Chapter 2

Special Relativity (I)

- The Galilean transformation
- The Michelson - Morley experiment
- The Lorentz transformation
- Relativity of simultanity
- Photographs
- Length Contraction
- Time Dilation
- Minkowski diagram
- Space - Time interval.
- Lorentz velocity transformation.
- Doppler effect
- The red shift of distant galaxis
- Twin paradox
- The pole and barn paradox

Special Relativity

Outline of Chapter 2 (Introduction)

Galilean transformation

$$\vec{u}' = \vec{u} - \vec{v}$$

velocity addition

Galilean invariance and momentum conservation.

Galilean principle of relativity

Maxwell equation.

$\frac{1}{\sqrt{\epsilon_0 \mu_0}} = c$ is valid in the rest system of ether

permeated all space

Michelson and Morley experiment

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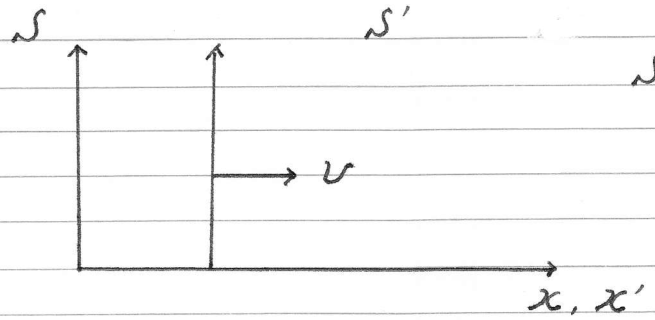
TOPIC	RELEVANT EQUATIONS AND REMARKS
1. Classical relativity	
Galilean transformation	$x' = x - vt \qquad y' = y \qquad z' = z$
Newtonian relativity	Newton's laws are invariant in all systems connected by a Galilean transformation.
2. Einstein's postulates	The laws of physics are the same in all inertial reference frames. The speed of light is c, independent of the motion of the source.
3. Relativity of simultaneity	Events simultaneous in one reference frame are not simultaneous in any other inertial frame.
4. Lorentz transformation	$x' = \gamma(x - vt) \qquad y' = y \qquad z' = z$ $t' = \gamma(t - vx/c^2)$ with $\gamma = (1 - v^2/c^2)^{-1/2}$
5. Time dilation	Proper time is the time interval τ between two events that occur at the same space point. If that interval is $\Delta t' = \tau$, then the time interval in S is $\Delta t = \gamma \Delta t' = \gamma\tau$, where $\gamma = (1 - v^2/c^2)^{-1/2}$
6. Length contraction	The proper length of a rod is the length L_p measured in the rest system of the rod. In S, moving at speed v with respect to the rod, the length measured is $L = L_p/\gamma$
7. Spacetime interval	All observers in inertial frames measure the same interval Δs between pairs of events in spacetime, where $(\Delta s)^2 = (c\,\Delta t)^2 - (\Delta x)^2$
8. Doppler effect	
Source/observer approaching	$f = \sqrt{\dfrac{1+\beta}{1-\beta}}\,f_0$
Source/observer receding	$f = \sqrt{\dfrac{1-\beta}{1+\beta}}\,f_0$

Chapter 2

The Basics of Relativity

The Galilean Transformation.



S, S' inertal frame of references

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

$\Rightarrow$ $\vec{u}\,' = \vec{u} - \vec{v}$ velocity addition formula.

$\vec{a}\,' = \vec{a}$

Newton's three laws

First law (the law of inertia)

No external force, S is an inertia frame of reference

$\Rightarrow$ $\vec{u} = \text{constant}$

$\downarrow$

$\vec{u}\,' = \text{constant}$

Second law

$\vec{F} = m\vec{a}$ in S frame

· $\vec{F} = \vec{F}\,'$

define forces by their effect on a standard calibrated spring balance

since observers in the S, S' frames will agree on the reading of the balance.

· In Newton's mechanics $m = m'$ (experimental fact)

$\vec{a}' = \vec{a}$

$$\vec{F} = m\vec{a} \Rightarrow \vec{F}' = m\vec{a}'$$

Third law clearly remains valid under $S \to S'$

Newton's laws are invariant under a Galilean transformation from S to S'

Momentum conservation

In S: $\quad m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{\bar{u}}_1 + m_2\vec{\bar{u}}_2$

$\downarrow$ under Galileam transformation.

$$m_1'\vec{u}_1' + m_2'\vec{u}_2' = m_1\vec{\bar{u}}_1' + m_2\vec{\bar{u}}_2'$$

Again, the momentum conservation is invariant under Galilean transformation.

$\Rightarrow$ No experiments obeying classical mechanics can ever tell us whether we, in frame S are moving and frame S' is at rest, or vice versa, or frame S and S' are both moving at different but constant velocity

To ask "Am I moving?" is strictly speaking quite meaningless, because nature precludes ever finding an answer.

$\downarrow$

Galilean principle of relativity.

Maxwell equation $\Rightarrow$ electromagnetic wave travelling (in vacuum) with speed $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$

If pulse is travelling with velocity $\vec{u}$ in S
it will be travelling with velocity $\vec{u}' = \vec{u} - \vec{v}$ in S'

$\Rightarrow$ If the speed is c in S
then the speed will be between $c-v$ and $c+v$ according to the direction of propagation.

$C = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ will not be valid in S'

⇒ the law is only valid in a particular frame S
↓
the rest system of ether.

ether is introduced to be the carrier of electromagnetic wave, it permeated all space.
↓
the earth itself is embedded in it.

⇒ two possible options

· the earth drags the ether along
↳ inconsistent with the observation of stellar aberration.

· the earth moves through the ether as it orbits the sun.

By measure the speed of light on earth along different directions
↓
we can find the relative velocity between the earth and ether rest system.
↓
Michelson - Morley experiment.

$c \sim 3 \cdot 10^8$ m/sec

The orbiting speed of the earth ~ 30 km/sec
↓
$\sim \frac{1}{10,000} c$
↓
great accuracy is needed.

Michelson - Morley experiment

Experimental set up

↓

schematic

$\vec{u}$ velocity of light relative to earth

$\vec{c}$ velocity of light relative to ether

$\vec{v}$ velocity of earth relative to ether

$$\underbrace{\vec{c} = \vec{u} + \vec{v}}_{\text{velocity addition rule}} \Rightarrow \vec{u} = \vec{c} - \vec{v}$$

Time to travel path ①

Path ② $\vec{u}$ is in the $\perp$ direction

$$u^2 = c^2 - v^2$$

$$u = \sqrt{c^2 - v^2}$$

Time to travel path ②

Phase difference $4\pi \frac{L}{\lambda} \beta^2$

$\beta = \frac{v}{c}$

$\beta^2 \sim 10^{-8}$

$L = 11\ m$

$\lambda = 5.5 \cdot 10^{-7}\ m$

2.5 rad

Measurement of the velocity of light

Invention of the interferometer

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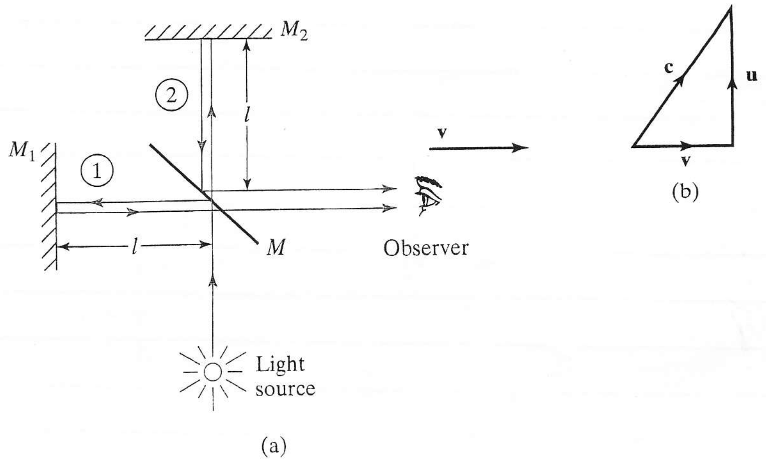
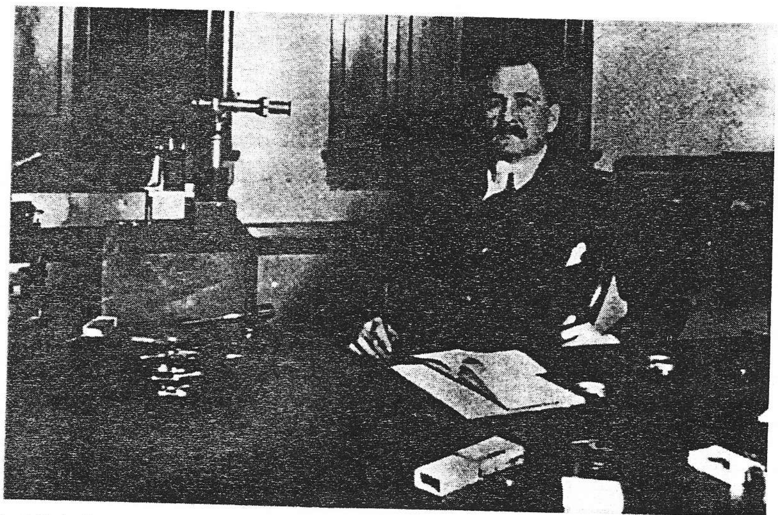
Michelson - Morley experiment



(a) Schematic diagram of the Michelson interferometer. M is a half-silvered mirror, M_1 and M_2 are mirrors. The vector $\mathbf{v}$ indicates the earth's velocity relative to the supposed ether frame. **(b)** The vector-addition diagram that gives the light's velocity $\mathbf{u}$, relative to the earth, as it travels from M to M_2 The velocity $\mathbf{c}$ relative to the ether is the vector sum of $\mathbf{v}$ and $\mathbf{u}$.



