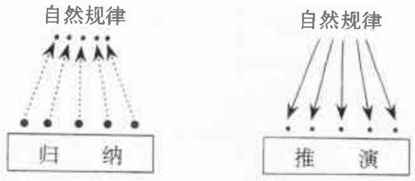
近代科學的思維方法



近代科学的两条寻求自然规律的方法。
归纳法用虚线表示，以示其难。推演法
用实线表示，以示其易

图 04e.2

归纳与推演都是近代科学中不可缺少的思维方法。为说明此点让我们看一下 Maxwell（1831—1879）创建 Maxwell 方程的历史。

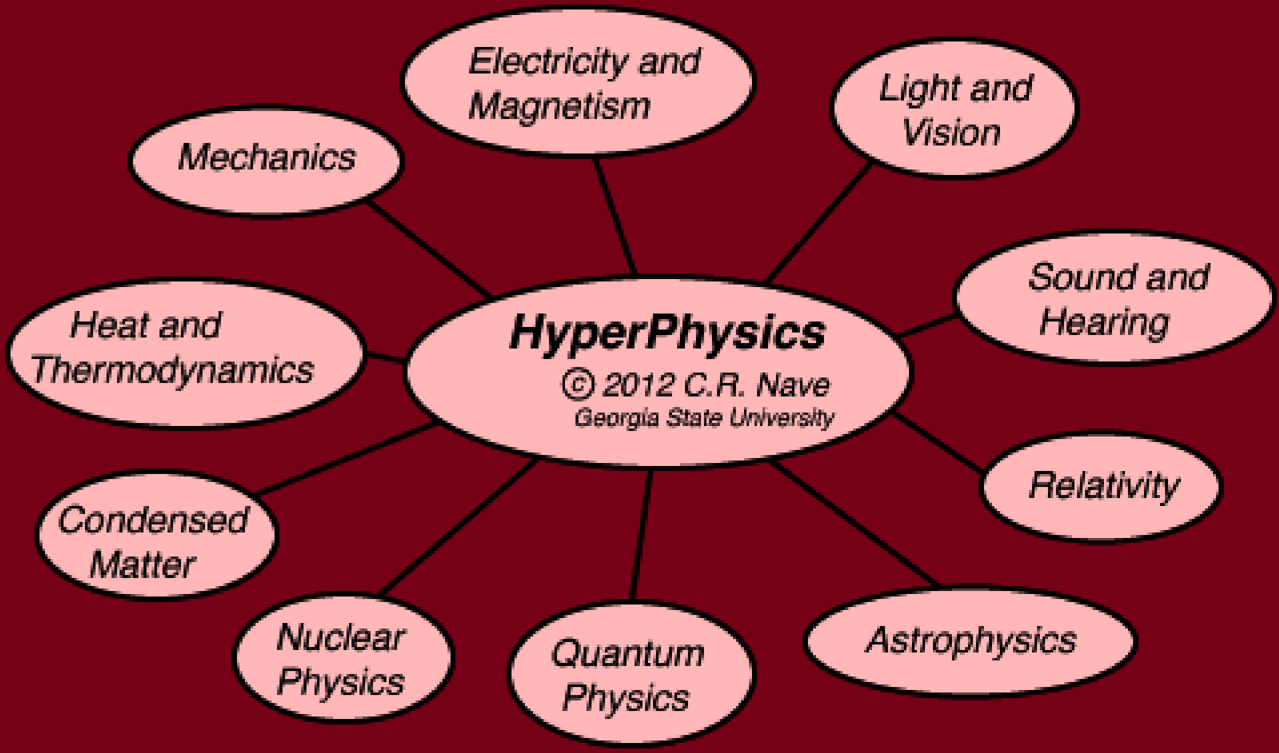
Maxwell 是 19 世纪最伟大的物理学家。他在 19 世纪中叶写了三篇论文，奠定了电磁波的准确结构，从而改变了人类的历史。20 世纪所发展出来的无线电、电视、网络通讯等等，统统都基于 Maxwell 方程式。他是怎样得到此划时代的结果的呢？

他的第一篇文章里面用的是归纳法，里面有这样一段话："我们必须认识到互相类似的物理学分支。就是说物理学中有不同的分支，可是它们的结构可以相互印证。"

他用这个观念来研究怎样写出电磁学方程式，以流体力学的一些方程式为蓝本。这种研究方法遵循了归纳法的精神。

几年以后，在后面的文章中他把用归纳法猜出的电磁方程式，运用推演法而得出新结论：这些方程式显示电磁可以以波的形式传播，其波速与当时已知的光速相符，所以"光即是电磁波"，这是划时代的推测，催生了 20 世纪的科技发展与人类今天的生活方式。

