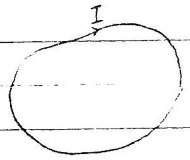
Chapter 32 Inductance and Circuit Oscillation

第二節 感應

1. 簡介：在本節中將討論自感、互感及磁場能量等觀念

2. 基本觀念



若一線圈上有電流 I 則在線圈附近會產生一磁場，其大小與 I 成正比。

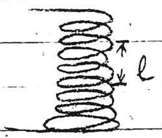
令由此電流 I 所引起的穿過由該線圈所包圍表面之磁通量為 Φ_I，Φ_I 應該與 I 成正比。

$$\Phi_I = LI \tag{1}$$

L 稱為自感。

我們現在討論一段螺線管之自感

令此段之長度為 l 其總圈數為 N



在線圈內之 $B = \dfrac{\mu_0 N I}{l}$，線圈之外之 $B=0$ (2)

通過每個線圈之磁通量為 $BA = \dfrac{\mu_0 N I}{l} A$ (3)

此處 A 是垂直於 $\vec{B}$ 場線圈之面積

通過此段螺線管之總磁通量 $=\phi_I = NBA = \dfrac{\mu_0 N^2 A}{l} I$ (4)

由自感之定義得知 $L = \dfrac{\mu_0 N^2 A}{l}$ (5)

$n = \dfrac{N}{l}$ 是單位長度中之線圈數，此式可寫成

$$L = \mu_0 n^2 l A \tag{6}$$

將上式與法那第一亨利定律與定義 (1) 合併得

$$V_L = -\frac{d}{dt}\Phi_I = -\frac{d}{dt}(LI) \tag{7}$$

若 L 不是時間的函數則 $V_L = -L\dfrac{dI}{dt}$ (8)

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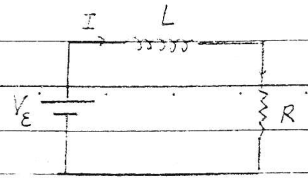
總號：



$$V_\varepsilon + V_L = IR \qquad (9)$$

$$V_\varepsilon - L\frac{dI}{dt} = IR \qquad (10)$$

$$V_\varepsilon = L\frac{dI}{dt} + IR \qquad (11)$$

兩邊乘以 I

$$V_\varepsilon I = LI\frac{dI}{dt} + I^2R \qquad (12)$$

我們現在從功和能的觀念來解釋以上公式的物理意義.

(i) $V_\varepsilon I$ 是由電池在單位時間給予該電路之能

(ii) I^2R 是在電阻中單位時間變成焦耳熱量

(iii) $LI\frac{dI}{dt}$ 是單位時間存入磁場之能量

$$\frac{dU_B}{dt} = LI\frac{dI}{dt} = \frac{d}{dt}\left(\frac{1}{2}LI^2\right) \qquad (13)$$

所以 $$U_B = \frac{1}{2}LI^2 \qquad (14)$$

即是磁場之能量

在螺線管中之磁場能量

$$U_B = \frac{1}{2}\mu_0 n^2 \underbrace{lA}_{V} I^2 \qquad (15)$$

因此它在單位体積中之磁能

$$u_B = \frac{1}{2}\mu_0 n^2 I^2 = \frac{1}{2\mu_0}|B|^2 \qquad (16)$$

此一結果雖然是由以上之特例導出, 但它在一般情形下仍成立.

當電磁場均存在時則電磁能密度是

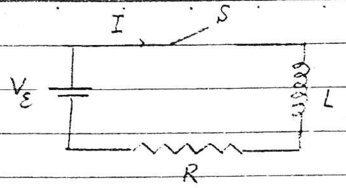
$$u = \frac{1}{2}\epsilon_0|\vec{E}|^2 + \frac{1}{2\mu_0}|\vec{B}|^2 \qquad (17)$$

若是在介質中則 $u = \frac{1}{2}\epsilon|\vec{E}|^2 + \frac{1}{2\mu}|\vec{B}|^2 \qquad (18)$

分類：
編號： 3
總號：

L - C 組成的線路

(1)



$$V_\varepsilon = RI + L\frac{dI}{dt} \tag{19}$$

$$t = 0^- \qquad I = 0 \tag{20}$$

$t = 0$ 開關閉上，$t = 0^+$，I 仍為 0. (21)

$$L\frac{dI}{dt} = V_\varepsilon - RI \tag{22}$$

$$\frac{dI}{I - V_\varepsilon/R} = -\frac{R}{L}\,dt \tag{23}$$

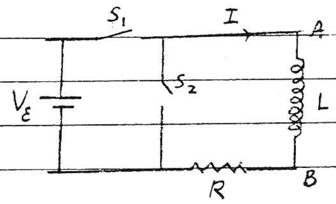
$$\int_0^I \frac{dI}{I - V_\varepsilon/R} = -\frac{R}{L}\int_0^t dt \tag{24}$$

$$\ln\,(I - V_\varepsilon/R)/(-V_\varepsilon/R) = -\frac{R}{L}t \tag{25}$$

$$\frac{I - V_\varepsilon/R}{-V_\varepsilon/R} = e^{-\frac{R}{L}t} \tag{26}$$

所以 $I = \frac{V_\varepsilon}{R}(1 - e^{-\frac{R}{L}t})$ (27)

(2)



$t = 0^-$ $I = I_0$ （S_1 開關閉開而 S_2 開了很久時間電流變成穩定）

$t = 0$，S_1 打開 S_2 關閉 $t = 0^+$ $I = I_0$

$$0 = RI + L\frac{dI}{dt} \tag{28}$$

因此 $I = Ae^{-\frac{R}{L}t}$， (29)

由於 $t = 0$，$I = I_0$，所以

$$I = I_0 e^{-\frac{Rt}{L}} \tag{30}$$

$$V_L = L\frac{dI}{dt} = V_A - V_B \tag{31}$$

$$= -RI_0 e^{-\frac{Rt}{L}} \tag{32}$$

此兩頁反了.

互感

分類：	
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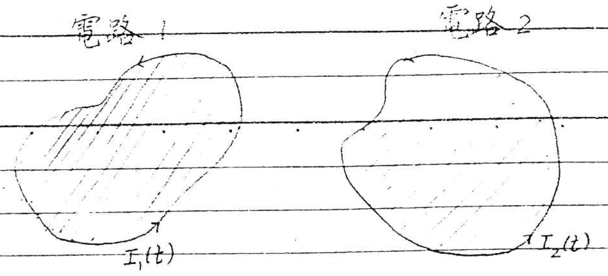
總號：	



$$\Phi_1 = \int_{S_1} \vec{B}_1 \cdot \hat{u}_{N1}\, dS + \int_{S_1} \vec{B}_2 \cdot \hat{u}_{N1}\, dS = \Phi_{11} + \Phi_{12} \tag{33}$$

$$\Phi_2 = \int_{S_2} \vec{B}_1 \cdot \hat{u}_{N2}\, dS + \int_{S_2} \vec{B}_2 \cdot \hat{u}_{N2}\, dS = \Phi_{21} + \Phi_{22} \tag{34}$$

$\vec{B}_1$，$\vec{B}_2$ 是由 I_1，I_2 所產生之 $\vec{B}$ 場

$$\Phi_{11} = L_1 I_1 \tag{35}$$

$$\Phi_{22} = L_2 I_2 \tag{36}$$

由於 Φ_{12} 是與 I_2 成正比

$$\Phi_{12} = M_{12} I_2 \tag{37}$$

同理

$$\Phi_{21} = M_{21} I_1 \tag{38}$$

M_{12}，M_{21} 稱為互感，與兩個線路之幾何形狀及相互位置有關

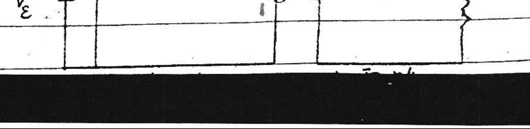
在第一線路上之感應電動勢是

$$V_1 = -L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt} \tag{39}$$

在第二線路上之感應電動勢是

$$V_2 = -M_{21} \frac{dI_1}{dt} - L_2 \frac{dI_2}{dt} \tag{40}$$

我們現在假設已求得自感及互感，我們討論以下之應用



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主電路之電路公式

$$V_\varepsilon - L_1\frac{dI_1}{dt} - M_{12}\frac{dI_2}{dt} = I_1 R_1 \qquad (41)$$

