Interference and Diffraction

Wave phenomenon

Key Concepts

Superposition Principle

Phase Relationship (Phase shift)

Coherence

Example : Water wave

Minimum displacement → lines of nodes → destructive interference

Maximum displacement → lines of antinodes → constructive interference

Interference Pattern

↓

stationary interference pattern

↓

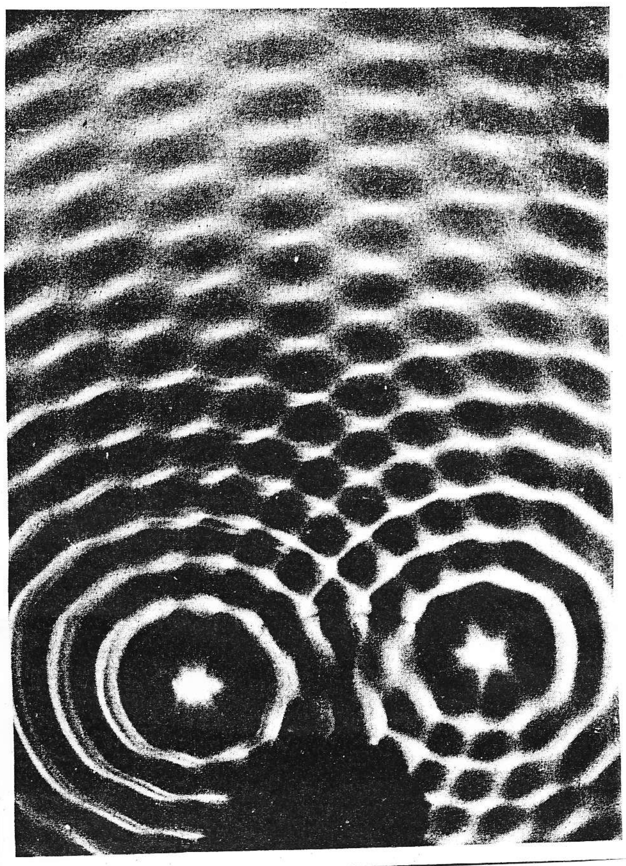
require the sources have the same frequency and there is definite phase relationship between them (maintain for a long time)

Sources have these properties

↓

can produce stationary interference pattern

mutually coherent



Interference of water waves from two conerent point sources. Photo courtesy of Professor T. A. Wiggins, Pennsylvania State University.

Demonstration: Microwave Interference

Last time: Microwaves (mw)

$$f_{mw} = 2\times10^{9}\ Hz \quad \lambda_{mw} = \frac{c}{f} = 15\,cm$$

This time: Visible (red) light:

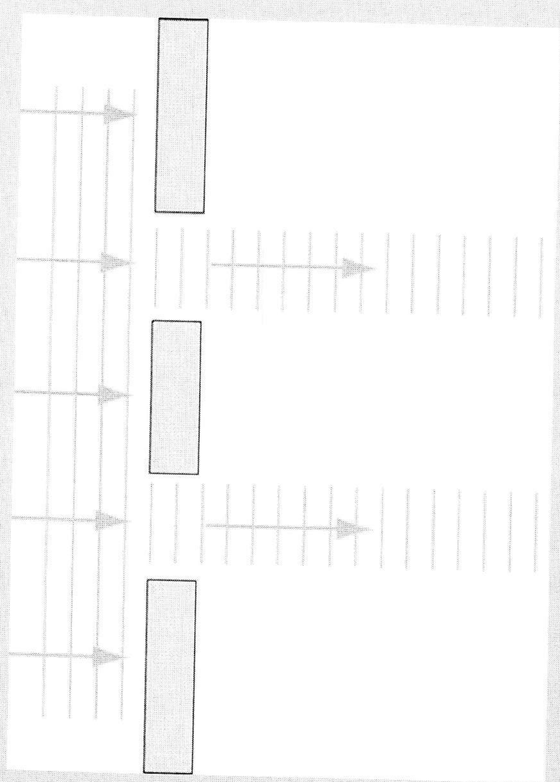
$$f_{red} = 4.6\times10^{14}\ Hz \quad \lambda_{red} = \frac{c}{f} = 6.54\times10^{-5}\,cm$$

How in the world do we measure 1/10,000 of a cm?

