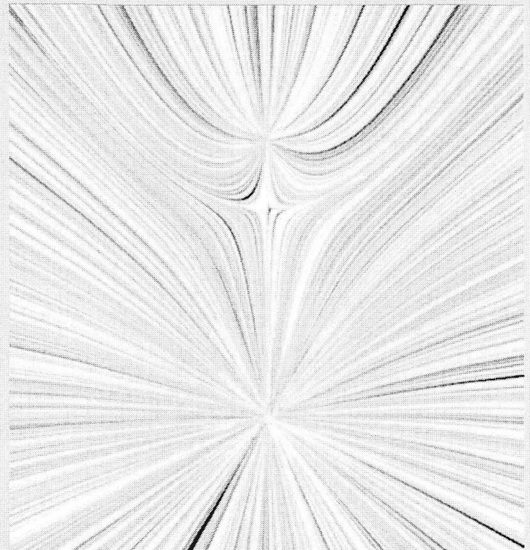
Concept Review / Overview

General Exam Suggestions

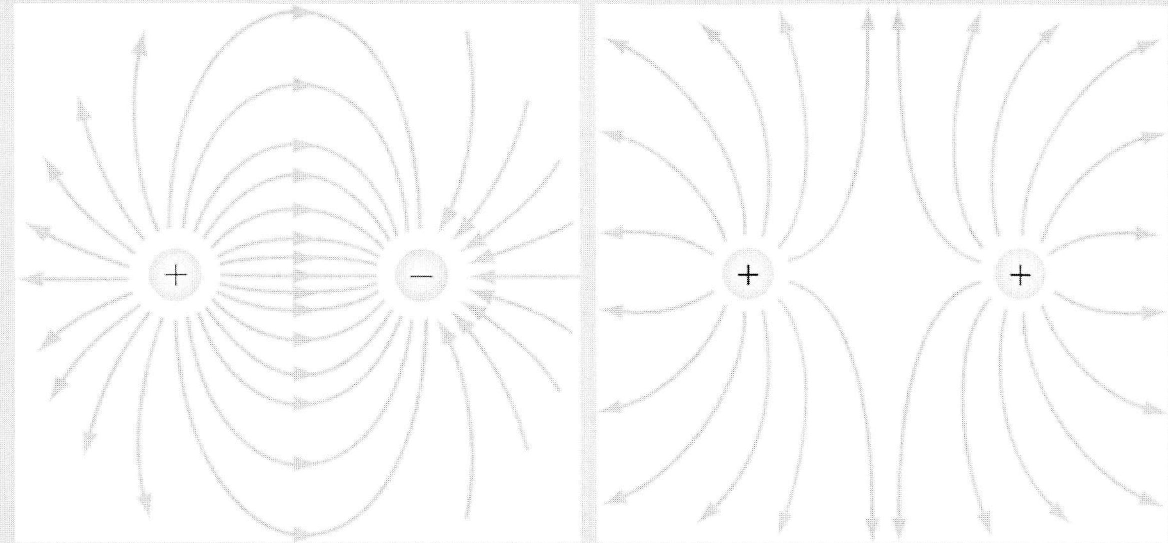
- You should be able to complete every problem
  - If you are confused, ask
  - If it seems too hard, think some more
  - Look for hints in other problems
  - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
  - Make sure that unknowns drop out of solution
- Don't forget units!

Fields



Grass Seeds

Know how to read

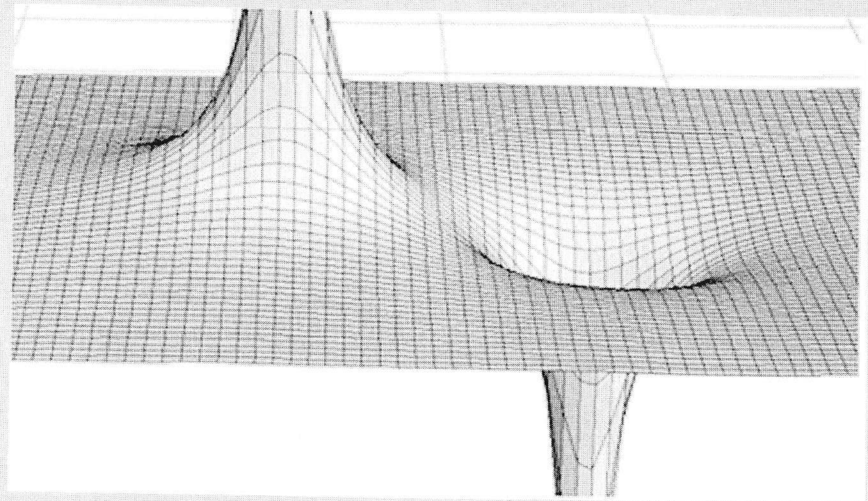


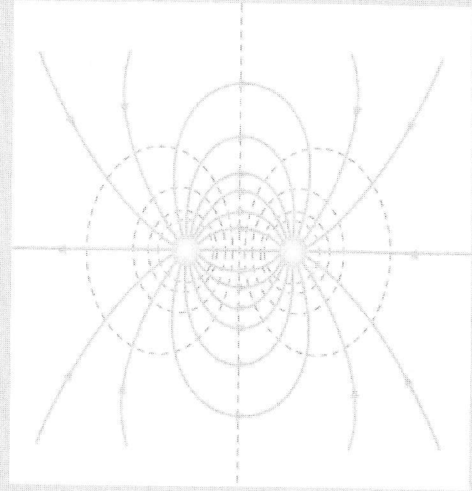
Field Lines

Know how to draw

- Field line density tells you field strength
- Lines have tension (want to be straight)
- Lines are repulsive (want to be far from other lines)
- Lines begin and end on sources (charges) or ∞

E Field and Potential: Creating





A point charge *q* creates a field and potential around it:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \; V = k_e \frac{q}{r}$$

Use superposition for systems of charges

They are related:

$$\vec{\mathbf{E}} = -\nabla V; \; \Delta V \equiv V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \; V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}; \; dV = k_e \frac{dq}{r}$$

Break charged object into small pieces, *dq*, and integrate

Continuous Sources: Charge Density

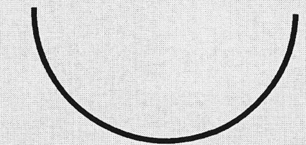
Charge Densities:

$$\lambda = \frac{Q}{L} \qquad \sigma = \frac{Q}{A} \qquad \rho = \frac{Q}{V}$$

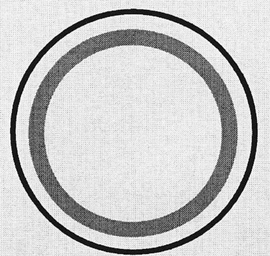
$$dQ = \lambda\, dL \qquad dQ = \sigma\, dA \qquad dQ = \rho\, dV$$

Don't forget your geometry:

$$dL = dx$$



$$dL = Rd\theta$$



$$dA = 2\pi r dr$$

$$dV_{cyl} = 2\pi r l dr$$

$$dV_{sphere} = 4\pi r^2 dr$$

E Field and Potential: Creating

Discrete set of point charges:

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}; \;\; V = k_e \frac{q}{r}$$

Add up from each point charge

Continuous charge distribution:

$$d\vec{\mathbf{E}} = k_e \frac{dq}{r^2}\hat{\mathbf{r}}; \;\; dV = k_e \frac{dq}{r}$$

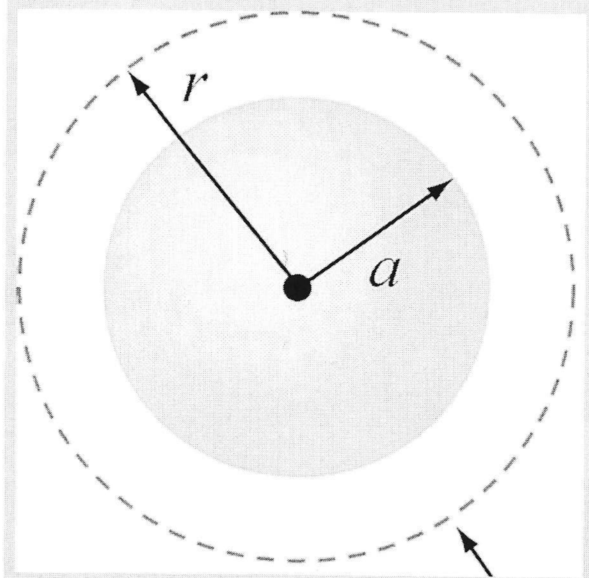
Break charged object into small pieces, *dq*, and integrate

