Chapter 31

1. Review of Dielectrics
2. Magnetism in Matter
3. Orbital Magnetic Dipole Moment
4. Spin Magnetic Moment
5. Paramagnetism
6. Diamagnetism.
7. Ferromagnetism

Chapter 31

Electric and Magnetic Properties of Matter

Dielectrics
↓
we have already discussed this problem in Chapter 25

A dielectric weaken the electric field between the plates of capacitor decreases because the dielectric produce a field opposite to the field produced by the plates

Molecular picture

q_f , q_b

q_f ↓ free change

q_b ↓ bounded charge

Dielectrics

Outline

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$\vec{P}$ = dipole moment/volume

$\uparrow$ definition

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\int \vec{D} \cdot d\vec{S} = q_{free}$$

$$\updownarrow$$

$$\nabla \cdot \vec{D} = \rho_{free}$$

$$\vec{P} \propto \vec{E} \quad \Rightarrow \quad \vec{P} = \chi_e \epsilon_0 \vec{E}$$

$$\begin{aligned} \vec{D} &= \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E} \\ &= (1+\chi_e)\epsilon_0 \vec{E} \\ &= \epsilon \vec{E} \end{aligned} \qquad K = \frac{\epsilon}{\epsilon_0} = 1+\chi_e$$

$$\nabla \cdot \epsilon \vec{E} = \rho_{free}$$

$$\Downarrow$$

$$\epsilon \nabla \cdot \vec{E} = \rho_{free} \quad \Rightarrow \quad \nabla \cdot \vec{E} = \frac{\rho_{free}}{\epsilon}$$

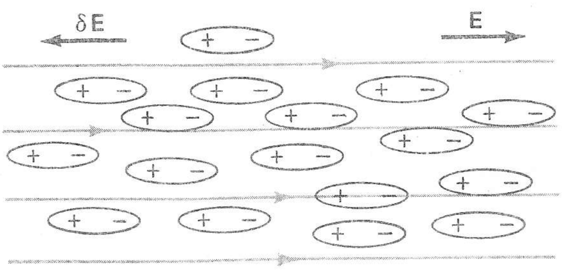
A dielectric weaken the electric field between the plates of capacitor decreases because the dielectric produce a field in a direction opposite to the field produced by the plates

The electric field is due to the electric dipole moment of the molecules in the dielectrics

Electric dipole moment

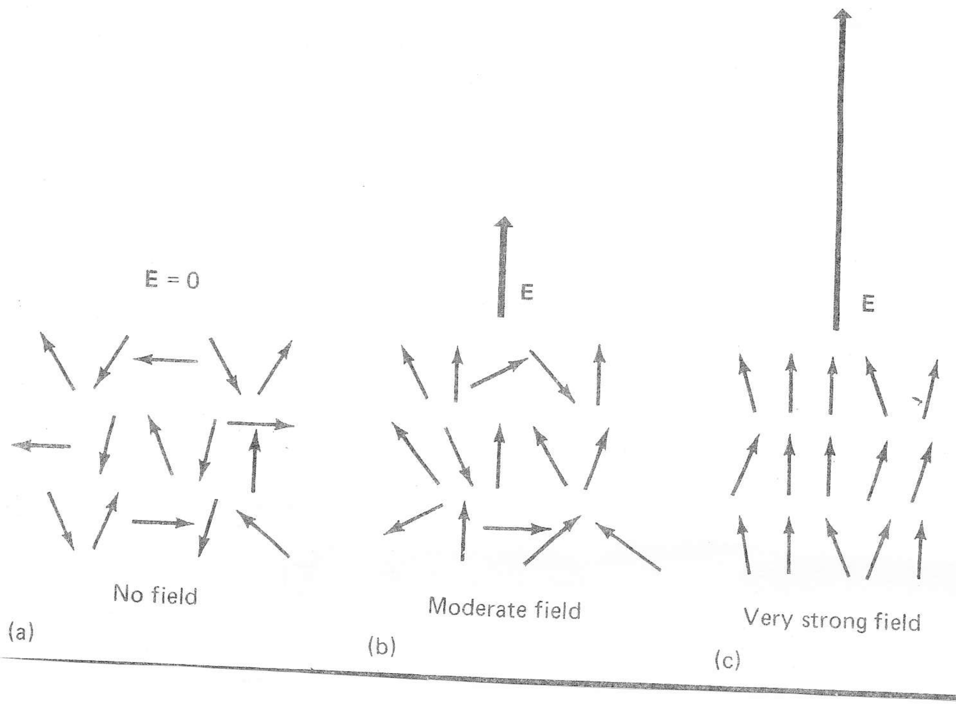
Induced dipole moment

(i)