上面的故事清楚地显示归纳与推演二者同时是近代科学的基本思维方法。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Light and Vision
Sound and Hearing
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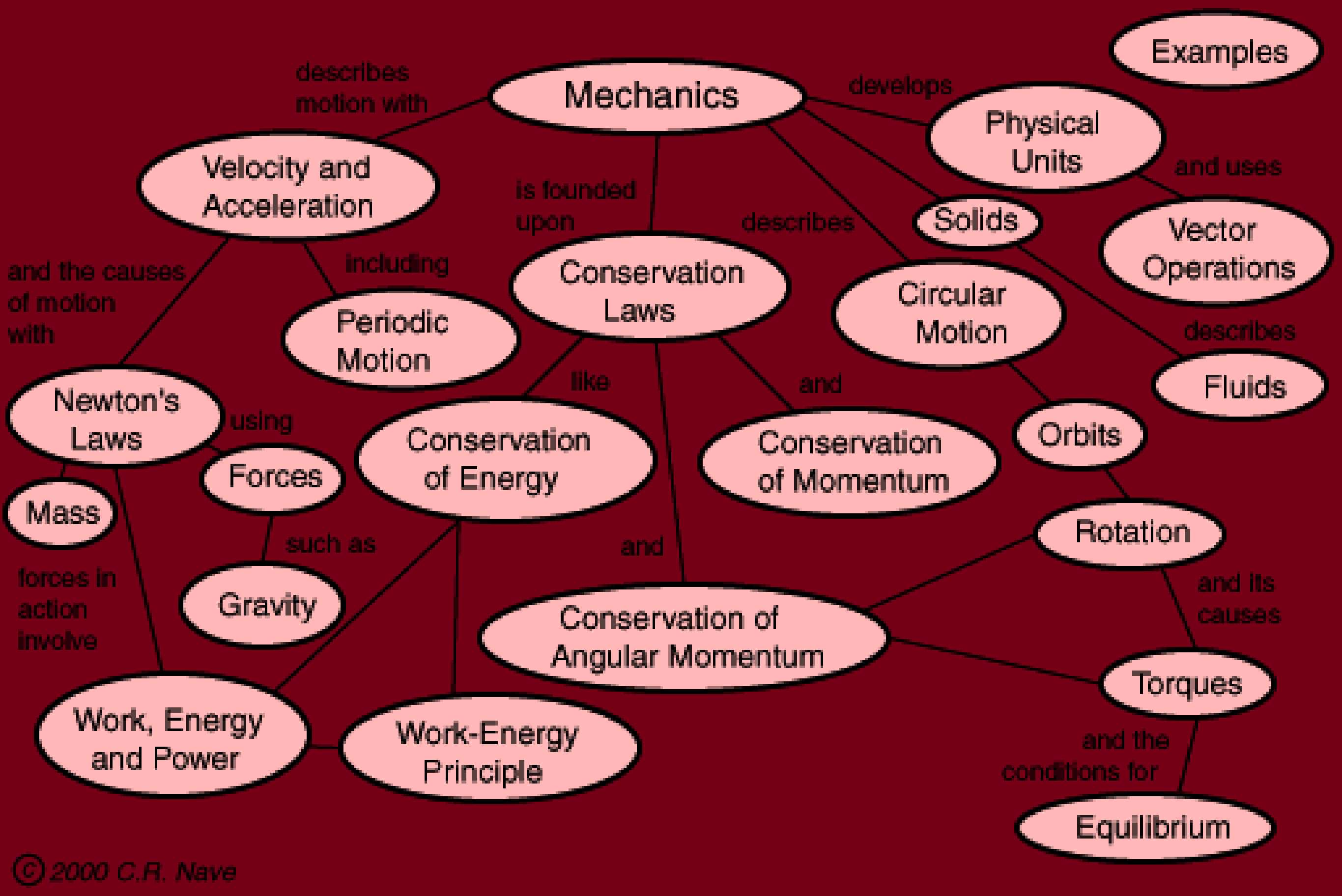
Quantum Physics
Astrophysics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describes motion with
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and uses
