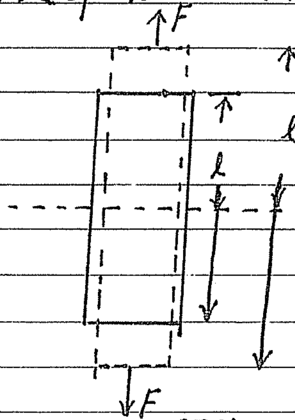
彈性力學

$$彈性係數 = \frac{應力}{應變}$$

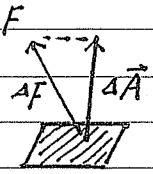
應力單位 $\frac{力}{面積}$

應變單位 無因次



$$S_n = \frac{F_n}{A}$$

↓ 正應力

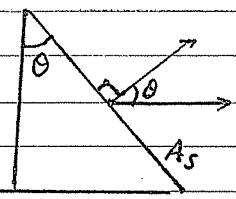


$\Delta\vec{A}$

$$\hat{n} = \frac{\Delta\vec{A}}{\Delta A}$$

$$S_n = \lim_{\Delta A \to 0} \frac{\Delta\vec{F}\cdot\hat{n}}{\Delta A}$$

$$S_t = \lim \frac{|\hat{n}\times(\vec{F}\times\hat{n})|}{\Delta A} = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A}$$



$$S_n' = F_n / A_s$$

$$S_t' = F_t / A_s$$

$$A_s \cos\theta = A$$

作用在整個表面上

平衡 作用物体上 外力 / 外力矩 和 = 0

$$e_l = \frac{\Delta l}{l}$$

↳ 縱應變

橫應变 $\frac{\Delta W}{W} = \frac{W' - W}{W} = e_t$

$$\sigma = -\frac{e_t}{e_l}$$

↓ Poisson constant

$$Y = \frac{S_n}{e_l}$$

↓ 楊氏係數

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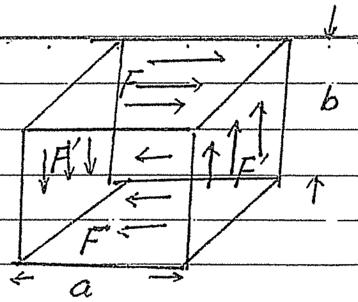
純切應力



$|\vec{F}|\, b = |F'|\, a$

↓

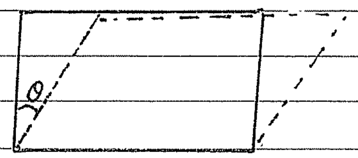
使該系統所受之 Torque 為 0

↓

$S_t = \frac{F_t}{A}$

↓

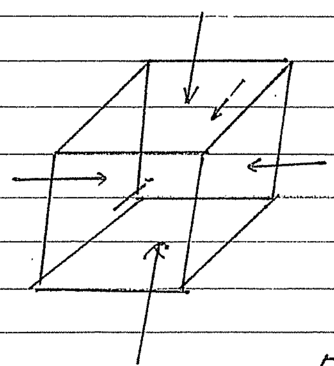
由各方面算均為一樣



$\mu = \frac{S_t}{\tan\theta}$

↓

can be measured using torsion pendulum



$p = \frac{F}{A}$ 應力

應變 $-\frac{\Delta V}{V}$

$$B = \frac{p}{-\frac{\Delta V}{V}}$$

先沿 z 方面加力

$a_z' = a(1+e_\ell)$, $a_y' = a(1+e_t)$, $a_x' = a(1+e_t)$

$e_\ell,\ e_t \ll 1$

同理可沿 x, y 方面加壓

$V' = a_x' a_y' a_z' \sim V(1+e_\ell+2e_t)^3$

$\sim V(1+6e_t+3e_\ell) \Rightarrow \frac{\Delta V}{V} = \frac{V'-V}{V} = \frac{-3p}{Y}(1-2\sigma)$

$\Rightarrow B = \frac{Y}{3(1-2\sigma)}$ 彈性係數間之關係

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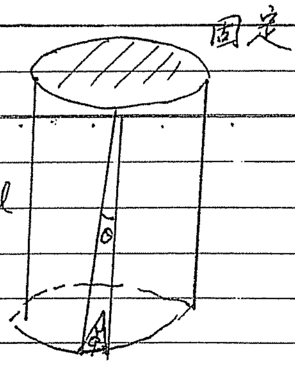

$$\theta = \frac{r\phi}{l}$$

$$\mu = \frac{S_t}{\theta}$$

$dF_t = S_t \, dA$ 　　對 r to r+dr 之切應

$= \mu\theta \, 2\pi r \, dr$

$d\tau = r \, dF_t$

$= \mu\theta \, 2\pi r^2 \, dr$

$\tau = \int_0^R \mu\theta \, 2\pi r^2 \, dr$ （$\theta = \frac{r\phi}{l}$）

$= \underbrace{\frac{\mu\pi}{2l}R^4}_{K}\phi$

$\tau \rightarrow -I\frac{d^2\phi}{dt^2}$

Release 　 restoring torque $= -\tau$

$$I\frac{d^2\phi}{dt^2} = -K\phi$$

Measure period $\Rightarrow K \xRightarrow[R,\,l]{\text{measure}} \mu$

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流体靜力學

↓

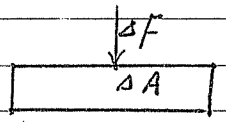
平衡是最基本之觀念

流体 → 不能抵抗切應力

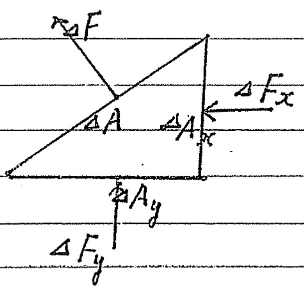
↓

有切應力，物質即會移動

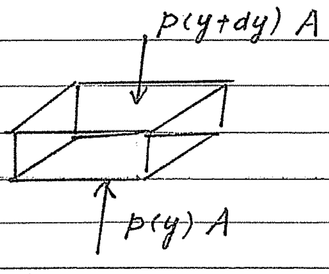
壓力之觀念



$$P = \lim \frac{\Delta F}{\Delta A}$$



平衡 $\Rightarrow \quad \frac{\Delta F}{\Delta A} = \frac{F_x}{\Delta A_x} = \frac{F_y}{\Delta A_y}$



平衡 $\quad p(y)A = p(y+dy)A + dW$ ↳ weight

$dW \rightarrow \rho A\, dy\, g$

$$\frac{p(y+dy) - p(y)}{dy} = -\rho g$$

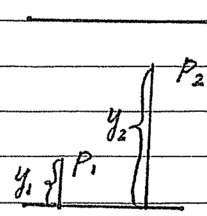
$$\Rightarrow \frac{dP}{dy} = -\rho g$$

$$\int_1^2 dp = \int_1^2 (-\rho g)\, dy$$

For constant ρg

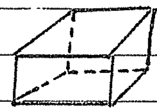
$$P_2 - P_1 = -\rho g y_2 + \rho g y_1$$

$$P_2 + \rho g y_2 = P_1 + \rho g y_1$$



流体動力學

Lagrange picture



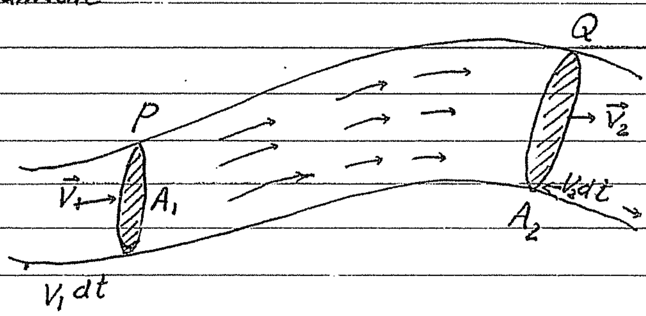
Euler picture

$$\rho(x, y, z, t)$$
$$\vec{V}(x, y, z, t)$$

Steady state $\quad \vec{V}(x, y, z, t) \rightarrow \vec{V}(x, y, z)$

(i) steady state
(ii) non-rotational
(iii) imcompressible
(iv) non-viscous

Streamline



In the time interval dt

$$dm_1 = \rho_1 \underbrace{A_1 \, v_1 \, dt}_{V} \quad \text{into region } PQ$$

$$dm_2 = \rho_2 A_2 v_2 \, dt \quad \text{out of region } PQ$$

No sink or source in region PQ

$$\Rightarrow \quad dm_1 = dm_2 \quad \text{質量守恒}$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$\rho_1 = \rho_2$

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

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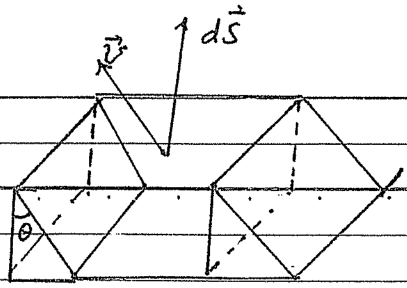
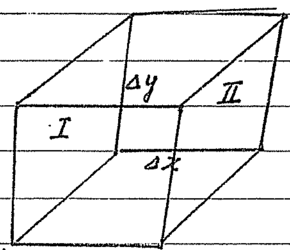


在 dt 時間內通過之質量

$$\rho \vec{v} \cdot d\vec{S} \; dt$$



在時間 dt 內

由 II 流出

$$\rho(x+\Delta x, y, z) \; v_x(x+\Delta x, y, z) \, \Delta y \Delta z \; dt$$

由 I 流入

$$\rho(x, y, z) \; v_x(x, y, z) \, \Delta y \Delta z$$

$$\frac{\rho(x+\Delta x, y, z) \; v_x(x+\Delta x, y, z) - \rho(x,y,z) \, v_x(x,y,z)}{\Delta x} \Delta x \Delta y \Delta z$$

$$\longrightarrow \left(\frac{\partial}{\partial x} \rho v_x\right) \Delta x \Delta y \Delta z \; dt$$

$$[\rho(x, y, z, t+\Delta t) - \rho(x,y,z,t)] \; \Delta x \Delta y \Delta z \, \Delta t$$

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = -\frac{\partial \rho}{\partial t}$$

↓

連續方程式

$$\frac{\partial \rho}{\partial t} = 0, \quad \rho = \text{constant}$$

$$\frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z = 0$$

↓

$$\nabla \cdot \vec{V} = 0$$

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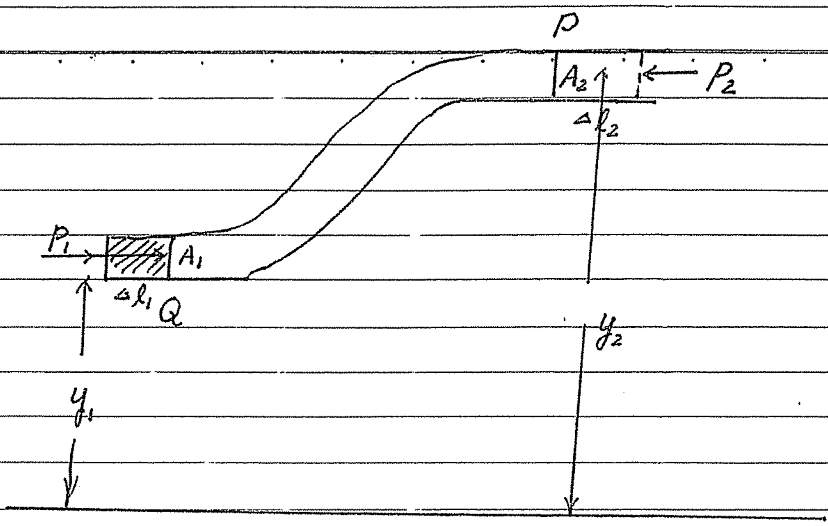

外力對該系統所作之功

Pressure

$$\underbrace{P_1 A_1}_{F} \; l_1 = P_1 \frac{\Delta m_1}{\rho_1} = P_1 \frac{\Delta m}{\rho}$$

$$-P_2 \frac{\Delta m}{\rho}$$

重力

$$-\Delta m \; g (y_2 - y_1)$$

$$W = -\Delta m \; g(y_2 - y_1) + (P_1 - P_2)\frac{\Delta m}{\rho}$$

$$\| $$

$$\frac{1}{2}\Delta m(v_2^2 - v_1^2)$$ 功能定理

$$\frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = -g y_2 + g y_1 + \frac{P_1 - P_2}{\rho}$$

$$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

↓

伯勞力定律

If $v_1 = v_2 = 0$, then

$$\Rightarrow P_1 + \rho g y_1 = P_2 + \rho g y_2$$

流体静力學的公式

分類：
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例題

1. 測量流体之速度

2. 飛機之起飛

3. 變化球

Chapter 15

Fluids

Fluid can not sustain a force that is tangential to its surface.

↓

substances that flows because it cannot sustain (withstand) a shearing stress.

Density

ρ

Pressure

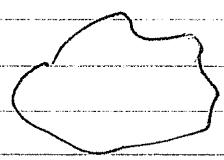
fluid

$$p = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$$

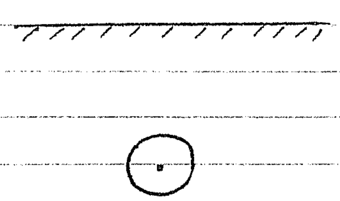
↓ at a point

$\Delta F \perp \Delta A$

independent of orientation.

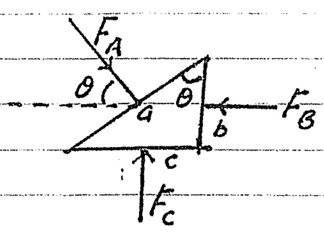


cut the material into many pieces until ΔA is a plane. ΔF is then the force perpendicular to ΔA.



Pressures measured on any direction are the same

depth d



Equilibrium

$F_B = F_A \cos\theta$

$F_C = F_A \sin\theta$

area ⊥ to F_A $\quad d \cdot a$

area ⊥ to F_B $\quad d \cdot b = a\cos\theta$

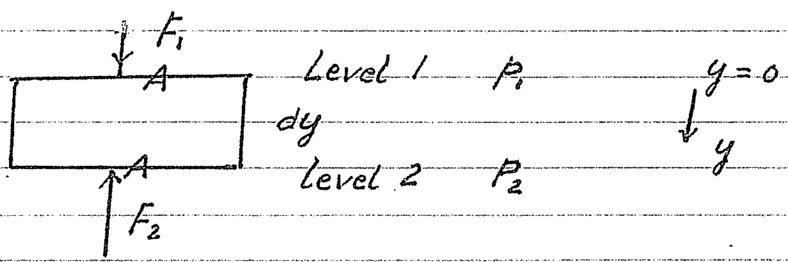
$$\frac{F_A}{\text{area} \perp \text{to } F_A} = \frac{F_A}{d \cdot a} = \frac{F_A \cos\theta}{d \cdot a\cos\theta} = \frac{F_B}{\text{area} \perp \text{to } F_B}$$

Similarily

$$\frac{F_C}{\text{area} \perp \text{to } C} = \frac{F_A}{\text{area} \perp \text{to } A}$$

Remember as area $\to 0$ mass inside $\to 0$

⇒ the total force acted on it must vanish.



Fluid at rest

$$\vec{F}_2 = \vec{F}_1 + \vec{W}$$

$$dP = (P_2 - P_1) = \frac{\vec{F}_2 - \vec{F}_1}{A} = \frac{\vec{W}}{A} = -\frac{\rho g \, dy A}{A}$$

$$dP = -\rho g \, dy$$

$$P_2 - P_1 = -\int_1^2 \rho g \, dy = -\rho g (y_2 - y_1)$$

for ρ = constant

In general ρ is a function of y

↓ integration is needed.

⇒ fluid mechanics

↓ hydrostatics.

Pascal's principle

A change in the pressure to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of the containg vessel.

Hydraulic Levels

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o}$$

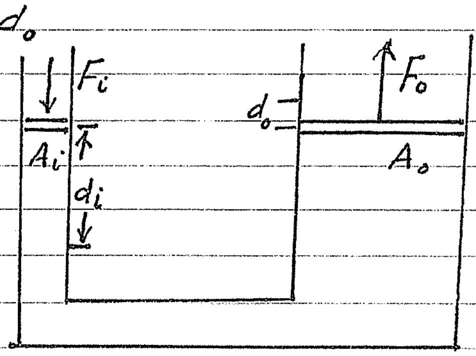
$i \rightarrow$ input
$o \rightarrow$ output

$$F_o = F_i \frac{A_o}{A_i} \qquad (\text{if } A_o > A_i \text{, then } F_o > F_i)$$

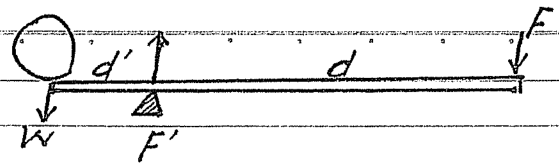
$$V = A_i d_i = A_o d_o \qquad (\text{incompressible})$$

$$d_o = \frac{A_i}{A_o} d_i \qquad (\text{if } A_o > A_i \text{, then } d_o < d_i)$$

$W_{in} = F_i d_i$
$\quad \parallel$ the same.
$W_{out} = F_o d_o$



Level



$F d = W d'$ (equilibrium)

$F' = F + W$

$F d > W d' \Rightarrow \tau$ (clockwise)

$F d < W d' \Rightarrow \tau$ (anti clock-wise)

Level

Work (input) $F s = F d \tan\theta$

Work (output) $W s' = W d' \tan\theta$

$\Rightarrow$ work (in) = work (out)

Suggested by Archimede.

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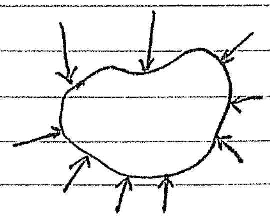
Archimedes' Principle



$\vec{B}$ = weight of the fluid through the center of mass system of the fluid. upward.

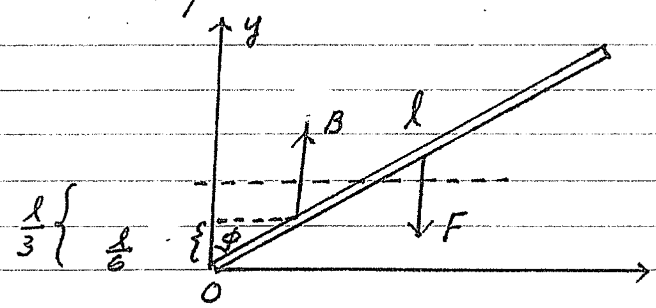
fluid at <u>rest</u>

force exerted on the fluid (particular pieces) due to the rest of the fluid.

Archimedes' principle

A body (fully or partially) immersed in a fluid is buoyed up by a force equal to the weight of the fluid that the body displaces.

Example 1



$\rho_{wood} = 0.45\ g/cm^3$

$\rho_{water} = 1\ g/cm^3$

F' is adjustable

To solve this equilibrium problem is to use the torque equation with O as the reference point

$\vec{B}$: $|\vec{B}| = \rho_{water} A \ell' g \hat{j}$ (up)

Acted at $\left(\frac{1}{6} \ell \tan\phi , \frac{1}{6} \ell \right)$

$\Uparrow$

$$\tan\phi = \frac{x_B}{\frac{1}{6}\ell} \qquad \ell' = \frac{\ell/3}{\cos\phi}$$

$\vec{F} = Mg = -\rho_{wood} A \ell g \hat{j}$

Acted at $\left(\frac{\ell}{2} \sin\phi , \frac{\ell}{2} \cos\phi \right)$

Torque from B

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{6}\ell\tan\phi & \frac{1}{6}\ell & 0 \\ 0 & \rho_{water}A\ell' g & 0 \end{vmatrix}$$

$$= \rho_{water} A \ell' g \cdot \frac{1}{6}\ell\tan\phi$$

$$= \rho_{water} A \frac{\ell}{3}\frac{1}{\cos\phi}\frac{1}{6}\ell\tan\phi$$

Torque from F

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\ell}{2}\sin\phi & \frac{\ell}{2}\cos\phi & 0 \\ 0 & -\rho_{wood}A\ell g & 0 \end{vmatrix}$$

$$= -\frac{\ell}{2}\sin\phi\,\rho_{wood}A\ell g$$

$$\Rightarrow \rho_{water} A \frac{\ell}{3}\frac{1}{\cos\phi}\frac{1}{6}\ell\tan\phi = \frac{\ell}{2}\sin\phi\,\rho_{wook} A\ell g$$

$$\Rightarrow \cos^2\phi = \frac{1}{9}\frac{\rho_{water}}{\rho_{wood}} = \frac{1}{9\times 0.45} \simeq \frac{1}{4}$$

$$\Rightarrow \cos\phi \simeq \frac{1}{2} \qquad \phi \cong 60^\circ$$

$$F' = (Mg - B)\hat{j} \qquad (\Sigma F_i = 0)$$

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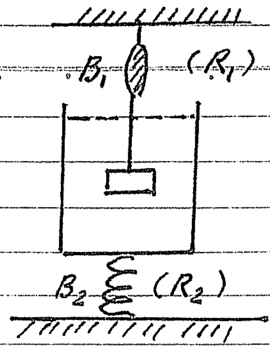
Example 2



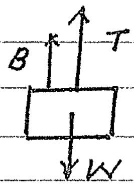
$W_c = 3\,Kg\,g$ (c: cup)

$W_{liquid} = 2\,Kg \cdot g$

Reading on B_1 $= 10\,Kg\,g$

Reading on B_2 $= 20\,Kg\,g$

Free force diagram

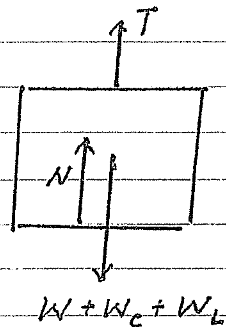


$T + F_B - W = 0$

T ↓ reading on B_1

$T + \overbrace{N - W}^{F_B} - W_c - W_L = 0$

N ↓ reading on B_2



W, F_B are the unknowns

$V_B = 0.1 \quad m^3$

$\underset{20kgg}{N} - \underset{3kgg}{W_c} - \underset{2kgg}{W_L} = F_B = \rho_{liquid} V_B\, g$

↓ solve for ρ_{liquid}

$F_B = 15\,Kg\,g = \rho_{liquid}\, 0.1\, m^3 \cdot g$

$\Rightarrow \rho_{liquid} = 150\ Kg/m^3$

$= 0.15\ gm/cm^3$

$W = T + F_B = 25\,Kg\,g$

$= \rho_{wood}\ 0.1\ m^3 g$

$\rho_{wood} = 0.25\ gm/cm^3$

Dynamics

Ideal fluid

- steady flow (laminar)
- nonsteady (turbulent)

- incompressible fluid
  ↓
  ρ is constant

- nonviscous flow
- irrotational flow

$$\rho_1 A_1 \underbrace{v_1 \Delta t}_{\Delta s} = \rho_2 A_2 v_2 \Delta t$$

mass going out in Δt

$$\Rightarrow \quad A_1 v_1 = A_2 v_2$$

↓
Equation of continuity

$$W = -\Delta m g(y_2 - y_1) + (p_1 - p_2)\frac{\Delta m}{\rho}$$

$$= \Delta K.E. = \frac{1}{2}\Delta m(\vec{v}_2^{\,2} - \vec{v}_1^{\,2})$$

$$\Rightarrow \quad p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

↓
Bernoulli's equation.

See P. 2 of the note.

the fundamental equation for all further discussion.

Fluid Mechanics

Apply the laws of mechanics to fluids

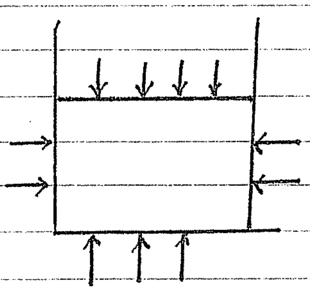
Fluids are substances that flow by the application of shear stress

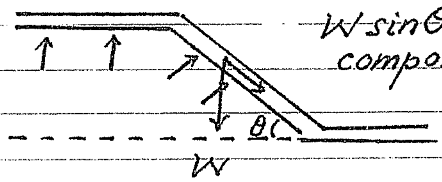
Liquids ~ practically incompressible
↓
fixed volume, varying shape

Gases ~ highly compressible (expandable)
↓
no characteristic volume.

Normal forces that may act on its boundary without flowing

Cannot resist tangential force
↓
will flow under a tangential force

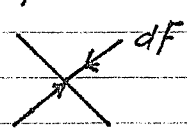




$W\sin\theta$ tangential component of weight of the fluid, act to set fluid to flow

Conditions of equilibrium
↓
its boundaries experience only normal forces.

Density, pressure, volume
↓
incompressible



film of zero thickness and mass and area da

$$p = \frac{dF}{da}$$

pressure at a point is defined.

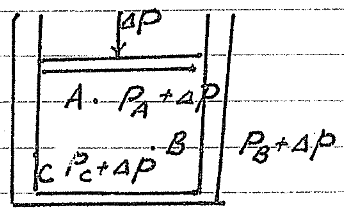
Pressure is a "scalar"

↓

See Notes.

Pascal (1633) principle

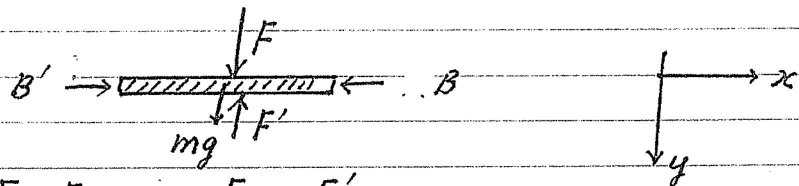
Pressure applied to a fluid is transmitted undiminished to all parts of the fluid and to the walls of the container enclosing it.



change in ΔP

See examples in the note
hydrau-level, level
⇒ Archimedes' principle

The variation of fluid pressure with depth



$$\Sigma F_y = F - F' + mg = 0$$

$$PA - P'A + \rho A \Delta y\, g = 0$$

$$dP = \rho g\, dy$$

$$P(y) - P(0) = \int_0^y \rho g\, dy \rightarrow \rho g y$$

Archimedes' principle

↓

examples.

Fluid dynamics

↓

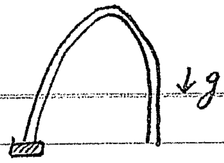
undergo acceleration.

Bernoulli's equation

Daniel Bernoulli (1700-1782)

↓

Swiss.



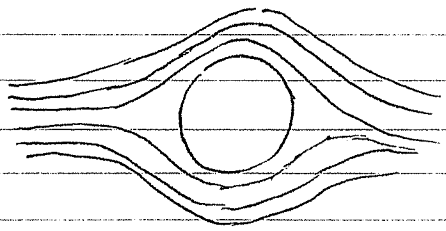
Steady-state ✓

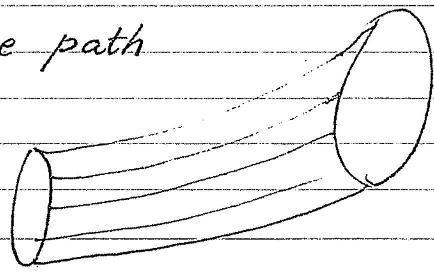
Incompressible ✓

Nonviscous ✓ — Ideal fluid.

Irrotational ✓

Streamline
↳ lines of flow
↳ mark the path





(1) No fluid flow ⊥ to the streamline

(2) Families of streamlines
↳ tube of flow
↓
no flow through the surface of the tube.
↓
Bernoulli's equation
↓
energy conservation

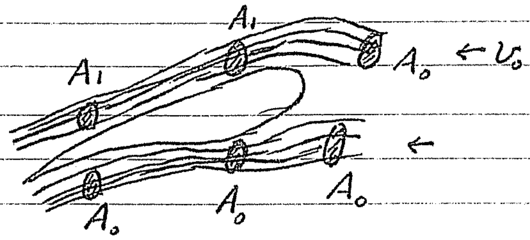
See the notes.

$$p + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}.$$

At the same height $h = \text{constant}$

$$\Rightarrow \quad p + \frac{1}{2}\rho v^2 = \text{constant}$$

Lifting of an airplane.



$$P_{below} = \text{constant} - \frac{1}{2}\rho v_0^2 = P_0$$

$$P_{above} = \text{constant} - \frac{1}{2}\rho v_1^2$$

$$= P_0 - \frac{1}{2}\rho(v_1^2 - v_0^2)$$

$$v_1 = \frac{A_0}{A_1}$$

$$\underbrace{P_{below} - P_{above}}_{\Delta P} = \frac{1}{2}\rho v_0^2\left[\left(\frac{A_0}{A_1}\right)^2 - 1\right]$$

$$F = \text{Area} \cdot \Delta P = \frac{1}{2}\rho v_0^2\left[\left(\frac{A_0}{A_1}\right)^2 - 1\right] \text{Area}$$

$$= \frac{1}{2}\rho v_0^2 k \text{ area}.$$

Put in the numbers $\rho_{air} = 1.3 \text{ Kg/m}^3$

$\frac{A_1}{A_0} = 0.8$ ✓ $\quad v = 120$ miles/hr $= 53.5$ m/sec. (the value one has to reach).

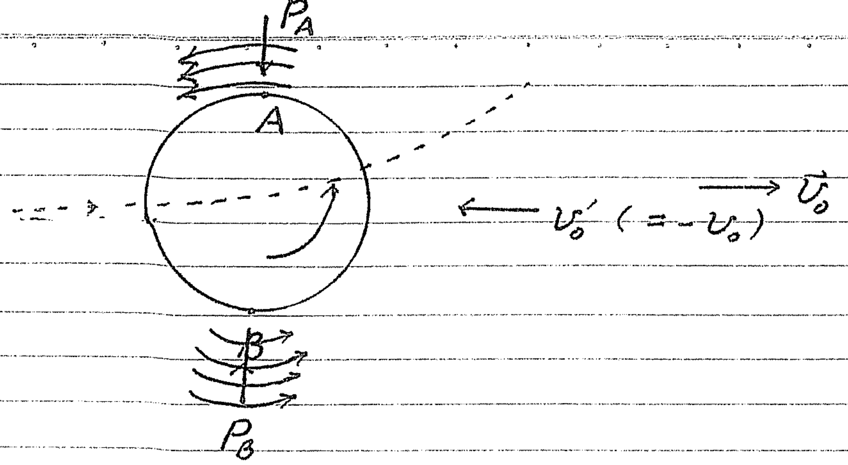
Area $= 20 \text{ m}^2$

$F \cong 20{,}900$ Newton.

↓

can lift ~ 2,100 Kg's light plane

Baseball



$v_A = v_0' + v_A'$ ($v_A' \to r\omega_0$)

$v_B = v_0' - v_B'$

$v_A > v_B$, $P_A < P_B$

⇒ a force on the ball toward the direction indicated.
↓ curve ball.

drag along a rotating layer of air
↕
friction between the air and surface of the ball.
↓
complicated.
ideal fluid is just a crude approximation.

atomospheric friction is responsible for the curvature of the ball path.

⇒ a pitcher could never throw a curve in a vacuum.

Surface tension and capillary attraction

Surface Tension

Cause.

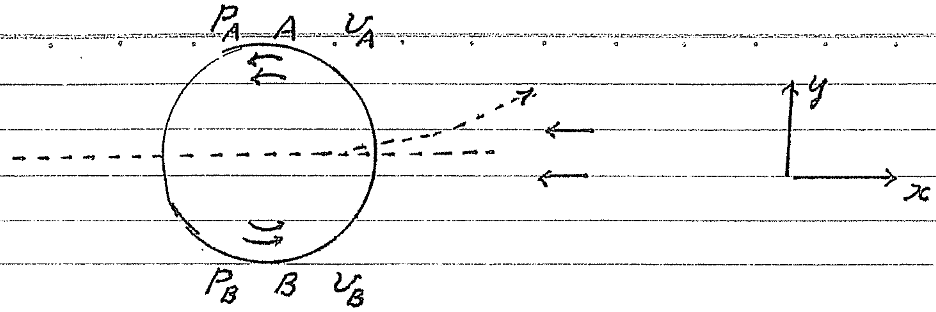
intermolecular forces acting on molecules at or near a liquid surface, different from those which act on molecules deep in the interior of the liquid.

Capillary Attraction

Cause:

intermolecular attractive forces between fluid molecules and the molecules of the capillary tubing are stronger those between the fluid molecules themselves

Curve Ball



Friction between air and the ball
⇒ drag the air around the ball
⇒ $v_A > v_B$ ⇒ $P_A < P_B$ ⇒ push the ball from B toward A
⇒ curve ball

Throw the ball by a right handed pitcher
⇒ act at B with force toward x
⇒ produce a torque along $-\hat{j} \times \hat{i} = \hat{i} \times \hat{j} = \hat{k}$ direction as indicated.

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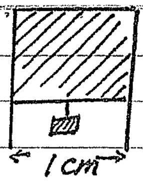
Surface Tension



增加表面 1 cm² 所需做的功

$= 2S$
↳ 表面張力

Measurement : see the graph

Reason: molecular at the surface has higher energy than interior

⇒ create surface needs work

Capillary attraction
↓
between the molecules of the wall and molecules of the liquid

第十一章 彈性學及流体力學

在本章中我們將討論能改變形狀物体的力學，這些問題相當複雜，我們將只討論最簡單的情形。

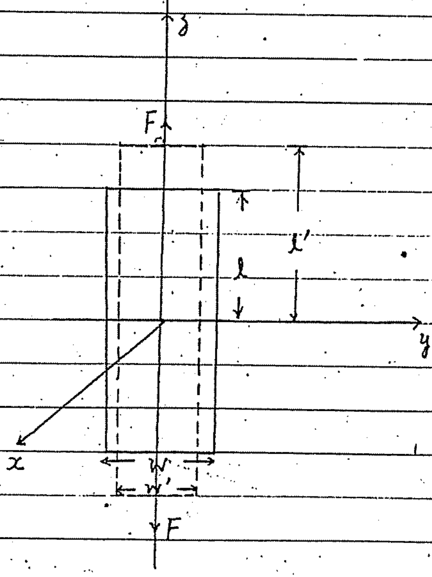
第一章 靜彈性學

1. 簡介：剛体只是一個理想的物体。通常物体在受外力時會改變其形狀。當一固体受一使其整体維持平衡之外力時，若外力不太大，則該物体會稍微改變其形狀。同時當外力消除時，該物体恢復原狀。靜彈性學討論在此種情況下外力與物体變形間之關係。

2. 基本觀念：

當外力不大時，一物体之應變與其所受之應力成正比，它們之間的比例常數則稱為彈性係數。

$$彈性係數 = \frac{應力}{應變} \qquad (1)$$



(1) 正應力

我們考慮長為 l，寬為 w，厚也為 w 之固体。

當我們在沿 z 軸方向施一力 F，則其正應力為 $S_n = \frac{F_n}{A}$ [2,3,4] (2)

在沿應力方向其長度之改變為 $\Delta l = l' - l$

我們定義其縱應變為

$$e_l = \frac{\Delta l}{l} \qquad (3)$$

由實驗觀察所得，在垂直方向它縮了 $\Delta w = w' - w$，我們定義橫應變為

國立清華大學研究室紀錄

$$e_t = \frac{\Delta W}{W} = \frac{W'-W}{W} \qquad (4)$$

一般而言 縱應變与橫應變之比是一負的常數。此一常數与物性有關。我們定義

$$\sigma = -\frac{e_t}{e_l} \qquad (5)$$

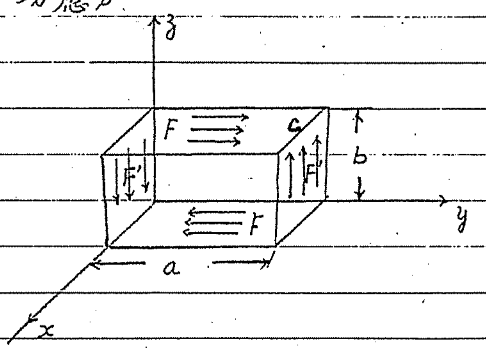
為帕松常數。

正應力与縱應變成正比，因此

$$Y = \frac{S_n}{e_l} \qquad (6)$$

Y 稱為楊氏係數

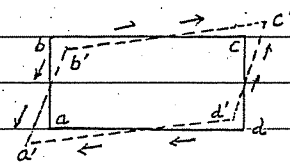
(2) 純切應力

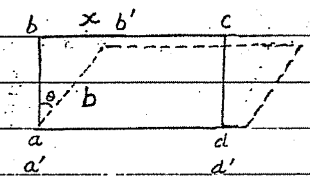


當一物体受力如圖所示，而且 $|\vec{F}|b = |\vec{F}'|a$ 時，我們稱之為純切應力之情形。

此時之切應變力

$$S_t = \frac{F_t}{A} \qquad (7)$$





我們定義切應變為

$$\tan\theta = \frac{x}{b} \qquad (8)$$

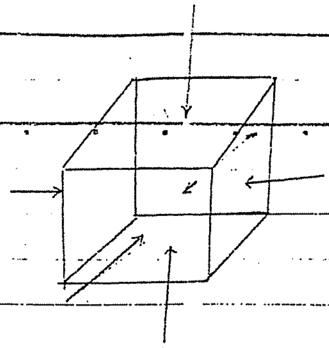
當 θ 小時 $\theta \simeq \tan\theta = \frac{x}{b}$ (9)

切應力与切應變成正比

$$\mu = \frac{S_t}{\tan\theta} \qquad (10)$$

μ 稱為切變係數或扭轉係數

(3) 壓縮。



當一物体受力如圖所示，則該物体所受之壓力[6]

$$P = \frac{F}{A} \qquad (11)$$

F 是垂直表面之力，A 是表面之面積

壓力即是此問題之應力。

其應變為 $-\frac{\Delta V}{V}$，此處 V 是該物体之体積，而 ΔV 是体積之變化。

應力與應變成正比

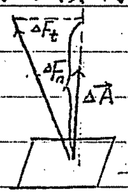
$$B = -\frac{P}{\frac{\Delta V}{V}} \qquad (12)$$

B 稱為体積彈性係數。其倒数 $k = \frac{1}{B}$ 稱為壓縮係数。

討論

1. 應力的單位是力/面積，應變則無因次

2. 我們定義表面為平面之面積向量其大小即為其面積，其方向是垂直於面積所在之平面。而對所討論之物体而言是向外的，若其表面不是一平面時，則我們將表面分成很多小塊，直到夠小到表面可以看成平面時再以上法定義表示此一小面積的面積向量 $\Delta\vec{A}$。將所有小塊的面積向量以向量的方法加起來即得到代表該表面之面積之向量。

3. 

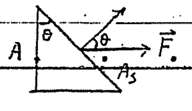
若此小塊面積上所受之力為 $\Delta\vec{F}$，當 ΔA 夠小時 $\Delta\vec{F}$ 是一定向量。我們先定義一沿 $\Delta\vec{A}$ 方向之單位向量為 $\hat{n} = \frac{\Delta\vec{A}}{\Delta A}$

正應力之定義為

$$S_n = \lim_{\Delta A \to 0} \frac{\Delta\vec{F} \cdot \hat{n}}{\Delta A} = \lim_{\Delta A \to 0} \frac{\Delta F_n}{\Delta A} \qquad (13)$$

切應力之定義為

$$S_t = \lim_{\Delta A \to 0} \frac{|\hat{n} \times (\Delta\vec{F} \times \hat{n})|}{\Delta A} = \lim_{\Delta A \to 0} \frac{\Delta F_t}{\Delta A} \qquad (14)$$



$$S_n' = F_n / A_s \qquad S_t' = F_t / A_s \qquad (15)$$

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$A_s = A/\cos\theta$, $F_n = F\cos\theta$, $F_t = F\sin\theta$ (16)

因此 $S_n' = \frac{F}{A}\cos^2\theta$, $S_t' = \frac{F}{A}\cos\theta\sin\theta$ (17)

4. 我們強調 (一) 力是施於物体兩面的整個表面上 (二) 因為此物体之整体仍維持平衡，因此作用該物体上之外力及外力矩之和均必須為零。

5 此一條件為滿足該物体所受外力矩和為零所要求者。由此一條件我們很容易的看出在各面上之 S_t 均相等。

6. 壓力 $P = -S_n$

應用

1. 首先我們將找出在各向同性的物体中 σ, Y 与 B 之關係

取一每邊為 a 之正方体，若在 z 方向加一應力 $S_n = -P$

$a_z' = a(1+e_\ell)$, $a_y' = a(1+e_t)$, $a_x' = a(1+e_t)$, $e_\ell = S_n/Y$, $-\frac{e_t}{e_\ell} = \sigma$ (18)

再在 x 方向加一應力 $S_n = -P$

$a_z' = a(1+e_\ell)(1+e_t) \simeq a(1+e_\ell+e_t)$, $a_y' \simeq a(1+2e_t)$, $a_x' = a(1+e_\ell+e_t)$ (19)

再在 y 方向加一應力 $S_n = -P$

$a_z' \simeq a(1+e_\ell+e_t)(1+e_t) \simeq a(1+e_\ell+2e_t)$

$a_y' \simeq a(1+2e_t)(1+e_\ell) \simeq a(1+e_\ell+2e_t)$

$a_x' \simeq a(1+e_\ell+e_t)(1+e_t) \simeq a(1+e_\ell+2e_t)$ (20)

因此 $V' = a_x' a_y' a_z' \cong V(1+6e_t+3e_\ell)$ (21)

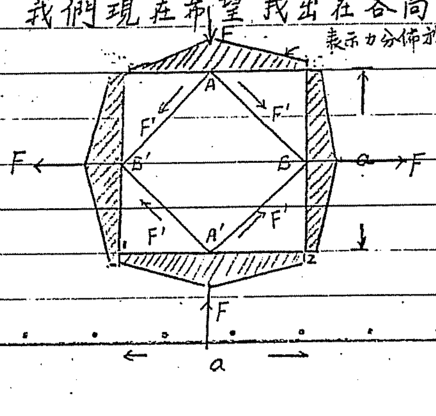
由於 $e_\ell \ll 1$, $e_t \ll 1$，所以我們在上式中將 $e_t e_\ell$, $e_t e_t$ 及 $e_\ell e_t^2$ 等項略去不計。

$$\frac{\Delta V}{V} = \frac{V'-V}{V} = 3(e_\ell + 2e_t) = 3e_\ell(1-2\sigma)$$

$$= -\frac{3P}{Y}(1-2\sigma) \quad (22)$$

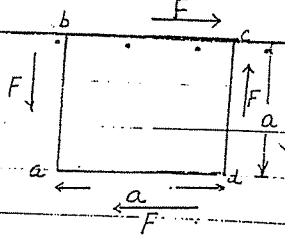
因此 $B = \frac{Y}{3(1-2\sigma)}$ (23)

2. 我們現在希望找出在各向同性的物体中 σ, Y 与 μ 之關係

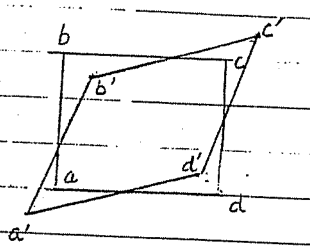


由 AB'A'B 之觀點來看，它可以受如圖所示之正應力但也可以看成是受 F' 之切應力。利用此兩種方法計算的結果應該相同，由此可找出 σ, Y 与 μ 之關係

(a) 由切應力之觀點來看

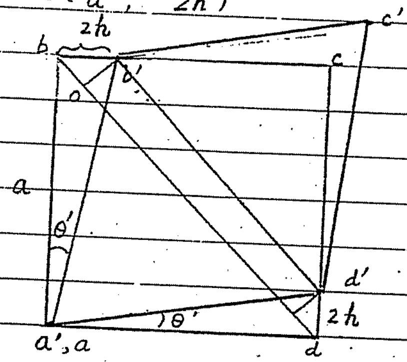


$a(0,0) \rightarrow a'(-h, -h)$
$b(0,a) \rightarrow b'(h, a-h)$
$c(a,a) \rightarrow c'(a+h, a+h)$
$d(a,0) \rightarrow d'(a-h, h)$



我們將 $a'd'c'b'$ 作 − $a'a$ 之平移使 a' 與 a 重合，此時

b' 之座標為 $(2h, 0)$
c' 之座標為 $(a+2h, a+2h)$
d' 之座標為 $(a, 2h)$



顯然易見的 $2\theta' = \theta$ $\tan\theta$ 是應變

$$\frac{b'd' - \sqrt{2}a}{\sqrt{2}a} = \frac{\Delta AA'}{AA'} = \frac{-2\cdot ob}{\sqrt{2}a} = \frac{-2\sqrt{2}h}{\sqrt{2}a} = -\frac{2h}{a} = -\tan\theta' \simeq -\theta'$$
$$= -\frac{1}{2}\theta \simeq -\frac{1}{2}\tan\theta \qquad (24)$$

$$\frac{\Delta AA'}{AA'} = -\frac{1}{2}\tan\theta \qquad (25)$$

由於 $\mu = \dfrac{S_t}{\tan\theta}$ (26)

$$\frac{\Delta AA'}{AA'} = -\frac{1}{2}\frac{S_t}{\mu} \qquad (27)$$

(b) 由正應力之觀念來看

由於垂直方向之力 $AA' \rightarrow a(1+e_\ell)$ = (28)

由於 $Y = \frac{S_n}{e_\ell}$ 此處 S_n 為負的 $S_n = -|S_n|$ (29)

$$e_\ell = -\frac{|S_n|}{Y}$$

$$AA' \rightarrow a\left(1-\frac{S_n}{Y}\right) \qquad (30)$$

由於水平方向之力 AA' 由 $a\left(1-\frac{S_n}{Y}\right) \rightarrow a\left(1-\frac{S_n}{Y}\right)(1+e_t)$

$$\simeq a\left(1-\frac{S_n}{Y}+e_t\right) \qquad (31)$$

由於 $\sigma = -\frac{e_t}{e_\ell}$，上式可寫成

$$\simeq a\left(1-\frac{|S_n|}{Y}-\sigma e_\ell\right) \qquad (32)$$

此時 $S_n = |S_n|$

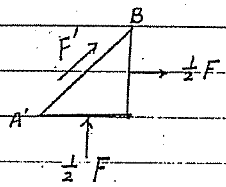
經此兩正應力 AA' 由 $a \rightarrow a\left(1-\frac{|S_n|}{Y}-\sigma\frac{|S_n|}{Y}\right)$，所以

$$\frac{\Delta AA'}{AA'} = -\frac{|S_n|}{Y}(1+\sigma) \qquad (33)$$

比較 (28), (33) 兩式得

$$\frac{1}{2}\frac{S_t}{\mu} = \frac{|S_n|}{Y}(1+\sigma) \qquad (34)$$

現在我們要找出 S_t 及 $|S_n|$ 間之關係



$$F' = \frac{1}{\sqrt{2}}F \qquad (35)$$

$A'B$ 之面積是 1,2 底面 A 之 $\sqrt{2}$ 倍，也即是 $\sqrt{2}A$

$$|S_n| = \frac{F}{A} \qquad (36)$$

$$S_t = \frac{F'}{\sqrt{2}\cdot A} = \frac{F}{A} = |S_n| \qquad (37)$$

代入 (34) 式得

$$\frac{1}{2\mu} = \frac{1}{Y}(1+\sigma) \qquad (38)$$

也即是 $\mu = \frac{Y}{2(1+\sigma)}$ (39)

3. 我們現在來討論拉力的問題

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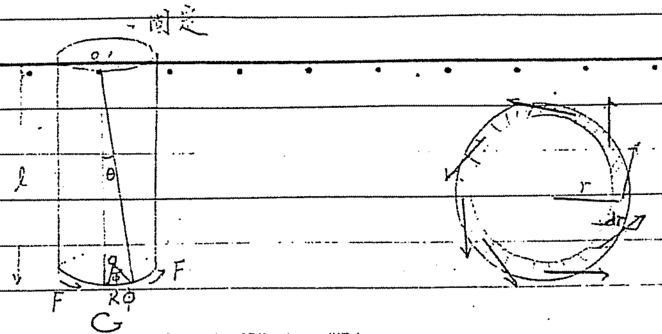

在底部加一力距 τ，我們想了解 τ 及 θ 之關係

我們首先討論底面半徑為 r 到 $r+dr$ 之間之圓環其面積 $dA = 2\pi r\,dr$

其應變為 $\theta = \frac{r\phi}{l}$ (40)

利用應變及應力之間的關係

$$\mu = \frac{S_t}{\theta} \Rightarrow S_t = \mu\theta \quad (41)$$

而此一圓環上所受之切力是 $dF_t = S_t\, dA$ (42)

$$= \mu\theta\, 2\pi r\, dr \quad (43)$$

因此，對 O 点，此一圓環所受之力距是沿z軸，其大小為

$$d\tau = r\, dF_t \quad (44)$$

$$= \mu\theta\, 2\pi r^2 dr \quad (45)$$

因此在底部所受之總力距是

$$\tau = \int_0^R \mu\theta\; 2\pi r^2\, dr = \frac{\mu\phi}{l}\cdot 2\pi \int_0^R r^3\, dr$$

$$= \underbrace{\frac{\mu\pi}{2l}R^4}_{K}\phi \quad (46)$$

若除去外力距時此物体受一回復力距

$$\tau = -\frac{\mu\pi}{2l}R^4\phi = -K\phi \quad (47)$$

$$\tau = I\frac{d^2\phi}{dt^2} \quad (48)$$

因此

$$I\frac{d^2\phi}{dt^2} = -K\phi \quad (49)$$

$$\phi = A\sin(\omega_0 t + \alpha) \qquad \omega_0 = \sqrt{\frac{K}{I}} \quad (50)$$

而其振動週期為 $P = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I_0}{K}}$. 從量度其週期可求得 K，由 K

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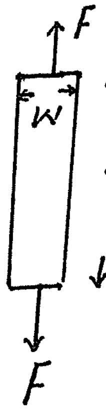
總號：

則可利用第(46)式求得 μ

Definition



應力 ∝ 應變

$\frac{F}{A} \Rightarrow \frac{\text{force}}{\text{area}}$

↓

stress 應力 → $\tau_\perp$

longitudal strain $\frac{\Delta L}{L} = e_\ell$

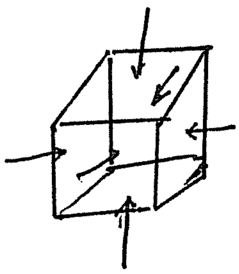
transverse strain $\frac{\Delta W}{W} = e_t$

$$\tau_\perp = Y \frac{\Delta L}{L} = Y e_\ell$$

↓

Young's modulus

$$\sigma = \left| \frac{e_t}{e_\ell} \right| \qquad \sigma = - \frac{e_t}{e_\ell}$$



$\frac{F}{A}$ stress

$$e_V = \frac{\Delta V}{V}$$

$$\tau_\perp = -B \frac{\Delta V}{V}$$ B = Bulk modulus

$B > 0$, $\tau_\perp > 0 \rightarrow \Delta V < 0$

These modulus and ratio are related by

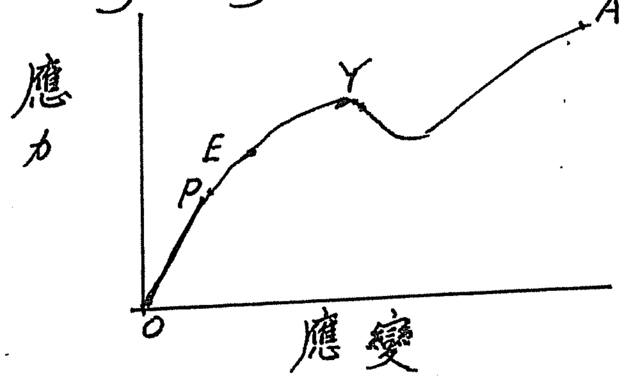
$$B = \frac{1}{3} \frac{Y}{1-2\sigma}$$

↓

Problem 59 of Chapter 11.