Polarization of randomly oriented dipole molecules by an externally applied field

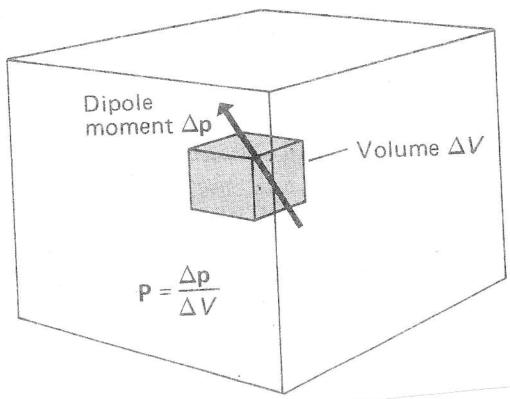
(ii)



Remarks:

- Both case show that the polariz is due to the external applied.
- Meaning of dielectric strength.
- Temperature dependence
  ↓
  competition with thermal motion
- Frequency Dependence of of oscillation field
  ↓
  related to the detail molecular structure

Polarization Vector



Induced elementary dipole moments arising from the application of an external electric field. The total electric field within the substance is the sum of the external field and the fields arising from the induced dipoles.

$\vec{p}_i$ small molecular dipole moment $\vec{p}_i = q\vec{d}$

ΔV

$$\Delta\vec{p} = \sum_{i=1}^{n} \vec{p}_i = \text{dipole moment in the volume}$$

$$\vec{P} = \lim_{\Delta V \to 0} \frac{\Delta \vec{p}}{\Delta V} = \frac{d\vec{p}}{dV}$$

↓

polarization vector

The existence of the polarization $\vec{P}$

↓

additional electric field which partially the original field

It is difficult to calculate the new field by superposing the field produced by each small volume element within the dielectric.

↓

"model" are used

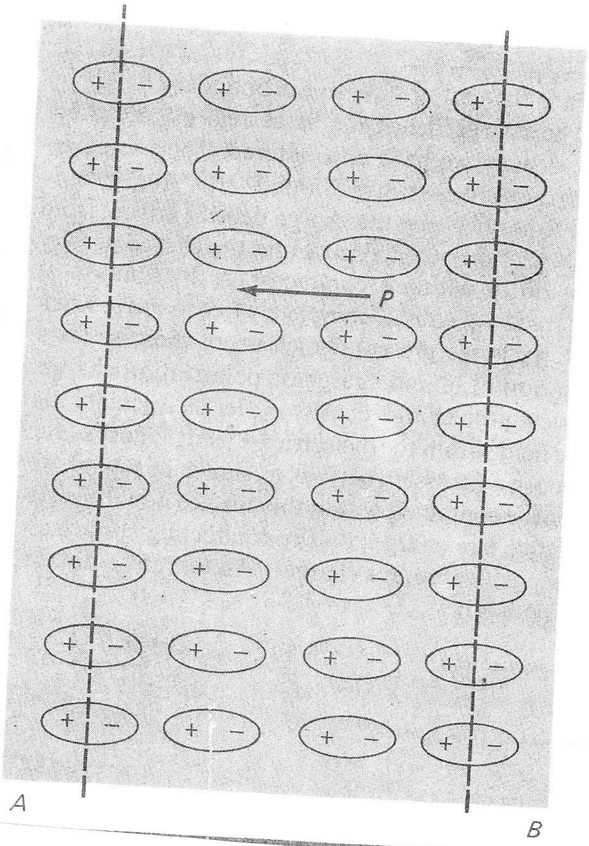
Assumptions

Neglect the interaction

$\vec{P}$ is uniform throughout

Use parallel plate as an example

$$\sigma_b = P$$



In a uniformly polarized dielectric, the charges of adjacent interior dipoles cancel each other; the net field produced is solely due to the unbalanced surface charge distribution that appears on the exterior surfaces of the material.