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and the conditions for
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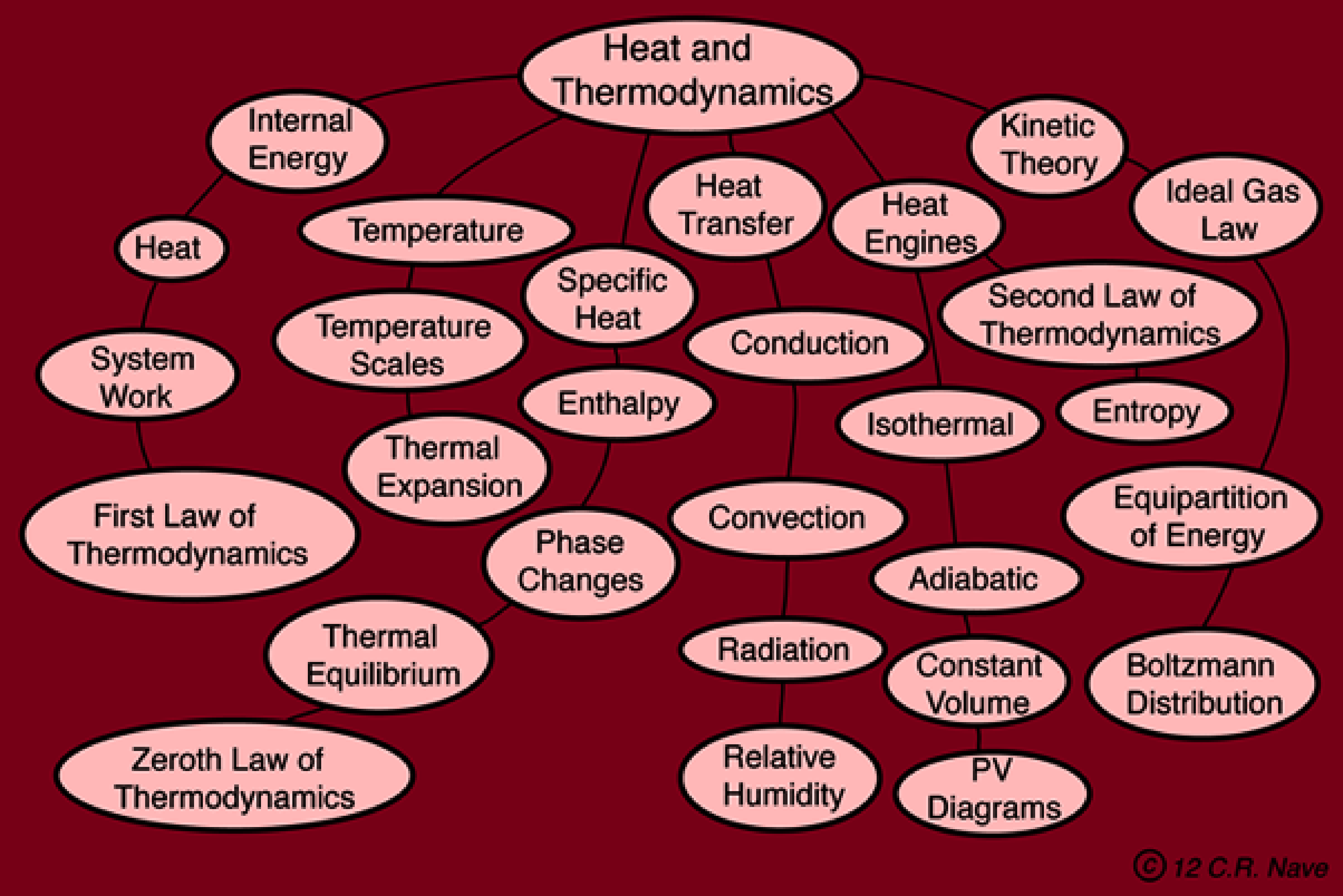
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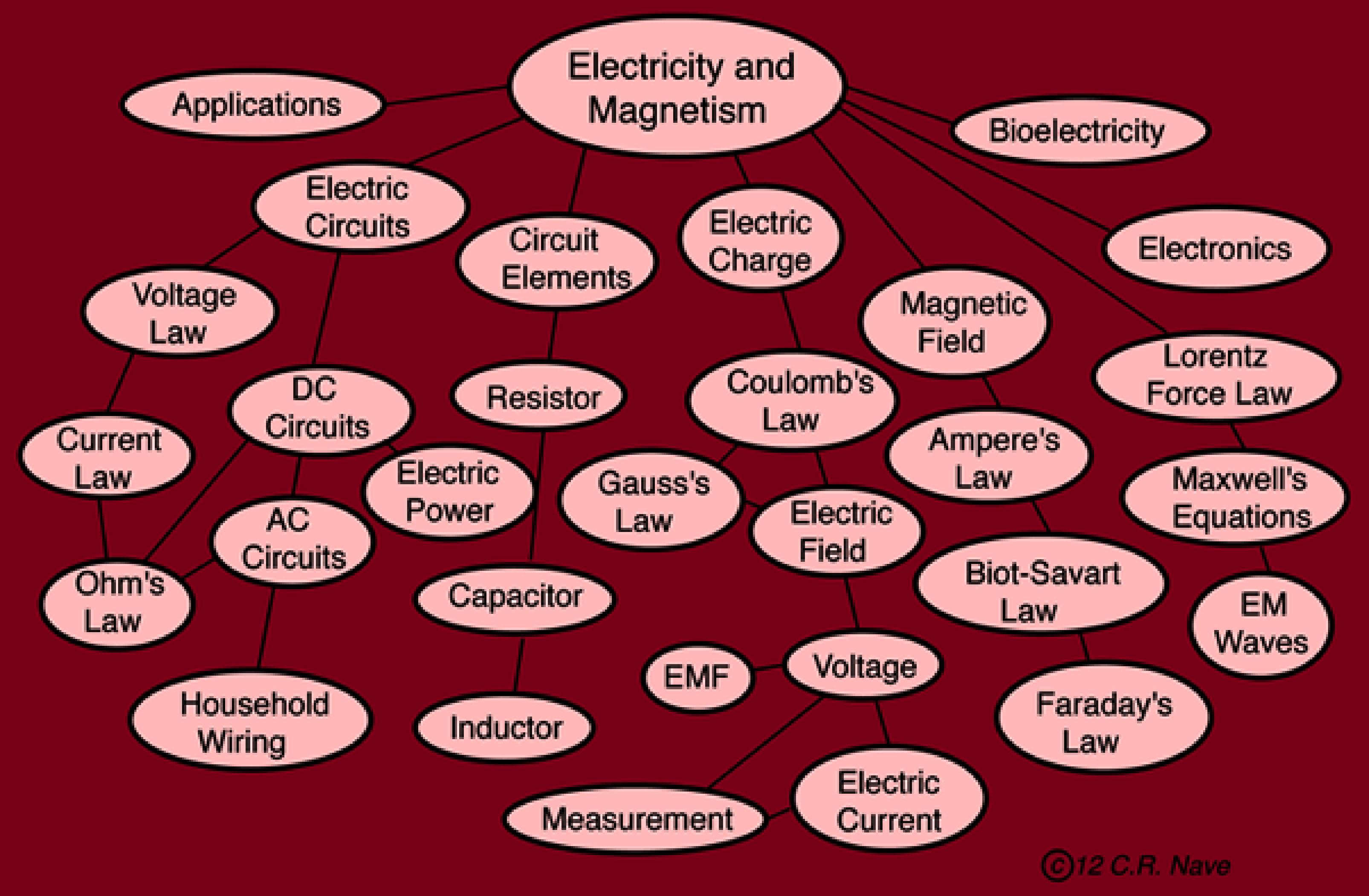
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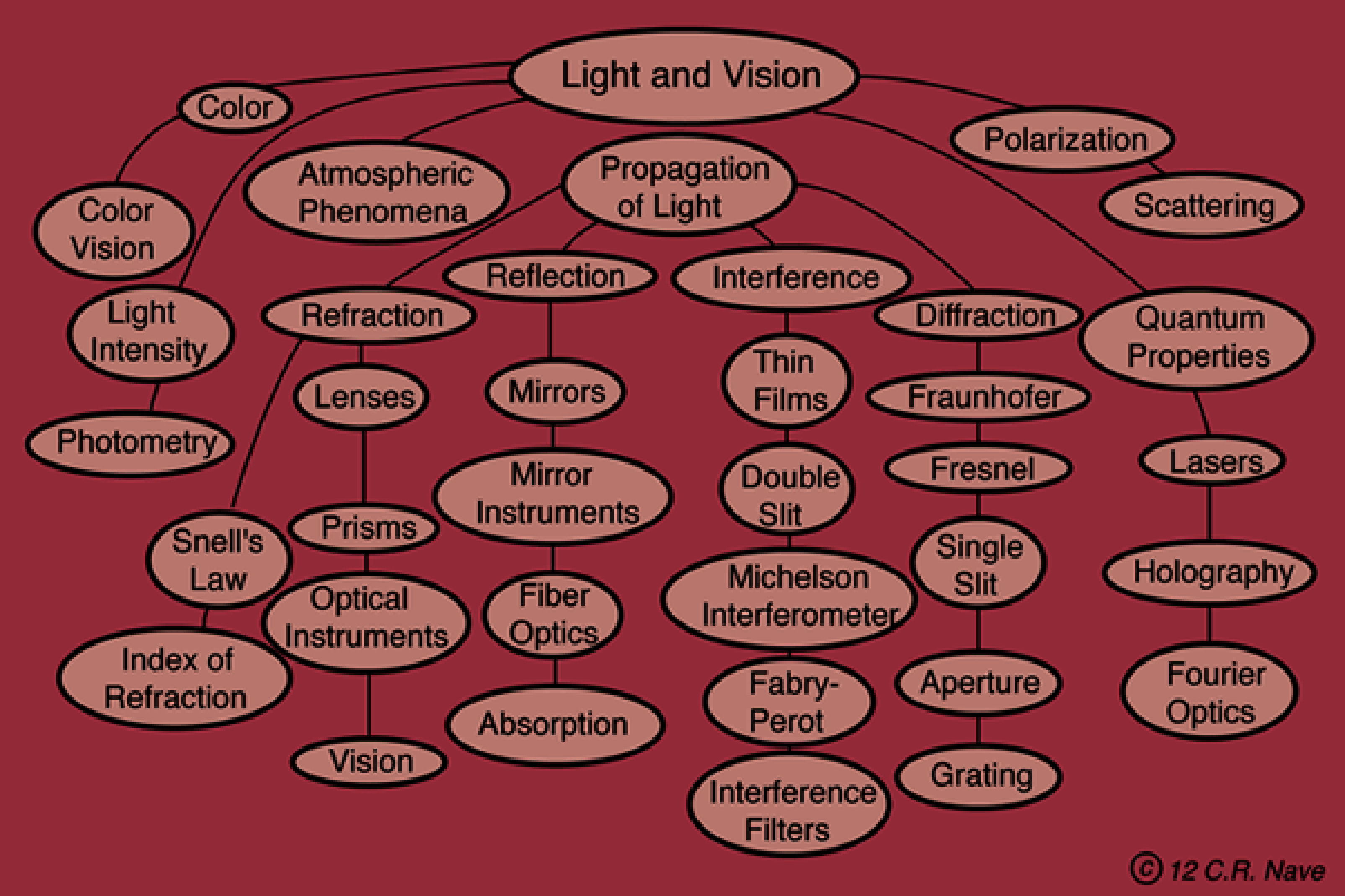

