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Comments on Last Lecture

$\nabla \times \vec{E} = 0$

Point Charge at the origin

$\downarrow$

Q

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad \vec{E}(x,y,z)$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{x}{(x^2+y^2+z^2)^{3/2}} & \frac{y}{(x^2+y^2+z^2)^{3/2}} & \frac{z}{(x^2+y^2+z^2)^{3/2}} \end{vmatrix}$$

x component

(i) $$\frac{\partial}{\partial y}\frac{z}{(x^2+y^2+z^2)^{3/2}} = z\left(-\frac{3}{2}\frac{2y}{(x^2+y^2+z^2)^{5/2}}\right)$$

$-$(i) $$-\frac{\partial}{\partial z}\frac{y}{(x^2+y^2+z^2)^{3/2}} = -y\left(-\frac{3}{2}\frac{2z}{(x^2+y^2+z^2)^{5/2}}\right)$$

$= 0$

At $r = 0$, due to spherical symmetry, there is no preferred direction.

$\nabla \times \vec{E}$ at $r = 0$ must be a vector $\Rightarrow$ it has a direction

Since at $r = 0$, there is no preferred direction; $\nabla \times \vec{E}$ at $r = 0$ must vanish.

$\nabla \times \vec{E} = 0$ vanish everywhere.

For a charge Q_i located at $\vec{r}_i$, then $\vec{E}_{P,i}(x, y, z) = \vec{E}_i(\vec{r})$

$$= \frac{1}{4\pi\epsilon_0} \frac{Q}{|\vec{r} - \vec{r}_i|^3} (\vec{r} - \vec{r}_i)$$

One can show $\nabla \times \vec{E}_i = 0$

Electric field produced by charges $Q_1 \cdots Q_N$ located at $\vec{r}_1 \cdots \vec{r}_N$ is given by

$$\vec{E} = \sum_{i=1}^{N} \vec{E}_i$$

$$\nabla \times \vec{E} = \nabla \times \sum_{i=1}^{N} \vec{E}_i = \sum_{i=1}^{N} \nabla \times E_i = 0$$

$\Rightarrow$ For electrostatics $\nabla \times \vec{E} = 0$

$$\Rightarrow \oint \vec{E} \cdot \underset{d\vec{l}}{\underset{\shortparallel}{d\vec{r}}} = 0$$

In electrostatics, $\vec{E}$ is conservative

$\nabla \times \vec{E} = 0 \quad \Rightarrow \vec{E} = -\nabla V$

Lines of Force

Useful as an aid of visualizing the electric fields

introduced by Faraday

Lines of forces are imaginary lines drawn through a region of space so that at every point it is tangent to the direction of $\vec{E}$ at that point.

The lines of forces are drawn in such a way that the number of lines per unit cross sectional area perpendicular (to the line) is proportional to the magnitude of $\vec{E}$

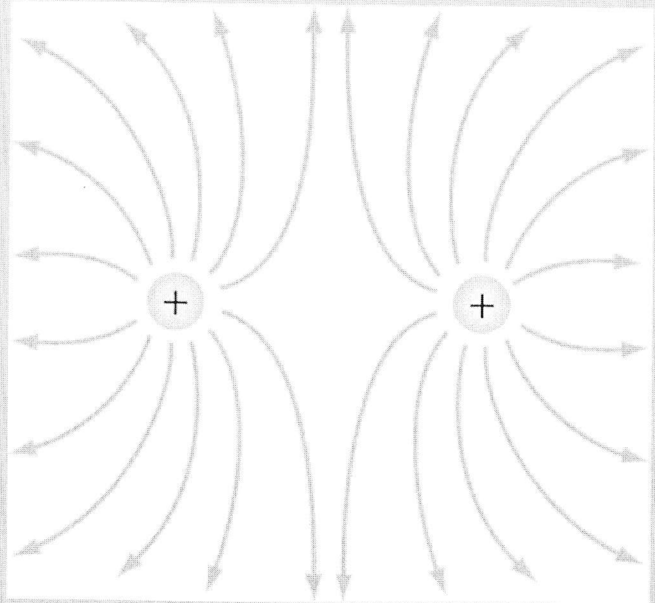
At any one point, the electric field can have only one direction $\rightarrow$ lines of force never intersect.

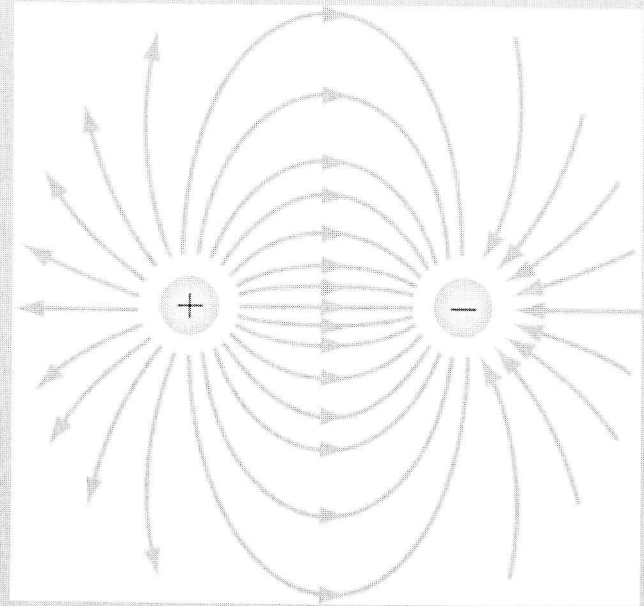
The lines of force radiate from positive charge.