Symmetric charged object:

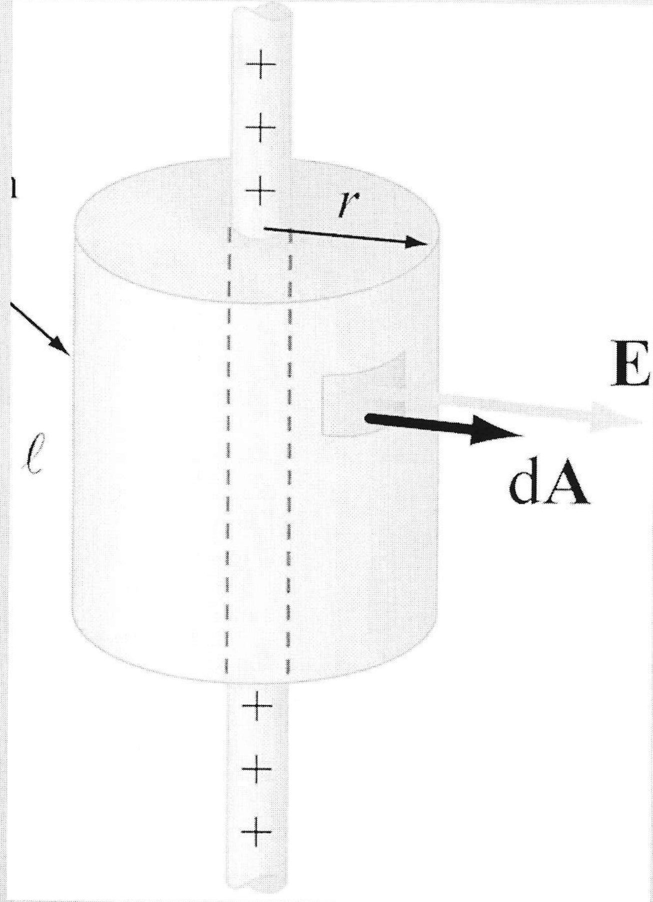
$$\oint\!\!\!\oint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}; \;\; \Delta V \equiv -\int \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}}$$

Use Gauss' law to get E everywhere, then integrate to get V

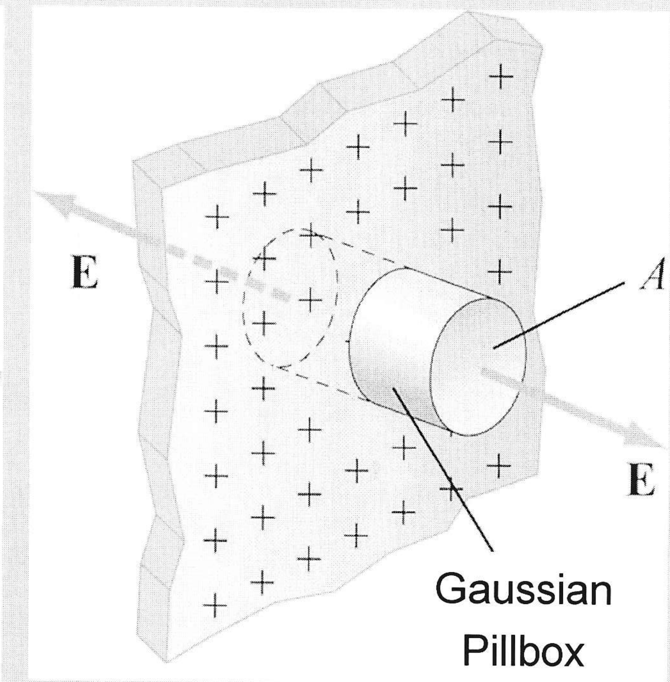
Gauss's Law: $\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_0}$



Spherical Symmetry



Cylindrical Symmetry



Planar Symmetry

E Field and Potential: Effects

If you put a charged particle, *q*, in a field:

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$$

To move a charged particle, *q*, in a field:

$$W = \Delta U = q\Delta V$$

Conductors in Equilibrium

Conductors are equipotential objects:

1) E = 0 inside

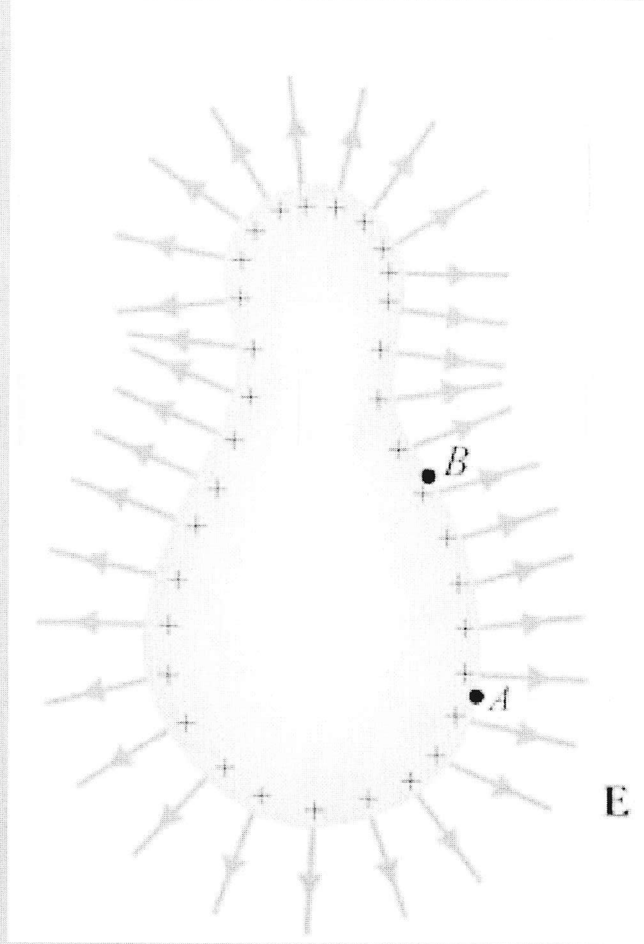
2) Net charge inside is 0

3) E perpendicular to surface

4) Excess charge on surface

$$E = \sigma / \varepsilon_0$$

5) Shielding – inside doesn't "talk" to outside



Capacitors

Capacitance

$$C = \frac{Q}{|\Delta V|}$$

To calculate:

1) Put on arbitrary ±Q
2) Calculate E
3) Calculate ΔV

In Series & Parallel

$$\frac{1}{C_{eq,\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq,\text{parallel}} = C_1 + C_2$$

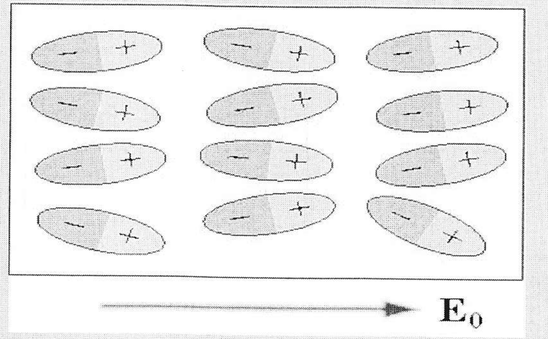
Energy

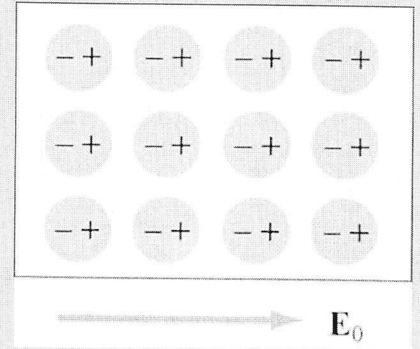
$$U = \frac{Q^2}{2C} = \frac{1}{2} Q|\Delta V| = \frac{1}{2} C|\Delta V|^2 = \iiint u_E \, d^3r = \iiint \frac{\varepsilon_o E^2}{2} d^3r$$