Albert A. Michelson made the first accurate measurement of the speed of light. Above, in his own handwriting, is the value as recorded on page 107 of his laboratory records of the 1878 experiment. (*Below*) Michelson in his laboratory. [*Courtesy of American Institute of Physics, Niels Bohr Library.*]

In the moving frame (the rest system of the interformeter)

L is the distance

Velocity in the rest system of the ether c

Velocity of the moving frame when the light is moving along the direction of v $c - v$

velocity of the moving frame relative the ether rest system

Time it takes to travel toward the mirror

$$= \frac{L}{c-v}$$

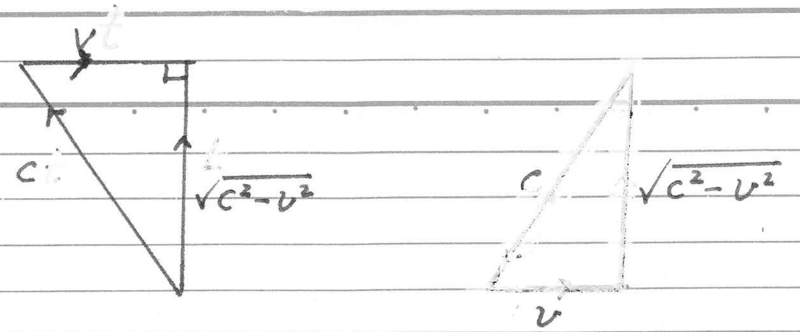
Time it takes to travel back from the mirror

$$= \frac{L}{c+v}$$

Total time it takes

$$= \frac{L}{c-v} + \frac{L}{c+v} = L\left(\frac{2c}{c^2-v^2}\right)$$

$$= \frac{2L/c}{1-\beta^2} \qquad \beta \equiv \frac{v}{c}$$



In the moving frome, the time travels to the mirror

$$(ct)^2-(vt)^2=L^2$$

$$\Rightarrow \quad \frac{L}{\sqrt{c^2-v^2}}$$

$$\text{Total time} = \frac{2L}{c\sqrt{1-\beta^2}}$$

$$\Delta T = \frac{2L}{c}\left(\frac{1}{1-\beta^2}-\frac{1}{\sqrt{1-\beta^2}}\right)$$

$$\simeq \frac{L}{c}\beta^2$$

(binomial expansion) valid for $\beta \ll 1$

Rotate the apparatus by 90°

$$\text{Total time difference} = \frac{2L}{c}\beta^2 = \Delta T_{tot}$$

$$\text{Phase Difference} = 2\pi \frac{\Delta T_{tot}}{T}$$

T: period

$$= \frac{2\pi c}{\lambda}\Delta T_{tot}$$

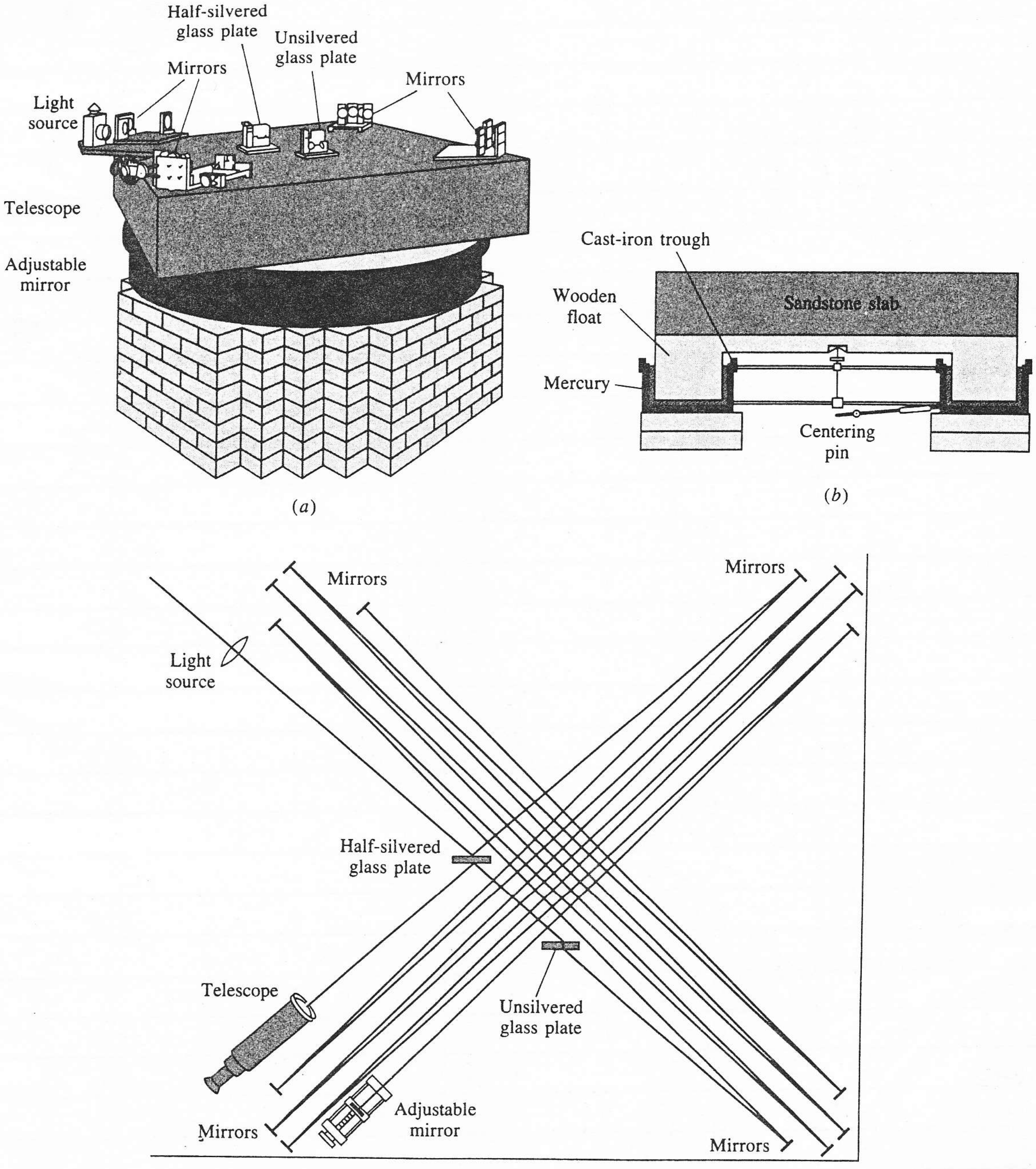
$$= \frac{4\pi L}{\lambda}\beta^2$$

$L = 11\ m$, $\lambda = 5.5 \cdot 10^{-7} m$ (yellow light)

$\beta^2 \sim 10^{-8}$

$\Rightarrow$ phase difference ~ 2.5 radians

The displacement to be expected was 0.4 fringe.
The actual displacement was certainly less than a twentieth part of this, and probably less than a fortieth part.



Drawing of the apparatus used by Michelson and Morley in 1887. (*a*) The light source, mirrors, and telescope are all mounted on a 5-ft square stone slab, which floats on a pool of mercury (*b*). This permits slow rotation of the entire apparatus without introducing strains. (*c*) A top view of the apparatus shows the additional mirrors used by Michelson and Morley to increase the optical path length.

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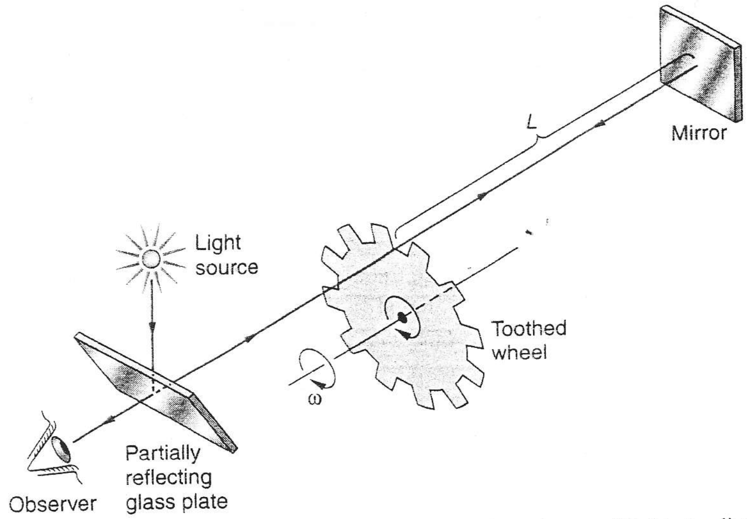

Fig. 1-5 Fizeau measured the speed of light in 1849 by aiming a beam of light at a distant mirror through the gap between two teeth in a wheel, in effect changing the light beam into pulses. A light pulse traveling at speed c would take $2L/c$ seconds to go from the wheel to the mirror and back to the wheel, If, during that time, rotation of the wheel moved a tooth into the light's path, the observer could not see the light. But if the angular velocity ω were such that the pulse arrived back at the wheel coincident with the arrival of the next gap, the observer saw the light.

Page 107.

The numbers in column 2 of the preceding table are thus translated:

3 = good
2 = fair
1 = poor

These numbers do not however, represent the relative weights.

Mean result	299728
Correction for temp. (P. 51)	+ 12
Velocity of Light in air	299740
* Correction for vacuo	+ 88
Velocity of Light in vacuo	299828 Kilometers per second.

* Should be + 80

The simplest explanation of this negative result.

- ether does not exist, electromagnetic wave can propagate in vacuum.
- the speed of light in vacuum, as measured in any inertial reference frame, is c regardless of the motion of the light source relative to that reference frame.

$\downarrow$

this can be accomplished if $S \rightarrow S'$ is given by a Lorentz transformation instead of Galilean transformation.

The Lorentz transformation

S S' inertial frame

x, y, z, t x', y', z', t'

The light pulse travel in vacuum is considered

At $t = t' = 0$ $S \overset{\vec{u}}{\leftrightarrow} S'$ $\vec{u}$ is along the x direction

A light pulse is emitted from the origin

The light pulse is propagating at speed c in both frame
(The space is isotropic)

The location of the wavefront in the two reference frame frames is given by

$$r = ct \qquad r^2 = x^2 + y^2 + z^2$$
$$r' = ct' \qquad r'^2 = x'^2 + y'^2 + z'^2$$

$S \xrightarrow{\vec{u}} S'$ $\vec{u}$ is along the x direction

$$y' = y \quad , \quad z' = z$$

$$x' = ax + bt$$
$$t' = fx + gt$$

The transformation must be linear in the variables to ensure the basic equivalents of all inertial reference

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Lorentz transformation

$$x' = ax + bt$$

$$t' = fx + gt$$

· Solve for a, b

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2$$

equation for f, g

$$(ax+bt)^2 - c^2(fx+gt)^2 = x^2 - c^2t^2$$

Look for coefficient of x^2, t^2, xt

f, g

frames. If terms involving x^2 or t^2 or higher powers of these variables appeared in the transformations, motion at constant in S would transform into accelerated motion in S'

$$x' = a\left[x + \frac{b}{a}t\right]$$

At time t, the origin of S' is at $x = ut$

$$0 = a\left[ut + \frac{b}{a}t\right] \Rightarrow \frac{b}{a} = -u$$

$$\Rightarrow \quad x' = a(x - ut)$$

$\updownarrow$ The inverse transformation must be identical to $x' = a(x - ut)$ except for a change in sign of u.

$$x = a(x' + ut')$$

To determine a, set $y = y' = z = z' = 0$

$$x' = ct' = a(ct - ut) = act(1-\beta)$$

$$t' = at(1-\beta)$$

$$x = ct = a(ct' + ut') = act'(1+\beta)$$

$$= ac\,(at)\,(1-\beta^2)$$

$$\Rightarrow \quad a^2(1-\beta^2) = 1 \quad \Rightarrow \quad a = \frac{1}{\sqrt{1-\beta^2}} = \gamma$$

The transformation for x and x', t and t' are then

$$x' = \gamma(x - ut), \qquad x = \gamma(x' + ut')$$
$$y' = y, \quad z' = z$$
$$t' = \gamma\left(t - \frac{u}{c^2}x\right), \qquad t = \gamma\left(t' + \frac{u}{c^2}x'\right)$$

with $\gamma = \dfrac{1}{\sqrt{1-\beta^2}}$, $\beta = \dfrac{u}{c}$

This is the Lorentz transformation

It can be shown that Maxwell equation is invariant under Lorentz transformation

⇒ Conflict between classical (Newtonian) mechanics
and
electricity and magnetism.