The above results are valid below the proportional limit

In a larger region



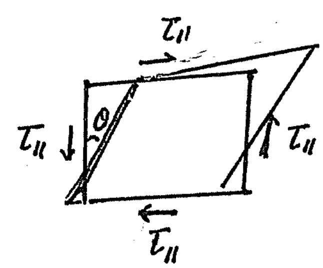
OP — proportional region

E — elastic limit
↓
beyond it, even if the external stress is removed the body will not return to its original form

Y — yielding point, even without increase stress the length can be increased

A — breaking

Sheer modulus



keep the body in equilibrium need 4 $\tau_{\parallel}$ as indicated

stress $\tau_{\parallel}$

strain θ

$$\tau_{\parallel} = G\,\theta$$

θ ↓ or $\tan\theta$, $\sin\theta$ (since θ is small)

G ↓ sheer modulus

All these modulus depends on the properties of the material
Rough models are given in the textbook

第五章 连续体力学

在上一章我们讨论了刚体的运动，刚体是不能形变的。用质点组的观点来说，就是内部质点之间没有相对运动。本章将讨论连续体的力学，连续体包括弹性体和流体（液体和气体），它们的共同特点是其内部质点之间可以有相对运动，宏观地看，连续体可以有形变或非均匀流动。处理连续体的办法是不再把它看成一个个离散的质点，而是取“质元”，即有质量的体积元。为此我们要引进“密度”的概念，密度 ρ 是单位体积内的质量，从而体积为 dV 的质元具有质量 $dm=\rho dV$. 在连续体力学中，力不再看成是作用在一个个离散的质点上，而看成是作用在质元的表面上，因而需要引进作用在单位面积上的力，即“应力”的概念。

§1. 固体的弹性

1.1 应力和应变

本章以前各节采用的是刚体模型，把固体的一切形变都忽略了，在本节里我们将讨论固体的弹性。在外力作用下，在弹性体内同时产生相应的形变（应变）和弹性恢复力（应力）。

应力（stress）是物体中各部分之间相互作用的内力。讨论物体中某处的内力，就得设想在该处有一假想的截面 ΔS（见图 5 – 1），把两边的物质 1 和 2 分开。面元 ΔS 的取向任意，设被此面元分开的两部分物质之间的作用力和反作用力分别为 $\Delta \boldsymbol{f}$ 和 $-\Delta \boldsymbol{f}$，则在此截面上的应力定义为如下矢量：

$$\boldsymbol{\tau} = \lim_{\Delta S\to 0}\frac{\Delta \boldsymbol{f}}{\Delta S} = \frac{d\boldsymbol{f}}{dS} \qquad (5.1)$$

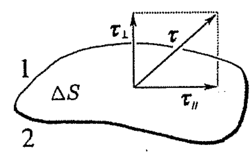


图 5 – 1 应 力

在固体中一个截面上的应力一般不与此截面垂直，我们可以将它分解为法向分量 $\tau_\perp$ 和切向分量 $\tau_{/\!/}$，前者称为*正应力*（压力或张力），后者称为*剪应力*。一般说来，应力不仅与截面 ΔS 的位置有关，并且随它的取向而异。在 MKS 制中应力的单位为牛顿每平方米（N/m^2），称为“帕斯卡（pascal）”，简称“帕”，符号为 Pa. 应力的量纲为$[\tau]=ML^{-1}T^{-2}$.

固体的应变有两种基本形式：与纯正应力相对应的*体应变*（bulk strain）和纯剪应力相对应的*剪应变*（shear strain）。

在静止的流体中只有各向同性的正应力，一般是压力，称为“静水压（static hydraulic pressure）”。在弹性体上加以静水压时，其体积 V 将发生

变化 ΔV,体应变 $\varepsilon_{体}$ 定义为体积的相对变化：

$$\varepsilon_{体} = \frac{\Delta V}{V}. \quad (5.2)$$

在弹性限度内正应力 $\tau_{\perp}$ 与体应变 $\varepsilon_{体}$ 成正比：

$$\tau_{\perp} = K\varepsilon_{体} = K\frac{\Delta V}{V}, \quad (5.3)$$

K 称为体弹性模量，其倒数 $\kappa = 1/K$ 称为压缩系数。

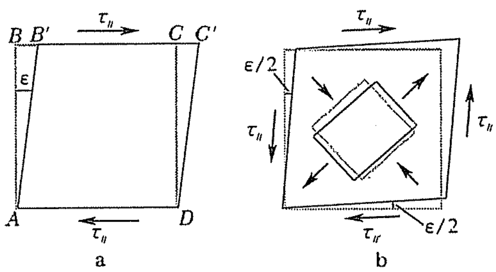


图 5－2 剪应变

为了讨论剪应变 $\varepsilon_{剪}$，我们设想从弹性体中隔离出一方块体(见图 5－2)，如果在这方块体上下底面加一对大小相等、方向相反的切向应力 $\tau_{/\!/}$，则弹性体会发生如图 5－2a 所示的形变。不过仅仅有这一对切向力，它们构成的力偶矩将使块体倾翻，不能平衡。所以在块体左右两侧面上还得加一对力矩相反的切向应力，使之能够平衡。这时块体将发生如图 5－2b 所示的形变，方块变成菱形。这便是剪应变。由此可以看出，剪应变也可看作是沿对角方向压缩的形变。剪应变的大小 $\varepsilon_{剪}$ 用平行截面间相对滑动位移 $\overline{BB'}$ 与它们之间垂直距离 $\overline{AB}$ 之比来表征(见图 5－2a)，这比值就是 $\angle BAB'$ 的大小 ε，这角度称为剪变角：

$$\varepsilon_{剪} = \frac{\overline{BB'}}{\overline{AB}} = \varepsilon. \quad (5.4)$$

在弹性限度内切向应力 $\tau_{/\!/}$ 与剪应变 $\varepsilon_{剪}$ 成正比：

$$\tau_{/\!/} = G\varepsilon_{剪} = G\frac{\overline{BB'}}{\overline{AB}} = G\varepsilon. \quad (5.5)$$

系数 G 称为剪变模量。

1.2 直杆的拉伸或压缩

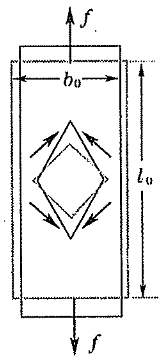


图 5－3
拉伸压缩形变

如图 5－3 所示，在直杆两端加上与杆平行的力 f 拉伸或压缩时，杆的长度 l_0 将有改变(拉伸时 $\Delta l > 0$，压缩时 $\Delta l < 0$)，此种应变 ε 以长度的相对增量 $\Delta l / l_0$ 来表征。设杆的截面积为 S，则其两端的应力为 $\tau = f/S$. 在弹性限度内应力 τ 与应变 ε 成正比：

$$\tau = Y\varepsilon = Y\frac{\Delta l}{l_0} \quad (5.6)$$

系数 Y 称为杨氏模量。胡克(R. Hooke) 于 17 世纪 70 年代末研究并发表了弹性杆拉伸压缩形变的规律(5.6)，现在除此式外，人们把所有应力、应变

成比例的规律[如(5.3)和(5.5)式]统称为胡克定律。

直杆在发生纵向形变的同时,总伴有横向形变(见图5-3):纵向拉伸时横向收缩,纵向压缩时横向膨胀。设杆的横向线度原来为 b_0,改变量为 Δb,则横向应变为

$$\varepsilon_{横} = \frac{\Delta b}{b_0}, \tag{5.7}$$

一般说来,$\varepsilon_{横}$ 比纵向应变 ε 在绝对值上小3-4倍,二者绝对值之比叫做泊松比,记作 σ,

$$\sigma = \left|\frac{\varepsilon_{横}}{\varepsilon}\right|, \tag{5.8}$$

它是一个小于1/2的无量纲量。

表5-1 固体的弹性模量和泊松比

材料	$K/10^{10}$Pa	$G/10^{10}$Pa	$Y/10^{10}$Pa	σ
铝	7.8	2.5	6.8	0.355
黄铜	13.9	3.8	10.5	0.374
铜	16.1	4.6	12.6	0.37
金	16.9	2.85	8.1	0.42
电解铁	16.7	8.2	21	0.29
铅	3.6	0.54	1.51	0.43
镁	3.6	1.62	4.23	0.306
铂	14.2	6.4	16.8	0.303
银	10.4	2.7	7.5	0.38
不锈钢	16.4	7.57	19.7	0.30
熔凝石英	3.7	3.12	7.3	0.17
聚苯乙烯	0.41	0.133	0.36	0.353

单向的拉伸或压缩形变中除了体形变外,还包含着剪切形变。如图5-3所示,在杆内取一个各边与轴线成45°的正方形,当杆被拉伸时,它被拉成菱形,即发生了剪切形变。所以,我们已经引入的三个弹性模量 K、G、Y 和泊松比 σ 之间是有联系的。根据弹性理论可以证明,在这四个参量中只有两个是独立的,其中 Y 和 σ 可用 K 和 G 表示出来:

$$\begin{cases} Y = \dfrac{9GK}{3K+G}, & (5.9) \\ \sigma = \dfrac{3K-2G}{2(3K+G)}. & (5.10) \end{cases}$$

当然也可以反过来,用 Y 和 σ 来表示 K 和 G:

$$\begin{cases} K = \dfrac{Y}{3(1-2\sigma)}, & (5.11) \\ G = \dfrac{Y}{2(1+\sigma)}. & (5.12) \end{cases}$$

由于所有弹性模量都只能是正的,故泊松比 σ 不可能大于1/2.

因为应变 ε 是无量纲量，以上各弹性模量的量纲都与应力相同，单位皆为 $\mathrm{Pa=N/m^2}$. 此外，从(5.9)到(5.12)各式还可以看出，弹性模量 K、G、Y 具有相同的数量级。表 5－1 给出一些材料的弹性模量和泊松比的数值，一个引人注目的现象是，尽管各种材料的软硬可以差别很大，它们的弹性模量却差不多同为 10^{10} Pa(约 10^5 大气压)的数量级。个中的奥秘需要用量子理论来解释，这里不便多谈了。[18]

1.3 梁的弯曲

先考虑矩形截面的梁，设其高为 h，宽为 b，两端的支撑力 N_1、N_2 与全部载荷(包括自重)P 平衡，梁本身则发生弯曲形变(见图 5－4a)。为简单计，设载荷集中在中点，于是 $N_1=N_2=P/2$. 用一个假想的面 $O'O''$ 把梁从中间分开，成为对称的左右两段。由图可以看出，两段各受到一个方向彼此相反的力偶矩 $Pl/4$ (l 为梁长)，此力偶矩由什么来平衡?回答这个问题要分析弯曲形变的特点和横截面 $O'O''$ 上的应力分布。

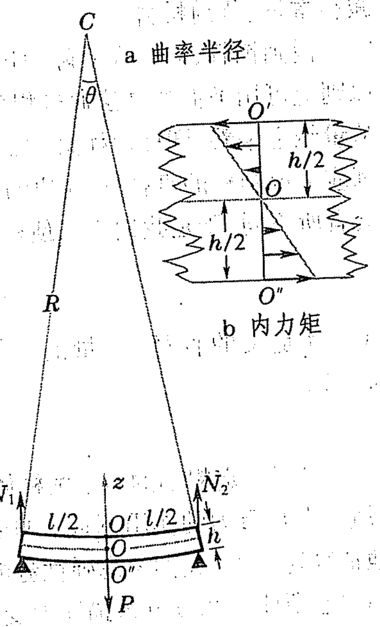


图 5－4 梁的弯曲

为了分析形变的特点，设想将梁分成上下许多层。当梁向下弯曲时，上层受到压缩，下层受到拉伸，中间有个无应力的中性层。在 $O'O''$ 面上的内应力分布将如图 5－4b 所示，上挤下拉，形成一个力偶矩 $M_内$，与外力矩 $Pl/4$ 平衡。现在我们来计算这个内力矩 $M_内$。

如图 5－4a 所示，设弯曲的梁的曲率半径为 R，曲率中心在 C 点，梁对 C 所张的圆心角为 $\theta=l/R$. 在截面 $O'O''$ 上取 z 轴沿高度方向，以中性层处 O 为原点，坐标为 z 处一层的长度为 $\theta(R-z)=l(R-z)/R=l-lz/R$，即 $\Delta l=-lz/R$；应变为 $\varepsilon=\Delta l/l=-z/R$，按照胡克定律，应力 $\tau=-Yz/R$(负应力代表压力，正应力代表张力)。高度为 $\mathrm{d}z$ 的一层横截面积为 $\mathrm{d}S=b\,\mathrm{d}z$，作用在其上的总力为 $\mathrm{d}f=\tau\,\mathrm{d}S=-Ybz\,\mathrm{d}z/R$，对 O 点的力矩为 $\mathrm{d}M=z\,\mathrm{d}f=-Ybz^2\mathrm{d}z/R$，故总力偶矩为

$$M_{内}=\int_{(横截面)}\mathrm{d}M=-\frac{Yb}{R}\int_{-h/2}^{h/2}z^2\mathrm{d}z=-2\frac{Yb}{R}\int_0^{h/2}z^2\,\mathrm{d}z=-\frac{Ybh^3}{12R},\quad(5.13)$$

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[18] 可参看赵凯华. 定性与半定量物理学. 北京：高等教育出版社，1992. 153

负号代表左边半段所受的力偶矩是逆时针方向的。在平衡时 $M_{内}=M_{外}=Pl/4$，此时梁的曲率为

$$\kappa=\frac{1}{R}=\frac{12M_{外}}{Ybh^3}. \tag{5.14}$$

外力偶矩是载荷造成的。上式表明，在一定的载荷下梁的弯曲程度（曲率）与宽度的一次方和高度的三次方成反比。由此可见，为了提高梁的抗弯能力，增加其高度比增加其宽度有效得多。此外，梁的中性层部分对抗弯的总力偶矩 $M_{内}$ 贡献不大，取去或减少这部分的材料，对梁的抗弯能力不会有显著影响。工程上广泛采用工字钢、空心钢管等构件，既能保证安全可靠，又能减轻重量、节约材料。

说到钢管，人们在实际中还经常使用圆形截面的梁。对于圆截面，我们原则上可用类似的办法通过积分来运算其内力偶矩，不过积分要复杂一点。从量纲上来看，圆柱形横梁 $M_{内}$ 的表达式应和(5.13)差不多，其中的 b 和 h 都应换成圆柱的直径 d，前边的无量纲系数 $1/12=0.083$ 与几何形状有关，会有所不同。于是我们可以预料，对于圆柱型横梁有

$$M_{内}\propto\frac{Yd^4}{R}.$$

式中 d 是梁的直径。定量的计算表明，上式中的数值系数应为 $\pi/64=0.049$，即

$$M_{内}=\frac{\pi Yd^4}{64R}. \tag{5.15}$$

对于一定粗细的实心圆柱体，竖起来的时候其高度 l 有个临界值 l_c，超过它，在自重力的作用下直立的姿态不再是稳定的，它开始弯折。为了定性地估算这个 l_c，我们考虑弹性势能和重力势能的变化。我们仍用柱长 l 对曲率中心所张的角度 θ 来描述形变，按(5.15)式

$$M_{内}=\frac{\pi Yd^4}{64R}=\frac{\pi Yd^4}{64l}\theta,$$

从而弹性势能的增量为

$$\Delta E_{p弹}=\int_0^\theta M_{内}\,d\theta=\frac{\pi Yd^4}{128l}\theta^2, \tag{5.16}$$

弯折时柱的重心下降量为（见图5-5）

$$\Delta h=\frac{l}{2}-R\sin\frac{\theta}{2}=\frac{l}{2}-\frac{R\theta}{2}\left[1-\frac{1}{3!}\left(\frac{\theta}{2}\right)^2\right]=\frac{l\theta^2}{48}.$$

柱体的重量为 $W=\pi d^2 l\rho g/4$，故重力势能的改变为

$$\Delta E_{p重}=-W\Delta h=-\frac{\pi d^2l^2\rho g}{192}\theta^2, \tag{5.17}$$

式中负号表示 $E_{p重}$ 减少。当 $|\Delta E_{p重}|\geqslant\Delta E_{p弹}$ 时，直立柱体失稳，故 l_c 由 $|\Delta E_{p重}|=\Delta E_{p弹}$ 决定。从(5.16) 和(5.17) 式

$$\frac{\pi Y d^4}{128 l_c}\theta^2=\frac{\pi d^2 l_c^2\rho g}{192}\theta^2$$

即

$$l_c=\left(\frac{3Yd^2}{2\rho g}\right)^{1/3}\propto d^{2/3}. \qquad (5.18)$$

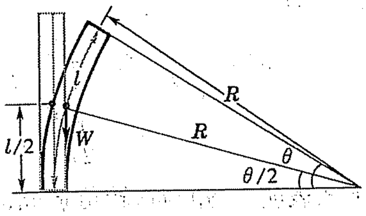


图5－5 直立圆柱的弯折

此式表明，一圆柱体在自重力作用下能抗弯折的临界高度并没有它的直径 d 增长得快。例如，当直径加倍时，其临界高度只增大 $2^{2/3}=1.59$ 倍。

把(5.18) 式运用到树木的高度与粗细关系的问题上，是饶有兴味的。当然，树不是光杆，其上还有树冠；此外，决定树高的因素也未必就是它的抗弯能力。图5－6给出一些北美洲树木的 l-d 关系的数据。上面那条实线代表抗弯折临界高度，纵横坐标都是按对数标度的，此直线的斜率等于2/3. 虚线的方程也具有 $l=Cd^{2/3}$ 的形式，为拟合那些实际的分散数据点，这里取 $C=34.9$. 看来没有数据点出现在那条代表理论极限的实线之上，且拟合曲线的斜率也接近 2/3. 以上结果加强了我们这样的信念，即抗弯折强度是决定树木高粗比的关键因素。

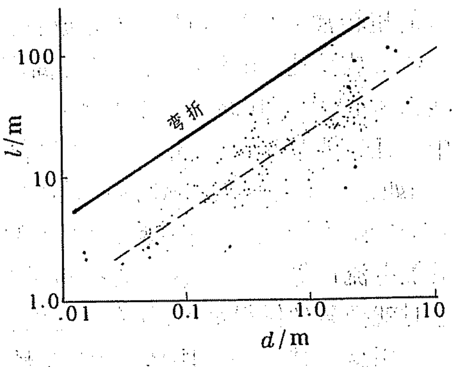


图5－6 一些树木长粗比的数据

1.4 柱的扭转

如图5－7所示，长度为 l 的圆柱体两端受到一对大小相等、方向相反的力偶矩 $\pm M_外$ 时，将发生扭转形变。圆柱体两端面相对转过的角度 φ 叫做**扭转角**。设圆柱体的半径为 R，则其表面上各点转过的弧长为 $R\varphi$，表面上的每根母线都倾斜一个角度，柱面上的“正方形”面元变成了“菱形”。所以，扭转形变本质上是剪切形变，不过距柱轴不同距离的地方剪变角是不同的。我们设想把圆柱体分割为半径不同的薄层，半径为 r 的薄层上剪变角为 $\varepsilon(r)=r\varphi/l$（见图5－7），圆柱表面的剪变角为 $\varepsilon(R)=R\varphi/l$. 考虑半径从 r 到 $r+\mathrm{d}r$ 的薄层，其横截面积 $\mathrm{d}S=2\pi r\,\mathrm{d}r$. 在横截面上的应力是切向的，设为 $\tau_{/\!/}(r)$，按胡克定律有 $\tau_{/\!/}(r)=G\varepsilon(r)=Gr\varphi/l$，作用在这薄层横截面上的力为 $\mathrm{d}f=\tau_{/\!/}(r)\,\mathrm{d}S=2\pi G\varphi r^2\mathrm{d}r/l$，此力对柱轴的力偶矩为

$dM = r\,df = 2\pi G\varphi\, r^3 dr/l$. 在整个横截面上总的力偶矩为

$$M = \int_{(\text{横截面})} dM = \frac{2\pi G\varphi}{l}\int_0^R r^3 dr = \frac{\pi G R^4}{2l}\varphi = D\varphi, \qquad (5.19)$$

上式表明，力矩与扭转角 φ 成正比，比例系数

$$D = \frac{\pi G R^4}{2l} \qquad (5.20)$$

称为圆柱体的*扭转常量*。机械中的传动轴、旋进的螺丝钉都需要有一定的抗扭能力。上式表明，圆柱体的扭转常量与半径的四次方成正比，与长度的一次方成反比。由此可见，为了提高圆柱体的抗扭能力，加大半径比减小长度有效得多。在物理实验中往往需要相反的情况，为了增加仪器的灵敏度（例如第二章4.3节中描写的厄缶实验，第七章2.3节中描写的卡文迪许实验，以及实验室中常用的灵敏电流计等），希望悬丝的扭转系数愈小愈好。这时就把悬丝做得很细，并有一定的长度。

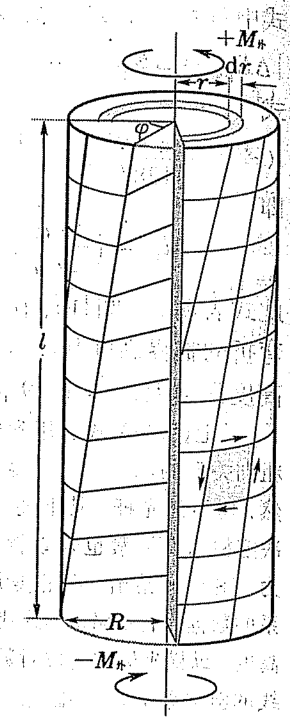


图 5－7 柱的扭转

用悬丝挂着一个刚体，使它在悬丝的弹性恢复力矩的作用下绕铅垂轴线来回扭动（见图 5－8）。这种装置叫做*扭摆*。设扭摆的角位移为 φ，弹性势能的增加等于抵抗弹性力矩所作的功。(5.19) 式中的 M 就是抵抗弹性力矩的外力矩，故以平衡位置 $\varphi=0$ 为参考点的弹性势能为

$$U(\varphi) = \int_0^{\varphi} M\,d\varphi = D\int_0^{\varphi}\varphi\,d\varphi = \frac{1}{2}D\varphi^2. \qquad (5.21)$$

利用(4.45)式我们还可求出扭摆的周期公式来。由上式知 $U_0'' = [d^2U(\varphi)/d\varphi^2]_{\varphi=0} = D$，代入(4.45)式得

$$T = 2\pi\sqrt{\frac{I}{D}}, \qquad (5.22)$$

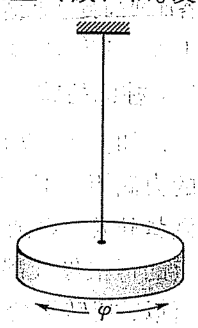


图 5－8 扭 摆

式中 I 为转动惯量。扭摆的周期提供了一种测量刚体转动惯量的方法。▌

1.5 相似性原理

尺度大小的变换叫做“标度变换”，通常遇到的物理系统是不具有标度变换下的不

变性的，即几何上相似的物体并不见得在物理性质上也相似。然而在工程技术中作模型试验时，不能把模型做得总与实物一样大小。怎样保证缩小了的模型与实物在物理上保持相似性？我们知道，无量纲的方程是没有尺度问题的，把物理方程无量纲化以后，就可以适用于一切尺度。所以，在物理上相似的条件是有关的无量纲组合量具有相同的数值。

现在我们来考虑工程上弹性结构（如桥梁桁架）的模拟问题。如前所述，各向同性建筑材料的的弹性性能由两个参量来表征。在这里我们选杨氏模量 Y 和泊松比 σ，前者的量纲为 $ML^{-1}T^{-2}$，后者无量纲。如果此机构是在重力下达到平衡的，则单位体积的重量 ρg 将是一个重要的参量。加上特征长度 L 和负载力 P，共五个参量。除原有的一个无量纲量 σ 外，稍加分析我们即可发现，在剩下的四个量中只有两个的量纲彼此独立，我们还可以找到另外两个无量纲量

$$\Pi_1 = \frac{P}{L^3\rho g}, \qquad \Pi_2 = \frac{Y}{L\rho g},$$

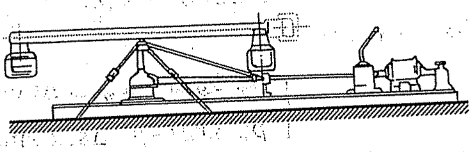


图 5－9 模型试验用的离心机

若模型采用与实物相同的材料来制做，则 Y、σ、ρ 不变，重力加速度 g 通常也是不变的。令 P 按正比于 L^3 的比例缩小，可保证 Π_1 不变，但怎样才能在 L 缩小时保证 Π_2 不变？从上式看来好像没什么办法了。实际上出路尚有一条，加大 g！把模型装在离心机上甩，用惯性离心力来模拟重力，以增大有效的 g. 实际中正是这样做的（参见图 5－9）。

Chapter 16

1. Introduction

    Definitions

        Incompressible
        Density
        Pressure
            ↳ Pascal principle
                Statics

2. 阿基米德原理

    Application
    Dynamics

3. 伯勞尼定理
    連續方程式

4. 表面張力和其他.

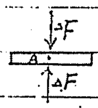
第二節 流体靜力學

1. 簡介 我們在此節中將討論在平衡狀況下之流体

2. 基本觀念

流体是能流動的物質。它包括液体及氣体。氣体和液体在壓縮下卻很不相同。氣体能容易被壓縮而液体卻幾乎不能被壓縮。

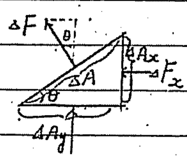
由於流体能流動，因此在流体中任何一處若受一切應力時流体即會移動。因此在平衡之液体中表面力必須垂直於表面。



我們定義在 A 点的壓力是將包圍 A 点之表面在單位面積所受的力，也即是

$$P = \lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A} \quad (1)$$

此處 ΔF 是表面 ΔA 所受的力之大小。它的方向是和表面相垂直。

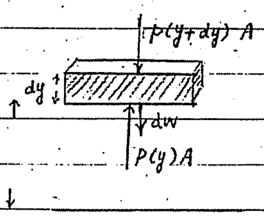


此一塊液体位於平衡情況，因此

$$\Delta F \sin\theta = \Delta F_x \quad , \quad \Delta F \cos\theta = \Delta F_y$$
$$\Delta A \sin\theta = \Delta A_x \quad , \quad A \cos\theta = \Delta A_y \quad (2)$$

由 (2) 式得 $$\frac{F}{\Delta A} = \frac{F_x}{\Delta A_x} = \frac{F_y}{\Delta A_y} \quad (3)$$

因此它在各方向的大小都是一樣。它是一個純量。但在液体中不同点之壓力可以不同。我們現在討論在重力場中壓力和高度之關係



因為該液体是在平衡狀態之下，因此

$$P(y)\,A = P(y+dy)\,A + dW \quad (4)$$

dW 是該立方体之重量 $= \rho A\, dy\, g \quad (5)$

此處 ρ 是該液体之密度[2]

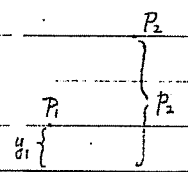
由 (4), (5) 式得

$$p(y) = p(y+dy) + \rho g\, dy \quad (6)$$

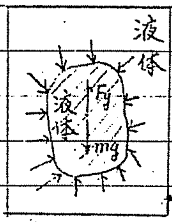
$$\frac{p(y+dy)-p(y)}{dy} = -\rho g \Rightarrow \frac{dp}{dy} = -\rho g \quad (7)$$

$$\int_1^2 dp = -\rho g \int_{y_1}^{y_2} dy \quad (8)$$ [3]

$$p_2 - p_1 = -\rho g y_2 + \rho g y_1 \Rightarrow p_2 + \rho g y_2 = p_1 + \rho g y_1 \quad (9)$$ [4]



阿基米得原理[5]：一物体浸於液体中受一向上之浮力，其大小等於它排出液体之重量。



考慮液体中一部分如圖所示。由於此液体是在平衡情況下，它所受週圍之力之和必需與該部分液体之体重相等，方向相反而且通過該部分液体的質心。若以同樣形狀的其他的物体來代替此部分之液体，則其所受之壓力與原來相同。因此，流体對該物体所施之力（浮力）與原來液体之重量相等，方向相反，而且通過原來那部分液体之質心。[6]

3. 討論

(1) $\Delta A \to 0$，因此這一塊液体之質量也趨近於零。壓力通常是位置的函數。

(2) 當物質是均勻時 密度之定義為 $\rho = \frac{M}{V}$。當物質為不均勻時，在一點 P 附近為取一小体積 ΔV，其中之質量為 ΔM。在 P 點密度之定義為 $\rho = \lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V} = \frac{dM}{dV}$。它通常也是位置之函數。

(3) 由 (7) 式到 (8) 式中我們假設 (i) ρ 不是 p 之函數 (ii) ρ, g 均是常數而將它們提出積分以外。應用到氣体時有時 ρ 是 p 之函數，若是液体並不均勻時 ρ 可以是位置之函數或者當深度很大 g 並非常數則第 (8) 式即不適用。

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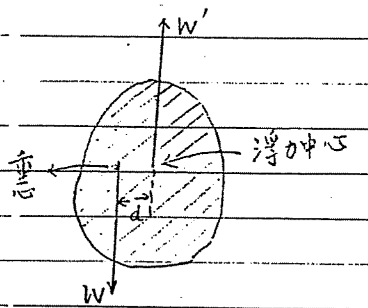
(4) 我們此處之壓力是絕對壓力。有時也用計示壓力 $P_g = P - P_a$ 此處 P_a 是大氣壓力。

(5) 此公式只在物体在液体中靜止時方成立。若物体在液体中有相對速度時，其淨向上之力与該物体上下，流体粒子速度之分佈有関，此時之力稱為白努利力或動升力。

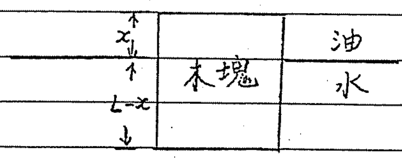
(6) 浮力不一定作用於物体之重心。當物体之質量之分佈不均匀常會發生如下之情形



因此對重心之力距為 $\tau = W'd$.

4. 應用

(1)



設 油之密度為 $0.6\ gm/cm^3$，水之密度為 $1\ gm/cm^3$，木塊之密度為 $0.9\ gm/c$

求該木塊浮於何處

該木塊所受之浮力為 $B = \rho_{water} A^2 (L-x) g + \rho_{oil} A^2 x g$ (10)

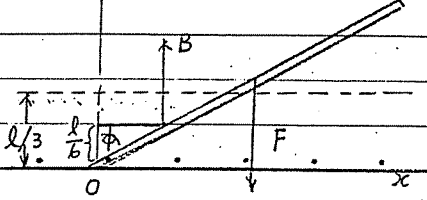
該木塊所受之重力為 $W = \rho_{wood} A^2 L$ (11)
（wood：木塊）

平衡條件 $B = W$ (12)

$$(\rho_{water} - \rho_{oil})\, x = (\rho_{water} - \rho_{wood}) L \quad (13)$$

將給的值代入上式得 $x = 0.25\, L$ (14)

(2)



一木桿之密度為 $0.45\ g/cm^3$ 長度為 l 以絞鏈固定於深度為 $\frac{l}{3}$ 之水池之底部。水之密度

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為 $1 gm/cm^3$。問該木桿與垂直方向之夾角為何時，它才能保持平衡？

浸在水中木桿之長度為 ℓ'。由圖中可看出

$$\cos\phi = \frac{\ell/3}{\ell'} \Rightarrow \ell' = \frac{\ell/3}{\cos\phi} \qquad (15)$$

該木桿所受之浮力大小為 $B = \rho_{water} A \ell' g \hat{j}$ （向上） (16)

（A：木桿之截面）

以絞鏈之位置為原點，則浮力作用於 $(x_B, y_B) = (\frac{1}{6}\ell\tan\phi, \frac{1}{6}\ell)$ (17)

該木桿所受之重力之大小為 $W = -\rho_{wood} A \ell g \hat{j}$（向下） (18)

作用於 $(x_F, y_F) = (\frac{\ell}{2}\sin\phi, \frac{\ell}{2})$

對絞鏈點，該木桿所受之力距 $\vec{\tau} = (x_B B - x_F W)\hat{k}$ (19)

因為木桿保持平衡，$\vec{\tau} = 0$ 也即是

$$x_B B = x_F W \qquad (20)$$

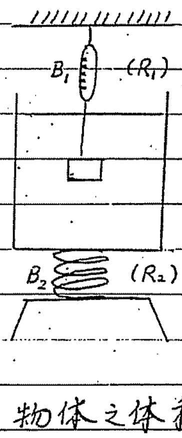
也即是 $\frac{1}{6}\ell\tan\phi\ \rho_{water} A \frac{\ell/3}{\cos\phi} g = \frac{\ell}{2}\sin\phi \cdot \rho_{wood} A \ell g$ (21)

$$\cos^2\phi = \frac{1}{9}\frac{\rho_{water}}{\rho_{wood}} \qquad (22)$$

代入題給之 $\rho_{water}, \rho_{wood}$ 得 $\cos^2\phi \simeq \frac{1}{4}$ 也即是 $\phi \simeq 60°$ (23)

絞鏈對木桿所施之力 $\vec{f} = (W - B)\hat{j}$ (24)

(3)



一物体掛在一彈簧秤 B_1 上。而該物体浸於裝滿液体之杯中而該杯置於另一彈簧秤 B_2。若杯子之重量為 $W_c = 3\,Kg \cdot g$，液体之重量為 $2Kg \cdot g$，在 B_1 上之讀數為 $R_1 = 10\ Kg\ g$，在 B_2 上之讀數 $R_2 = 20\ Kg\ g$。

若物体之体積 $V_B = 0.1\ m^3$，求該物体之密度

考慮物体為一系統

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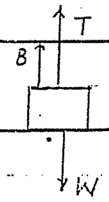
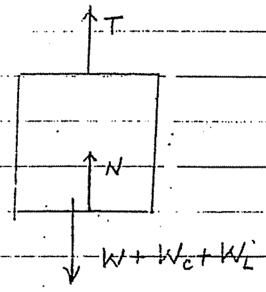


$$T + F_B - W = 0$$ T 即是 B_1 之讀數 (25)

以物塊加上杯子為考慮之系統



$$T + N - W - W_c - W_L = 0$$ N 即是 B_2 之讀數 (26)

由 (25) ; (26) 兩式得

$$N - W_c - W_L = F_B = \rho_{Liquid} V_B g \quad (27)$$

$$\rho_{Liquid} = 150 \ Kg/m^3 = 0.15 \ gm/cm^3 \quad (28)$$

$$W = 25 \ Kg \ g \quad (29)$$

$$\rho_{Wood} = \frac{25}{15} \cdot 0.15 \ gm/cm^3 = 0.25 \ gm/cm^3 \quad (30)$$

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第三節 流体動力學

1. 簡介 我們在此節中將討論白努利定理及其應用

2. 基本觀念

對流体運動的描述有兩種方法

(1) 拉格郎吉方法 將流体分成小塊，叫做"流体粒子"然後描述這些"粒子"的運動。這種方法与粒子力學相似。它仍是由在 t_0 時"流体粒子"的位置來求該"流体粒子"在時間 t 之位置。

(2) 奧尤勒方法[1] 他用流体在 x,y,z 點時間 t 時之密度 $\rho(x,y,z,t)$，速度 $\vec{v}(x,y,z,t)$ 來描述該流体。

流体運動的分類

(a) 若 $\vec{v}(x,y,z,t)=\vec{v}(x,y,z)$ 則稱為穩流[2]，否則則稱為不穩流。

(b) 將一自由浮動的小槳置於流体中，若它轉動，則稱為旋轉流[3]，否則稱為非旋轉流

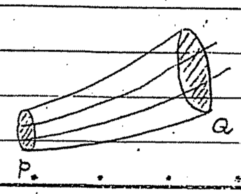
(c) 若流体能被壓縮則稱為可壓縮流体，否則稱為不可壓縮流。在不可壓縮流中密度 ρ 為常數[4]。

(d) 若流体中有黏度則稱為黏流，否則則稱為非黏流[5]。

我們將討論穩定，非旋轉，不可壓縮及非黏的流動。

由於 $\vec{v}$ 只是位置的函數，所有到達一定點 P 之質點其速度均為 $\vec{v}_P$。若將一個經過 P 點質點之途程繪出，則所有通過 P 點之質點也必沿此一曲線運動。此一曲線則稱為流線[6]

在穩流中我們選出一束流線構成一流管[7]如圖所示



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A_1，A_2 分別是此流管在 P 點及 Q 點垂直於流線之截面

令在 P 點之密度為 ρ_1，速度為 v_1，在 Q 點之密度為 ρ_2，速度為 v_2

在 dt 時間內由 P 點進入 PQ 管區之質量 $dm_1 = \rho_1 A_1 v_1 dt$。 (1)

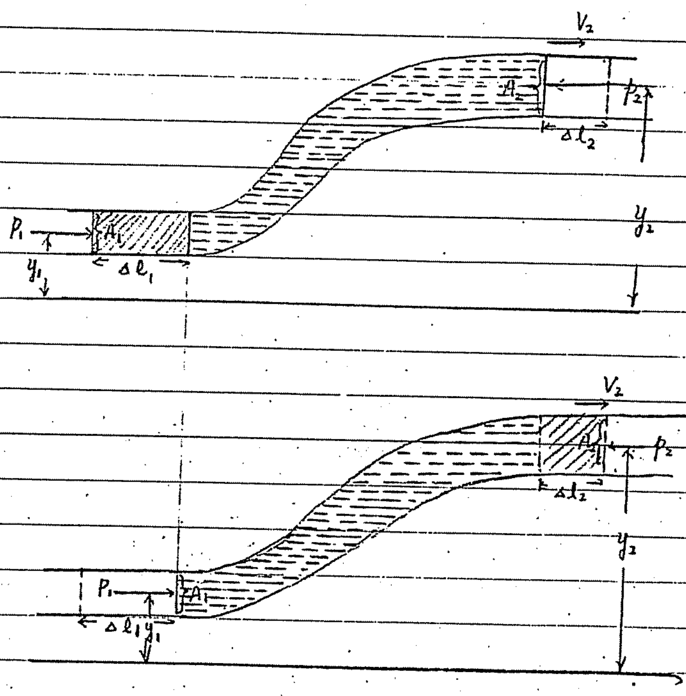
在 dt 時間內由 Q 點離開 PQ 管區之質量 $dm_2 = \rho_2 A_2 v_2 dt$。 (2)

由於流体不能由管壁流出而在 PQ 管區中質量既不能產生也不能消失，因此

$$dm_1 = dm_2 \quad (3)$$

也即是 $\rho_1 A_1 v_1 = \rho A_2 v_2$ [8] (4)

以上的公式稱為連續公式。[9,10] 若 $\rho_1 = \rho_2$，則 $A_1 v_1 = A_2 v_2$ (5)



我們將以斜線及虛線部分為系統。

首先我們討論外力對該系統所作之功

(1) P_1 對該系統所作之功為 $P_1 A_1 \Delta l_1 = P_1 \frac{\Delta m_1}{\rho} = P_1 \frac{\Delta m}{\rho}$，$\Delta m_1 = \Delta m_2$ (6)

P_2 對該系統所作之功為 $-P_2 A_1 \Delta l_2 = -P_2 \frac{\Delta m_2}{\rho} = -P_2 \frac{\Delta m}{\rho}$ (7)

(2) 重力對該系統所作之功為 $-\Delta mg(y_2 - y_1)$ (8)

外力對該系統所作之和為 $W = -\Delta mg(y_2 - y_1) + (P_1 - P_2)\frac{\Delta m}{\rho}$ (9)

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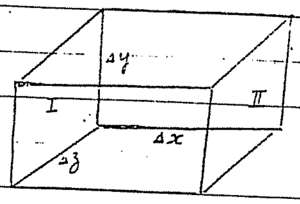
由圖可看出 V' 与 12345673 所構成之体積 V 相等。因此在時間 dt 内通過 $d\vec{S}$ 表面液体之質量為 $\rho|\vec{v}|\cos\theta\, dS = \rho\vec{v}\cdot d\vec{S}$



在時間 dt 内由第II面流出液体之質量是

$$\rho(x+\Delta x, y, z)\, v_x(x+\Delta x, y, z)\, \Delta t\, \Delta y \Delta z$$

由第I面流入之液体是

$$\rho(x, y, z)\, v_x(x, y, z)\, \Delta t\, \Delta y \Delta z$$

因此沿 X 軸在時間 Δt 中流出長方体液体的質量是

$$\frac{\rho(x+\Delta x, y, z)\, v_x(x+\Delta x, y, z) - \rho(x, y, z)\, v_x(x, y, z)}{\Delta x}\, \Delta x \Delta y\, dz\, \Delta t \qquad (15)$$

同理沿 y 軸在時間 Δt 中流出長方体液体的質量是

$$\frac{\rho(x, y+\Delta y, z)\, v_y(x, y+\Delta y, z) - \rho(x, y, z)\, v_y(x, y, z)}{\Delta y}\, \Delta x \Delta y \Delta z\, \Delta t \qquad (16)$$

沿 z 軸在時間 Δt 中流出長方体液体的質量是

$$\frac{\rho(x, y, z+\Delta z)\, v_z(x, y, z+\Delta z) - \rho(x, y, z)\, v_z(x, y, z)}{\Delta z}\, \Delta x \Delta y \Delta z\, \Delta t \qquad (17)$$

在時間 Δt 中 在長方体之質量之變化是 $(\rho(x, y, z, t+\Delta t) - \rho(x, y, z, t))\, \Delta x \Delta y \Delta z$ (18)

若在此長方体中沒有液体源及槽 則 $(15) + (16) + (17) = -(18)$ (19)

兩邊除以 Δt 後 取 $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$, $\Delta t \to 0$ 則得

$$\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = -\frac{\partial \rho}{\partial t} \qquad (20)$$

此公式即是連續方程式的微分形式。

若 ρ = 常數，則上式可寫成

$$\frac{\partial}{\partial x} v_x + \frac{\partial}{\partial y} v_y + \frac{\partial}{\partial z} v_z = 0 \qquad (21)$$

也即是 $\nabla\cdot\vec{V} = 0$. (22)

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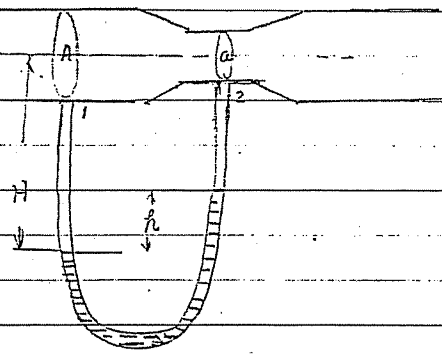
(10) 此處之連續公式仍是來自質量守恒。在電磁學中由電荷守恒可以導出類似之連續方程式。

(11) 沿同一流管時方能應用此一公式。

4 應用

(1) 汾丘里量度計 用來量液体之流速



由白努利公式得

$$p_1 + \frac{1}{2}\rho V_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$$

由於 $y_1 = y_2$

$$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

由連續公式得

$$\rho A v_1 = \rho a v_2$$

$$\therefore v_2 = \frac{A}{a} v_1$$

由 U 形管 (靜力學)

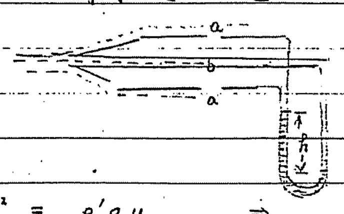
$$p_1 + \rho g H = p_2 + \rho g(H - h) + \rho' g h$$

$$p_1 - p_2 = (\rho' - \rho) g h$$

$$= \frac{1}{2}\rho\left(\left(\frac{A}{a}\right)^2 v_1^2 - v_1^2\right)$$

因此 $v_1 = a\sqrt{\dfrac{2(\rho'-\rho)gh}{\rho(A^2 - a^2)}}$

(2) 皮氏管 用來量氣体之流速



$$p_a + \frac{1}{2}\rho V^2 = p_b$$

$$p_a + \rho' g h = p_b$$

$\Rightarrow \frac{1}{2}\rho V^2 = \rho' g y \Rightarrow v = \sqrt{\dfrac{2gh\rho'}{\rho}}$ 即是氣体之速度

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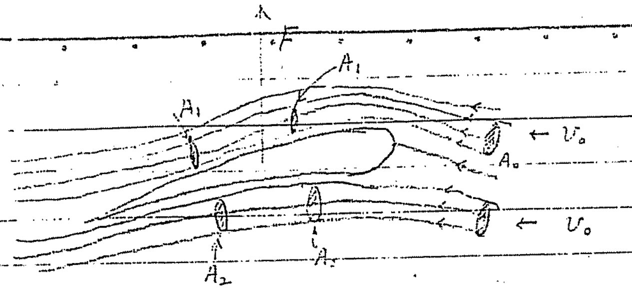
(3) 飛機的升力



$A_2 = A_0 \quad , \quad A_1 < A_0$

$p + \frac{1}{2}\rho v^2 = C$

$P_{below} = C - \frac{1}{2}\rho v_0^2 = P_0$

$P_{above} = C - \frac{1}{2}\rho v_1^2 = P_0 - \frac{1}{2}\rho(v_1^2 - v_0^2)$

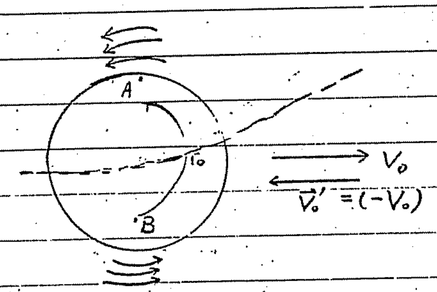
由連續方程式 $v_1 = \frac{A_0}{A_1} v_0$

$\Delta p = \frac{1}{2}\rho v_0^2 \left[\left(\frac{A_0}{A_1}\right)^2 - 1 \right]$

F (向上之升力) $= \frac{1}{2}\rho v_0^2 \left[\left(\frac{A_0}{A_1}\right)^2 - 1 \right] E = \frac{1}{2} k E v_0^2 \rho$

E：機翼之面積　k：$\left[\left(\frac{A_0}{A_1}\right)^2 - 1 \right]$

(4) 棒球投球之問題



由上面來看，由一投手所投出之球，其質量中心之初速為 V_0，同時使球以角速度 ω_0 旋轉。當它旋轉，該球使一層空氣隨着轉動如圖所示。為了看得更清楚一點我們可以在球的質心系統中討論此一問題。此時球除了轉動外是靜止的而空氣以 $V_0' = -V_0$ 之速度進。由於旋轉的關係在被球帶動的一層空氣在 A 点速度之大小為 $v_0 + r\omega_0$。在 B 点速度之大小為 $v_0 - r\omega_0$，也即是 $V_A > V_B$。由白努利定理得知 $P_A < P_B$，因此球向 A 方向偏斜如圖所示。因此球的軌跡是一曲線。投手投了一個曲球"。此種 $\vec{v}_0$，$\vec{\omega}_0$ 顯然的是比較易於由一沿 $\vec{v}_0$ 方向作用於 B 点之衝力所產生

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也就是說此球大約是由一右手投手所投出。

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FLUID MECHANICS

11

11.1 Introduction

We shall now attempt to apply the laws of mechanics to fluids. No change needs to be made in Newton's laws themselves, but there is the problem of understanding in precise terms how fluids behave and how we may most effectively apply Newton's laws to them.