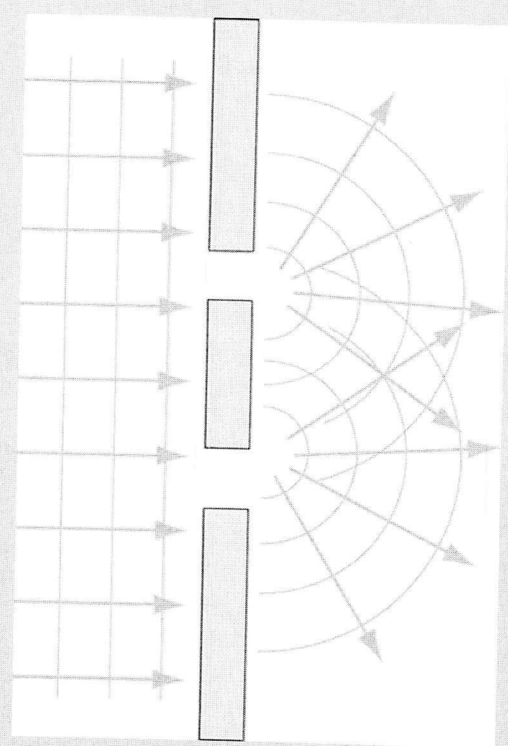
We Use Interference

This is also how we know that light is a wave phenomena

Interference: The difference between waves and bullets

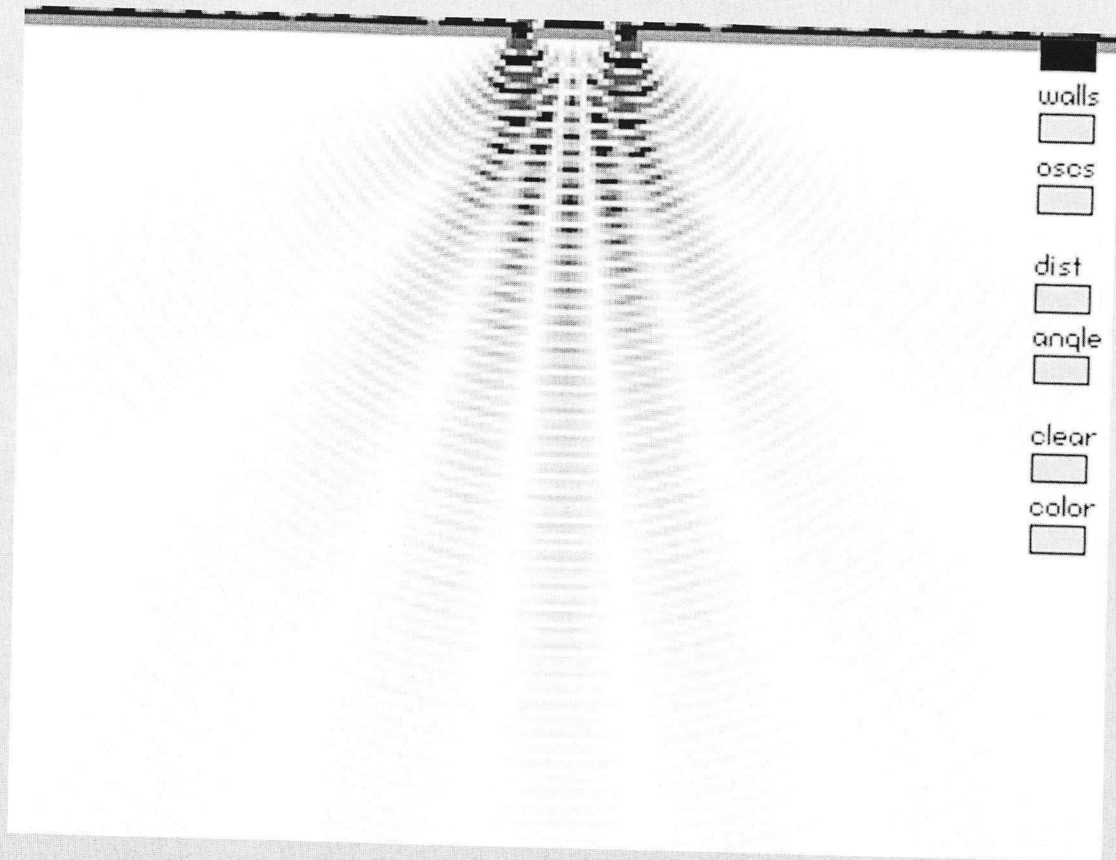


No Interference:
if light were made up of bullets



Interference: If light is a wave we see spreading and addition and subtraction

Interference: The difference between waves and bullets



Link to interference applet

Examples

Two sodium vapor lamps

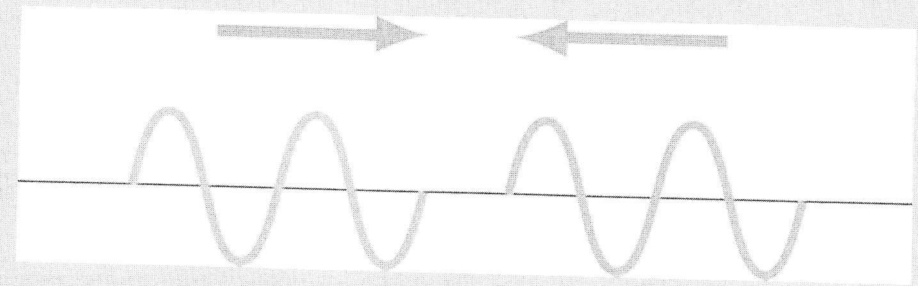
Ordinary fluorescent lamps
↓
not monochromatic

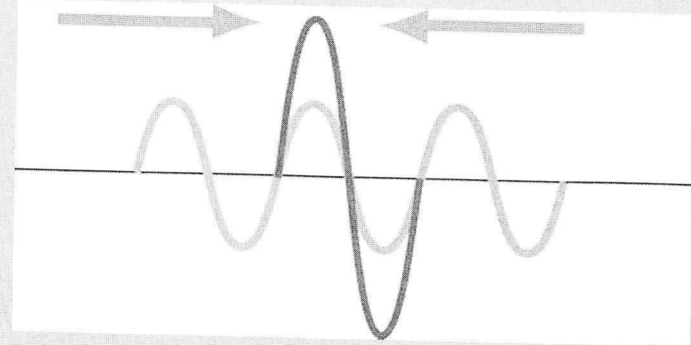
Laser

Interference

Interference: Combination of two or more waves to form composite wave – use superposition principle.

Waves can add *constructively* or *destructively*



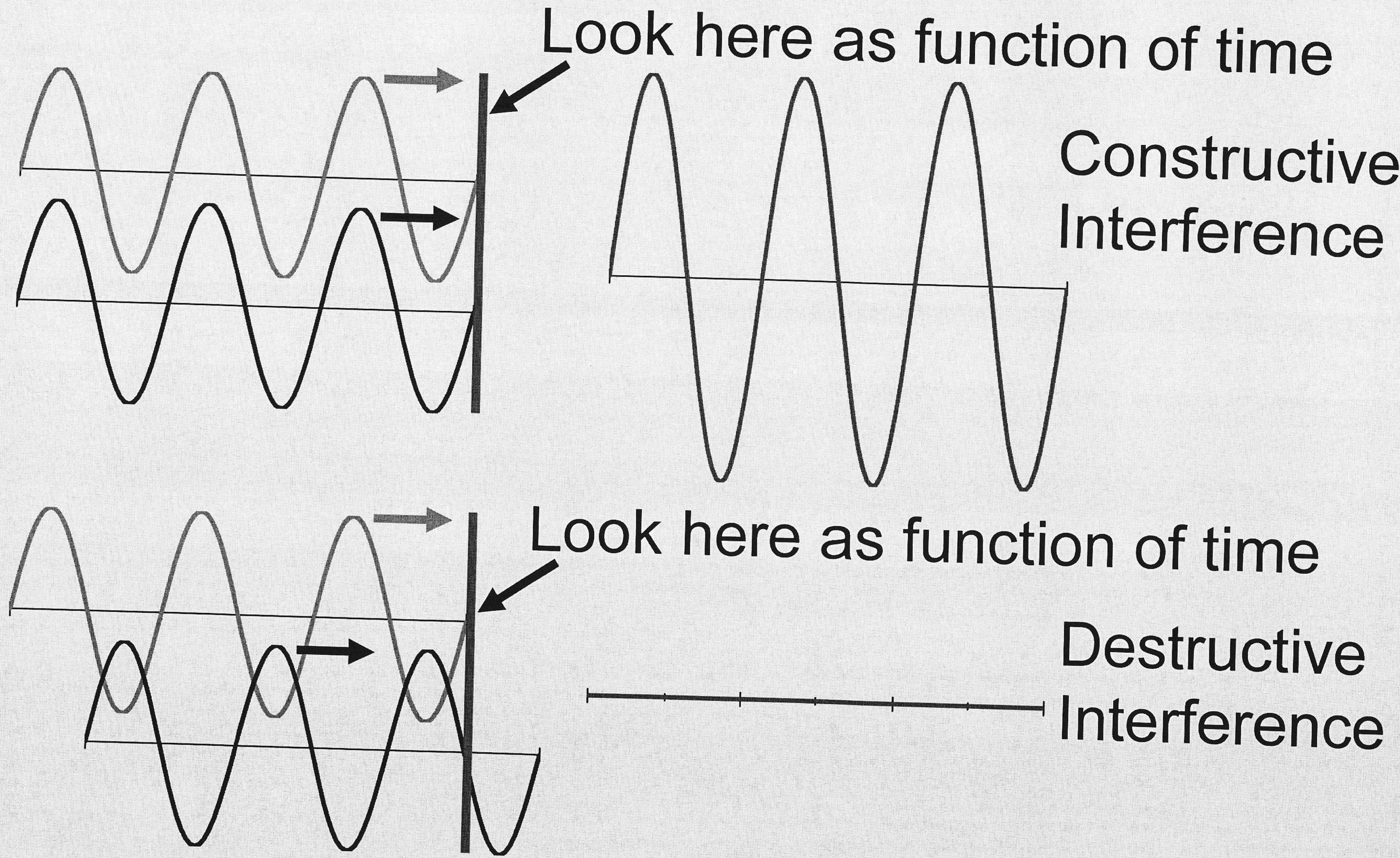


Conditions for interference:

1. **Coherence**: the sources must maintain a constant phase with respect to each other
2. **Monochromaticity**: the sources consist of waves of a single wavelength

Interference – Phase Shift

Consider two traveling waves, moving through space:



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Interference – Phase Shift

What can introduce a phase shift?

1. From different, out of phase sources
2. Sources in phase, but travel different distances
    1. Thin films
    2. Microwave Demonstration
    3. Double-slit or Diffraction grating

Review of Principle of Superposition (Electromagnetic Wave

1. At any given point and at any time, the electric (or magnetic) field due to a single wave passing through the point is independent of all the other waves that may be present.
2. The total electric (or magnetic) field at the point in question is the vector sum of the electric (or magnetic) fields of all the individual waves present.