$$q_b = q\, \underset{\substack{\downarrow \\ \text{number of molecule per unit volume}}}{n}\, A\Delta \qquad \sigma_b = qn\Delta$$

$$P = q\Delta n$$

$$\Rightarrow \quad P = \sigma_b$$

dimensional analysis

$$\left(\frac{QL}{L^3} = \frac{Q}{L^2} \right)$$

In general case

$$\boxed{\sigma_b = \vec{P}\cdot\hat{n}}$$

$$\oint (\epsilon_0 \vec{E} + \vec{P}) \cdot \hat{n} \; dS = q_f$$

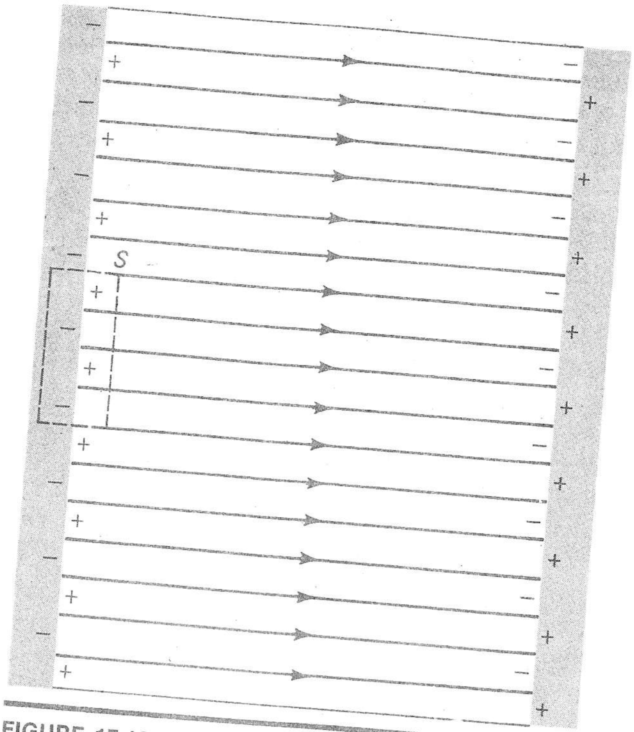


FIGURE 17.16. Gaussian surface, containing both free and bound polarization charge, used in the calculation of the field within a dielectric.

Using the charge density σ_b, let us calculate the electric field within the dielectric by using Gauss's law. We choose a Gaussian surface, as shown in Fig. 17.16. The charge contained within this surface now consists of negative *free* charges on the conducting surface as well as positive bound charges. We therefore obtain from Gauss's law

$$\oint_s \mathbf{E} \cdot \mathbf{n} \, da = EA = \left(\frac{\sigma_f - \sigma_b}{\varepsilon_0}\right) A$$

or

$$E = \frac{\sigma_f - \sigma_b}{\varepsilon_0} \tag{17.4.10}$$

[4] It is easy to see why this is so by noting that if the polarization vector is *parallel* to the boundary, there is no surface charge there at all. It is, therefore, only the component of the polarization vector *normal* to the surface (that is, $\mathbf{p} \cdot \mathbf{n}$) that is effective in generating a surface distribution of bound charge

Thus, for a given free surface charge density on the plates of the capacitor, the presence of the dielectric brings about a *reduction* of the electric field. Substituting $\sigma_b = \mathbf{P} \cdot \mathbf{n} = P$, we can rewrite the above equation as

$$\sigma_f = \varepsilon_0 E + P \tag{17.4.11}$$

The vector combination $\varepsilon_0 \mathbf{E} + \mathbf{P}$ is called the *displacement vector* $\mathbf{D}$. As we see from the simple case above, this vector is directly related to the free charge density. In fact, since we can regard $\sigma_b A/\varepsilon_0 = (\mathbf{P} \cdot \mathbf{n})A/\varepsilon_0$ as the integral of $(\mathbf{P} \cdot \mathbf{n})/\varepsilon_0$ over the Gaussian surface, we can *restate* Gauss's law in terms of $\mathbf{D}$ in the simple form