First of all, what do we mean by the term *fluids?* Fluids are substances that can be made to *flow* by the proper application of forces. Generally speaking, fluids can be classified as *liquids* or *gases*. Liquids are practically incompressible and, therefore, can be regarded as having a fixed volume, even though their shape may change as, for example, when they are poured from one container to another. Gases are highly compressible (or, looking at it the other way round, highly expandable) and, therefore, have no characteristic volume; they simply expand to fill any container in which they may be placed.

Well, then, what makes a fluid flow, and under what conditions will it flow? The answer to this is that any fluid body will support *normal* forces that may act on its boundaries, without flowing, and can be in equilibrium under the action of a number of such normal forces. But a fluid cannot resist *tangential*, *shearing*, or *bending* forces; as soon as such forces are exerted on a fluid, it will flow in response to them. The situation, which is illustrated by Fig. 11.1, has a resemblance to that at a frictionless interface between an object and a plane which supports it; there can be normal forces at such an interface, but no tangential ones. The difference is, of course, that in the case of a rigid body on a frictionless surface, this condition applies only at the surface, while for a fluid it holds for *every point* within the substance.

A necessary condition for a fluid body to be in equilibrium, therefore, is that its boundaries experience *only normal forces*. Water in a pond or in a pan or other container, or a compressed gas within a cylinder are examples of such situations.

On the atomic scale, the difference between solids, liquids, and gases is attributable entirely to the interplay between the attractive forces that exist between individual atoms, ions, or molecules and to the random motion these particles acquire as a result of their thermal energy. In a *solid*, at very low temperatures, the thermal motion of atoms or molecules is very slight and excites only small vibrations in the rigid crystalline lattice of atoms, molecules, or ions. With increasing temperature, the thermal motions increase in energy until they are strong enough occasionally to tear apart the bonds between neighboring atoms or molecules; atoms or molecules are now still held together by long-range cohesive forces, but can move more or less freely past one another. The solid has now melted and has become a *liquid*. Finally, as the temperature is increased still further, the average thermal kinetic energy of each atom or molecule becomes greater than the average attractive potential energy that binds it to the other particles. The atoms or molecules now fly apart and are constrained only by the walls of whatever container they may be in. The substance has now vaporized and exists as a *gas*.

It is clear that the *same* substance may exist at different temperatures as solid, liquid, or gas and that the transition temperatures between these states depend primarily on the strength of the cohesive forces between individual atoms, molecules, or ions. We shall postpone the detailed description of thermal relations between solids, liquids, and gases to a later chapter, but the student will perhaps more readily appreciate on an intuitive level the difference between the properties of these *states of aggregation* in the light of this simple discussion.

11.2 Basic Properties of Fluids: Pascal's Principle; Pressure, Volume, and Density

In studying the mechanics of particles and rigid bodies, we found it most useful to work with the masses of individual objects, the forces acting on individual bodies, and the linear dimensions of such bodies. In the case of fluids, due to their special properties, it is generally more convenient to speak instead in terms of density, pressure, and volume.

We are already familiar with volume. We have also occasionally worked with the density ρ, defined as mass per unit volume. For an incompressible fluid, the density is constant throughout, but this is not generally true for a compressible fluid, though it may be approximately true under certain restricted conditions. The earth's atmosphere is an example of variable density in a compressible fluid; in this instance, the density decreases with altitude, in response to decreasing atmospheric pressure. Over a restricted range of altitudes, however, this variation can often be neglected.

The concept of *pressure* may best be understood by considering an infinitesimal element of area da somewhere in the fluid, as illustrated in Fig. 11.2. It may in some cases be helpful to envision a tiny piece of paper or plastic film of zero thickness and mass and area da to represent such a surface area element, but this is not absolutely necessary. In any case, there may be forces exerted by the fluid on such an area element. Due to the properties of fluids, these forces, at least at equilibrium, will be normal to the element, since fluids cannot sustain tangential forces without flowing. The pressure on such an element of area is defined as the magnitude of the force acting on it divided by the area of the element. This may be written as

$$\boxed{P = \frac{dF}{da}} \qquad (11.2.1)$$

or as an expression for the force, in the form

$$\boxed{dF = P\,da} \qquad (11.2.2)$$

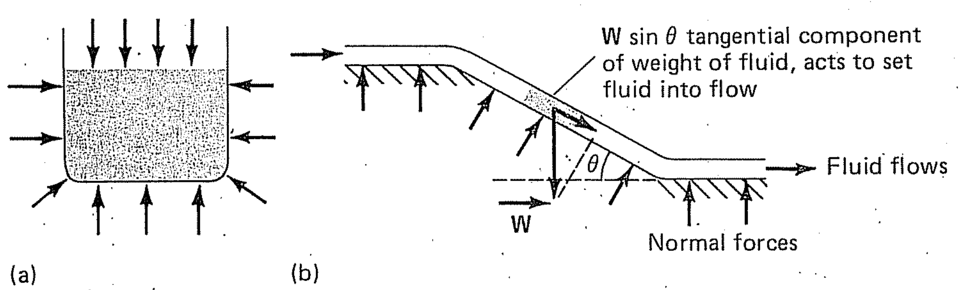


FIGURE 11.1. (a) Fluids in equilibrium experience only forces normal to their boundaries. (b) The action of tangential forces is to cause fluid flow.

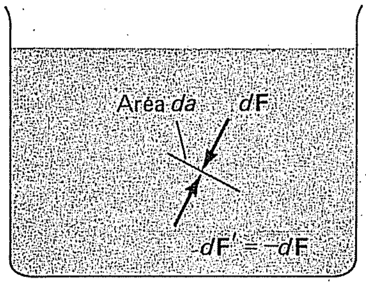


FIGURE 11.2. Equilibrium of forces within fluids, illustrating the definition of pressure as the ratio of force to area.

The total force on an area of finite size can be obtained by integrating this over the finite area:

$$F = \int_s P\,da \tag{11.2.3}$$

It is important to understand that pressure, as ordinarily defined, is a *scalar* rather than a vector; it is the ratio of the *magnitude* of a force to an area. In Fig. 11.2, it is clear that if the element da is to be in equilibrium, there can be no resultant force on it. The force $d\mathbf{F}$ acting on one side of it must, therefore, always be accompanied by an equal and opposite force $d\mathbf{F}'$ on the other side. In defining the pressure, of course, since only the magnitude of the force is used to define P, it makes no difierence whether the magnitude of $d\mathbf{F}$ or $d\mathbf{F}'$ is employed, since they are equal.

In Eq. (11.2.3), the pressure P over an area of finite size may vary from point to point. In general, therefore, the pressure P *cannot* be written outside the integral. This can be done *only if the pressure is constant over the entire surface in question.* This condition is often satisfied in cases of particular interest, and when it is, (11.2.3) becomes

$$F = P\int_s da = Pa \tag{11.2.4}$$

or

$$P = \frac{F}{a} \tag{11.2.5}$$

It is, however, only under the conditions set forth above that we may resort to the simple idea that "pressure is force divided by area" or "force is pressure times area."

In the metric system, density is expressed in units of g/cm^3 and kg/m^3, while pressure is expressed as dynes/cm^2 or newtons/m^2. It is easy to show that 1 g/cm^3 $\doteq$ 10^3 kg/m^3 and that 1 dyne/cm^2 = 0.1 newton/m^2. In the English system, the unit of pressure is the lb/ft^2 or, more commonly, lb/in.^{2}. Also in common use is the *bar*, equal to 10^6 dynes/cm^2, and the *standard atmosphere*, equal to 1.013×10^6 dynes/cm^2, or 14.7 lb/in.^{2}. One atmosphere is the average pressure exerted by the earth's atmosphere at sea level at a temperature of 0°C.

The basic properties of a fluid in equilibrium, discussed in the previous section from a rather different point of view, can be expressed concisely in a statement often referred to as *Pascal's principle*, since it was first set forth by the French mathematician and philosopher Blaise Pascal (1623–1662). Pascal's principle states:

Pressure applied to a fluid is transmitted undiminished to all parts of the fluid and to the walls of the container enclosing it.

For example, when a fluid is enclosed in a vessel fitted with a piston, as illustrated in Fig. 11.3, certain pressures P_A, P_B, and P_C will exist at points A, B, and C. If now a sudden change in pressure ΔP is caused by the application of a force $\mathbf{F}$ on the piston, Pascal's principle tells us that the pressures at A, B, and C immediately take on the values $P_A + \Delta P$, $P_B + \Delta P$, and $P_C + \Delta P$. At point C, the fluid is kept in equilibrium by a normal force exerted by the wall of the container. This normal force per unit area also experiences an immediate increase of magnitude ΔP.

Another consequence of Pascal's principle is exhibited by Fig. 11.4. In this illustration, four different containers are shown, each filled to the same level with the same fluid. The arrows represent the forces exerted per unit area (thus the pressures exerted) by the fluid on the container walls at various points. These forces arise ultimately from the weight forces acting on each volume element of the fluid which transmits force, hence pressure, to all parts of the fluid beneath and thence to the container walls. We can easily understand how this occurs by imagining, in Fig. 11.3, that the force $\mathbf{F}$ could equally well arise from the weight force associated with a layer of fluid on top of that shown instead of from an external force applied to a piston. In the slightly different situation

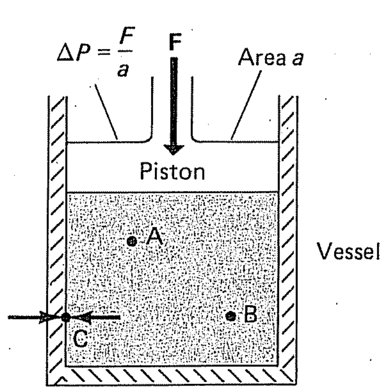


FIGURE 11.3. Pascal's principle. Forces acting on one part of a fluid are transmitted to the interior of the fluid and to its boundaries.

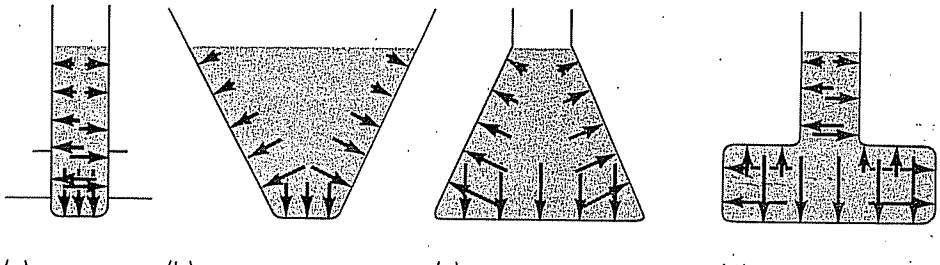


FIGURE 11.4. Weight forces exerted by fluids on containing vessels, as predicted by Pascal's principle.

shown in Fig. 11.4, it is apparent from Newton's third law that the forces exerted by the fluid on the container walls must be equal and opposite those exerted by the walls of the container on the fluid. If the fluid is in equilibrium, however, the latter forces must be everywhere normal to the walls of the vessel. Otherwise, they would have tangential components that would cause the fluid to flow. The forces the fluid exerts on the container, shown in Fig. 11.4, which are equal and opposite, then must likewise be normal to the container wall as shown in the drawing. Pascal's principle, along with Newton's laws, allows us in this way to understand how the pressure of higher layers of fluid are transmitted to those below, and also that these pressures generate *normal* forces on the container walls as illustrated in Fig. 11.4.

Though Pascal's principle applies in general only at equilibrium, it is frequently useful even in non-equilibrium situations. Though we shall not try to show why this is true in a rigorous mathematical way, the argument that follows will serve as a rough qualitative explanation. In the case of compressible fluids, forces are transmitted by longitudinal pressure waves in the fluid which propagate with the velocity of sound from the point of application of the force. Therefore, there is a time lag corresponding to the travel time of a longitudinal wave from the point of application of a force to the point where the pressure it ultimately creates is observed. Pascal's principle neglects this time lag and, therefore, can be applied rigorously only in situations of equilibrium, where the dynamic state of the fluid is independent of time. There are, nevertheless, many important cases where the effect of this time lag is negligible for practical purposes and where Pascal's principle can be used even in problems involving accelerated motion of compressible fluids to derive results which, though not exact, are very good approximations. For a perfectly incompressible fluid, the bulk modulus $(-V\,\Delta P/\Delta V)$ is infinite, since ΔV is identically zero. The wave velocity, equal to $\sqrt{B/\rho}$, becomes infinite in this limit, and hence, for such a substance there is no time lag and Pascal's principle is always valid.

11.3 The Variation of Fluid Pressure with Depth

Let us now try to calculate in detail how the pressure in a fluid in equilibrium changes as a function of depth. Consider the shaded element of fluid of thickness dy in Fig. 11.5. As usual in mechanics problems dealing with situations of equilibrium, we may proceed by isolating the fluid element, writing all the individual forces acting on the element, and setting their vector sum equal to zero. The forces **F**, **F**′, **A**, and **A**′ are forces exerted by the rest of the fluid against this element; they arise from the pressure in the fluid. The forces **A** and **A**′, which are equal in magnitude but opposite in direction, serve merely to maintain the system in equilibrium along the x-direction and are of no particular interest otherwise. There are three forces which have y-components, and since the system is in equilibrium these y-components must sum to zero. Therefore,

$$\sum F_y = F - F' + mg = 0 \tag{11.3.1}$$

Now let the pressure at depth y be P, and at depth $y + dy$ let it be $P + dP$. Then, since the pressure over the top surface of the shaded element is constant, as is the pressure over the bottom surface, $F = Pa$ and $F' = (P + dP)a$, where a is the area of the surface of the element. Also, since the mass of the element can be written as the density ρ times the volume $a\,dy$, Eq.

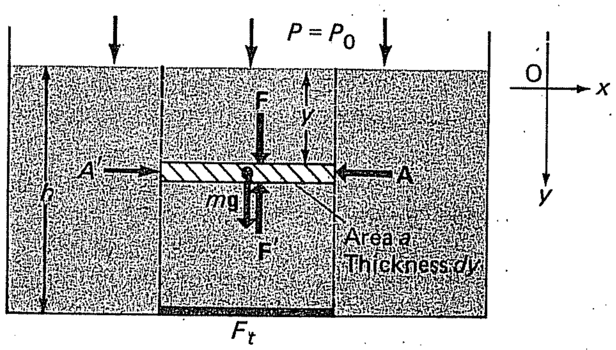


FIGURE 11.5. System of forces acting on a fluid element of infinitesimal thickness, in equilibrium.

(11.3.1) can be expressed as

$$Pa - (P + dP)a + \rho g a\, dy = 0 \tag{11.3.2}$$

or

$$\boxed{dP = \rho g\, dy} \tag{11.3.3}$$

This is the general expression relating a differential pressure variation dP to the depth change dy.

To understand what finite pressure change occurs over a finite distance y, we must integrate. If we are dealing with an *incompressible* fluid, ρ is *constant* and does not change as the pressure (or the depth) varies. Therefore, ρ may be written outside the integral. We may then integrate (11.3.3) from $y = 0$, at which point P has the value P_0 and expresses whatever atmospheric or other pressure exists at the surface of the fluid, to the depth y, at which point the pressure is P. Thus,

$$\int_{P_0}^{P} dP = \rho g \int_0^y dy \tag{11.3.4}$$

or

$$P - P_0 = \rho g y \tag{11.3.5}$$

$$\boxed{P = P_0 + \rho g y} \tag{11.3.6}$$

This simple result tells us that the pressure change $P - P_0$ in an incompressible fluid increases linearly with depth. It is an important finding and one that we shall make use of constantly. The quantity P is often referred to as the *absolute* pressure, and the quantity $P - P_0$ is sometimes called the *gauge pressure*, since it is this pressure that is measured by any instrument calibrated to read zero at atmospheric pressure.

In a *compressible* fluid, such as a gas, the density ρ is no longer constant but changes as the pressure (or the depth) varies. In this case, the specific form of the dependence of density upon pressure must be known before (11.3.3) can be integrated; now, needless to say, the quantity ρ may no longer be written outside the integral. Under these circumstances, the variation of pressure with depth may be quite complex. Though we may examine one or more such instances in detail at a later point, at the present we shall confine our pressure-versus-depth investigations to incompressible substances.

Let us now inquire into the total force F_t exerted on that part of the bottom of the container directly below the area a. This can most easily be found by multiplying both sides of (11.3.6) by a and setting y equal to the total depth of fluid h in the resulting equation:

$$Pa = P_0 a + (\rho a h) g \tag{11.3.7}$$

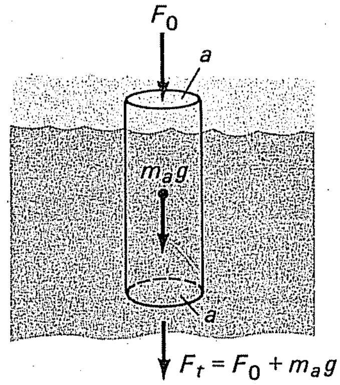


FIGURE 11.6. Forces acting on a column of fluid, in equilibrium.

Now, Pa is the force F_t on the bottom of the vessel, while $P_0 a$ is simply the force exerted by the atmosphere on the surface. The quantity ah is simply the volume of the fluid in a column having surface area a and height h (the boundaries of this column are the vertical dotted lines in Fig. 11.5). The quantity $\rho a h$ is, therefore, the mass of the fluid within that column. We may write, then,

$$\boxed{F_t = F_0 + m_a g} \tag{11.3.8}$$

where F_t is the force on the bottom, F_0 is the atmospheric force on the surface, and m_a is the mass of the fluid within the column. The force on any area of the bottom, then, is the atmospheric force on the corresponding area of the surface plus the *weight* of the fluid within the column standing over that area of the bottom. The situation is illustrated in Fig. 11.6. This result, by the way, computed only for incompressible fluids here, is true even for compressible substances. For example, the force exerted by the earth's atmosphere upon 1 cm^2 of the earth's surface at sea level is equal to the weight of atmosphere in a column of base area 1 cm^2 that extends upward as far as the atmosphere extends. Since a pressure of one standard atmosphere equals 1.013×10^6 dynes/cm^2, the weight of air in such a column is 1.013×10^6 dynes, corresponding to a mass of $(1.013 \times 10^6)/980$, or 1033 grams. This calculation, of course, neglects the variation of g over the upward extent of the atmosphere; but since most of the earth's atmosphere is within a few miles of the earth's surface, the error involved is quite small.

The discovery that gases such as air have appreciable weight and that the atmosphere exerts a definite and measureable pressure is usually attributed to the German scientist von Guericke, who in 1654 demonstrated that atmospheric pressure upon the two halves of an evacuated sphere a few feet in diameter gave rise to such strong forces that the two halves could not be separated by two teams of horses. Torricelli, an Italian scientist who in 1643 invented the barometer and first accurately measured the atmosphere's pressure, is also credited with this discovery. We shall investigate both these developments in the series of examples which follows. In these examples, we shall neglect the variation of the atmospheric pressure P_0 with height above the earth's surface. Near the earth's surface, the atmospheric pressure decreases by only 1 percent in an ascent of about 100 meters. Such a small variation can safely be ignored when the differences in height involved are on the scale of a few meters. But it should be understood that this variation must sometimes be accounted for in situations where height differences of hundreds of meters are encountered.

EXAMPLE 11.3.1

A swimming pool is rectangular in shape, having a length of 75 ft, a width of 40 ft, and a depth of 6 ft. Find (a) the gauge pressure at the bottom of the pool, (b) the total force on the bottom of the pool due to the water in it, and (c) the total force on one of the ends of the pool, whose dimensions are 40 ft by 6 ft. What is the absolute pressure on the bottom of the pool under normal sea level atmospheric conditions? The density of water is 62.4 lb/ft^3, or 1.940 slugs/ft^3.

The gauge pressure on the bottom follows directly from Eq. 11.3.5:

$$P_g = P - P_0 = \rho g y = (1.94)(32.2)(6)$$
$$= 375 \text{ lb/ft}^2, \text{ or } 2.604 \text{ lb/in.}^2$$

It is important to note that the quantity ρ is the *mass* per unit volume, while ρg represents the *weight* per unit volume. Since the pool is of constant depth, the pressure on the bottom is everywhere the same. Under these circumstances, we may use (11.2.4) to find

$$F = P_g a = (375)(75)(40) = 1{,}125{,}000 \text{ lb}$$

To find the force on one of the 40-ft by 6-ft pool ends we can no longer use (11.2.4), because the gauge pressure $P - P_0$ is not constant but varies from zero at the surface of the water to 375 lb/ft^2 at the bottom. We must now resort to (11.2.3) to express the gauge pressure as a function of depth and integrate over the area of the pool end. To see how this is done, let us refer to Fig. 11.7. In this diagram, the width of the pool is represented by w and the depth by h. The shaded element of area is of width w and height dy, which approaches zero in the limit of integration so that the pressure variation across it is, in the limit, zero. The force dF on the shaded element of area is given by

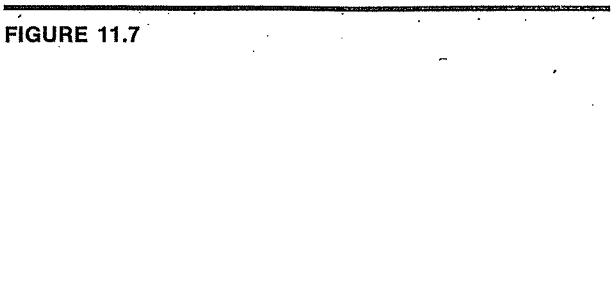


FIGURE 11.7

$$dF = P_g\, da = \rho g y w\, dy \tag{11.3.9}$$

This must be integrated from $y = 0$ to $y = h$, which gives

$$F = \int dF = \rho g w \int_0^h y\, dy \tag{11.3.10}$$

or

$$F = \rho g w\left[\tfrac{1}{2}y^2\right]_0^h = \tfrac{1}{2}\rho g w h^2 \tag{11.3.11}$$

Substituting the proper numerical values of the quantities in (11.3.11),

$$F = (\tfrac{1}{2})(1.94)(32.2)(40)(6)^2 = 45{,}000 \text{ lb}$$

The absolute pressure on the pool bottom is simply the sum of the gauge pressure and the atmospheric pressure P_0, which equals 14.7 lb/in.^{2}, or 2120 lb/ft^2. Then,

$$P = P_g + P_0 = 375 + 2120$$
$$= 2495 \text{ lb/ft}^2, \text{ or } 17.3 \text{ lb/in.}^2$$

Why is it that the bottom of the pool "feels" only the gauge pressure $P - P_0$ rather than the absolute pressure P? The atmospheric pressure P_0 is exerted not only *downward* on the pool bottom (transmitted there undiminished, according to Pascal), but also *upward* on the other side of the pool bottom, from below. These two pressures give rise to equal and opposite vertical forces on either side of the pool bottom, which add to zero. But there is water on only one side of the pool bottom, and the pressure it generates, which is $P - P_0$, or P_g, is *unbalanced*. Similar considerations apply to other systems on which atmospheric pressures act, including our own bodies. Since atmospheric pressure acts on us from within as well as without, we experience resultant pressure forces only when these two pressures become appreciably different, as in an ascending airplane or elevator. This point is illustrated also by the following example.

EXAMPLE 11.3.2

Two hollow hemispheres whose inside diameter is 2.0 ft can be fitted together and sealed, so that the air can be pumped out of the hollow sphere so formed with a vacuum pump, as illustrated in Fig. 11.8. How much force is required to separate the hemispheres when both the inside of the spherical enclosure and the outside surroundings are at atmospheric pressure (14.7 lb/in.^{2}, or 2120 lb/ft^2)? How much force is required to separate the hemispheres when all the air

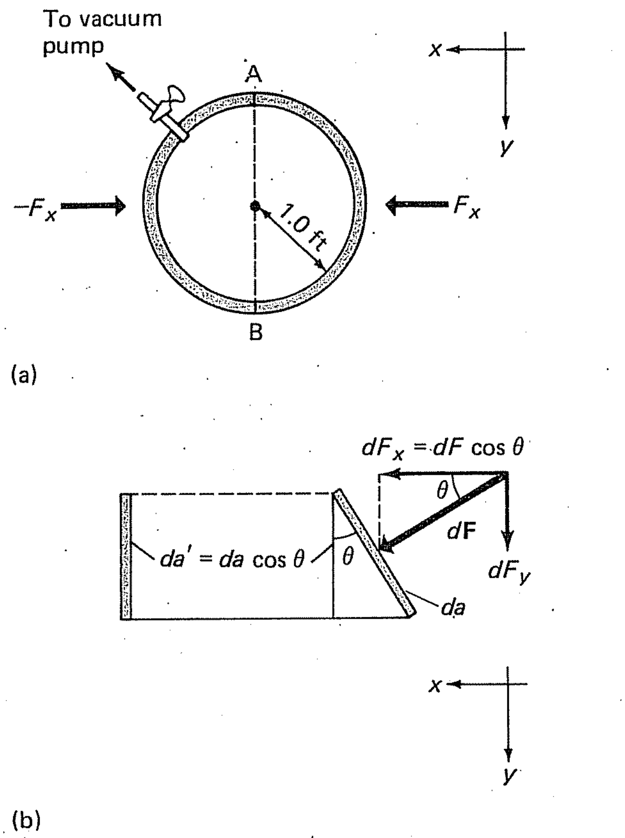


FIGURE 11.8. Magdeburg hemispheres, illustrating forces acting on an area element of the container wall.

is pumped out of the interior, reducing the absolute pressure there to zero, while the outside remains at atmospheric pressure?

A sectional view of the hemispheres is shown in Fig. 11.8a, and the forces acting on an area element da are illustrated in Fig. 11.8b. The external force on this element arising from atmospheric pressure, $d\mathbf{F}$, acts normally to the surface, as shown. If the inside of the sphere is at atmospheric pressure, there is an equal but oppositely directed force acting on the inside of the area element; the resultant of these two forces is zero. The situation is the same on all surface elements, so the net force is zero everywhere. Under these circumstances, no force is needed to separate the hemispheres.

Now suppose the inside of the sphere to be evacuated; the only forces then acting are those shown in Fig. 11.8b. It is only the x-component of the force $d\mathbf{F}$ which is effective in forcing the two halves of the sphere together. The x-component of $d\mathbf{F}$ can be written as

$$dF_x = dF \cos\theta = \frac{dF}{da}(da \cos\theta) = P(da \cos\theta) \tag{11.3.12}$$

since dF/da is the atmospheric pressure P. This can, in turn, be stated in the form

$$dF_x = P\, da' \tag{11.3.13}$$

where da' is the area da *projected* onto the plane AB of Fig. 11.8a. This can be integrated over the right-hand hemisphere to find the total force F_x which holds it against the left-hand one; the force components dF_y, of course, sum to zero over the hemisphere in such an integration. Since the pressure of the atmosphere is essentially constant over the surface, the pressure P may be written outside the integral during the integration. Integrating (11.3.13), then,

$$F_x = P \int_s da' = Pa' \tag{11.3.14}$$

where a' is the area of the hemisphere projected on the plane AB. Since this is simply the area of a circle having the same radius as the sphere, or πr^2, F_x can be written as

$$F_x = \pi r^2 P = (3.1416)(1)^2(2120) = 6660 \text{ lb}$$

The same result can be arrived at more simply by noting that each hemisphere can be considered as supporting a circular column of atmosphere of area $a' = \pi r^2$ which exerts a pressure of 2120 lb/ft^2 on its base. We preferred, however, to do the calculation in a more complex but rigorous fashion to show precisely how the atmospheric forces act and why the projected area a' rather than the total area a must be used.

This example illustrates Otto von Guericke's experiment of the so-called *Magdeburg hemispheres* which demonstrated in 1654 the forces exerted by the earth's atmosphere and which we referred to earlier.

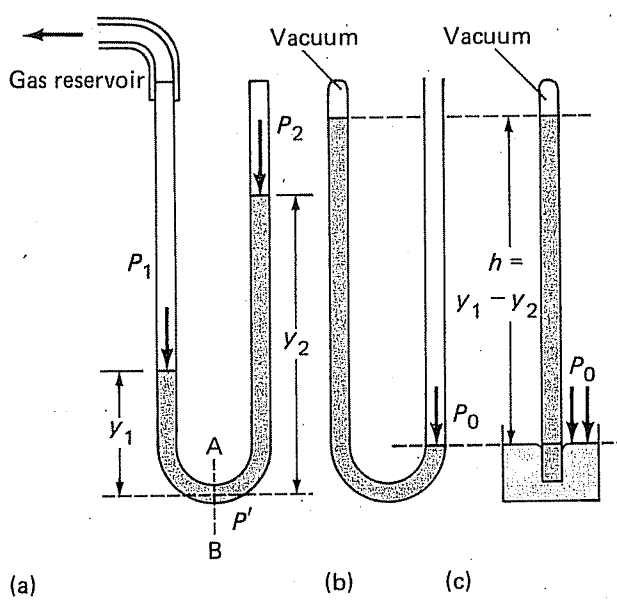


FIGURE 11.9. (a) Open U-tube manometer. (b) Closed U-tube manometer. (c) Torricelli's barometer.

Von Guericke was, at the time, mayor of the city of Magdeburg. Can you imagine, nowadays, the mayor of Pittsburgh or Baltimore concerning himself with such doings?

EXAMPLE 11.3.3

Consider the open U-tube *manometer* shown in Fig. 11.9. A gas at pressure P_1 is connected to the left side, while the right side is at another pressure P_2 (usually atmospheric, in which case $P_2 = P_0$, but not necessarily so). Find the relation between the liquid heights y_2 and y_1 and the pressure difference at equilibrium. What happens when the left side is evacuated? Assume the fluid in the U-tube to be incompressible.

Consider the plane AB which intersects the tube at its lowest point. Because the system is in equilibrium, the pressure on the left side of the plane from the column of fluid in the left side of the tube must be the same as that on the right side due to the fluid in that column. If it were not so, the fluid would move. Call this common pressure at the lowest point P'. Then, according to (11.3.5), for the column of fluid on the left,

$$P' - P_1 = \rho g y_1 \qquad (11.3.15)$$

while for the column on the right

$$P' - P_2 = \rho g y_2 \qquad (11.3.16)$$

If we now subtract (11.3.15) from (11.3.16), we will eliminate P', obtaining

$$\boxed{P_1 - P_2 = \rho g(y_2 - y_1)} \qquad (11.3.17)$$

which is the desired result. If the pressure on the surface of the liquid on the right is atmospheric pressure, then $P_2 = P_0$ and

$$P_1 - P_0 = \rho g(y_2 - y_1) \qquad (11.3.18)$$

If, now, the left side of the tube is evacuated, P_1 becomes zero, and (11.3.18) becomes

$$\boxed{P_0 = \rho g(y_1 - y_2) = \rho g h} \qquad (11.3.19)$$

where h is the difference between the levels y_1 and y_2. This gives a way of *measuring* atmospheric pressure in terms of the height of a fluid column, and the device so arranged is called a *liquid barometer*, as shown in Fig. 11.9b. In this device, which was first invented by Evangelista Torricelli in 1648, the pressure of the earth's atmosphere in the open tube or reservoir is balanced by that due to the column of liquid in the closed tube. In practice, instead of evacuating and sealing off one side of a U-tube, as shown in Fig. 11.9b, it is usually more convenient to entirely fill a barometer tube of sufficient length which is already sealed at one end, invert it, and release the open end while keeping it immersed in a reservoir of the fluid. This is the way in which the familiar *mercury barometer* shown in Fig. 11.9c is usually arranged.

There are many liquids which could be used to construct a barometer. One important requirement is that the *vapor pressure* of any such liquid be very small, so as to allow as good a vacuum as possible in the portion of the tube above the fluid. Mercury is particularly suitable in this respect, since its vapor pressure at room temperature is quite negligible in comparison to atmospheric pressure. Also, mercury is very dense and allows a column height which is quite convenient in the laboratory. A mercury barometer such as the one shown in Fig. 11.9c exhibits a column height of 76.0 cm at sea level under average conditions of atmospheric pressure. Since the density of mercury is 13.6 g/cm^3, Eq. (11.3.19) allows us to establish that the average atmospheric pressure should be

$$P_0 = \rho g h = (13.6)(980)(76) = 1.013 \times 10^6 \text{ dynes/cm}^2$$

Fluctuations in atmospheric pressure arising from meteorologic conditions are easily measured by noting the height of the mercury column in a barometer of this sort. If the barometric fluid were water instead of mercury, the column height would be 13.6 times greater because water is 13.6 times less dense than mercury. A water barometer would, therefore, have a column of height 10.34 meters under average atmospheric pressure. Water is not a very good barometric fluid, because, in addition to the inconvenient column height, its vapor pressure is too high.

The widespread use of the mercury barometer and manometer has led to the use of a unit of pressure related to the height of the mercury column in such a device. One may define a unit of pressure as the pressure exerted by a column of mercury 1 millimeter in height. This unit is called the *torr* (after Torricelli), or the *millimeter of mercury*. It is easy to see that 1 torr (1 mm Hg) = (13.6)(980)(0.1) = 1333 dynes/cm^2. The torr is not a very good unit in some respects, but its use has become widespread, particularly in the measurement of low pressures in vacuum systems and in measuring the barometric pressure of the atmosphere. It is, therefore, in line with common terminology to refer to the pressure of the atmosphere under standard conditions as 760 torr, or 760 mm Hg, or to refer, for example, to the tiny residual pressure inside an electronic tube as 10^{-6} torr, or 10^{-6} mm Hg. One even occasionally sees pressures expressed in *inches of mercury*, but this practice, fortunately, is becoming less common that it once was, despite the fact that there are millions of inexpensive household barometers calibrated this way.

EXAMPLE 11.3.4

A rowboat has a flat bottom whose area is 60 ft^2. The sides of the boat are perpendicular to the plane of the

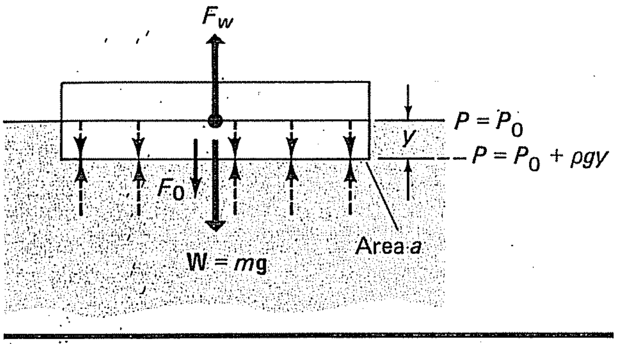


FIGURE 11.10. System of forces acting in equilibrium on a boat.

bottom and have a uniform height of 1.5 ft. The boat weighs 480 pounds. Find (a) the height of the water line of the unloaded boat above the bottom, (b) the distance the boat sinks when four passengers whose total weight is 640 pounds step aboard, and (c) the maximum weight the boat can carry before sinking. The loads are assumed to be uniformly distributed in all cases.

This example illustrates how a boat works, a subject not well understood even by many persons who have spent most of their lives afloat. Consider first the unloaded boat floating at equilibrium in the water, as shown in cross section in Fig. 11.10. The atmospheric pressure at the surface of the water is P_0; this same pressure is exerted by the atmosphere downward on the top surface of the boat's bottom. There is an upward force on the lower surface of the bottom of the boat, the surface that is in contact with the water, at a distance y beneath the surface. Since the pressure at this depth is greater than P_0, in view of the law of pressure versus depth expressed by Eq. (11.3.6), the upward force exerted by the water on the boat's bottom (which we shall learn to refer to in the next section as the *total buoyant force*) is greater than the downward force caused by atmospheric pressure acting inside. But there is also the boat's weight, which acts downward, too, and which, of course, in equilibrium, makes up the difference. If we call the downward force due to atmospheric pressure F_0, the total upward force of buoyancy due to water pressure on the boat's bottom F_w, and the weight force W, Newton's first law tells us that in equilibrium,

$$\sum F_y = F_w - F_0 - W = 0 \qquad (11.3.20)$$

Since the pressures involved in generating F_w and F_0 are both uniform over the boat's bottom, we may write

$$F_0 = P_0 a \qquad (11.3.21)$$

$$F_w = Pa = (P_0 + \rho g y)a \qquad (11.3.22)$$

where a is the area of the bottom of the boat and y is its depth beneath the surface. Inserting these values into (11.3.20),

$$\rho g y a = W$$

or

$$y = \frac{W}{\rho g a} \qquad (11.3.23)$$

Inserting the values of the quantities on the right-hand side given in the initial statement of the problem, we find, using $\rho g = 62.4$ lb/ft^3, that

$$y = \frac{480}{(62.4)(60)} = 0.128 \text{ ft}$$

When the passengers step aboard, of course, the total weight becomes 640 pounds greater; otherwise the argument is the same. Now,

$$y' = \frac{480 + 640}{(62.4)(60)} = 0.299 \text{ ft}$$

The boat, then, has sunk a distance $y' - y = 0.171$ ft. The question of how much of a load the boat will carry before it sinks can be settled by setting y equal to 1.5 ft in (11.3.23) and solving for W. The total weight is then found to be

$$W = \rho g a y = (62.4)(60)(1.5) = 5616 \text{ lb}$$

This represents the weight not only of the cargo but also of the boat itself. The weight of cargo which can be carried is 5616 − 480, or 5136 lb.

The important thing to realize in this example is that the buoyant force, which supports the boat and its cargo, arises because the upward pressure of the water on the bottom of the boat *is greater than the downward pressure of the atmosphere inside.* This is so because as one descends beneath the surface of an incompressible liquid, the pressure rises steadily in view of the pressure-versus-depth law (11.3.6); this law, in turn, you will recall, has its origins in Pascal's principle and Newton's first law. If there were no increase in pressure with depth, there would be no buoyant force, and no boat would float. This example illustrates clearly the origin of buoyant forces. In the next section, we shall examine in detail the action of such forces on objects immersed or floating in fluids.

11.4 Buoyant Forces and Archimedes' Principle

Anyone who has ever swum or owned a boat has experienced very directly the buoyant forces exerted by fluids on objects which are immersed in them or floating on them. In Example 11.3.4, we examined how these forces arise in a specific example involving a rowboat. It is now time to look at the situation in a more general way and try to arrive at some idea of how

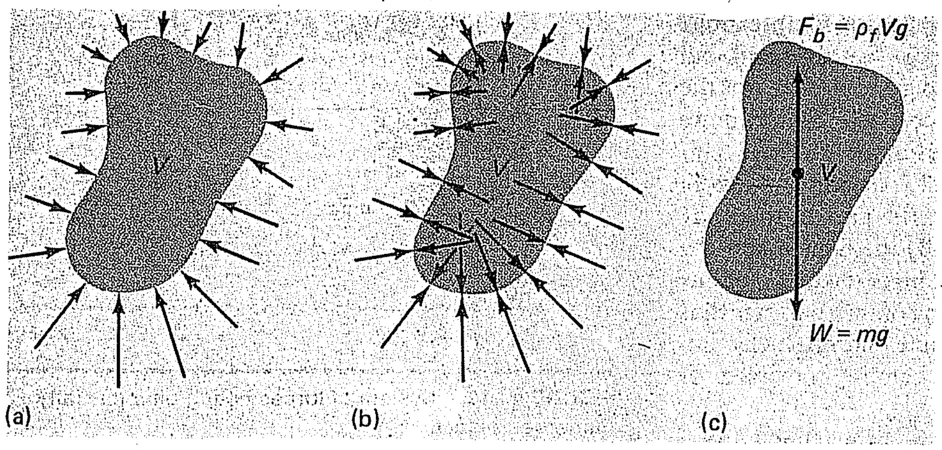


FIGURE 11.11. System of forces acting on an irregular, immersed object. (a) Forces exerted by the fluid on the object. (b) Forces exerted in equilibrium on a volume of fluid of the same size and shape as the immersed object, and forces exerted by this volume of fluid on its surroundings. (c) Equivalent system of forces acting on immersed body, with the forces shown at (a) replaced by the equivalent buoyant force $\mathbf{F}_b$.

to express buoyant forces acting on any object in a fluid.

Consider first a body of irregular shape which is totally immersed in a fluid, as shown in Fig. 11.11a. It is important to realize from the outset that such an object is *not necessarily in equilibrium*, for the net buoyant force exerted by the liquid may be greater than, less than, or equal to the body's mass. In the first instance, the body will sink; in the second, it will rise to the surface; and only in the final case will it remain in equilibrium. In any event, when immersed, any body will experience forces due to the pressures acting on it; a diagram showing the general appearance of these forces is shown in Fig. 11.11a. Since fluids can exert only forces normal to the boundaries, the forces must be perpendicular to the surface of the object at all points; and since the pressure within the fluid increases with increasing depth, the magnitude of the forces increases with depth. The fact that the upward forces acting on the bottom of the object are greater than the downward forces acting on the top is what causes the net buoyant force in the first place. The force vectors in Fig. 11.11a will be seen to display both these characteristics.