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Grating
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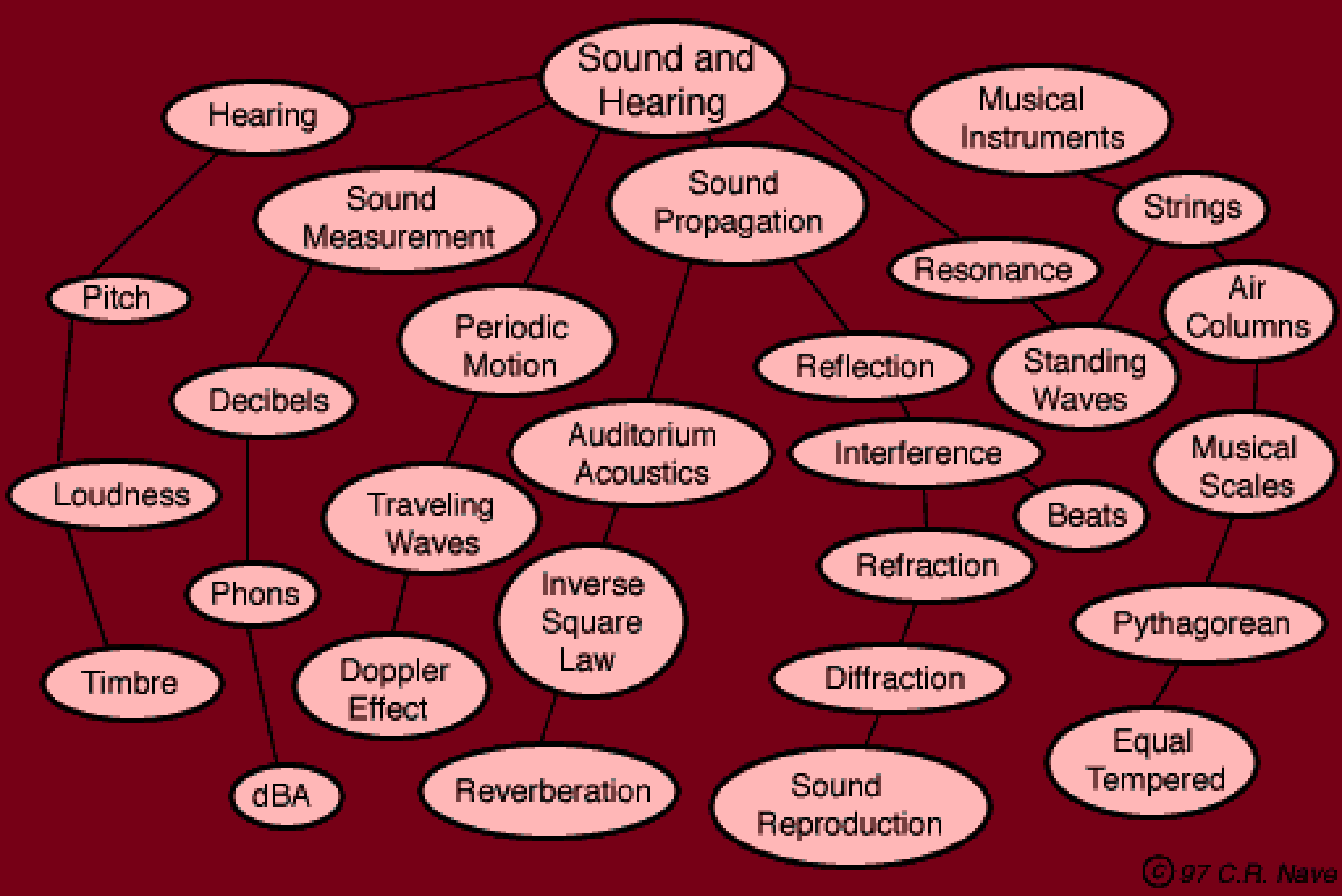
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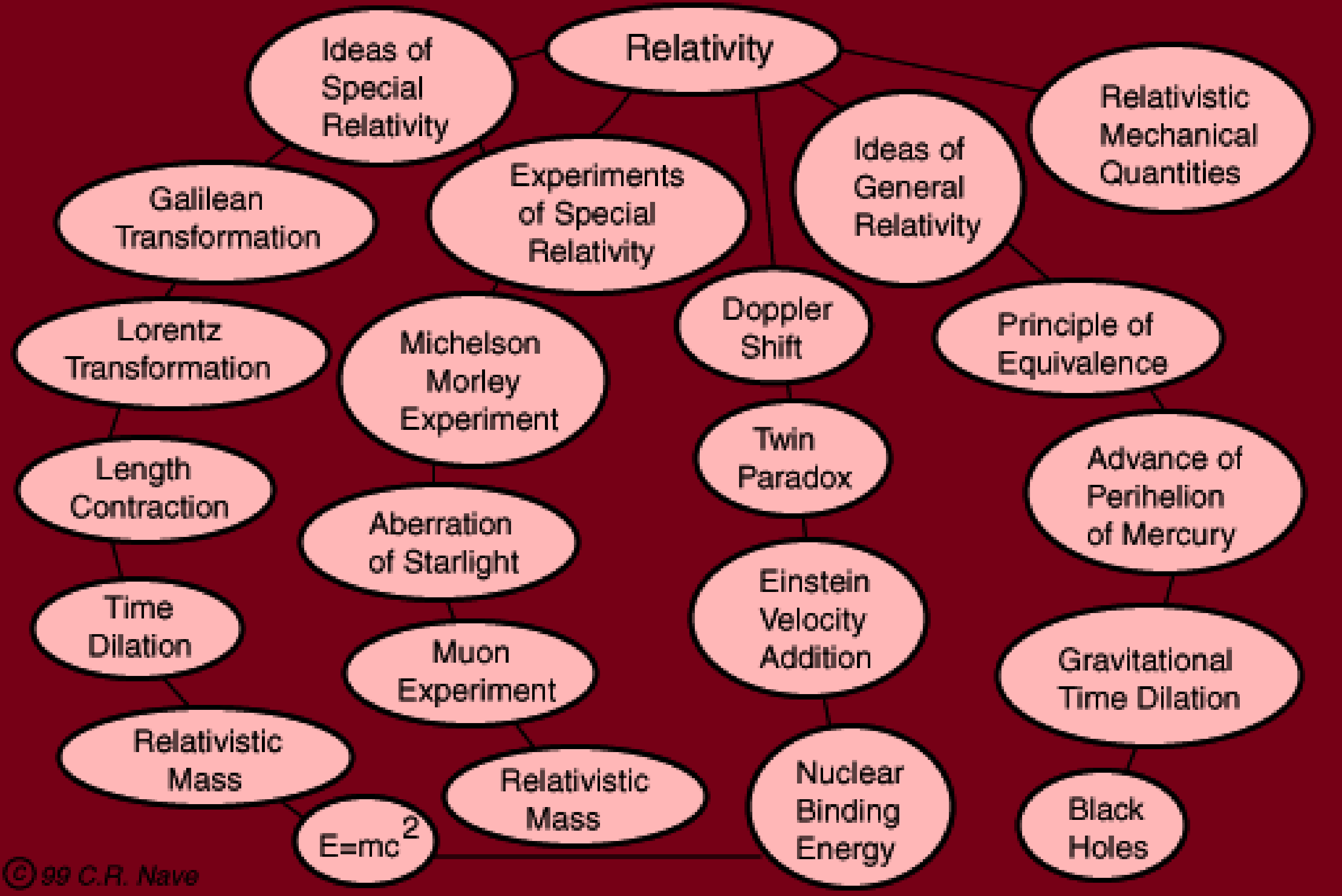
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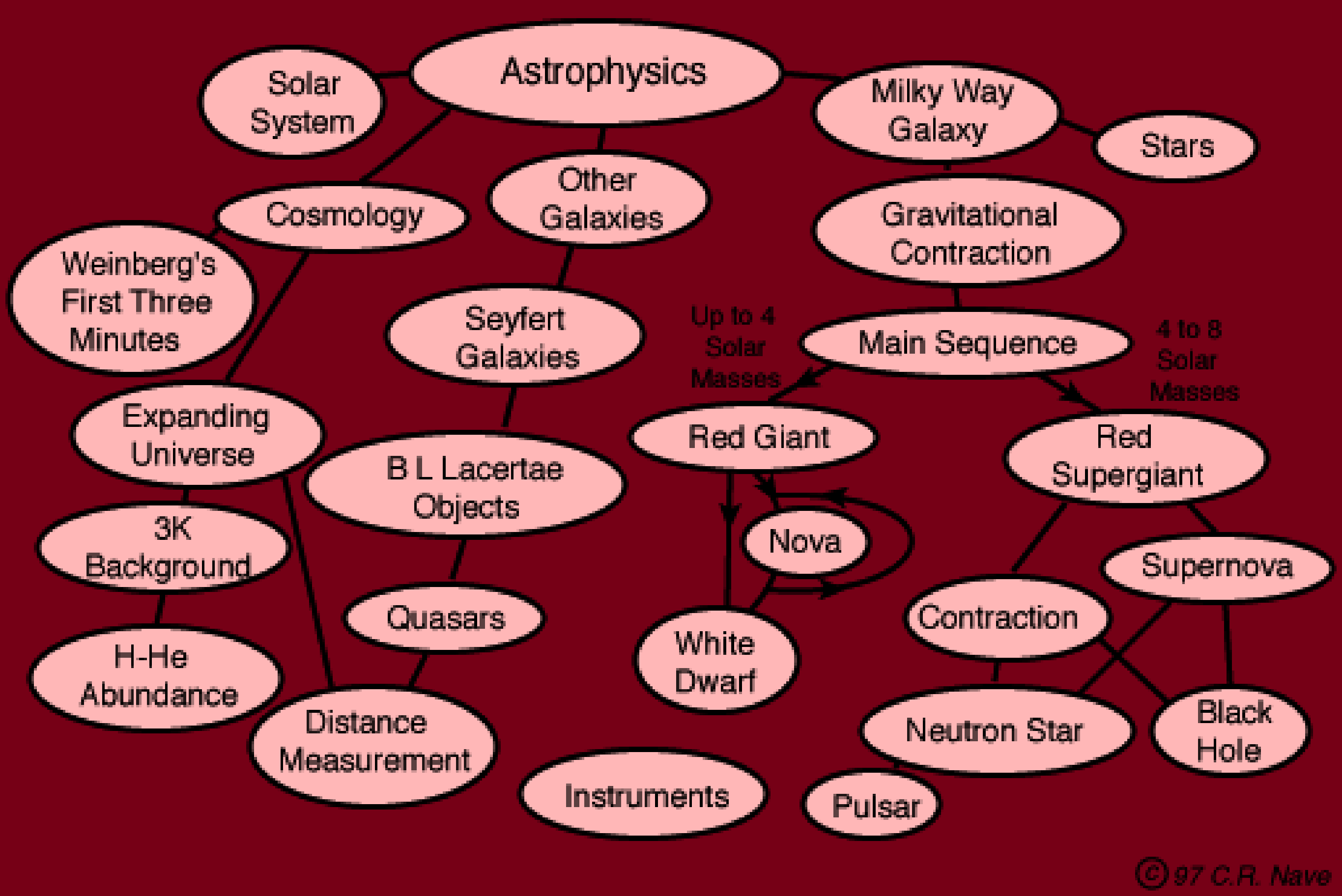