$$\boxed{\oint_s (\varepsilon_0 \mathbf{E} + \mathbf{P}) \cdot \mathbf{n} \, da = \oint_s \mathbf{D} \cdot \mathbf{n} = q_f} \tag{17.4.12}$$

where q_f is the total amount of *free* charge within the Gaussian surface. This is the form in which we ordinarily use Gauss's law when dielectric materials are present. It should be noted that the charge on the left side represents only the free charge within the Gaussian surface; effects due to the distribution of bound charges are completely accounted for by the inclusion of the polarization term in the integral. We shall return to a somewhat more detailed discussion of the displacement vector $\mathbf{D}$ in a subsequent section.

From the above discussion, we see that in a

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\oint \vec{D} \cdot \hat{n}\, dS = q_f$$

$$\downarrow$$

$$\int \nabla \cdot \vec{D}\, d^3v = \int \rho_f\, d^3v$$

$$\Downarrow$$

$$\boxed{\nabla \cdot \vec{D} = \rho_f}$$

first Maxwell equation in material

Linear Dielectric

$$\vec{P} = \chi \epsilon_0 \vec{E}$$

χ $\downarrow$ electric susceptibility

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \chi \epsilon_0 \vec{E}$$

$$= \underbrace{\epsilon_0 (1+\chi)}_{\epsilon} \vec{E}$$

$$\epsilon = \epsilon_0 (1+\chi) \quad \rightarrow \quad \text{electric susceptibility}$$

$$\downarrow$$

$$k = \frac{\epsilon}{\epsilon_0} = 1 + \chi$$

k $\downarrow$ dielectric constant

$$\nabla \cdot \vec{D} = \rho_f$$

$$\vec{D} = \epsilon \vec{E}$$

$$\epsilon\, \nabla \cdot \vec{E} = \rho_f$$

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\epsilon}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

in vacuum

Magnetic Material (Chapter 31)

Outline

$\vec{M}$ = magnetic dipole moment / volume

$$\oint_c \vec{H} \cdot d\vec{l} = I_{free}$$

$$\vec{H} = \frac{B}{\mu_0} - \vec{M}$$

$\vec{H}$ → magnetic intensity

$$\Rightarrow \quad \nabla \times \vec{H} = \vec{J}_{free}$$ by using Stoke's theorem

$$\vec{M} = \chi_m \vec{H}$$

χ_m → magnetic susceptibility

$$\vec{H} = \frac{B}{\mu_0} - \chi_m \vec{H}$$

$$\frac{B}{\mu_0} = (1+\chi_m)\vec{H}$$

$$\vec{B} = \underbrace{\mu_0 (1+\chi_m)}_{\mu} \vec{H}$$

$$\mu = \mu_0 (1+\chi_m)$$

$$\frac{\mu}{\mu_0} = (1+\chi_m) = \kappa_m$$

κ_m → magnetic permeability

$\chi_m > 0$ paramagnetic substance

$\chi_m < 0$ diamagnetic substance

Magnetic Susceptibilities

Substance	χ_m
Aluminum	2.3×10^{-5}
Bismuth	-1.7×10^{-4}
Copper	-1.0×10^{-5}
Gold	-3.6×10^{-5}
Lead	-1.7×10^{-5}
Magnesium	1.2×10^{-5}
Platinum	2.9×10^{-4}
Silver	-2.6×10^{-5}
Water	-0.88×10^{-5}
$CrK(SO_4)_2 \cdot 12H_2O$	2.32×10^{-5}
$Cu(SO_4) \cdot 5H_2O$	1.43×10^{-5}
$Gd_2(SO_4)_3 \cdot 8H_2O$	2.21×10^{-4}
MnF_2	4.59×10^{-4}
$CoCl_2$	3.38×10^{-4}
$FeCl_2$	3.10×10^{-4}
$FeCl_3$	2.40×10^{-4}
$NiCl_2$	1.71×10^{-4}
Iron (soft)	~ 5000