It will be noted that these rules are no more nor less than those previously given for the superposition of electrostatic and magnetic fields. If there are two sources, as shown in Fig. 26.3, for example, then the total electric and magnetic fields $\mathbf{E}_t(x, y, z, t)$ and $\mathbf{B}_t(x, y, z, t)$ at a point (x, y, z) at time t will be

$$\mathbf{E}_t(x, y, z, t) = \mathbf{E}_1(x, y, z, t) + \mathbf{E}_2(x, y, z, t) \tag{26.2.1}$$

and

$$\mathbf{B}_t(x, y, z, t) = \mathbf{B}_1(x, y, z, t) + \mathbf{B}_2(x, y, z, t) \tag{26.2.2}$$

The Poynting vector can now be written as

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}_t \times \mathbf{B}_t) \tag{26.2.3}$$

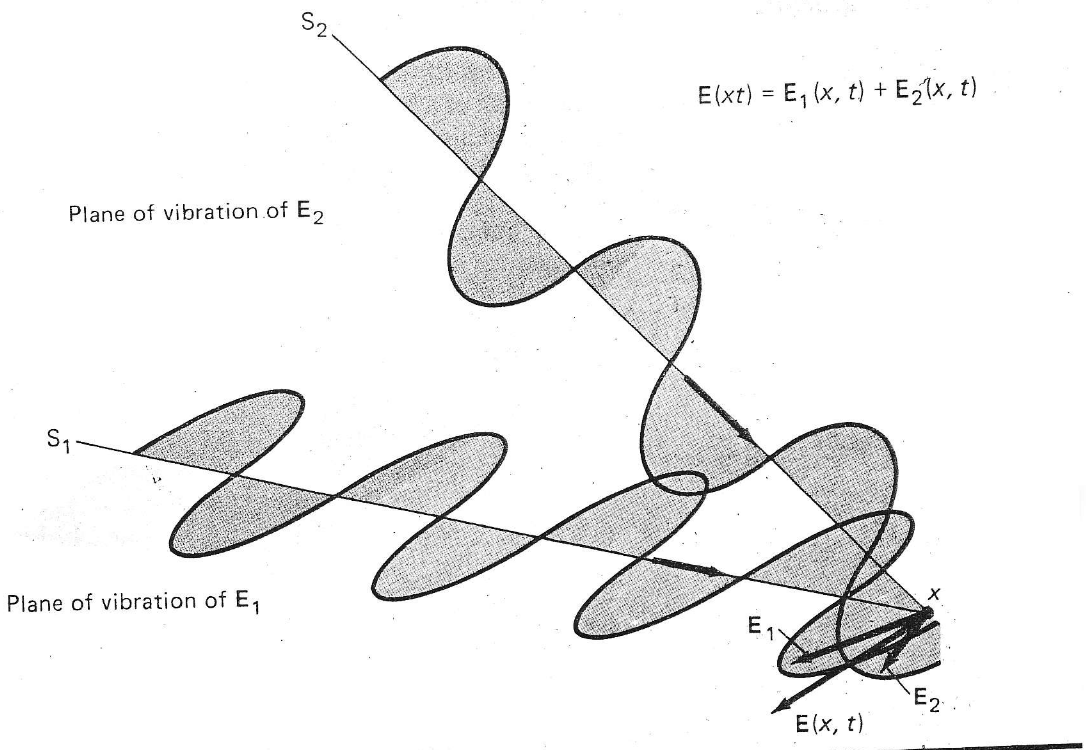
where $\mathbf{E}_t$ and $\mathbf{B}_t$ are the total fields given by (26.2.1) and (26.2.2). We have already seen in Eq. (23.5.17) that the time average of the magnitude of the Poynting vector, which represents the *intensity* of light at the point, is

$$\bar{S} = \overline{E_t B_t}/\mu_0 = \overline{E_t^2}/c\mu_0 \tag{26.2.4}$$

As a consequence, the average intensity will always be proportional to the average of the *square* of the electric field, that is,

$$\bar{S} \quad \overline{E_t^2(x, y, z, t)} \tag{26.2.5}$$

The relative intensity at different points can, therefore, always be determined simply by studying the average of the square of the electric field at the location in question. If the absolute intensity is needed, it can be obtained by multiplying by the factor $1/c\mu_0$ in (26.2.4).



Interference of two coherent electromagnetic waves. The magnetic vectors are omitted in the interest of clarity.

Young's Double Slit Experiment

Thomas Young 1801

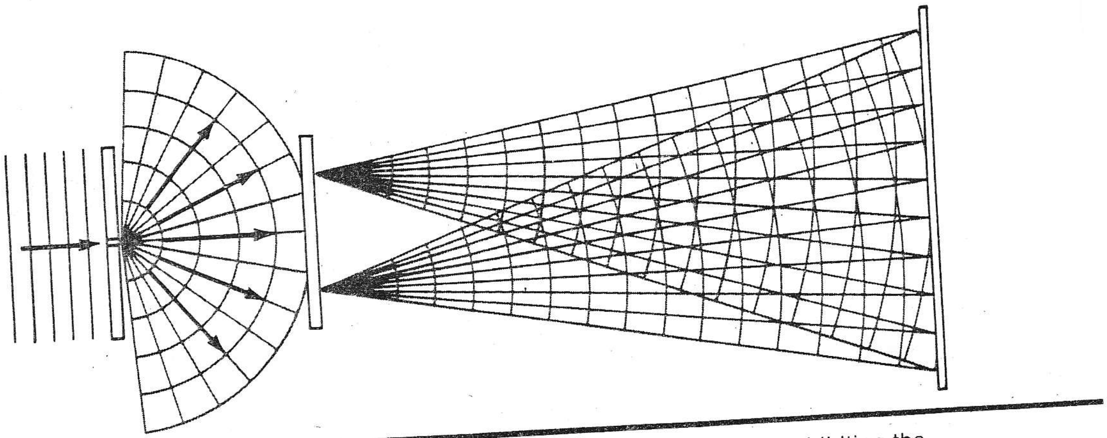
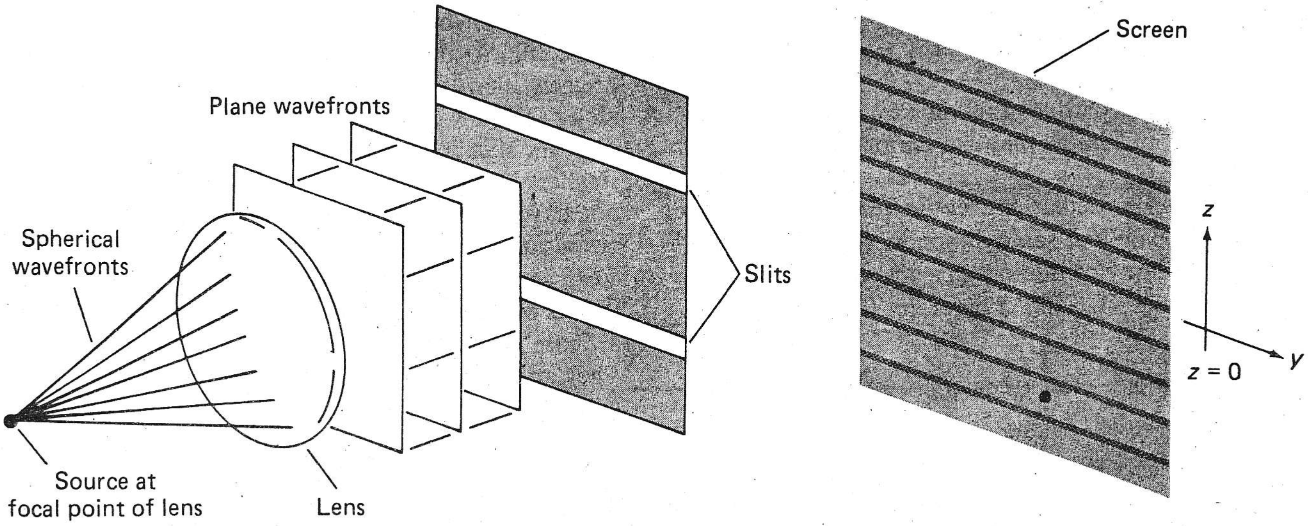


Diagram of rays and wavefronts in Young's experiment exhibiting the interference of light from two coherent point sources.



. Another view of Young's double-slit interference experiment.