次電路之電路公式

$$-L_2\frac{dI_2}{dt} - M_{21}\frac{dI_1}{dt} = I_2 R_2 \qquad (42)$$

此是一組耦合的微分方程式

Chapter 32

Inductance

Self inductance

Add to the lecture of last time

$$L = \frac{N\Phi_B}{I}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$\downarrow$

emf

Class 24: Outline

Hour 1:

Inductance & LR Circuits

Hour 2:

Energy in Inductors

Last Time:
Faraday's Law
Mutual Inductance

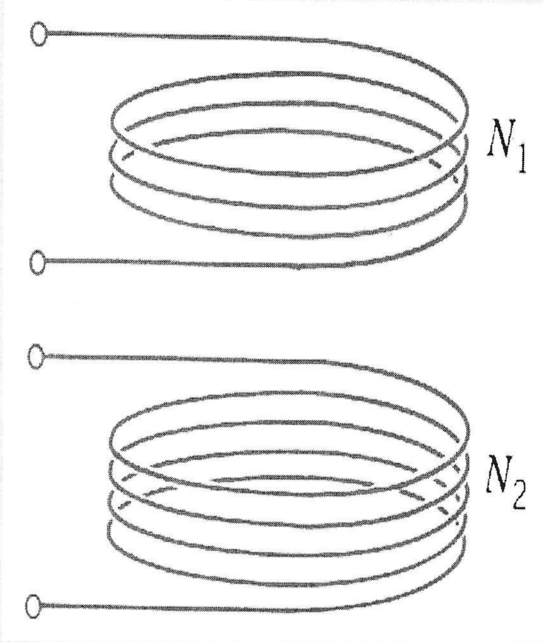
Faraday's Law of Induction

$$\boxed{\varepsilon = -N \frac{d\Phi_B}{dt}}$$

Changing magnetic flux *induces* an EMF

Lenz: Induction ***opposes*** change

Mutual Inductance



A current I_2 in coil 2, induces some magnetic flux Φ_{12} in coil 1. We define the flux in terms of a "mutual inductance" M_{12}:

$$N_1\Phi_{12} \equiv M_{12}I_2$$

$$\rightarrow M_{12} = \frac{N_1\Phi_{12}}{I_2}$$

$$M_{12} = M_{21} = M$$

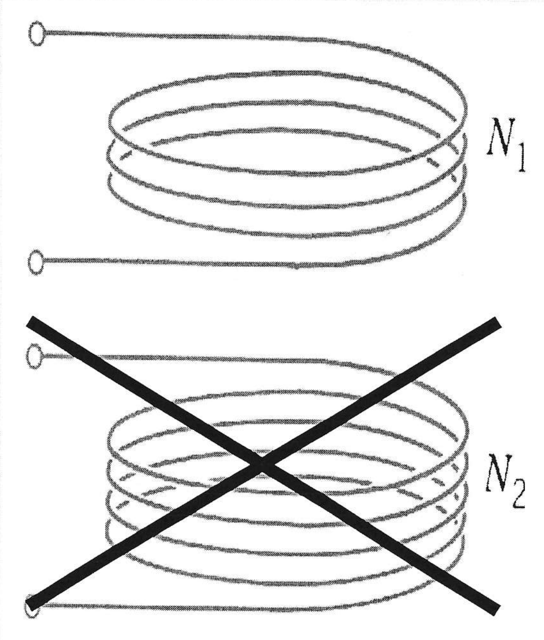
$$\boxed{\mathcal{E}_{12} \equiv -M\frac{dI_2}{dt}}$$

You need AC currents!

Demonstration: Remote Speaker

This Time:
Self Inductance

Self Inductance



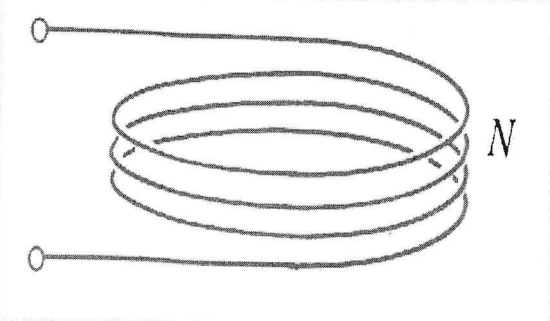
What if we forget about coil 2 and ask about putting current into coil 1? There is "self flux":

$$N_1 \Phi_{11} \equiv M_{11} I_1 \equiv L I$$

$$\rightarrow L \equiv \frac{N\Phi}{I}$$

$$\boxed{\varepsilon \equiv -L \frac{dI}{dt}}$$

Calculating Self Inductance



$$\boxed{L = \frac{N\Phi}{I}}$$

Unit: Henry

$$1\ \mathrm{H} = 1\ \frac{\mathrm{V \cdot s}}{\mathrm{A}}$$

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

Group Problem: Solenoid

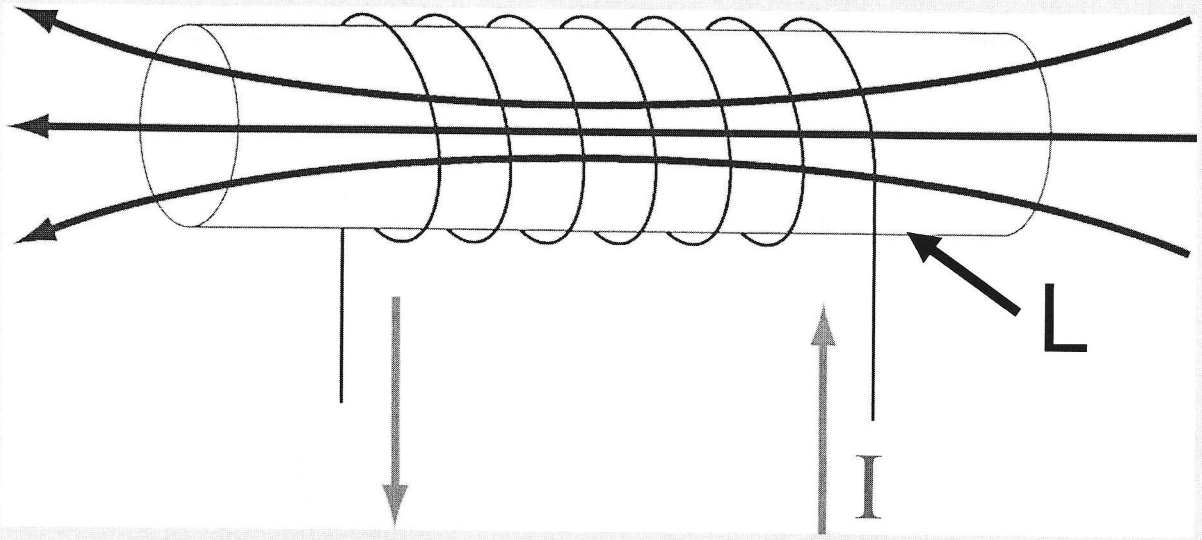
Calculate the self-inductance L of a solenoid (n turns per meter, length ℓ, radius R)

REMEMBER

1. Assume a current I is flowing in your device
2. Calculate the B field due to that I
3. Calculate the flux due to that B field
4. Calculate the self inductance (divide out I)