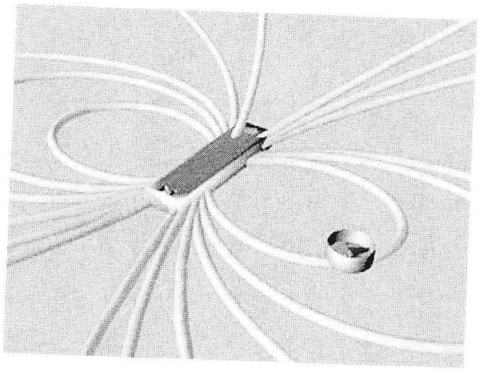


Figure 9.5.4 A bar magnet in Earth's magnetic field

9.6 Magnetic Materials

The introduction of material media into the study of magnetism has very different consequences as compared to the introduction of material media into the study of electrostatics. When we dealt with dielectric materials in electrostatics, their effect was *always* to reduce $\vec{\mathbf{E}}$ below what it would otherwise be, for a given amount of "free" electric charge. In contrast, when we deal with magnetic materials, their effect can be one of the following:

(i) reduce $\vec{\mathbf{B}}$ below what it would otherwise be, for the same amount of "free" electric current (*diamagnetic* materials);

(ii) increase $\vec{\mathbf{B}}$ a little above what it would otherwise be (*paramagnetic* materials);

(iii) increase $\vec{\mathbf{B}}$ *a lot* above what it would otherwise be (*ferromagnetic* materials).

Below we discuss how these effects arise.

9.6.1 Magnetization

Magnetic materials consist of many permanent or induced magnetic dipoles. One of the concepts crucial to the understanding of magnetic materials is the average magnetic field produced by many magnetic dipoles which are all aligned. Suppose we have a piece of material in the form of a long cylinder with area A and height L, and that it consists of N magnetic dipoles, each with magnetic dipole moment $\vec{\boldsymbol{\mu}}$, spread uniformly throughout the volume of the cylinder, as shown in Figure 9.6.1.

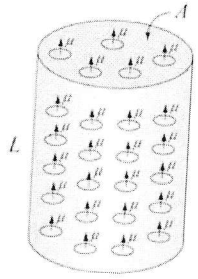


Figure 9.6.1 A cylinder with N magnetic dipole moments

We also assume that all of the magnetic dipole moments $\vec{\boldsymbol{\mu}}$ are aligned with the axis of the cylinder. In the absence of any external magnetic field, what is the average magnetic field due to these dipoles alone?

To answer this question, we note that each magnetic dipole has its own magnetic field associated with it. Let's define the magnetization vector $\vec{\mathbf{M}}$ to be the net magnetic dipole moment vector per unit volume:

$$\vec{\mathbf{M}} = \frac{1}{V}\sum_i \vec{\boldsymbol{\mu}}_i \tag{9.6.1}$$

where V is the volume. In the case of our cylinder, where all the dipoles are aligned, the magnitude of $\vec{\mathbf{M}}$ is simply $M = N\mu / AL$.

Now, what is the average magnetic field produced by all the dipoles in the cylinder?

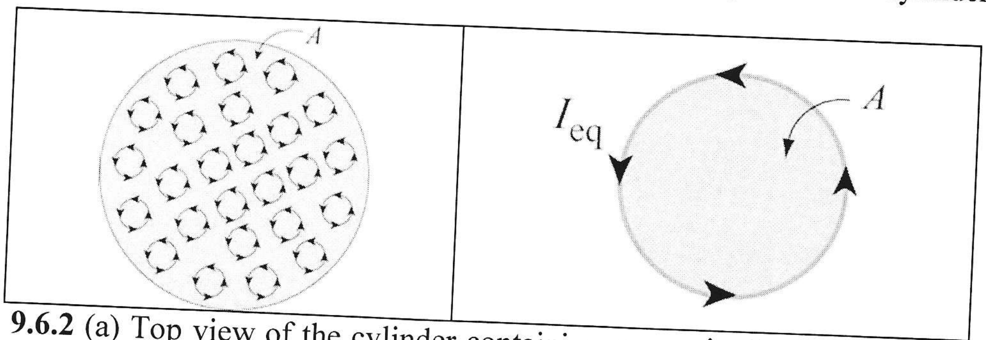


Figure 9.6.2 (a) Top view of the cylinder containing magnetic dipole moments. (b) The equivalent current.