Now suppose that the object is removed and the volume it formerly occupied is filled with fluid identical to that surrounding the volume, as shown in Fig. 11.11b. The system of forces acting upon the dotted volume of fluid formerly occupied by the immersed body is no different from what it was when the body was there; the surrounding fluid does not know (nor care!) whether its pressure forces are exerted upon an immersed object or upon a volume of fluid put there in its place. But there is one difference; when the body is replaced with an identical chunk of fluid, the system now *has to be in equilibrium.* If it were not, then there would be a flow of fluid, which clearly cannot take place under the conditions of Fig. 11.11b. Since the volume of fluid occupying the dotted volume previously filled by the body is in equilibrium, the vector sum of all forces acting on it must be zero. The forces acting on the fluid in this region are, first, the resultant of all the pressure forces exerted on it by the surrounding fluid (which, *in toto*, represents the net buoyant force itself) and, second, the weight of the fluid in the dotted volume. Since the sum of these two forces is zero, the net buoyant force must be equal in magnitude, but opposite in direction, to the weight force associated with the fluid in this volume.

Now let us put the immersed object back into the fluid in its former location. The net buoyant force, which is, after all, due to the surroundings, is again no different from what it always was. But now, we know that under the conditions of Fig. 11.11b it had to be an upward force equal to the weight of fluid occupying the same volume as the object itself. Therefore, that is what it is when the object itself is in place, as in Fig. 11.11a or 11.11c. The basic elements of the situation are shown in Fig. 11.11c, in which both the object's weight force and the resultant buoyant force are illustrated.

The result of all this can be stated in the following way:

A body immersed in a fluid experiences a resultant upward buoyant force equal in magnitude to the weight of displaced fluid.

This statement is called *Archimedes' principle*, and expresses essentially all there is to know about the buoyant force. It is important to note that in deriving Archimedes' principle, it was not necessary to assume that the fluid in question was incompressible. Archimedes' principle is, therefore, very general and applies not only to liquids but to gases as well. It is the buoyant force of the earth's atmosphere which allows a balloon or blimp to rise, for example.

In the case of an object only partially immersed, buoyant forces arise only from the part of the body below the surface of the liquid. By the same token, the volume of fluid displaced is no longer the total volume of the object but only the volume of that part which is immersed. Archimedes' principle, nevertheless, applies equally well to bodies totally immersed or partially immersed.

There are three ways for a body in a fluid to be in equilibrium. First, the density of the body may be precisely equal to that of the fluid. The weight of displaced fluid is then exactly equal to the body's own weight, and the buoyant force is precisely equal and opposite the weight force. The object will then remain in equilibrium while totally immersed, under the action of weight force and buoyant force alone; a submerged submarine is a good example.

Secondly, a body less dense than the fluid in which it is placed will come to equilibrium floating on the surface only partially immersed. Suppose, for example, the body shown in Fig. 11.11c were less dense than the surrounding fluid and it was released from rest while totally immersed. The upward buoyant force would be equal to the weight of displaced fluid, in this case $\rho_f V_g$, while the weight force would be $mg = \rho V_g$, where ρ is the density of the object and ρ_f is the density of the fluid. Since in this case ρ is less than ρ_f, the buoyant force will exceed the weight force and the body will undergo acceleration upward, eventually arriving at the surface. When it arrives there, part of its volume will rise *above* the surface, leaving the object only *partially* immersed. As this happens, the buoyant force will diminish, because the volume of fluid displaced by the body is no longer equal to the total volume of the object, but only to the volume of that part beneath the level of the fluid surface. The object will continue to rise out of the fluid until the weight of the volume of displaced fluid (which is now less than the total volume of the body) is equal to the body's own weight. At this point the weight force and the buoyant force will again be equal and opposite, and equilibrium will now be attained with the object floating on the surface only partially immersed, as shown in Fig. 11.12. In this condition, the buoyant force will be g times the mass of displaced fluid, hence $\rho_f V_{imm} g$, where V_{imm} is the volume of the part of the body beneath the level of the surface. Since the body is in equilibrium under the action of buoyant and weight forces alone,

$$F_b - W = \rho_f V_{imm} g - mg = 0 \tag{11.4.1}$$

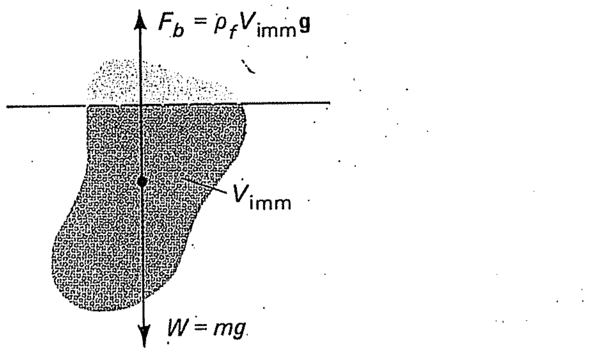


FIGURE 11.12. System of forces acting on a partially immersed object.

$$V_{imm} = \frac{m}{\rho_f} \tag{11.4.2}$$

Finally, there is the possibility that *external* forces may be introduced to hold an object in equilibrium either partially or wholly immersed. The magnitude and direction of any such external force can always be determined by the condition of force equilibrium in conjunction with Archimedes' principle. These three possibilities will be discussed in detail in the series of examples which follow. Before beginning this series, the reader may find it instructive to return to Example 11.3.4 and rework it using Archimedes' principle.

EXAMPLE 11.4.1

Work out the results of Example 11.3.4 using Archimedes' principle.

This is easily enough accomplished, but first, referring to Fig. 11.10, it is important to realize that the total upward force F_w exerted by the water on the bottom of the boat is a *gross* buoyant force, attributable to the total pressure $P_0 + \rho g y$ instead of the pressure difference $\rho g y$, rather than the *resultant* buoyant force F_b referred to in Archimedes' principle. The force F_w is opposed by the downward force F_0 arising from atmospheric pressure, and it is the *difference* between these two quantities which represents the resultant buoyant force F_b of Archimedes' principle and which is to be equated to the weight of displaced fluid. The situation will be clearly illustrated by Figs. 11.10 and 11.13. From these drawings, it is evident that since the boat is in equilibrium,

$$\sum F_y = F_b - W = 0 \tag{11.4.3}$$

This is the same as Eq. (11.3.20), except that $F_w - F_0$ is identified as the net buoyant force F_b. But now, according to Archimedes' principle, the net buoyant force equals the weight of water displaced, whence

$$F_b = \text{weight of displaced water} = \rho V_g = \rho a y g \tag{11.4.4}$$

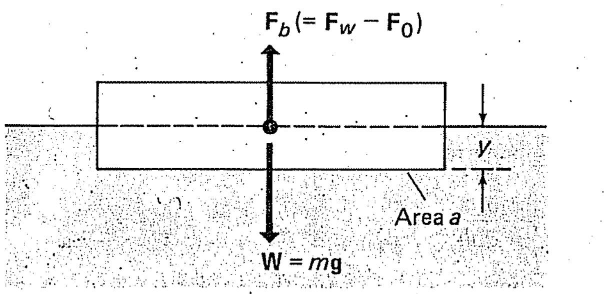


FIGURE 11.13

Therefore,

$$W = \rho a y g \qquad y = \frac{W}{\rho a g}$$

which is the same as Eq. (11.3.23). The numerical results asked for in the exercise all follow directly from this equation, just as before.

EXAMPLE 11.4.2

An engineer is assigned the task of designing a spherical balloon which will have a gross lifting capacity of 4900 newtons. This corresponds to a mass of 500 kg, which may include the mass of the balloon itself. The balloon is to be filled with hydrogen whose density is $\rho_{H_2} = 0.090$ kg/m³, while the density of air is $\rho_{air} =$ 1.293 kg/m³. What is the minimum radius such a balloon can have to lift this total load? If the balloon itself is made of flexible plastic of density $\rho = 900$ kg/m³ and thickness 0.100 cm, what will be the net lifting capacity or maximum payload? What will the payload be if the balloon is filled with helium of density $\rho_{He} = 0.180$ kg/m³?

Let us first neglect the mass of the balloon and deal with the lifting capacity of the gas alone. As shown in Fig. 11.14, for the gas to lift a mass of 500 kg, corresponding to a weight force of (500)(9.8) newtons, the buoyant force must *exceed* the weight of the gas itself by that amount; in other words,

$$F_b - m_{H_2} g = F_l = 4900 \text{ N} \qquad (11.4.5)$$

where F_l is the lifting force of the gas and m_{H_2} is the mass of the hydrogen in the balloon, which is equal, in turn, to $\rho_{H_2} V$. The buoyant force is equal to the weight of the displaced air, which is $\rho_{air} Vg$. Substituting these values into (11.4.5), we find

$$Vg(\rho_{air} - \rho_{H_2}) = F_l \qquad (11.4.6)$$

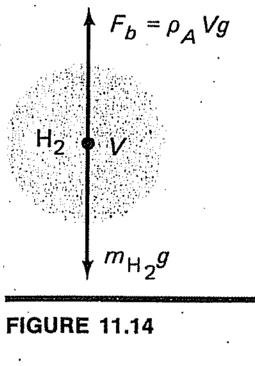


FIGURE 11.14

Since the volume is $\frac{4}{3}\pi r^3$, where r is the radius, this may be written as

$$\tfrac{4}{3}\pi r^3 g(\rho_{air} - \rho_H) = F_l$$

from which

$$r = \left(\frac{F_l}{\frac{4}{3}\pi g(\rho_{air} - \rho_H)}\right)^{1/3} = \left(\frac{(4900)}{(\frac{4}{3})(3.1416)(9.8)(1.293 - 0.090)}\right)^{1/3} = 4.63 \text{ m} \qquad (11.4.7)$$

If the skin of the balloon is made of plastic 0.1 cm ($=10^{-3}$ m) thick and of density $\rho = 900$ kg/m³, its mass m_b will be given by

$$m_b = \rho V = \rho a d \qquad (11.4.8)$$

where a is the surface area and d the thickness. Since $a = 4\pi r^2$ for a sphere,

$$m_b = 4\pi r^2 \rho d = (4)(3.1416)(4.63)^2(900)(10^{-3}) = 242 \text{ kg} \qquad (11.4.9)$$

The net lifting capacity, or payload, would be 500 − 242 = 258 kg.

If the balloon were to be filled with helium, the buoyant force would still be given by

$$F_b = \rho_{air} Vg \qquad (11.4.10)$$

but now the weight of lifting gas would be

$$W_{He} = \rho_{He} Vg \qquad (11.4.11)$$

The available lifting force would now be

$$\begin{aligned} F_b - W_{He} &= Vg(\rho_{air} - \rho_{He}) = \tfrac{4}{3}\pi r^3 g(\rho_{air} - \rho_{He}) \\ &= (\tfrac{4}{3})(3.1416)(4.63)^3(9.8)(1.293 - 0.180) \\ &= 4535 \text{ N} \end{aligned} \qquad (11.4.12)$$

Such a force could lift a mass of (4535/9.8) = 463 kg. Since the plastic skin has a mass of 242 kg, the net mass which could be lifted under these circumstances is 463 − 242 = 221 kg.

EXAMPLE 11.4.3

A vessel contains a layer of water, $\rho_2 = 1.00$ g/cm³, upon which is floating a layer of oil, $\rho_1 = 0.800$ g/cm³. A cylindrical object of unknown density ρ whose base area is a and whose height is h is dropped into the vessel, finally coming to equilibrium floating on the oil–water interface, immersed in the water to a depth $\frac{2}{3}h$, as shown in Fig. 11.15. What is the density of the object?

In this case, the object is partly immersed in the

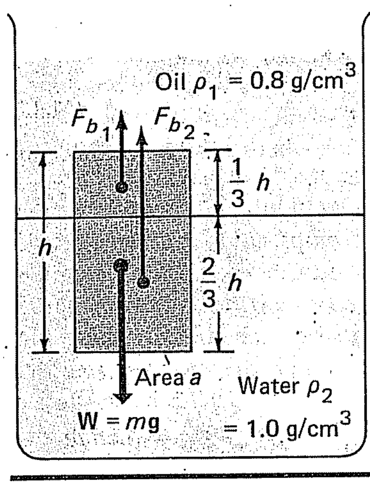


FIGURE 11.15

water and partly in the oil. There are two separate buoyant forces, one from the part of the body in the water, the other from the part in the oil, as shown in the diagram. Since the system is in equilibrium,

$$\sum F_y = F_{b1} + F_{b2} - mg = 0 \tag{11.4.13}$$

The buoyant force F_{b1} due to the oil is equal to the weight of the displaced oil, namely, the weight density $\rho_1 g$ times the volume of the object immersed in the oil, which is one third the total volume. The force F_{b2} may be calculated in the same way:

$$F_{b1} = \tfrac{1}{3}\rho_1 gV = \tfrac{1}{3}\rho_1 gah \tag{11.4.14}$$

$$F_{b2} = \tfrac{2}{3}\rho_2 gV = \tfrac{2}{3}\rho_2 gah \tag{11.4.15}$$

Substituting these values into Eq. (11.4.13) and expressing the total mass m of the object as $\rho V\,(=\rho ah)$, we find

$$\tfrac{1}{3}\rho_1 gah + \tfrac{2}{3}\rho_2 gah - \rho ahg = 0 \tag{11.4.16}$$

Canceling gah throughout and solving for ρ, we obtain

$$\rho = \tfrac{1}{3}\rho_1 + \tfrac{2}{3}\rho_2 = (\tfrac{1}{3})(0.800) + (\tfrac{2}{3})(1.00) = 0.933 \text{ g/cm}^3 \tag{11.4.17}$$

Somewhat related to this example is the question of why we always neglect the buoyant force of the atmosphere in any problem involving an object floating partly immersed in a liquid and partly in the air, as we did in Example 11.4.1 and the explanatory material leading to Eq. (11.4.2). The answer lies in the fact that the atmosphere's density is so small that the buoyant force arising from it is usually quite negligible in comparison with that of the liquid. Suppose, for example, in this exercise, instead of oil ($\rho_1 = 0.800$ g/cm^3) we were dealing with an upper layer of air ($\rho_1 = 0.001293$ g/cm^3) and that the other conditions were the same, including the fact that the floating object was two thirds immersed in the water. (This would mean that its density is no longer 0.933; can you figure out what it would be?) The buoyant forces due to the two fluids are still given by (11.4.14) and (11.4.15), and the *ratio* of the buoyant forces arising from the two fluids is

$$\frac{F_{b1}}{F_{b2}} = \frac{\frac{1}{3}\rho_1 gah}{\frac{2}{3}\rho_2 gah} = \frac{1}{2}\frac{\rho_1}{\rho_2} \tag{11.4.18}$$

Now when ρ_1 and ρ_2 have the values 0.8 and 1.0, as in the original statement of the example, this ratio amounts to 0.4, which is quite large. But if $\rho_1 = 0.001293$, as for air, the ratio of the buoyant force of air to the buoyant force of the water is only 6.46×10^{-4}, which is so small as to be of no consequence in most problems.

EXAMPLE 11.4.4

A certain object has a mass of 250 g and a density of 2.70 gm/cm^3. When "weighed" while immersed in a liquid of unknown density by the arrangement shown in Fig. 11.16a, it is found that balance is obtained when 180 g is placed on the right-hand side of the laboratory balance. What is the density of the fluid in the container?

The forces acting on the object while immersed are shown in Fig. 11.16b. The tension T in the string is equal to the weight force associated with a mass of 180 g. Therefore, $T = (180)(980) = 176{,}400$ dynes. The net buoyant force F_b, according to Archimedes' principle, must be

$$F_b = \rho gV \tag{11.4.19}$$

where ρ is the density of the fluid (*not* the density of the object, remember!). According to Newton's first law, then

$$\sum F_y = T + F_b - mg = 0 \tag{11.4.20}$$

or, in view of Eq. (11.4.19),

$$\rho gV = mg - T \tag{11.4.21}$$

Let us now divide both sides of this equation by mg,

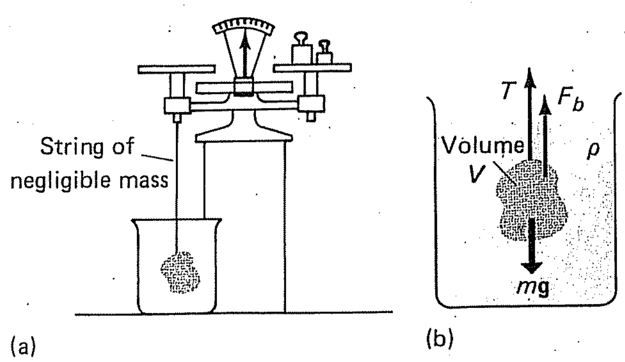


FIGURE 11.16

which can also be written as $\rho_0 gV$, where ρ_0 is the object's density:

$$\frac{\rho gV}{mg} = \frac{\rho gV}{\rho_0 gV} = \frac{mg - T}{mg}$$

or

$$\rho = \rho_0 \frac{mg - T}{mg} \tag{11.4.22}$$

In this case, $\rho_0 = 2.70$, $mg = (250)(980)$ dynes, and $T = (180)(980)$ dynes. Substituting these values into (11.4.22) gives the answer $\rho = 0.756$.

EXAMPLE 11.4.5

An object of mass 180 g, but of *unknown* density, is "weighed" in water (density $\rho_w = 1.00$ g/cm^3), the weight so obtained corresponding to a balancing mass of 150 g, and "weighed" again in a liquid of unknown density ρ_f, a balancing mass of 144 g being needed this time. What is the density of the second fluid? What is the density of the object?

Let the density of the object be ρ and its volume V. Then, from Newton's first law, the two "weighings" give

$$T_1 + F_{b1} - mg = 0 \tag{11.4.23}$$

$$T_2 + F_{b2} - mg = 0 \tag{11.4.24}$$

where, according to Archimedes' principle,

$$F_{b1} = \rho_w gV \tag{11.4.25}$$

$$F_{b2} = \rho_f gV \tag{11.4.26}$$

Equations (11.4.23) and (11.4.24) may now be written

$$T_1 + \rho_w gV - mg = 0 \tag{11.4.27}$$

$$T_2 + \rho_f gV - mg = 0 \tag{11.4.28}$$

In these equations there are two unknowns, V and ρ_f. One may most easily eliminate V by multiplying the first equation through by ρ_f, the second by ρ_w, and then subtracting the second equation from the first. The resulting expression can be solved for ρ_f to give

$$\rho_f = \rho_w \frac{mg - T_2}{mg - T_1} \tag{11.4.29}$$

Also, one may solve (11.4.27) for V, the result being

$$V = \frac{mg - T_1}{\rho_w g} \tag{11.4.30}$$

Now, the density of the object is simply $\rho = m/V$, with V given above. Then,

$$\rho = \frac{m}{V} = \frac{\rho_w mg}{mg - T_1}$$

We are given that $\rho_w = 1.00$ g/cm^3,

$mg = (180)(980)$ dynes

$T_1 = (150)(980)$ dynes

and

$T_2 = (144)(980)$ dynes

Substituting these values into (11.4.29) and (11.4.31), it is easy to establish that $\rho_f = 1.20$ and $\rho = 6.00$.

11.5 Fluid Dynamics and Bernoulli's Equation

Up to this point, we have discussed only situations involving fluids in equilibrium. It is, of course, also important to understand the mechanics of fluids which may undergo acceleration. This branch of mechanics is referred to as *fluid dynamics.* The general study of fluid dynamics is a complex and difficult undertaking, involving much advanced mathematics; it is a task we shall not attempt here, except to sketch roughly some of the problems involved and some of the possible areas of application. It is possible, however, to understand many of the important features of fluid dynamics in certain restricted cases by the development and use of a simple relation based on the work–energy theorem and the energy conservation principle, called *Bernoulli's equation* after its originator the Swiss scientist Daniel Bernoulli (1700–1782).

In developing Bernoulli's equation, we shall restrict ourselves to the case of steady-state, incompressible, nonviscous fluid flow. By *steady-state flow* we mean that at every point within the flowing fluid medium the velocity of the fluid does not change with time. This does not mean that the fluid is unaccelerated; each droplet of liquid may undergo acceleration as it moves from point to point. At a fixed *location*, however, the velocity of the fluid flowing past remains at all times the same in magnitude and direction. A fountain or a waterfall are common examples of this type of flow. In the case of a fountain, illustrated in Fig. 11.17, in which a stream of water is projected upward and then falls back to the level of the nozzle, the velocity of the water droplets is zero when they reach their maximum height, and this state of affairs does not change with time. But the acceleration of each droplet is *not* zero here; indeed, it has the value g at this point and, for that matter, at all other points along the path taken by the stream.

Under conditions of steady-state flow, a steady pattern of *lines of flow* marking the paths taken by fluid particles within the stream is set up. These flow lines can be made visible by dropping tiny particles such as confetti into a liquid, or by injecting a series of thin streams of ink at various points. In a gas flow,

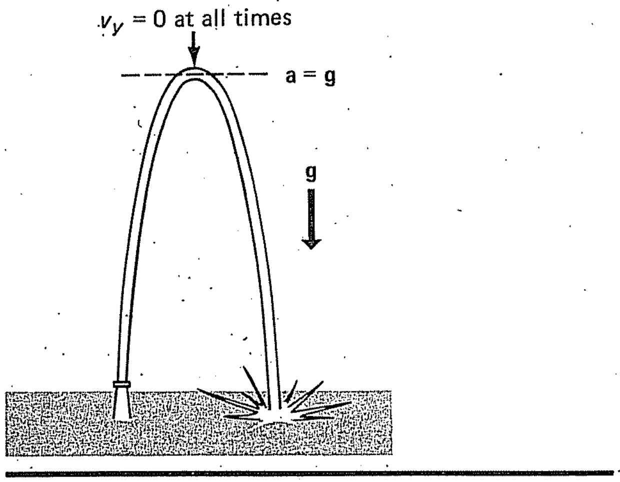
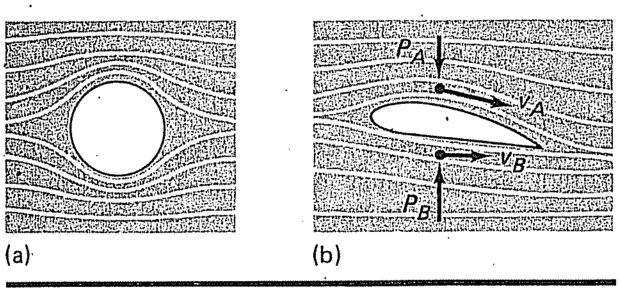


FIGURE 11.17. Motion of a projected stream of liquid.

the lines of flow, or *streamlines* as they are often called, can be seen if tiny smoke jets are provided at a number of points. Some situations involving steady-state flows in which the streamlines have been rendered visible in this way are illustrated in Fig. 11.18. There are two important observations to make in regard to the lines of flow, or streamlines. First and most obvious, since the direction of fluid flow is along the streamlines, there is no fluid flow perpendicular to them. Secondly, families of streamlines can be used to define *tubes of flow* within the fluid, as shown in Fig. 11.19. Since there can be no flow across the surface of such a tube of flow, the fluid flow is entirely along and within the tube.

In deriving Bernoulli's equation, we shall confine ourselves to the case of *incompressible* fluids. It is quite possible to modify the derivation to allow for compressibility, though for our purposes this introduces unnecessary complications. It turns out also that the incompressible Bernoulli equation gives a good *approximate* description of the steady-state flow of compressible fluids in cases where the flow velocity is everywhere small compared to the velocity of sound. We shall, therefore, find ourselves using the incompressible Bernoulli equation even in gases, though in doing so we have to be careful to avoid cases of high-velocity flow.



(a) (b)

FIGURE 11.18. Streamlines illustrating steady fluid flow around a spherical obstacle (a) and an airfoil (b).

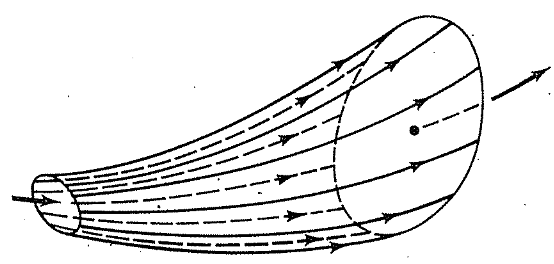


FIGURE 11.19. Tube of flow bounded by streamlines.

Finally, we shall have to concern ourselves with fluid flow situations which are *nonviscous*. By *viscosity* we mean the property exhibited by fluids of generating internal forces of friction or resistance when one layer of fluid is made to move across another parallel layer with finite relative velocity. We are familiar with the fact that when a liquid such as water or ethyl alcohol is stirred with rapid rotation in a cylindrical can, it may remain in rotation for a very long time before all evidence of the original motion ceases. In the case of a heavy oil, however, such rotational motion, if excited, falls off very rapidly when stirring is ceased. It is the internal viscous forces between fluid layers moving at different velocities that are responsible for the damping of the motion in both cases. In both cases, it is evident that the kinetic energy of rotation which was excited by stirring is dissipated (actually transformed into heat energy) by the action of these forces.

Viscous forces, therefore, have the characteristics of *frictional* forces in fluids. These forces, which arise from interactions between the molecules of the substance itself, are quite weak in the case of water and alcohol. In a given time interval, say a second, they may transform only a small fraction of the initial kinetic energy into heat. For such substances, it may be quite reasonable to completely ignore the effect of viscosity in turning mechanical energy into heat, just as it is possible to neglect the effect of sliding friction in many mechanics problems of the types studied in earlier chapters. On the other hand, if we are dealing with a very viscous fluid such as lubricating oil or glycerine, a very large fraction of the kinetic energy may be dissipated in a similar time interval; now the viscous forces are large and cannot be neglected.

We shall not try to treat viscous flow in this chapter, but shall confine ourselves to situations in which viscous friction is unimportant. Finally, we shall also exclude from our treatment any situation in which internal rotations of the fluids, or *vortex* effects, are important. If these effects are absent, the flow pattern is said to be *irrotational*, and we shall consider only instances of this type in deriving Bernoulli's equation.

Now, having restricted ourselves to a fairly narrow and simple range of fluid flow conditions—steady-state, incompressible, nonviscous, and irrota-

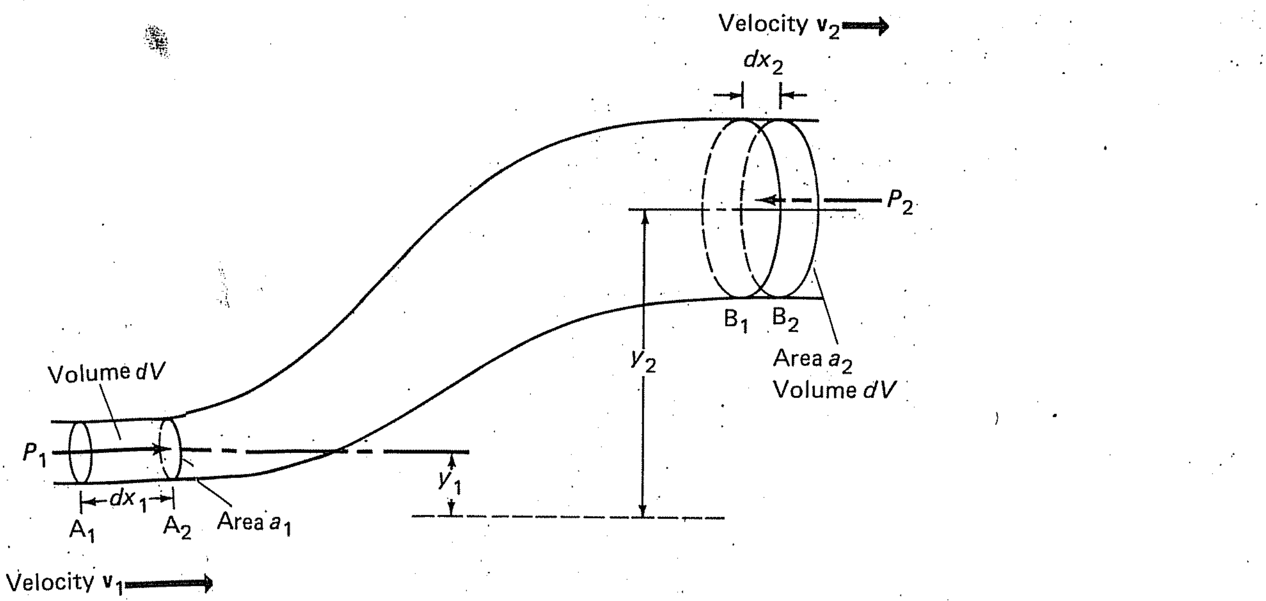


FIGURE 11.20. Sysn of forces acting on fluid confined within a tube of variable cross section.

tional—we are readꜰo derive Bernoulli's equation. Consider the motioıf a fluid in a pipe of variable cross-sectional area, illustrated in Fig. 11.20. In particular, let us inveıate the motion of an element of fluid which at timeextends from point A_1 along the pipe to B_1 and wh, at a time dt later, extends from point A_2 to B_2, ving moved a distance dx_1, at speed dx_1/dt, from ꜰo B_1, and, at the other end, a distance dx_2, at speex$_2/dt$, from A_2 to B_2. Let the cross-sectional areathe tube be a_1 from A_1 to B_1 and a_2 from A_2 to We shall assume that the section A_1B_1 is at heighabove the earth's surface and that the section A_2s at height y_2. Since the fluid is incompressible, tme volume dV is swept out at either end, and theans that the velocities v_1 and v_2 and the areas nd a_2 are *related.* The connection between themıs from the relation

$$dV = a_1\,dx_1 = a_2\,dx_2 \tag{11.5.1}$$

or, dividing by dt and notıat $v_1 = dx_1/dt$, $v_2 = dx_2/dt$,

$$\frac{dV}{dt} = a_1 v_1 = a_2 v_2 \tag{11.5.2}$$

Bernoulli's equation is equence of energy conservation and the workɜy theorem. The work–energy theorem states e work done on any body by the resultant foting on it must equal its change in kinetic enerı must, therefore, compute the work done by als acting on the element of fluid and also its cha kinetic energy. The forces acting are of two kinssure forces P_1 and P_2 at the two ends and gravitı forces arising from the weight of the fluid in the element. As the fluid element moves from its initial to its final position, the work done by the forces attributable to pressures P_1 and P_2 are $P_1 a_1\,dx_1$ and $-P_2 a_2\,dx_2$, the minus sign arising from the fact that the pressure on the element acts in the $-x$ direction on the right-hand end of the fluid element A_1B_1 or A_2B_2. Recalling Eq. (11.5.1), the net work done by the pressure forces is then

$$dW_p = P_1 a_1\,dx_1 - P_2 a_2\,dx_2 = (P_1 - P_2)\,dV \tag{11.5.3}$$

Work is also done on the element by gravitational forces; in effect, as the element of fluid moves from its initial position A_1B_1 to its final position the effect is to raise a volume of fluid dV, hence a mass of fluid $dm = \rho\,dV$, through a height $y_2 - y_1$. The amount of work done by gravity is therefore

$$dW_g = -(dm)g(y_2 - y_1) = -\rho g(y_2 - y_1)\,dV \tag{11.5.4}$$

Finally, the change in kinetic energy of the element, dU_k, can be calculated by observing that the element A_1B_1 has lost its kinetic energy $\frac{1}{2}(dm)v_2^2$ and the element A_2B_2 has acquired kinetic energy $\frac{1}{2}(dm)v_2^2$. The velocities of all *other* points in the fluid column remain unchanged and, therefore, give no contribution to the change in kinetic energy. The total change in kinetic energy during the process may, therefore, be written as

$$\begin{aligned} dU_k &= \tfrac{1}{2}(dm)v_2^2 - \tfrac{1}{2}(dm)v_1^2 \\ &= \tfrac{1}{2}\rho(v_2^2 - v_1^2)\,dV \end{aligned} \tag{11.5.5}$$

According to the work–energy theorem, the work done on the fluid by the resultant force, which can be expressed as the sum of the work contnoutions of the individual forces, equals the change in kinetic energy of the element. Hence, we may write

$$dW_p + dW_g = dU_k \tag{11.5.6}$$

or, from Eqs. (11.5.3), (11.5.4), and (11.5.5),

$$(P_1 - P_2)\,dV - \rho g(y_2 - y_1)\,dV = \tfrac{1}{2}\rho(v_2^2 - v_1^2)\,dV \tag{11.5.7}$$

Canceling dV throughout and rearranging the terms, this can be written finally as

$$\boxed{P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2} \tag{11.5.8}$$

This is Bernoulli's equation. Note that if the fluid is at equilibrium ($v_1 = v_2 = 0$), it reduces to what should now be the familiar result

$$P_1 - P_2 = \rho g(y_2 - y_1) \tag{11.5.9}$$

which was set forth previously as Eq. (11.3.7).

The derivation was carried out for a fluid within a pipe or tube, and it might, therefore, be thought that the results so obtained would not apply to freely flowing fluids which are not so constrained. *This is not so*, however, because the same derivation could also have been applied to the flow of a certain portion of an unconstrained fluid enclosed by one of the *tubes of flow* of the system! Since no fluid ever flows across the boundary of any tube of flow, the derivation would have been equally valid for such a situation. After all, it makes no difference whether a certain portion of a flowing fluid is constrained by the physical presence of the walls of a pipe or merely by the rest of a larger fluid system surrounding it and flowing along with it.

Bernoulli's equation, (11.5.8), tells us that the quantity $P + \rho g y + \frac{1}{2}\rho v^2$ is the same at any two points in the flow system, which implies that its value is the same everywhere in the system. The quantity $\rho g y$ represents the potential energy of the fluid per unit volume at a given point, and the quantity $\frac{1}{2}\rho v^2$ likewise represents the kinetic energy per unit volume at that point. Bernoulli's equation may then be interpreted as stating that the sum of the pressure, the potential energy per unit volume, and the kinetic energy per unit volume is everywhere the same—hence is *conserved.* Since the derivation of Bernoulli's equation is based on the principle of energy conservation, it is hardly surprising that it has the form of a conservation law.

One of the important physical effects predicted by Bernoulli's equation can be understood in simple terms by considering the case where there is no appreciable difference in gravitational potential energy throughout the flowing fluid. We may then set both y_1 and y_2 equal to zero, and (11.5.8) becomes

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \tag{11.5.10}$$

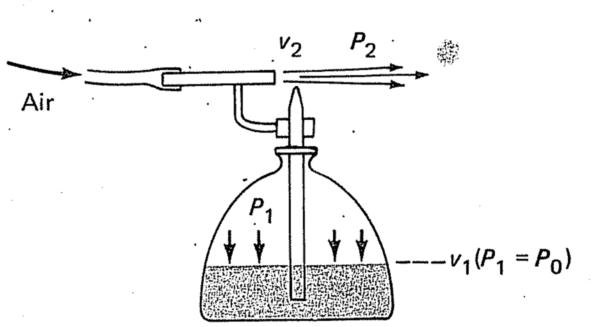


FIGURE 11.21. An atomizer.

$$P_2 - P_1 = \tfrac{1}{2}\rho(v_1^2 - v_2^2) \tag{11.5.11}$$

In this equation, suppose that v_2 is greater than v_1; then $v_1^2 - v_2^2$ is negative, as is $P_2 - P_1$. This means than P_1 is greater than P_2 or, turning it around, that P_2 is smaller than P_1. In other words, when gravitational effects are unimportant, an *increase* in the flow velocity of a fluid is inevitably accompanied by a *decrease* in pressure. This is sometimes called the *Venturi effect* and is responsible for the action of atomizers, automotive carburetors, wind tunnels, and many other devices, as well as for the curvature of the path of a rapidly spinning baseball and for the aerodynamic lift of an airplane wing.

The example of an atomizer is shown in Fig. 11.21. In this device, a large flow velocity v_2 is created by a stream of air across the nozzle, the flow velocity being zero or very small elsewhere, in particular at the surface of the liquid in the reservoir. According to Bernoulli's equation, then, the pressure in the region of the nozzle must be less than that experienced in the reservoir, where atmospheric pressure prevails. Accordingly, the atmospheric pressure in the reservoir forces the liquid to rise in the tube leading to the nozzle (just as in a fluid barometer) and finally to emerge from the nozzle orifice to be broken up by and carried away in the air stream.

The case of the spinning baseball is shown in

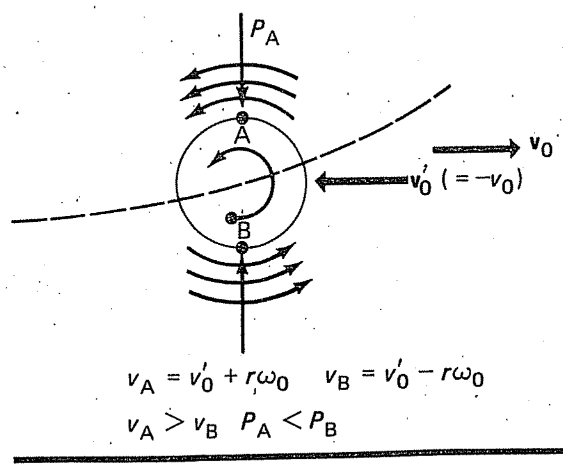


FIGURE 11.22. Motion of a spinning baseball.

Fig. 11.22. In this illustration, we are looking down from above on a baseball thrown by a pitcher at the far right. The pitcher has imparted an initial spin angular velocity ω_0 to the ball and, at the same time, a linear velocity v_0 to the center of mass. As the ball spins, it drags along a rotating layer of air as shown by the arrows. Let us temporarily move with the center of mass of the baseball and try to account for the flow velocities in its vicinity. In doing this, it is as if the baseball is motionless (except for spin) in a wind tunnel which sends a stream of air at it from right to left with velocity v_0', equal to $-v_0$, as shown in the diagram. The layr of air dragged by the surface of the ball has lineavelocity $r_0\omega_0$ at point B and $-r_0\omega_0$ at point A, where r_0 is the ball's radius. The total magnitude of flowelocity with respect to the center of the ball is then $v_0' + r_0\omega_0$ at A and $v_0' - r_0\omega_0$ at B. Since the velocity at B is less than at A, the pressure on the ball, according Bernoulli's equation, is greater at B than at A. Thmeans that there will be a net force which will defl the ball upward in the diagram or sideways to a per in the position of the pitcher. It is evident from allis that atmospheric friction is responsible for the vature of the ball's path. A pitcher, no matter ho'ood he is, could never throw a curve in a vacuum!

Somewhat the e analysis applies to the problem of the dynamift of an airplane wing. The streamlines about the s section of such an airfoil are shown in Fig. 11.18he airfoil is so shaped and positioned in the airstre at an appropriate angle of attack, that a pattern ow is produced in which the velocity v_A near the r surface of the wing is necessarily greater than tlocity v_B near the lower surface. According to Berni's equation, this means that the pressure P_B on undersurface must be greater than the pressure n the top surface and that a resultant upward aenamic lift is created. It is not entirely obvious howairfoil is to be shaped or positioned so that this rel between the velocities is obtained; there are sflow velocities and attack angles for which it is n as, for example, in "stall" conditions. A more detiscussion of aerodynamic lift in airfoils is some beyond the scope of this work and will not be in here.

The following series of ees will illustrate more explictly how Bernoulli's ion may be applied to various instances of sttate fluid flow problems.

EXAMPLE 11.5.1

An incompressible fluid is flowin left to right through a cylindrical pipe such asne shown in Fig. 11.23. The density of the fluid slug/ft^3. Its velocity at the input end is 5 ft/sec,e pressure there is 25 lb/in.^{2} The output end i below the

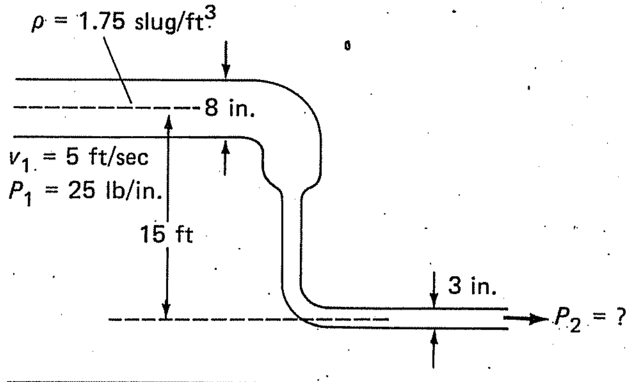


FIGURE 11.23

input end. What is the pressure at the output end?

Bernoulli's equation tells us that

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 \qquad (11.5.12)$$

We know P_1, ρ, y_1, y_2, and v_2; neither P_2 nor v_2 are given. However, since the fluid is incompressible, v_2 must be related to v_1 by (11.5.2):

$$v_2 = \frac{a_1}{a_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{(\frac{1}{3})^2}{(\frac{1}{8})^2}(5) = 35.6 \text{ ft/sec}$$

Substituting this value along with $P_1 = 25$ lb/in.^{2} = 3600 lb/ft^2, $v_1 = 5$ ft/sec, $\rho = 1.75$ slug/ft^3, $y_1 = 15$ ft, and $y_2 = 0$ into (11.5.12), we obtain

$$3600 + (1.75)(32.2)(15) + (0.5)(1.75)(25) = P_2 + 0 + (0.5)(1.75)(35.6)^2$$

$P_2 = 3358$ lb/ft^2, or 23.3 lb/in.^{2}

EXAMPLE 11.5.2

A *Venturi flowmeter* is illustrated in Fig. 11.24. It consists of a constricted tubular section of frontal area

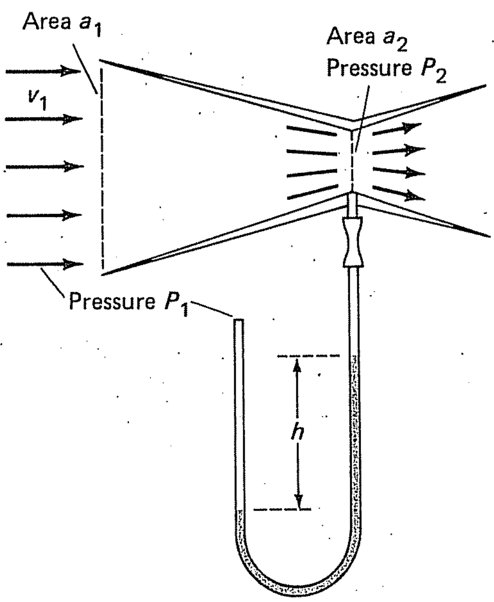


FIGURE 11.24. The Venturi flowmeter.

a_1 and throat area a_2. It is inserted into a stream of gas or liquid flowing at velocity v_1, which is to be determined, and a small orifice in the throat is connected to a pressure gauge, shown as an open U-tube manometer in the figure. A pressure P_2 is measured on the manometer. Find the equation relating the flow velocity v_1 which is to be measured to the known areas a_1 and a_2 and the measured pressure difference $P_1 - P_2$. What is the flow velocity of a stream of air ($\rho = 1.293 \times 10^{-3}$ g/cm³) in a situation where the frontal area a_1 is 100 cm, the throat area a_2 is 20 cm², and the height h measured on an open tube mercury manometer 25 cm?

We begin again with Bernoulli's equation:

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 \qquad (11.5.13)$$

In this situation, $y_1 = y_2 = 0$, and the throat velocity v_2 can be expressed in terms of the stream velocity v_1 by the relation (11.5.2), which states that $a_1 v_1 = a_2 v_2$, or

$$v_2 = \frac{a_1}{a_2} v_1 \qquad (11.5.14)$$

Bernoulli's equation then reduces to

$$P_1 + 0 + \tfrac{1}{2}\rho v_1^2 = P_2 + 0 + \tfrac{1}{2}\rho \frac{a_1^2}{a_2^2} v_1^2$$

which upon rearranging may be expressed as

$$P_1 - P_2 = \tfrac{1}{2}\rho v_1^2 \left(\frac{a_1^2}{a_2^2} - 1\right)$$

from which

$$v_1^2 = \frac{P_1 - P_2}{\tfrac{1}{2}\rho\left(\frac{a_1^2}{a_2^2} - 1\right)} \qquad (11.5.15)$$

For an open-tube mercury manometer, using the notation of this example, the pressure difference, according to (11.3.17), is

$$P_1 - P_2 = \rho_{Hg} g h \qquad (11.5.16)$$

where ρ_{Hg} is the density of mercury, equal to 13.6 g/cm³. Equation (11.5.15) now becomes

$$v_1^2 = \frac{2\rho_{Hg} g h}{\rho\left(\frac{a_1^2}{a_2^2} - 1\right)} \qquad (11.5.17)$$

For an air stream where $\rho = 1.293 \times 10^{-3}$ g/cm³ and where $h = 25$ cm, $a_1 = 100$ cm², and $a_2 = 20$ cm², this gives

$$v_1^2 = \frac{(2)(13.6)(980)(25)}{(1.293 \times 10^{-3})(5^2 - 1)} = 21.47 \times 10^6 \text{ cm}^2/\text{sec}^2$$

$$v_1 = 4634 \text{ cm/sec, or } 46.34 \text{ m/sec}$$

EXAMPLE 11.5.3

A cylindrical tank 6 ft in diameter rests atop a platform 20 ft high, as shown in Fig. 11.25. Initially, the tank is filled with water ($\rho = 1.938$ slug/ft³) to a depth h_0 equal to 10 ft. A plug whose area is 1.00 in.² is removed from an orifice in the side of the tank at the very bottom. With what velocity does the water flow initially from this orifice? What is the velocity of the stream initially as it strikes the ground? How long will it take to empty the tank entirely?

Let us first of all assign dimensions and values of pressure, velocity, and height to the system as shown in Fig. 11.25. We shall work with gauge pressures throughout; this is permissible when using Bernoulli's equation, provided it is done consistently. What one *cannot* do is use gauge pressure on one side of the equation and absolute pressure on the other, so beware of doing that!

At the very top surface of the liquid, P, v, and y have the values P_1, v_1, and y_1, and in the stream which is just outside the orifice at the bottom of the tank, they have the values P_2, v_2, and y_2. According to Bernoulli's equation,

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 \qquad (11.5.18)$$

Now, obviously, the gauge pressure P_1 is zero; but would you believe that P_2 is also zero? Well, it is, and the reasoning goes like this. Inside the tank, in the water *behind* the orifice, the pressure is not zero but rather $\rho g h_0$; but all this pressure is transformed to kinetic energy in the trip through the orifice. Outside the orifice, the only pressure on the now rapidly moving stream is that of the atmosphere, hence the gauge pressure there is once more zero! This argument neglects, as usual, the slight rise in atmospheric pressure encountered in descending through the distance h_0, which in this problem is only 10 ft.