The conflict is resolved by Einstein's special theory of relativity
with
two basic postulates

- Principle of relativity
  The laws of nature are the same in all inertial reference frames.
  The principle of relativity is valid for all natural events

- The speed of light in vacuum, as measured in any inertial reference frame, is c regardless of the motion of the light soure relative to that reference frame

⇒ all laws of physics must be invariant under Lorentz transformation

↓

Newtonian mechanics must be modified

↓

relativistic mechanics

Two topics of importance:

- $S \xrightarrow{u} S'$ is related to each other through Lorentz transformation.
  ⇒ consequences of the Lorentz transformations
  and
  experimental evidences of relativistic kinematics.
  ↓
  Second half of Chapter 2

- Relativistic mechanics (dynamics)
  ↓
  Chapter 3

分類：
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Galileo Galilei

(1564–1642, Italian)

Considered by many the father of modern science, Galileo understood the importance of experiment and theory and was a master of both. Although he did not discover the telescope, he improved it and was the first to use it as a tool of astronomy, discovering the mountains on the moon, phases of Venus, moons of Jupiter, stars of the Milky Way, and sunspots and rotation of the sun. Among his many contributions to mechanics, he established the law of inertia and proved that gravity accelerates all bodies equally and that the period of a small-amplitude pendulum is independent of the amplitude. He understood clearly that the laws of mechanics hold in all unaccelerated frames, arguing that inside an enclosed cabin it would be impossible to detect the uniform motion of a ship. This argument appeared in his *Dialogue on the Two Chief World Systems* and was used to show that the earth could perfectly well be moving in orbit around the sun without our being aware of it in everyday life. For publishing this book, he was found guilty of heresy by the Holy Office of the Inquisition, and his book was placed on the Index of Prohibited Books — from which it was not removed until 1835.

Albert Michelson

(1852–1931, American)

Michelson devoted much of his career to increasingly accurate measurements of the speed of light, and in 1907 he won the Nobel Prize in physics for his contributions to optics. His failure to detect the earth's motion relative to the supposed ether is probably the most famous "unsuccessful" experiment in the history of science.

Hendrik Lorentz

(1853–1928, Dutch)

Lorentz was the first to write down the equations we now call the Lorentz transformation, although Einstein was the first to interpret them correctly. He also preceded Einstein with the length contraction formula (though, again, he did not interpret it correctly). He was one of the first to suggest that electrons are present in atoms, and his theory of electrons earned him the 1902 Nobel Prize in physics.

References:

Taylor, Zafiratos, and Dubson

Modern Physics for Scientists and Engineers" Second Edition

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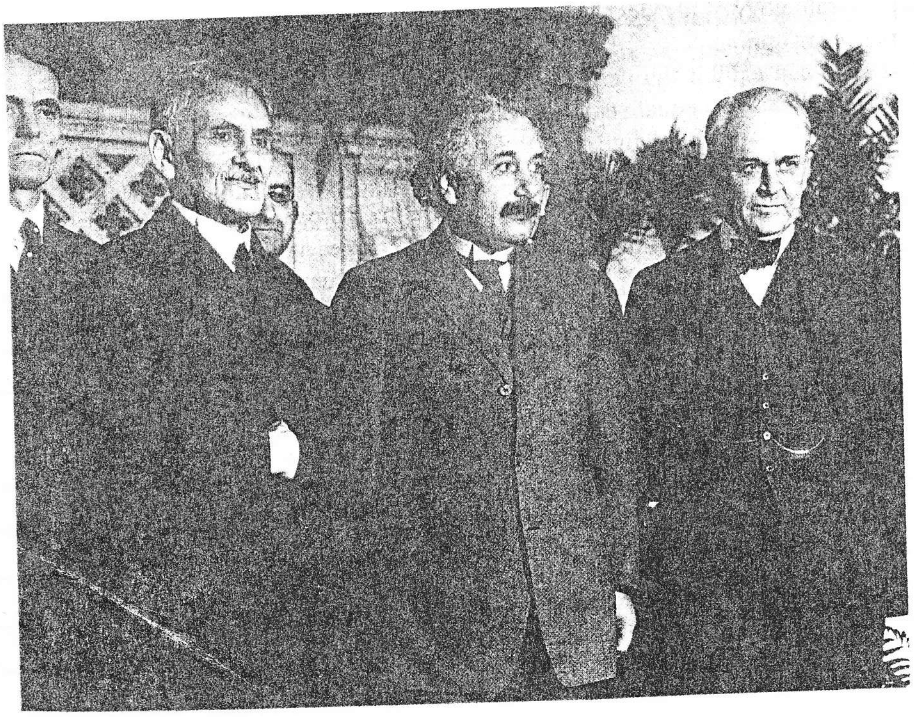

Figure 1.6 Albert A. Michelson, Albert Einstein, and Robert A. Millikan at the California Institute of Technology in 1931. (*AP/Wide World Photos.*)

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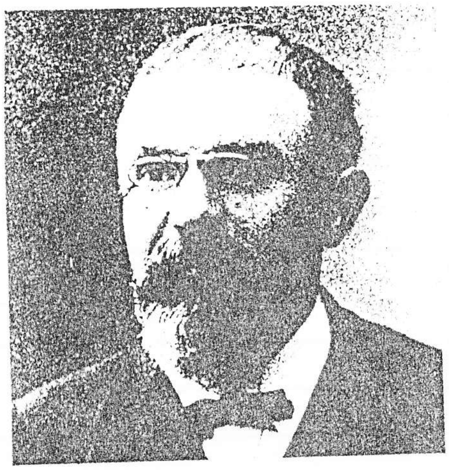

Jules Henri Poincaré, 1854-1912. Known for his contributions to many branches of pure mathematics, Poincaré devoted the majority of his efforts to mathematical dynamics. Among the first to accept the fact that the classical analytical methods of Euler and Lagrange had serious limitations, he revived geometrical methods. The results were revolutionary for dynamics, and gave birth to topology and global analysis as well. These branches of pure mathematics are very active yet.

photo courtesy of the Library of Congress, Washington, D.C.. U.S.A.

Albert Einstein

(1879–1955, German–Swiss–American)

Like all scientific theories, relativity was the work of many people. Nevertheless, Einstein's contributions outweigh those of anyone else by so much that the theory is quite properly regarded as his. As we will see in Chapter 4, he also made fundamental contributions to quantum theory, and it was for these that he was awarded the 1921 Nobel Prize in physics. The exotic ideas of relativity and the gentle, unpretentious persona of its creator excited the imagination of the press and public, and Einstein became the most famous scientist who ever lived. Asked what his profession was, the aged Einstein once answered, "photographer's model."

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The relativity theory arose from necessity, from serious and deep contradictions in the old theory from which there seemed no escape. The strength of the new theory lies in the consistency and simplicity with which it solves all these difficulties, using only a few very convincing assumptions.

Albert Einstein and Leopold Infeld, *The Evolution of Physics* (1938)

(a) (b) (c)

1

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Length contraction and time dilation

Muon decay.

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Length contraction

The proper length of a rod is the length L_p measured in the rest system of the rod.

In S, moving at speed v with respect to the rod, the length measured is

$$L = L_p / \gamma$$

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Time dilation   Proper time is the time interval τ

between two events occur at the same space point

In S' moving with respect to S

$$\Delta t' = \gamma \Delta t' = \gamma \tau \qquad \gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$$

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Relativity of simultaneity

According to Newtonian mechanics, two simultaneous events that occur at two different locations in S will also be simultaneous in a reference frame S' moving with velocity u relative to S

(In Galilean transformation , $t = t'$)

In special theory of relativity, two events that are simultaneous in one inertial reference frame are not simultaneous in another reference frame that is moving with respect to the first.

The reason for this dependence of simultaneity on the reference frames is that the information that an event has taken place is transmitted to the two observers at a finite, albeit very high speed.
The situation is illustrated in the following figure.
In reference frame S, the two light flashes appear simultaneous to the observer at rest midway between A and B.
The observer in S' approaches B and recedes from A at speed u. The signal from B reaches this observer before the signal from A

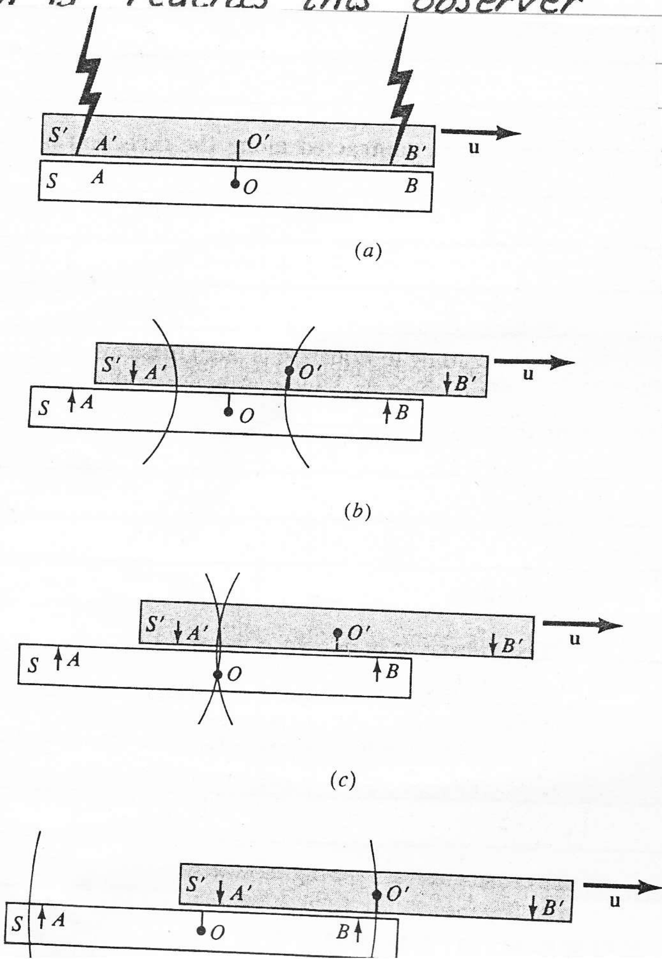

Illustration of the relativity of simultaneity. Light flashes strike at *A* and *B*. To observer *O*, the two flashes appear simultaneous. Observer *O'*, in a reference frame moving to the right with velocity **u**, records the flash from *B* before recording the flash from *A*; in this observer's reference frame, the two events are not simultaneous.

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In S two events A and B occur at

$$x_A = -X \quad , \quad x_B = X$$
$$t_A = t_B = T$$

In S'

$$t_A' = \gamma\left(T - \frac{-u}{c^2}X\right) = \gamma T + \gamma\left(\frac{u}{c^2}\right)X$$

$$t_B' = \gamma T - \gamma\left(\frac{u}{c^2}\right)X$$

$\Rightarrow$ to the observer in S' the two events are not simultaneous; event B occurs before event A

Simultaneity and Clock Synchronization

An important consequence of the special relativity we have been discussing is the question: How do two observers in different inertial frames synchronize their clocks? The process of clock synchronization requires some sort of signal being sent and this signal can, at most, travel at the speed of light. By synchronization of clocks, we are not referring to the time dilation of times observed by two observers. We are discussing the following issue: Suppose two clocks, separated by some distance, have been synchronized by observer O. Do the clocks also appear synchronized to observer O′ moving at a velocity u with respect to O?

Fig. 9.7 demonstrates a simple mechanism that an observer O could use to synchronize two clocks. O places a light source in the middle of the two clocks that are separated by a distance L. The two clocks start when they receive a pulse of light. Since the light pulse takes the same time to reach the clocks, they will be synchronized.