Figure 9.6.2(a) depicts the small current loops associated with the dipole moments and the direction of the currents, as seen from above. We see that in the interior, currents flow in a given direction will be cancelled out by currents flowing in the opposite direction in neighboring loops. The only place where cancellation does not take place is near the edge of the cylinder where there are no adjacent loops further out. Thus, the average current in the interior of the cylinder vanishes, whereas the sides of the cylinder appear to carry a net current. The equivalent situation is shown in Figure 9.6.2(b), where there is an equivalent current I_{eq} on the sides.

The functional form of I_{eq} may be deduced by requiring that the magnetic dipole moment produced by I_{eq} be the same as total magnetic dipole moment of the system. The condition gives

$$I_{\text{eq}} A = N\mu \tag{9.6.2}$$

or

$$I_{\text{eq}} = \frac{N\mu}{A} \tag{9.6.3}$$

Next, let's calculate the magnetic field produced by I_{eq}. With I_{eq} running on the sides, the equivalent configuration is identical to a solenoid carrying a surface current (or current per unit length) K. The two quantities are related by

$$K = \frac{I_{\text{eq}}}{L} = \frac{N\mu}{AL} = M \tag{9.6.4}$$

Thus, we see that the surface current K is equal to the magnetization M, which is the average magnetic dipole moment per unit volume. The average magnetic field produced by the equivalent current system is given by (see Section 9.4)

$$B_M = \mu_0 K = \mu_0 M \tag{9.6.5}$$

Since the direction of this magnetic field is in the *same* direction as $\vec{\mathbf{M}}$, the above expression may be written in vector notation as

$$\vec{\mathbf{B}}_M = \mu_0 \vec{\mathbf{M}} \tag{9.6.6}$$

This is exactly opposite from the situation with electric dipoles, in which the average electric field is anti-parallel to the direction of the electric dipoles themselves. The reason is that in the region interior to the current loop of the dipole, the magnetic field is in the *same* direction as the magnetic dipole vector. Therefore, it is not surprising that after a large-scale averaging, the average magnetic field also turns out to be parallel to the average magnetic dipole moment per unit volume.

Notice that the magnetic field in Eq. (9.6.6) is the *average* field due to all the dipoles. A very different field is observed if we go close to any one of these little dipoles.

Let's now examine the properties of different magnetic materials

9.6.2 Paramagnetism

The atoms or molecules comprising paramagnetic materials have a permanent magnetic dipole moment. Left to themselves, the permanent magnetic dipoles in a paramagnetic material never line up spontaneously. In the absence of any applied external magnetic field, they are randomly aligned. Thus, $\vec{\mathbf{M}} = \vec{\mathbf{0}}$ and the average magnetic field $\vec{\mathbf{B}}_M$ is also zero. However, when we place a paramagnetic material in an external field $\vec{\mathbf{B}}_0$, the dipoles experience a torque $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}_0$ that tends to align $\vec{\boldsymbol{\mu}}$ with $\vec{\mathbf{B}}_0$, thereby producing a net magnetization $\vec{\mathbf{M}}$ parallel to $\vec{\mathbf{B}}_0$. Since $\vec{\mathbf{B}}_M$ is parallel to $\vec{\mathbf{B}}_0$, it will tend to *enhance* $\vec{\mathbf{B}}_0$. The total magnetic field $\vec{\mathbf{B}}$ is the sum of these two fields:

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_M = \vec{\mathbf{B}}_0 + \mu_0 \vec{\mathbf{M}} \qquad (9.6.7)$$

