Anisotropic inflation reexamined

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1410.xxxx, Atsushi Naruko, Eiichiro Komatsu & MY

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

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Introduction

Inflation

Starobinsky, Sato, Guth

The Universe rapidly expanded thanks to the vacuum energy density and goes into the almost De Sitter phase.

But, the inflation must end to (re)heat the Universe.

Time translational invariance must be broken.

In fact, the spectral index of the curvature perturbations deviates from unity at 5 sigma level, which confirms the violation of time translational invariance.

$$n_s - 1 = -2\epsilon - \eta, \quad \epsilon \equiv -H^2/\dot{H}, \quad \eta \equiv \dot{\epsilon}/(H\epsilon).$$

Planck 2013 results. XXII

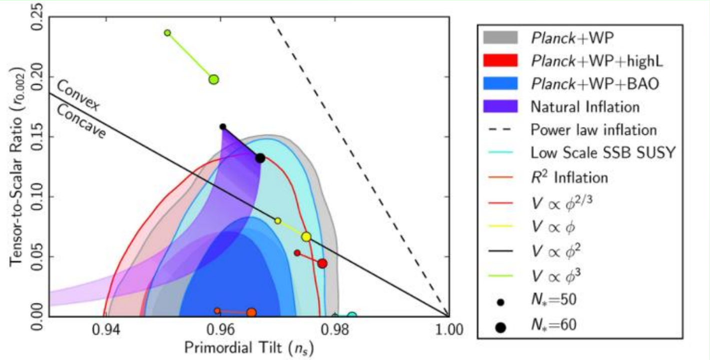


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

De Sitter spacetime not only has time translational invariance but also is homogeneous and isotropic.

How much can the isotropy be broken during inflation ?

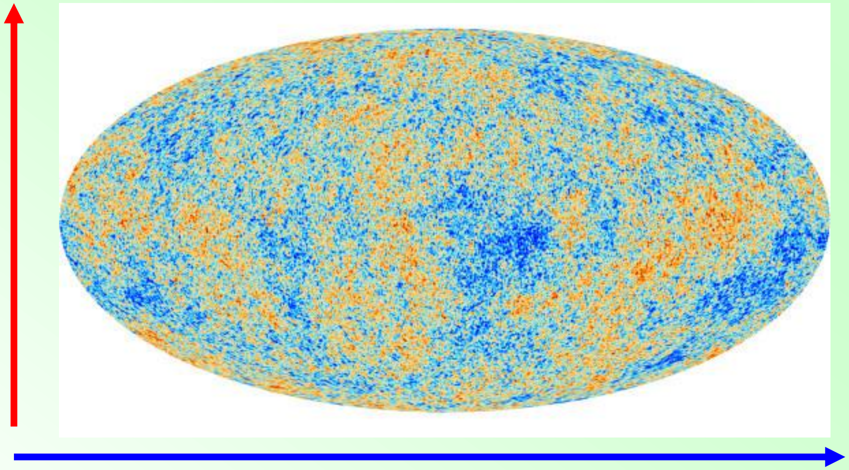
Observational constraint on statistical anisotropies

Powerspectrum of the primordial curvature perturbations can depend not only on the magnitude of a momentum but also on the its direction, if the Universe is not exactly isotropic .

$$P(\boldsymbol{k}) = P(k)\left[1 + g_*(\boldsymbol{k}\cdot\boldsymbol{v})^2\right]. \quad g_* = 0.002 \pm 0.016$$

(Kim & Komatsu)

(consistent with isotropy)



Amplitude can be different for red and blue directions.

- **Is it possible to realize anisotropic Universe in the inflationary Universe ?**
- **How is the anisotropy of the perturbations (g_*) related to that of the background (that is, breaking of the rotational symmetry of the background) ?**

Cosmic no hair conjecture

Gibbons & Hawking, Wald

Given the cosmological constant and matter which satisfies

- **Dominant energy condition :** $\rho \geq 0, \quad \rho \geq |P|.$
- **Strong energy condition :** $\rho + 3P \geq 0, \quad \rho + P \geq 0.$

⇒ **The Universe approaches the de Sitter state and hence will be isotropized.**

However, inflaton potential is not exactly cosmological "constant" because, otherwise, it does not roll along the potential and does not end.

⇒ **In a realistic model of inflation, anisotropy with at most the slow-roll order may be allowed.**

Anisotropic inflation

Anisotropic inflation

Watanabe, Kanno, Soda

$$S = \int \mathrm{d}x^4 \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right].$$

(non-trivial kinetic term)

- f(φ) breaks the conformal invariance of vector field.
  With conformal invariance, vector field is decoupled from cosmic expansion and no interesting effects are generated.
  (Note that FLRW is conformal to Minkowski.)