The height y_1 is equal to $h_0 + H$, while y_2 equals H. The velocities v_1 and v_2 are again related by

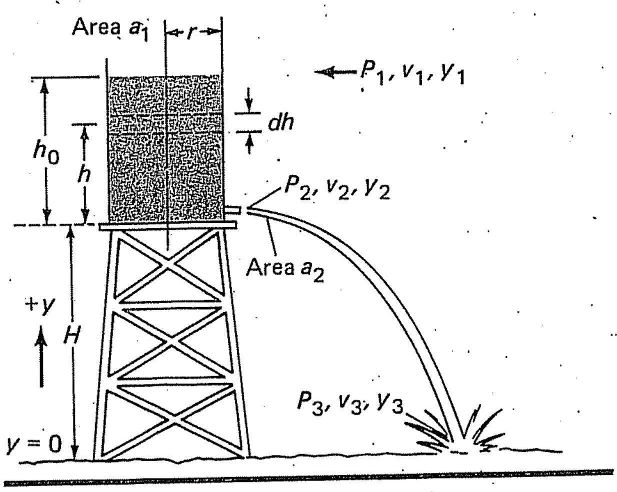


FIGURE 11.25

$a_1 v_1 = a_2 v_2$. In this case, however, the area a_1 is $\pi r^2 = (\pi)(3)^2 = 9\pi = 28.27 \text{ ft}^2 = 4072 \text{ in.}^2$, while $a_2 = 1.00 \text{ in.}^2$ This means that v_2 is some 4000 times larger than v_1, and under these circumstances the term $\frac{1}{2}\rho v_1^2$ will be some 1,600,000 times less than $\frac{1}{2}\rho v_2^2$. We might then just as well neglect it altogether and set v_1 equal to zero. Putting all this information into Eq. (11.5.18), we find

$$0 + \rho g(h_0 + H) + 0 = 0 + \rho gH + \tfrac{1}{2}\rho v_2^2$$
$$v_2 = \sqrt{2gh_0} = \sqrt{(2)(32.2)(10)} = 25.4 \text{ ft/sec} \tag{11.5.19}$$

Once more assigning values for P, v, and y as P_2, v_2, and y_2 just outside the tank orifice and P_3, v_3, and y_3 as the stream hits the ground, according to Bernoulli's equation,

$$P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 = P_3 + \rho g y_3 + \tfrac{1}{2}\rho v_3^2 \tag{11.5.20}$$

But now, $P_2 = P_3 = 0$, $y_2 = H$, $y_3 = 0$, and, from Eq. (11.5.19), $v_2^2 = 2gh_0$. Putting all this into (11.5.20), we obtain

$$0 + \rho gH + \rho g h_0 = 0 + 0 + \tfrac{1}{2}\rho v_3^2$$
$$v_3 = \sqrt{2g(h_0 + H)} = \sqrt{(2)(32.2)(30)} = 44.0 \text{ ft/sec} \tag{11.5.21}$$

In finding how long it takes to empty the tank, the time required for the depth to change from h to zero is required, and this is closely related to the velocity v_1 which we found we could neglect for the purpose of determining v_2. We must now, however, start worrying about v_1. As always in the case of an incompressible fluid,

$$a_1 v_1 = a_2 v_2 \tag{11.5.22}$$

Now suppose the height h of the liquid in the tank decreases by the amount dh during a time interval dt. The velocity v_1 must now be given by

$$v_1 = -\frac{dh}{dt} \tag{11.5.23}$$

the minus sign being necessary because dh/dt is negative (h is decreasing with time) while the fluid speed v_1 is essentially positive, since it is the magnitude of a velocity vector. Substituting this value for v_1 into (11.5.22), we obtain

$$-\frac{dh}{dt} = \frac{a_2}{a_1} v_2 = \frac{a_2}{a_1}\sqrt{2gh} \tag{11.5.24}$$

recalling from (11.5.19) that $v_2 = \sqrt{2gh}$. We shall now have to integrate this equation to find the relationship between the depth h and the time. Multiplying the equation by dt and dividing by $\sqrt{h}$, we may express (11.5.24) in the form

$$-\frac{dh}{\sqrt{h}} = \frac{a_2}{a_1}\sqrt{2g}\, dt \tag{11.5.25}$$

This may now be integrated between time $t = 0$, when the depth has the initial value h_0 and a later time t, when the depth is h. Accordingly,

$$-\int_{h_0}^{h} h^{-1/2}\, dh = \frac{a_2}{a_1}\sqrt{2g}\int_0^t dt$$
$$-[2\sqrt{h}]_{h_0}^{h} = 2(\sqrt{h_0} - \sqrt{h}) = \frac{a_2}{a_1}\sqrt{2g}\, t \tag{11.5.26}$$

and finally, solving for t,

$$t = \frac{a_1}{a_2}\frac{\sqrt{2}(\sqrt{h_0} - \sqrt{h})}{\sqrt{g}} \tag{11.5.27}$$

This equation gives the elapsed time t as a function of the fluid depth h. Initially, the depth h_0 is 10 ft; when the tank is empty, the depth is zero. Setting $h = 0$ in (11.5.27), we find

$$t = \frac{a_1}{a_2}\sqrt{\frac{2h_0}{g}} = \frac{(4072)}{(1.00)}\frac{(2)(10)}{[(32.2)]^{1/2}} = 3209 \text{ sec, or } 53.5 \text{ min}$$

EXAMPLE 11.5.4

A small fixture is attached to the tank orifice of the previous example to direct the stream upward at an angle θ without affecting its speed or cross-sectional area. What is the maximum height h' attained by the stream?

The situation is now that illustrated in Fig. 11.26. From Bernoulli's equation, we know that

$$P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2 = P_4 + \rho g y_4 + \tfrac{1}{2}\rho v_4^2 \tag{11.5.28}$$

But $P_2 = P_4 = 0$, $y_2 = H$, and from the results of the preceding example, $v_2^2 = 2gh_0$. We may also write $v_2^2 = v_{2x}^2 + v_{2y}^2 = v_2^2\cos^2\theta + v_2^2\sin^2\theta$. Bernoulli's equation now reads

$$\rho gH + \tfrac{1}{2}\rho v_2^2\cos^2\theta + \tfrac{1}{2}\rho v_2^2\sin^2\theta = \rho g h' + \tfrac{1}{2}\rho v_4^2 \tag{11.5.29}$$

In this equation, we know neither h' nor v_4. We do know, however, that since the pressure is everywhere

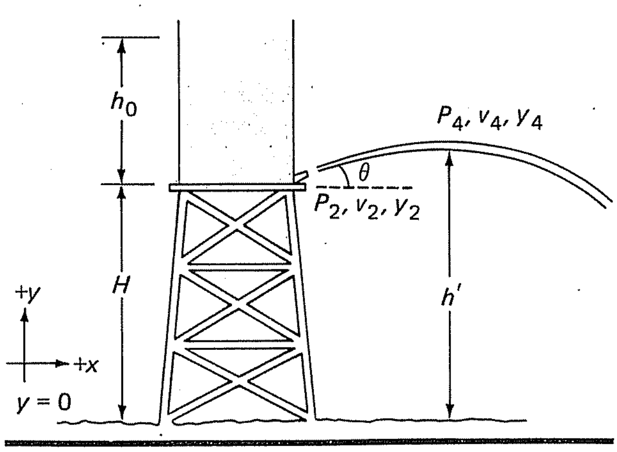


FIGURE 11.26

zero in the stream of fluid emitted by the orifice, the only force on the water in that stream is the force of gravity acting on each individual droplet. Under these circumstances, each droplet of the stream executes *projectile motion;* there is, therefore, no x-component of acceleration for the liquid in the stream because the resultant force has only a y-component. The x-component of velocity for the liquid in the stream is, therefore, *constant*, and this, in turn, tells us that

$$v_4 = v_{2x} = v_2 \cos\theta \tag{11.5.30}$$

Substituting this value for v_4 into Eq. (11.5.29) along with the value $2gh_0$ for $v_2{}^2$ and solving for h', it is easy to obtain

$$h' = H + h_0 \sin^2\theta \tag{11.5.31}$$

It is evident that the jet of fluid emitted by the tank nozzle can never rise higher than the level of fluid in the tank, since the largest value $\sin^2\theta$ can have is unity, attained when $\theta = 90°$, in which case $h' = H + h_0$, the stream then rising just to the inside fluid level.

EXAMPLE 11.5.5

A cylindrical tank 1.2 m in diameter is filled to a depth of 0.3 m with water ($\rho = 1000\ \text{kg/m}^3$). The space above the water is occupied by air, compressed to a gauge pressure of $1.00 \times 10^5\ \text{N/m}^2$. A plug is removed from an orifice in the bottom of the tank having an area of 2.5 cm^2. What is the initial velocity of the stream which flows through this orifice? What is the upward force experienced by the tank when the plug is removed?

The situation is illustrated in Fig. 11.27. From Bernoulli's equation,

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1{}^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2{}^2 \tag{11.5.32}$$

In this example, just as in Example 11.5.3, the velocity v_1 may be neglected because the area a_2 is so much smaller than a_1. The pressure P_1 is equal to $1.00 \times 10^5\ \text{N/m}^2$, while P_2 is zero; also $y_1 = h$, while $y_2 = 0$. Putting all this into (11.5.32), we find

$$P_1 + \rho g h + 0 = 0 + 0 + \tfrac{1}{2}\rho v_2{}^2$$

$$v_2{}^2 = \frac{2(P_1 + \rho g h)}{\rho} = \frac{(2)[(10^5) + (10^3)(9.8)(0.3)]}{10^3}$$

$$= 205\ \text{m}^2/\text{sec}^2$$

$$v_2 = 14.35\ \text{m/sec} \tag{11.5.33}$$

To find the initial upward thrust on the tank, we may note that before the plug is removed the initial momentum of the system is zero. Since there are no *external* forces acting on the system (tank plus water), the momentum remains zero after the plug is removed. But in time interval dt, the liquid acquires

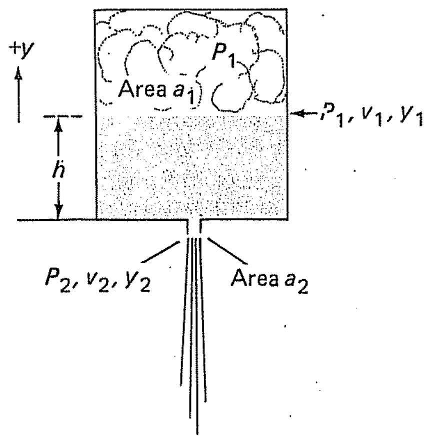


FIGURE 11.27

a negative y-component of momentum equal to

$$dp_{\text{liq}} = -v_2\, dm \tag{11.5.34}$$

where dm is the mass which squirts from the orifice in time dt. This quantity can be expressed as the product of the density times the volume emitted in time dt, the latter in turn being equal to the area a_2 times the distance the stream travels in time dt, which is $v_2\, dt$. We may write, therefore,

$$dm = \rho a_2 v_2\, dt \tag{11.5.35}$$

Substituting this into Eq. (11.5.34), we find

$$dp_{\text{liq}} = -\rho a_2 v_2{}^2\, dt \tag{11.5.36}$$

The momentum imparted to the tank is just the negative of this, in view of the fact that the total final momentum must be zero. But the momentum imparted to the tank in time dt may also be equated to the *impulse* of the force acting on the tank, according to the impulse–momentum theorem. We may write, therefore,

$$dp_{\text{tank}} = \rho a_2 v_2{}^2\, dt = F\, dt$$

whereupon it is evident that the force F must be given by

$$F = \rho a_2 v_2{}^2 = (10^3)(2.5 \times 10^{-4})(14.35)^2 = 51.5\ \text{N}$$

11.6 Surface Tension and Capillary Attraction

We have already had occasion to mention the phenomenon of viscosity, or internal friction, in fluids, and we have seen that this property has its origin in the forces between individual molecules of the substance. There are other unique properties of liquids attributable to these intermolecular forces, notably the phenomena of *surface tension* and *capillary attraction.* While these effects, like viscosity, do not alter the basic laws of fluid mechanics, they do alter the way they must be applied in certain particular situations. Therefore, though not treating these subjects in great detail, we shall offer a brief description of each, trying to indicate under what circumstances they may be important and when they may safely be neglected.

The phenomenon of surface tension arises from the fact that the intermolecular forces acting on molecules at or near a liquid surface differ from those which act on molecules deep in the interior of the liquid. Deep inside the liquid, the forces acting on each individual molecule are exerted equally in all directions, while a molecule at the surface experiences no force whatsoever from outside. A surface molecule, when displaced slightly from its equilibrium position toward the outside of the liquid, therefore, experiences strong resultant forces which tend to return it to its original position. This gives the surface of a liquid somewhat the character of a stressed elastic membrane, like the surface of an inflated balloon, except, of course, that it is self-healing when punctured!

The force necessary to rupture the surface "skin" of any liquid can be measured conveniently by determining experimentally how much upward force is required to pull a wire loop free of the surface, as illustrated in Fig. 11.28a. From the force diagram shown there, the force of surface tension T_s can be expressed in terms of the maximum force F required to free the loop from the liquid and the weight of the loop as

$$\sum F_y = F - mg - T_s = 0 \qquad (11.6.1)$$

or

$$T_s = F - mg \qquad (11.6.2)$$

Now, the force T_s is clearly proportional to the *length* of the loop that has to be pulled through the surface and also to the *strength of the intermolecular forces* that have to be overcome. The latter, of course, depends in detail upon just what forces are exerted by whatever molecules are present in the conditions of temperature, pressure, etc., which prevail, in other words, upon the substance itself and its ambient conditions. In any case, the force T_s can be written as a constant of proportionality times the *length* of fluid surface ruptured:

$$T_s = \gamma l \qquad (11.6.3)$$

All the information about the strength of the intermolecular forces is contained in the proportionality constant γ, called the *specific surface tension parameter*, which will be different for each liquid. This parameter is very difficult to calculate from the fundamental properties of the molecules, but it is very easy to *measure* by the arrangement shown in Fig. 11.28;

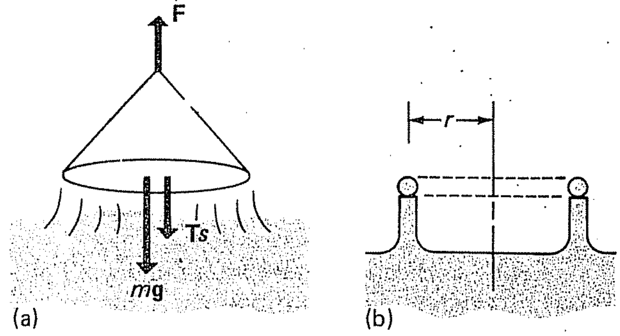


FIGURE 11.28. System of forces acting on a wire loop being pulled upward through the surface of a liquid.

in that diagram, the length of surface to be broken is given by $l = 2(2\pi r) = 4\pi r$, whence from Eqs. (11.6.2) and (11.6.3),

$$\gamma l = 4\pi r\gamma = F - mg$$

$$\gamma = \frac{F - mg}{4\pi r} \qquad (11.6.4)$$

It should be noted that the length of surface to be ruptured is in this case *twice* the circumference of the loop, since a surface film on the inside as well as the outside of the wire loop has to be broken. This is illustrated in Fig. 11.28b.

The surface tension parameter γ provides us with an easily measured index of the strength of intermolecular forces. From (11.6.3), it is evident that γ has the dimensions of force per unit length, dynes/cm or newtons/m in the metric system or pounds/ft in the English system. Because of surface tension, a pin or a razor blade can, with care, be supported by the surface of a liquid much less dense than the metal from which these objects are fashioned. In these cases, the surface tension force is much greater than the Archimedean buoyant force. Surface tension is also responsible for the fact that small quantities of liquids assume the form of spherical droplets, because the stressed surface "skin" tends to contract, molding the liquid into a form having minimum surface area for its volume, that is, into a *sphere*. Surface tension is also important in understanding the behavior of bubbles and soap films.

There are situations in which surface tension forces are quite important and others in which they are negligible. The surface tension of water will support a razor blade but not a battleship. Surface tension forces are wholly responsible for the shape of a raindrop but have nothing to do with the shape of Lake Erie. The difference lies in the different ways in which surface tension forces and weight forces vary as a function of the linear dimensions of the fluid mass in question. The perimeter of a fluid body varies as the first power of its linear extent, its area varies as the square of the linear extent, while the volume (or mass) changes as the cube. Forces that are directly proportional to an object's mass, therefore, fall off much more rapidly as the dimensions of the object decrease than do forces proportional to the surface area or linear dimensions. The surface tension force is an example of this latter type, while weight forces are of the first kind.

Let us consider, for example, a raindrop. For a spherical volume of water of radius, r, the surface tension force holding the two hemispheric halves together is the surface tension parameter γ times the circumference of the sphere:

$$F_s = \gamma l = 2\pi r\gamma \tag{11.6.5}$$

The weight force, on the other hand, is given by

$$W = mg = \rho Vg = \tfrac{4}{3}\pi r^3 \rho g \tag{11.6.6}$$

It will be equal to the external normal force experienced by the drop when it rests on a rigid surface. For a spherical water droplet of radius 0.1 cm, for which $\rho = 1.00$ g/cm^3 and $\gamma = 81$ dynes/cm, we find from (11.6.5) and (11.6.6) that the surface tension force holding the droplet together is 50.9 dynes, while the external normal force exerted by the surface on which it rests, equal to its weight, is 4.1 dynes. In this case, the external normal force is practically negligible in comparison with the surface tension force; it is, therefore, the latter that determines the form of the droplet and renders it practically spherical.

Now consider a spherical volume of water whose radius is 10 cm. The surface tension force holding the two halves together is now, according to Eq. (11.6.5), 100 times larger than before, or 5090 dynes. But, from (11.6.6), the external normal force exerted on the volume by its surroundings is proportional to r^3, so it will be 10^6 times larger than before and thus have the value of 4.1×10^6 dynes. In this case, the force attributable to surface tension is almost 1000 times *less* than the normal force from the surface on which the volume of liquid rests. The surface tension force now is clearly insufficient to maintain the spherical form of the liquid volume. So the liquid spreads out to ultimately take the shape of whatever container it happens to be in. Now the container furnishes the normal force necessary to establish equilibrium with the fluid's weight force, and the relatively small surface tension force is of little or no importance.

If the liquid is supported only by a perfectly flat surface, a circular puddle will form which will spread until its circumference is so large that the surface tension force is again sufficient to allow the system to reach equilibrium. When this occurs, the surface tension force on the upper half of the puddle (which acts downward) will just balance the normal force exerted by the lower half of the puddle on the upper half, as illustrated in Fig. 11.29. The normal force N is half the weight of a sphere of water of radius 10 cm and thus of volume $V = \tfrac{4}{3}\pi(10)^3 = 4189$ cm^3. The surface tension force F_s is γ times the circumference of the puddle, $2\pi r$, where r is the radius. Equating these quantities, we find

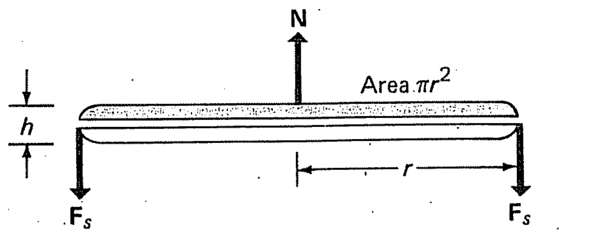


FIGURE 11.29. System of forces acting on fluid in a circular puddle.

$$N = \tfrac{1}{2}\rho g V = \gamma 2\pi r$$

$$r = \frac{\rho g V}{4\pi\gamma} = \frac{(1.00)(980)(4189)}{(4)(3.1416)(81)} = 4033 \text{ cm, or } 40.33 \text{ m} \tag{11.6.7}$$

The resulting puddle is, then, over 80 m (264 ft) in diameter! The corresponding depth of fluid, h, can be obtained from

$$v = \pi r^2 h = 4189 \text{ cm}^3$$

$$h = \frac{V}{\pi r^2} = \frac{(4189)}{(3.1416)(4033)^2} = 8.20 \times 10^{-5} \text{ cm} \tag{11.6.8}$$

The puddle so formed is clearly very large and very shallow. The quantitative accuracy of these numbers should not be taken very seriously, since there are many practical factors that interfere with actually observing the formation of such a puddle in reality. The numbers, however, do illustrate the relative weakness of surface tension forces in situations where relatively large volumes or masses of fluids are involved.

Closely related to surface tension is the phenom-

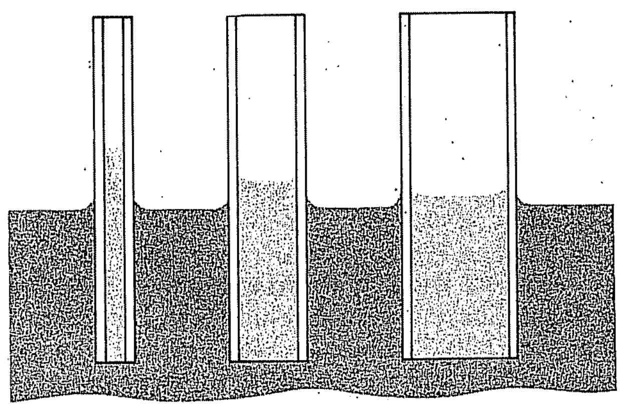


FIGURE 11.30. Capillary rise of a liquid in tubes of various radii.

enon of capillary attraction. When liquids confined within thin tubes, or capillaries, are allowed to reach equilibrium with a free liquid outside, as shown in Fig. 11.30, it is found that the level of the fluid within the tube is slightly higher than the free outside level, and the smaller the radius of the tube, the higher the level within. The difference in levels depends also upon the liquid used and the composition of the capillary tubing; in some cases, such as mercury within glass, the liquid surface within the capillary may be lower rather than higher. Associated with the capillary rise is the formation of a curved liquid surface (or "meniscus") within the tube, the level of the fluid rising toward the edge of the capillary where liquid and tube wall meet, though in the mercury–glass system the curvature is in the other direction.

This phenomenon is also caused by intermolecular attraction, but now between the molecules of the liquid and those of the capillary wall rather than between the molecules of the liquid alone. Consider the situation illustrated in Fig. 11.31. From this drawing, one may easily infer that the forces of intermolecular attraction responsible for the capillary rise are those which exist where the liquid surface meets the capillary wall, denoted by F_s. Elsewhere within the capillary, of course, there are also forces of intermolecular attraction between molecules of liquid and those in the capillary wall, such as F and F' in Fig. 11.31. But these forces act horizontally, as shown, and cannot support the liquid column within the capillary. We need, therefore, only consider the surface tension force F_s as supporting the liquid column. Indeed, it is only the vertical component of this force, $F_s \cos\theta$, where θ is the contact angle at which the liquid intersects the tube wall, that supports the column within the capillary.

Now, F_s is equal to the surface tension γ times the circumference of the tube:

$$F_s = 2\pi r\gamma \qquad (11.6.9)$$

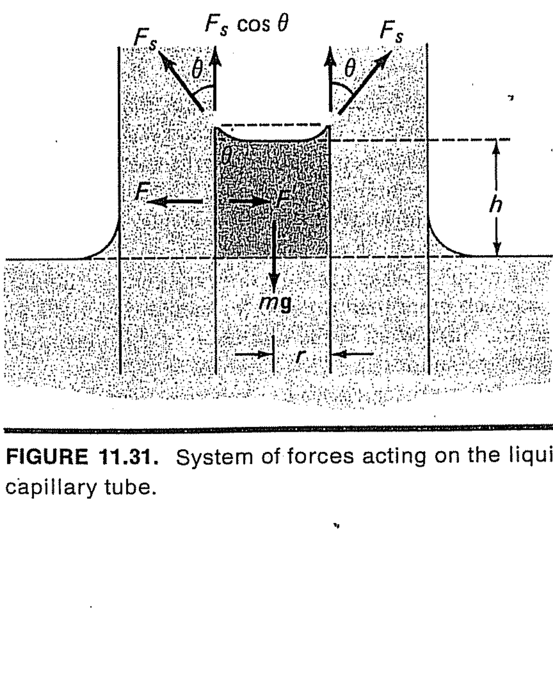


FIGURE 11.31. System of forces acting on the liquid in a capillary tube.

where r is the inside radius of the capillary. Also, since the system is in equilibrium,

$$\sum F_y = F_s \cos\theta - mg = 0 \qquad (11.6.10)$$

where m represents the mass of the shaded liquid column in Fig. 11.31. Neglecting the small amount of fluid above the level h at the edges of the column, this can be written as

$$m = \rho V = \rho\pi r^2 h \qquad (11.6.11)$$

Inserting the values of m and F_s as given by (11.6.11) and (11.6.9) into (11.6.10), we obtain

$$2\pi r\gamma \cos\theta - \pi\rho g r^2 h = 0$$

$$h = \frac{2\gamma \cos\theta}{\rho g r} \qquad (11.6.12)$$

Thus, we see that the capillary rise h is proportional to the surface tension γ and to the cosine of the contact angle and inversely proportional to the capillary radius and to the density of the fluid. For water in a glass capillary ($\rho = 1.00$ g/cm^3, $\gamma = 81$ dynes/cm, $\theta = 25.5°$) of radius 0.05 cm,

$$h = \frac{(2)(81)(0.9026)}{(1.00)(980)(0.05)} = 2.98 \text{ cm}$$

The strength of the intermolecular attraction between liquid molecules and those of the capillary wall enters into this calculation through the *contact angle* θ. This quantity has a definite, measurable value for each pair of substances. For example, for water in a glass capillary, $\theta = 25.5°$. If the force of attraction between molecules of liquid and capillary is the same as between the molecules of liquid themselves, then $\theta = 90°$ and $\cos\theta = 0$, and there will be no capillary effect at all. If the forces of attraction between liquid and wall molecules are stronger than the forces of attraction between liquid molecules, then $\theta < 90°$ and $\cos\theta$ is positive, resulting in a capillary rise. This is the situation encountered with water in a glass tube. If the forces of attraction between individual liquid molecules are stronger than those existing between fluid molecules and capillary molecules, the contact angle will be greater than 90° and $\cos\theta$ will be negative, leading to a capillary *depression*. This effect is observed with mercury in glass capillaries.

SUMMARY

Fluids such as liquids and gases are substances that flow under the action of applied forces. Gases are highly compressible, while liquids are practically incompressible.

The pressure exerted by a liquid or on a liquid is the force per unit area it exerts or experiences. The density of a liquid is its mass per unit volume. Pascal's principle states that

pressure applied to a fluid is transmitted undiminished to all parts of the fluid and to the walls of its container.

For incompressible fluids, the variation of pressure with depth is given by

$$P = P_0 + \rho g y$$

where P_0 is the surface pressure and y is the depth.

When a body is partially or totally immersed in a fluid, the fluid exerts an upward force upon it which is referred to as buoyant force. Archimedes' principle states that the net buoyant force on a body is equal to the weight of the fluid displaced by the object.

The dynamics of steady-state, nonviscous, irrotational fluid flow are described by Bernoulli's equation, which states that, at any two points within a tube of flow,

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

Surface tension and capillary attraction are effects that arise from the action of intermolecular attractive forces. Near the surface of a liquid, these forces tend to pull fluid molecules that are displaced from equilibrium toward the outside of the liquid back into the fluid; this gives the surface of the liquid the character of a self-healing stressed elastic membrane. The surface tension parameter γ is the force necessary to rupture a unit length of fluid surface. Surface tension is responsible for the spherical form of liquid droplets and for the forces that allow a pin or a razor blade to float on the surface of a fluid despite the fact that the weight exceeds the Archimedean buoyant force.

Capillary attraction is responsible for the rise of a fluid within a fine capillary with respect to the fluid level outside. It arises because the intermolecular attractive forces between fluid molecules and the molecules of the capillary tubing are stronger than those between the fluid molecules themselves.

QUESTIONS

1. Explain why a helium-filled balloon soars toward the sky while an air-filled one drops to the ground.
2. Can you think of any way of using Archimedes' principle to determine the weight of your head without removing it?
3. An ice cube is placed in a glass of water. The fluid now completely fills the glass. Describe what happens when the ice cube melts.
4. Pressure is expressed fundamentally in units of force per unit area. What is meant, then, when we describe pressures in units such as "millimeters of mercury"?
5. It is always easier to float in the ocean than in a swimming pool. Can you explain why?
6. Is it true that when an object floats, the fraction submerged depends on the ratio of densities of the fluid and the object?
7. Under what conditions does the pressure in a fluid increase linearly with depth?
8. A body sinking to the bottom of the ocean after a time achieves a certain constant terminal velocity. What forces act on it while it sinks?
9. What methods can be used to alter the depth at which a submarine cruises?
10. A plastic bag is weighed in air when empty and also when filled with air at atmospheric pressure. Are the two weights different? Explain. The two weighings are now repeated under vacuum. Are the weights equal to each other in this case?
11. When a truck passes your car on the highway, your car experiences a force. Explain this on the basis of Bernoulli's equation.
12. A fluid flows through a funnel the upper part of which is conical in shape. How does the fluid speed vary with distance from the upper surface of the fluid?
13. In steady streamline flow at any given location, the velocity vector **v** does not change with time. Does that mean that elements of fluid are unaccelerated?
14. A solid cube suspended from a spring balance under vacuum has a weight W. When it is completely submerged in liquid of density ρ, its weight is $W/2$. Find the volume of the cube.
15. The purity of gold can be determined by weighing it in air and also in water. Describe how this procedure works.
16. Why is it that a towel dries you off after a shower much more efficiently than a sheet of waxed paper having the same dimensions?

PROBLEMS

1. Show that (a) $1\ \text{g/cm}^3 = 10^3\ \text{kg/m}^3$, (b) $1\ \text{kg/m}^3 = 1.939 \times 10^{-3}\ \text{slug/ft}^3$, (c) $1\ \text{dyne/cm}^2 = 0.1\ \text{N/m}^2$, (d) $1\ \text{lb/in.}^2 = 68{,}900\ \text{dynes/cm}^2$.
2. In Fig. 11.4, vessels (a) and (b) have the same base area, as do vessels (c) and (d). The pressure at the bottom of each vessel is the same, since they are all filled with the same fluid to the same level. This means that the force exerted by the liquid on the base of (a) and (b) is the same, and, therefore, each vessel should exert the same downward force on the surface supporting it. The same remarks apply to vessels (c) and (d). But this is saying that (a) and (b) should have the same weight, as should (c) and (d). However, isn't it clear from the drawing that (b) should weigh much more than (a), and (c) should weigh much more than (d)? What is the resolution of this apparent hydrostatic paradox?
3. All the air is evacuated from a spherical glass bulb of radius 0.12 m. (a) Taking normal atmospheric pressure to be $1.013 \times 10^6\ \text{dyne/cm}^2$ or $1.013 \times 10^5\ \text{newtons/meter}^2$, find the total force exerted by the atmosphere on its sur-

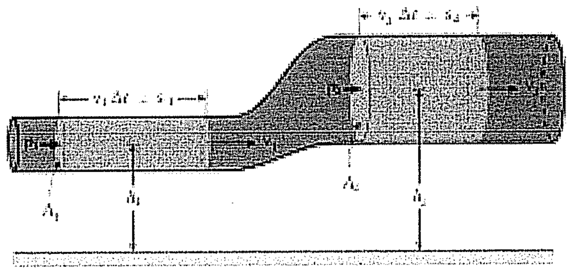
Fluid mechanics

From Wikipedia, the free encyclopedia

Fluid mechanics is the study of fluids and the forces on them. (Fluids include liquids, gases, and plasmas.) Fluid mechanics can be divided into fluid statics, the study of fluids at rest; fluid kinematics, the study of fluids in motion; and fluid dynamics, the study of the effect of forces on fluid motion. It is a branch of continuum mechanics, a subject which models matter without using the information that it is made out of atoms, that is, it models matter from a macroscopic viewpoint rather than from a microscopic viewpoint. Fluid mechanics, especially fluid dynamics, is an active field of research with many unsolved or partly solved problems. Fluid mechanics can be mathematically complex. Sometimes it can best be solved by numerical methods, typically using computers.

d di i li

Continuum mechanics



Laws

Conservation of mass
Conservation of momentum
Conservation of energy
Entropy inequality

Solid mechanics

Solids
Stress · Deformation
Compatibility
Finite strain · Infinitesimal strain
Elasticity (linear) · Plasticity
Bending · Hooke's law
Failure theory
Fracture mechanics
Frictionless/Frictional Contact mechanics

Fluid mechanics

Fluids
Fluid statics · Fluid dynamics
Surface tension
Navier–Stokes equations

Viscosity:
Newtonian, Non-Newtonian

dynamics (CFD), is devoted to this approach to solving fluid mechanics problems. Also taking advantage of the highly visual nature of fluid flow is particle image velocimetry, an experimental method for visualizing and analyzing fluid flow.

	Viscoelasticity
	Smart fluids: Magnetorheological Electrorheological Ferrofluids
	Rheometry · Rheometer
Scientists	Bernoulli · Cauchy · Hooke Navier · Newton · Stokes

Contents

Brief history

Main article: History of fluid mechanics

The study of fluid mechanics goes back at least to the days of ancient
G h A hi d i i d fl id i d b d

advancement in fluid mechanics began with Leonardo da Vinci (observation and experiment), Evangelista Torricelli (barometer), Isaac Newton (viscosity) and Blaise Pascal (hydrostatics), and was continued by Daniel Bernoulli with the introduction of mathematical fluid dynamics in *Hydrodynamica* (1738). Inviscid flow was further analyzed by various mathematicians (Leonhard Euler, d'Alembert, Lagrange, Laplace, Poisson) and viscous flow was explored by a multitude of engineers including Poiseuille and Gotthilf Heinrich Ludwig Hagen. Further mathematical justification was provided by Claude-Louis Navier and George Gabriel Stokes in the Navier–Stokes equations, and boundary layers were investigated (Ludwig Prandtl, Theodore von Kármán), while various scientists (Osborne Reynolds, Andrey Kolmogorov, Geoffrey Ingram Taylor) advanced the understanding of fluid viscosity and turbulence.

Relationship to continuum mechanics

Fluid mechanics is a subdiscipline of continuum mechanics, as illustrated in the following table.

<table>
<tr><td rowspan="4">Continuum mechanics
The study of the physics of continuous materials</td><td rowspan="2">Solid mechanics
The study of the physics of continuous materials with a defined rest shape.</td><td colspan="2">Elasticity
Describes materials that return to their rest shape after an applied stress.</td></tr>
<tr><td>Plasticity
Describes materials that permanently deform after a sufficient applied stress.</td><td rowspan="2">Rheology
The study of materials with both solid and fluid characteristics.</td></tr>
<tr><td rowspan="2">Fluid mechanics
The study of the physics of continuous materials which take the shape of their container.</td><td>Non-Newtonian fluids</td></tr>
<tr><td colspan="2">Newtonian fluids</td></tr>
</table>

In a mechanical view, a fluid is a substance that does not support shear stress; that is why a fluid at rest has the shape of its containing vessel. A fluid at rest has no shear stress.

Assumptions

Like any mathematical model of the real world, fluid mechanics makes some basic assumptions about the materials being studied. These assumptions are turned into equations that must be satisfied if the assumptions are to be held true. For example, consider an incompressible fluid in three dimensions. The assumption that mass is conserved means that for any fixed closed surface (such as a sphere) the rate of mass passing from *outside* to *inside* the surface must be the same as rate of mass passing the other way. (Alternatively, the mass *inside* remains constant, as does the mass *outside*). This can be turned into an integral equation over the surface.

Fluid mechanics assumes that every fluid obeys the following:

- Conservation of mass
- Conservation of energy
- Conservation of momentum
- The *continuum hypothesis*, detailed below.

Further, it is often useful (at subsonic conditions) to assume a fluid is incompressible – that is, the density of the fluid does not change.

Similarly, it can sometimes be assumed that the viscosity of the fluid is zero (the fluid is *inviscid*). Gases can often be assumed to be inviscid. If a fluid is viscous, and its flow contained in some way (e.g. in a pipe), then the flow at the boundary must have zero velocity. For a viscous fluid, if the boundary is not porous, the shear forces between the fluid and the boundary results also in a zero velocity for the fluid at the boundary. This is called the no-slip condition. For a porous media otherwise, in the frontier of the containing vessel, the slip condition is not zero velocity, and the fluid has a discontinuous velocity field between the free fluid and the fluid in the porous media (this is related to the Beavers and Joseph condition).

Continuum hypothesis

Main article: Continuum mechanics

continuous. That is, properties such as density, pressure, temperature, and velocity are taken to be well-defined at "infinitely" small points, defining a REV (Reference Element of Volume), at the geometric order of the distance between two adjacent molecules of fluid. Properties are assumed to vary continuously from one point to another, and are averaged values in the REV. The fact that the fluid is made up of discrete molecules is ignored.

The continuum hypothesis is basically an approximation, in the same way planets are approximated by point particles when dealing with celestial mechanics, and therefore results in approximate solutions. Consequently, assumption of the continuum hypothesis can lead to results which are not of desired accuracy. That said, under the right circumstances, the continuum hypothesis produces extremely accurate results.

Those problems for which the continuum hypothesis does not allow solutions of desired accuracy are solved using statistical mechanics. To determine whether or not to use conventional fluid dynamics or statistical mechanics, the Knudsen number is evaluated for the problem. The Knudsen number is defined as the ratio of the molecular mean free path length to a certain representative physical length scale. This length scale could be, for example, the radius of a body in a fluid. (More simply, the Knudsen number is how many times its own diameter a particle will travel on average before hitting another particle). Problems with Knudsen numbers at or above unity are best evaluated using statistical mechanics for reliable solutions.

Navier–Stokes equations

Main article: Navier–Stokes equations

The **Navier–Stokes equations** (named after Claude-Louis Navier and George Gabriel Stokes) are the set of equations that describe the motion of fluid substances such as liquids and gases. These equations state that changes in momentum (force) of fluid particles depend only on the external pressure and internal viscous forces (similar to friction) acting on the fluid. Thus, the Navier–Stokes equations describe the balance of forces acting at any given region of the fluid.

change of the variables of interest. For example, the Navier–Stokes equations for an ideal fluid with zero viscosity states that acceleration (the rate of change of velocity) is proportional to the derivative of internal pressure.

This means that solutions of the Navier–Stokes equations for a given physical problem must be sought with the help of calculus. In practical terms only the simplest cases can be solved exactly in this way. These cases generally involve non-turbulent, steady flow (flow does not change with time) in which the Reynolds number is small.

For more complex situations, such as global weather systems like El Niño or lift in a wing, solutions of the Navier–Stokes equations can currently only be found with the help of computers. This is a field of sciences by its own called computational fluid dynamics.

General form of the equation

The general form of the Navier–Stokes equations for the conservation of momentum is:

$$\rho\frac{D\mathbf{v}}{Dt} = \nabla\cdot\mathbb{P} + \rho\mathbf{f}$$

where

- ρ is the fluid density,
- $\frac{D}{Dt}$ is the substantive derivative (also called the material derivative),
- $\mathbf{v}$ is the velocity vector,
- $\mathbf{f}$ is the body force vector, and
- $\mathbb{P}$ is a tensor that represents the surface forces applied on a fluid particle (the stress tensor).

Unless the fluid is made up of spinning degrees of freedom like vortices, $\mathbb{P}$ is a symmetric tensor. In general, (in three dimensions) $\mathbb{P}$ has the form:

Main article: Newtonian fluid

The constant of proportionality between the shear stress and the velocity gradient is known as the viscosity. A simple equation to describe Newtonian fluid behaviour is

$$\tau = -\mu \frac{dv}{dy}$$

where

τ is the shear stress exerted by the fluid ("drag")
μ is the fluid viscosity – a constant of proportionality
$\frac{dv}{dy}$ is the velocity gradient perpendicular to the direction of shear.

For a Newtonian fluid, the viscosity, by definition, depends only on temperature and pressure, not on the forces acting upon it. If the fluid is incompressible and viscosity is constant across the fluid, the equation governing the shear stress (in Cartesian coordinates) is

$$\tau_{ij} = \mu \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$$

where

τ_{ij} is the shear stress on the i^{th} face of a fluid element in the j^{th} direction
v_i is the velocity in the i^{th} direction
x_j is the j^{th} direction coordinate.

If a fluid does not obey this relation, it is termed a non-Newtonian fluid, of which there are several types.

Among fluids, two rough broad divisions can be made: ideal and non-ideal fluids. An ideal fluid really does not exist, but in some calculations, the

$$\mathbb{P} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

where

- σ are normal stresses,
- τ are tangential stresses (shear stresses).

The above is actually a set of three equations, one per dimension. By themselves, these aren't sufficient to produce a solution. However, adding conservation of mass and appropriate boundary conditions to the system of equations produces a solvable set of equations.

Newtonian versus non-Newtonian fluids

A **Newtonian fluid** (named after Isaac Newton) is defined to be a fluid whose shear stress is linearly proportional to the velocity gradient in the direction perpendicular to the plane of shear. This definition means regardless of the forces acting on a fluid, it *continues to flow*. For example, water is a Newtonian fluid, because it continues to display fluid properties no matter how much it is stirred or mixed. A slightly less rigorous definition is that the drag of a small object being moved slowly through the fluid is proportional to the force applied to the object. (Compare friction). Important fluids, like water as well as most gases, behave — to good approximation — as a Newtonian fluid under normal conditions on Earth.[1]

By contrast, stirring a non-Newtonian fluid can leave a "hole" behind. This will gradually fill up over time – this behaviour is seen in materials such as pudding, oobleck, or sand (although sand isn't strictly a fluid). Alternatively, stirring a non-Newtonian fluid can cause the viscosity to decrease, so the fluid appears "thinner" (this is seen in non-drip paints). There are many types of non-Newtonian fluids, as they are defined to be something that fails to obey a particular property — for example, most fluids with long molecular chains can react in a non-Newtonian manner.[1]

One can group real fluids into Newtonian and non-Newtonian. Newtonian fluids agree with Newton's law of viscosity. Non-Newtonian fluids can be either plastic, bingham plastic, pseudoplastic, dilatant, thixotropic, rheopectic, viscoelatic.

See also

- Aerodynamics
- Applied mechanics
- Secondary flow
- Bernoulli's principle
- Communicating vessels

Notes

1. ^ *a* *b* Batchelor (1967), p. 145.

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External links

Books.html)
- Fluid Mechanics webpage by Falkovich (http://www.weizmann.ac.il/complex/falkovich/fluid-mechanics)
- Annual Review of Fluid Mechanics (http://arjournals.annualreviews.org/loi/fluid)
- CFDWiki (http://www.cfd-online.com/Wiki/Main_Page) – the Computational Fluid Dynamics reference wiki.
- Educational Particle Image Velocimetry – resources and demonstrations (http://www.interactiveflows.com/downloads/)
- prof. DSc Ivan Antonov (http://www.mfantonov.110mb.com) , Technical University of Sofia, Bulgaria.

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Categories: Fluid mechanics

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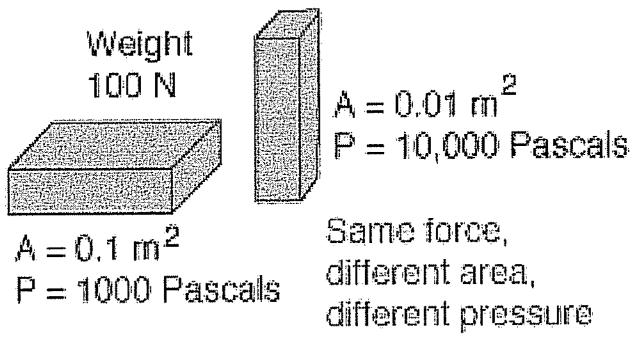
Pressure

Pressure is defined as force per unit area. It is usually more convenient to use pressure rather than force to describe the influences upon fluid behavior. The standard unit for pressure is the Pascal, which is a Newton per square meter.

For an object sitting on a surface, the force pressing on the surface is the weight of the object, but in different orientations it might have a different area in contact with the surface and therefore exert a different pressure.

$$Pressure = \frac{Force}{Area} = \frac{F}{A}$$



Pressure calculation.

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Pressure concepts

There are many physical situations where pressure is the most important variable. If you are peeling an apple, then pressure is the key variable: if the knife is sharp, then the area of contact is small and you can peel with less force exerted on the blade. If you must get an injection, then pressure is the most important variable in getting the needle through your skin: it is better to have a sharp needle than a dull one since the smaller area of contact implies that less force is required to push the needle through the skin.

When you deal with the pressure of a liquid at rest, the medium is treated as a continuous distribution of matter. But when you deal with a gas pressure, it must be approached as an average pressure

Pressure in a fluid can be seen to be a measure of energy per unit volume by means of the definition of work. This energy is related to other forms of fluid energy by the Bernoulli equation.

$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{F \cdot d}{A \cdot d} = \frac{W}{V} = \frac{Energy}{Volume}$$

Pressure as Energy Density

Pressure in a fluid may be considered to be a measure of energy per unit volume or energy density. For a force exerted on a fluid, this can be seen from the definition of pressure:

$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{F \cdot d}{A \cdot d} = \frac{W}{V} = \frac{Energy}{Volume}$$

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Bernoulli concepts

The most obvious application is to the hydrostatic pressure of a fluid, where pressure can be used as energy density alongside kinetic energy density and potential energy density in the Bernoulli equation.

The other side of the coin is that energy densities from other causes can be conveniently expressed as an effective "pressure". For

example, the energy density of solvent molecules which leads to

i i d ti Th d it

pressure.

HyperPhysics***** Mechanics ***** Fluids | R Nave | Go Back

Fluid Kinetic Energy

The kinetic energy of a moving fluid is more useful in applications like the Bernoulli equation when it is expressed as kinetic energy per unit volume

$$\frac{Kinetic\ energy}{Volume} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\rho v^2$$

When the kinetic energy is that of fluid under conditions of laminar flow through a tube, one must take into account the velocity profile to evaluate the kinetic energy. Across the cross-section of flow, the kinetic energy must be calculated using the average of the velocity squared , which is not the same as squaring the average velocity. Expressed in terms of the maximum velocity v_m at the center of the flow, the kinetic energy is

$$\frac{KE}{V} = \frac{1}{2}\rho(v^2)_{avg} = \frac{1}{2}\rho v_{rms}^2 = \frac{1}{2}\rho\frac{v_m^2}{3}$$

Show more detail

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Bernoulli concepts

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Fluid Potential Energy

The potential energy of a moving fluid is more useful in applications like the Bernoulli equation when is expressed as potential energy per unit volume

$$\frac{Potential\ energy}{Volume} = \frac{mgh}{V} = \rho g h$$

The energy density of a fluid can be expressed in terms of this potential energy density along with kinetic energy density and fluid pressure.