Dielectrics

Dielectrics locally *weaken* the electric field

$$E = \frac{E_0}{\kappa}; \quad \kappa \geq 1$$





Inserted into a capacitor: $C = \kappa C_0$

$$C = \frac{Q}{|\Delta V|}$$

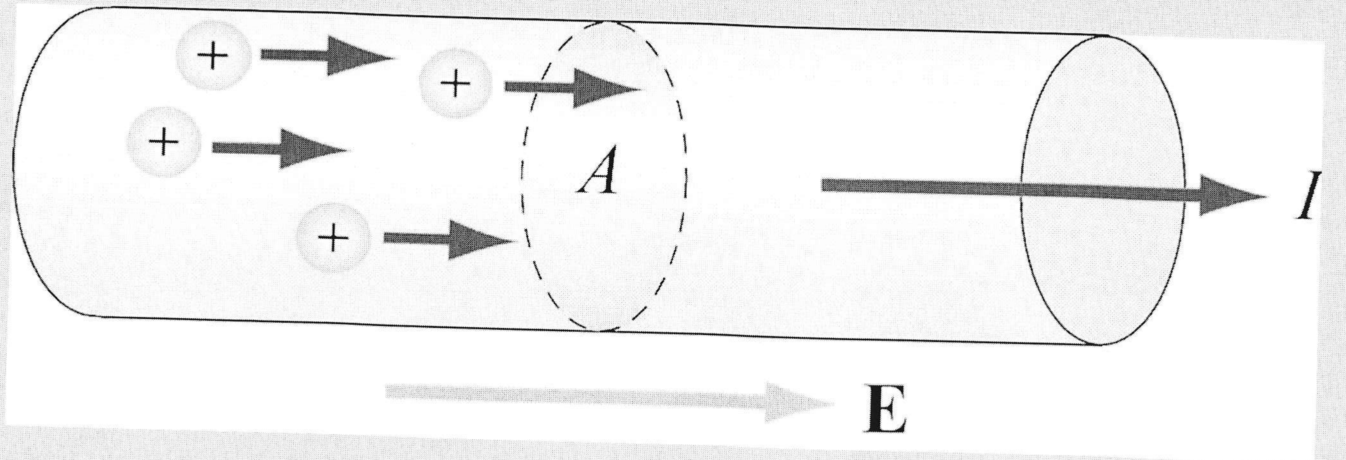
Hooked to a battery? Q increases

Not hooked up? V decreases

Part 2 Topics

- DC Circuits
  - Current & Ohm's Law (Macro- and Microscopic)
  - Power
  - Kirchhoff's Loop Rules
  - Charging/Discharging Capacitor (RC Circuits)
- Magnetic Fields
  - Force due to Magnetic Field (Lorentz Force)
  - Magnetic Dipoles
  - Generating Magnetic Fields
    - Biot-Savart Law & Ampere's Law

Current & Ohm's Law



$$I = \frac{dQ}{dt}$$

$$\vec{\mathbf{J}} \equiv \frac{I}{A}\hat{\mathbf{I}}$$

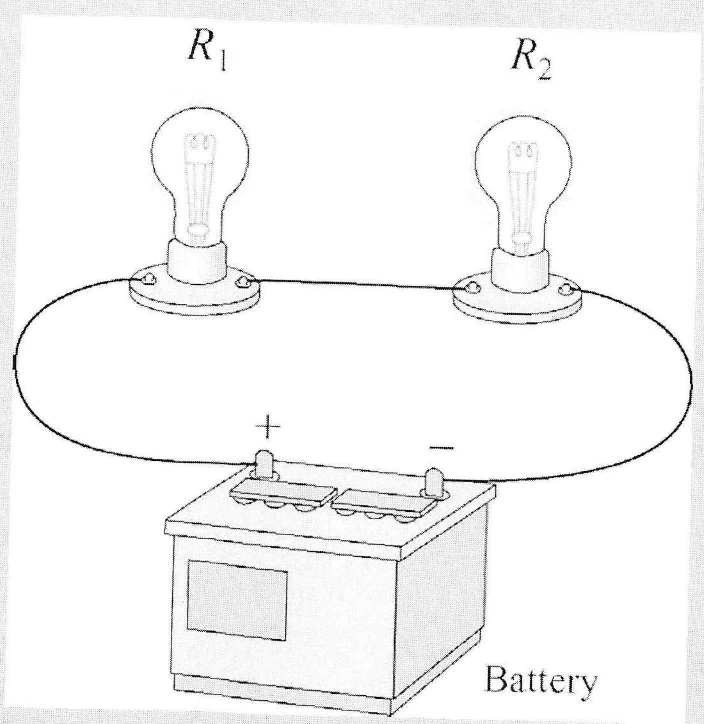
<u>Ohm's Laws</u>

$$\vec{\mathbf{E}} = \rho\vec{\mathbf{J}} = \left(1/\sigma\right)\vec{\mathbf{J}}$$

$$\boxed{\Delta V = IR}$$

$$R = \frac{\rho \ell}{A}$$

Series vs. Parallel

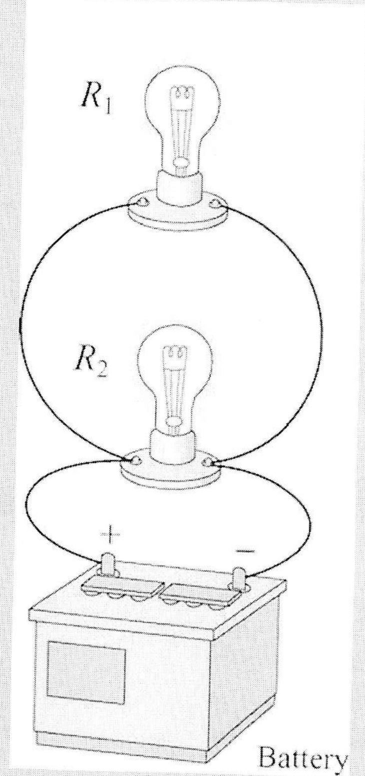


$$R_s = R_1 + R_2$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$C_P = C_1 + C_2$$