- No ghost and/or tachyonic instabilities different from the following models:

$$\begin{cases} \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - V(A_\mu A^\mu), \\ \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \lambda(A_\mu A^\mu - m^2), \\ \mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(m^2 - \frac{R}{6}\right)A_\mu A^\mu. \end{cases}$$

Vector potential (Ford)

Lorentz violation (Ackerman et al.)

Non-minimal coupling (Golovnev et al.)

$$c = \hbar = M_G^2 = 1/(8\pi G) = 1$$

Behavior of vector field

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} \right].$$

(non-trivial kinetic term)

- special coupling : $f(\phi) = e^{2c \int \mathrm{d}\phi \frac{U}{U_\phi}} \simeq e^{2c \int \mathrm{d}\phi \left(-\frac{\mathrm{d}\alpha}{\mathrm{d}\phi} \right)} \propto e^{-2c\alpha}$

(c : constant, e^α : scale factor)

⇨ **Vector field on large scales :** $0 = \frac{1}{\sqrt{-g}} \partial_\nu \left(\sqrt{-g} f^2 F^{\mu\nu} \right) = \frac{1}{e^{3\alpha}} \partial_t \left(f^2 e^\alpha \dot{A} \right).$

⇨ $\dot{A} \propto \frac{1}{e^\alpha f^2} \propto e^{(4c-1)\alpha}.$

⇨ **If 4c-1 > 0, vector field (E) on large scales can survive, that is, there can be a homogeneous (background) vector field.**

⇨ **Bianchi (homogeneous but anisotropic) Universe can be realized.**

Background

Bianchi type I Universe

- Bianchi I metric (homogeneous but anisotropic) :

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2\alpha(t)}\left[e^{-4\beta(t)}\,\mathrm{d}x^2 + e^{2\beta(t)}\left(\mathrm{d}y^2 + \mathrm{d}z^2\right)\right].$$

(shear σ ~ dβ/ dα)

- homogeneous vector field : $A_\mu = (0, A_x(t), 0, 0)\,.$

- EOM of vector field : $0 = \frac{1}{\sqrt{-g}}\partial_\nu\left(\sqrt{-g}f^2F^{\mu\nu}\right) = \frac{1}{e^{3\alpha}}\partial_t\left(f^2e^{\alpha+4\beta}\,\dot{A}_x\right)\delta^\mu_x.$

Integration

$\Rightarrow$ $\dot{A}_x = \frac{C_A}{f^2e^{\alpha+4\beta}}.$ (C_A : const)

Background EOMs

$$\begin{cases} 3H^2 - 3\dot{\beta}^2 = \frac{1}{2}\dot{\phi}^2 + U(\phi) + \frac{1}{2}\mathcal{V}\,, \\ \dot{H} + 3H^2 = U(\phi) + \frac{1}{6}\mathcal{V}\,, \\ \ddot{\beta} + 3H\dot{\beta} = \frac{1}{3}\mathcal{V}\,, \\ \ddot{\phi} + 3H\dot{\phi} + U_\phi = U_\phi^{\vec{A}} = \frac{f_\phi}{f}\mathcal{V}\,. \end{cases}$$

Quantities related to vector field can be written by single quantity V:

$$\mathcal{V} = \frac{f^2 \dot{A}_x^2}{e^{2(\alpha - 2\beta)}} = \frac{C_A^2}{f^2 e^{4(\alpha+\beta)}}\,.$$

$$\Rightarrow \quad \rho^{\vec{A}} = \frac{1}{2}\mathcal{V}\,, \quad P^{\vec{A}} = \frac{1}{6}\mathcal{V}\,, \quad (\pi^{\vec{A}})^x{}_x = -\frac{2}{3}\mathcal{V}\,, \quad (\pi^{\vec{A}})^a{}_b = \frac{1}{3}\mathcal{V}\,\delta^a{}_b\,.$$

Attractor solution ???

Watanabe, Kanno, Soda

$U(\phi) = \frac{1}{2}m^2\phi^2\,, \quad f(\phi) = e^{\frac{1}{2}c\phi^2}$ as a concrete example.

$$I \equiv \frac{\mathcal{V}}{m^2} = \frac{c-1}{c^2}\left[1 + \left(\frac{c-1}{c^2}\frac{1}{I_*} - 1\right) e^{-4(c-1)(\alpha-\alpha_*)}\right]^{-1}$$

If ….. is subdominant, I converges to (c-1) / c^2.
This is an attractor solution.