$$L = N\Phi / I$$

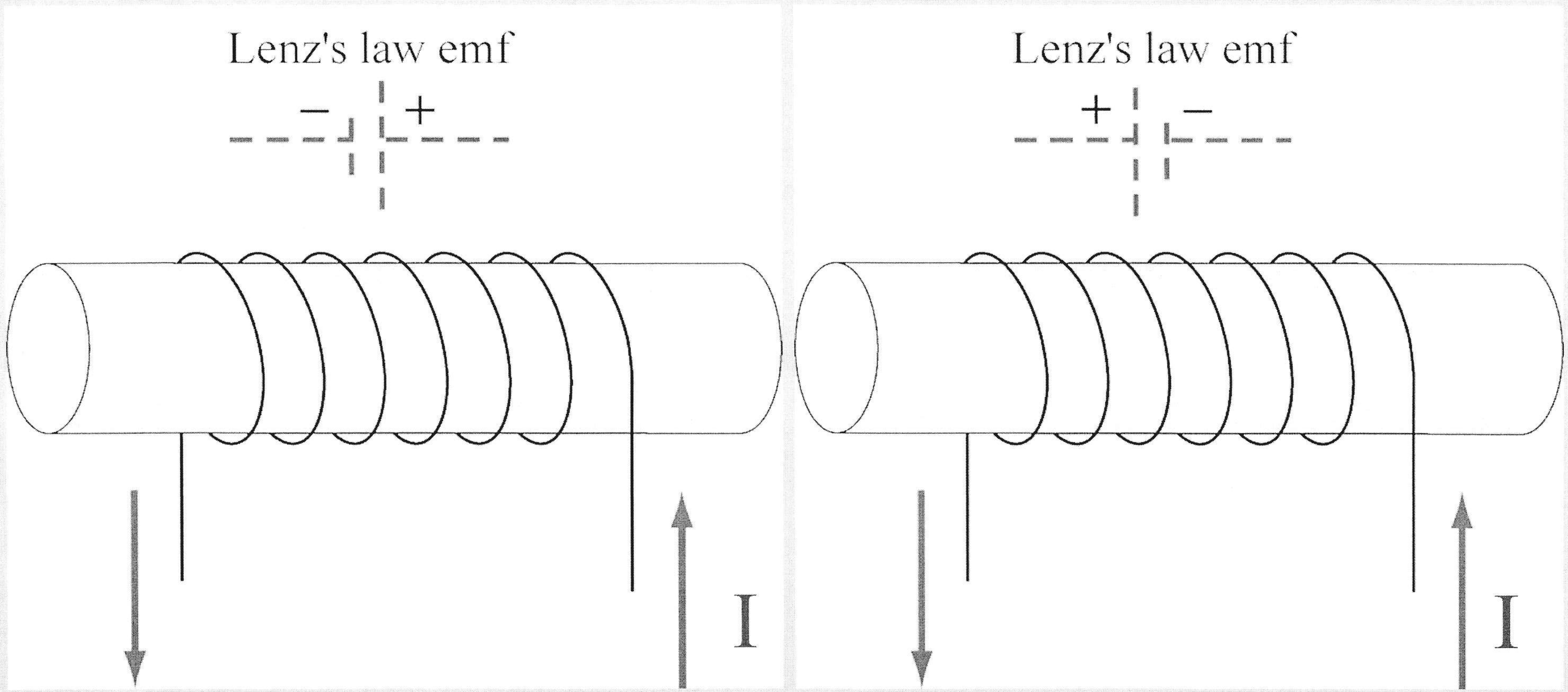
Inductor Behavior



$$\mathcal{E} = -L\frac{dI}{dt}$$

Inductor with constant current does nothing

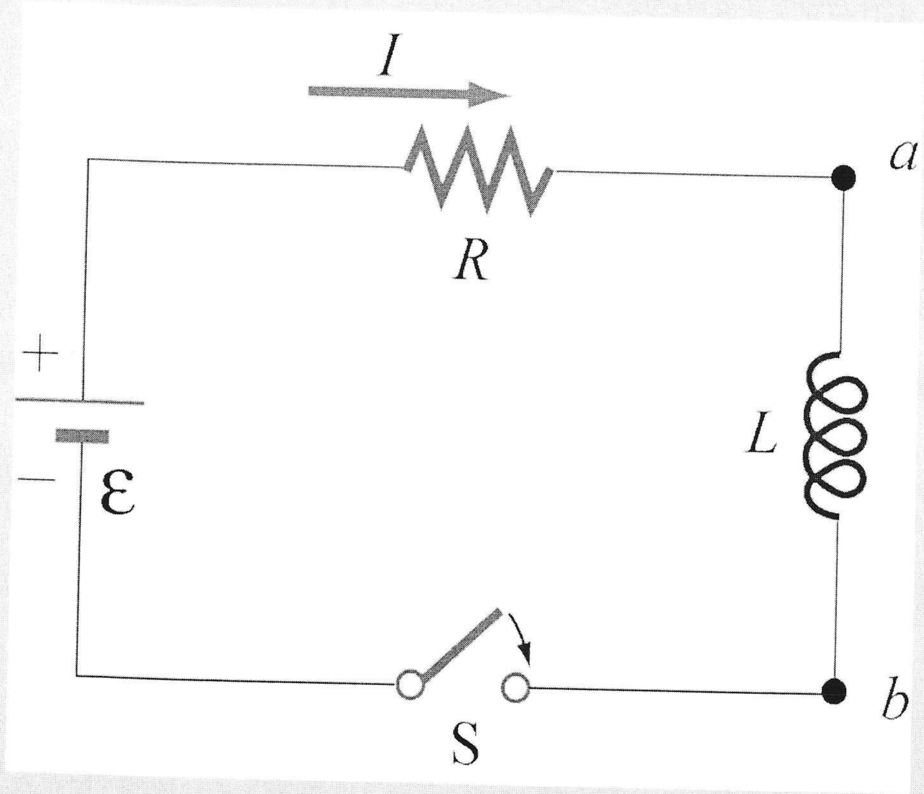
Back EMF $\varepsilon = -L\dfrac{dI}{dt}$

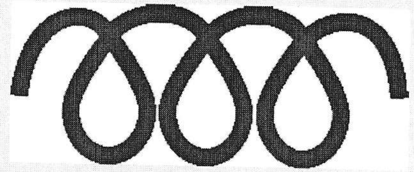


$$\frac{dI}{dt} > 0,\ \varepsilon_L < 0 \qquad \frac{dI}{dt} < 0,\ \varepsilon_L > 0$$

Inductors in Circuits

Inductor: Circuit element which exhibits self-inductance



Symbol: 

When traveling in direction of current:

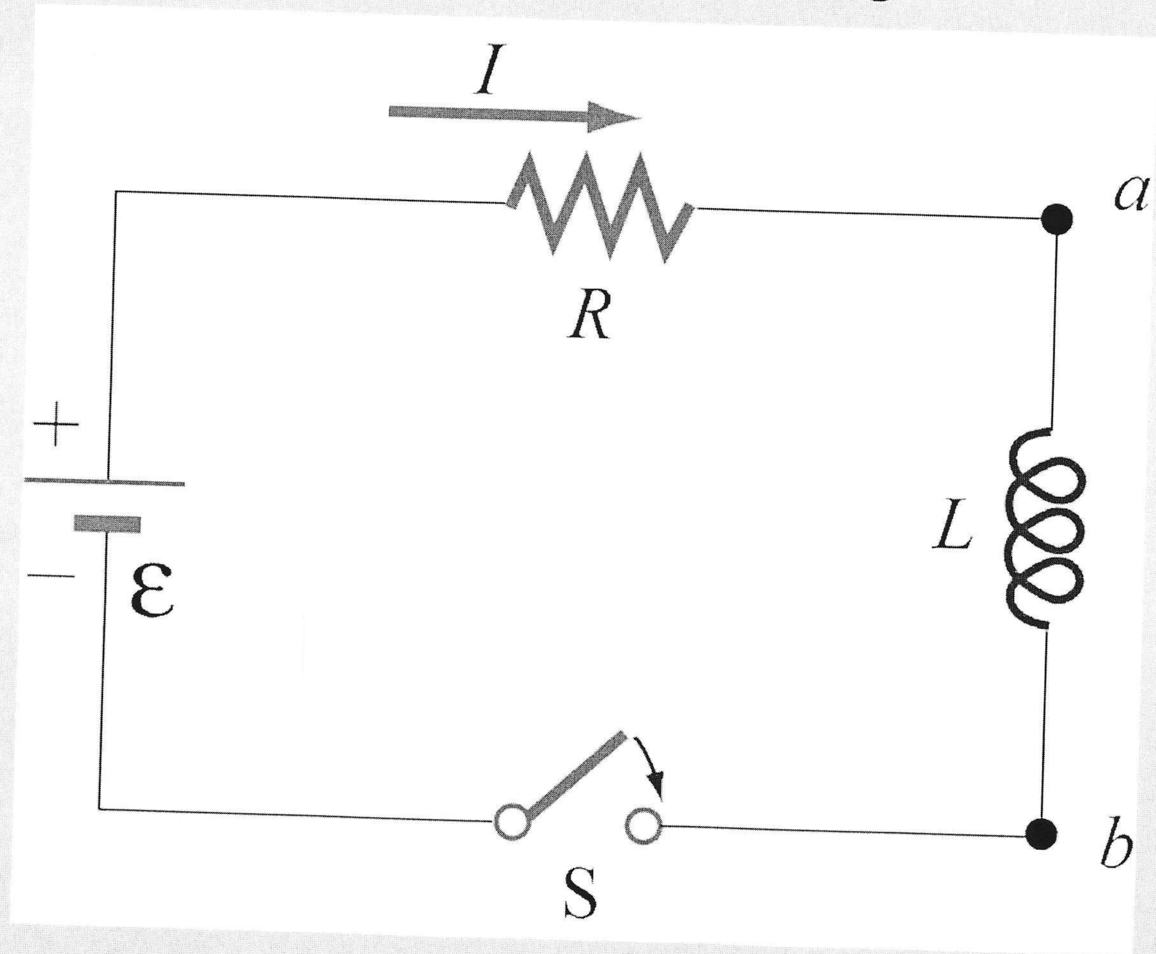
$$\mathcal{E} = -L\frac{dI}{dt}$$

Inductors hate change, like steady state

They are the opposite of capacitors!