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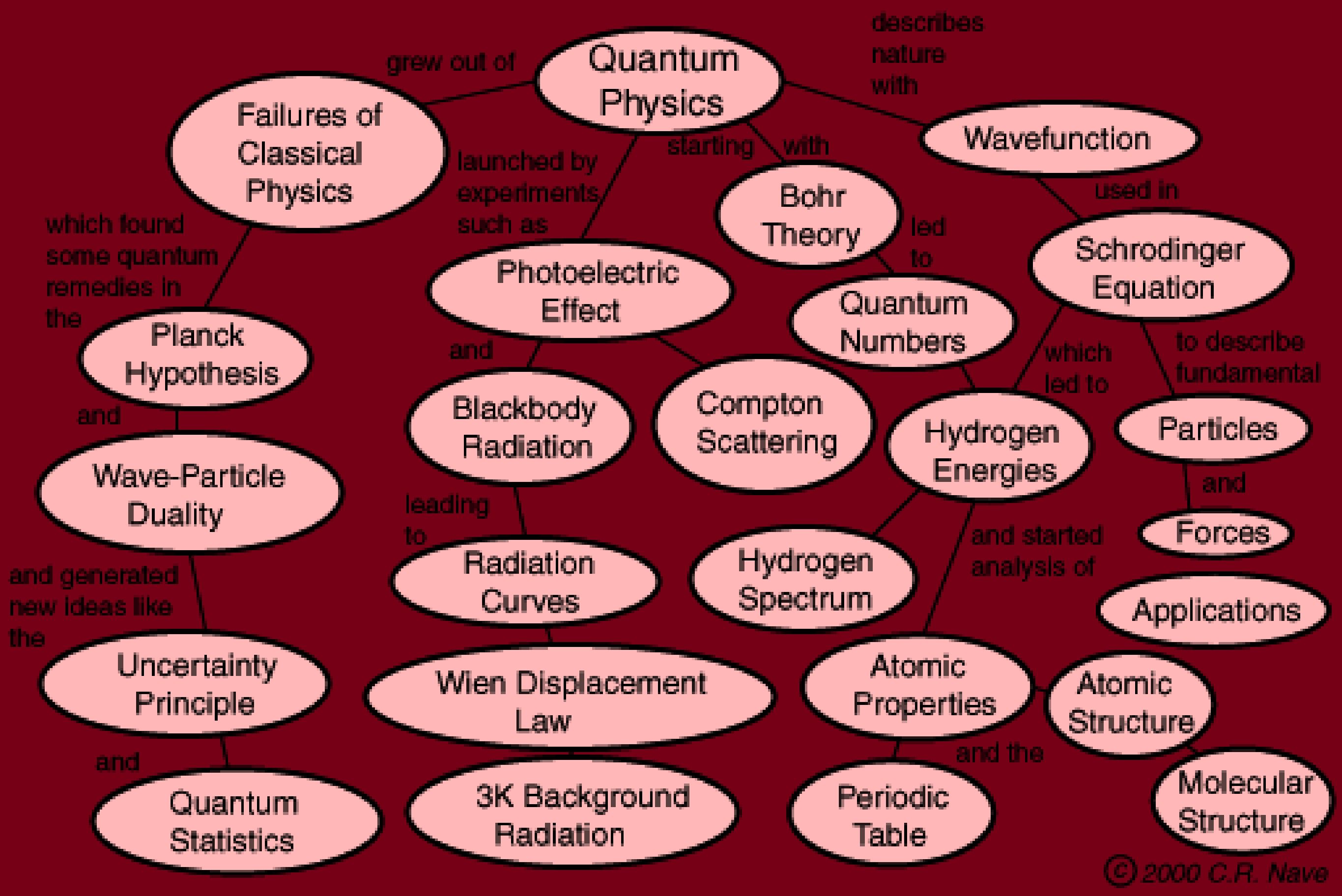
Quantum Physics
grew out of
Failures of Classical Physics
which found some quantum remedies in the
Planck Hypothesis
and
Wave-Particle Duality
and generated new ideas like the
Uncertainty Principle
and
Quantum Statistics
launched by experiments such as
Photoelectric Effect
and
Blackbody Radiation
Compton Scattering