Let us consider an observer O′ moving with a velocity u along the distance between the clocks. In frame O the coordinate and time of the pulse reaching clock

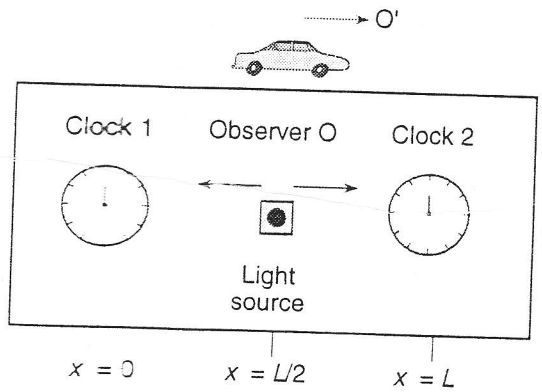


Figure 9.7: Observer O uses a light source placed at the middle of two clocks to synchronize the two clocks. Observer O′ sees that clock 2 starts earlier than clock 1.

1 is

$$x_1 = 0$$
$$t_1 = \frac{L}{2c} \tag{9.31}$$

The coordinate and time for clock 2 starting is

$$x_2 = L$$
$$t_2 = \frac{L}{2c} \tag{9.32}$$

Using the Lorentz transformation, the signal is received at clock 1 at a time

$$t'_1 = \frac{t_1(u/c^2)x_1}{\sqrt{1-u^2/c^2}} = \frac{L/2c}{\sqrt{1-u^2/c^2}} \tag{9.33}$$

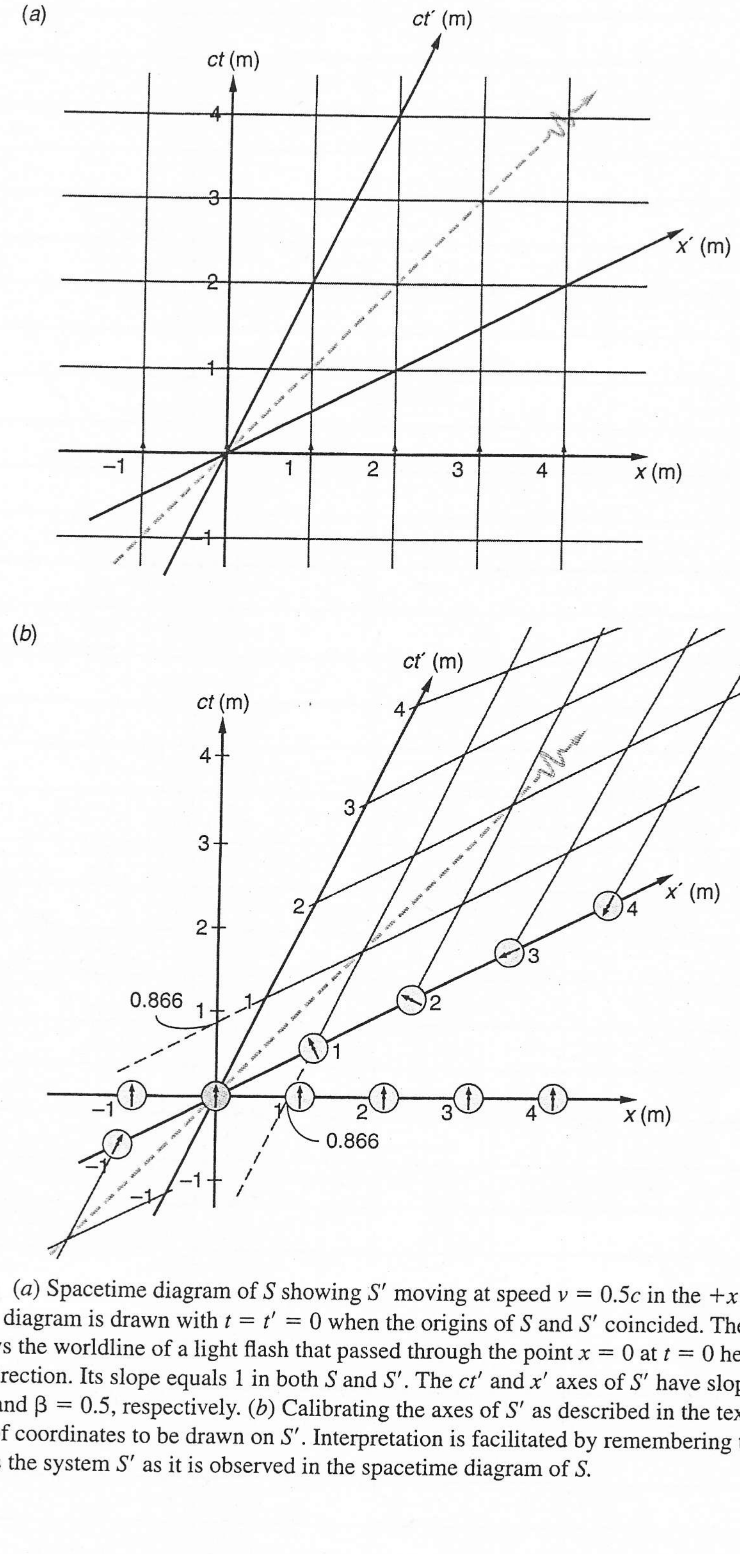
The time the second clock starts (for observer O′) is

$$t'_2 = \frac{t_2 - (u/c^2)x_2}{\sqrt{1-u^2/c^2}} = \frac{L/2c - (u/c^2)L}{\sqrt{1-u^2/c^2}} \tag{9.34}$$

We see that t'_2 is less than t'_1, so that observer O′ sees clock 2 as starting earlier than clock 1. The clocks are thus not synchronized for O′. The time difference is

$$\Delta t' = t'_1 - t'_2 = \frac{uL/c^2}{\sqrt{1-u^2/c^2}} \tag{9.35}$$

We see from this exercise that two events which occur simultaneously in one frame are not simultaneous in another frame, unless they occur at the same point in space.



(*a*) Spacetime diagram of S showing S' moving at speed $v = 0.5c$ in the $+x$ direction. The diagram is drawn with $t = t' = 0$ when the origins of S and S' coincided. The dashed line shows the worldline of a light flash that passed through the point $x = 0$ at $t = 0$ heading in the $+x$ direction. Its slope equals 1 in both S and S'. The ct' and x' axes of S' have slopes of $1/\beta = 2$ and $\beta = 0.5$, respectively. (*b*) Calibrating the axes of S' as described in the text allows the grid of coordinates to be drawn on S'. Interpretation is facilitated by remembering that (*b*) shows the system S' as it is observed in the spacetime diagram of S.

Length Contraction.

Consider the measurement of length.

The length of an object is defined as the difference between the coordinates of its two ends
If the object is moving, it is, of course, essential that the coordinates of the endpoints be measured at the same instant.

Assume a rod rests along the x axis in S

$$x_2 - x_1 = L_0$$

x_2, x_1 are coordinates of its endpoints in S

Going to S'

$$x_2 = \gamma(x_2' + u t_2')$$
$$x_1 = \gamma(x_1' + u t_1')$$

$$\Rightarrow \quad L_0 = x_2 - x_1 = \gamma[(x_2' - x_1') + u(t_2' - t_1')]$$

To measure length in S' $\Rightarrow$ require $t_2' = t_1'$

then $L' = x_2' - x_1'$

$$\Rightarrow \quad L_0 = \gamma L'$$

L': length in S'

$$L' = \frac{L_0}{\gamma} \qquad \gamma \geq 1$$

$$\Rightarrow \quad L' \leq L_0$$

$\Rightarrow$ Moving objects are contracted along the direction of motion

length contraction.

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編號： 2-20
總號：

Hermann Minkowski

From the book "Hilbert"

分類: 15
編號: 2-16
總號:

Time dilation

In S, the time interval is

$t_2 - t_1$ at the same position, namely,

$x_2 = x_1$

the particle is at rest

In S' $x_1' \neq x_2'$

$$t_2' = \gamma\left(t_2 - \frac{u}{c^2} x_2\right)$$

$$t_1' = \gamma\left(t_1 - \frac{u}{c^2} x_1\right)$$

$$\Rightarrow \quad t_2' - t_1' = \gamma(t_2 - t_1)$$

$$\Delta T' = \gamma \Delta T_0 \geq \Delta T_0$$

time dilation.

Evidence of time dilation and length contraction.

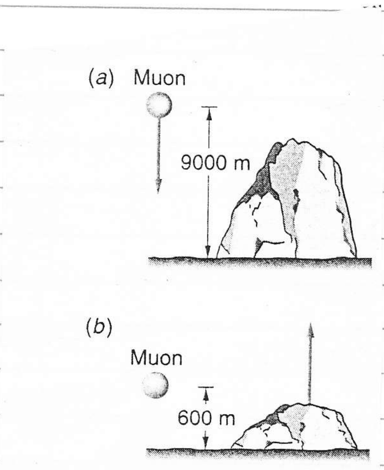
A beam of $\mu^{\pm}$ → unstable particles with half-live $= 2.2 \cdot 10^{-6}$ sec
(half of particle will decay in the rest system of μ)

If the beam is accelerated to $0.99c$ in the lab system S.

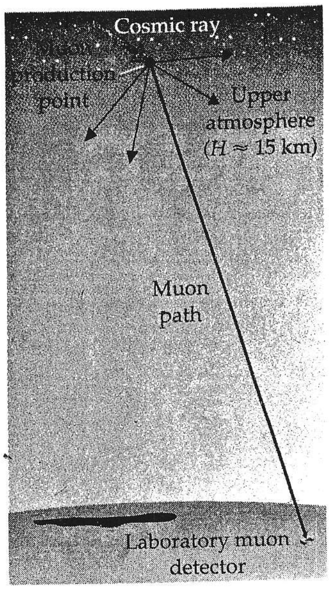
- Without time dilation, after a distance
$$\Delta x = v t_{1/2} = 0.99c \cdot 2.2 \cdot 10^{-6}\,\text{sec} = 650\ \text{m}$$
half of the μ in the beam would have decayed

- With time dilation, half of the μ beam would have decayed after travelling a distance
$$\Delta x = v \gamma t_{1/2} = 650\,\text{m} \cdot \frac{1}{\sqrt{1-(0.99)^2}} = 4.62\ \text{km}$$

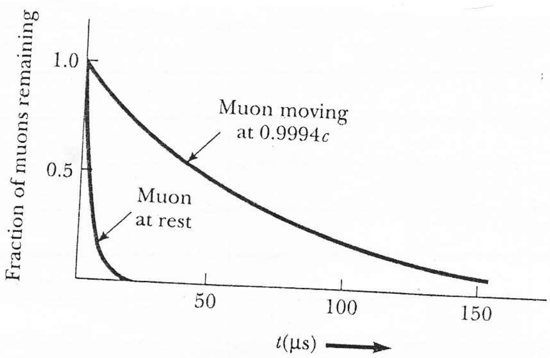
It is easy to accelerate μ to $0.99c$
↓
the observation support the validity of time dilation.



Although muons are created high above Earth, and their mean lifetime is only about 2 μs when at rest, many appear at Earth's surface. (*a*) In Earth's reference frame, a typical muon moving at 0.998*c* has a mean lifetime of 30 μs and travels 9000 m in this time. (*b*) In the reference frame of the muon, the distance traveled by Earth is only 600 m in the muon's lifetime of 2 μs.



High-energy muons are decay products of particles produced in high-energy collisions of cosmic rays with molecules of Earth's upper atmosphere. These muons travel distances on the order of 15 km to detectors on Earth's surface, which means that they live very much longer than expected from a calculation of their proper lifetime.