Let us now state clearly the assumptions that are made in the present analysis:

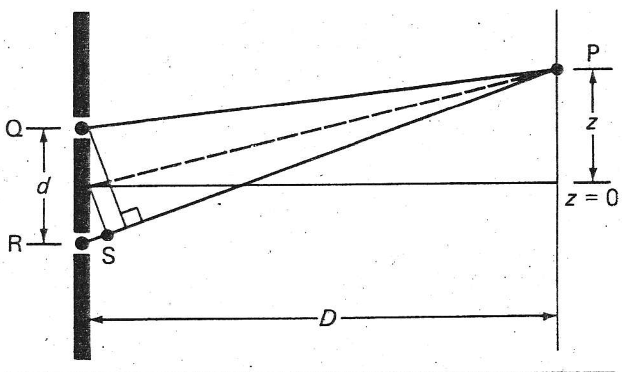
1. The primary source is assumed to be a *monochromatic*[2] source.
2. The two slits act as secondary sources of monochromatic light that are *mutually coherent*. In this case, they are exactly in phase, because the geometry has been arranged so that both slits always lie on the same wavefront.
3. The width of the slits is narrow enough to avoid interference of light coming from different portions of a single slit.
4. The screen containing the slits is a distance D from the observation screen, and D is much larger than the separation d between the slits.

Let us now calculate the electric field at an observation point P quite distant from the slits. At any time t, the electric fields at each of the secondary slits will be identical and are assumed to be given by $\mathbf{E}_0 \cos \omega t$. These oscillating fields propagate outward and will eventually pass through P. Since the waves are cylindrical, the amplitudes will fall off as $1/\sqrt{r}$, where r is the distance from the source to the observation point. The distances QP and RP are very nearly equal, and, therefore, it may be assumed that the *amplitudes of the received waves are practically identical.* In fact, since the directions of propagation of the two waves are almost parallel, the individual electric fields can be assumed to have amplitudes $\mathbf{E}_1$ and $\mathbf{E}_2$ that are identical in *both* magnitude and direction. These vectors, which are illustrated in Fig. 26.7, are, therefore, assumed to be parallel at point P. This assumption is good, however, *only* when the distance d between the slits is small in comparison with the separation D between the slits and the screen.

On the other hand, the *path difference* $r_1 - r_2$ between the distances that each wave travels before striking the screen introduces a significant *difference in phase* between the two waves as they arrive at point P. It is this phase difference that is, in fact, responsible for most of the important characteristics of the observed interference pattern. The wave field observed at point P as a result of emission from the source at R can be written

$$E_1 = E_0 \cos(kr_1 - \omega t) \tag{26.3.1}$$

where E_0 is the amplitude of the electric field associated with the wave and k represents the common magnitude of the vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ shown in Fig. 26.7. These propagation vectors have the same magnitude because light of the same wavelength is emitted from both sources. In much the same way, we can write the electric field associated with the radiation from



. Geometry of Young's experiment.

[2] In Young's original experiment, of course, sunlight was used. Though visible interference effects can be observed with white-light sources such as this, they are invariably much sharper and more clearly defined when monochromatic light sources are employed.

the slit at Q as

$$E_2 = E_0 \cos(kr_2 - \omega t) \tag{26.3.2}$$

In writing the equations this way, we assume that the waves are *in phase* as they are emitted from the two sources.

Since the oscillating electric vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ have practically the same direction, they may be superposed simply by adding their magnitudes as given by (26.3.1) and (26.3.2). Therefore, the total field E will be

$$E = E_0[\cos(kr_1 - \omega t) + \cos(kr_2 - \omega t)] \tag{26.3.3}$$

From elementary trigonometry, however, we know that

$$\cos a + \cos b = 2 \cos \tfrac{1}{2}(a + b) \cos \tfrac{1}{2}(a - b) \tag{26.3.4}$$

This allows us to write (26.3.3) as

$$E = 2E_0 \cos \tfrac{1}{2}k(r_1 - r_2) \cos[\tfrac{1}{2}k(r_1 + r_2) - \omega t]$$

or

$$E = \left(2E_0 \cos \frac{\delta}{2}\right) \cos(k\bar{r} - \omega t) \tag{26.3.5}$$

where

$$\delta = k(r_1 - r_2) = 2\pi \frac{r_1 - r_2}{\lambda} \tag{26.3.6}$$

and

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) \tag{26.3.7}$$

From this, it is apparent that $\bar{r}$ represents the *average* of the two distances r_1 and r_2, which is essentially equal to the distance PT in Fig. 26.7. Since $(r_1 - r_2)/\lambda$ represents the path difference between the two waves, expressed in wavelengths, it is also evident from (26.3.6) that δ is simply the *difference in phase* between E_1 and E_2 at point P.

Equation (26.3.5) can now readily be seen to represent a wave disturbance of the original wavelength and frequency whose amplitude is given by

$$|E| = 2E_0 \cos(\delta/2) \tag{26.3.8}$$

From Fig. 26.7, however, it is easily seen that the path difference $r_1 - r_2$ is practically equal to $d \sin \theta$ so long as $d \ll D$. According to (26.3.6), then, the phase difference can be expressed as

$$\delta = \frac{2\pi d \sin \theta}{\lambda} \tag{26.3.9}$$

At point P_0, in the center of the screen, the angle θ is zero and the distances r_1 and r_2 to the slits are equal. The phase difference δ is, therefore, also zero, and since $\cos(\delta/2)$ is then equal to unity, the wave amplitude as given by (26.3.8) has the maximum possible value $2E_0$. At this point, therefore, the screen will be brightly illuminated. As we move upward, however, along the z-direction, the angle θ increases slowly, as does its sine. However, since λ is ordinarily much smaller than the distance d between the slits, we do not have to go far before the quantity $(2\pi d/\lambda) \sin \theta$ attains the value π, which means that in (26.3.8), $\cos(\delta/2)$ is *zero*. At this point, the total wave amplitude is zero, and the screen will be *dark*. There is now a phase difference $\delta = 180°$ between the two waves. They are, in other words, out of step by half a wavelength and, therefore, interfere destructively. If we move upward still further, θ becomes still larger, and the phase difference increases further until, at length, when the path difference equals a full wavelength, constructive interference again occurs and another bright region is encountered.

It is evident, therefore, that as we move along the z-direction in Fig. 26.7, we must traverse, alternately, bright and dark regions corresponding to path differences of even and odd numbers of half-wavelengths. An *interference pattern* of alternating bright and dark bands, therefore, appears on the screen. The spacing of these bands depends upon the light wavelength, the distance between the slits, and the spacing between the slits and the screen.

When the magnitude of the electric field amplitude at a given point is a maximum, $\cos(\delta/2)$ must be either $+1$ or -1. The condition for *constructive interference* is, therefore,

$$\cos(\delta/2) = \pm 1$$

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(r_1 - r_2) = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$$

or

$$|r_1 - r_2| = n\lambda \tag{26.3.10}$$

where

$$n = 0, 1, 2, 3, \ldots$$

Similarly, the electric field amplitude will have a minimum value of zero whenever $\cos \delta/2$ vanishes. Thus, in the case of *destructive interference*, we must have $\cos \delta/2 = 0$, or

$$\frac{\delta}{2} = \frac{\pi}{\lambda}(r_1 - r_2) = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$$