PRS Question:
Closing a Switch

LR Circuit

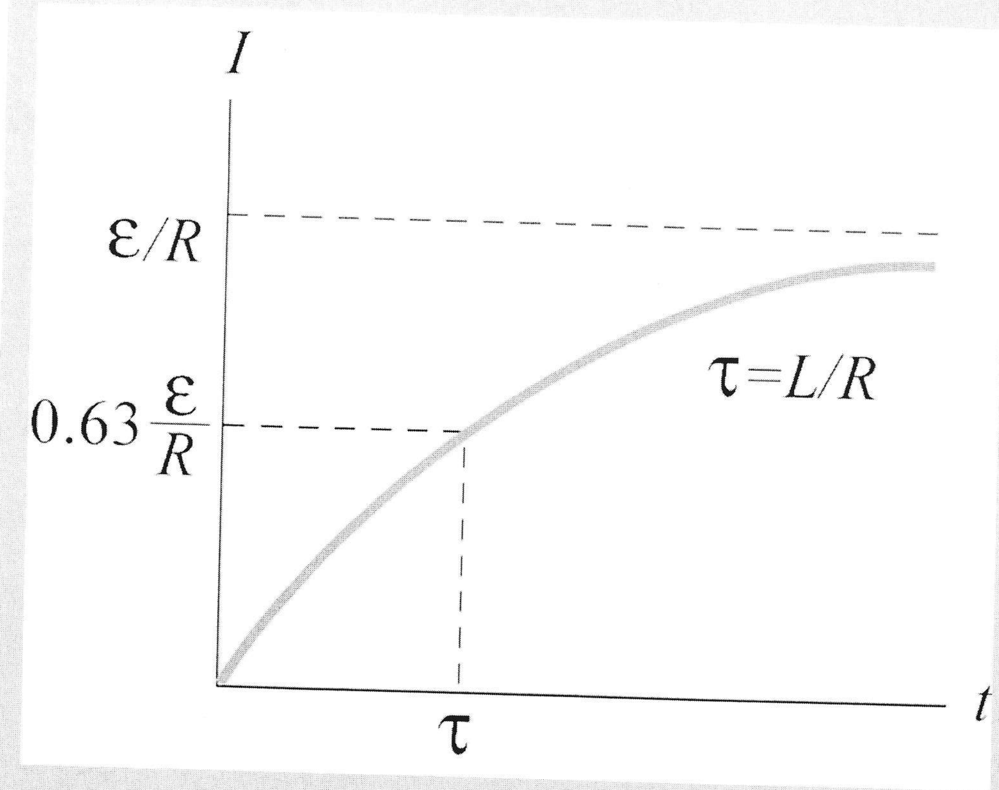


$$\sum_i V_i = \varepsilon - IR - L\frac{dI}{dt} = 0$$

LR Circuit

$$\varepsilon - IR - L\frac{dI}{dt} = 0 \quad \Rightarrow \quad \frac{L}{R}\frac{dI}{dt} = -\left(I - \frac{\varepsilon}{R}\right)$$

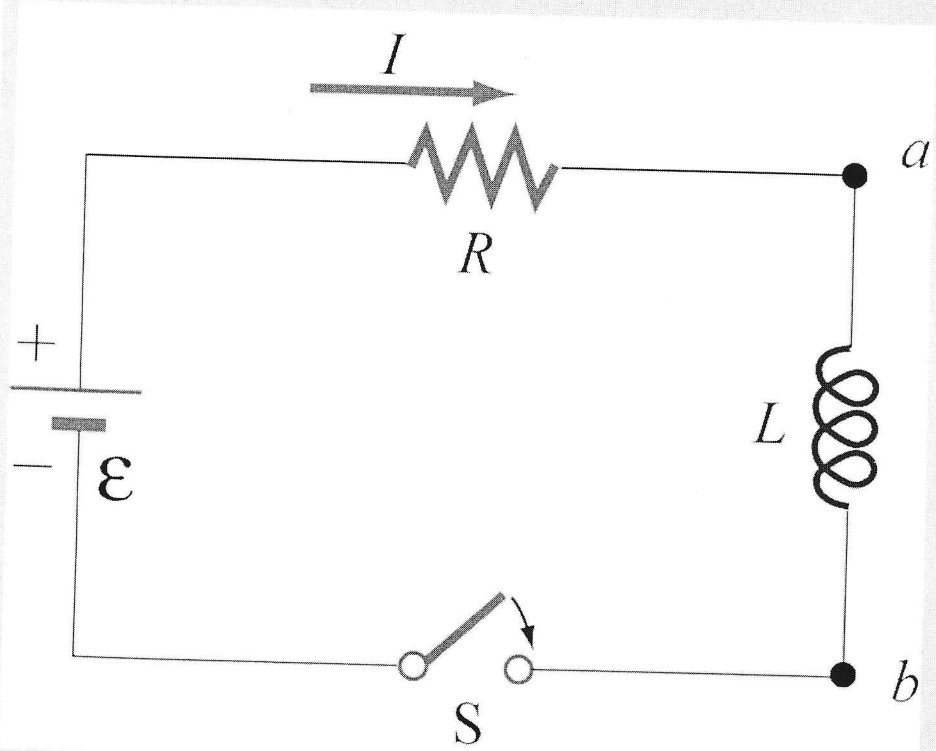
Solution to this equation when switch is closed at t = 0:

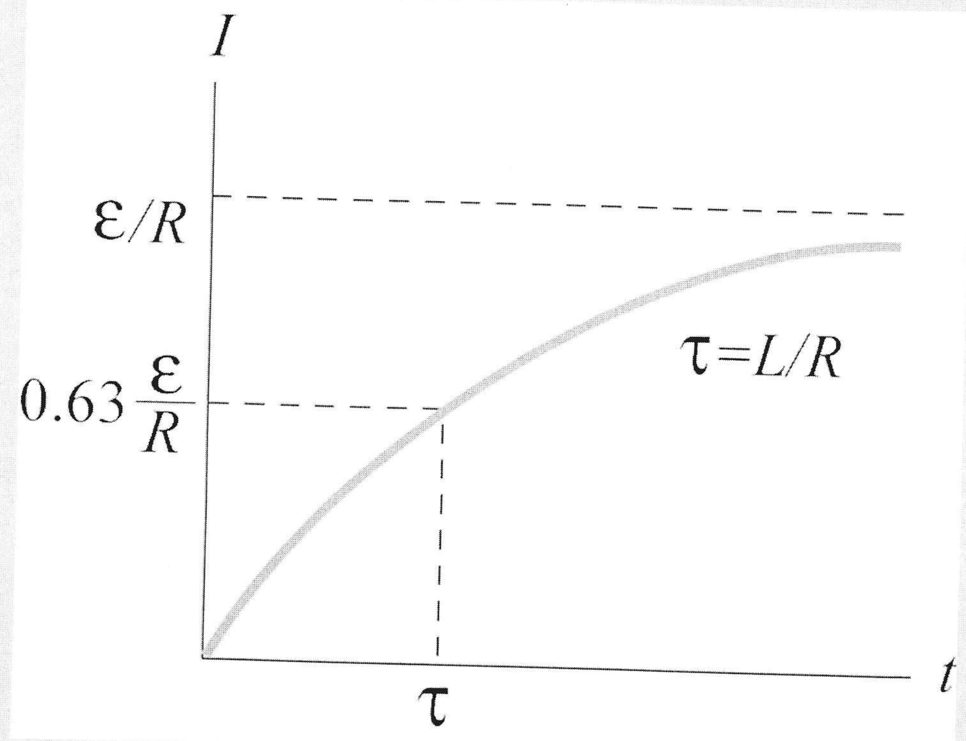


$$\boxed{I(t) = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$$

$$\tau = \frac{L}{R} : \text{LR time constant}$$

LR Circuit

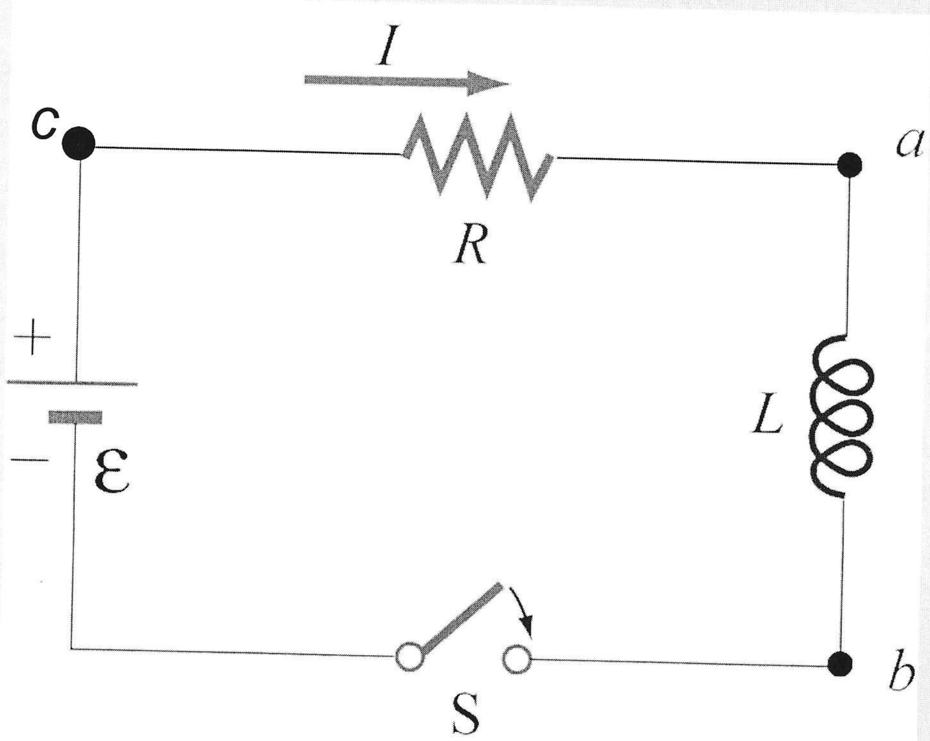


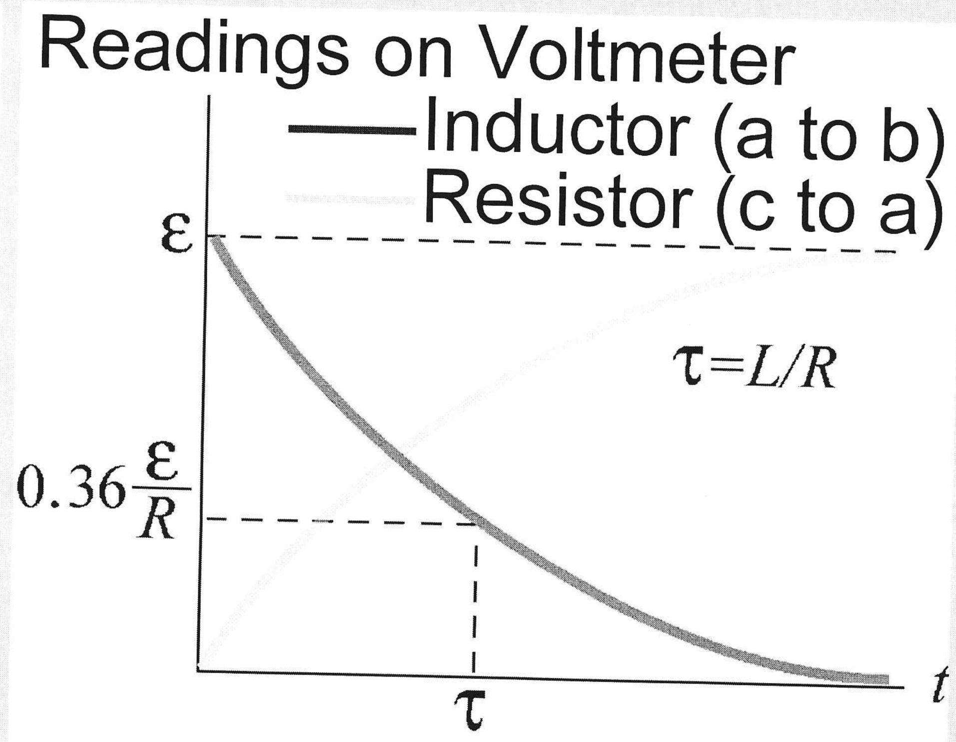


t=0⁺: Current is trying to change. Inductor works as hard as it needs to to stop it

t=∞: Current is steady. Inductor does nothing.

LR Circuit



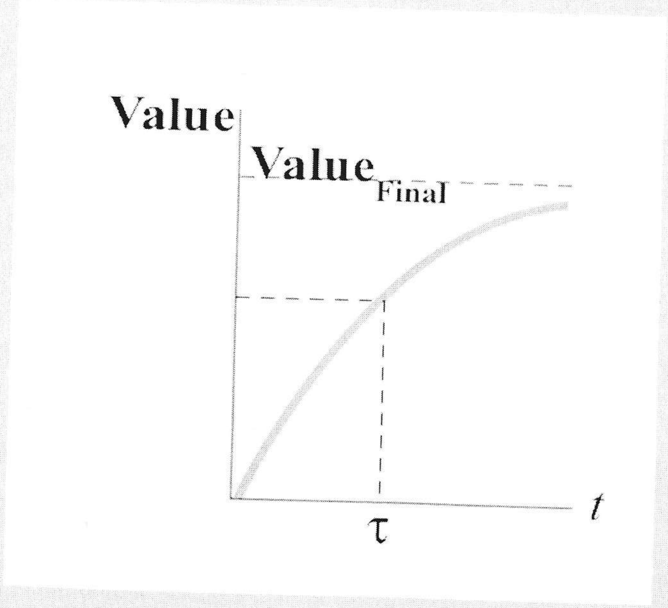


t=0⁺: Current is trying to change. Inductor works as hard as it needs to to stop it

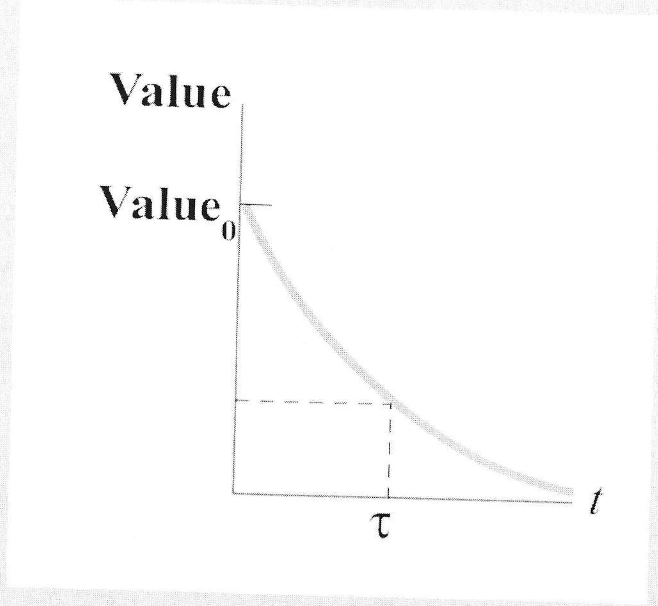
t=∞: Current is steady. Inductor does nothing.

General Comment: LR/RC

All Quantities Either:



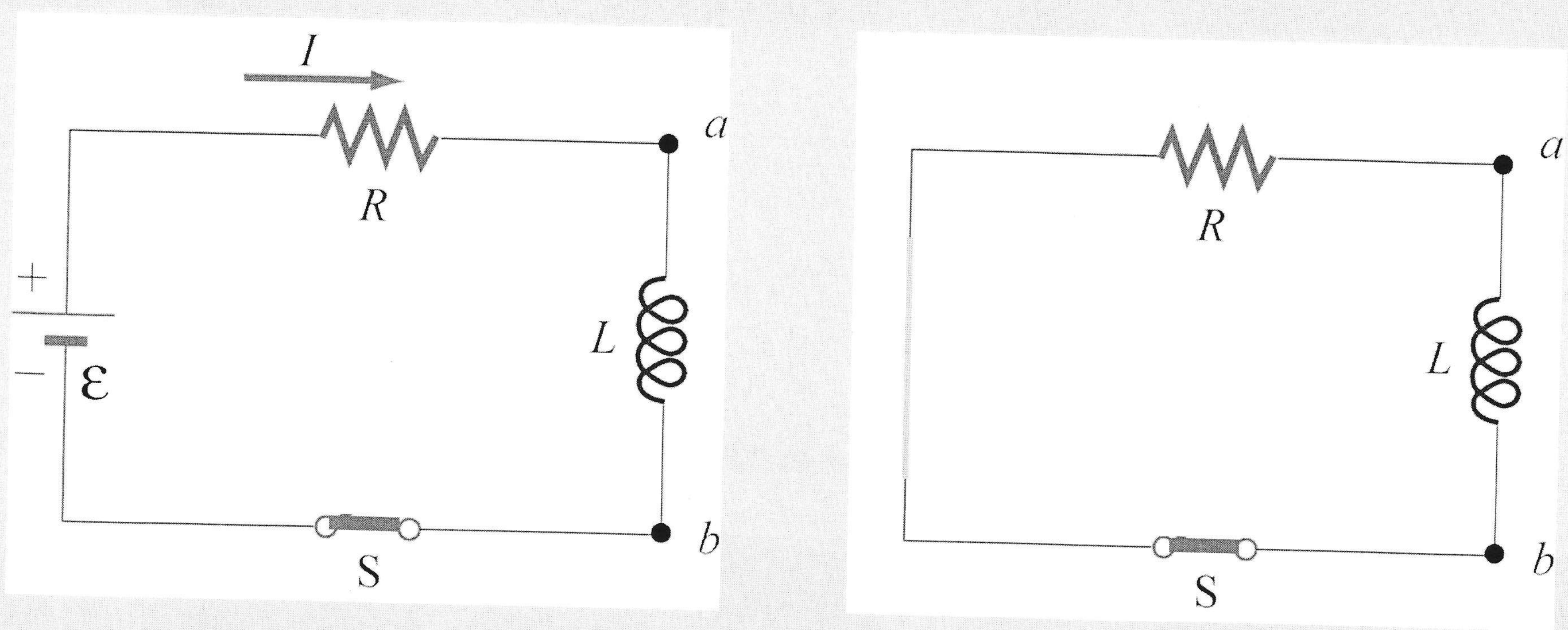
$$\text{Value}(t) = \text{Value}_{\text{Final}}\left(1 - e^{-t/\tau}\right)$$



$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

τ can be obtained from differential equation (prefactor on d/dt) e.g. τ = L/R or τ = RC

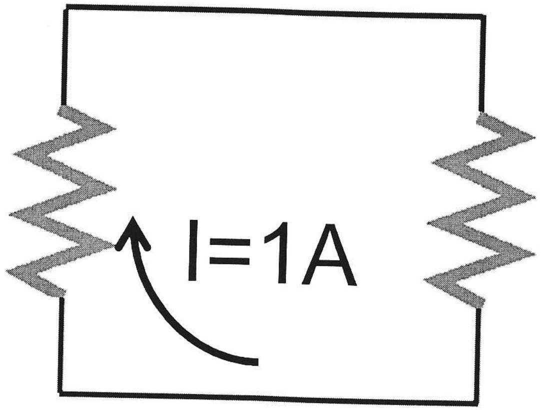
Group Problem: LR Circuit



1. What direction does the current flow just after turning off the battery (at t=0+)? At t=∞?
2. Write a differential equation for the circuit
3. Solve and plot I vs. t and voltmeters vs. t

PRS Questions:
LR Circuit & Problem…

Non-Conservative Fields



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$

E is no longer a conservative field – Potential now meaningless

This concept
(& next 3 slides)
are complicated.
Bare with me and try not to
get confused

Kirchhoff's Modified 2nd Rule

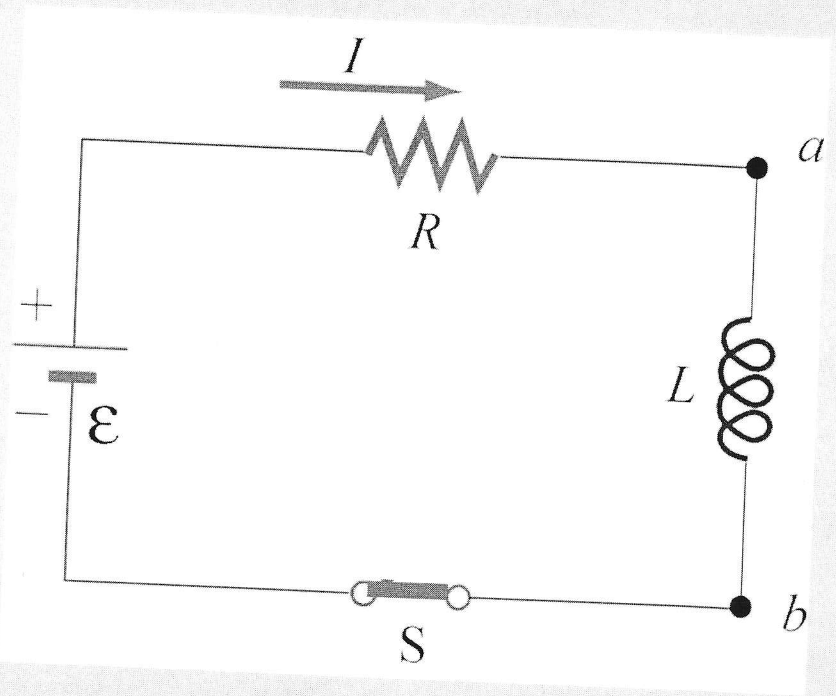
$$\sum_i \Delta V_i = -\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = +N \frac{d\Phi_B}{dt}$$

$$\Rightarrow \sum_i \Delta V_i - N \frac{d\Phi_B}{dt} = 0$$

If all inductance is 'localized' in inductors then our problems go away – we just have:

$$\sum_i \Delta V_i - L \frac{dI}{dt} = 0$$

Ideal Inductor



BUT, EMF generated in an inductor is not a voltage drop across the inductor!

$$\varepsilon = -L\frac{dI}{dt}$$

$$\Delta V_{\text{inductor}} \equiv -\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Because resistance is 0, E must be 0!

Conclusion:
Be mindful of physics
Don’t think too hard doing it

Demos: Breaking circuits with inductors

Internal Combustion Engine

See figure 1:

http://auto.howstuffworks.com/engine3.htm

Ignition System

The Distributor:

http://auto.howstuffworks.com/ignition-system4.htm

(A) High Voltage Lead
(B) Cap/Rotor Contact
(C) Distributor Cap
(D) To Spark Plug

(A) Coil connection
(B) Breaker Points
(D) Cam Follower
(E) Distributor Cam

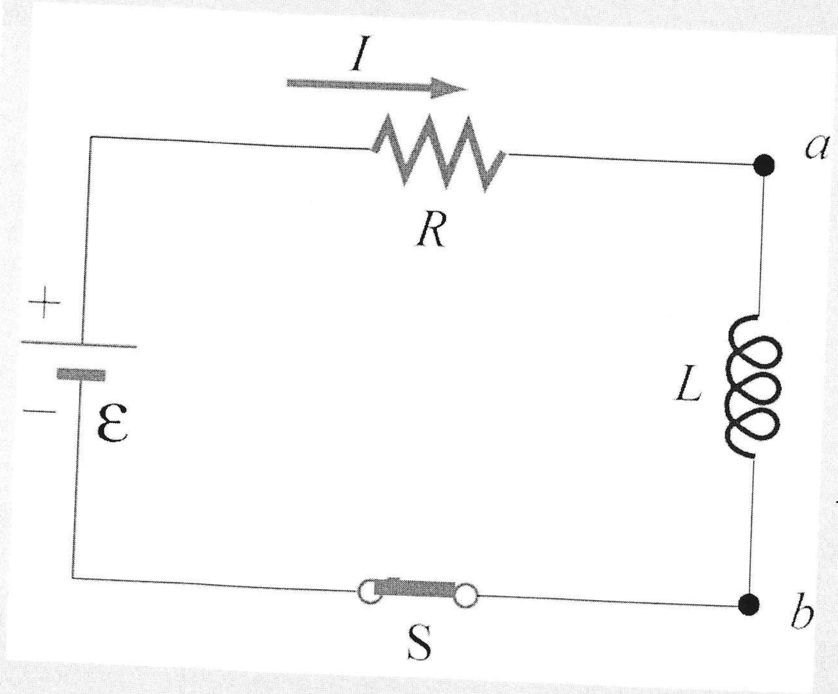
Modern Ignition

See figure:

http://auto.howstuffworks.com/ignition-system.htm

Energy in Inductor

Energy Stored in Inductor



$$\mathcal{E} = +IR + L\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + LI\frac{dI}{dt}$$

$$I\mathcal{E} = I^2R + \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$$

Battery Supplies

Resistor Dissipates

Inductor Stores

Energy Stored in Inductor

$$\boxed{U_L = \frac{1}{2} L I^2}$$

But where is energy stored?