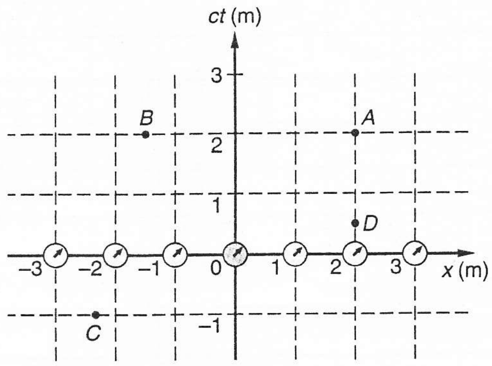


Decay curves for muons traveling at a speed of $0.9994c$ and for muons at rest.

Appendix

Space-time diagram. (Minkowski diagram)
↓
a simple way to display events

Limit our attention to one space and to time.
(two of the space dimensions (y and z) are suppressed.



Spacetime diagram for an inertial reference frame S. Two of the space dimensions (y and z) are suppressed. The units on both the space and time axes are the same, meters. A meter of time means the time required for light to travel 1 meter, i.e., 3.3×10^{-9} s.

Particles in spacetime

Particles moving in space trace out a line in the space-time diagram ⇒ worldline of the particle.

Analyzing events in inertial systems that are in relative motion can be accomplished more easily in spacetime diagrams.

ct' axis $\Rightarrow$ $x' = 0$

$\Rightarrow x' = \gamma(x - ut) = 0$

$\Rightarrow x = ut = \beta ct \Rightarrow ct = \frac{1}{\beta} x$

the slope of the ct' axis is $\frac{1}{\beta}$

x' axis $\Rightarrow$ $ct' = 0$

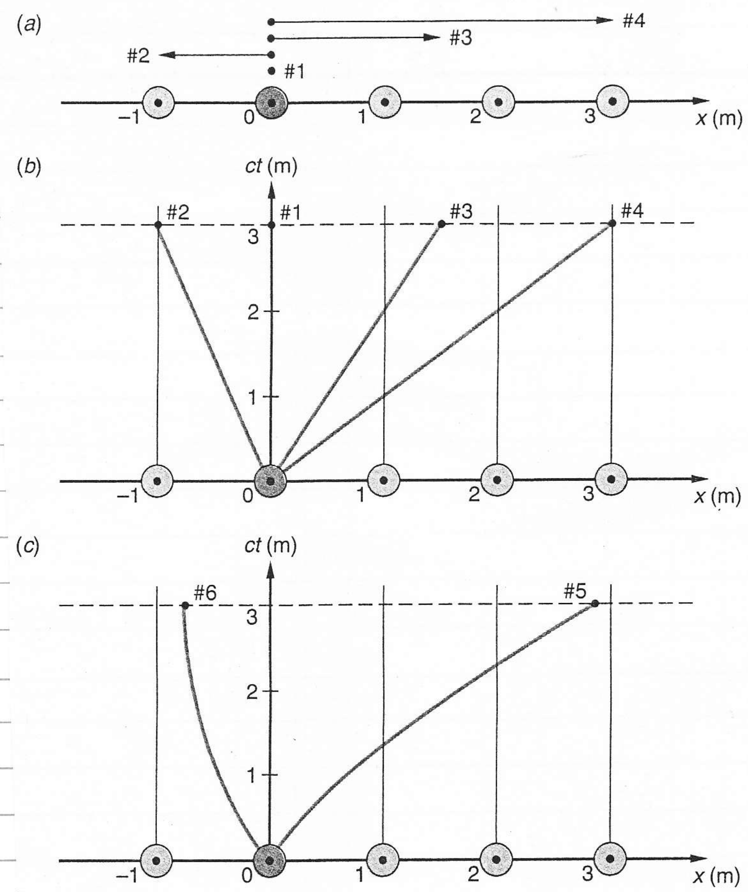
$\Rightarrow t' = \gamma \cdot (t - \frac{ux}{c^2}) = 0$

$\Rightarrow t = \frac{ux}{c} \Rightarrow ct = \frac{u}{c} x = \beta x$

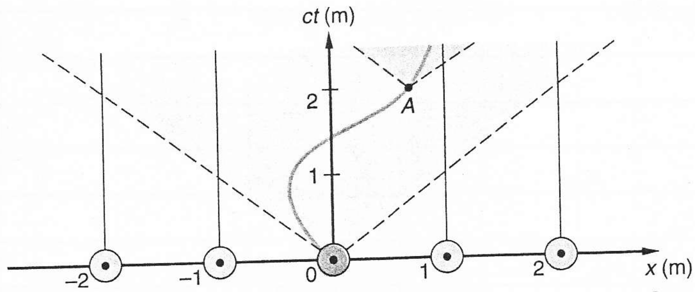
the slope of the x' axis as measured by an observer is β.

Note: x and x' is still parallel in space. Here we are plotting the space-time diagram.

分類：
編號： 2-21
總號：



(*a*) The space trajectories of four particles with various constant speeds. Note that particle 1 has a speed of zero and particle 2 moves in the $-x$ direction. The worldlines of the particles are straight lines. (*b*) The worldline of particle 1 is also the *ct* axis, since that particle remains at $x = 0$. The constant slopes are a consequence of the constant speeds. (*c*) For accelerating particles 5 and 6 [not shown in (*a*)], the worldlines are curved, the slope at any point yielding the instantaneous speed.



The speed-of-light limit to the speeds of particles limits the slopes of worldlines for particles that move through $x = 0$ at $ct = 0$ to the shaded area of spacetime, i.e., to slopes < -1 and $> +1$. The dashed lines are worldlines of light flashes moving in the $-x$ and $+x$ directions. The curved worldline of the particle shown has the same limits at every instant. Notice that the particle's speed = 1/slope.

分類：
編號：
總號：

Event can be represented by a point (x, t) in the space-time diagram.

ct

give the right dimension

A point particle

At rest

Moving in the $x-t$ plane with constant velocity

With acceleration

World line

The space-time interval

②

Invariant under Lorentz transformation

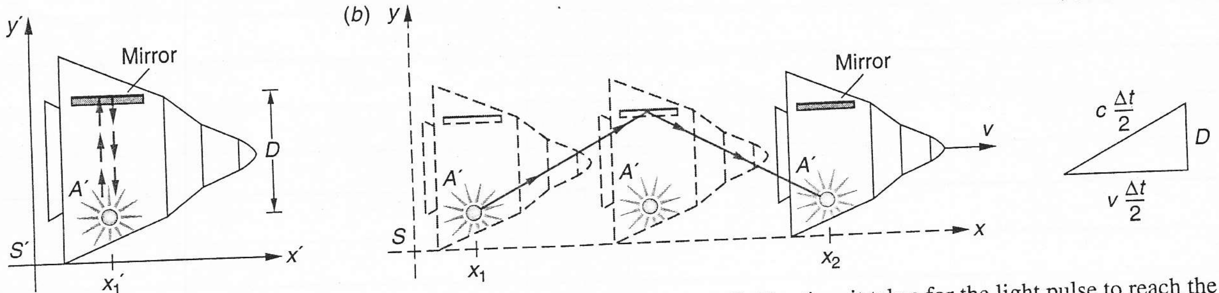
①

The coordinates

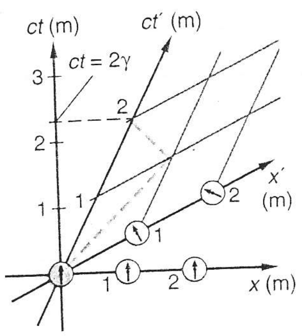
Lorentz velocity transformation

分類：	
編號：	2- 22
總號：	

Illustration of time dilation



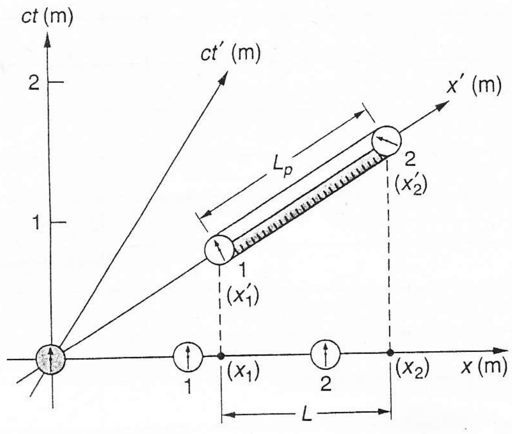
(*a*) Observer A' and the mirror are in a spaceship at rest in frame S'. The time it takes for the light pulse to reach the mirror and return is measured by A' to be $2D/c$. (*b*) In frame *S*, the spaceship is moving to the right with speed *v*. If the speed of light is the same in both frames, the time it takes for the light to reach the mirror and return is longer than $2D/c$ in *S* because the distance traveled is greater than $2D$. (*c*) A right triangle for computing the time Δt in frame *S*.



Spacetime diagram illustrating time dilation. The dashed line is the worldline of a light flash emitted at $x' = 0$ and reflected back to that point by a mirror at $x' = 1$ m. $\beta = 0.5$.

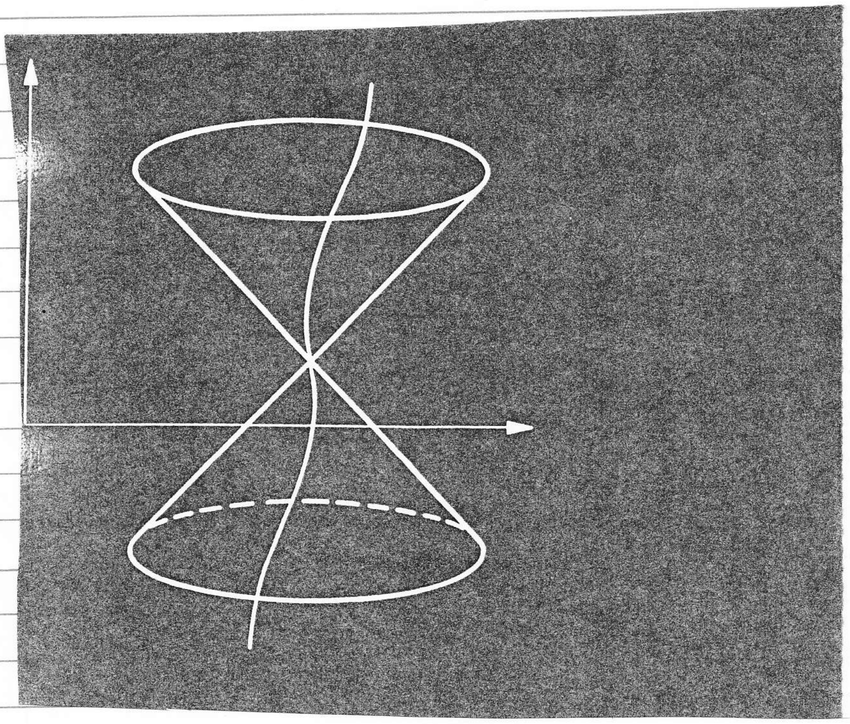
分類：	
編號：	2-25
總號：	

Illustration in length contraction in space-time diagram.