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Bernoulli concepts

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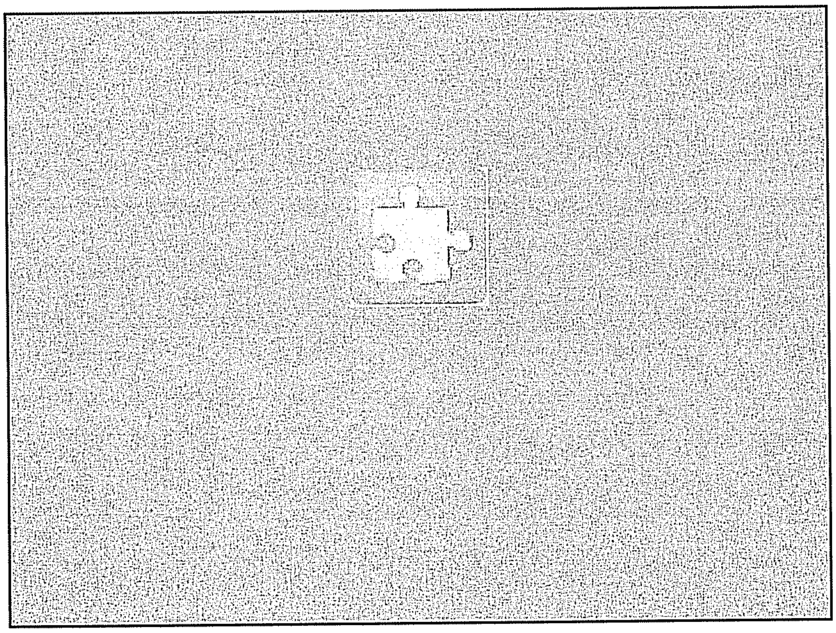
Pascal's Principle

Pascal's principle : Pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid, as well as to the walls of the container.

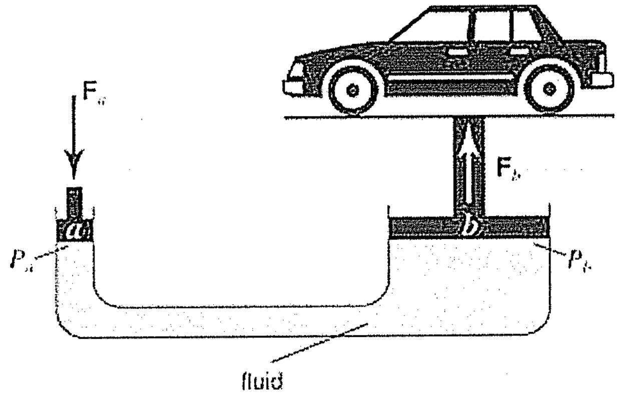
A common application of this is a hydraulic lift used to raise a car off the ground so it can be repaired at a garage. A small force applied to a small-area piston is transformed to a large force at a large-area piston. If a car sits on top of the large piston, it can be lifted by applying a relatively small force to the smaller piston, the ratio of the forces being equal to the ratio of the areas of the pistons.



play | pause | <<step | step>> | reset

Add mass to the small cylinder

Are you getting something for nothing here? Absolutely not. The hydraulic lift is very similar to a lever, where a small force applied through a large distance can move a heavy object a small distance. The work required to lift the heavy object equals the work done by the small force. A lever uses a bar and a fulcrum to transform the work - the fluid does that in the hydraulic lift

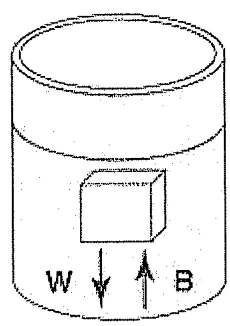


Pascal's principle is used to easily lift a car.

Let the subscripts and denote the quantities at each piston. The press equal; therefore, $P_a = P_b$. Substitute the expression for pressure in term force and area to obtain $f_a / f_b = (b^2 / a^2)$. Substitute π 2 for the are circle, simplify, and solve for : =( )(2/ 2). Because the fo exerted at point is multiplied by the square of the ratio of the radii and a modest force on the small piston can lift a relatively larger weight on

Water commonly provides partial support for any object placed in it. The force on an object placed in a fluid is called the According the magnitude of a buoyant force on a complete partially submerged object always equals the weight of the fluid displace object.

Archimedes' principle can be verified by a nonmathematical argument. C the cubic volume of water in the container of water shown in Figure _ . T volume is in equilibrium with the forces acting on it, which are the weight buoyant force; therefore, the downward force of the weight ( ) must be balanced by the upward buoyant force ( ), which is provided by the rest water in the container.



Concept Questions Archimedes Principle

8.01t

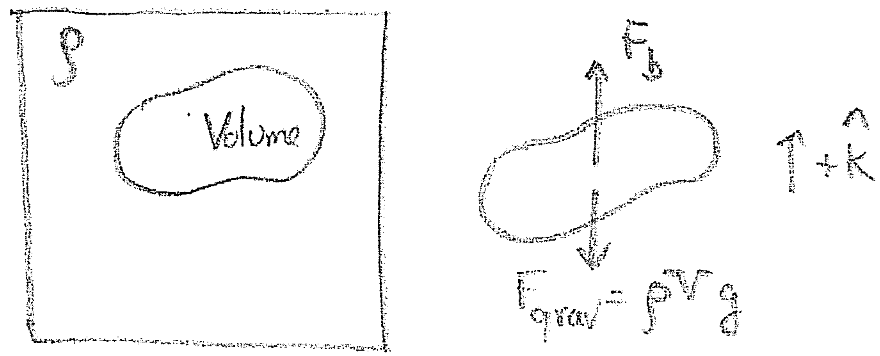
Nov 24, 2004

Pascal's Law

- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel

Archimedes' Principle

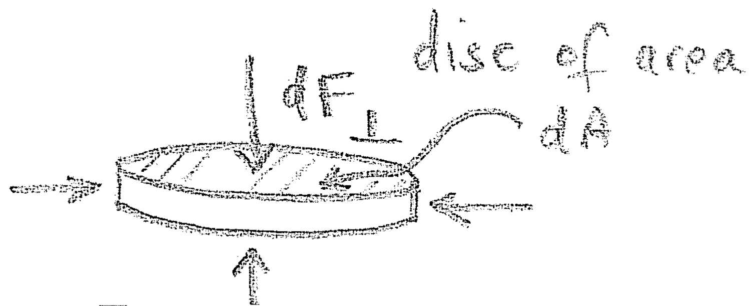
- The buoyant force in a liquid is equal in magnitude to the gravitational force on the displaced volume of liquid



$$F_b = \rho g V$$

Mass Density and Pressure

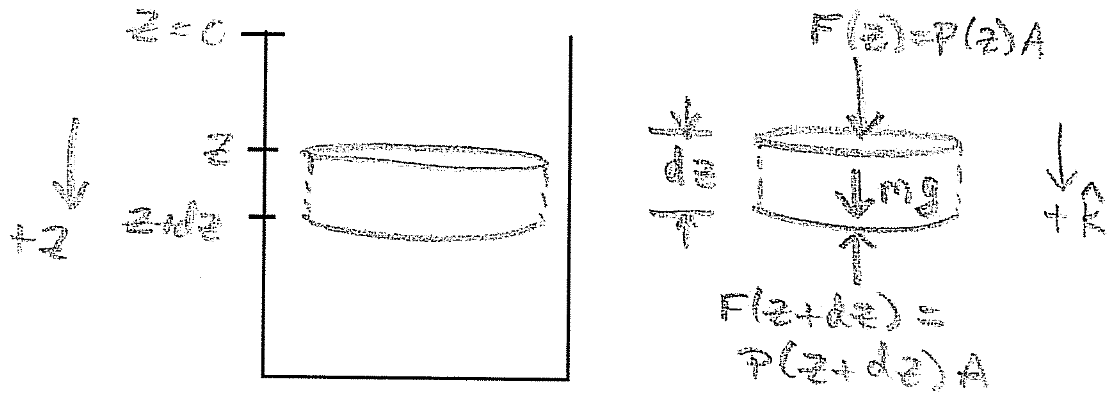
- Mass density = ρ = Mass/volume
- Pressure $P = \dfrac{dF_\perp}{dA}$



- Units: pascal $[Pa] = [N \cdot m^{-2}]$
- bar: $1\,bar = 10^5\,Pa$
- atmosphere $1\,atm = 1.013 \times 10^5\,Pa$

Pressure and Depth

- Fluid with one end open to atmosphere pressure P_0 . Consider a slice of the fluid of cross sectional area A at a depth z and thickness dz.



- Newton's Second Law

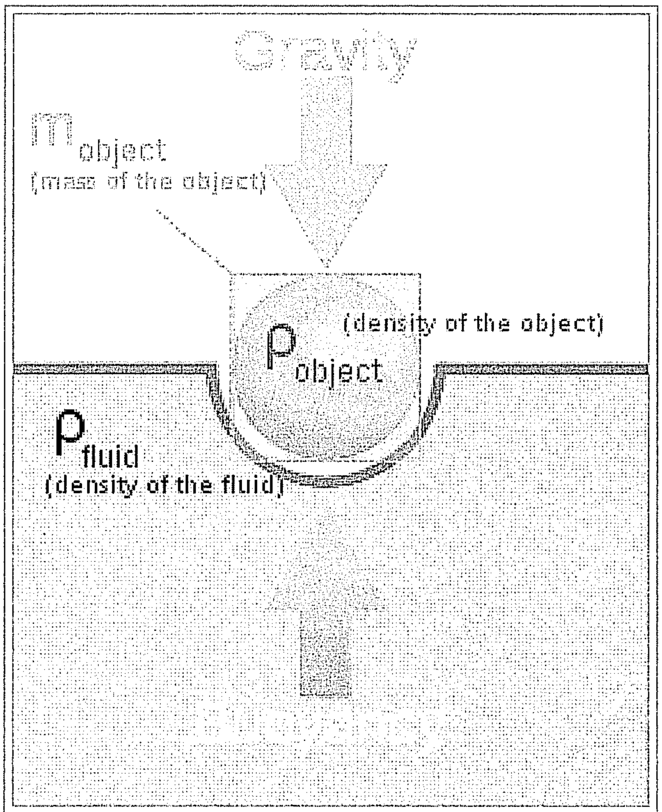
$$A\left(P\left(z\right)-P\left(z+\Delta z\right)\right)+\rho gA\Delta z=0$$

Pressure and Depth

- Fluid Equation $$\frac{P(z+\Delta z)-P(z)}{\Delta z}=\rho g$$
- Definition of derivative $$\frac{dP}{dz}=\lim_{\Delta z\to 0}\frac{P(z+\Delta z)-P(z)}{\Delta z}$$
- Differential equation $$\frac{dP}{dz}=\rho g$$
- Integration $$\int_{P_0}^{P(z)} dP=\int_{z=0}^{z}\rho g dz'$$
- Solution $$P(z)-P_0=\rho g z$$

Buoyancy

From Wikipedia, the free encyclopedia



The forces at work in buoyancy

In physics, **buoyancy** (/ˈbɔɪ.ənsi/) is a force exerted by a fluid that opposes an object's weight. In a column of fluid, pressure increases with depth as a result of the weight of the overlying fluid. Thus a column of fluid, or an object submerged in the fluid, experiences greater pressure at the bottom of the column than at the top. This difference in pressure results in a net force that tends to accelerate an object upwards. The magnitude of that force is proportional to the difference in the pressure between the top and the bottom of the column, and is also equivalent to the weight of the fluid that would otherwise occupy the column. For this reason, an object whose density is greater than that of the fluid in which it is submerged tends to sink. If the object is either less dense than the liquid or is shaped appropriately (as in a boat), the force can keep the object afloat. This can occur only in a reference frame which either has a gravitational field or is accelerating due to a force other than gravity defining a "downward" direction (that is, a non-inertial reference frame). In a situation of fluid statics, the net upward buoyancy force is equal to the magnitude of the weight of fluid displaced by the body.[1]

Contents

Archimedes' principle

Main article: Archimedes' principle

Archimedes' principle is named after Archimedes of Syracuse, who first discovered this law in 212 B.C.[2] His treatise, *On floating bodies*, proposition 5 states:

> Any floating object displaces its own weight of fluid.
>
> — Archimedes of Syracuse[3]

For more general objects, floating and sunken, and in gases as well as liquids (i.e. a fluid), Archimedes' principle may be stated thus in terms of forces:

> Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.
>
> — Archimedes of Syracuse

with the clarifications that for a sunken object the volume of displaced fluid is the volume of the object and for a floating object on a liquid the weight of

More tersely: **Buoyancy = weight of displaced fluid.**

Archimedes' principle does not consider the surface tension (capillarity) acting on the body,[4] but this additional force modifies only the amount of fluid displaced, so the principle that *Buoyancy = weight of displaced fluid* remains valid.

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). In simple terms, the principle states that the buoyancy force on an object is going to be equal to the weight of the fluid displaced by the object, or the density of the fluid multiplied by the submerged volume times the gravitational constant, g. Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Suppose a rock's weight is measured as 10 newtons when suspended by a string in a vacuum with gravity acting upon it. Suppose that when the rock is lowered into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyancy force: 10 − 3 = 7 newtons. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water.

Assuming Archimedes' principle to be reformulated as follows,

$$\text{apparent immersed weight} = \text{weight} - \text{weight of displaced fluid}$$

then inserted into the quotient of weights, which has been expanded by the mutual volume

$$\frac{\text{density}}{\text{density of fluid}} = \frac{\text{weight}}{\text{weight of displaced fluid}},$$

yields the formula below. The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{\text{density of object}}{[illegible]} = \frac{\text{weight}}{[illegible]}$$

(This formula is used for example in describing the measuring principle of a dasymeter and of hydrostatic weighing.)

Example: If you drop wood into water, buoyancy will keep it afloat.

Example: A helium balloon in a moving car. In increasing speed or driving a curve, the air moves in the opposite direction of the car's acceleration. The balloon however, is pushed due to buoyancy "out of the way" by the air, and will actually drift in the same direction as the car's acceleration.

Forces and equilibrium

This is the equation to calculate the pressure inside a fluid in equilibrium. The corresponding equilibrium equation is:

$$\mathbf{f} + \operatorname{div} \sigma = 0$$

where **f** is the force density exerted by some outer field on the fluid, and σ is the stress tensor. In this case the stress tensor is proportional to the identity tensor:

$$\sigma_{ij} = -p\delta_{ij}.$$

Here δ_{ij} is the Kronecker delta. Using this the above equation becomes:

$$\mathbf{f} = \nabla p.$$

Assuming the outer force field is conservative, that is it can be written as the negative gradient of some scalar valued function:

$$\mathbf{f} = -\nabla\Phi.$$

Then:

$$\nabla(p + \Phi) = 0 \Longrightarrow p + \Phi = \text{constant}.$$

Therefore, the shape of the open surface of a fluid equals the equipotential plane of the applied outer conservative force field. Let the z-axis point downward. In this case the field is gravity, so $\Phi = -\rho_f g z$ where g is the

the pressure inside the fluid, when it is subject to gravity, is

$$p = \rho_f g z.$$

So pressure increases with depth below the surface of a liquid, as z denotes the distance from the surface of the liquid into it. Any object with a non-zero vertical depth will have different pressures on its top and bottom, with the pressure on the bottom being greater. This difference in pressure causes the upward buoyancy forces.

The buoyancy force exerted on a body can now be calculated easily, since the internal pressure of the fluid is known. The force exerted on the body can be calculated by integrating the stress tensor over the surface of the body which is in contact with the fluid:

$$\mathbf{B} = \oint \sigma \, d\mathbf{A}$$

The surface integral can be transformed into a volume integral with the help of the Gauss divergence theorem:

$$\mathbf{B} = \int \operatorname{div} \sigma \, dV = -\int \mathbf{f} \, dV = -\rho_f \mathbf{g} \int dV = -\rho_f \mathbf{g} V$$

where V is the measure of the volume in contact with the fluid, that is the volume of the submerged part of the body. Since the fluid doesn't exert force on the part of the body which is outside of it.

The magnitude of buoyancy force may be appreciated a bit more from the following argument. Consider any object of arbitrary shape and volume V surrounded by a liquid. The force the liquid exerts on an object within the liquid is equal to the weight of the liquid with a volume equal to that of the object. This force is applied in a direction opposite to gravitational force, that is of magnitude:

$$B = \rho_f V_{\text{disp}} \, g,$$

where ρ_f is the density of the fluid, V_{disp} is the volume of the displaced body of liquid, and g is the gravitational acceleration at the location in question.

shape, the force the liquid exerts on it must be exactly the same as above. In other words the "buoyancy force" on a submerged body is directed in the opposite direction to gravity and is equal in magnitude to

$$B = \rho_f V g.$$

The net force on the object must be zero if it is to be a situation of fluid statics such that Archimedes principle is applicable, and is thus the sum of the buoyancy force and the object's weight

$$F_{\text{net}} = 0 = mg - \rho_f V_{\text{disp}} g$$

If the buoyancy of an (unrestrained and unpowered) object exceeds its weight, it tends to rise. An object whose weight exceeds its buoyancy tends to sink. Calculation of the upwards force on a submerged object during its accelerating period cannot be done by the Archimedes principle alone; it is necessary to consider dynamics of an object involving buoyancy. Once it fully sinks to the floor of the fluid or rises to the surface and settles, Archimedes principle can be applied alone. For a floating object, only the submerged volume displaces water. For a sunken object, the entire volume displaces water, and there will be an additional force of reaction from the solid floor.

In order for Archimedes' principle to be used alone, the object in question must be in equilibrium (the sum of the forces on the object must be zero), therefore;

$$mg = \rho_f V_{\text{disp}} g,$$

and therefore

$$m = \rho_f V_{\text{disp}}.$$

showing that the depth to which a floating object will sink, and the volume of fluid it will displace, is independent of the gravitational field regardless of geographic location.

(Note: If the fluid in question is seawater, it will not have the same density (ρ) at every location. For this reason, a ship may display a

It can be the case that forces other than just buoyancy and gravity come into play. This is the case if the object is restrained or if the object sinks to the solid floor. An object which tends to float requires a tension restraint force T in order to remain fully submerged. An object which tends to sink will eventually have a normal force of constraint N exerted upon it by the solid floor. The constraint force can be tension in a spring scale measuring its weight in the fluid, and is how apparent weight is defined.

If the object would otherwise float, the tension to restrain it fully submerged is:

$$T = \rho_f V g - m g.$$

When a sinking object settles on the solid floor, it experiences a normal force of:

$$N = m g - \rho_f V g.$$

It is common to define a *buoyancy mass* m_b that represents the effective mass of the object as can be measured by a gravitational method. If an object which usually sinks is submerged suspended via a cord from a balance pan, the reference object on the other dry-land pan of the balance will have mass:

$$m_b = m_o \cdot \left(1 - \frac{\rho_f}{\rho_o}\right)$$

where m_o is the true (vacuum) mass of the object, and ρ_o and ρ_f are the average densities of the object and the surrounding fluid, respectively. Thus, if the two densities are equal, $\rho_o = \rho_f$, the object is seemingly weightless, and is said to be neutrally buoyant. If the fluid density is greater than the average density of the object, the object floats; if less, the object sinks.

Another possible formula for calculating buoyancy of an object is by finding the apparent weight of that particular object in the air (calculated in Newtons), and apparent weight of that object in the water (in Newtons). To find the force of buoyancy acting on the object when in air, using this particular information, this formula applies:

The final result would be measured in Newtons.

Air's density is very small compared to most solids and liquids. For this reason, the weight of an object in air is approximately the same as its true weight in a vacuum. The buoyancy of air is neglected for most objects during a measurement in air because the error is usually insignificant (typically less than 0.1% except for objects of very low average density such as a balloon or light foam).

Stability

A floating object is stable if it tends to restore itself to an equilibrium position after a small displacement. For example, floating objects will generally have vertical stability, as if the object is pushed down slightly, this will create a greater buoyancy force, which, unbalanced by the weight force, will push the object back up.

Rotational stability is of great importance to floating vessels. Given a small angular displacement, the vessel may return to its original position (stable), move away from its original position (unstable), or remain where it is (neutral).

Rotational stability depends on the relative lines of action of forces on an object. The upward buoyancy force on an object acts through the center of buoyancy, being the centroid of the displaced volume of fluid. The weight force on the object acts through its center of gravity. A buoyant object will be stable if the center of gravity is beneath the center of buoyancy because any angular displacement will then produce a 'righting moment'.

Compressible fluids and objects

The atmosphere's density depends upon altitude. As an airship rises in the atmosphere, its buoyancy decreases as the density of the surrounding air decreases. In contrast, as a submarine expels water from its buoyancy tanks, it rises because its volume is constant (the volume of water it displaces if it is fully submerged) while its mass is decreased.

As a floating object rises or falls, the forces external to it change and, as all objects are compressible to some extent or another, so does the object's volume. Buoyancy depends on volume and so an object's buoyancy reduces if it is compressed and increases if it expands.

If an object at equilibrium has a compressibility less than that of the surrounding fluid, the object's equilibrium is stable and it remains at rest. If, however, its compressibility is greater, its equilibrium is then unstable, and it rises and expands on the slightest upward perturbation, or falls and compresses on the slightest downward perturbation.

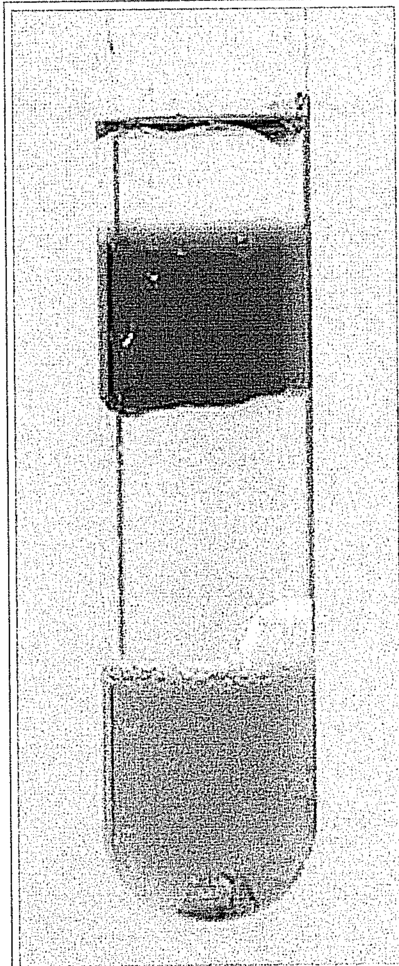
Submarines rise and dive by filling large tanks with seawater. To dive, the tanks are opened to allow air to exhaust out the top of the tanks, while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has neutral buoyancy and will remain at that depth.

The height of a balloon tends to be stable. As a balloon rises it tends to increase in volume with reducing atmospheric pressure, but the balloon's cargo does not expand. The average density of the balloon decreases less, therefore, than that of the surrounding air. The balloon's buoyancy decreases because the weight of the displaced air is reduced. A rising balloon tends to stop rising. Similarly, a sinking balloon tends to stop sinking.

Density

A pound coin floats in mercury due to the buoyancy force upon it.

If the weight of an object is less than the weight of the displaced fluid when fully submerged, then the object has an average density that is less than the fluid and when fully submerged will experience a buoyancy force greater than its own weight. If the fluid has a surface, such as water in a lake or the sea, the object will float and settle at a level where it displaces the same weight of fluid as the weight of the object. If the object is i d i th fl id h exactly the same density as the fluid, then its buoyancy equals its weight. It will remain submerged in the fluid, but it will neither sink nor float, although a disturbance in either direction will cause it to drift away from its position. An object with a higher average density than the fluid will never experience more buoyancy than weight and it will sink. A ship will float even though it may be made of steel (which is much denser than water), because it encloses a volume of air (which is much less dense than water), and the resulting shape has an average density less than that of the water.



A density column containing some common liquids and solids. From top: baby oil, rubbing alcohol (with red food coloring), vegetable oil, wax, water (with blue food coloring), and aluminum.

Beyond Archimedes' principle

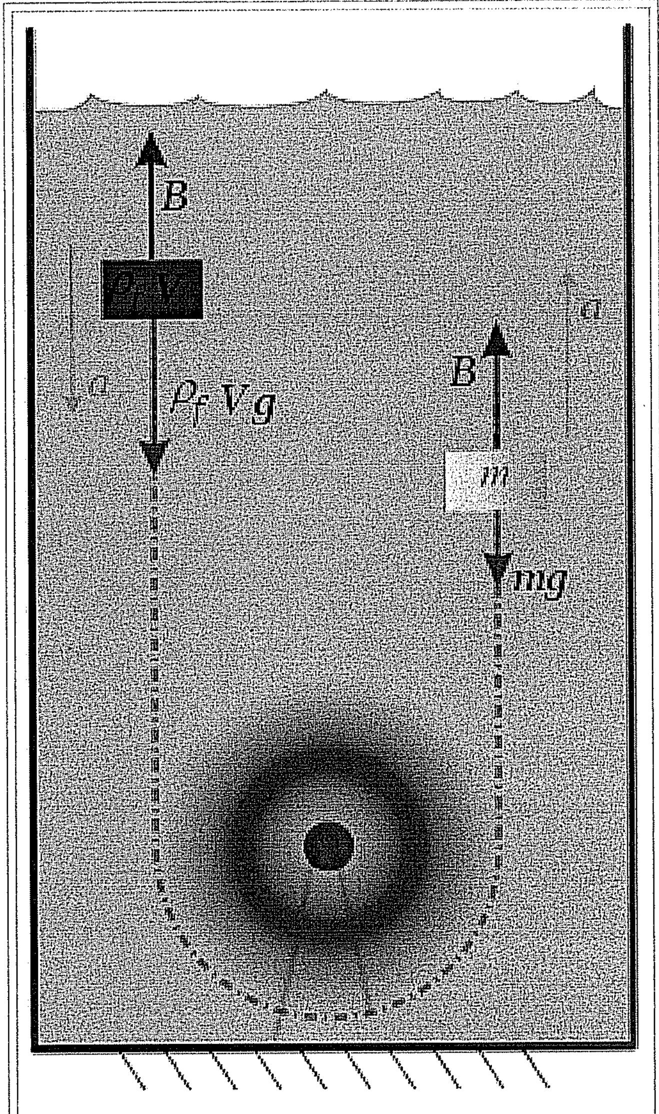
Archimedes' principle is a fluid statics concept. In its simple form, it applies when the object is not accelerating relative to the fluid. To examine the case when the object is accelerated by buoyancy and gravity, the fact that the displaced fluid itself has inertia as well must be considered.[5]

This means that both the buoyant object and a parcel of fluid (equal in volume to the object) will experience the same magnitude of buoyancy force because of Newton's third law, and will experience the same acceleration, but in opposite directions, since the total volume of the system is unchanged. In each case, the difference between magnitudes of the buoyancy force and the force of gravity is the net force, and when divided by the relevant mass, it will yield the respective acceleration through Newton's second law. All acceleration measures are relative to the reference frame of the undisturbed background fluid.

Atwood's machine analogy

The system can be understood by analogy with a suitable modification of fluid and the buoyant object, as shown in the diagram right.

- The solid object is represented by the gray object
- The fluid being displaced is represented by dark blue object
- Undisturbed background fluid is analogous to the inextensible massless cord
- The force of buoyancy is analogous to the tension in the cord
- The solid floor of the body of fluid is analogous to the pulley, and reverses the direction of the buoyancy force, such that both the solid object and the displaced fluid experience their buoyancy force upward.



Atwood's Machine Analogy for dynamics of buoyant objects in vertical motion. The displaced parcel of fluid is indicated as the dark blue rectangle, and the buoyant solid object is indicated as the gray object. The acceleration vectors (a) in this visual depict a positively buoyant object which naturally accelerates upward, and upward acceleration of the object is our sign convention.

Results

It is important to note that this simplification of the situation completely ignores drag and viscosity, both of which come in to play to a greater extent as speed increases, when considering the dynamics of buoyant objects. The following simple formulation makes the assumption of slow speeds such that drag and viscosity are not significant. It is difficult to carry out such an experiment in practice with speeds close to zero, but if measurements of acceleration are made as quickly as possible after release from rest the equations below give

A system consists of a well-sealed object of mass m and volume V which is fully submerged in a uniform fluid body of density ρ_f and in an environment of a uniform gravitational field g. Under the forces of buoyancy and gravity alone, the "dynamic buoyancy force" B acting on the object and its upward acceleration a are given by:

Buoyancy force

$$B = \frac{2gm\rho_f V}{m + \rho_f V}$$

Upward acceleration

$$a = \frac{g(\rho_f V - m)}{m + \rho_f V}$$

Derivations of both of these equations originates from constructing a system of equations by means of Newton's second law for both the solid object and the displaced parcel of fluid. An equation for upward acceleration of the object is constructed by dividing the net force on the object ($B - mg$) by its mass m. Due to the mechanical coupling, the object's upward acceleration is equal in magnitude to the downward acceleration of the displaced fluid, an equation constructed by dividing the net force on the displaced fluid ($B - \rho_f V g$) by its mass $\rho_f V$.

Should other forces come in to play in a different situation (such as spring forces, human forces, thrust, drag, or lift), it is necessary for the solver of problem to re-consider the construction of Newton's second law and the mechanical coupling conditions for both bodies, now involving these other forces. In many situations turbulence will introduce other forces that are much more complex to calculate.

In the case of neutral buoyancy, m is equal to $\rho_f V$. Thus B reduces to mg and the acceleration is zero. If the object is much denser than the fluid, then B approaches zero and the object's upward acceleration is approximately $-g$, i.e. it is accelerated downward due to gravity as if the fluid were not present. As an example, a pellet of osmium falling through air will initially accelerate at 99.98% of g downward, though this will reduce as speed increases. Similarly, if the fluid is much denser than the object, then B approaches $2mg$

98.5% g.

See also

- Buoy
- Brunt–Väisälä frequency
- Buoyancy compensator (diving)
- Buoyancy compensator (aviation)
- Cartesian diver
- Dasymeter
- Diving weighting system
- Hydrostatics
- Hull (ship)
- Hydrometer
- Hydrostatic weighing
- Lighter than air
- Naval architecture
- Plimsoll line
- Pontoon
- Quicksan
- Salt fingering
- Submarin
- Swim bladder
- Thrust
- Air

References

1. ^ Note: In the absence of surface tension, the mass of fluid displaced is equal to the submerged volume multiplied by the fluid density. High repulsive surface tension will cause the body to float higher than expected, though the same total volume will be displaced, but at a greater distance from the object. Where there is doubt about the meaning of "volume of fluid displaced", this should be interpreted as the overflow from a full container when the object is floated in it, or as the volume of the object below the average level of the fluid.
2. ^ Acott, Chris (1999). "The diving "Law-ers": A brief resume of their lives." (http://archive.rubicon-foundation.org/5990) . *South Pacific Underwater Medicine Society journal* **29** (1). ISSN 0813-1988 (http://www.worldcat.org/issn/0813-1988) . OCLC 16986801 (http://www.worldcat.org/oclc/16986801) . http://archive.rubicon-foundation.org/5990. Retrieved 2009-06-13..
3. ^ "The works of Archimedes" (http://www.archive.org/stream/worksofarchimede00arch#page/256/mode/2up) . p. 257. http://www.archive.org/stream/worksofarchimede00arch#page/256/mode/2up. Retrieved 11 March 2010. "Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid

hydrophobic particles to congregate at specific points on a wave" (http://www.weizmann.ac.il/home/fnfal/papers/Natfloat.pdf) (PDF). 2005-06-23. http://www.weizmann.ac.il/home/fnfal/papers/Natfloat.pdf.
5. ^ Smid, Thomas: "The Dynamics of Buoyant Objects" (http://www.physicsmyths.org.uk/buoyancy.htm) *Physicsmyths.org.uk*

External links

- Falling in Water (http://www.newton.dep.anl.gov/askasci/phy99/phy99x88.htm)
- Archimedes' Principle (http://www.juliantrubin.com/bigten/archimedesprinciple.html) – background and experiment
- BuoyancyQuest (http://www.buoyancyquest.com) (a website featuring buoyancy control videos)

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Categories: Fundamental physics concepts | Underwater diving | Introductory physics | Ship construction | Airship technology

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Surface tension

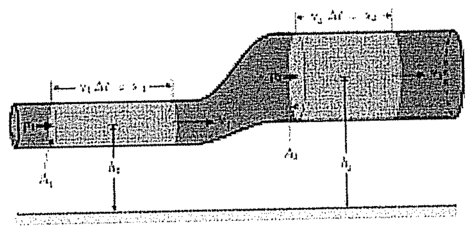
From Wikipedia, the free encyclopedia

Surface tension is a property of the surface of a liquid that allows it to resist an external force. It is revealed, for example, in floating of some objects on the surface of water, even though they are denser than water, and in the ability of some insects (e.g. water striders) to run on the water surface. This property is caused by cohesion of similar molecules, and is responsible for many of the behaviors of liquids.

Surface tension has the dimension of force per unit length, or of energy per unit area. The two are equivalent—but when referring to energy per unit of area, people use the term surface energy—which is a more general term in the sense that it applies also to solids and not just liquids.

In materials science, surface tension is used for either surface stress or surface free energy.

Continuum mechanics



Laws

Conservation of mass
Conservation of momentum
Conservation of energy
Entropy inequality

Solid mechanics

Solids
Stress · Deformation
Compatibility
Finite strain · Infinitesimal strain
Elasticity (linear) · Plasticity
Bending · Hooke's law
Failure theory
Fracture mechanics
Frictionless/Frictional Contact mechanics

Fluid mechanics

Fluids
Fluid statics · Fluid dynamics
Surface tension
Navier–Stokes equations

Viscosity:
Newtonian, Non-Newtonian

Rheology

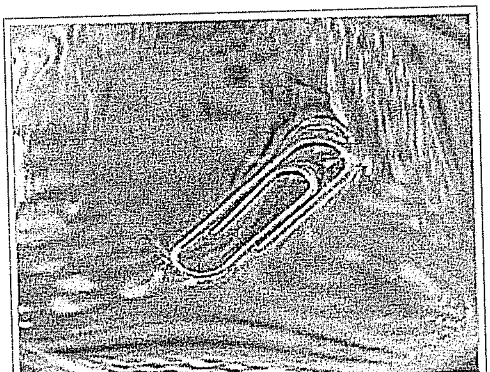
Viscoelasticity

Smart fluids:
Magnetorheological
Electrorheological
Ferrofluids

Contents

Bernoulli · Cauchy · Hooke
Navier · Newton · Stokes

Causes



Surface tension prevents the paper clip from submerging.

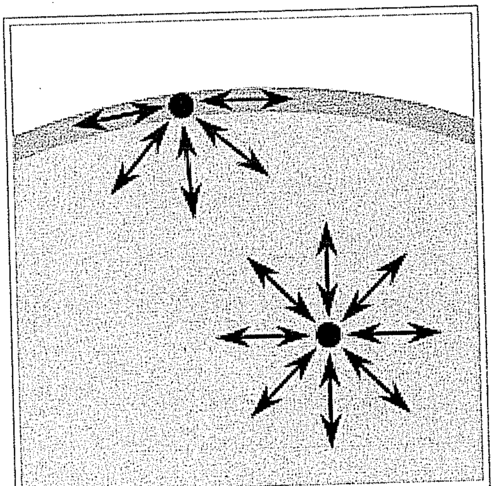


Diagram of the forces on two molecules of liquid

The cohesive forces among the liquid molecules are responsible for this phenomenon of surface tension. In the bulk of the liquid, each molecule is pulled equally in every direction by neighboring liquid molecules, resulting in a net force of zero. The molecules at the surface do not have other molecules on all sides of them and therefore are pulled inwards This creates some internal pressure and forces liquid surfaces to contract to the

Surface tension is responsible for the shape of liquid droplets. Although easily deformed, droplets of water tend to be pulled into a spherical shape by the cohesive forces of the surface layer. In the absence of other forces, including gravity, drops of virtually all liquids would be perfectly spherical. The spherical shape minimizes the necessary "wall tension" of the surface layer according to Laplace's law.

Another way to view it is in terms of energy. A molecule in contact with a neighbor is in a lower state of energy than if it were alone (not in contact with a neighbor). The interior molecules have as many neighbors as they can possibly have, but the boundary molecules are missing neighbors (compared to interior molecules) and therefore have a higher energy. For the liquid to minimize its energy state, the number of higher energy boundary molecules must be minimized. The minimized quantity of boundary molecules results in a minimized surface area.[1]

As a result of surface area minimization, a surface will assume the smoothest shape it can (mathematical proof that "smooth" shapes minimize surface area relies on use of the Euler–Lagrange equation). Since any curvature in the surface shape results in greater area, a higher energy will also result. Consequently the surface will push back against any curvature in much the same way as a ball pushed uphill will push back to minimize its gravitational potential energy.

Effects in everyday life

Water

Several effects of surface tension can be seen with ordinary water:

A. Beading of rain water on the surface of a waxy surface, such as a leaf. Water adheres weakly to wax and strongly to itself, so water clusters into drops. Surface tension gives them their near-spherical shape, because a sphere has the smallest possible surface area to volume ratio.

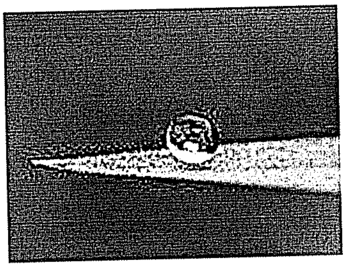
B. Formation of drops occurs when a mass of liquid is stretched. The animation shows water adhering to the faucet gaining mass until it is stretched to a point where the surface tension can no longer bind it to the faucet. It then separates and surface tension forms the drop into a sphere. If a stream of water were running from the faucet, the stream would break up into drops during its fall. Gravity stretches the stream, then surface tension pinches it into spheres.[2]

C. Floatation of objects denser than water occurs when the object is nonwettable and its weight is small enough to be borne by the forces arising from surface tension.[1] For example, water striders use surface tension to walk on the surface of a pond. The surface of the water behaves like an elastic film: the insect's feet cause indentations in the water's surface, increasing its surface area.[3]

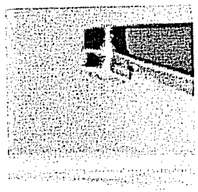
i il d (i hi d li id) i d b i i

tension", but its physics are the same.

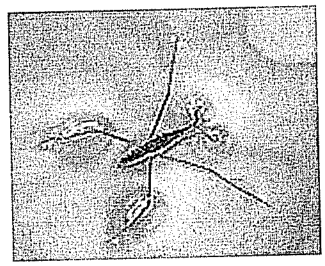
E. Tears of wine is the formation of drops and rivulets on the side of a glass containing an alcoholic beverage. Its cause is a complex interaction between the differing surface tensions of water and ethanol; it is induced by a combination of surface tension modification of water by ethanol together with ethanol evaporating faster than water.



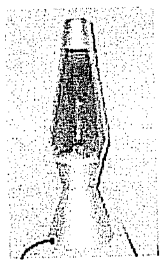
A. Water beading on a leaf



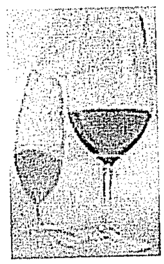
B. Water dripping from a tap



C. Water striders stay atop the liquid because of surface tension



D. Lava lamp with interaction between dissimilar liquids; water and liquid wax



E. Photo showing the "tears of wine" phenomenon.

Surfactants

Surface tension is visible in other common phenomena, especially when surfactants are used to decrease it:

- Soap bubbles have very large surface areas with very little mass. Bubbles in pure water are unstable. The addition of surfactants, however, can have a stabilizing effect on the bubbles (see Marangoni effect). Notice that surfactants actually reduce the surface tension of water by a factor of three or more.
- Emulsions are a type of solution in which surface tension plays a role. Tiny fragments of oil suspended in pure water will spontaneously assemble themselves

(or vice versa).

Basic physics

Two definitions

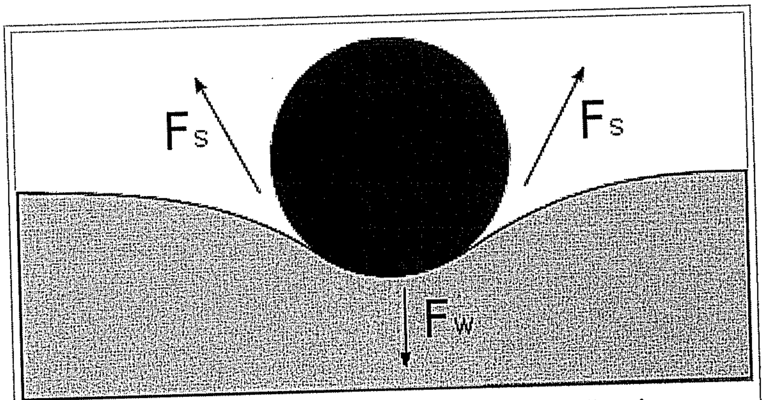


Diagram shows, in cross-section, a needle floating on the surface of water. Its weight, F_w, depresses the surface, and is balanced by the surface tension forces on either side, F_s, which are each parallel to the water's surface at the points where it contacts the needle. Notice that the horizontal components of the two F_s arrows point in opposite directions, so they cancel each other, but the vertical components point in the same direction and therefore add up[1] to balance F_w.

Surface tension, represented by the symbol γ is defined as the force along a line of unit length, where the force is parallel to the surface but perpendicular to the line. One way to picture this is to imagine a flat soap film bounded on one side by a taut thread of length, L. The thread will be pulled toward the interior of the film by a force equal to $2\gamma L$ (the factor of 2 is because the soap film has two sides, hence two surfaces).[4] Surface tension is therefore measured in forces per unit length. Its SI unit is newton per meter but the cgs unit of dyne per cm is also used.[5] One dyn/cm corresponds to 0.001 N/m.

An equivalent definition, one that is useful in thermodynamics, is work done per unit area. As such, in order to increase the surface area of a mass of liquid by an amount, δA, a quantity of work, $\gamma\delta A$, is needed.[4] This work is stored as potential energy. Consequently surface tension can be also measured in SI system as joules per square meter and in the cgs system as ergs per cm^2. Since mechanical systems try to find a state of minimum potential energy, a free droplet of liquid naturally assumes a spherical shape, which has the minimum surface area for a given volume.

The equivalence of measurement of energy per unit area to force per unit length can be proven by dimensional analysis.[4]

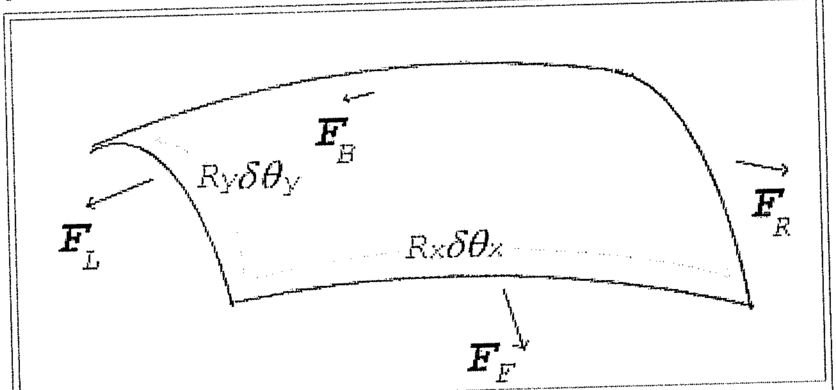
Surface curvature and pressure

If no force acts normal to a tensioned surface, the surface must remain flat. But if the pressure on one side of the surface differs from pressure on the other side, the pressure difference times surface area results in a normal force. In order for the surface tension forces to cancel the force due to pressure, the surface must be curved. The diagram shows how surface curvature of a tiny patch of surface leads to a net component of surface tension forces acting normal to the center of the patch. When all the forces are balanced, the resulting equation is [6]

$$\Delta p = \gamma \left(\frac{1}{R_x} + \frac{1}{R_y} \right)$$

where:

- Δp is the pressure difference.
- γ is surface tension.
- R_x and R_y are radii of curvature in each of the axes that are parallel to the surface.



Surface tension forces acting on a tiny (differential) patch of surface. $\delta\theta_x$ and $\delta\theta_y$ indicate the amount of bend over the dimensions of the patch. Balancing the tension forces with pressure leads to the Young–Laplace equation

The quantity in parentheses on the right hand side is in fact (twice) the mean curvature of the surface (depending on normalisation).

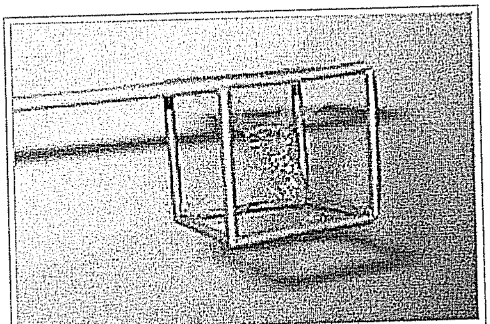
Solutions to this equation determine the shape of water drops, puddles, menisci, soap bubbles, and all other shapes determined by surface tension (such as the shape of the impressions that a water strider's feet make on the surface of a pond).

The table below shows how the internal pressure of a water droplet increases with decreasing radius. For not very small drops the effect is subtle, but the pressure difference becomes enormous when the drop sizes approach the molecular size. (In the limit of a single molecule the concept becomes meaningless.)

Δp for water drops of different radii at STP				
Droplet radius	1 mm	0.1 mm	1 μm	10 nm
Δp (atm)	0.0014	0.0144	1.436	143.6

Liquid surface

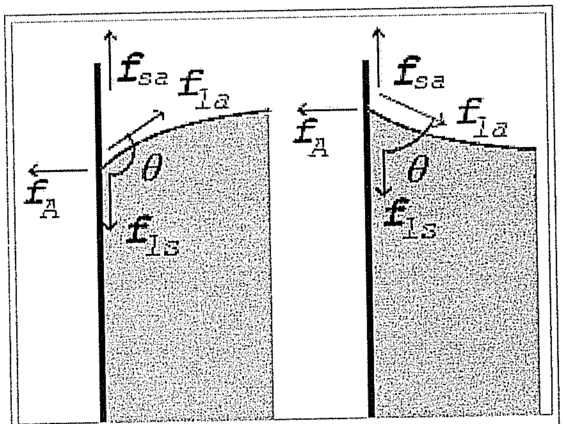
To find the shape of the minimal surface bounded by some arbitrary shaped frame using strictly mathematical means can be a daunting task. Yet by fashioning the frame out of wire and dipping it in soap-solution, a locally minimal surface will appear in the resulting soap-film within seconds.[4][7]


Minimal surface

The reason for this is that the pressure difference across a fluid interface is proportional to the mean curvature, as seen in the Young-Laplace equation. For an open soap film, the pressure difference is zero, hence the mean curvature is zero, and minimal surfaces have the property of zero mean curvature.