Example: Solenoid

Ideal solenoid, length l, radius R, n turns/length, current I:

$$B = \mu_0 n I \qquad L = \mu_o n^2 \pi R^2 l$$

$$U_B = \tfrac{1}{2} L I^2 = \tfrac{1}{2}\left(\mu_o n^2 \pi R^2 l\right) I^2$$

$$U_B = \underbrace{\left(\frac{B^2}{2\mu_o}\right)}_{\text{Energy Density}} \underbrace{\pi R^2 l}_{\text{Volume}}$$

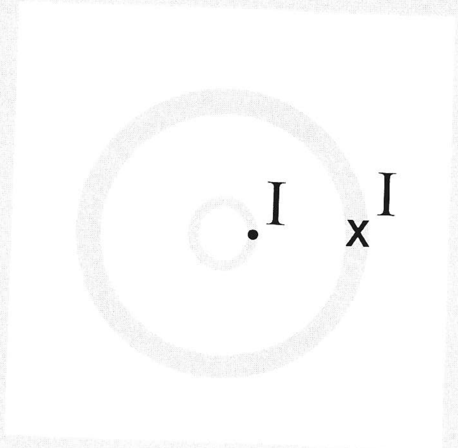
Energy Density

Energy is stored in the magnetic field!

$$\boxed{u_B = \frac{B^2}{2\mu_o}}$$: Magnetic Energy Density

$$u_E = \frac{\varepsilon_o E^2}{2}$$: Electric Energy Density

Group Problem: Coaxial Cable



Inner wire: r=a

Outer wire: r=b

1. How much energy is stored per unit length?
2. What is inductance per unit length?

HINTS: This does require an integral
The EASIEST way to do (2) is to use (1)

Back to Back EMF

PRS Question: Stopping a Motor

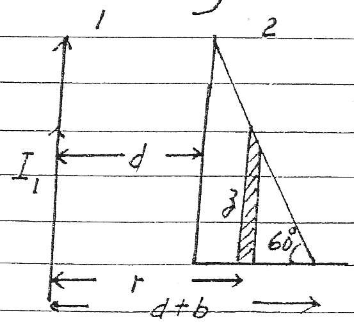
Mutual Inductance

Definition

Examples of calculating mutual inductance

Examples of Calculating Mutual Inductance

(1)



$\vec{B}$ produced by an infinite long wire

$$z = (d+b-r)\tan 60^\circ$$

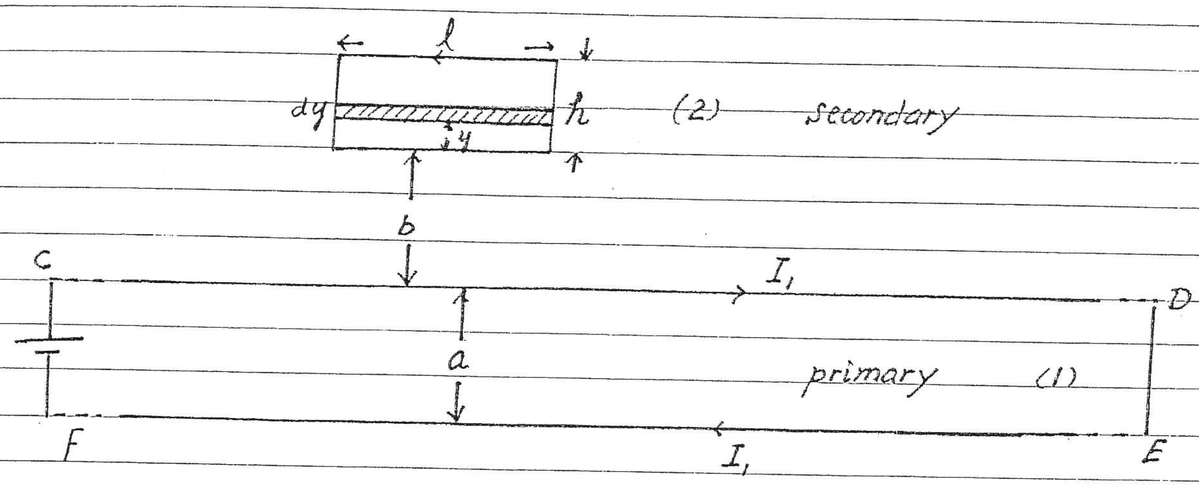
$$\Phi_{12} = \int \vec{B}_1 \cdot d\vec{S}_2$$

$$= \int \frac{\mu_0 I_1}{2\pi r} z \, dr$$

$$= \frac{\sqrt{3}\,\mu_0 I_1}{2\pi} \int_d^{d+b} \frac{(d+b)-r}{r}\, dr$$

$$= \underbrace{\frac{\sqrt{3}\,\mu_0}{2\pi} \left[(d+b) \ln\left(1+\frac{b}{d}\right) - b \right]}_{M_{12}} I_1$$

(2)



The $\vec{B}$ field through the secondary loop can be considered as superposition of $\vec{B}_{CD}$ and $\vec{B}_{EF}$

$\vec{B}_{CD}$ ↓ produced by the wire CD ↓ out of the page choosing to be the z - direction

$\vec{B}_{EF}$ ↓ produced by the wire EF ↓ into the page along $-z$ direction

$$\vec{B}_1 = \left[\frac{\mu_0 I_1}{2\pi(y+b)} - \frac{\mu_0 I}{2\pi(y+a+b)} \right] \hat{k}$$ at the shaded region

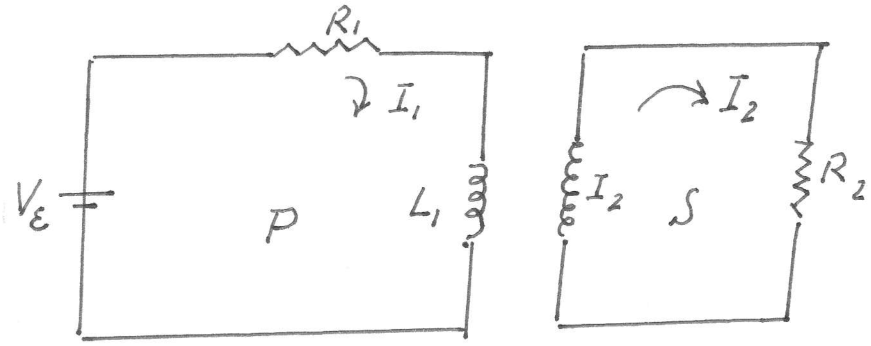
$$d\vec{S}_2 = l\, dy\, \hat{k}$$

$$\Rightarrow \Phi_{12} = \int \vec{B}_1 \cdot d\vec{S}_2$$

$$= \frac{\mu_0 I_1 l}{2\pi} \int_0^h \left(\frac{1}{y+b} - \frac{1}{y+a+b} \right) dy$$

$$= \frac{\mu_0 I_1 l}{2\pi} \left[\ln\left(1+\frac{h}{b}\right) - \ln\left(1+\frac{h}{a+b}\right) \right]$$

$$\Rightarrow M_{12} = \underbrace{\frac{\mu_0 l}{2\pi} \left[\ln\left(1+\frac{h}{b}\right) - \ln\left(1+\frac{h}{a+b}\right) \right]}_{\text{Geometrical in nature}}$$



$$V_\varepsilon - L_1 \frac{dI_1}{dt} - M_{12} \frac{dI_2}{dt} = I_1 R_1 \qquad P$$

$$-L_2 \frac{dI_2}{dt} - M_{21} \frac{dI_1}{dt} = I_2 R_2 \qquad S$$

I_1, I_2 are coupled

Given the above equations

and

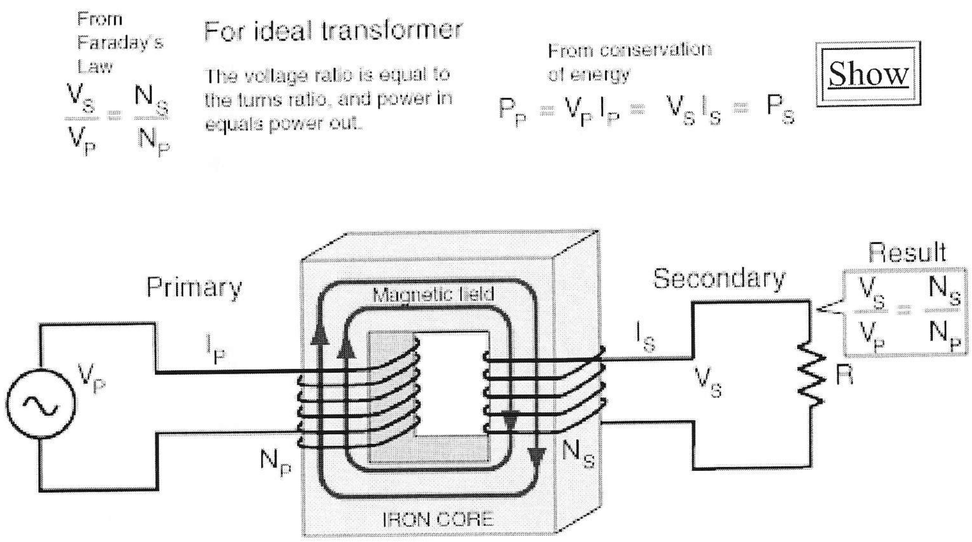
the initial conditions

$\Rightarrow$ Solve for $I_1(t)$, $I_2(t)$.