Series

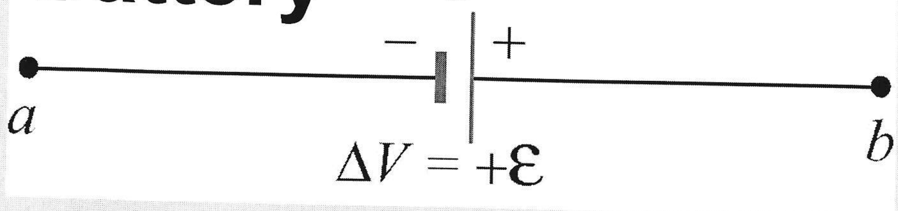
- Current same
- Voltages add

Parallel

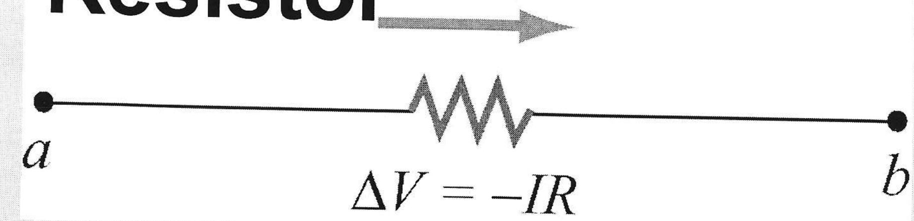
- Currents add
- Voltages same

Current, Voltage & Power

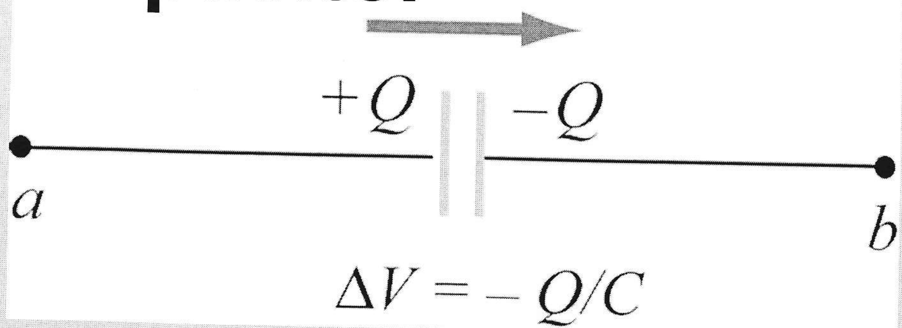
Battery



Resistor



Capacitor



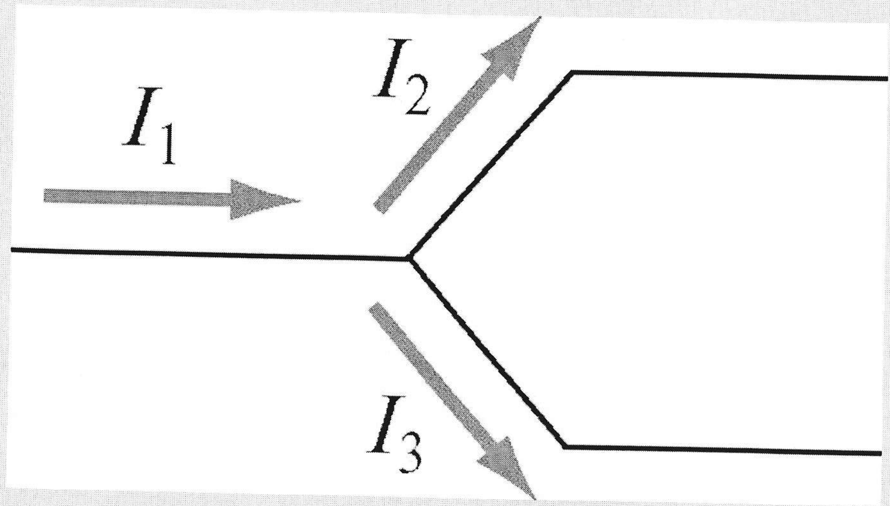
$$P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon$$

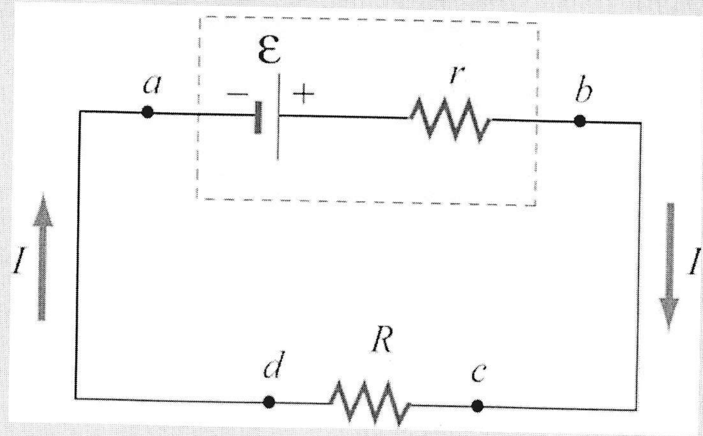
$$P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

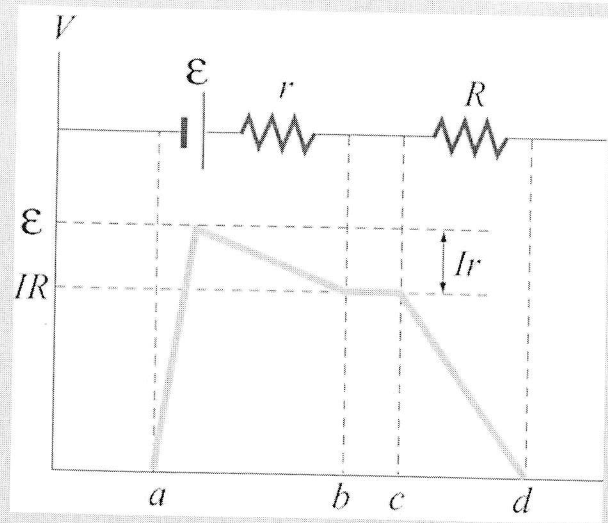
$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C}$$

$$= \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

Kirchhoff's Rules



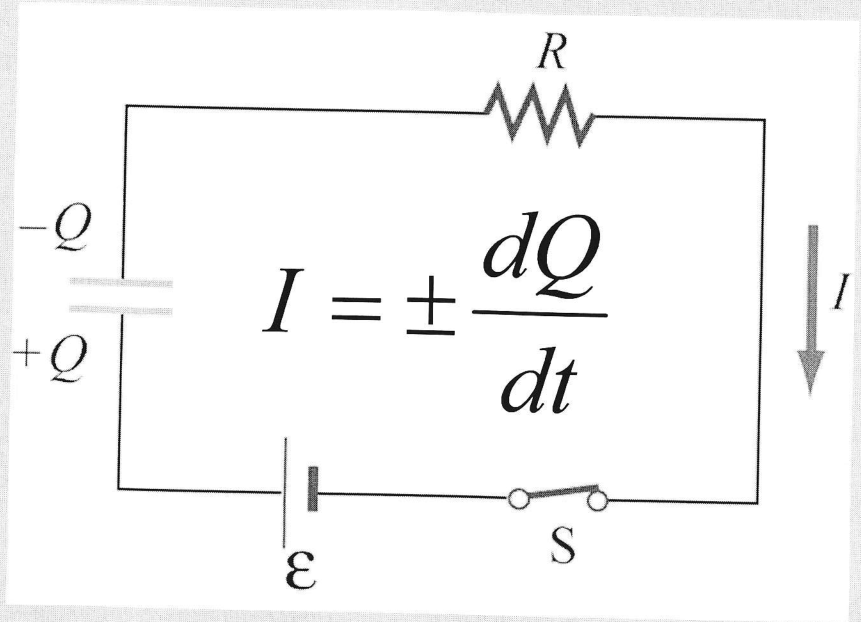




$$I_1 = I_2 + I_3$$

$$\Delta V = -\oint_{\text{Closed Path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

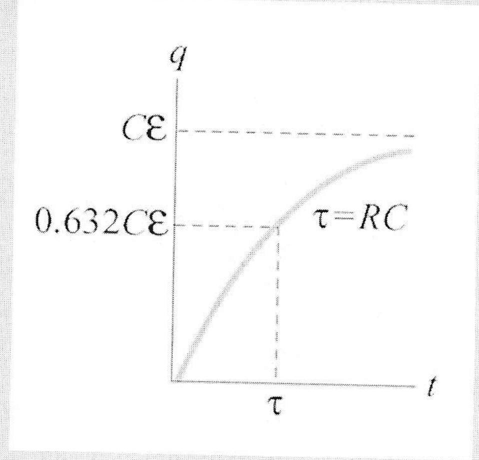
(Dis)Charging A Capacitor



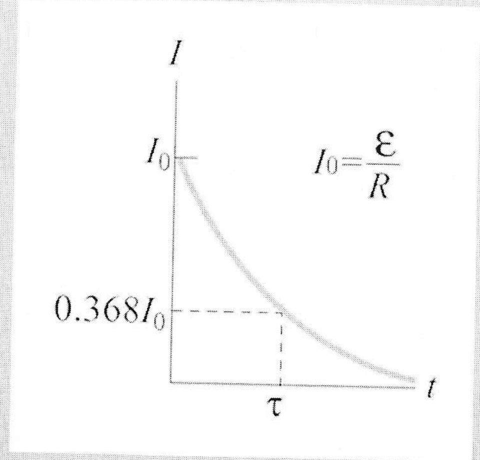
$$I = \pm \frac{dQ}{dt}$$

$$\sum_i \Delta V_i = \varepsilon - \frac{Q}{C} - IR = 0$$

$$\underbrace{C\mathcal{E}}_{Q_{final}} - Q - \underbrace{RC}_{\tau}\frac{dQ}{dt} = 0$$