Note that this solution exists only if $e^{-4(c-1)(\alpha-\alpha_*)} \ll 1, \;\; c-1 = \mathcal{O}(1).$

But, the current constraint on g$_*$ yields $I = \frac{c-1}{c^2} \sim c-1 \lesssim 10^{-8} \times \left(\frac{|g_*|}{10^{-3}}\right)\left(\frac{N_k}{60}\right)^{-2}.$

Thus, unfortunately, the attractor solution is not realized in our Universe.
So, we have to consider another class of solution (perturbative solution).

$$I = \frac{\mathcal{V}}{m^2} = I_* e^{-4(c-1)(\alpha-\alpha_*)}.$$

Perturbations

(Linear) Perturbations in 2D symmetry

Bianchi Universe has less symmetry than FLRW Universe

(3D ➔ 2D (y-z) rotational symmetry)

$$A_\mu = (0, A_x(t), 0, 0) \,.$$

➡ **Only two types of perturbations : 2D scalars & 2D vectors**

- **2D scalars : δg_{00}, δg_{0x}, 1 of δg_{0a}, δg_{xx}, 1 of δg_{xa}, 2 of δg_{ab} (metric)**
  $\delta\varphi$, δA_0, δA_x, 1 of δA_a (matter)

(7+4=11 components)

(a = y or z)

- **2D vectors : 1 of δg_{0a}, 1 of δg_{xa}, 1 of δg_{ab} (metric)**
  1 of δA_a (matter)

(3+1=4 components)

- **General covariance : 4 = 3S + 1V gauge d.o.f & 4 = 3S + 1V constraint eqs.**
- **U(1) gauge symmetry: 1 = 1 S gauge d.o.f. & 1 = 1S constraint eq.**

➡ **3 = 11 – 6 – 2 2D scalars & 2 = 4 – 2 2D vectors**

2D scalar perturbations

$$\begin{cases} \delta g_{\mu\nu} = \begin{pmatrix} -2A & e^{2(\alpha-2\beta)}B_x & e^{2(\alpha+\beta)}B_y & 0 \\ e^{2(\alpha-2\beta)}B_x & 2e^{2(\alpha-2\beta)}C & 0 & 0 \\ e^{2(\alpha+\beta)}B_y & 0 & 2e^{2(\alpha+\beta)}C & 0 \\ 0 & 0 & 0 & -2e^{2(\alpha+\beta)}C \end{pmatrix} \\ \delta\phi\,, \qquad \delta A_\mu = (\delta A_t\,, 0\,, \delta A_y\,, 0) \end{cases}$$

(A, Bx, By, δAt are non-dynamical and can be integrated out.)

3 canonically normalized variables :

$$\delta\phi \;\to\; \varphi \equiv e^{\alpha+\beta}\,\delta\phi\,, \qquad C \;\to\; \mathcal{G} \equiv \sqrt{2}e^{\alpha+\beta}\,C\,, \qquad \delta A_y \;\to\; \mathcal{A} \equiv f\,\frac{k_x}{\tilde{k}}\,\delta A_y$$

(3D scalar) **(3D tensor)**

$$\left(\tilde{k} \equiv \sqrt{k_x^2 + (e^{-3\beta}k_y)^2}\right)$$

$$\Rightarrow \quad S^{\rm scalar} = \int {\rm d}t\,\frac{{\rm d}^3k}{(2\pi)^3}\,{\rm e}^{\alpha-2\beta}\,\big(\underbrace{\mathcal{L}^{\varphi\varphi} + \mathcal{L}^{\mathcal{G}\mathcal{G}} + \mathcal{L}^{\mathcal{A}\mathcal{A}}}_{\text{Auto-correlation}} + \underbrace{\mathcal{L}^{\varphi\mathcal{G}} + {\rm L}^{\varphi\mathcal{A}} + \mathcal{L}^{\mathcal{G}\mathcal{A}}}_{\text{Cross-correlation}}\big)\,,$$

Anisotropy

$$S^{\mathrm{scalar}} = \int \mathrm{dt}\, \frac{\mathrm{d}^3\mathrm{k}}{(2\pi)^3}\, \mathrm{e}^{\alpha-2\beta} \left(\mathcal{L}^{\varphi\varphi} + \mathcal{L}^{\mathcal{G}\mathcal{G}} + \mathcal{L}^{\mathcal{A}\mathcal{A}} + \mathcal{L}^{\varphi\mathcal{G}} + \mathrm{L}^{\varphi\mathcal{A}} + \mathcal{L}^{\mathcal{G}\mathcal{A}} \right),$$