or

$$|r_1 - r_2| = (n + \tfrac{1}{2})\lambda \tag{26.3.11}$$

where

$$n = 0, 1, 2, 3, \ldots$$

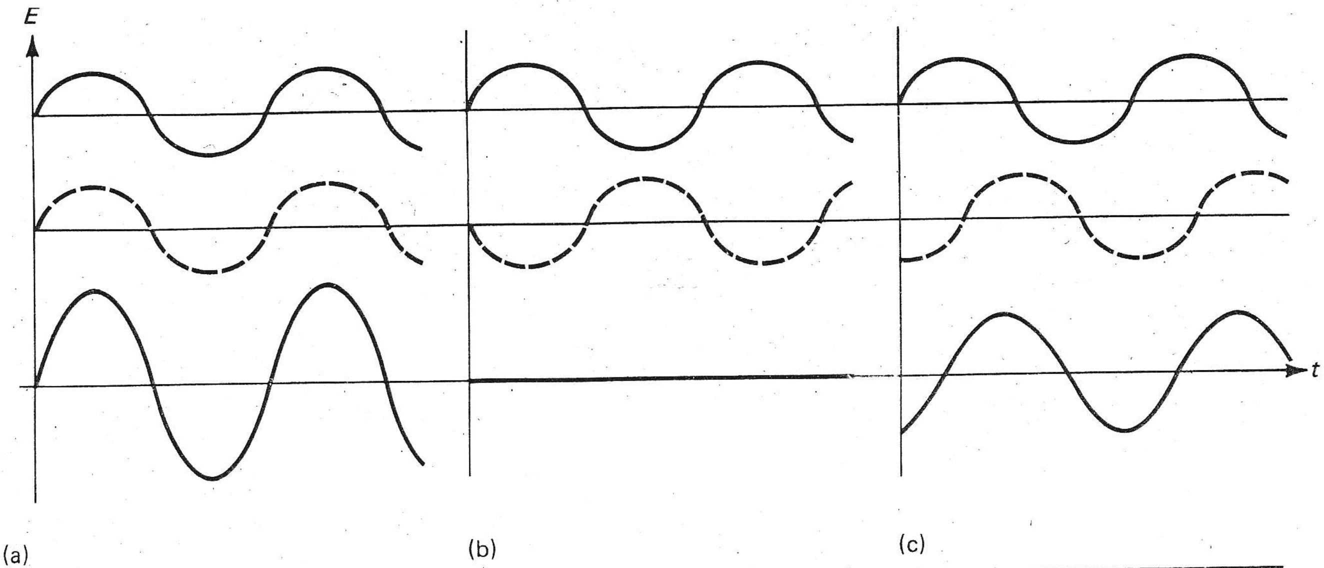


FIGURE 26.8. Superposition of plane waves (a) in phase, (b) 180° out of phase, and (c) with an intermediate phase difference.

Thus, we see that if the path difference between the two waves is an integral multiple of the wavelength, the waves arrive at P in phase and reinforcement occurs. On the other hand, if the path difference is an odd multiple of $\lambda/2$, they arrive out of phase and interfere destructively. In Fig. 26.8, the addition of waves in phase and out of phase is illustrated. An intermediate case corresponding to neither a maximum nor a minimum is also shown.

The *intensity* of light at the screen is proportional to the *square* of the electric field. From (26.3.5), we find this to be

$$E^2 = 4E_0^2 \cos^2(\delta/2) \cos^2(k\bar{r} - \omega t) \qquad (26.3.12)$$

For the three cases mentioned in Fig. 26.8, we also illustrate the square of the electric field. This is drawn in Fig. 26.9. Since E^2 oscillates very rapidly, its time variation will not be detected. Instead, typical instruments for measuring light intensity will respond only to the *time average* of the square of the electric field. Since the average of the $\cos^2$ function is $\frac{1}{2}$, as discussed earlier, the average of (26.3.5) will be

$$\overline{E^2} = 2E_0^2 \cos^2(\delta/2) \qquad (26.3.13)$$

This function is also plotted for the three cases drawn in Figs. 26.8 and 26.9. From this, we see that for constructive interference the average of the square of the amplitude is $2E_0^2$ as illustrated in Fig. 26.10. This is *four* times as large as the corresponding average intensity obtained with one slit covered! Therefore, constructive two-slit interference quadruples the light intensity at points where the waves arrive in phase. Of course, when they are out of phase, the average intensity will be zero. If there is some other phase difference, an intensity smaller than the maximum will be observed.

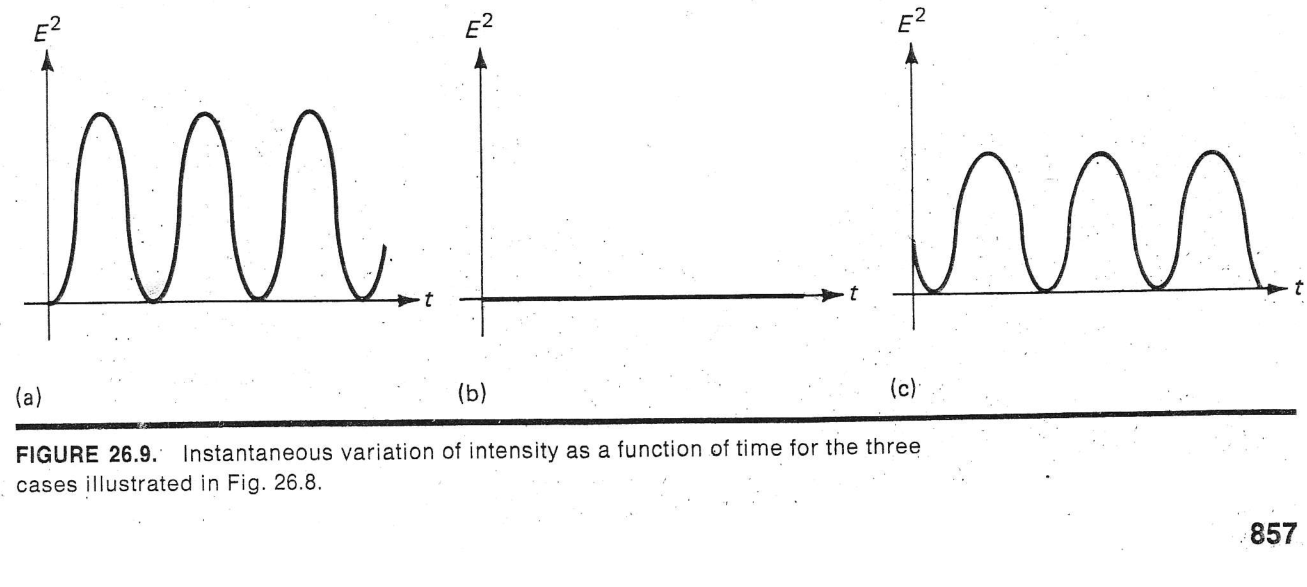


FIGURE 26.9. Instantaneous variation of intensity as a function of time for the three cases illustrated in Fig. 26.8.

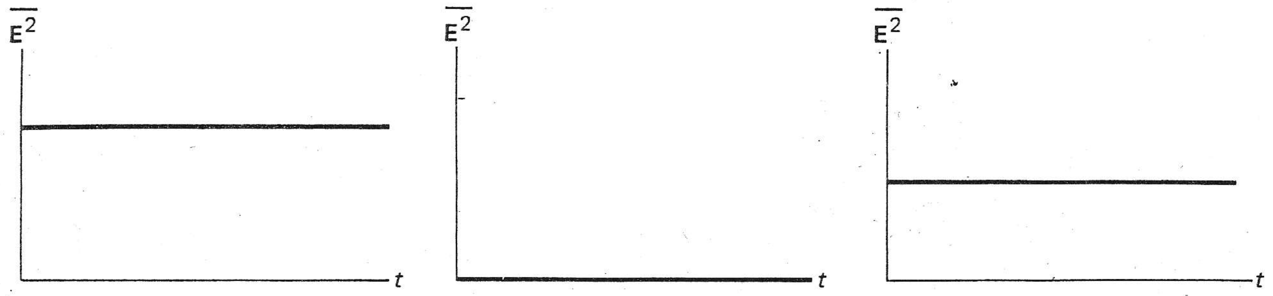


FIGURE 26.10. Time averaged intensity for the three cases illustrated in Figs. 26.8 and 26.9.