Main article: Contact angle

The surface of any liquid is an interface between that liquid and some other medium.[note 1] The top surface of a pond, for example, is an interface between the pond water and the air. Surface tension, then, is not a property of the liquid alone, but a property of the liquid's interface with another medium. If a liquid is in a container, then besides the liquid/air interface at its top surface, there is also an interface between the liquid and the walls of the container. The surface tension between the liquid and air is usually different (greater than) its surface tension with the walls of a container. And where the two surfaces meet, their geometry must be such that all forces balance.[4][6]



Forces at contact point shown for contact angle greater than 90° (left) and less than 90° (right)

Where the two surfaces meet, they form a contact angle, θ, which is the angle the tangent to the surface makes with the solid surface. The diagram to the right shows two examples. Tension forces are shown for the liquid-air interface, the liquid-solid interface, and the solid-air interface. The example on the left is where the difference between the liquid-solid and solid-air surface tension, $\gamma_{ls} - \gamma_{sa}$, is less than the liquid-air surface tension, γ_{la}, but is nevertheless positive, that is

$$\gamma_{la} > \gamma_{ls} - \gamma_{sa} > 0$$

In the diagram, both the vertical and horizontal forces must cancel exactly at the contact point, known as equilibrium. The horizontal component of f_{la} is canceled by the adhesive force, f_A.[4]

$$f_A = f_{la} \sin\theta$$

The more telling balance of forces, though, is in the vertical direction. The vertical component of f_{la} must exactly cancel the force, f_{ls}.[4]

$$f_{ls} - f_{sa} = -f_{la} \cos\theta$$

Since the forces are in direct proportion to their respective surface tensions, we also have:[6]

$$\gamma_{ls} - \gamma_{sa} = -\gamma_{la} \cos\theta$$

where

- γ_{ls} is the liquid-solid surface tension,
- γ_{la} is the liquid-air surface tension,

Liquid	Solid	Contact angle
water ethanol diethyl ether carbon tetrachloride glycerol acetic acid	soda-lime glass lead glass fused quartz	0°
water	paraffin wax	107°
	silver	90°
	soda-lime glass	29°

- θ is the contact angle, where a concave meniscus has contact angle less than 90° and a convex meniscus has contact angle of greater than 90°.[4]

mercury	soda-lime glass	140°
Some liquid-solid contact angles[4]		

This means that although the difference between the liquid-solid and solid-air surface tension, $\gamma_{ls} - \gamma_{sa}$, is difficult to measure directly, it can be inferred from the liquid-air surface tension, γ_{la}, and the equilibrium contact angle, θ, which is a function of the easily measurable advancing and receding contact angles (see main article contact angle).

This same relationship exists in the diagram on the right. But in this case we see that because the contact angle is less than 90°, the liquid-solid/solid-air surface tension difference must be negative:

$$\gamma_{la} > 0 > \gamma_{ls} - \gamma_{sa}$$

Special contact angles

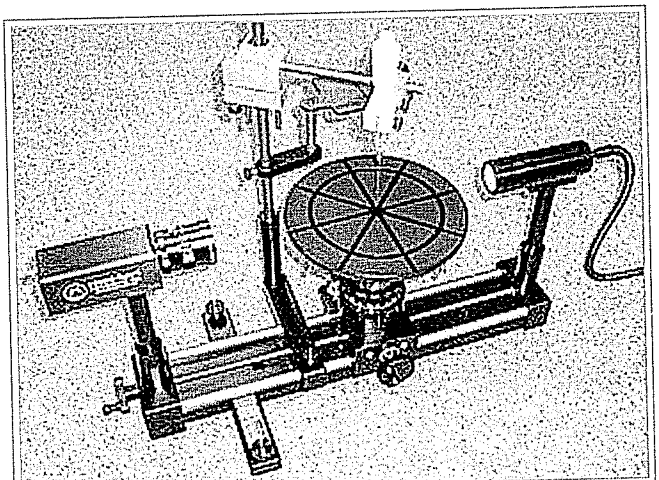
Observe that in the special case of a water-silver interface where the contact angle is equal to 90°, the liquid-solid/solid-air surface tension difference is exactly zero.

Another special case is where the contact angle is exactly 180°. Water with specially prepared Teflon approaches this.[6] Contact angle of 180° occurs when the liquid-solid surface tension is exactly equal to the liquid-air surface tension.

$$\gamma_{la} = \gamma_{ls} - \gamma_{sa} > 0 \qquad \theta = 180^\circ$$

Methods of measurement

Because surface tension manifests itself in various effects, it offers a number of paths to its measurement. Which method is optimal depends upon the nature of the liquid being measured, the conditions under which its tension is to be measured, and the stability of its surface when it is deformed.



Surface tension can be measured using the pendant drop method on a goniometer.

- Du Noüy Ring method: The traditional method used to measure surface or interfacial tension. Wetting properties of the surface or interface have little influence on this measuring technique. Maximum pull exerted on the ring by the surface is measured.[8]

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(http://www.planeandpilotmag.com/component/zine/article/289.html) Retrieved on 2009-11-26

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34. ^ Batchelor, G.K. (1967). Sections 3.5 and 5.1
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36. ^ Weltner, Klaus; Ingelman-Sundberg, Martin. "Physics of Flight – reviewed" (http://user.uni-frankfurt.de/~weltner/Flight/PHYSIC4.htm) . http://user.uni-frankfurt.de/~weltner/Flight/PHYSIC4.htm. "The conventional explanation of aerodynamical lift based on Bernoulli's law and velocity differences mixes up *cause* and *effect*. The faster flow at the upper side of the wing is the consequence of low pressure and not its cause."

Notes

1. ^ Clancy, L.J., *Aerodynamics*, Section 5.5 ("When a stream of air flows past an airfoil, there are local changes in flow speed round the airfoil, and consequently changes in static pressure, in accordance with Bernoulli's Theorem. The distribution of pressure determines the lift, pitching moment and form drag of the airfoil, and the position of its centre of pressure.")

Further reading

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- Lamb, H. (1993). *Hydrodynamics* (6th ed.). Cambridge University Press. ISBN 9780521458689. Originally published in 1879; the 6th extended edition appeared first in 1932.
- Chanson, H. (2009). *Applied Hydrodynamics: An Introduction to Ideal and Real Fluid Flows* (http://www.uq.edu.au/~e2hchans/reprints/book15.htm) . CRC Press, Taylor & Francis Group. ISBN 978-0-415-49271-3. http://www.uq.edu.au/~e2hchans/reprints/book15.htm.

External links

- Interactive animation demonstrating Bernoulli's principle (http://home.earthlink.net/~mmc1919/venturi.html)
- Denver University – Bernoulli's equation and pressure measurement (http://mysite.du.edu/~jcalvert/tech/fluids/bernoul.htm)
- Millersville University – Applications of Euler's equation (http://www.millersville.edu/~jdooley/macro/macrohyp/eulerap/eulap.htm)
- Nasa – Beginner's guide to aerodynamics (http://www.grc.nasa.gov/WWW/K-12/airplane/bga.html)
- Misinterpretations of Bernoulli's equation – Weltner and Ingelman-Sundberg (http://user.uni-frankfurt.de/~weltner/Mis6/mis6.html)
- Video demonstration of levitating ping pong ball using Bernoulli principle (http://www.physics.org//interact/physics-to-go/bernoulli-balls/index.html)

Retrieved from "http://en.wikipedia.org/w/index.php?title=Bernoulli%27s_principle&oldid=469369736"

Categories: Aerodynamics | Equations of fluid dynamics | Fluid dynamics | Principles |

frankfurt.de/~weltner/Mis6/mis6.html) , http://user.uni-frankfurt.de/~weltner/Mis6/mis6.html

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8. ^ Mulley, Raymond (2004). *Flow of Industrial Fluids: Theory and Equations*. CRC Press. ISBN 0849327679., 410 pages. See pp. 43–44.
9. ^ Chanson, Hubert (2004). *Hydraulics of Open Channel Flow: An Introduction*. Butterworth-Heinemann. ISBN 0750659785., 650 pages. See p. 22.
10. ^ Oertel, Herbert; Prandtl, Ludwig; Böhle, M.; Mayes, Katherine (2004). *Prandtl's Essentials of Fluid Mechanics*. Springer. pp. 70–71. ISBN 0387404376.
11. ^ "Bernoulli's Equation" (http://www.grc.nasa.gov/WWW/K-12/airplane/bern.htm) . NASA Glenn Research Center. http://www.grc.nasa.gov/WWW/K-12/airplane/bern.htm. Retrieved 2009-03-04.
12. ^ ***a b*** Clancy, L.J., *Aerodynamics*, Section 3.5.
13. ^ Clancy, L.J. *Aerodynamics*, Equation 3.12
14. ^ ***a b*** Batchelor, G.K. (1967), p. 383
15. ^ Clarke C. and Carswell B., *Astrophysical Fluid Dynamics*
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17. ^ Van Wylen, G.J., and Sonntag, R.E., (1965), *Fundamentals of Classical Thermodynamics*, Section 5.9, John Wiley and Sons Inc., New York
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22. ^ Resnick, R. and Halliday, D. (1960), *Physics*, Section 18–5, John Wiley & Sons, Inc., New York (" [streamlines] are closer together above the wing than they are below so that Bernoulli's principle predicts the observed upward dynamic lift.")
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About.com Physics

Contact Angle & Capillarity - Liquid in a Vertical Tube

By Andrew Zimmerman Jones, About.com Guide

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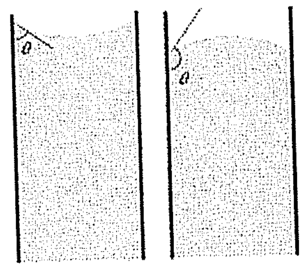
(Continued from Page 2)

Contact Angle

Surface tension occurs during a gas-liquid interface, but if that interface comes in contact with a solid surface - such as the walls of a container - the interface usually curves up or down near that surface. Such a concave or convex surface shape is known as a *meniscus*



Determining the contact angle, theta, of liquid in a vertical tube.
Public domain

The contact angle, *theta*, is determined as shown in the picture to the right.

The contact angle can be used to determine a relationship between the liquid-solid surface tension and the liquid-gas surface tension, as follows:

$$gamma_{ls} = - gamma_{lg} \cos theta$$

where

- $gamma_{ls}$ is the liquid-solid surface tension
- $gamma_{lg}$ is the liquid-gas surface tension
- *theta* is the contact angle

One thing to consider in this equation is that in cases where the meniscus is convex (i.e. the contact angle is greater than 90 degrees), the cosine component of this equation will be negative which means that the liquid-solid surface tension will be positive.

If, on the other hand, the meniscus is concave (i.e. dips down, so the contact angle is less than 90 degrees), then the cos *theta* term is positive, in which case the relationship would result in a *negative* liquid-solid surface tension!

What this means, essentially, is that the liquid is adhering to the walls of the container and is working to maximize the area in contact with solid surface, so as to minimize the overall potential energy.

Capillarity

Another effect related to water in vertical tubes is the property of capillarity, in which the surface of liquid becomes elevated or depressed within the tube in relation to the surrounding liquid. This, too, is related to the contact angle observed.

$y = (2\ gamma_{lg} \cos theta) / (dgr)$

where

- y is the vertical displacement (up if positive, down if negative)
- $gamma_{lg}$ is the liquid-gas surface tension
- *theta* is the contact angle
- d is the density of the liquid
- g is the acceleration of gravity
- r is the radius of the capillary

NOTE: Once again, if *theta* is greater than 90 degrees (a convex meniscus), resulting in a negative liquid-solid surface tension, the liquid level will go down compared to the surrounding level, as opposed to rising in relation to it.

Capillarity manifests in many ways in the everyday world. Paper towels absorb through capillarity. When burning a candle, the melted wax rises up the wick due to capillarity. In biology, though blood is pumped throughout the body, it is this process which distributes blood in the smallest blood vessels which are called, appropriately, *capillaries*.

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Surface Tension

By Andrew Zimmerman Jones, About.com Guide

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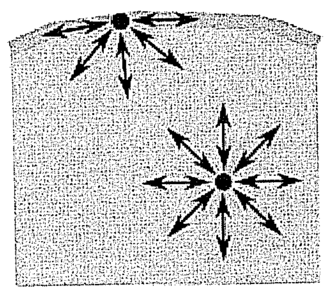
Surface tension is a phenomenon in which the surface of a liquid, where the liquid is in contact with gas, acts like a thin elastic sheet. This term is typically used only when the liquid surface is in contact with gas (such as the air). If the surface is between two liquids (such as water and oil), it is called "interface tension."


The forces acting on a liquid that cause surface tension.

Causes of Surface Tension

Various intermolecular forces, such as Van der Waals forces, draw the liquid particles together. Along the surface, the particles are pulled toward the rest of the liquid, as shown in the picture to the right.

Surface tension (denoted with the Greek variable) is defined as the ratio of the surface force to the length along which the force acts:

= /

Units of Surface Tension

Surface tension is measured in SI units of N/m (newton per meter), although the more common unit is the cgs unit dyn/cm (dyne per centimeter).

In order to consider the thermodynamics of the situation, it is sometimes useful to consider it in terms of work per unit area. The SI unit in that case is the J/m^2 (joules per meter squared). The cgs unit is erg/cm^2.

These forces bind the surface particles together. Though this binding is weak - it's pretty easy to break the surface of a liquid after all - it does manifest in many ways.

Examples of Surface Tension

Drops of water. When using a water dropper, the water does not flow in a continuous stream, but rather in a series of drops. The shape of the drops is caused by the surface tension of the water. The only reason the drop of water isn't completely spherical is because of the force of gravity pulling down on it. In the absence of gravity, the drop would minimize the surface area in order to minimize tension, which would result in a perfectly spherical shape.

Insects walking on water. Several insects are able to walk on water, such as the water strider. Their legs are formed to distribute their weight, causing the surface of the liquid to become depressed, minimizing the potential energy to create a balance of forces so that the strider can move

Needle (or paper clip) floating on water. Even though the density of these objects are greater than water, the surface tension along the depression is enough to counteract the force of gravity pulling down on the metal object. Click on the picture to the right, then click "Next," to view a force diagram of this situation or try out the Floating Needle trick for yourself.

Other Surface Tension Topics

- Pressure Inside a Bubble
- Contact Angle & Capillarity - Liquid in a Vertical Tube
- Surface Tension Physics Experiments & Tricks
- Experimental Surface Tension Values

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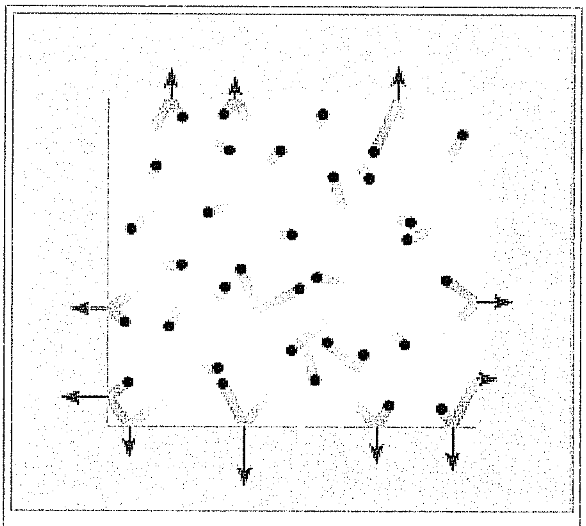
Pressure

From Wikipedia, the free encyclopedia

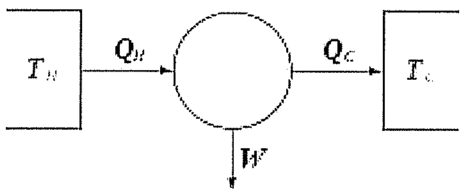
Pressure (the symbol: *P*) is the force per unit area applied in a direction perpendicular to the surface of an object. Gauge pressure is the pressure relative to the local atmospheric or ambient pressure.



Pressure as exerted by particle collisions inside a closed container.

Contents

Thermodynamics



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Definition

Pressure is the effect of a force applied to a surface. Pressure is the amount of force acting per unit area.[1][2]

Formula

Mathematically:

$$P = \frac{F}{A} \text{ or } P = \frac{dF_n}{dA}$$

where:

- P is the pressure,
- F is the normal force,
- A is the area of the surface area on contact

Pressure is a scalar quantity. It relates the vector surface element (a vector normal to the surface) with the normal force acting on it. The pressure is the scalar proportionality constant that relates the two normal vectors:

$$d\mathbf{F}_n = -P\,d\mathbf{A} = -P\,\mathbf{n}\,dA$$

The minus sign comes from the fact that the force is considered towards the surface element, while the normal vector points outward.

It is incorrect (although rather usual) to say "the pressure is directed in such or such direction". The pressure, as a scalar, has no direction. It is the force given by the previous relationship to the quantity that has a direction, not the pressure. If we change the orientation of the surface element, the direction of the normal force changes accordingly, but the pressure remains the same.

Pressure is transmitted to solid boundaries or across arbitrary sections of fluid *normal to* these

Quasistatic · Polytropic
Free expansion
Reversibility · Irreversibility
Endoreversibility

Cycles:
Heat engines · Heat pumps
Thermal efficiency

System properties

Property diagrams
Intensive and extensive properties

State functions:
Temperature / Entropy (intro.) †
Pressure / Volume †
Chemical potential / Particle no. †
(† Conjugate variables)
Vapor quality
Reduced properties

Process functions:
Work · Heat

Material properties

Specific heat capacity $c = \frac{T}{N}\frac{\partial S}{\partial T}$

Compressibility $\beta = -\frac{1}{V}\frac{\partial V}{\partial p}$

Thermal expansion $\alpha = \frac{1}{V}\frac{\partial V}{\partial T}$

Property database

Equations

Carnot's theorem
Clausius theorem
Fundamental relation
Ideal gas law
Maxwell relations

Table of thermodynamic equations

thermodynamics, and it is conjugate to volume.

Units

The SI unit for pressure is the pascal (Pa), equal to one newton per square meter (N/m^2 or $kg{\cdot}m^{-1}{\cdot}s^{-2}$). This special name for the unit was added in 1971;[3] before that, pressure in SI was expressed simply as N/m^2.

Non-SI measures such as pounds per square inch and *bars* are used in some parts of the world, primarily in the United States of America. The cgs unit of pressure is the barye (ba), equal to 1 $dyn{\cdot}cm^{-2}$ or 0.1 Pa. Pressure is sometimes expressed in grams-force/cm^2, or as kg/cm^2 and the like without properly identifying the force units. But using the names kilogram, gram, kilogram-force, or gram-force (or their symbols) as units of force is expressly forbidden in SI. The technical atmosphere (symbol: at) is 1 kgf/cm^2 (14.223 psi).

Since a system under pressure has potential to perform work on its surroundings, pressure is a measure of potential energy stored per unit volume measured in $J{\cdot}m^{-3}$, related to energy density.

Some meteorologists prefer the hectopascal (hPa) for atmospheric air pressure, which is equivalent to the older unit millibar (mbar). Similar pressures are given in kilopascals (kPa) in most other fields, where the hecto- prefix is rarely used. The inch

Internal energy	$U(S,V)$
Enthalpy	$H(S,p) = U + pV$
Helmholtz free energy	$A(T,V) = U - TS$
Gibbs free energy	$G(T,p) = H - TS$

History and culture

Philosophy:
Entropy and time · Entropy and life
Brownian ratchet
Maxwell's demon
Heat death paradox
Loschmidt's paradox
Synergetics

History:
General · Heat · Entropy · Gas laws
Perpetual motion
Theories:
Caloric theory · Vis viva
Theory of heat
Mechanical equivalent of heat
Motive power
Publications:
"An Experimental Enquiry Concerning ... Heat"
"On the Equilibrium of Heterogeneous Substances"
"Reflections on the Motive Power of Fire"

Timelines of:
Thermodynamics · Heat engines

Art:
Maxwell's thermodynamic surface

Education:
Entropy as energy dispersal

Scientists

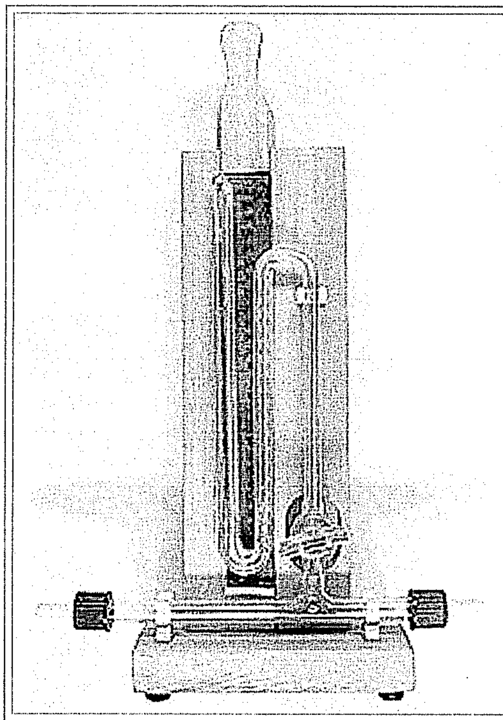
Daniel Bernoulli
Sadi Carnot
Benoît Paul Émile Clapeyron

measure underwater pressure in decibars (dbar) because an increase in pressure of 1 dbar is approximately equal to an increase in depth of 1 meter. Scuba divers often use a manometric rule of thumb: the pressure exerted by 10 meters depth of water is approximately equal to one atmosphere. The increase in pressure at 34 feet of fresh water or 33 feet of sea water is one atm.

Constantin Carathéodory
Pierre Duhem
Josiah Willard Gibbs
James Prescott Joule
James Clerk Maxwell
Julius Robert von Mayer
William Rankine
John Smeaton
Georg Ernst Stahl
Benjamin Thompson
William Thomson, 1st Baron Kelvin
John James Waterston

The standard atmosphere (atm) is an established constant. It is approximately equal to typical air pressure at earth mean sea level and is defined as follows:

standard atmosphere = 101,325 Pa = 101.325 kPa = 1,013.25 hPa.



Mercury column

Because pressure is commonly measured by its ability to displace a column of liquid in a manometer, pressures are often expressed as a depth of a particular fluid (e.g., centimeters of water, mm or inches of mercury). The most common choices are mercury (Hg) and water; water is nontoxic and readily available, while mercury's high density allows a shorter column (and so a smaller manometer) to be used to measure a given pressure. The pressure exerted by a column of liquid of height h and density ρ is given by the hydrostatic pressure equation $p = \rho gh$. Fluid density and local gravity can vary from one reading to another depending on local factors, so the height of a fluid column does not define pressure precisely. When millimeters of mercury or inches of mercury are quoted today, these units are not based on a physical column of mercury; rather, they have been given precise definitions that can be expressed in terms of SI units. One mmHg (millimeter of mercury) is equal to one torr. The water-based units still depend on the density of water, a measured, rather than defined, quantity. These *manometric units* are still encountered in many fields. Blood pressure is measured in millimeters of mercury in most of the world, and lung pressures in centimeters of water are still common.

confusion, for example 'kPaa', 'psia'. However, the US National Institute of Standards and Technology recommends that, to avoid confusion, any modifiers be instead applied to the quantity being measured rather than the unit of measure[4] For example, "P_g = 100 psi" rather than "P = 100 psig".

Differential pressure is expressed in units with 'd' appended; this type of measurement is useful when considering sealing performance or whether a valve will open or close.

Presently or formerly popular pressure units include the following:

- atmosphere (atm)
- manometric units:
  - centimeter, inch, and millimeter of mercury (torr)
  - Height of equivalent column of water, including millimeter (mm H_2O), centimeter (cm H_2O), meter, inch, and foot of water
- customary units:
  - kip, ton-force (short), ton-force (long), pound-force, ounce-force, and poundal per square inch
  - ton-force (short), and ton-force (long) per square inch
- non-SI metric units:
  - bar, decibar, millibar
  - kilogram-force, or kilopond, per square centimeter (technical atmosphere)
  - gram-force and tonne-force (metric ton-force) per square centimeter
  - barye (dyne per square centimeter)
  - kilogram-force and tonne-force per square meter
  - sthene per square meter (pieze)

Pressure units

	pascal	**bar**	**technical atmosphere**	**standard atmosphere**	**torr**	**pound per square inch**
	Pa	**bar**	**at**	**atm**	**Torr**	**psi**
1 Pa	≡ 1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}
1 bar	10^5	≡ 10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744
1 at	0.980665×10^5	0.980665	≡ 1 kp/cm^2	0.96784	735.56	14.223
1 atm	1.01325×10^5	1.01325	1.0332	$\equiv p_0$	760	14.696

1 psi	6.895×10^3	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	$\equiv 1\ lb_F/in^2$

Examples

As an example of varying pressures, a finger can be pressed against a wall without making any lasting impression; however, the same finger pushing a thumbtack can easily damage the wall. Although the force applied to the surface is the same, the thumbtack applies more pressure because the point concentrates that force into a smaller area. Pressure is transmitted to solid boundaries or across arbitrary sections of fluid *normal to* these boundaries or sections at every point. Unlike stress, pressure is defined as a scalar quantity.

Another example is of a common *knife*. If we try to cut a fruit with the flat side it obviously won't cut. But if we take the thin side, it will cut smoothly. The reason is that the flat side has a greater surface area (less pressure) and so it does not cut the fruit. When we take the thin side, the surface area is reduced and so it cuts the fruit easily and quickly. This is one example of a practical application of pressure.

The gradient of pressure is called the force density. For gases, pressure is sometimes measured not as an **absolute pressure**, but relative to atmospheric pressure; such measurements are called **gage pressure** (also spelled *gauge pressure*).[5] An example of this is the air pressure in an automobile tire, which might be said to be "220 kPa/32 psi", but is actually 220 kPa/32 psi above atmospheric pressure. Since atmospheric pressure at sea level is about 100 kPa/14.7 psi, the absolute pressure in the tire is therefore about 320 kPa/46.7 psi. In technical work, this is written "a gage pressure of 220 kPa/32 psi". Where space is limited, such as on pressure gauges, name plates, graph labels, and table headings, the use of a modifier in parentheses, such as "kPa (gage)" or "kPa (absolute)", is permitted. In non-SI technical work, a gage pressure of 32 psi is sometimes written as "32 psig" and an absolute pressure as "32 psia", though the other methods explained above that avoid attaching characters to the unit of pressure are preferred.[6]

Gauge pressure is the relevant measure of pressure wherever one is interested in the stress on storage vessels and the plumbing components of fluidics systems. However, whenever equation-of-state properties, such as densities or changes in densities, must be calculated, pressures must be expressed in terms of their absolute values. For instance, if the atmospheric pressure is 100 kPa, a gas (such as helium) at 200 kPa (gage) (300 kPa [absolute]) is 50% denser than the same gas at 100 kPa (gage) (200 kPa [absolute]). Focusing on gage values, one might erroneously conclude the first sample had twice the density of the second one.

In a static gas, the gas as a whole does not appear to move. The individual molecules of the gas, however, are in constant random motion. Because we are dealing with an extremely large number of molecules and because the motion of the individual molecules is random in every direction, we do not detect any motion. If we enclose the gas within a container, we detect a pressure in the gas from the molecules colliding with the walls of our container. We can put the walls of our container anywhere inside the gas, and the force per unit area (the pressure) is the same. We can shrink the size of our "container" down to an infinitely small point, and the pressure has a single value at that point. Therefore, pressure is a scalar quantity, not a vector quantity. It has magnitude but no direction sense associated with it. Pressure acts in all directions at a point inside a gas. At the surface of a gas, the pressure force acts perpendicular (at right angle) to the surface.

A closely related quantity is the stress tensor σ, which relates the vector force **F** to the vector area **A** via

$$\mathbf{F} = \sigma \mathbf{A}$$

This tensor may be divided up into a scalar part (pressure) and a traceless tensor part shear. The shear tensor gives the force in directions *parallel* to the surface, usually due to viscous or frictional forces. The stress tensor is sometimes called the pressure tensor, but in the following, the term "pressure" will refer only to the scalar pressure.

According to the theory of general relativity, pressure increases the strength of a gravitational field (see stress-energy tensor) and so adds to the mass-energy cause of gravity. This effect is unnoticeable at everyday pressures but is significant in neutron stars, although it has not been experimentally tested.[7]

Types

Fluid pressure

Fluid pressure is the pressure at some point within a fluid, such as water or air.

Fluid pressure occurs in one of two situations:

1. an open condition, called "open channel flow"
   a. the ocean, or
   b. swimming pool, or
   c. the atmosphere.
2. a closed condition, called closed conduits
   a. water line, or
   b gas line

non-moving conditions (even in the ocean where there are waves and currents), because the motions create only negligible changes in the pressure. Such conditions conform with principles of fluid statics. The pressure at any given point of a non-moving (static) fluid is called the **hydrostatic pressure**.

Closed bodies of fluid are either "static", when the fluid is not moving, or "dynamic", when the fluid can move as in either a pipe or by compressing an air gap in a closed container. The pressure in closed conditions conforms with the principles of fluid dynamics.

The concepts of fluid pressure are predominantly attributed to the discoveries of Blaise Pascal and Daniel Bernoulli. Bernoulli's equation can be used in almost any situation to determine the pressure at any point in a fluid. The equation makes some assumptions about the fluid, such as the fluid being ideal[8] and incompressible.[8] An ideal fluid is a fluid in which there is no friction, it is inviscid,[8] zero viscosity.[8] The equation is written between any two points in a system that contain the same fluid.

$$\frac{p}{\gamma} + \frac{v^2}{2g} + z = \frac{p}{\gamma} + \frac{v^2}{2g} + z$$ [9]

where:

p = pressure of the fluid
$\gamma = \rho g$ = density·acceleration of gravity = specific weight of the fluid.[8]
v = velocity of the fluid
g = acceleration of gravity
z = elevation
$\frac{p}{\gamma}$ = pressure head
$\frac{v^2}{2g}$ = velocity head

Applications

- Artesian well
- Blood pressure
- Hydraulic head
- Plant cell turgidity
- Pythagorean cup

Explosion or deflagration pressures

Negative pressures

While **pressures** are, in general, positive, there are several situations in which negative pressures may be encountered:

- When dealing in relative (gauge) pressures. For instance, an absolute pressure of 80 kPa may be described as a gauge pressure of −21 kPa (i.e., 21 kPa below an atmospheric pressure of 101 kPa).
- When attractive forces (e.g., van der Waals forces) between the particles of a fluid exceed repulsive forces. Such scenarios are generally unstable since the particles will move closer together until repulsive forces balance attractive forces. Negative pressure exists in the transpiration pull of plants, and is used to suction water even higher than the ten meters that it rises in a pure vacuum.
- The Casimir effect can create a small attractive force due to interactions with vacuum energy; this force is sometimes termed "vacuum pressure" (not to be confused with the negative *gauge pressure* of a vacuum).
- Depending on how the orientation of a surface is chosen, the same distribution of forces may be described either as a positive pressure along one surface normal, or as a negative pressure acting along the opposite surface normal.
- In the cosmological constant.

Stagnation pressure

Stagnation pressure is the pressure a fluid exerts when it is forced to stop moving. Consequently, although a fluid moving at higher speed will have a lower static pressure, it may have a higher stagnation pressure when forced to a standstill. Static pressure and stagnation pressure are related by the Mach number of the fluid. In addition, there can be differences in pressure due to differences in the elevation (height) of the fluid. See Bernoulli's equation (note: Bernoulli's equation only applies for incompressible, inviscid flow).

The pressure of a moving fluid can be measured using a Pitot tube, or one of its variations such as a Kiel probe or Cobra probe, connected to a manometer. Depending on where the inlet holes are located on the probe, it can measure static pressures or stagnation pressures.

Surface pressure

There is a two-dimensional analog of pressure – the lateral force per unit length applied on a line perpendicular to the force.

Surface pressure is denoted by π and shares many similar properties with three-constant temperature.

$$\pi = \frac{F}{l}.$$

Pressure of an ideal gas

Main article: Ideal gas law

In an ideal gas, molecules have no volume and do not interact. Pressure varies linearly with temperature, volume, and quantity according to the ideal gas law,

$$P = \frac{nRT}{V}$$

where:

P is the absolute pressure of the gas
n is the amount of substance
T is the absolute temperature
V is the volume
R is the ideal gas constant.

Real gases exhibit a more complex dependence on the variables of state.[10]

Vapor pressure

Main article: Vapor pressure

Vapor pressure is the pressure of a vapor in thermodynamic equilibrium with its condensed phases in a closed system. All liquids and solids have a tendency to evaporate into a gaseous form, and all gases have a tendency to condense back to their liquid or solid form.

The atmospheric pressure boiling point of a liquid (also known as the normal boiling point) is the temperature at which the vapor pressure equals the ambient atmospheric pressure. With any incremental increase in that temperature, the vapor pressure becomes sufficient to overcome atmospheric pressure and lift the liquid to form vapor bubbles inside the bulk of the substance. Bubble formation deeper in the liquid requires a higher pressure, and therefore higher temperature, because the fluid pressure increases above the atmospheric pressure as the depth increases.

The vapor pressure that a single component in a mixture contributes to the total

Liquid pressure or pressure at depth

Used with liquid columns of constant density or at a depth within a substance (example: pressure at 20 km depth in the Earth).

$$P = \rho g h$$

where:

P is Pressure
g is gravity at the surface of overlaying material
ρ is density of liquid or overlaying material
h is height of liquid or depth within a substance

See also

- Atmospheric pressure
- Blood pressure
- Boyle's Law
- Combined gas law
- Conversion of units
- Critical point (thermodynamics)
- Dynamic pressure
- Hydraulics
- Internal pressure
- Kinetic theory
- Microphone
- Orders of magnitude (pressure)
- Partial pressure
- Pressure measurement
- Pressure sensor
- Sound pressure
- Spouting can
- Timeline of temperature and pressure measurement technology
- Units conversion by factor-label
- Vacuum
- Vacuum pump
- Vertical pressure variation

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1. ^ Giancoli, Douglas G. (2004). *Physics: principles with applications*. Upper Saddle River, N.J.: Pearson Education. ISBN 0-13-060620-0.
2. ^ Note the upper case *P* is also used for power.
3. ^ 14th Conference of the International Bureau of Weights and Measures (http://www.bipm.fr/en/convention/cgpm/14/pascal-siemens.html)
4. ^ "Rules and Style Conventions for Expressing Values of Quantities"

5. ^ The preferred spelling varies by country and even by industry. Further, both spellings are often used *within* a particular industry or country. Industries in British English-speaking countries typically use the "gauge" spelling. Many of the largest American manufacturers of pressure transducers and instrumentation use the spelling "gage pressure" in their most formal documentation (*Honeywell-Sensotec's* FAQ page (http://www.sensotec.com/pressurefaq.shtml) and Fluke Corporation's product search page (http://us.fluke.com/usen/Home/Search.asp?txtSearchBox=%22gage+pressure%22&x=0&y=0)).
6. ^ NIST, *Rules and Style Conventions for Expressing Values of Quantities* (http://physics.nist.gov/Pubs/SP811/sec07.html#7.4) , Sect. 7.4.
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External links

- *Introduction to Fluid Statics and Dynamics* (http://www.physnet.org/modules/pdf_modules/m48.pdf) on Project PHYSNET (http://www.physnet.org/)
- Pressure being a scalar quantity (http://www.grc.nasa.gov/WWW/K-12/airplane/pressure.html)
- Online pressure converter for 52 different pressure units (http://www.coleparmer.com/Techinfo/converters/commpressure.asp)

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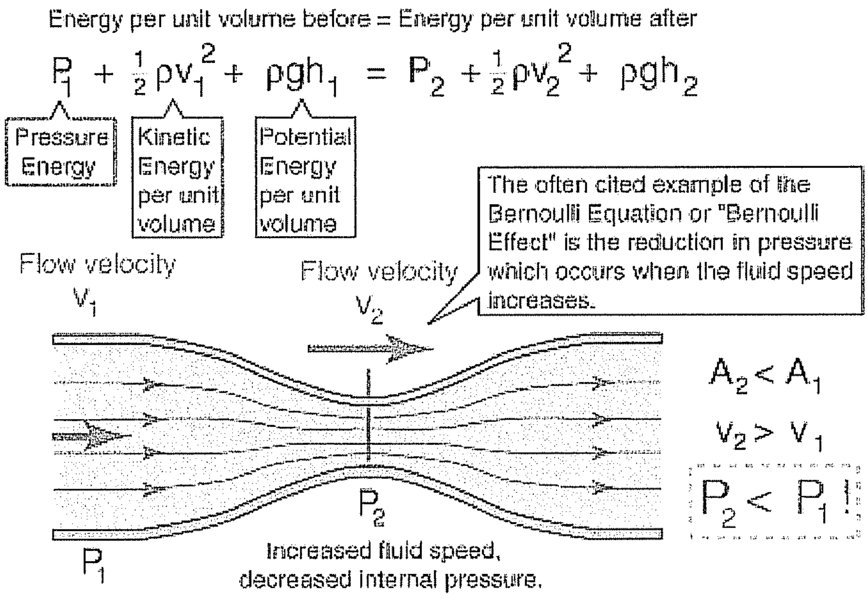
Bernoulli Equation

The Bernoulli Equation can be considered to be a statement of the conservation of energy principle appropriate for flowing fluids. The qualitative behavior that is usually labeled with the term "Bernoulli effect" is the lowering of fluid pressure in regions where the flow velocity is increased. This lowering of pressure in a constriction of a flow path may seem counterintuitive, but seems less so when you consider pressure to be energy density. In the high velocity flow through the constriction, kinetic energy must increase at the expense of pressure energy.



Bernoulli calculation

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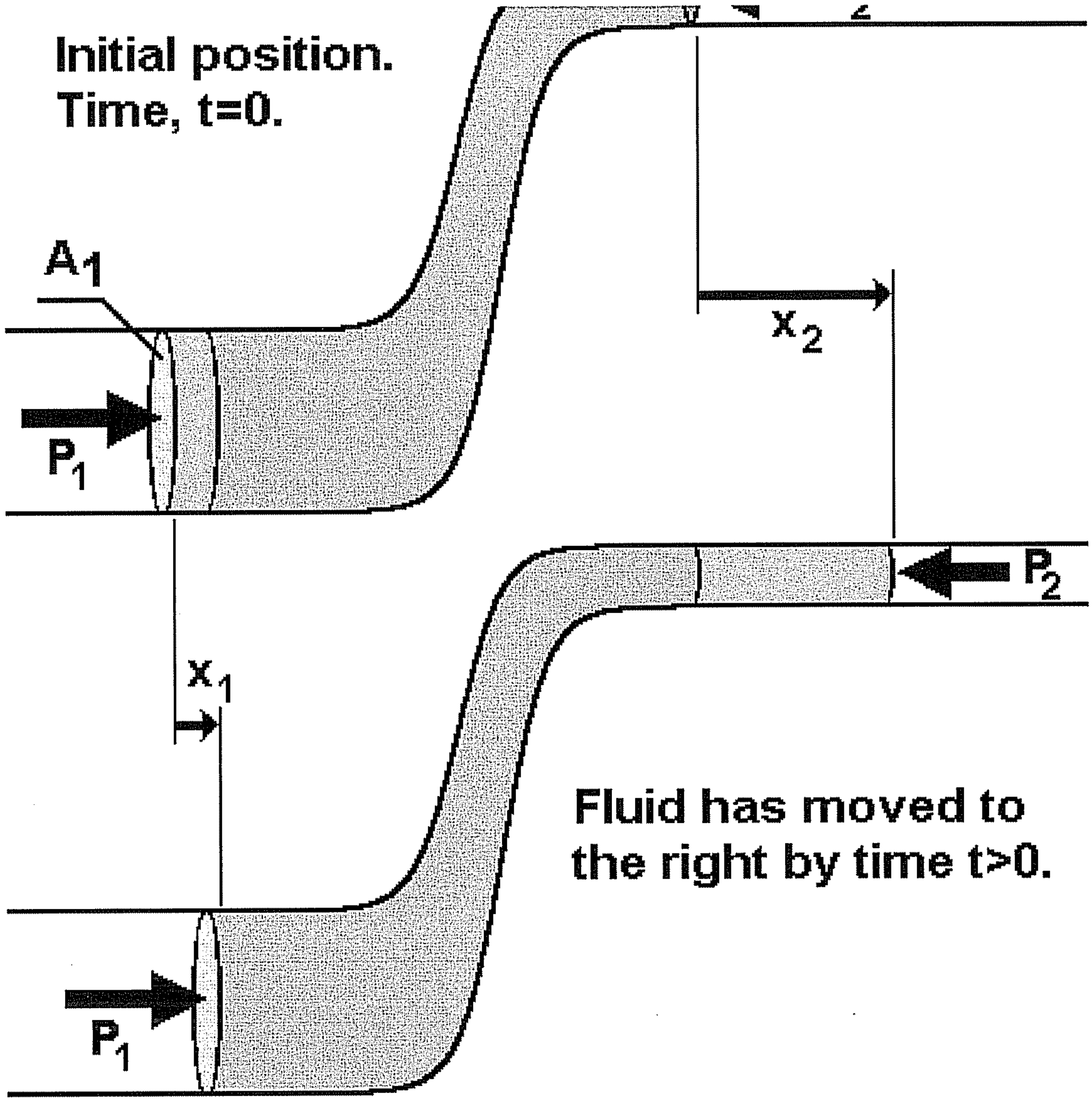
Bernoulli concepts

Bernoulli Calculation

The calculation of the "real world" pressure in a constriction of a tube is difficult to do because of viscous losses, turbulence, and the assumptions which must be made about the velocity profile (which affect the calculated kinetic energy). The model calculation



We examine a fluid section of mass m traveling to the right as shown in the schematic above. The net work done in moving the fluid is

$$W = W_1 + W_2 = F_1 x_1 - F_2 x_2 . \qquad \textbf{Eq.(1)}$$

where F denotes a force and an x a displacement. The second term picked up its negative sign because the force and displacement are in opposite directions.

Pressure is the force exerted over the cross-sectional area, or $P = F/A$. Rewriting this as $F = PA$ and substituting into Eq.(1) we find that

$$\Delta W = P_1 A_1 x_1 - P_2 A_2 x_2 . \qquad \textbf{Eq.(2)}$$

The displaced fluid volume V is the cross-sectional area A times the thickness x. This volume remains constant for an incompressible fluid, so

$$V = A_1 x_1 = A_2 x_2 . \qquad \textbf{Eq.(3)}$$

Using Eq.(3) in Eq.(2) we have

$$\Delta W = (P_1 - P_2) V. \qquad \textbf{Eq.(4)}$$

$$P + \rho g h + \rho v^2/2 = \text{constant.} \qquad \text{Eq.(11)}$$

Equation (11) is commonly referred to as Bernoulli's equation. Keep in mind that this expression was restricted to incompressible fluids and smooth fluid flows.

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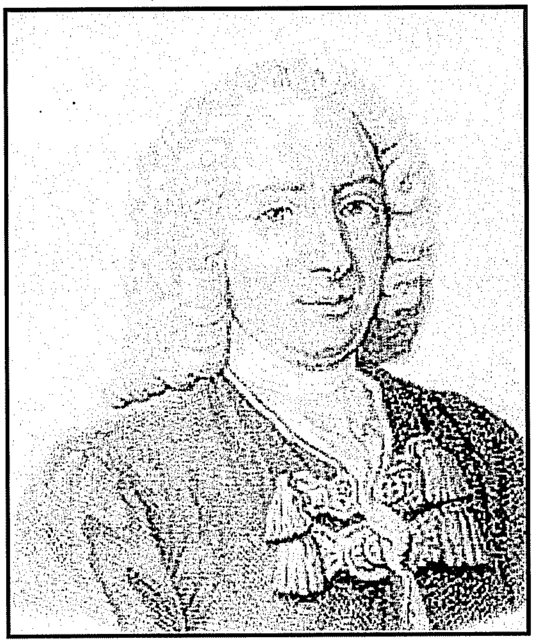
The Bernoulli Effect

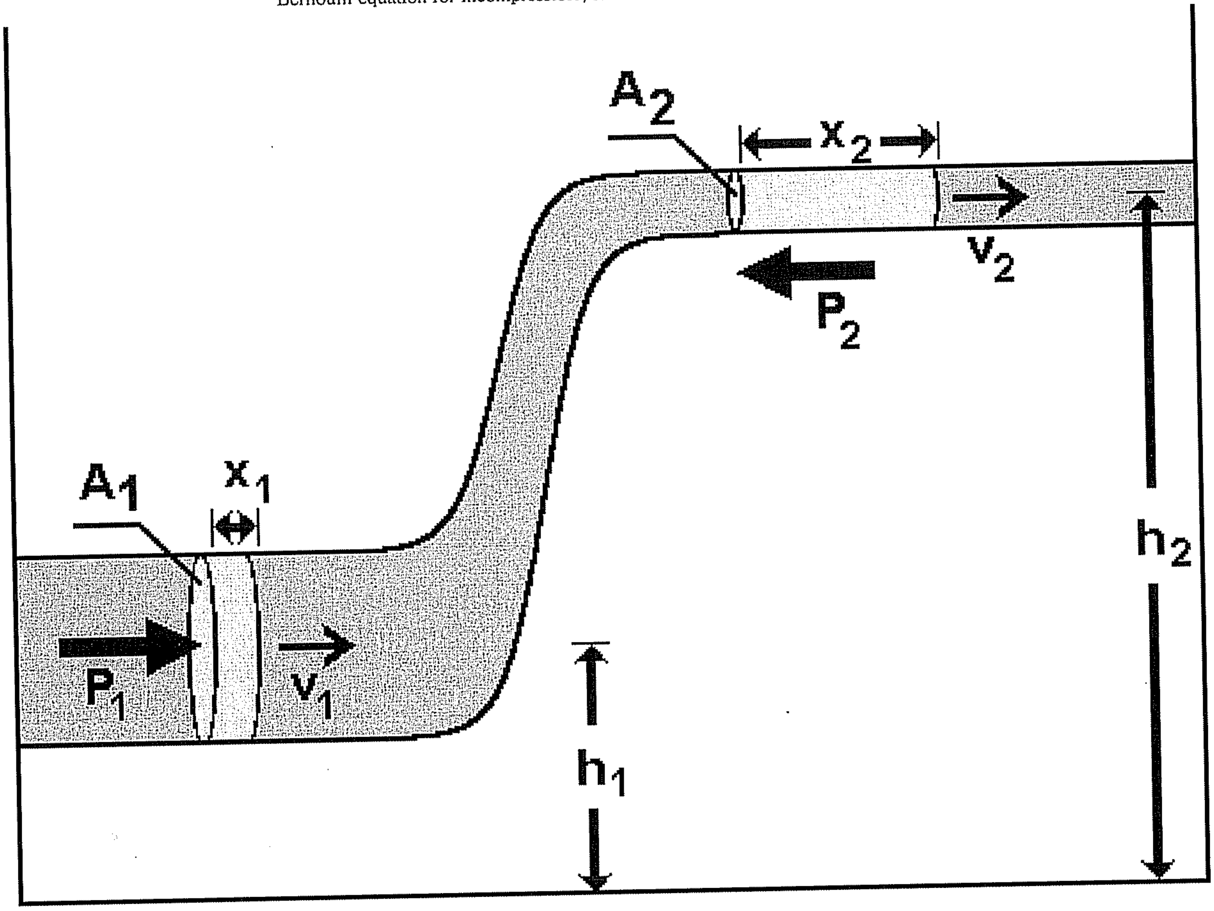
The Bernoulli Equation for an Incompressible, Steady Fluid Flow



In 1738 Daniel Bernoulli (1700-1782) formulated the famous equation for fluid flow that bears his name. The Bernoulli Equation is a statement derived from conservation of energy and work-energy ideas that come from Newton's Laws of Motion.