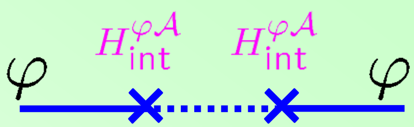
$$\begin{cases} \mathcal{L}^{\varphi\varphi} = \frac{1}{2}|\dot{\varphi}|^2 - \frac{1}{2}\frac{\tilde{k}^2}{e^{2(\alpha-2\beta)}}|\varphi|^2 + \frac{1}{2}H^2(2 + 2\epsilon_H + 3\eta_H + \delta m^2_{\varphi\varphi})|\varphi|^2 \\ \mathcal{L}^{\varphi\mathcal{A}} = \sqrt{6}cH\sqrt{I}\,\sin\theta(\dot{\varphi} - H\varphi)\mathcal{A}^* + (\mathrm{c.c.}) \quad \left(\mathcal{L}^{\mathcal{G}\varphi} \propto \sqrt{\epsilon_H} I \ll \mathcal{L}^{\mathcal{G}\mathcal{A}} \propto \sqrt{\epsilon_H I} \ll \mathcal{L}^{\varphi\mathcal{A}} \propto \sqrt{I} \right) \end{cases}$$

$$\left(\delta m^2_{\varphi\varphi} = 2\left[12c^2\sin^2\theta - (4c^2 + c + 1) \right] I + \mathcal{O}(\epsilon_H I), \quad \sin\theta \equiv e^{-3\beta}k_y/\tilde{k}. \right)$$

$$I\,(t) \le \mathcal{O}(\epsilon_H)\,, \quad \left| \frac{\dot{I}}{HI} \right| \le \mathcal{O}(\epsilon_H) \iff |c-1| \le \mathcal{O}(\epsilon_H)\,.$$

(almost scale invariant)

$$P_{\varphi\varphi}(\boldsymbol{k}) = \underset{\mathcal{L}^{\varphi\varphi}}{\underset{\uparrow}{P^{\mathrm{iso}}_{\varphi\varphi}}}(k) + \underset{\mathcal{L}^{\varphi\mathcal{A}}}{\underset{\uparrow}{P^{\mathrm{aniso}}_{\varphi\varphi}}}(\boldsymbol{k}) = P(k)\left[1 + g_*\sin^2\theta \right].$$



$$\langle \mathrm{in}|\varphi_{\mathbf{k}}\varphi_{\mathbf{p}}|\mathrm{in}\rangle = i^2\int_{\tau_i}^{\tau} \mathrm{d}\tau_2 \int_{\tau_i}^{\tau_2} \mathrm{d}\tau_1\, \langle 0 \left| \left[H^{\varphi\mathcal{A}}_{\mathrm{int}}(\tau_1)\,, \left[H^{\varphi\mathcal{A}}_{\mathrm{int}}(\tau_2)\,, \varphi_{\mathbf{k}}(\tau)\varphi_{\mathbf{p}}(\tau) \right] \right] \right| 0 \rangle\,.$$

$\mathbf{g}_*$

$$P_{\varphi\varphi}(\boldsymbol{k}) = P^{\rm iso}_{\varphi\varphi}(k) + P^{\rm aniso}_{\varphi\varphi}(\boldsymbol{k}) = P(k)\left[1 + g_* \sin^2\theta\right].$$

$$\Rightarrow \quad g_* \simeq -24\, I_* \times \left(\frac{e^{N_k\delta} - 1}{2^{\nu_\varphi - \nu_A}\,\delta}\right)^2 \sim -\frac{24 I_*}{\delta^2}.$$

(unless δ is extremely small accidentally)

$$\begin{cases} \delta = 2\left(\dfrac{\epsilon_H + 4\eta_H}{3} - 2(c-1)\right), \\ \nu^\varphi - \nu^A = \dfrac{2\epsilon_H + 5\eta_H}{3} - 2(c-1). \end{cases}$$

$\Rightarrow$ **shear ~ anisotropy ~** $\epsilon_H I \leq 10^{-8}\epsilon_H \left(\dfrac{g_*}{10^{-3}}\right)\left(\dfrac{\delta}{10^{-2}}\right)^2.$

$g_* = 0.002 \pm 0.016$ **(Kim & Komatsu)**

breaking of rotational sym. <<< **breaking of time translational sym.**

shear $= \mathcal{O}(10^{-8}\epsilon)$ $\qquad$ $n_s - 1 = \mathcal{O}(\epsilon)$

Summary

- We have reexamined anisotropic inflation model.
- We found a new perturbative (background) solution instead of the attractor solution, which unfortunately is ruled out from the current constraint on g_*.
- Based on this perturbative solution, we gave a new formula of g_*.
- We found that the magnitude of the breaking of rotational symmetry during inflation is much smaller than that of time translational symmetry.

謝謝