity.case:

T. Katsuragawa and S. Nojiri, “Noether current from surface term, Virasoro algebra and black hole entropy in bigravity,” Phys. Rev. D **87** (2013) 10, 104032 [arXiv:1304.3181 [hep-th]],

T. Katsuragawa, “Properties of Bigravity Solutions in a Solvable Class,” Phys. Rev. D **89** (2014) 124007 [arXiv:1312.1550 [hep-th]].

Entropy is the sum of the contributions from two metric sectors corresponding to $g_{\mu\nu}$ and $f_{\mu\nu}$.

Summary

- Proposition of a new theory describing massive spin two particle.

- The coupling of the theory with gravity
  $\rightarrow$ a new kind of bimetric gravity or bigravity.
  - The field of the massive spin two particle plays the role of the cosmological constant.
    * The conditions of no ghost is not changed from those in the Fierz-Pauli case.
    * Accelerating expansion (inflation or dark energy).
    * (anti-)de Sitter-Schwarzschild (Kerr) space-time is an exact solution.

- Can massive spin two particle be dark matter?