leading to
Radiation Curves
Wien Displacement Law
3K Background Radiation
starting with
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led to
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Hydrogen Spectrum
and started analysis of
Atomic Properties
and the
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Molecular Structure
describes nature with
Wavefunction
used in
Schrodinger Equation
which led to
to describe fundamental
Particles
and
Forces
Applications
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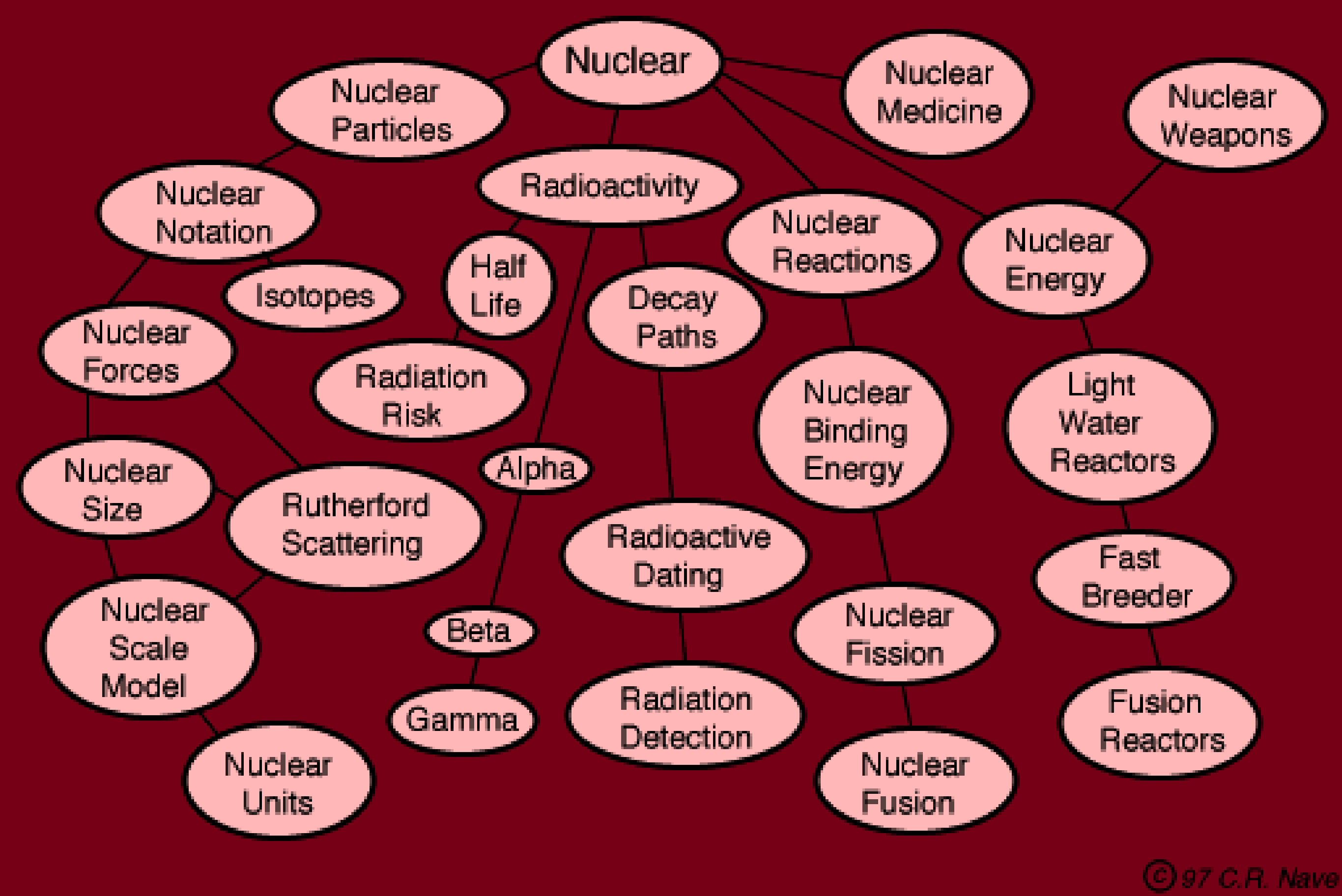
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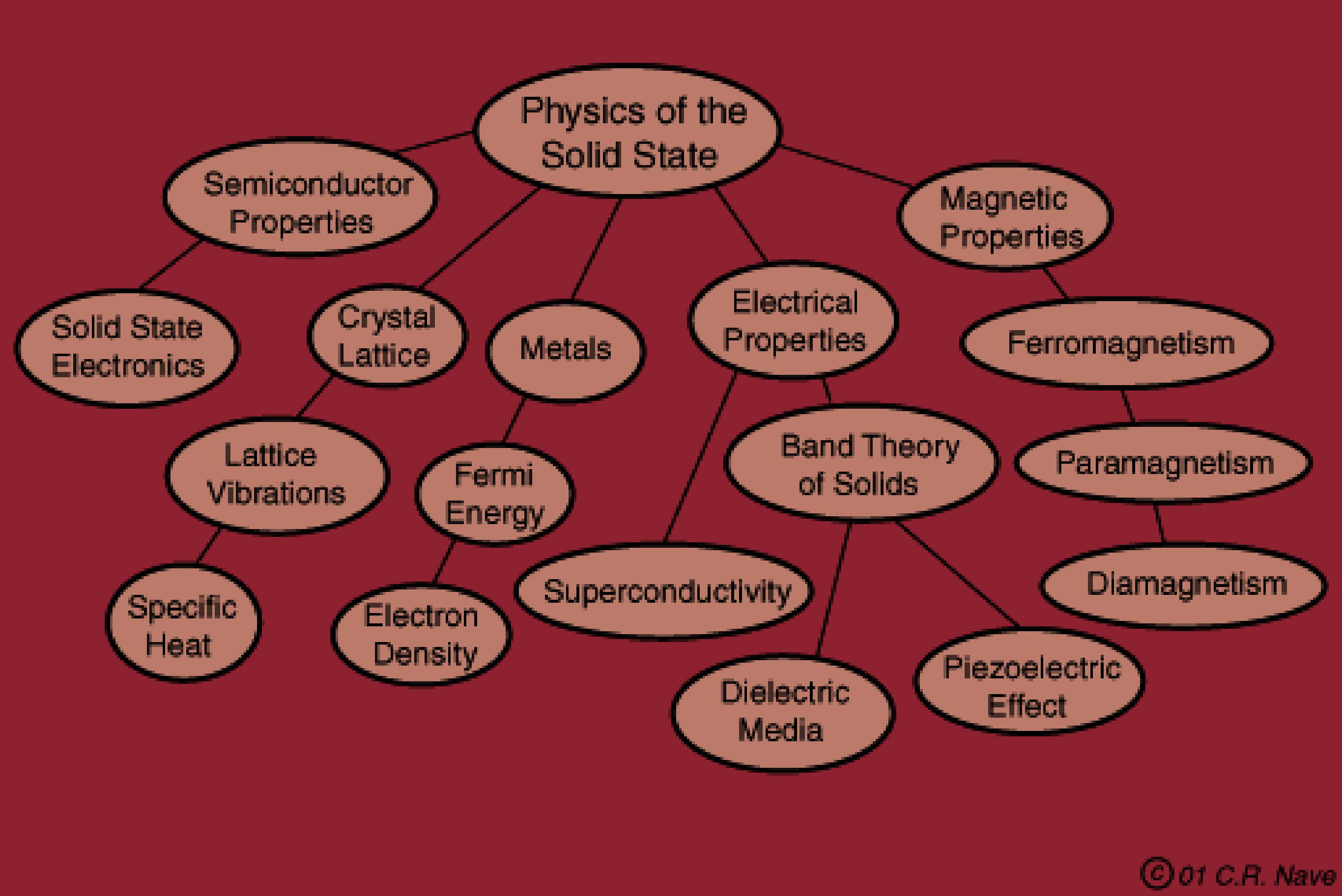
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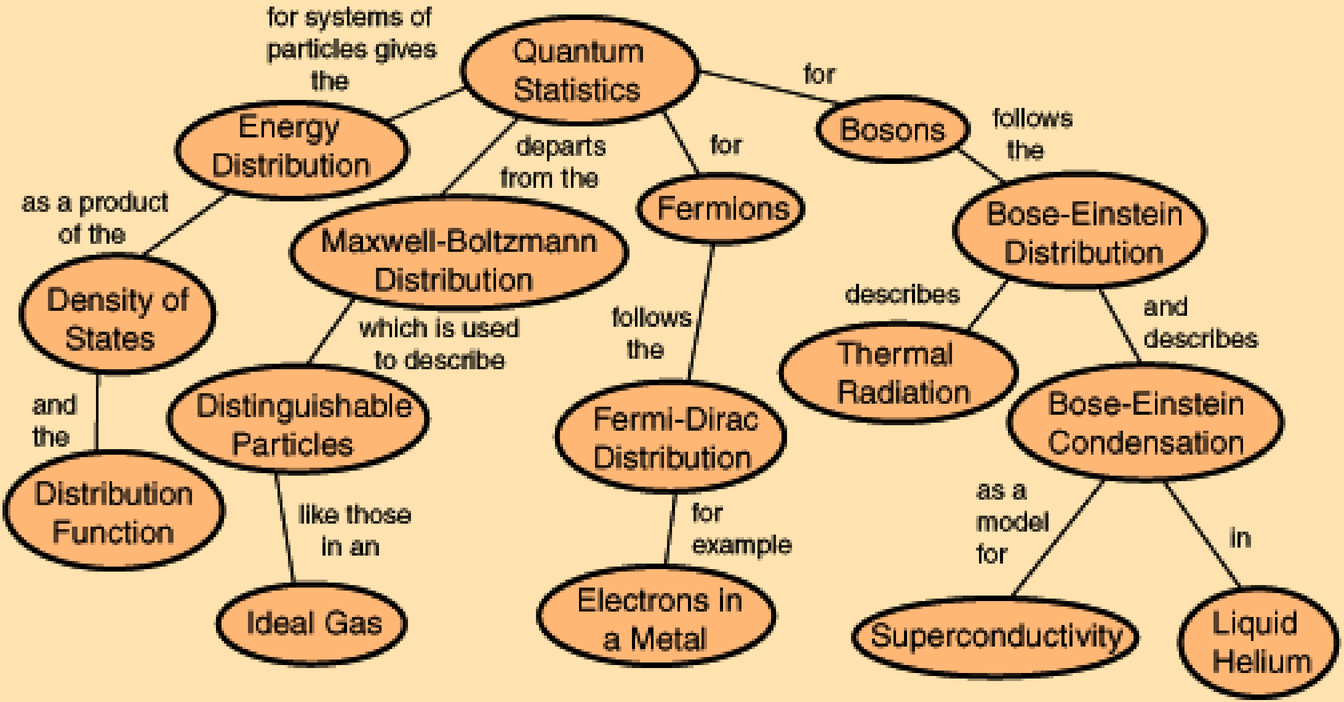