A measuring rod, a meterstick in this case, lies at rest in S' between $x_2' = 2$ m and $x_1' = 1$ m. System S' moves with $\beta = 0.62$ relative to S. Since the rod is in motion, S must measure the locations of the ends of the rod x_2 and x_1 simultaneously in order to have made a valid length measurement. L is obviously shorter than L_p. By direct measurement from the diagram (use a millimeter scale) $L/L_p = 0.78 = 1/\gamma$.

分類:	20
編號:	2-27
總號:	



The space time interval.

$$(\Delta S)^2 = (C\Delta t)^2 - [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]$$

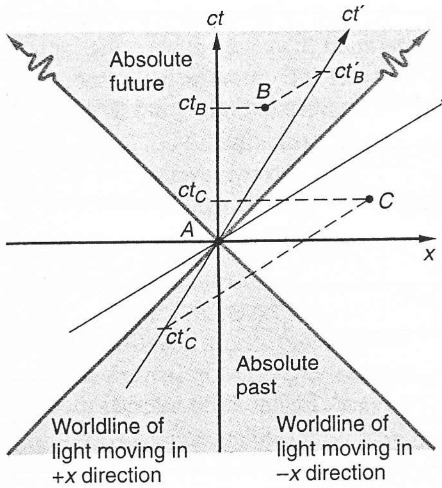
↳ space-time interval.

It is invariant under Lorentz transformation.

$(\Delta S)^2 > 0$ time like

$(\Delta S)^2 < 0$ space like

$(\Delta S)^2 = 0$ light like.



The relative temporal order of events for pairs characterized by timelike intervals, such as A and B, is the same for all inertial observers. Events in the upper shaded area will all occur in the future of A; those in the lower shaded area occurred in A's past. Events whose intervals are spacelike, such as A and C, can be measured as occurring in either order, depending on the relative motion of the frames. Thus, C occurs after A in S, but before A in S'.

Lorentz velocity transformation.

$$x' = \gamma(x - ut)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{u}{c^2}x)$$

$$v_x' = \frac{dx'}{dt'}, \quad v_y' = \frac{dy'}{dt'}, \quad v_z' = \frac{dz'}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

$$dx' = \frac{dx - u\,dt}{\sqrt{1 - (\frac{u}{c})^2}} \qquad dy' = dy, \quad dz' = dz$$

$$dt' = \frac{dt - \frac{u}{c^2}dx}{\sqrt{1 - u^2/c^2}}$$

$$v_x' = \frac{dx'}{dt'} = \frac{dx - u\,dt}{dt - \frac{u}{c^2}dx}$$

$$= \frac{v_x - u}{1 - \frac{v_x u}{c^2}}$$

$$v_y' = \frac{v_y}{\gamma[1 - \frac{v_x u}{c^2}]}$$

$$v_z' = \frac{v_z}{\gamma[1 - \frac{v_x u}{c^2}]}$$

- When v_x, u are both $\ll c$ ⇒ the denominator approaches unity

  non-relativistic limit.

  $$v_x' = v_x - u$$

  corresponds to the Galilean velocity transformation.

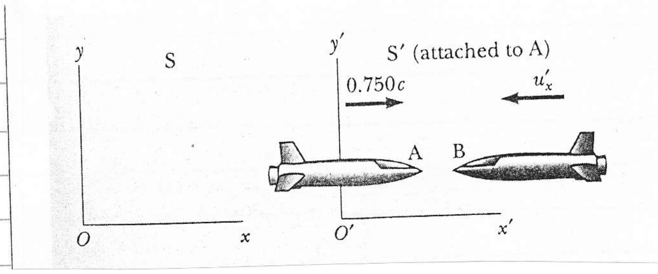
- When $v_x = c$

  $$v_x' = \frac{c - u}{1 - \frac{cu}{c^2}} = c$$

⇒ an object moving with a speed c relative to an observer S also have a speed c to an observer in S' independent of relative motion of S and S' ⇒ consistent with Einstein's second postulate.

分類：	22
編號：	2-29
總號：	

Two spaceship A and B are moving in opposite directions. An observer on Earth measures the speed of A to be 0.75 C and the speed of B to be 0.85 C
Find the velocity of B with respect to A



S frame → observer on earth

S' frame → attach to spacecraft A

$S \longrightarrow S'$ $\quad u = 0.75C$

$v_x = -0.85C$

$$\Rightarrow \quad v_x' = \frac{v_x - u}{1 - \frac{v_x u}{c^2}}$$

velocity of B relative to A

$$= \frac{-0.85c - 0.75c}{1 - \frac{(-0.85c)(0.75c)}{c^2}}$$

$$= -0.9771\,c$$

- The negative sign ⇒ the spaceship B is moving in the negative x direction.
- Note the result is less than C.
  ⇒ a body whose speed in less than C in one frame of reference must have a speed less than C in any other frame.

[If the Galilean velocity transformation were used, we would find $u_x' = u_x - v = -0.85C - 0.75C = -1.6C$, which is greater than C]

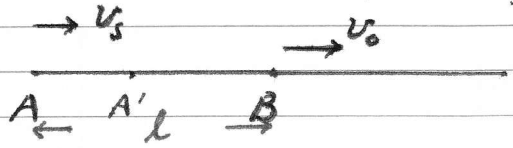
分類:	23
編號:	2-34
總號:	

Doppler Effect

Source of the wave and the observer may be in relative motion with respect to the medium in which the wave propagates, the frequency of the wave observed is different from the frequency of the source.

↓

Doppler effect



v_o = velocity of the observer relative to the medium.

v_s = velocity of the source relative to the medium.

$t=0$ Source at A, observer at B

$AB = l$

source emit a wave that reaches the observer at t

$$vt = l + v_o t \qquad v = \text{velocity of the wave with respect to the medium}$$

$$\Rightarrow \quad t = \frac{l}{v - v_o}$$

At time τ, the source is at A', the wave reaches the observer at time t'

Point of emission A', point of receiving B'

Distance travelled $l - v_s t + v_o t'$

$$v(t' - \tau) = l - v_s t + v_o t'$$

$$\Rightarrow \quad t' = \frac{l + (v - v_s)\tau}{v - v_o}$$

Source time interval τ number of wave emitted $= \nu\tau$

Observer time interval $\tau' = t' - t$, number of wave received $= \nu'\tau'$

$$\nu\tau' = \nu\tau \quad \Rightarrow \quad \nu' = \frac{\tau}{\tau'}\nu$$

$$\tau' = t' - t = \frac{l + (v - v_s)\tau}{v - v_o} - \frac{l}{v - v_o} = \frac{v - v_s}{v - v_o}\tau$$

$$\Rightarrow \quad \nu' = \frac{\tau}{\tau'} = \frac{v - v_o}{v - v_s}\nu$$

ν = frequency emitted by the source

ν = frequency observed by the observer.

分類： 24
編號： 2-35
總號：

$$\nu' = \frac{v - v_o}{v - v_s}\,\nu = \frac{1 - v_o/v}{1 - v_s/v}\,\nu$$

$$\simeq \left(1 - \frac{v_o}{v}\right)\left(1 + \frac{v_s}{v}\right)\nu \qquad (\text{assuming } v_s, v_o \ll v)$$

$$\simeq \left[1 - \frac{v_o - v_s}{v}\right] = \left(1 - \frac{v_{os}}{v}\right)\nu$$

$$v_{os} = v_o - v_s$$

$v_{os} > 0$ observer is moving away from the source
$\nu' < \nu$

$v_{os} < 0$ observer is approaching the source.
$\nu' > \nu$

For the case $v_o = 0$ (receiver is at rest)
$v_s > 0$

$$\Rightarrow \quad \nu' = \frac{1}{1 - u/v}$$

$u =$ relative speed of approach of receiver

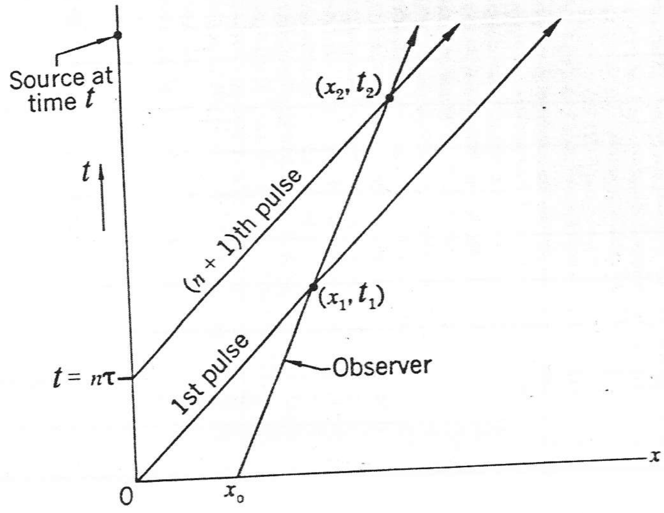
receiver is stationary
and
the source approaches the receiver

For the case $v_s = 0$, the reciever is approach the source, $v_o < 0$

$$\Rightarrow \quad \nu' = \left(1 + \frac{u}{v}\right)\nu$$

The apparent lack of symmetry between these two equations $\Rightarrow$ they cannot be valid for electromagnetic wave.

Now we study the relativistic case



Space-time diagram to illustrate the processes of emission and reception of light signals when source and receiver are in relative motion.

分類:	25
編號:	2-36
總號:	

Emitter is located at the origin of reference frame S
Observer moves relative to S at velocity u.
↓
at rest in S'

Each emitted pulse travels to S with speed c.

First pulse is sent out at $t=0$ where the observer is at $x=x_0$
$(n+1)$ pulse is sent out at $t=n\tau$.
↓
this will have convered n periods of vibration
↓
measured frequency of the source in S is $\nu = \frac{1}{\tau}$

(x_1, t_1) intersections of the world line of the observer
(x_2, t_2) with the world line of the first, $(n+1)$th pulse respectively (in S)

$$x_1 = ct_1 = x_0 + ut_1$$
$$x_2 = c(t_2 - n\tau) = x_0 + ut_2$$
$$\Rightarrow \quad t_2 - t_1 = \frac{cn\tau}{c-u}$$
$$x_2 - x_1 = \frac{ucn\tau}{c-u}$$

Measured in S'

$$\underbrace{t_2' - t_1'}_{= n\tau'} = \gamma\left[(t_2 - t_1) - \frac{u}{c^2}(x_2 - x_1)\right]$$
(by the Lorentz transformation

$$= \gamma\left(\frac{cn\tau}{c-u} - \frac{u}{c^2}\frac{ucn\tau}{c-u}\right)$$

Since this time interval covers n period.

$$\tau' = \frac{\gamma c\tau}{c-u}\left(1 - \frac{u^2}{c^2}\right) = \frac{\gamma(1-\beta^2)}{1-\beta}\tau$$
$$= \gamma(1+\beta)\tau = \left(\frac{1+\beta}{1-\beta}\right)^{\frac{1}{2}}\tau$$
$$\Downarrow$$
$$\nu' = \left(\frac{1-\beta}{1+\beta}\right)^{\frac{1}{2}}\nu$$

This is the relativistic Dopper effect.

$\beta > 0$ receding $\nu' < \nu \Rightarrow \lambda' > \lambda$ redshift
$\beta < 0$ approaching $\nu' > \nu \Rightarrow \lambda' < \lambda$ blueshift.

分類:	26
編號:	2-37
總號:	

Astronomers define the redshift of light from astronomical sources by the expression

$$z = \frac{\nu_0 - \nu}{\nu}$$

ν_0 = frequency measured in the frame of the star or galaxy

ν = frequency measured at the receiver on Earth.