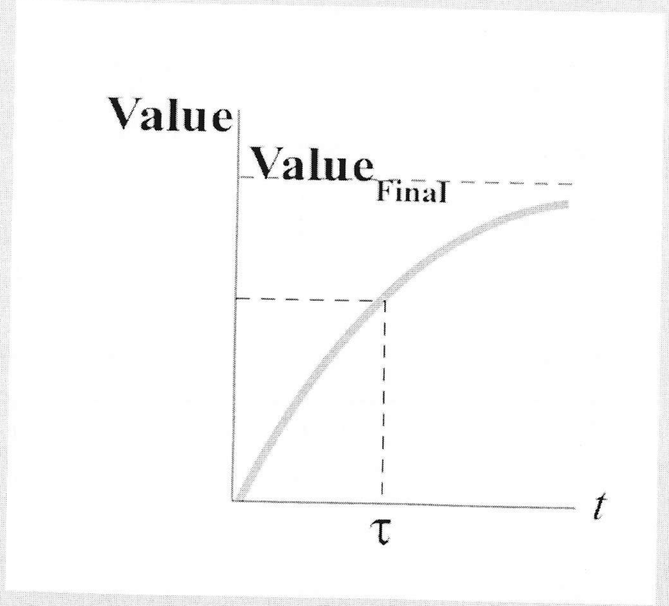
$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$



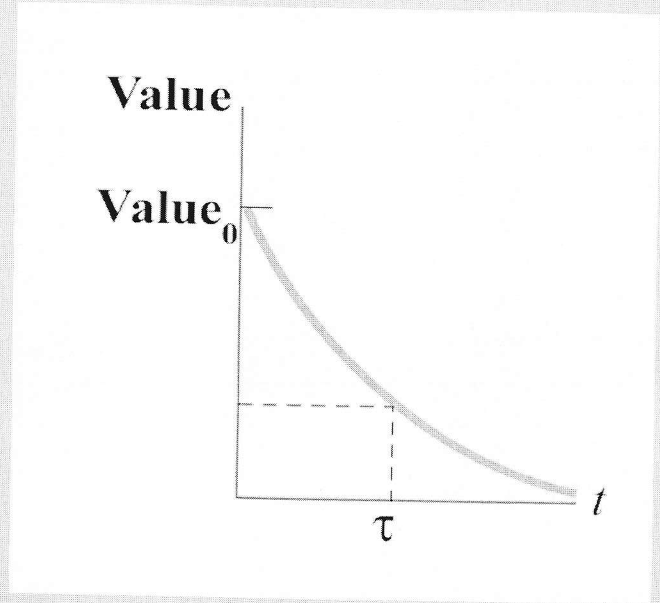
$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

General Comment: RC

All Quantities Either:



$$\text{Value}(t) = \text{Value}_{\text{Final}}\left(1 - e^{-t/\tau}\right)$$



$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

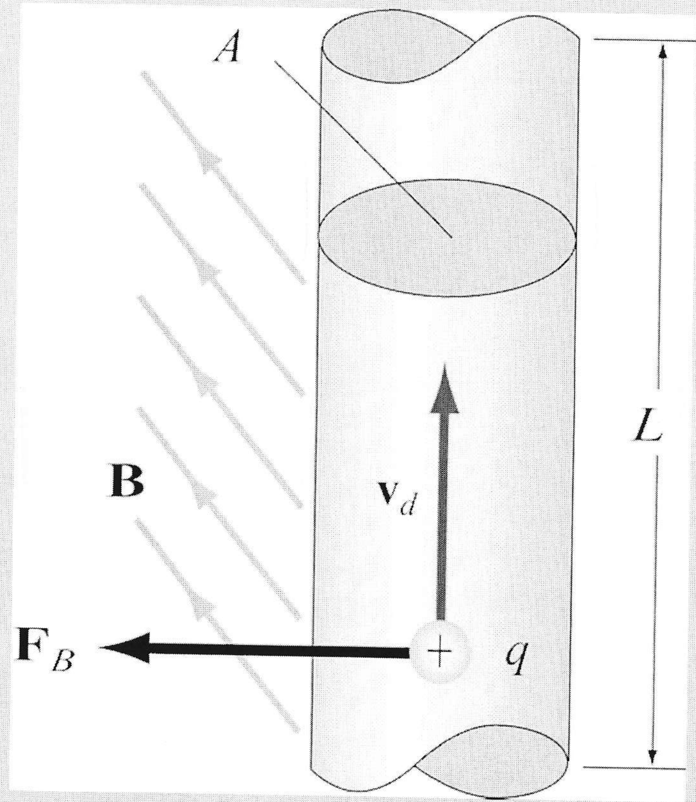
τ can be obtained from differential equation (prefactor on d/dt) e.g. $\tau = RC$

Right Hand Rules

1. Torque: Thumb = torque,
   Fingers show rotation
2. Feel: Thumb = I,
   Fingers = B,
   Palm = F
3. Create: Thumb = I
   Fingers (curl) = B
4. Moment: Fingers (curl) = I
   Thumb = Moment (=B inside loop)

Magnetic Force

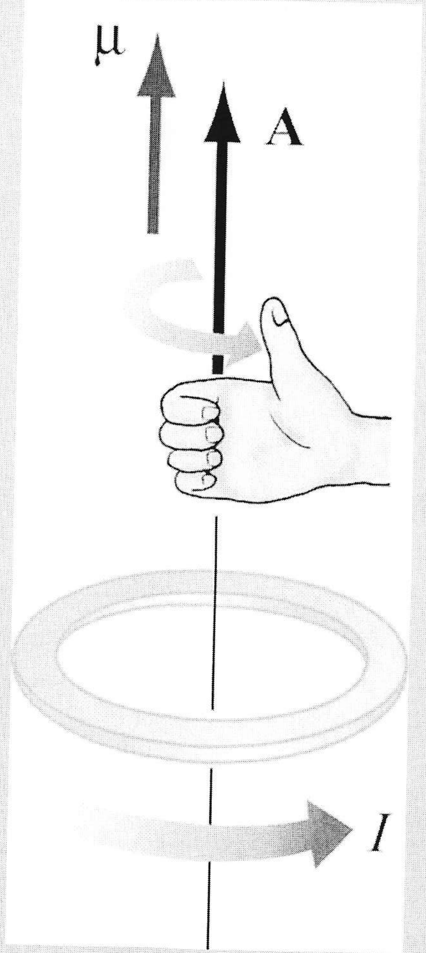
$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$



$$d\vec{\mathbf{F}}_B = I d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

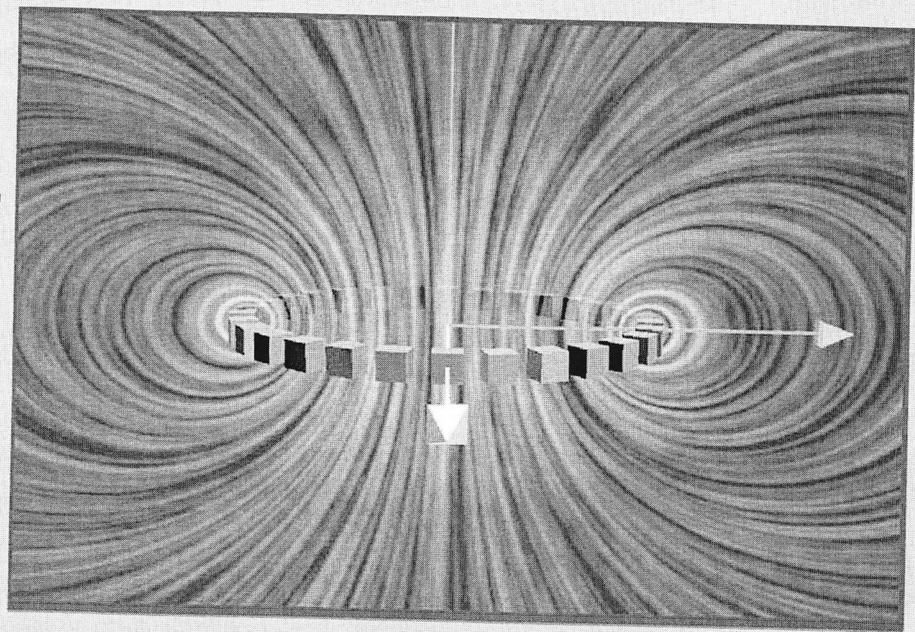
$$\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)$$