Note how different this is than in the case of dielectric materials. In both cases, the torque on the dipoles causes alignment of the dipole vector parallel to the external field. However, in the paramagnetic case, that alignment *enhances* the external magnetic field, whereas in the dielectric case it *reduces* the external electric field. In most paramagnetic substances, the magnetization $\vec{\mathbf{M}}$ is not only in the same direction as $\vec{\mathbf{B}}_0$, but also linearly proportional to $\vec{\mathbf{B}}_0$. This is plausible because without the external field $\vec{\mathbf{B}}_0$ there would be no alignment of dipoles and hence no magnetization $\vec{\mathbf{M}}$. The linear relation between $\vec{\mathbf{M}}$ and $\vec{\mathbf{B}}_0$ is expressed as

$$\vec{\mathbf{M}} = \chi_m \frac{\vec{\mathbf{B}}_0}{\mu_0} \qquad (9.6.8)$$

where χ_m is a dimensionless quantity called the *magnetic susceptibility*. Eq. (10.7.7) can then be written as

$$\vec{\mathbf{B}} = (1 + \chi_m)\vec{\mathbf{B}}_0 = \kappa_m \vec{\mathbf{B}}_0 \qquad (9.6.9)$$

where

$$\kappa_m = 1 + \chi_m \qquad (9.6.10)$$

is called the *relative permeability* of the material. For paramagnetic substances, $\kappa_m > 1$, or equivalently, $\chi_m > 0$, although χ_m is usually on the order of 10^{-6} to 10^{-3}. The *magnetic permeability* μ_m of a material may also be defined as

$$\mu_m = (1 + \chi_m)\mu_0 = \kappa_m \mu_0 \qquad (9.6.11)$$

Paramagnetic materials have $\mu_m > \mu_0$.

9.6.3 Diamagnetism

In the case of magnetic materials where there are no permanent magnetic dipoles, the presence of an external field $\vec{\mathbf{B}}_0$ will induce magnetic dipole moments in the atoms or molecules. However, these induced magnetic dipoles are anti-parallel to $\vec{\mathbf{B}}_0$, leading to a magnetization $\vec{\mathbf{M}}$ and average field $\vec{\mathbf{B}}_M$ anti-parallel to $\vec{\mathbf{B}}_0$, and therefore a *reduction* in the total magnetic field strength. For diamagnetic materials, we can still define the magnetic permeability, as in equation (8-5), although now $\kappa_m < 1$, or $\chi_m < 0$, although χ_m is usually on the order of -10^{-5} to -10^{-9}. Diamagnetic materials have $\mu_m < \mu_0$.

9.6.4 Ferromagnetism

In ferromagnetic materials, there is a strong interaction between neighboring atomic dipole moments. Ferromagnetic materials are made up of small patches called *domains*, as illustrated in Figure 9.6.3(a). An externally applied field $\vec{\mathbf{B}}_0$ will tend to line up those magnetic dipoles parallel to the external field, as shown in Figure 9.6.3(b). The strong interaction between neighboring atomic dipole moments causes a *much stronger* alignment of the magnetic dipoles than in paramagnetic materials.

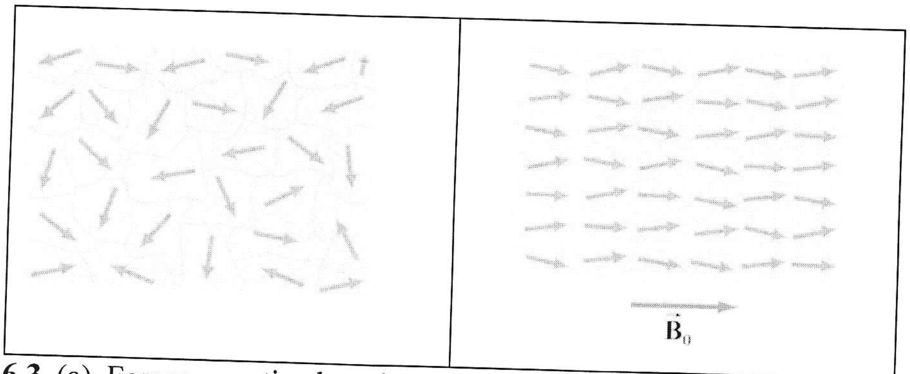


Figure 9.6.3 (a) Ferromagnetic domains. (b) Alignment of magnetic moments in the direction of the external field $\vec{\mathbf{B}}_0$.