Simple Applications

Transformer

A transformer makes use of Faraday's law and the ferromagnetic properties of an iron core to efficiently raise or lower AC voltages. It of course cannot increase power so that if the voltage is raised, the current is proportionally lowered and vice versa.



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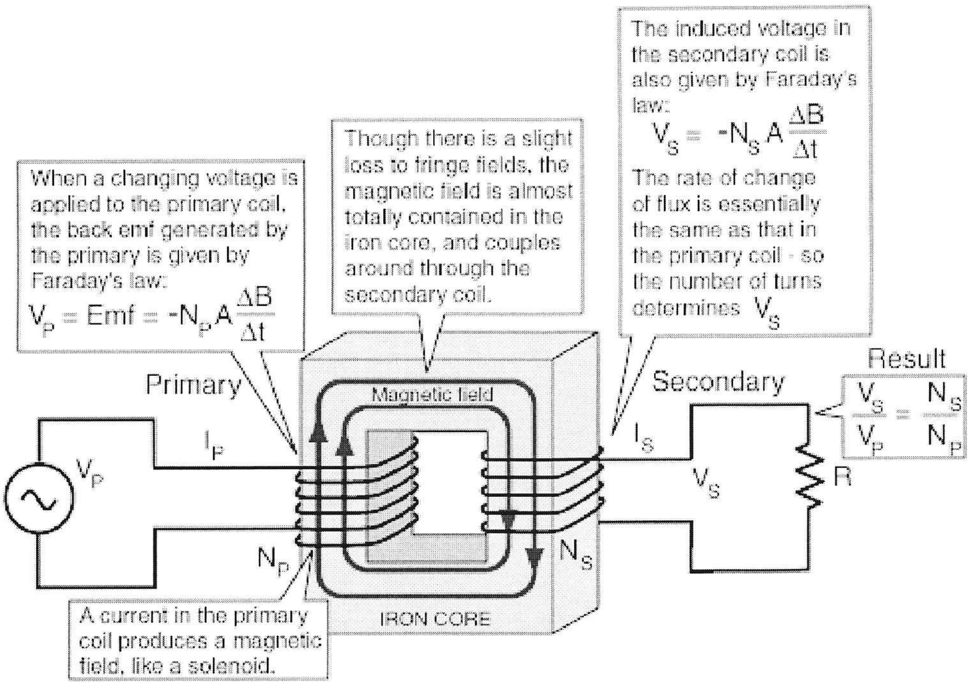
Magnetic



Faraday's law	Solenoid magnetic field	Calculation

force

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Ideal Transformer Calculation

Primary

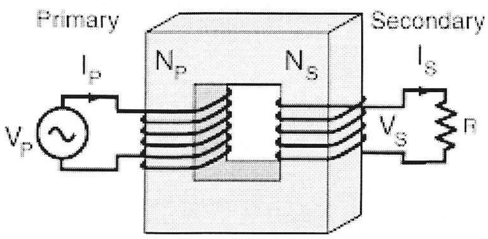
Primary voltage = V_P = volts.

Primary current = I_P = amps

Primary turns =

For an ideal transformer:

N_P =
turns



Secondary

Secondary voltage =
V_S =
volts

Secondary current =
I_S =
amps

Secondary turns =
N_S =
turns

Load resistor =
R =
ohms

Power

Power used = P_P = P_S =
watts

The ideal transformer neglects losses to resistive heating in the primary coil and assumes ideal coupling to the secondary (i.e., no magnetic losses).

More realistic treatment:	Numerical example
	Calculation with primary losses

Notes: For this exploratory calculation, you may enter data for any parameter except the power. Then click on the active text for the parameter you wish to calculate; values will not be forced to be consistent until you do. For primary calculations, the voltage and number of turns in the secondary will be considered to be established and vice versa. Default values will be entered for unspecified parameters, but all values except the power may be changed.

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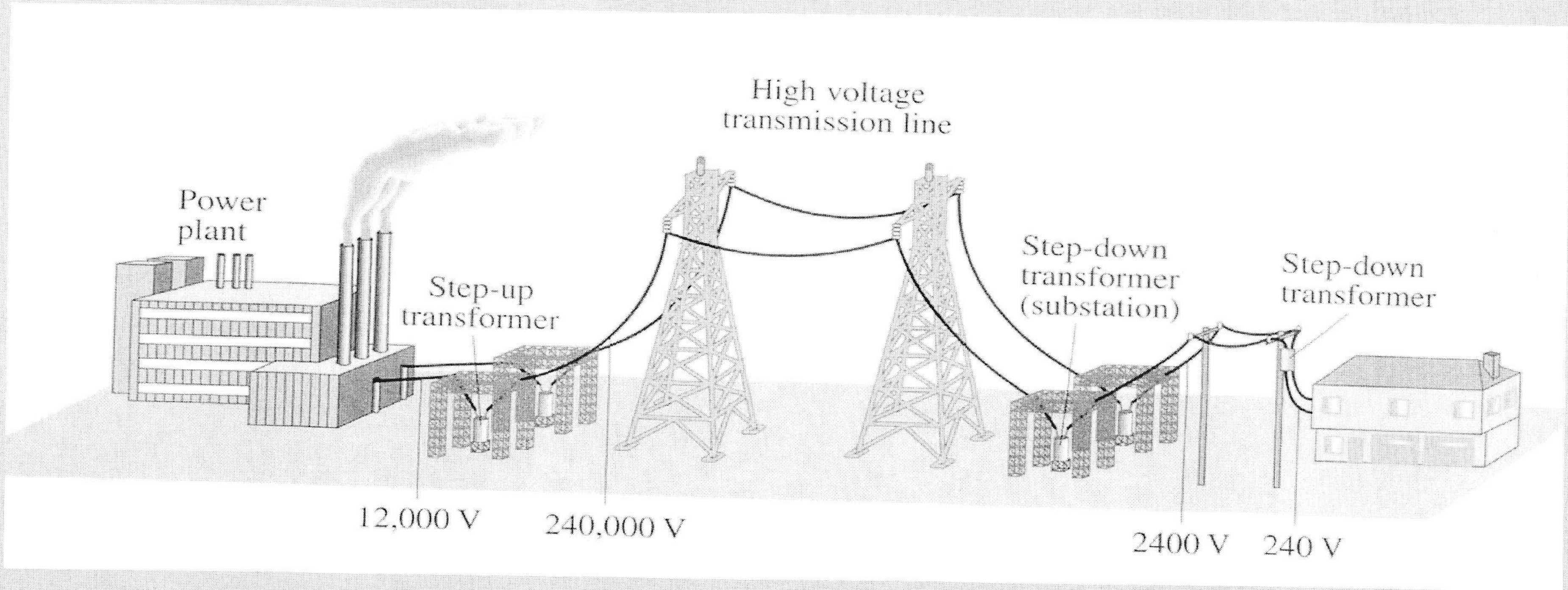
Simple Applications

Transmission of Electric Power



Power loss can be greatly reduced if transmitted at high voltage

Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $$I = \frac{P}{V} = \frac{1.2 \times 10^5 W}{2.4 \times 10^2 V} = 500A$$

83% loss!!

$$P_L = I^2 R = (500A)^2 (0.40\Omega) = 100kW$$

(b) $$I = \frac{P}{V} = \frac{1.2 \times 10^5 W}{2.4 \times 10^4 V} = 5.0A$$

0.0083% loss

$$P_L = I^2 R = (5.0A)^2 (0.40\Omega) = 10W$$