Magnetic Dipole Moments



$$\vec{\boldsymbol{\mu}} \equiv IA\hat{\mathbf{n}} \equiv I\vec{\mathbf{A}}$$

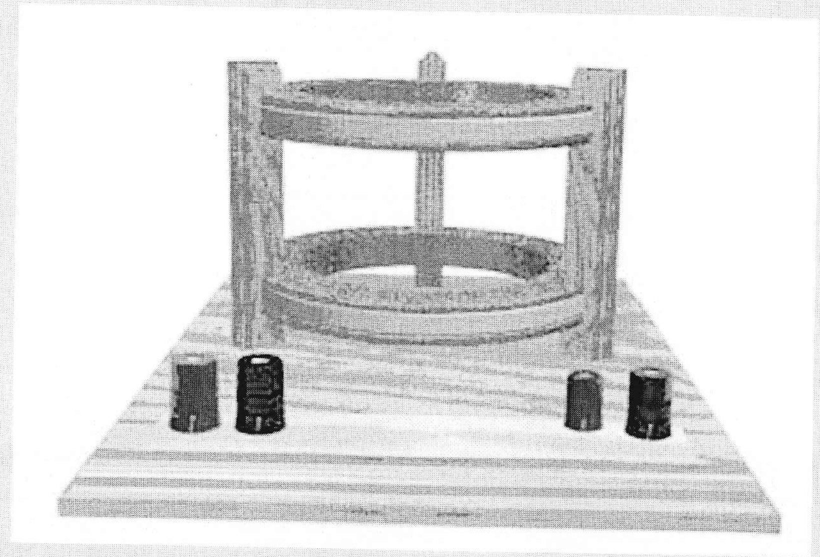
Generate:



Feel:

1) Torque to align with external field
2) Forces as for bar magnets

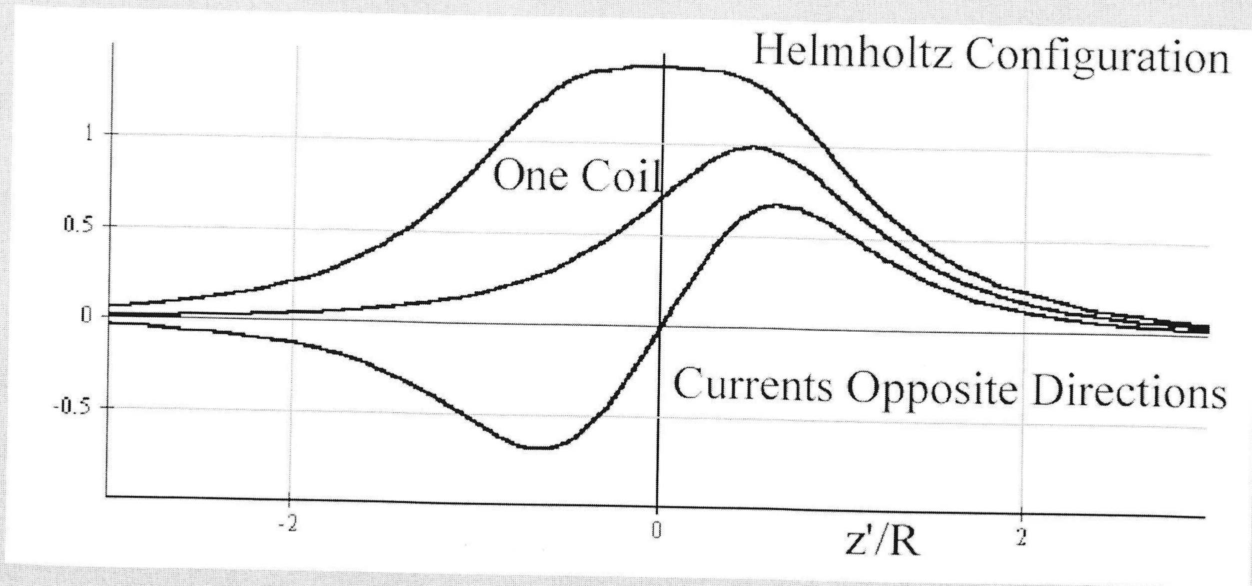
Helmholtz Coil



Common Concept Question

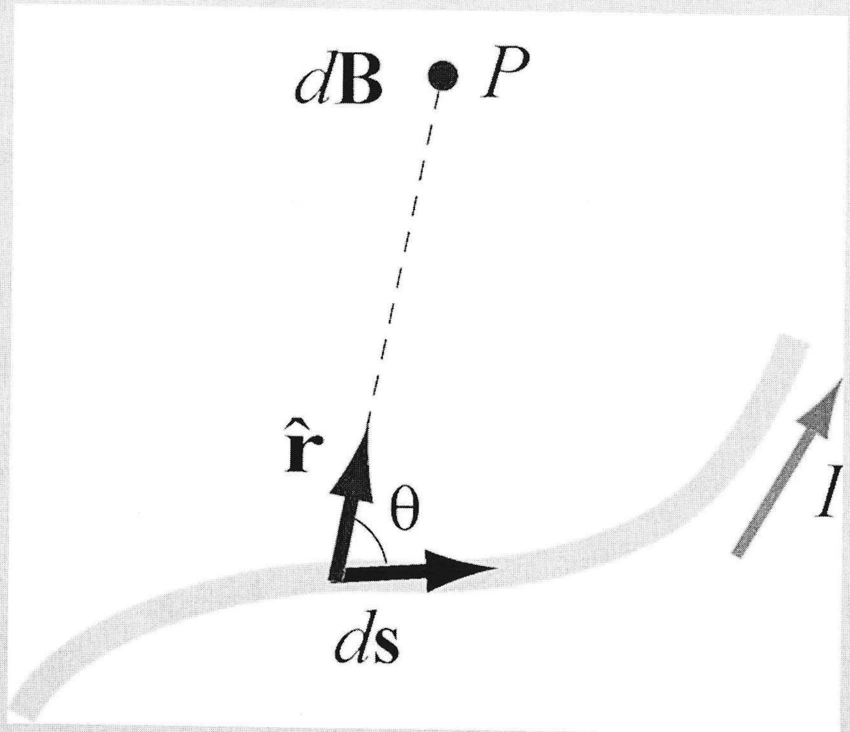
Parallel (Helmholtz) makes uniform field (torque, no force)

Anti-parallel makes zero, non-uniform field (force, no torque)



The Biot-Savart Law

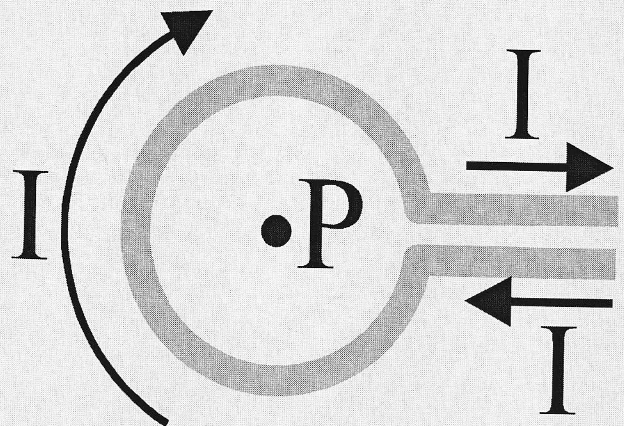
Current element of length ds carrying current I (or equivalently charge q with velocity v) produces a magnetic field:

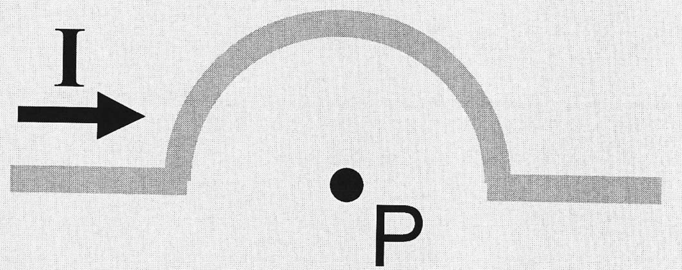


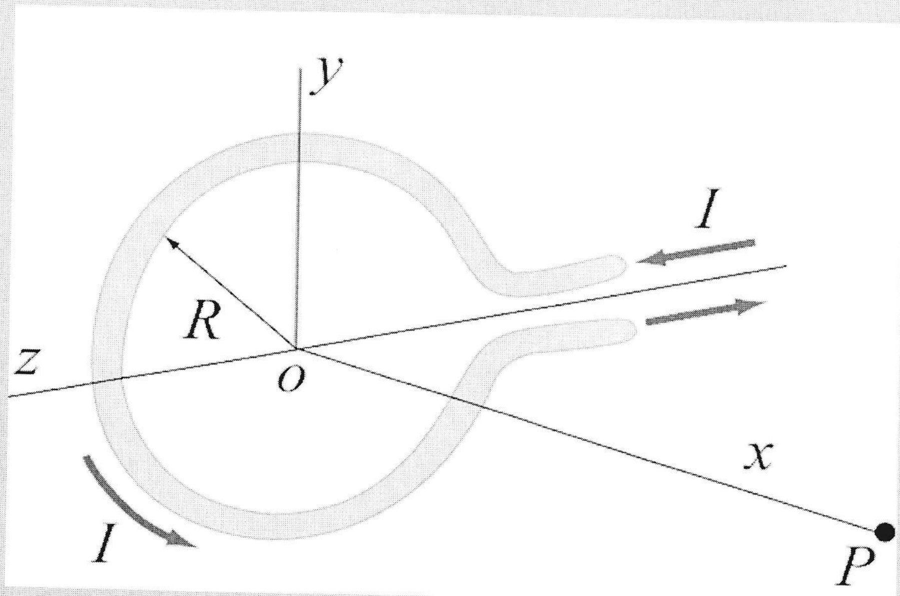
$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi} \frac{q\,\vec{\mathbf{v}} \, \mathrm{x} \, \hat{\mathbf{r}}}{r^2}$$

$$\mathbf{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

Biot-Savart: 2 Problem Types



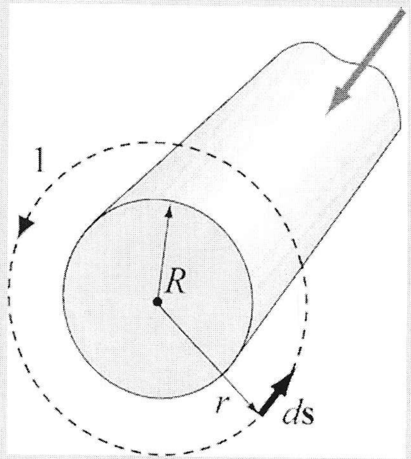


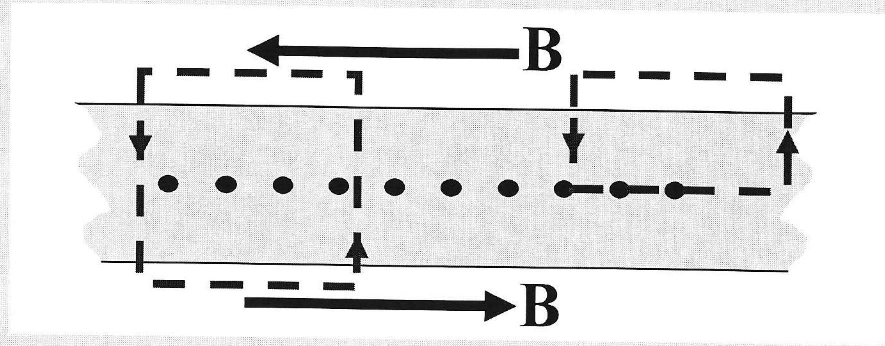


Notice that r is the same for every point on the loop. You don't really need to integrate (except to find path length)

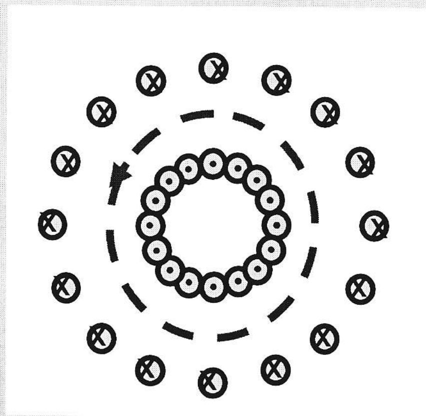
Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$

Long Circular Symmetry



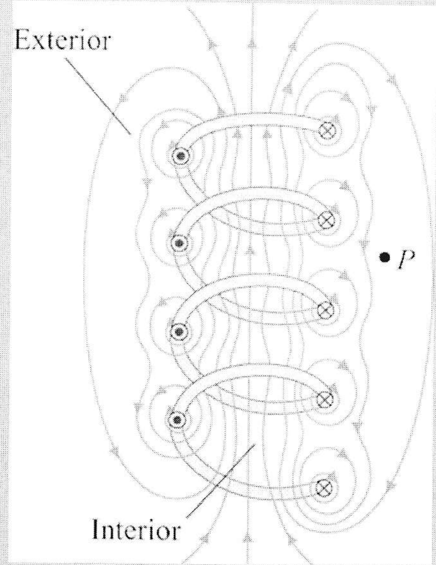


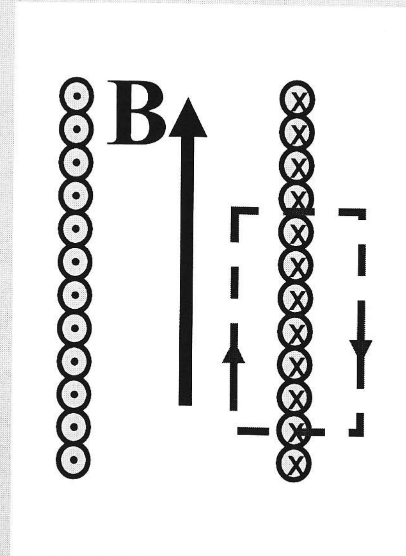
(Infinite) Current Sheet



Torus/Coax

Solenoid = 2 Current Sheets

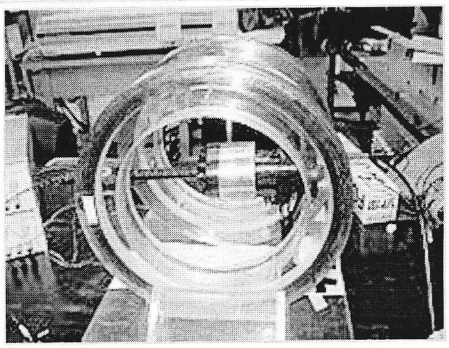




Helmholtz coil

From Wikipedia, the free encyclopedia

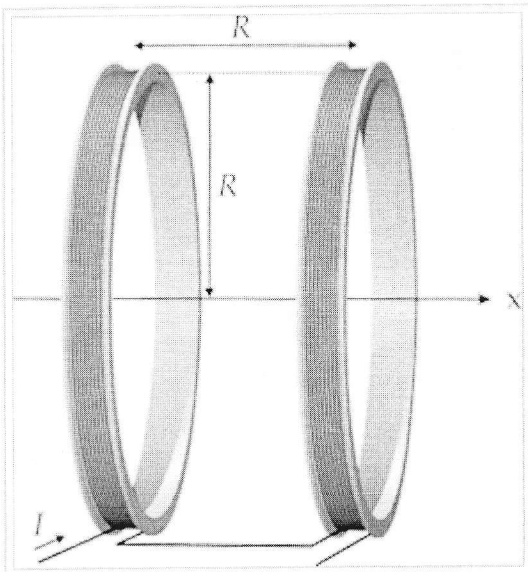
A **Helmholtz coil** is a device for producing a region of nearly uniform magnetic field. It is named in honor of the German physicist Hermann von Helmholtz.