The number of lines radiate from positive charge is proportional to the charge Q. Lines of forces move toward negative charge.

Electric Field Lines

1. Direction of field line at any point is tangent to field at that point
2. Field lines point away from positive charges and terminate on negative charges
3. Field lines never cross each other





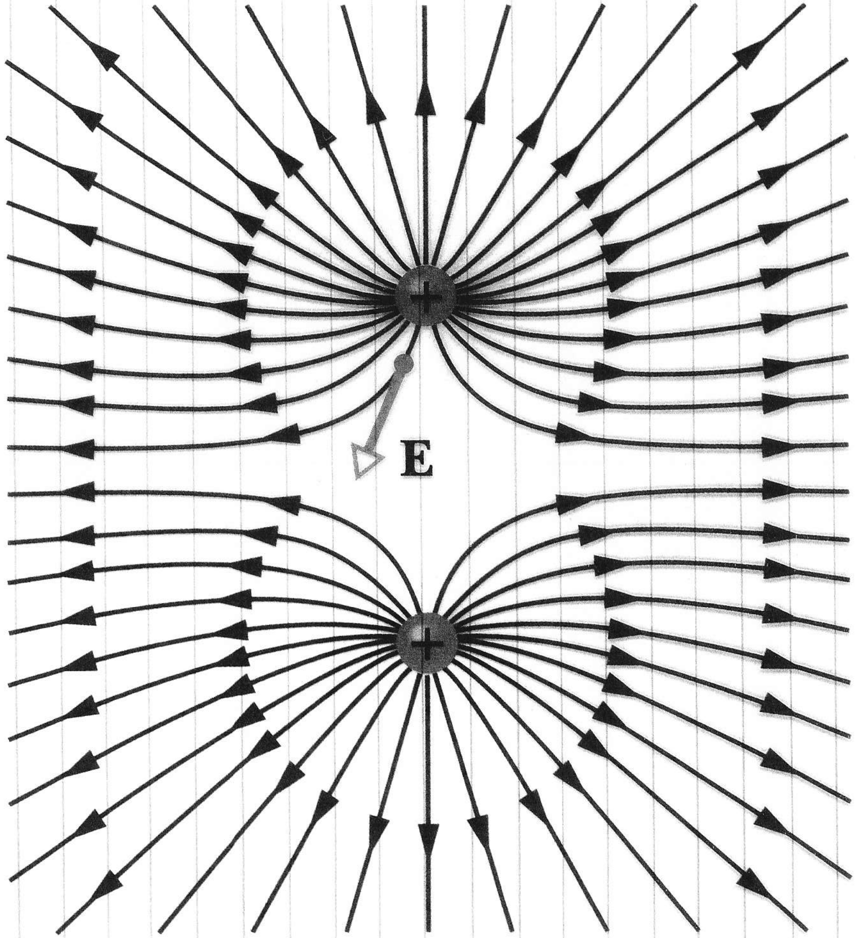


Figure 23-4

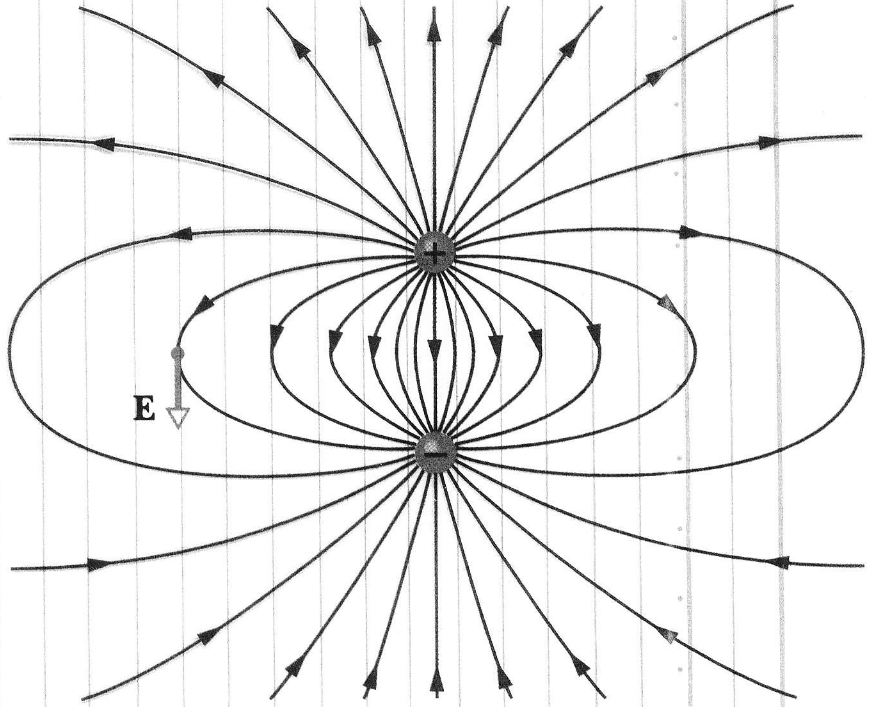
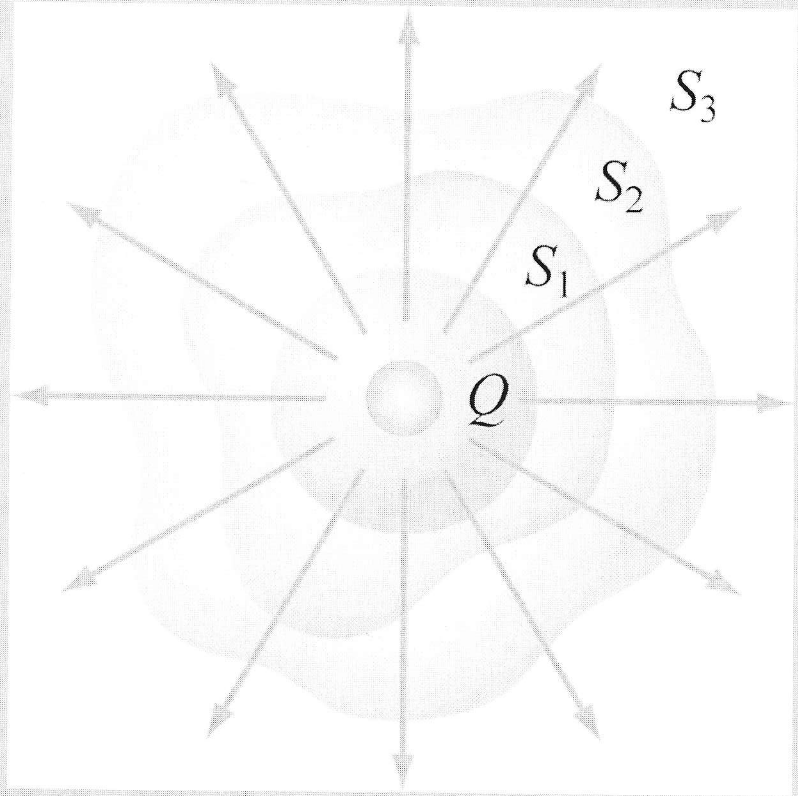


Figure 23-5

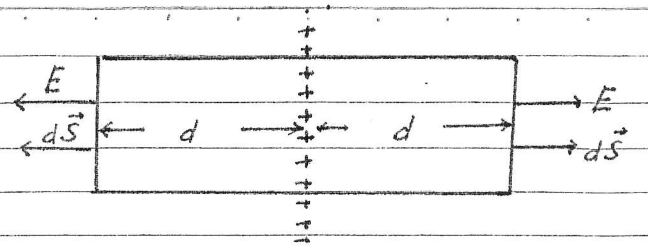
Gauss's Law – The Idea

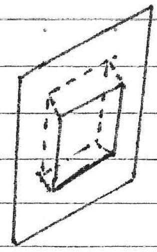


The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

Gauss' Law and Superposition principle

Uniform surface charge on an infinite sheet





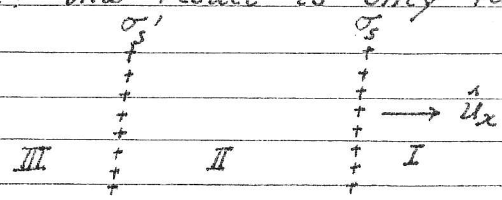
From symmetry consideration, the direction $\vec{E}$ are as shown in the figure $\Rightarrow$ only the front and back surfaces will contribute to $\oint \vec{E}\cdot d\vec{S}$

With equal distance d, the $|\vec{E}|$ at the front and back surfaces are equal

$$\oint \vec{E}\cdot d\vec{S} = 2|\vec{E}|A = \frac{1}{\epsilon_0}\underbrace{\sigma_s}_{\text{charge per unit area}} A$$

$$\Rightarrow |\vec{E}| = \underbrace{\frac{\sigma_s}{2\epsilon_0}}_{\text{independent of } d}$$

Note: this result is only valid for infinite sheet



Use principle of superposition

Region I: $\vec{E} = \dfrac{\sigma_s + \sigma_s'}{2\epsilon_0}\hat{u}_x$

Region II: $\vec{E} = \dfrac{-\sigma_s + \sigma_s'}{2\epsilon_0}\hat{u}_x$

Region III: $\vec{E} = \dfrac{-\sigma_s - \sigma_s'}{2\epsilon_0}\hat{u}_x$

Charging

How Do You Charge Objects?

- Friction
- Transfer (touching)
- Induction