Exercise : Show

$$\beta = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

Example : Quaser 2000-300 has a measured $z = 3.78$

$\downarrow$

it is receding from Earth at $0.91c$

A more detailed discussion of the redshift of distant galaxies is given in Appendix.

In Appendix we derive

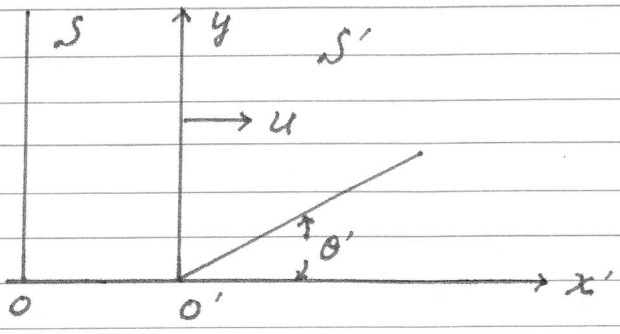
$$\nu' = \frac{\nu(1 - \beta\cos\theta)}{\sqrt{1-\beta^2}}$$

$\downarrow$

including the treatment of transverse. Doppler effect

Appendix

$$\nu' = \frac{\nu(1-\beta\cos\theta)}{\sqrt{1-\beta^2}}$$



Consider a train of plane monochromatic light wave of unit amplitude emitted from a source at the origin of the S' frame

$$\cos 2\pi \left[\frac{x'\cos\theta' + y'\sin\theta'}{\lambda'} - \nu' t' \right] \qquad (A)$$

In the S frame, the wavefronts will still be plane.

$$\cos 2\pi \left[\frac{x\cos\theta + y\sin\theta}{\lambda} - \nu t \right] \qquad (B)$$

[for the Lorentz transformation is linear, a plane transforms into a plane]

Lorentz transformation

$$x' = \frac{x - ut}{\sqrt{1-\beta^2}}, \quad t' = \frac{t - \frac{u}{c^2}x}{\sqrt{1-\beta^2}} \qquad \begin{array}{l} y' = y \\ z' = z \end{array}$$

Substitute into (A)

$$\Rightarrow \cos 2\pi \left[\frac{1}{\lambda'} \frac{(x-ut)}{\sqrt{1-\beta^2}} \cos\theta' + \frac{y\sin\theta'}{\lambda'} - \nu' \frac{[t - \frac{u}{c^2} x]}{\sqrt{1-\beta^2}} \right]$$

Rearrange terms

$$\cos 2\pi \left[\frac{\cos\theta' + \beta}{\lambda'\sqrt{1-\beta^2}} x + \frac{y\sin\theta'}{\lambda'} - \frac{(\beta\cos\theta' + 1)\nu' t}{\sqrt{1-\beta^2}} \right]$$

Compare with (B)

$$\Rightarrow \frac{\cos\theta}{\lambda} = \frac{\cos\theta' + \beta}{\lambda'\sqrt{1-\beta^2}}$$

$$\frac{\sin\theta}{\lambda} = \frac{\sin\theta'}{\lambda'}$$

$$\nu = \frac{\nu'(1+\beta\cos\theta')}{\sqrt{1-\beta^2}} \qquad (C)$$

The inverse transformation of (c) is clearly

$$\nu' = \frac{\nu(1 - \beta\cos\theta)}{\sqrt{1-\beta^2}}$$

this is equation (2-37)
of the textbook

[$k^\mu x_\mu$ is invariant is the key requirement]

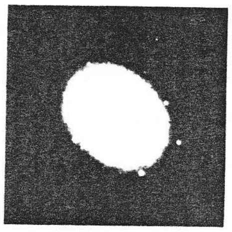
Appendix

The red shift of distant galaxies

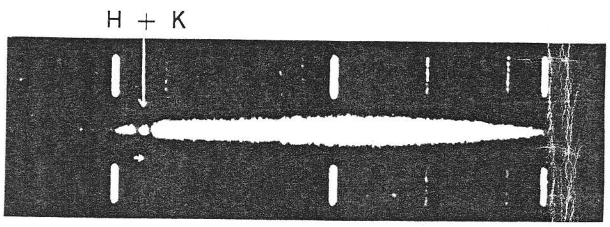
A. EVIDENCE OF AN EXPANDING UNIVERSE[a]

It is extremely difficult to photograph the spectra of distant galaxies. They are so faint that they cannot be observed visually, even with the largest telescopes. The correct aim of the telescope must therefore be calculated from the nebula's position in a photograph. Then the telescope has to be kept directed at the object for hours to photograph its spectrum which, in the end, may only measure 2 by 1 mm. The spectrum obtained is like that of an average star (spectral type G)—continuous, with absorption lines. But the absorption lines are very indistinct, forming as they do the average for all objects in the entire galaxy. By 1917 Slipher had managed to photograph the spectra of 15 spiral nebulae. He was surprised to find that the absorption lines in 13 of these spectra were displaced toward the red. This would suggest that the galaxies were moving away from us. From the shift observed it was possible to calculate that they were receding at 400 miles/sec (640 km/sec), on the average. In 1919 Hubble discovered that all external galaxies whose spectra had been photographed and distances determined are moving away from us at velocities proportional to their distances from us. This was once more very amply proved within the next few years by the observational material which Humason amassed using the Hooker and Hale telescopes. (Photographs from the Mount Wilson and Palomar Observatories.)

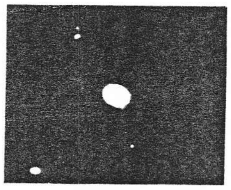
[a] *Comments from Ernst and De Vries, Atlas of the Universe, Thomas Nelson, London*, 1961.



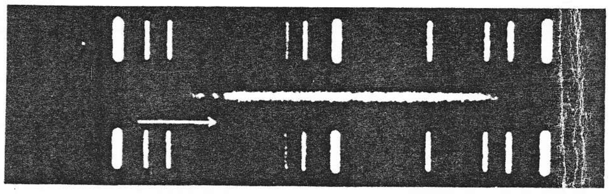
Virgo



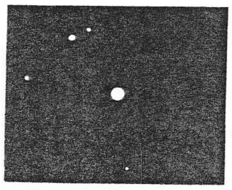
750 mi/sec



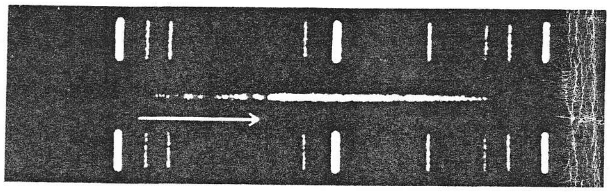
Ursa Major



9300 mi/sec

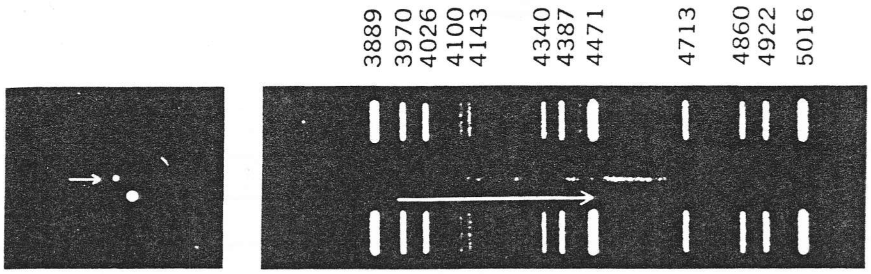


Corona Borealis

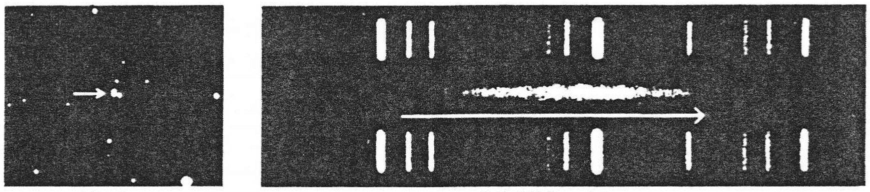


13,400 mi/sec

Comparison Lines of H and He
(wavelengths as marked)



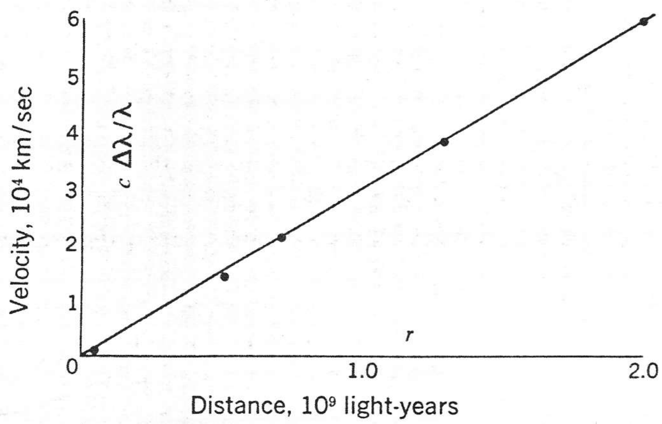
Boötes 24,400 mi/sec



Hydra 38,000 mi/sec

B. HUBBLE'S LAW

The distances in the table are based on a 1958 revision of the distance scale. Velocities were calculated simply as $c\,\Delta\lambda/\lambda$, with no account taken of relativistic modification of the Doppler formula.



Galaxy in:	*Velocity, $\times 10^4$ km/sec*	*Distance, light-years*
Virgo	0.12	0.4×10^8
Ursa Major	1.40	5.0×10^8
Corona Borealis	2.14	7.0×10^8
Boötes	3.90	1.3×10^9
Hydra	6.10	2.0×10^9

分類： 29
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The Twin Paradox

Homer and Ulysses are identical twins. Ulysses travels at a constant high speed to a star beyond our solar system and returns to Earth while his twin Homer remains at home. When the traveler Ulysses returns home, he finds his twin brother much aged compared to himself—in agreement, we shall see, with the prediction of relativity. The paradox arises out of the contention that the motion is relative and either twin could regard the other as the traveler, in which case each twin should find the other to be younger than he and we have a logical contradiction—a paradox. Let's illustrate the paradox with a specific example. Let Earth and the destination star be in the same inertial frame S. Two other frames S' and S'' move relative to S at $v = +0.8c$ and $v = -0.8c$, respectively. Thus $\gamma = 5/3$ in both cases. The spaceship carrying Ulysses accelerates quickly from S to S', then coasts with S' to the star, again accelerates quickly from S' to S'', coasts with S'' back to Earth, and brakes to a stop alongside Homer.

It is easy to analyze the problem from Homer's point of view on Earth. Suppose, according to Homer's clock, Ulysses coasts in S' for a time interval $\Delta t = 5$ y and in S'' for an equal time. Thus Homer is 10 y older when Ulysses returns. The time interval in S' between the events of Ulysses' leaving Earth and arriving at the star is shorter because it is proper time. The time it takes to reach the star by Ulysses' clock is

$$\Delta t' = \frac{\Delta t}{\gamma} = \frac{5\text{ y}}{5/3} = 3\text{ y}$$

Since the same time is required for the return trip, Ulysses will have recorded 6 y for the round trip and will be 4 y younger than Homer upon his return.