In Fig. 26.11, a photograph showing double-slit interference is given. Note the pattern of light and dark areas, which are referred to as *interference fringes*. The central bright fringe arises from the condition of (26.3.10) for $n = 0$. Successive bright fringes for $n = 1, 2, \ldots$ correspond to the additional constructive interference possibilities allowed by (26.3.10). The dark spots occur at locations which satisfy (26.3.11).

EXAMPLE 26.3.1

A double-slit interference pattern occurs on a screen very far from the slits. Obtain an expression giving the angles at which maxima and minima of intensity occur. Also calculate the distances z at which the fringes occur. See Fig. 26.7 for the geometry.

If $d \ll D$ and θ is assumed to be a small angle, the path difference $r_1 - r_2$ is approximately given by

$$r_1 - r_2 = d \sin\theta$$

where d is the distance between the slits. The conditions for maxima and minima can, therefore, be stated as follows:

Maxima

$$|d \sin\theta| = n\lambda$$

Minima

$$|d \sin\theta| = (n + \tfrac{1}{2})\lambda \tag{26.3.15}$$

Now, from Fig. 26.7,

$$\sin\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2 + D^2}} \tag{26.3.16}$$

Therefore, the conditions can be stated in terms of the distance z from the center of the screen. If $z \ll D$, which is usually the case, Eq. (26.3.16) can be written in the approximate form

$$\sin\theta \cong \frac{z}{D} \tag{26.3.17}$$

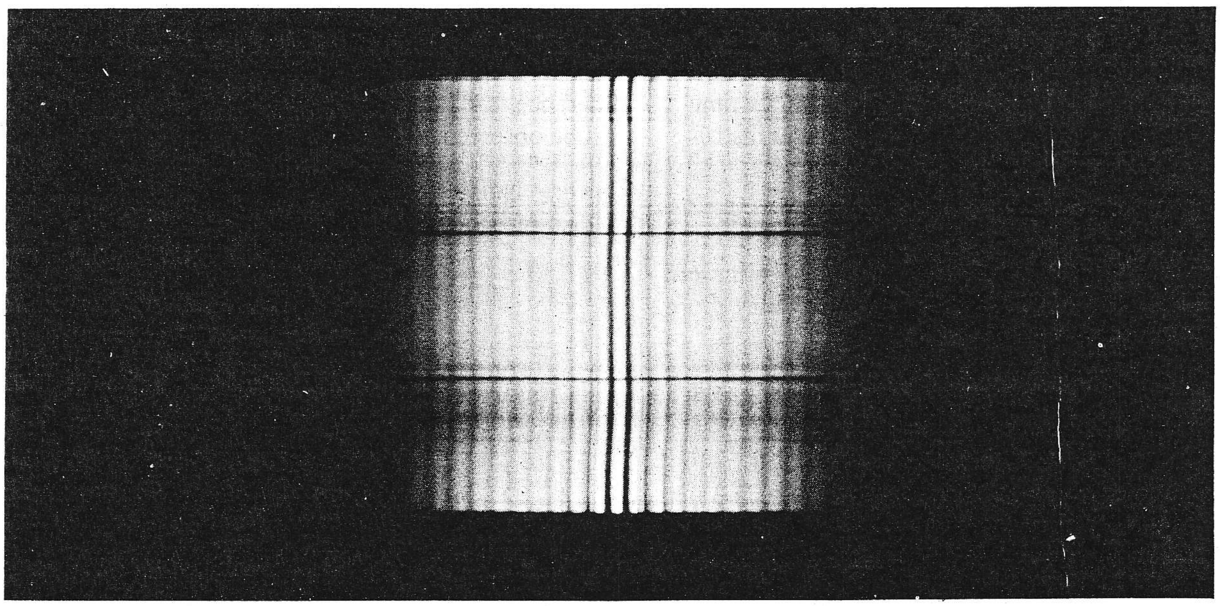


FIGURE 26.11. Photograph of double-slit interference fringes.

We may then write the interference conditions (26.3.10) and (26.3.11) as follows:

Maxima

$$|z| = \frac{n\lambda D}{d} \qquad (26.3.18)$$

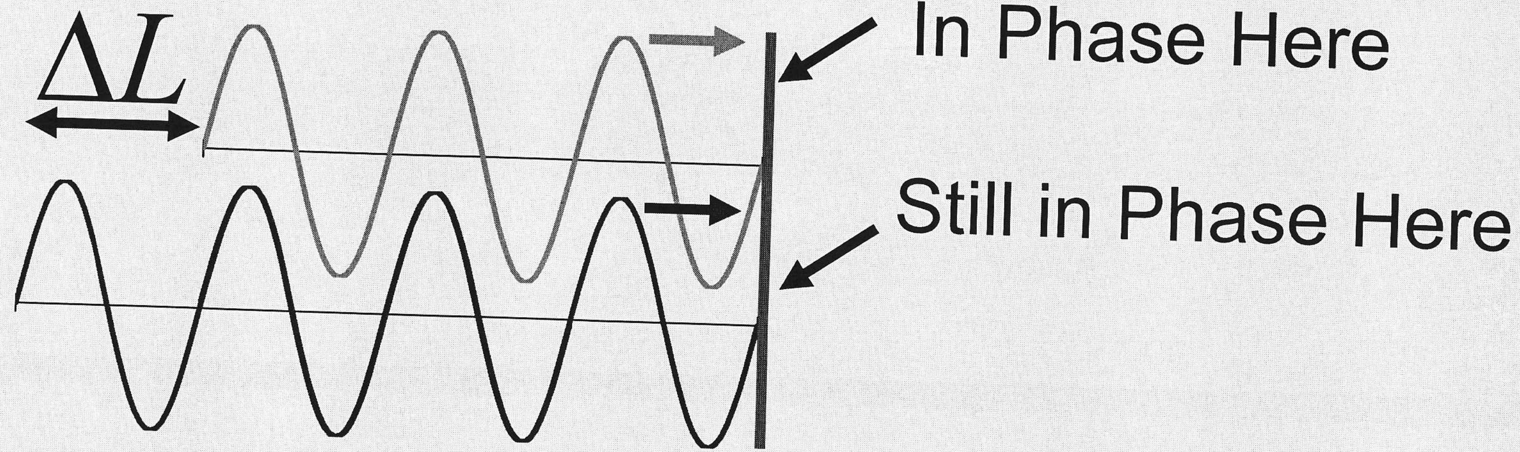
Minima

$$|z| = \frac{(n + \frac{1}{2})\lambda D}{d} \qquad (26.3.19)$$

Equations (26.3.18) and (26.3.19) show that the fringes are uniformly spaced on the screen so long as the angle θ is small.

Phase Shift

Extra Path Length



$$\Delta L = m\lambda \quad (m=0, \pm1, \pm2\ldots)$$

$$\Downarrow$$

Constructive Interference

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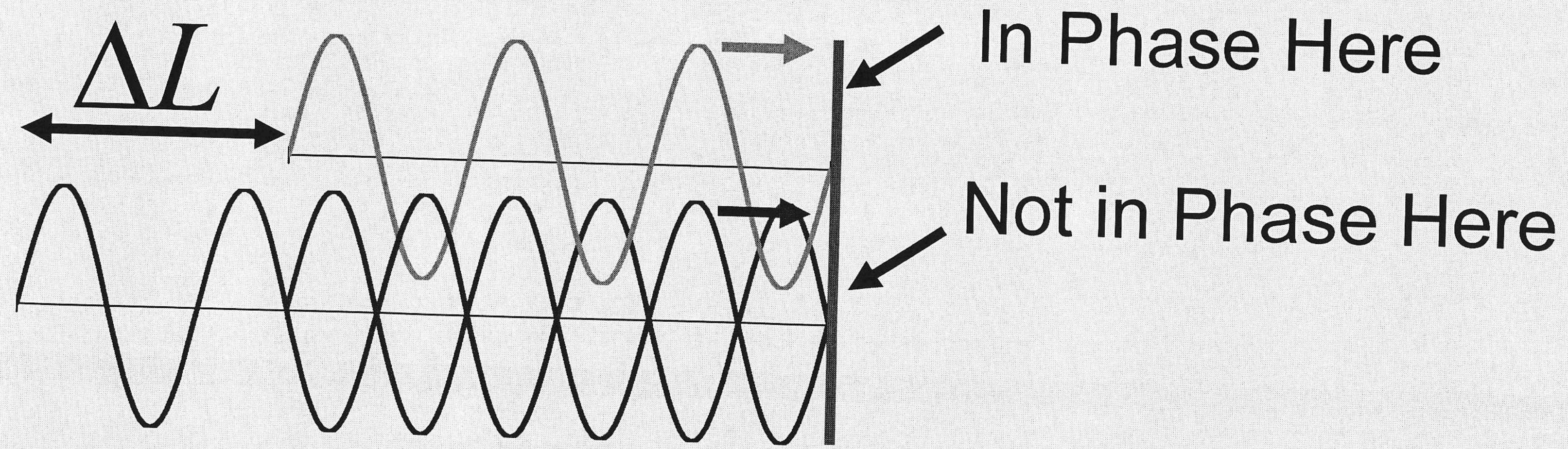
Extra Path Length