An important and highly useful special case is where friction is ignored and the fluid is incompressible. This is not as unduly restrictive as it might first seem. The absence of friction means that the fluid flow is steady. That is, the fluid does not stick to the pipe sides and has no turbulence. Most common liquids such as water are nearly incompressible, which meets the second condition.

Consider the case of water flowing though a smooth pipe. Such a situation is depicted in the figure below. We will use this as our working model and obtain Bernoulli's equation employing the work-energy theorem and energy conservation.



The energy change between the initial and final positions is given by

$$\Delta E = E_2 - E_1 = (U_2 + K_2) - (U_1 + K_1),$$

or **Eq.(5)**

$$\Delta E = (mgh_2 + mv_2^2/2) - (mgh_1 + mv_1^2/2).$$

Here, the the kinetic energy $K = mv^2/2$ where m is the fluid mass and v is the speed of the fluid. The potential energy $U = mgh$ where g is the acceleration of gravity, and h is average fluid height.

The work-energy theorem says that the net work done is equal to the change in the system energy. This can be written as

$$\Delta W = \Delta E. \qquad \textbf{Eq.(6)}$$

Substitution of Eq.(4) and Eq.(5) into Eq.(6) yields

$$(P_1 - P_2)V = (mgh_2 + mv_2^2/2) - (mgh_1 + mv_1^2/2). \qquad \textbf{Eq.(7)}$$

Dividing Eq.(7) by the fluid volume, V gives us

$$P_1 - P_2 = (\rho gh_2 + \rho v_2^2/2) - (\rho gh_1 + \rho v_1^2/2) \qquad \textbf{Eq.(8)}$$

where

$$\rho = m/V \qquad \textbf{Eq.(9)}$$

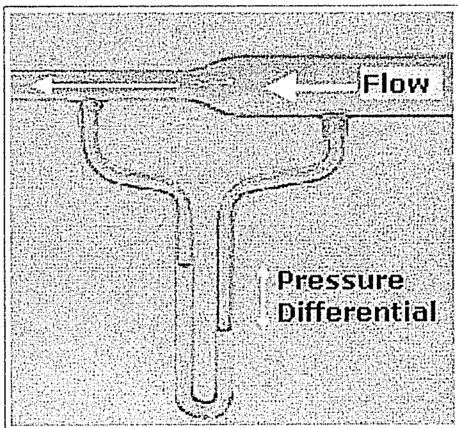
is the fluid mass density. To complete our derivation, we reorganize Eq.(8).

$$P_1 + \rho gh_1 + \rho v_1^2/2 = P_2 + \rho gh_2 + \rho v_2^2/2. \qquad \textbf{Eq.(10)}$$

Bernoulli's principle

From Wikipedia, the free encyclopedia

In fluid dynamics, **Bernoulli's principle** states that for an inviscid flow, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.[1][2] Bernoulli's principle is named after the Dutch-Swiss mathematician Daniel Bernoulli who published his principle in his book *Hydrodynamica* in 1738.[3]



A flow of air into a venturi meter. The kinetic energy increases at the expense of the fluid pressure, as shown by the difference in height of the two columns of water.

Bernoulli's principle can be applied to various types of fluid flow, resulting in what is loosely denoted as **Bernoulli's equation**. In fact, there are different forms of the Bernoulli equation for different types of flow. The simple form of Bernoulli's principle is valid for incompressible flows (e.g. most liquid flows) and also for compressible flows (e.g. gases) moving at low Mach numbers. More advanced forms may in some cases be applied to compressible flows at higher Mach numbers (see the derivations of the Bernoulli equation).

Bernoulli's principle can be derived from the principle of conservation of energy. This states that, in a steady flow, the sum of all forms of mechanical energy in a fluid along a streamline is the same at all points on that streamline. This requires that the sum of kinetic energy and potential energy remain constant. Thus an increase in the speed of the fluid occurs proportionately with an increase in both its dynamic pressure and kinetic energy, and a decrease in its static pressure and potential energy. If the fluid is flowing out of a reservoir the sum of all forms of energy is the same on all streamlines because in a reservoir the energy per unit volume (the sum of pressure and gravitational potential $\rho g h$) is the same everywhere.[4]

Bernoulli's principle can also be derived directly from Newton's 2nd law. If a small volume of fluid is flowing horizontally from a region of high pressure to a region of low pressure, then there is more pressure behind than in front. This gives a net force on the volume, accelerating it along the streamline.[5][6]

Fluid particles are subject only to pressure and their own weight. If a fluid is flowing horizontally and along a section of a streamline, where the speed increases it can only be because the fluid on that section has moved from a region of higher pressure to a region of lower pressure; and if its speed decreases, it can only be because it has moved from a region of lower pressure to a region of higher pressure. Consequently, within a fluid flowing horizontally, the highest speed occurs where the pressure is lowest, and the lowest speed occurs where the pressure is highest.

Contents

Incompressible flow equation

In most flows of liquids, and of gases at low Mach number, the mass density of a fluid parcel can be considered to be constant, regardless of pressure variations in the flow. For this reason the fluid in such flows can be considered to be incompressible and these flows can be described as incompressible flow. Bernoulli performed his experiments on liquids and his equation in its original form is valid only for incompressible flow. A common form of Bernoulli's equation, valid at any arbitrary point along a streamline where gravity is constant, is:

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant} \qquad \textbf{(A)}$$

where:

v is the fluid flow speed at a point on a streamline,
g is the acceleration due to gravity,
z is the elevation of the point above a reference plane, with the positive z-direction pointing upward – so in the direction opposite to the gravitational acceleration,
p is the pressure at the chosen point, and
ρ is the density of the fluid at all points in the fluid.

For conservative force fields, Bernoulli's equation can be generalized as:[7]

$$\frac{v^2}{2} + \Psi + \frac{p}{\rho} = \text{constant}$$

where Ψ is the force potential at the point considered on the streamline. *E.g.* for the Earth's gravity $\Psi = gz$.

The following two assumptions must be met for this Bernoulli equation to apply:[7]

- the flow must be incompressible – even though pressure varies the density must remain

By multiplying with the fluid density ρ, equation (**A**) can be rewritten as:

$$\tfrac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

or:

$$q + \rho g h = p_0 + \rho g z = \text{constant}$$

where:

$q = \tfrac{1}{2}\rho v^2$ is dynamic pressure,

$h = z + \dfrac{p}{\rho g}$ is the piezometric head or hydraulic head (the sum of the elevation z and the pressure head)[8][9] and

$p_0 = p + q$ is the **total pressure** (the sum of the static pressure p and dynamic pressure q).[10]

The constant in the Bernoulli equation can be normalised. A common approach is in terms of **total head** or **energy head** H:

$$H = z + \frac{p}{\rho g} + \frac{v^2}{2g} = h + \frac{v^2}{2g},$$

The above equations suggest there is a flow speed at which pressure is zero, and at even higher speeds the pressure is negative. Most often, gases and liquids are not capable of negative absolute pressure, or even zero pressure, so clearly Bernoulli's equation ceases to be valid before zero pressure is reached. In liquids – when the pressure becomes too low – cavitation occurs. The above equations use a linear relationship between flow speed squared and pressure. At higher flow speeds in gases, or for sound waves in liquid, the changes in mass density become significant so that the assumption of constant density is invalid.

Simplified form

In many applications of Bernoulli's equation, the change in the $\rho g z$ term along the streamline is so small compared with the other terms it can be ignored. For example, in the case of aircraft in flight, the change in height z along a streamline is so small the $\rho g z$ term can be omitted. This allows the above equation to be presented in the following simplified form:

$$p + q = p_0$$

where p_0 is called total pressure, and q is dynamic pressure.[11] Many authors refer to the pressure p as static pressure to distinguish it from total pressure p_0 and dynamic pressure q. In *Aerodynamics*, L.J. Clancy writes: "To distinguish it from the total and dynamic pressures, the actual pressure of the fluid, which is associated not with its motion but with its state, is often referred to as the static pressure, but where the term pressure alone is used it refers to this static pressure."[12]

equation:

$$\textit{static pressure} + \textit{dynamic pressure} = \textit{total pressure}$$ [12]

Every point in a steadily flowing fluid, regardless of the fluid speed at that point, has its own unique static pressure p and dynamic pressure q. Their sum $p + q$ is defined to be the total pressure p_0. The significance of Bernoulli's principle can now be summarized as *total pressure is constant along a streamline.*

If the fluid flow is irrotational, the total pressure on every streamline is the same and Bernoulli's principle can be summarized as *total pressure is constant everywhere in the fluid flow.*[13] It is reasonable to assume that irrotational flow exists in any situation where a large body of fluid is flowing past a solid body. Examples are aircraft in flight, and ships moving in open bodies of water. However, it is important to remember that Bernoulli's principle does not apply in the boundary layer or in fluid flow through long pipes.

If the fluid flow at some point along a stream line is brought to rest, this point is called a stagnation point, and at this point the total pressure is equal to the stagnation pressure.

Applicability of incompressible flow equation to flow of gases

Bernoulli's equation is sometimes valid for the flow of gases: provided that there is no transfer of kinetic or potential energy from the gas flow to the compression or expansion of the gas. If both the gas pressure and volume change simultaneously, then work will be done on or by the gas. In this case, Bernoulli's equation – in its incompressible flow form – can not be assumed to be valid. However if the gas process is entirely isobaric, or isochoric, then no work is done on or by the gas, (so the simple energy balance is not upset). According to the gas law, an isobaric or isochoric process is ordinarily the only way to ensure constant density in a gas. Also the gas density will be proportional to the ratio of pressure and absolute temperature, however this ratio will vary upon compression or expansion, no matter what non-zero quantity of heat is added or removed. The only exception is if the net heat transfer is zero, as in a complete thermodynamic cycle, or in an individual isentropic (frictionless adiabatic) process, and even then this reversible process must be reversed, to restore the gas to the original pressure and specific volume, and thus density. Only then is the original, unmodified Bernoulli equation applicable. In this case the equation can be used if the flow speed of the gas is sufficiently below the speed of sound, such that the variation in density of the gas (due to this effect) along each streamline can be ignored. Adiabatic flow at less than Mach 0.3 is generally considered to be slow enough.

Unsteady potential flow

The Bernoulli equation for unsteady potential flow is used in the theory of ocean surface waves and acoustics.

For an irrotational flow, the flow velocity can be described as the gradient $\nabla\varphi$ of a velocity potential φ. In that case, and for a constant density ρ, the momentum equations of the Euler equations can be integrated to:[14]

which is a Bernoulli equation valid also for unsteady – or time dependent – flows. Here $\partial\varphi/\partial t$ denotes the partial derivative of the velocity potential φ with respect to time t, and $v = |\nabla\varphi|$ is the flow speed. The function $f(t)$ depends only on time and not on position in the fluid. As a result, the Bernoulli equation at some moment t does not only apply along a certain streamline, but in the whole fluid domain. This is also true for the special case of a steady irrotational flow, in which case f is a constant.[14]

Further $f(t)$ can be made equal to zero by incorporating it into the velocity potential using the transformation

$$\Phi = \varphi - \int_{t_0}^{t} f(\tau)\,\mathrm{d}\tau, \text{ resulting in } \frac{\partial\Phi}{\partial t} + \tfrac{1}{2}v^2 + \frac{p}{\rho} + gz = 0.$$

Note that the relation of the potential to the flow velocity is unaffected by this transformation: $\nabla\Phi = \nabla\varphi$.

The Bernoulli equation for unsteady potential flow also appears to play a central role in Luke's variational principle, a variational description of free-surface flows using the Lagrangian (not to be confused with Lagrangian coordinates).

Compressible flow equation

Bernoulli developed his principle from his observations on liquids, and his equation is applicable only to incompressible fluids, and compressible fluids at very low speeds (perhaps up to 1/3 of the sound speed in the fluid). It is possible to use the fundamental principles of physics to develop similar equations applicable to compressible fluids. There are numerous equations, each tailored for a particular application, but all are analogous to Bernoulli's equation and all rely on nothing more than the fundamental principles of physics such as Newton's laws of motion or the first law of thermodynamics.

Compressible flow in fluid dynamics

For a compressible fluid, with a barotropic equation of state, and under the action of conservative forces,

$$\frac{v^2}{2} + \int_{p_1}^{p} \frac{d\tilde{p}}{\rho(\tilde{p})} + \Psi = \text{constant}^{[15]} \qquad \text{(constant along a streamline)}$$

where:

- p is the pressure
- ρ is the density
- v is the flow speed
- Ψ is the potential associated with the conservative force field, often the gravitational potential

In engineering situations, elevations are generally small compared to the size of the Earth, and the time scales of fluid flow are small enough to consider the equation of state as adiabatic. In this

$$\frac{v^2}{2} + gz + \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} = \text{constant}^{[16]} \qquad \text{(constant along a streamline)}$$

where, in addition to the terms listed above:

γ is the ratio of the specific heats of the fluid
g is the acceleration due to gravity
z is the elevation of the point above a reference plane

In many applications of compressible flow, changes in elevation are negligible compared to the other terms, so the term gz can be omitted. A very useful form of the equation is then:

$$\frac{v^2}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{p_0}{\rho_0}$$

where:

p_0 is the total pressure
ρ_0 is the total density

Compressible flow in thermodynamics

Another useful form of the equation, suitable for use in thermodynamics, is:

$$\frac{v^2}{2} + \Psi + w = \text{constant.}^{[17]}$$

Here w is the enthalpy per unit mass, which is also often written as h (not to be confused with "head" or "height").

Note that $w = \epsilon + \frac{p}{\rho}$ where ε is the thermodynamic energy per unit mass, also known as the specific internal energy.

The constant on the right hand side is often called the Bernoulli constant and denoted b. For steady inviscid adiabatic flow with no additional sources or sinks of energy, b is constant along any given streamline. More generally, when b may vary along streamlines, it still proves a useful parameter, related to the "head" of the fluid (see below).

When the change in Ψ can be ignored, a very useful form of this equation is:

$$\frac{v^2}{2} + w = w_0$$

where w_0 is total enthalpy. For a calorically perfect gas such as an ideal gas, the enthalpy is directly proportional to the temperature, and this leads to the concept of the total (or stagnation) temperature.

Wh h k i f f i hi h h h k i i d h fl

this rule is radiative shocks, which violate the assumptions leading to the Bernoulli equation, namely the lack of additional sinks or sources of energy.

Derivations of Bernoulli equation

Bernoulli equation for incompressible fluids

The Bernoulli equation for incompressible fluids can be derived by integrating the Euler equations, or applying the law of conservation of energy in two sections along a streamline, ignoring viscosity, compressibility, and thermal effects.

The simplest derivation is to first ignore gravity and consider constrictions and expansions in pipes that are otherwise straight, as seen in Venturi effect. Let the x axis be directed down the axis of the pipe.

Define a parcel of fluid moving through a pipe with cross-sectional area "A", the length of the parcel is "dx", and the volume of the parcel A dx. If mass density is ρ, the mass of the parcel is density multiplied by its volume $m = \rho\, A$ dx. The change in pressure over distance dx is "dp" and flow velocity $v = \mathrm{d}x \,/\, \mathrm{d}t$.

Apply Newton's Second Law of Motion Force F =mass . acceleration and recognizing that the effective force on the parcel of fluid is -A dp. If the pressure decreases along the length of the pipe, dp is negative but the force resulting in flow is positive along the x axis.

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = F$$

$$\rho A\,\mathrm{d}x\frac{\mathrm{d}v}{\mathrm{d}t} = -A\,\mathrm{d}p$$

$$\rho\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\mathrm{d}p}{\mathrm{d}x}$$

In steady flow the velocity is constant with respect to time, $v = v(x) = v(x(t))$, so v itself is not directly a function of time t. It is only when the parcel moves through x that the cross sectional area changes: v depends on t only through the cross-sectional position $x(t)$.

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}v = \frac{d}{\mathrm{d}x}\left(\frac{v^2}{2}\right).$$

With density ρ constant, the equation of motion can be written as

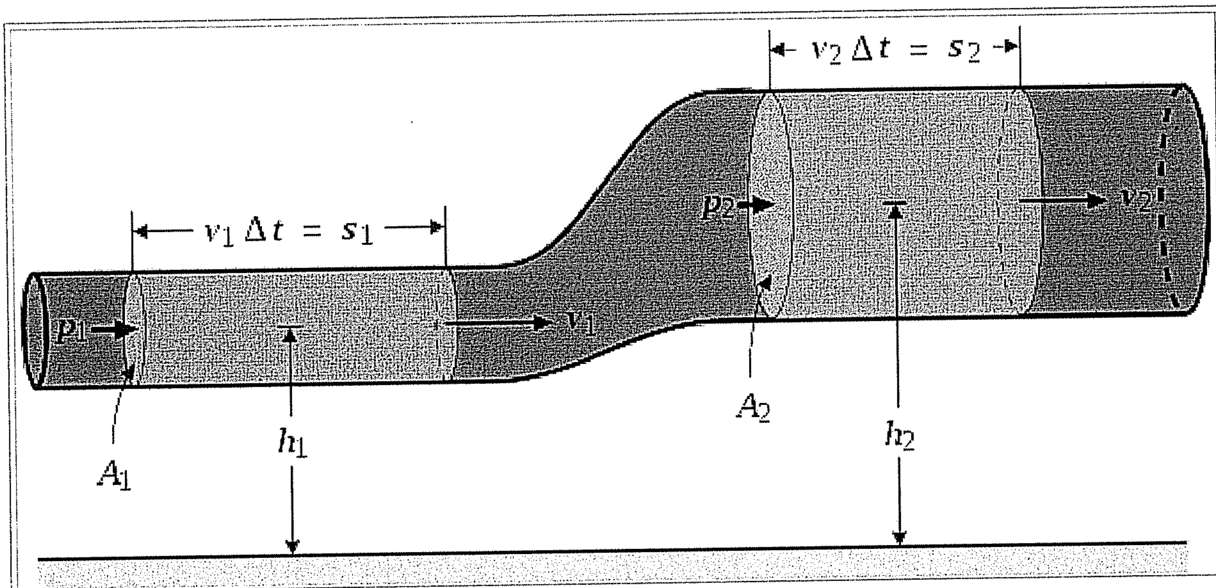
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\rho\frac{v^2}{2} + p\right) = 0$$

by integrating with respect to x

$$\frac{v^2}{2} + \frac{p}{\rho} = C$$

h Ci t t ti f d t th B lli t t It i t i 1

In the above derivation, no external work-energy principle is invoked. Rather, Bernoulli's principle was inherently derived by a simple manipulation of the momentum equation.



A streamtube of fluid moving to the right. Indicated are pressure, elevation, flow speed, distance (s), and cross-sectional area. Note that in this figure elevation is denoted as h, contrary to the text where it is given by z.

Another way to derive Bernoulli's principle for an incompressible flow is by applying conservation of energy.[18] In the form of the work-energy theorem, stating that[19]

> *the change in the kinetic energy* E_{kin} *of the system equals the net work* W *done on the system*;
>
> $$W = \Delta E_{\text{kin}}.$$

Therefore,

> *the work done by the forces in the fluid* = *increase in kinetic energy.*

The system consists of the volume of fluid, initially between the cross-sections A_1 and A_2. In the time interval Δt fluid elements initially at the inflow cross-section A_1 move over a distance $s_1 = v_1\,\Delta t$, while at the outflow cross-section the fluid moves away from cross-section A_2 over a distance $s_2 = v_2\,\Delta t$. The displaced fluid volumes at the inflow and outflow are respectively $A_1\,s_1$ and $A_2\,s_2$. The associated displaced fluid masses are – when ρ is the fluid's mass density – equal to density times volume, so $\rho\,A_1\,s_1$ and $\rho\,A_2\,s_2$. By mass conservation, these two masses displaced in the time interval Δt have to be equal, and this displaced mass is denoted by Δm:

$$\rho A_1 s_1 = \rho A_1 v_1 \Delta t = \Delta m,$$
$$\rho A_2 s_2 = \rho A_2 v_2 \Delta t = \Delta m.$$

The work done by the forces consists of two parts:

- The *work done by the pressure* acting on the areas A_1 and A_2

$$W_{\text{pressure}} = F_{1,\text{pressure}}\, s_1 - F_{2,\text{pressure}}\, s_2 = p_1 A_1 s_1 - p_2 A_2 s_2 = \Delta m\, \frac{p_1}{\rho} - \Delta m\, \frac{p_2}{\rho}$$

- The *work done by gravity*: the gravitational potential energy in the volume $A_1\, s_1$ is lost, and at the outflow in the volume $A_2\, s_2$ is gained. So, the change in gravitational potential energy $\Delta E_{\text{pot,gravity}}$ in the time interval Δt is

$$\Delta E_{\text{pot,gravity}} = \Delta m\, g z_2 - \Delta m\, g z_1.$$

Now, the work by the force of gravity is opposite to the change in potential energy, $W_{\text{gravity}} = -\Delta E_{\text{pot,gravity}}$: while the force of gravity is in the negative z-direction, the work —gravity force times change in elevation—will be negative for a positive elevation change $\Delta z = z_2 - z_1$, while the corresponding potential energy change is positive.[20] So:

$$W_{\text{gravity}} = -\Delta E_{\text{pot,gravity}} = \Delta m\, g z_1 - \Delta m\, g z_2.$$

And the total work done in this time interval Δt is

$$W = W_{\text{pressure}} + W_{\text{gravity}}.$$

The *increase in kinetic energy* is

$$\Delta E_{\text{kin}} = \frac{1}{2} \Delta m\, v_2^2 - \frac{1}{2} \Delta m\, v_1^2.$$

Putting these together, the work-kinetic energy theorem $W = \Delta E_{\text{kin}}$ gives:[18]

$$\Delta m\, \frac{p_1}{\rho} - \Delta m\, \frac{p_2}{\rho} + \Delta m\, g z_1 - \Delta m\, g z_2 = \frac{1}{2} \Delta m\, v_2^2 - \frac{1}{2} \Delta m\, v_1^2$$

or

$$\frac{1}{2} \Delta m\, v_1^2 + \Delta m\, g z_1 + \Delta m\, \frac{p_1}{\rho} = \frac{1}{2} \Delta m\, v_2^2 + \Delta m\, g z_2 + \Delta m\, \frac{p_2}{\rho}.$$

After dividing by the mass $\Delta m = \rho\, A_1\, v_1\, \Delta t = \rho\, A_2\, v_2\, \Delta t$ the result is:[18]

$$\frac{1}{2} v_1^2 + g z_1 + \frac{p_1}{\rho} = \frac{1}{2} v_2^2 + g z_2 + \frac{p_2}{\rho}$$

or, as stated in the first paragraph:

$$\frac{v^2}{2} + g z + \frac{p}{\rho} = C$$ **(Eqn. 1)**, Which is also Equation (A)

Further division by g produces the following equation. Note that each term can be described in the length dimension (such as meters). This is the head equation derived from Bernoulli's principle:

The middle term, z, represents the potential energy of the fluid due to its elevation with respec to a reference plane. Now, z is called the elevation head and given the designation $z_{\text{elevation}}$.

A free falling mass from an elevation $z > 0$ (in a vacuum) will reach a speed

$v = \sqrt{2gz}$, when arriving at elevation $z = 0$. Or when we rearrange it as a *head*: $h_v = \frac{v^2}{2g}$

The term $v^2 / (2g)$ is called the *velocity head*, expressed as a length measurement. It represents the internal energy of the fluid due to its motion.

The hydrostatic pressure p is defined as

$p = p_0 - \rho g z$, with p_0 some reference pressure, or when we rearrange it as a *head*:
$$\psi = \frac{p}{\rho g}$$

The term $p / (\rho g)$ is also called the *pressure head*, expressed as a length measurement. It represents the internal energy of the fluid due to the pressure exerted on the container.

When we combine the head due to the flow speed and the head due to static pressure with the elevation above a reference plane, we obtain a simple relationship useful for incompressible fluids using the velocity head, elevation head, and pressure head.

$$h_v + z_{\text{elevation}} + \psi = C \qquad \textbf{(Eqn. 2b)}$$

If we were to multiply Eqn. 1 by the density of the fluid, we would get an equation with three pressure terms:

$$\frac{\rho v^2}{2} + \rho g z + p = C \qquad \textbf{(Eqn. 3)}$$

We note that the pressure of the system is constant in this form of the Bernoulli Equation. If the static pressure of the system (the far right term) increases, and if the pressure due to elevation (the middle term) is constant, then we know that the dynamic pressure (the left term) must have decreased. In other words, if the speed of a fluid decreases and it is not due to an elevation difference, we know it must be due to an increase in the static pressure that is resisting the flow.

All three equations are merely simplified versions of an energy balance on a system.

Bernoulli equation for compressible fluids

The derivation for compressible fluids is similar. Again, the derivation depends upon (1) conservation of mass, and (2) conservation of energy. Conservation of mass implies that in the above figure, in the interval of time Δt, the amount of mass passing through the boundary defined by the area A_1 is equal to the amount of mass passing outwards through the boundary defined by the area A_2:

Conservation of energy is applied in a similar manner: It is assumed that the change in energy of the volume of the streamtube bounded by A_1 and A_2 is due entirely to energy entering or leaving through one or the other of these two boundaries. Clearly, in a more complicated situation such as a fluid flow coupled with radiation, such conditions are not met. Nevertheless, assuming this to be the case and assuming the flow is steady so that the net change in the energy is zero,

$$0 = \Delta E_1 - \Delta E_2$$

where ΔE_1 and ΔE_2 are the energy entering through A_1 and leaving through A_2, respectively.

The energy entering through A_1 is the sum of the kinetic energy entering, the energy entering in the form of potential gravitational energy of the fluid, the fluid thermodynamic energy entering, and the energy entering in the form of mechanical $p\,dV$ work:

$$\Delta E_1 = \left[\frac{1}{2}\rho_1 v_1^2 + \Psi_1 \rho_1 + \epsilon_1 \rho_1 + p_1\right] A_1 v_1 \,\Delta t$$

where $\Psi = gz$ is a force potential due to the Earth's gravity, g is acceleration due to gravity, and z is elevation above a reference plane.

A similar expression for ΔE_2 may easily be constructed. So now setting $0 = \Delta E_1 - \Delta E_2$:

$$0 = \left[\frac{1}{2}\rho_1 v_1^2 + \Psi_1 \rho_1 + \epsilon_1 \rho_1 + p_1\right] A_1 v_1 \,\Delta t - \left[\frac{1}{2}\rho_2 v_2^2 + \Psi_2 \rho_2 + \epsilon_2 \rho_2 + p_2\right] A_2 v_2 \,\Delta t$$

which can be rewritten as:

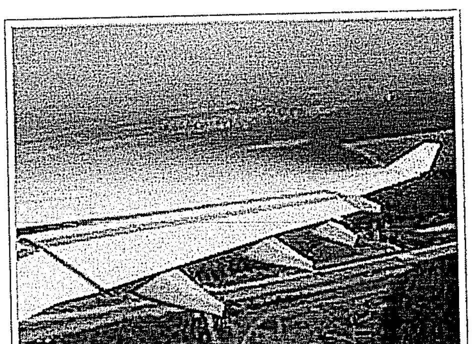
$$0 = \left[\frac{1}{2}v_1^2 + \Psi_1 + \epsilon_1 + \frac{p_1}{\rho_1}\right] \rho_1 A_1 v_1 \,\Delta t - \left[\frac{1}{2}v_2^2 + \Psi_2 + \epsilon_2 + \frac{p_2}{\rho_2}\right] \rho_2 A_2 v_2 \,\Delta t$$

Now, using the previously-obtained result from conservation of mass, this may be simplified to obtain

$$\frac{1}{2}v^2 + \Psi + \epsilon + \frac{p}{\rho} = \text{constant} \equiv b$$

which is the Bernoulli equation for compressible flow.

Real-world application



In modern everyday life there are many observations that can be successfully explained by application of Bernoulli's principle, even though no real fluid is entirely inviscid[21] and a small viscosity often has a large effect on the flow.

- Bernoulli's principle can be used to calculate the lift air flowing past the top surface of an aircraft wing is moving faster than the air flowing past the bottom surface, then Bernoulli's principle implies that the pressure on the surfaces of the wing will be lower above than below. This pressure difference results in an upwards lift force.[nb 1][22] Whenever the distribution of speed past the top and bottom surfaces of a wing is known, the lift forces can be calculated (to a good approximation) using Bernoulli's equations[23] – established by Bernoulli over a century before the first man-made wings were used for the purpose of flight. Bernoulli's principle does not explain why the air flows faster past the top of the wing and slower past the underside. To understand why, it is helpful to understand circulation, the Kutta condition, and the Kutta–Joukowski theorem.

Condensation visible over the upper surface of a wing caused by the fall in temperature accompanying the fall in pressure, both due to acceleration of the air.

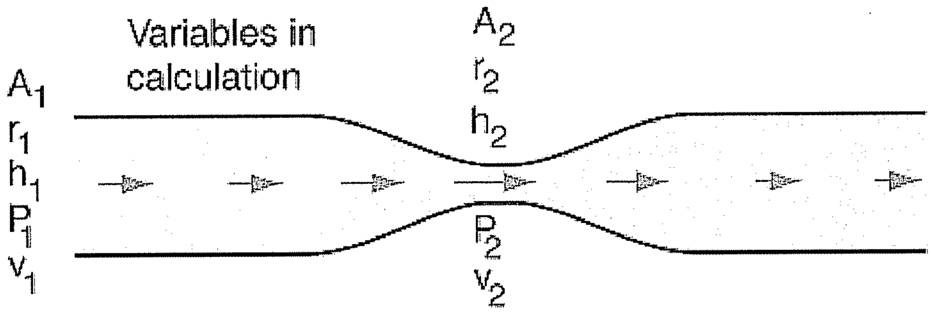
- The carburetor used in many reciprocating engines contains a venturi to create a region of low pressure to draw fuel into the carburetor and mix it thoroughly with the incoming air. The low pressure in the throat of a venturi can be explained by Bernoulli's principle; in the narrow throat, the air is moving at its fastest speed and therefore it is at its lowest pressure.

- The Pitot tube and static port on an aircraft are used to determine the airspeed of the aircraft. These two devices are connected to the airspeed indicator which determines the dynamic pressure of the airflow past the aircraft. Dynamic pressure is the difference between stagnation pressure and static pressure. Bernoulli's principle is used to calibrate the airspeed indicator so that it displays the indicated airspeed appropriate to the dynamic pressure.[24]

- The flow speed of a fluid can be measured using a device such as a Venturi meter or an orifice plate, which can be placed into a pipeline to reduce the diameter of the flow. For a horizontal device, the continuity equation shows that for an incompressible fluid, the reduction in diameter will cause an increase in the fluid flow speed. Subsequently Bernoulli's principle then shows that there must be a decrease in the pressure in the reduced diameter region. This phenomenon is known as the Venturi effect.

- The maximum possible drain rate for a tank with a hole or tap at the base can be calculated directly from Bernoulli's equation, and is found to be proportional to the square root of the height of the fluid in the tank. This is Torricelli's law, showing that Torricelli's law is compatible with Bernoulli's principle. Viscosity lowers this drain rate. This is reflected in the discharge coefficient, which is a function of the Reynolds number and the shape of the orifice.[25]
- In open-channel hydraulics, a detailed analysis of the Bernoulli theorem and its extension were recently (2009) developed.[26] It was proved that the depth-averaged specific energy reaches a minimum in converging accelerating free-surface flow over weirs and flumes (also[27][28]). Further, in general, a channel control with minimum specific energy in curvilinear flow is not isolated from water waves, as customary state in open-channel hydraulics.

viscous losses can be neglected, and assumes that the velocity profile follows that of theoretical laminar flow. Specifically, this involves assuming that the effective flow velocity is one half of the maximum velocity, and that the average kinetic energy density is given by one third of the maximum kinetic energy density.



Now if you can swallow all those assumptions, you can model* the flow in a tube where the volume flowrate is ℑ= cm^3/s and the fluid density is ρ = gm/cm^3. For an inlet tube area A_1= cm^2 (radius r_1 = cm), the geometry of flow leads to an effective fluid velocity of v_1 = cm/s. Since the Bernoulli equation includes the fluid potential energy as well, the height of the inlet tube is specified as h_1 = cm. If the area of the tube is constricted to A_2= cm^2 (radius r_1 = cm), then without any further assumptions the effective fluid velocity in the constriction must be v_2 = cm/s. The height of the constricted tube is specified as h_2 = cm.

The kinetic energy densities at the two locations in the tube can now be calculated, and the Bernoulli equation applied to constrain the process to conserve energy, thus giving a value for the pressure in the constriction. First, specify a pressure in the inlet tube:
Inlet pressure = P_1 = kPa = lb/in^2 = mmHg = atmos.
The energy densities can now be calculated. The energy unit for the CGS units used is the erg.

Inlet tube energy densities	Constricted tube energy densities
Kinetic energy density = erg/cm^3	Kinetic energy density = erg/cm^3
Potential energy density = erg/cm^3	Potential energy density = erg/cm^3
Pressure =	

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Bernoulli concepts

The pressure energy density in the constricted tube can now be finally converted into more conventional pressure units to see the effect of the constricted flow on the fluid pressure:

Calculated pressure in constriction =
P_2= kPa = lb/in^2 = mmHg = atmos.

This calculation can give some perspective on the energy involved in fluid flow, but it's accuracy is always suspect because of the assumption of laminar flow. For typical inlet conditions, the energy density associated with the pressure will be dominant on the input side; after all, we live at the bottom of an atmospheric sea which contributes a large amount of pressure energy. If a drastic enough reduction in radius is used to yield a pressure in the constriction which is less than atmospheric pressure, there is almost certainly some turbulence involved in the flow into that constriction. Nevertheless, the calculation can show why we can get a significant amount of suction (pressure less than atmospheric) with an "aspirator" on a high pressure faucet. These devices consist of a metal tube of reducing radius with a side tube into the region of constricted radius for suction.

*Note: Some default values will be entered for some of the values as you start exploring the calculation. All of them can be changed as a part of your calculation.

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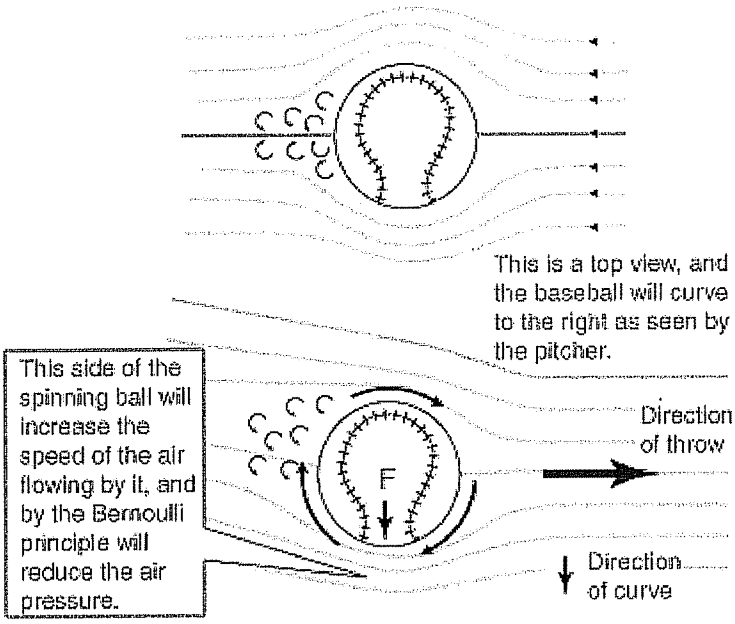
Curve of a Baseball

A non-spinning baseball or a stationary baseball in an airstream exhibits symmetric flow. A baseball which is thrown with spin will curve because one side of the ball will experience a reduced pressure. This is commonly interpreted as an application of the Bernoulli principle and involves the viscosity of the air and the boundary layer of air at the surface of the ball.



The roughness of the ball's surface and the laces on the ball are important! With a perfectly smooth ball you would not get enough interaction with the air.

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Bernoulli Equation

Bernoulli concepts

Reference Watts and Ferrer

There are some difficulties with this picture of the curving baseball. The Bernoulli equation cannot really be used to predict the amount of curve of the ball; the flow of the air is compressible, and you can't track the density changes to quantify the change in effective pressure. The experimental work of Watts and Ferrer with baseballs in a wind tunnel suggests another model which gives prominent attention to the spinning boundary layer of air around the baseball. On the side of the ball where the boundary layer is moving in the same direction as the free stream air speed, the boundary layer carries further around the ball before it separates into turbulent flow. On the side where the boundary layer is opposed by the free stream flow, it tends to separate prematurely. This gives a net deflection of the airstream in one direction behind the ball, and therefore a Newton's 3rd law reaction force on the ball in the opposite direction. This gives an effective force in the same direction indicated above.

Similar issues arise in the treatment of a spinning cylinder in an airstream, which has been shown to experience lift. This is the subject of the Kutta-Joukowski theorem. It is also invoked in the discussion of airfoil lift.

Airfoil

The air across the top of a conventional airfoil experiences constricted flow lines and increased air speed relative to the wing. This causes a decrease in pressure on the top according to the Bernoulli equation and provides a lift force. Aerodynamicists (see Eastlake) use the Bernoulli model to correlate with pressure measurements made in wind tunnels, and assert that when pressure measurements are made at multiple locations around the airfoil and summed, they do agree reasonably with the observed lift.

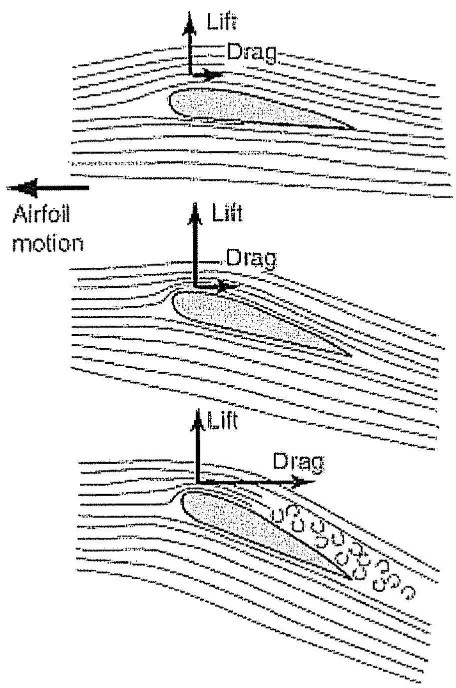

Illustration of lift force and angle of attack

Bernoulli vs Newton for airfoil lift

Airfoil terminology

Others appeal to a model based on Newton's laws and assert that the main lift comes as a result of the angle of attack. Part of the Newton's law model of part of the lift force involves attachment of the boundary layer of air on the top of the wing with a resulting downwash of air behind the wing. If the wing gives the air a downward force, then by Newton's third law, the wing experiences a force in the opposite direction - a lift. While the "Bernoulli vs Newton" debate continues, Eastlake's position is that they are really equivalent, just different approaches to the same physical phenonenon. NASA has a nice aerodynamics site at which these issues are discussed.

Increasing the angle of attack gives a larger lift from the upward component of pressure on the bottom of the wing. The lift force can be considered to be a Newton's 3rd law reaction force to the force exerted downward on the air by the wing.

At too high an angle of attack, turbulent flow increases the drag dramatically and will stall the aircraft.

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References Eastlake

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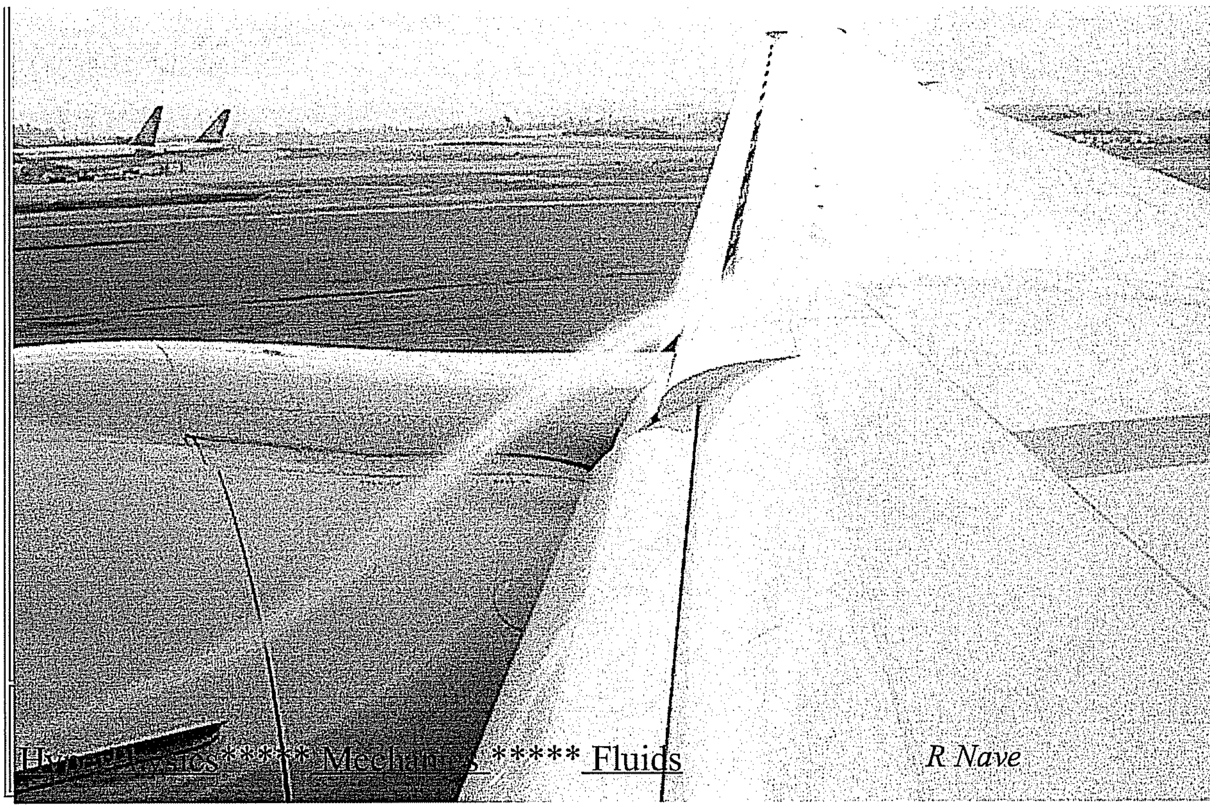

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A vapor trail over the wing helps visualize the air flow. Photo by Frank Starmer, used by permission.

Misunderstandings about the generation of lift

Main article: Lift (force)

Many explanations for the generation of lift (on airfoils, propeller blades, etc.) can be found; but some of these explanations can be misleading, and some are false.[29] This has been a source of heated discussion over the years. In particular, there has been debate about whether lift is best explained by Bernoulli's principle or Newton's laws of motion. Modern writings agree that both Bernoulli's principle and Newton's laws are relevant and either can be used to correctly describe lift.[30][31][32]

Several of these explanations use the Bernoulli principle to connect the flow kinematics to the flow-induced pressures. In cases of incorrect (or partially correct) explanations relying on the Bernoulli principle, the errors generally occur in the assumptions on the flow kinematics and how these are produced. It is not the Bernoulli principle itself that is questioned because this principle is well established.[33][34][35][36]

See also

- Terminology in fluid dynamics
- Navier–Stokes equations – for the flow of a viscous fluid
- Euler equations – for the flow of an inviscid fluid
- Hydraulics – applied fluid mechanics for liquids
- Venturi effect
- Inviscid flow

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4. ^ Streeter, V.L., *Fluid Mechanics*, Example 3.5, McGraw–Hill Inc. (1966), New York.
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Physics of Blood Pressure

Physical Meaning of Diastolic, Systolic, High, or Low Readings

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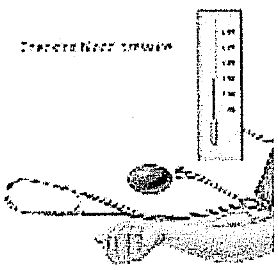

Taking Blood Pressure -

Blood pressure readings are a fundamental diagnostic tool for health care professionals. What is the physics behind blood pressure?

When visiting a clinic, a patient usually gets a blood pressure reading. High blood pressure, hypertension, can warn of impending cardiovascular disease. Excessively low blood pressure can prevent blood from flowing to higher portions of the body, including the brain. Pressure is a basic physical quantity. How does physics apply to blood pressure?

Pressure

The pressure on a surface is the total force acting on the surface divided by the surface's area.

Pressure is usually measured in newtons per square meter (often called pascals) or in pounds per square inch. Pressure is also often measured in millimeters of mercury (mm HG), a unit that originated from old-fashioned mercury barometers. The conversion factor is:

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1 mm Hg = 133 pascals = 0.02 pounds per square inch.

Blood pressure is the force the blood applies to the artery wall divided by the area of the wall. Higher blood pressure means the blood applies more force to each square meter of artery wall.

Read This Next

- Does Cutting Out Salt Really Reduce the Risk of Heart Disease?
- Understanding Pressure in Physics
- What High Blood Pressure Means in a Child

Medical personnel measure blood pressure in millimeters of mercury. A typical blood pressure reading of 120 mm Hg equals 16,000 newtons per square meter or 2.4 pounds per square inch. The blood applies a force of 16,000 newtons to every square meter of artery wall or 2.4 pounds to every square inch.

Diastolic and Systolic Pressure

A blood pressure reading is two numbers, for example 120/80. The higher number is the systolic pressure and is the maximum pressure on the artery walls during the pumping cycle. The lower number is the diastolic pressure is the lowest pressure during the cycle.

The heart pumps blood through the body. With every heartbeat the ventricles (lower heart chambers) contract, squeeze the blood out to the arteries, maximizing the blood pressure, producing the systolic reading. When the ventricles relax, the blood pressure is minimized, producing the diastolic reading. The heart refills with blood and the cycle repeats.

slowly decreased.

Eventually the pressure in the cuff is low enough that blood starts flowing again in the constricted artery. The healthcare worker listening to the artery with a stethoscope hears turbulence as the blood flow resumes, and reads the systolic blood pressure from the sphygmomanometer dial. When the turbulence stops, the pressure reading is the diastolic reading. The blood again flows smoothly because the pressure in the cuff is too low to constrict the blood flow during any part of the pumping cycle.

Blood pressure is taken on the arm at the same level as the heart because the pressure in a fluid varies with the height of the fluid.

Effects of High and Low Blood Pressure

If blood pressure is too high, the extra force on the artery walls eventually leads to increased risk of cardiovascular problems. If arteries are clogged by fatty deposits from high cholesterol, the narrow arteries force the heart to pump harder, leading to high blood pressure. Those with high blood pressure wanting details of the health consequences should consult their physicians.

Pressure in a fluid decreases with height, so there must be enough blood pressure to pump the blood upward to the brain. If the blood pressure is too low the heart cannot push the blood up to the brain and the person may faint.

Further Reading

Wilson, J.D., Buffa, A.J., and Lou, B., ., Pearson, 2007.

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