The enhancement of the applied external field can be considerable, with the total magnetic field inside a ferromagnet 10^3 or 10^4 times greater than the applied field. The permeability κ_m of a ferromagnetic material is not a constant, since neither the total field $\vec{\mathbf{B}}$ or the magnetization $\vec{\mathbf{M}}$ increases linearly with $\vec{\mathbf{B}}_0$. In fact the relationship between $\vec{\mathbf{M}}$ and $\vec{\mathbf{B}}_0$ is not unique, but dependent on the previous history of the material. The phenomenon is known as *hysteresis*. The variation of $\vec{\mathbf{M}}$ as a function of the externally applied field $\vec{\mathbf{B}}_0$ is shown in Figure 9.6.4. The loop $abcdef$ is a *hysteresis curve.*

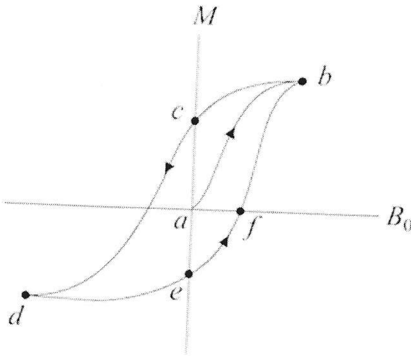


Figure 9.6.4 A hysteresis curve.

Moreover, in ferromagnets, the strong interaction between neighboring atomic dipole moments can keep those dipole moments aligned, even when the external magnet field is reduced to zero. And these aligned dipoles can thus produce a strong magnetic field, all by themselves, without the necessity of an external magnetic field. This is the origin of permanent magnets. To see how strong such magnets can be, consider the fact that magnetic dipole moments of atoms typically have magnitudes of the order of $10^{-23}\ \text{A}\cdot\text{m}^2$. Typical atomic densities are 10^{29} atoms/m^3. If all these dipole moments are aligned, then we would get a magnetization of order

$$M \sim (10^{-23}\ \text{A}\cdot\text{m}^2)(10^{29}\ \text{atoms/m}^3) \sim 10^6\ \text{A/m} \tag{9.6.12}$$

The magnetization corresponds to values of $\vec{\mathbf{B}}_M = \mu_0\vec{\mathbf{M}}$ of order 1 tesla, or 10,000 Gauss, just due to the atomic currents alone. This is how we get permanent magnets with fields of order 2200 Gauss.

9.7 Summary

- **Biot-Savart law** states that the magnetic field $d\vec{\mathbf{B}}$ at a point due to a length element $d\vec{\mathbf{s}}$ carrying a steady current I and located at $\vec{\mathbf{r}}$ away is given by

  $$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

  where $r = |\vec{\mathbf{r}}|$ and $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$ is the permeability of free space.

- The magnitude of the magnetic field at a distance r away from an infinitely long straight wire carrying a current I is

$$B = \frac{\mu_0 I}{2\pi r}$$

- The magnitude of the magnetic force F_B between two straight wires of length ℓ carrying steady current of I_1 and I_2 and separated by a distance r is

$$F_B = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$$

- **Ampere's law** states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed loop is proportional to the total steady current passing through any surface that is bounded by the close loop:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$$

- The magnetic field inside a **toroid** which has N closely spaced of wire carrying a current I is given by

$$B = \frac{\mu_0 N I}{2\pi r}$$

where r is the distance from the center of the toroid.

- The magnetic field inside a **solenoid** which has N closely spaced of wire carrying current I in a length of l is given by

$$B = \mu_0 \frac{N}{l} I = \mu_0 n I$$

where n is the number of number of turns per unit length.

- The properties of magnetic materials are as follows:

Materials	Magnetic susceptibility χ_m	Relative permeability $\kappa_m = 1 + \chi_m$	Magnetic permeability $\mu_m = \kappa_m \mu_0$
Diamagnetic	$-10^{-5} \sim -10^{-9}$	$\kappa_m < 1$	$\mu_m < \mu_0$
Paramagnetic	$10^{-5} \sim 10^{-3}$	$\kappa_m > 1$	$\mu_m > \mu_0$
Ferromagnetic	$\chi_m \gg 1$	$\kappa_m \gg 1$	$\mu_m \gg \mu_0$