The difficulty in this situation seems to be for Ulysses to understand why his twin aged 10 y during his absence. If we consider Ulysses as being at rest and Homer as moving away, Homer's clock should run slow and measure only $3/\gamma = 1.8$ y, and it appears that Ulysses should expect Homer to have aged only 3.6 y during the round trip. This is, of course, the paradox. Both predictions can't be right. However, this approach makes the incorrect assumption that the twins' situations are symmetrical and interchangeable. They are not. Homer remains in a single inertial frame, whereas Ulysses *changes* inertial frames, as illustrated in Figure 1-35*a*, the spacetime diagram for Ulysses' trip. While the turnaround may take only a minute fraction of the total time, it is absolutely essential if the twins' clocks are to come together again so that we can compare their ages (readings).

A correct analysis can be made using the invariant interval Δs from Equation 1-33 rewritten as

$$\left(\frac{\Delta s}{c}\right)^2 = (\Delta t)^2 - \left(\frac{\Delta x}{c}\right)^2$$

where the left side is constant and equal to $(\tau)^2$, the proper time interval squared, and the right side refers to measurements made in any inertial frame. Thus Ulysses, along each of his worldlines in Figure 1-35*a*, has $\Delta x = 0$ and, of course, measures $\Delta t = \tau = 3$ y, or 6 y for the round trip. Homer, on the other hand, measures

$$(\Delta t)^2 = (\tau)^2 + \left(\frac{\Delta x}{c}\right)^2$$

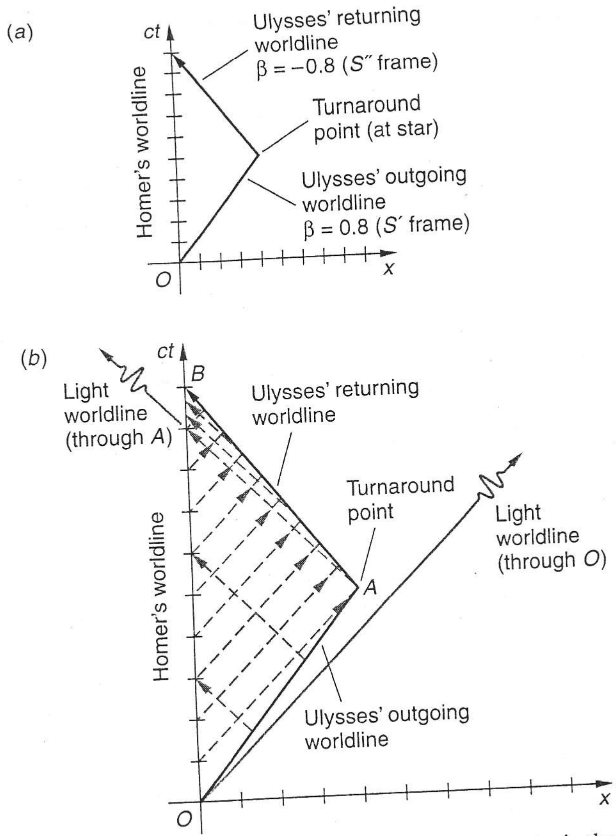

Fig. 1-35 (*a*) The spacetime diagram of Ulysses' journey to a distant star in the inertial frame in which Homer and the star are at rest. (*b*) Divisions on the *ct* axis correspond to years on Homer's clock. The broken lines show the paths (worldlines) of light flashes transmitted by each twin with a frequency of one/year on his clock. Note the markedly different frequencies at the receivers.

and since $(\Delta x/c)^2$ is always positive, he always measures $\Delta t > \tau$. In this situation $\Delta x = 0.8c\Delta t$, so

$$(\Delta t)^2 = (3\ \text{y})^2 + (0.8c\Delta t/c)^2$$

or

$$(\Delta t)^2(0.36) = (3)^2$$

$$\Delta t = \frac{3}{0.6} = 5\ \text{y}$$

or 10 y for the round trip, as we saw earlier. The reason that the twins' situations cannot be treated symmetrically is because the special theory of relativity can predict the behavior of accelerated systems, such as Ulysses at the turnaround, provided that in the formulation of the physical laws we take the view of an inertial, i.e., unaccelerated, observer such as Homer. That's what we have done. Thus, we cannot do the same analysis in the rest frame of Ulysses' spaceship because it does not remain in an inertial frame during the round trip; hence, it falls outside of the special theory, and no paradox arises. The laws of physics can be reformulated so as to be invariant for accelerated observers, which is the role of general relativity (see Chapter 2), but the result is the same: Ulysses returns younger that Homer by just the amount calculated.

The Pole and Barn Paradox

An interesting problem involving length contraction developed by E. F. Taylor and J. A. Wheeler[22] involves putting a long pole into a short barn. One version goes as follows. A runner carries a pole 10 m long toward the open front door of a small barn 5 m long. A farmer stands near the barn so that he can see both the front and the back doors of the barn, the latter being a closed swinging door, as shown in Figure 1-37*a*. The runner carrying the pole at speed v enters the barn and at some instant the farmer sees the pole completely contained in the barn and closes the front door, thus putting a 10-m pole into a 5-m barn. The minimum speed of the runner v that is necessary for the farmer to accomplish this feat may be computed from Equation 1-30, giving the relativistic length contraction $L = L_p/\gamma$, where L_p = proper length of the pole (10 m) and L = length of the pole measured by the farmer, to be equal to the length of the barn (5 m). Therefore, we have

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{L_p}{L} = \frac{10}{5}$$
$$1 - v^2/c^2 = (5/10)^2$$
$$v^2/c^2 = 1 - (5/10)^2 = 0.75$$
$$v = 0.866c \qquad \text{or} \qquad \beta = 0.866$$

A paradox seems to arise when this situation is viewed in the rest system of the runner. For him the pole, being at rest in the same inertial system, has its proper length of 10 m. However, the runner measures the length of the barn to be

$$L = L_p/\gamma = 5\sqrt{1 - \beta^2}$$
$$L = 2.5 \text{ m}$$

How can he possibly fit the 10-m pole into the length-contracted 2.5-m barn? The answer is that he can't, and the paradox vanishes, but how can that be? To understand the answer, we need to examine two events—the coincidences of both the front and back ends of the pole, respectively, with the rear and front doors of the barn—in the inertial frame of the farmer and in that of the runner.

These are illustrated by the spacetime diagrams of the inertial frame S of the farmer and barn (Figure 1-37*b*) and that of the runner S' (Figure 1-37*c*). Both diagrams are drawn with the front end of the pole coinciding with the front door of the barn at the instant the clocks are started. In Figure 1-37*b* the worldlines of the barn doors are, of course, vertical, while those of the two ends of the pole make an angle $\theta = \tan^{-1}(1/\beta) = 49.1°$ with the x axis. Note that in S the front of the pole reaches the rear door of the barn at $ct = 5 \text{ m}/0.866 = 5.8$ m *simultaneously* with the arrival of the back end of the pole at the front door; i.e., at that instant in S the pole is entirely contained in the barn.

In the runner's rest system S' it is the worldlines of the ends of the pole that are vertical, while those of the front and rear doors of the barn make angles of 49.1° with the $-x'$ axis (since the barn moves in the $-x'$ direction at v). Now we see that the rear door passes the front of the pole at $ct' = 2.5 \text{ m}/0.866 = 2.9$ m, but the front door of the barn doesn't reach the rear of the pole until $ct' = 10 \text{ m}/0.866 = 11.5$ m. Thus the first of those two events occurs *before* the second, and the runner never sees the pole entirely contained in the barn. Once again, the relativity of simultaneity is the

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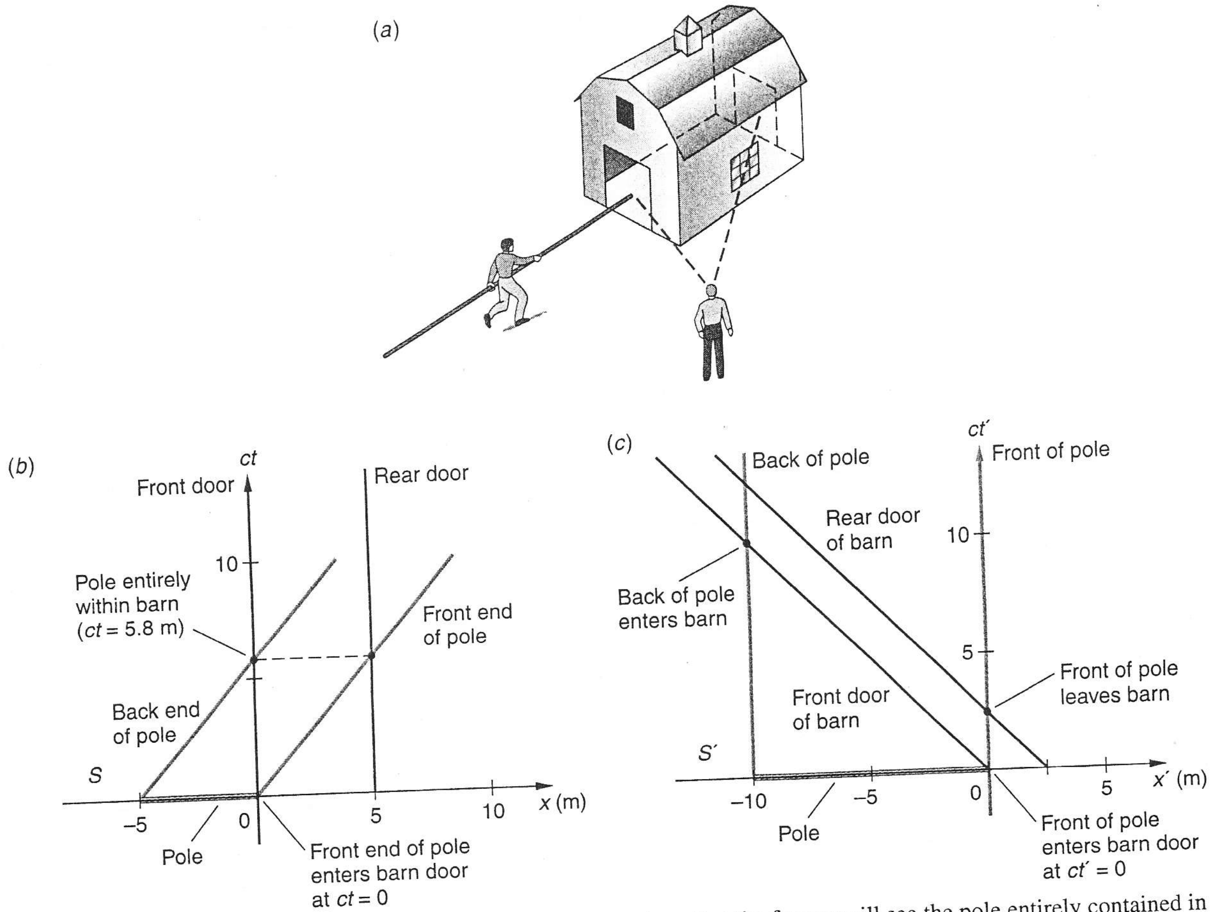

Fig. 1-37 (*a*) A runner carrying a 10-m pole moves quickly enough so that the farmer will see the pole entirely contained in the barn. The spacetime diagrams from the point of view of the farmer's inertial frame (*b*) and that of the runner (*c*). The resolution of the paradox is in the fact that the events of interest, shown by the large dots in each diagram, are simultaneous in *S*, but not in *S*′.

key—events simultaneous in one inertial frame are not simultaneous when viewed from another inertial frame.