$$\Delta L = \left(m + \tfrac{1}{2}\right)\lambda$$

$\Downarrow$

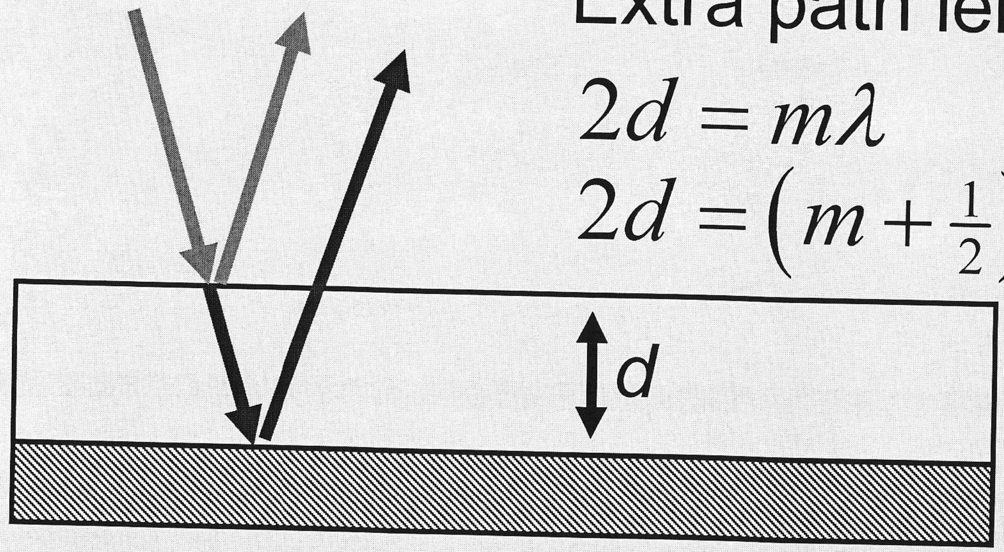
(m=0, ±1, ±2…)

Destructive Interference

Thin Film Interference - Iridescence

- Bubbles
- Butterfly Wings
- Oil on Puddles

Thin Film: Extra Path



Extra path length ~ $2d$

$$2d = m\lambda \Rightarrow \text{Constructive}$$

$$2d = \left(m + \tfrac{1}{2}\right)\lambda \Rightarrow \text{Destructive}$$

Oil on concrete, non-reflective coating on glass, etc.

Phase Shift = Extra Path?

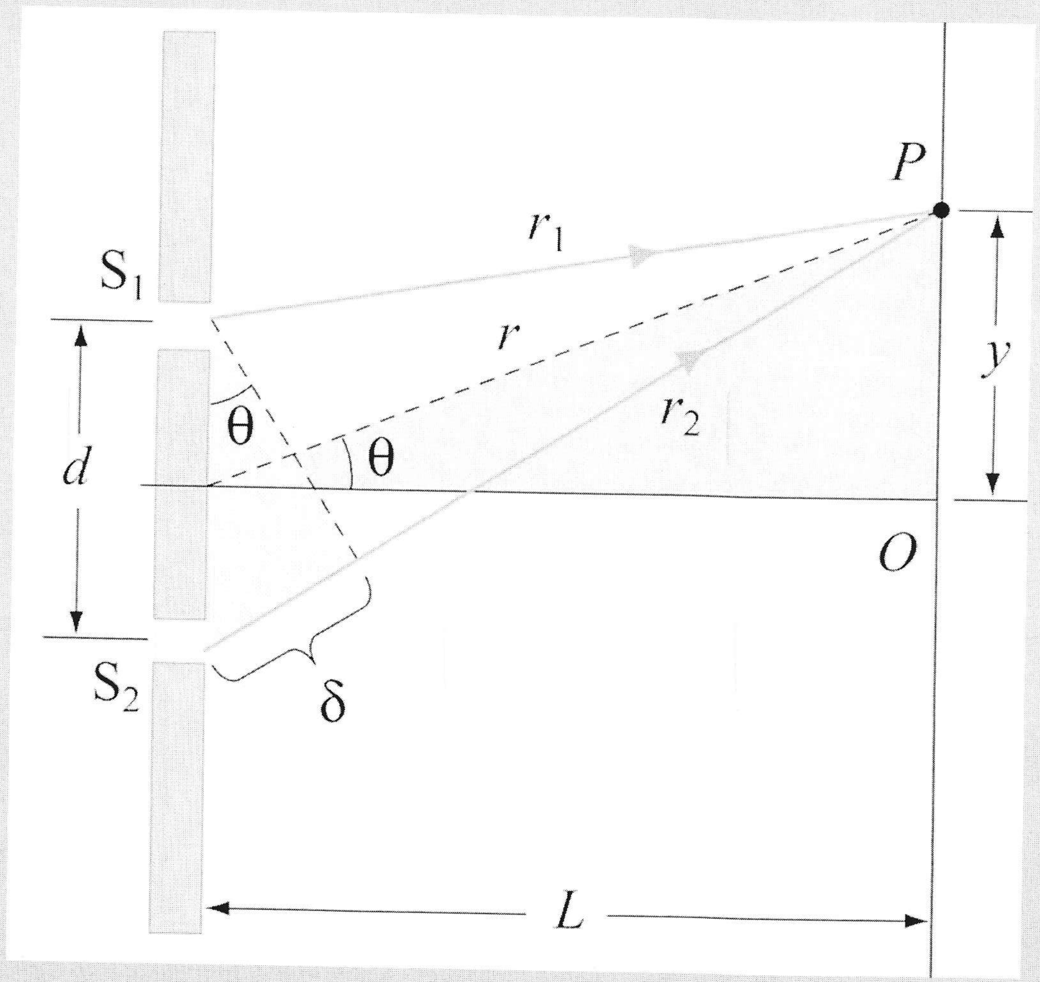
What is exact relationship between ΔL & ϕ?

$$\sin(k(x+\Delta L)) = \sin(kx + k\Delta L)$$

$$= \sin(kx + \frac{2\pi}{\lambda}\Delta L) \equiv \sin(kx+\varphi)$$

$$\boxed{\frac{\Delta L}{\lambda} = \frac{\phi}{2\pi}} = \begin{cases} m & \text{constructive} \\ m+\frac{1}{2} & \text{destructive} \end{cases}$$

Two In-Phase Sources: Geometry



Assuming $L \gg d$:

Extra path length

$$\delta = d\sin(\theta)$$

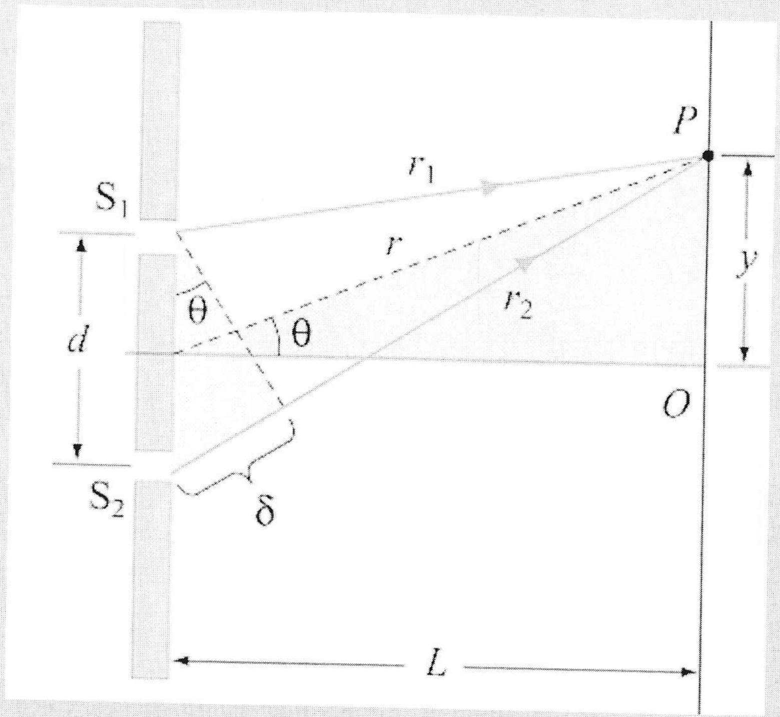
Assume $L \gg d \gg \lambda$

$$y = L\tan\theta \approx L\sin\theta$$

$\delta = d\sin(\theta) = m\lambda$ $\Rightarrow$ Constructive

$\delta = d\sin(\theta) = \left(m + \frac{1}{2}\right)\lambda$ $\Rightarrow$ Destructive

Interference for Two Sources in Phase



(1) Constructive: $\delta = m\lambda$

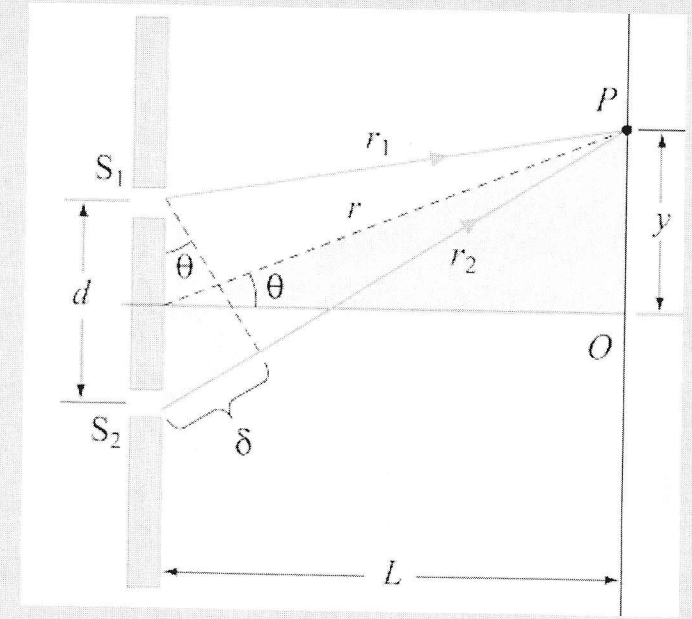
$$\sin\theta = \frac{\delta}{d} = \frac{m\lambda}{d} = \frac{y_{constructive}}{L}$$

$$y_{constructive} = m\frac{\lambda L}{d} \quad m = 0,1\ldots$$

(2) Destructive: $\delta = (m + 1/2)\lambda$

$$y_{destructive} = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \quad m = 0,1,\ldots$$

How we measure 1/10,000 of a cm



Question: How do you measure the wavelength of light?

Answer: Do the same experiment we just did (with light)

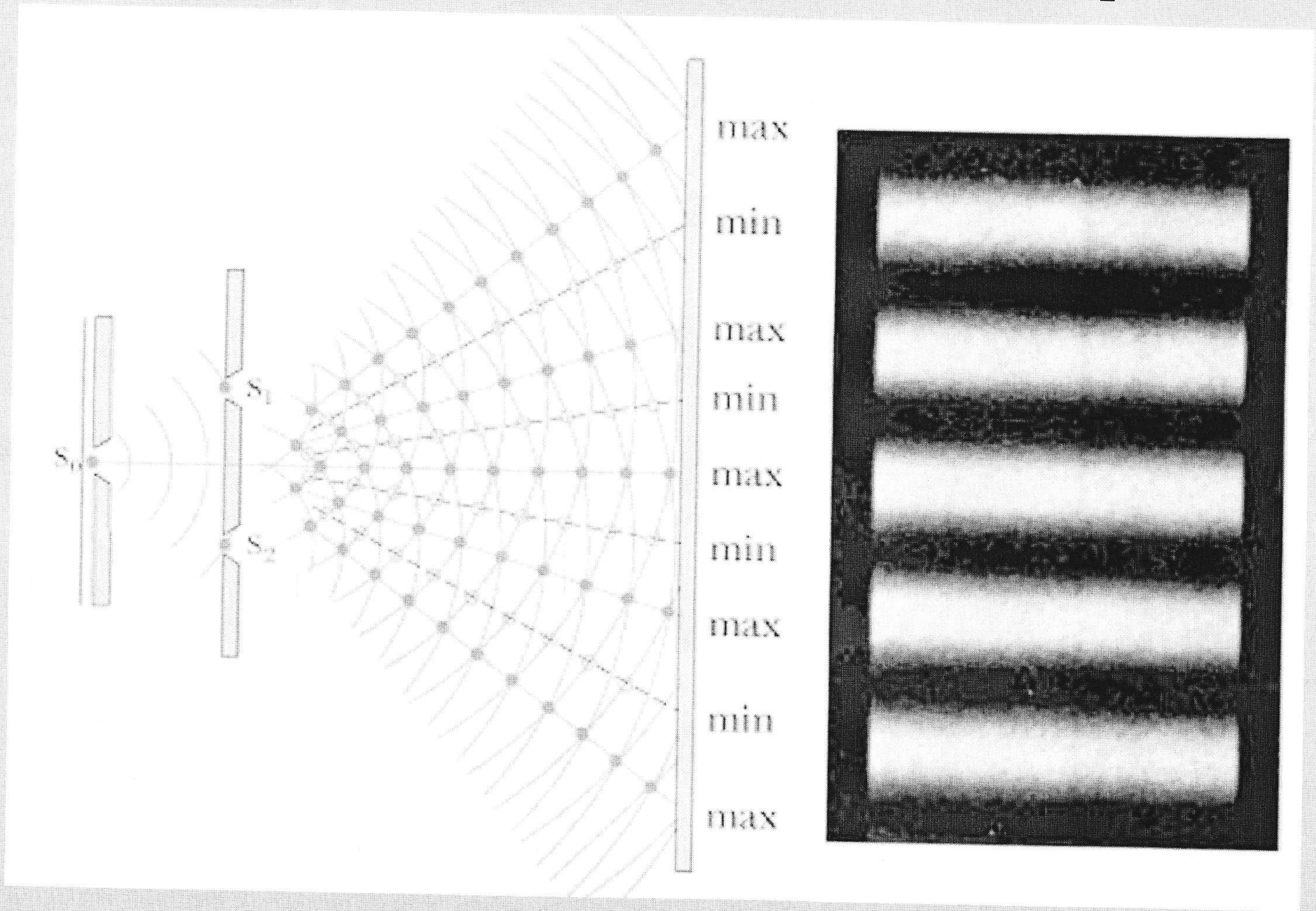
$$\text{First } y_{destructive} = \lambda L / 2d$$

λ is smaller by 10,000 times.

But d can be smaller (0.1 mm instead of 0.24 m)

So y will only be 10 times smaller – **still measurable**

Young's Double-Slit Experiment



Bright Fringes: Constructive interference

Dark Fringes: Destructive interference