A Helmholtz coil

Contents

Description



Helmholtz coil schematic drawing

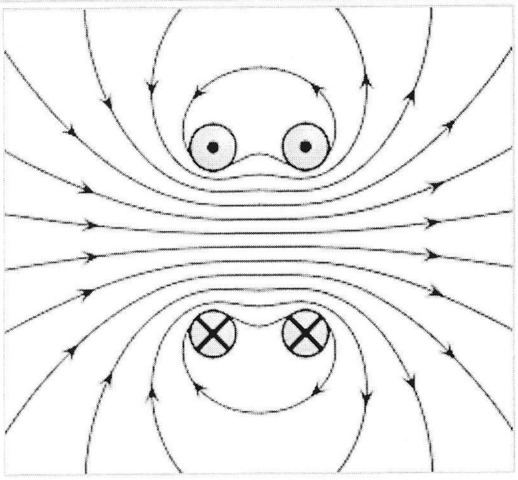
A Helmholtz pair consists of two identical circular magnetic coils that are placed symmetrically one on each side of the experimental area along a common axis, and separated by a distance h equal to the radius R of the coil. Each coil carries an equal electrical current flowing in the same direction.

Setting $h = R$, which is what defines a Helmholtz pair, minimizes the nonuniformity of the field at the center of the coils, in the sense of setting $\partial^2 B/\partial x^2 = 0$[1] (meaning that the first nonzero derivative is $\partial^4 B/\partial x^4$ as explained below), but leaves about 7% variation in field strength between the center and the planes of the coils. A slightly larger value of h reduces the difference in field between the center and the planes of the coils, at the expense of worsening the field's uniformity in the region near the center, as measured by $\partial^2 B/\partial x^2$.[2]

In some applications, a Helmholtz coil is used to cancel out the Earth's magnetic field, producing a region with a magnetic field intensity much closer to zero.[3]

Mathematics

The calculation of the exact magnetic field at any point in space is mathematically complex and involves the study of Bessel functions. Things are simpler along the axis of the coil-pair, and it is convenient to think about the Taylor series expansion of the field strength as a function of x, the distance from the central point of the coil-pair along the axis. By symmetry the odd order terms in the expansion are zero. By separating the coils so that charge $x = 0$ is an inflection point for each coil separately we can guarantee that the order x^2 term is also zero, and hence the leading non-uniform term is of order x^4. One can easily show that the inflection point for a simple coil is $R/2$ from the coil center along the axis; hence the location of each coil at $x = \pm R/2$

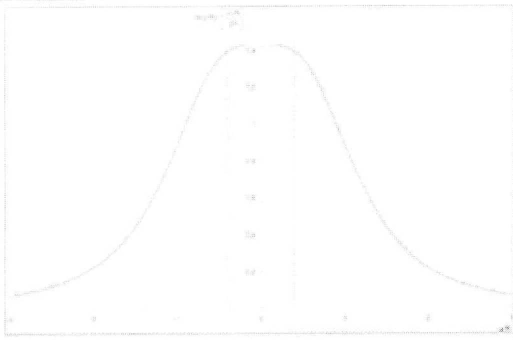


Magnetic field lines in a plane bisecting the current loops. Note the field is approximately uniform in between the coil pair. (In this picture the coils are placed one beside the other: the axis is horizontal)

A simple calculation gives the correct value of the field at the center point. If the radius is *R*, the number of turns in each coil is *n* and the current flowing through the coils is *I*, then the magnetic flux density, B at the midpoint between the coils will be given by

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$

μ_0 is the permeability of free space ($1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}$).



Magnetic field induction along the axis crossing the center of coils; $z = 0$ is the point in the middle of distance between coils.

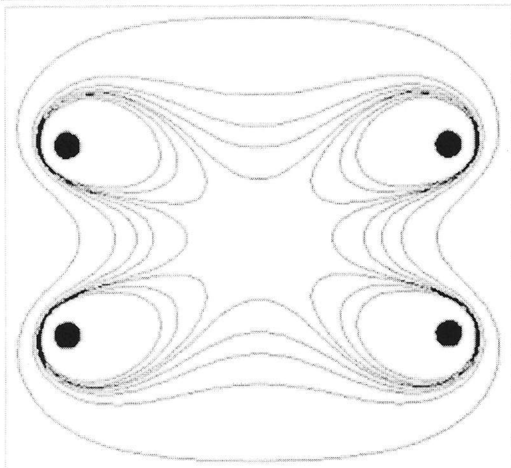
Derivation

Start with the formula for the on-axis field due to a single wire loop [1] (http://hyperphysics.phy-astr.gsu.edu/HBASE/magnetic/curloo.html#c3) (which is itself derived from the Biot-Savart law):

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Where:

μ_0 = the permeability constant =



Contours showing the magnitude of the

magnetic field near the coil pair. Inside the central 'octopus' the field is within 1% of its central value B_0. The five contours are for field magnitudes of $0.5B_0$, $0.8B_0$, $0.9B_0$, $0.95B_0$, and $0.99B_0$.

$$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} = 1.257 \times 10^{-6}\ \mathrm{T \cdot m/A}$$

I = coil current, in amperes
R = coil radius, in meters
x = coil distance, on axis, to point, in meters

However the coil consists of a number of wire loops, the total current in the coil is given by

nI = total current

Where:

n = number of wire loops in one coil

Adding this to the formula:

$$B = \frac{\mu_0 n I R^2}{2(R^2 + x^2)^{3/2}}$$

In a Helmholtz coil, a point halfway between the two loops has an x value equal to R/2, so let's perform that substitution:

$$B = \frac{\mu_0 n I R^2}{2(R^2 + (R/2)^2)^{3/2}}$$

There are also two coils instead of one, so let's multiply the formula by 2, then simplify the formula:

$$B = \frac{2\mu_0 n I R^2}{2(R^2 + (R/2)^2)^{3/2}}$$

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$

Maxwell coils

To improve the uniformity of the field in the space inside the coils, additional coils can be added around the outside. James Clerk Maxwell showed in 1873 that a third larger-diameter coil located midway between the two Helmholtz coils can reduce the variance of the field on the axis to zero up to the sixth derivative of position. This is sometimes called a Maxwell coil.

See also

- Maxwell coil
- Solenoid

References

1. ^ Helmholtz Coil in CGS units (http://www.purcellsolutions.com/2011/06/purcell-physics-problem-6-13-solution.html)
2. ^ Electromagnetism (http://www.lightandmatter.com/html_books/0sn/ch11/ch11.html)
3. ^ "Earth Field Magnetometer: Helmholtz coil" (http://www.circuitcellar.com/library/print/0606/Wotiz191/5.htm) by Richard Wotiz 2004

External links

- On-Axis Field of an Ideal Helmholtz Coil (http://www.netdenizen.com/emagnet/helmholtz/idealhelmholtz.htm)
- Axial field of a real Helmholtz coil pair (http://www.netdenizen.com/emagnet/helmholtz/realhelmholtz.htm)
- *Helmholtz-Coil Fields (http://demonstrations.wolfram.com/HelmholtzCoilFields/)* by Franz Kraft, The Wolfram Demonstrations Project.
- Complete derivation for **OFF-AXIS** field for a single current loop. Includes reduction to on-axis field as derived from the Biot-Savart Law. **See expression on Page 8 in this paper. Uses elliptic integrals.** (http://plasmalab.pbwiki.com/f/bfield.pdf)