Quantum Statistics
for systems of particles gives the
Energy Distribution
as a product of the
Density of States
and the
Distribution Function
departs from the
Maxwell-Boltzmann Distribution
which is used to describe
Distinguishable Particles
like those in an
Ideal Gas
for
Fermions
follows the
Fermi-Dirac Distribution
for example
Electrons in a Metal
for
Bosons
follows the
Bose-Einstein Distribution
describes
Thermal Radiation
and describes
Bose-Einstein Condensation
as a model for
Superconductivity
in
Liquid Helium

The Fermi-Dirac Distribution

The Fermi-Dirac distribution applies to fermions, particles with half-integer spin which must obey the Pauli exclusion principle. Each type of distribution function has a normalization term multiplying the exponential in the denominator which may be temperature dependent. For the Fermi-Dirac case, that term is usually written:

$$e^{-E_F/kT} \quad \text{where } E_F = \text{Fermi energy}$$

The significance of the Fermi energy is most clearly seem by setting T=0. At absolute zero, the probability is =1 for energies less than the Fermi energy and zero for energies greater than the Fermi energy. We picture all the levels up to the Fermi energy as filled, but no particle has a greater energy. This is entirely consistent with the Pauli exclusion principle where each quantum state can have one but only one particle.

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

Explain the symbols

Distribution functions	Numerical example

Fermi level in solids	Fermi level in band theory of solids

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Applied statistics concepts

HyperPhysics***** Quantum Physics

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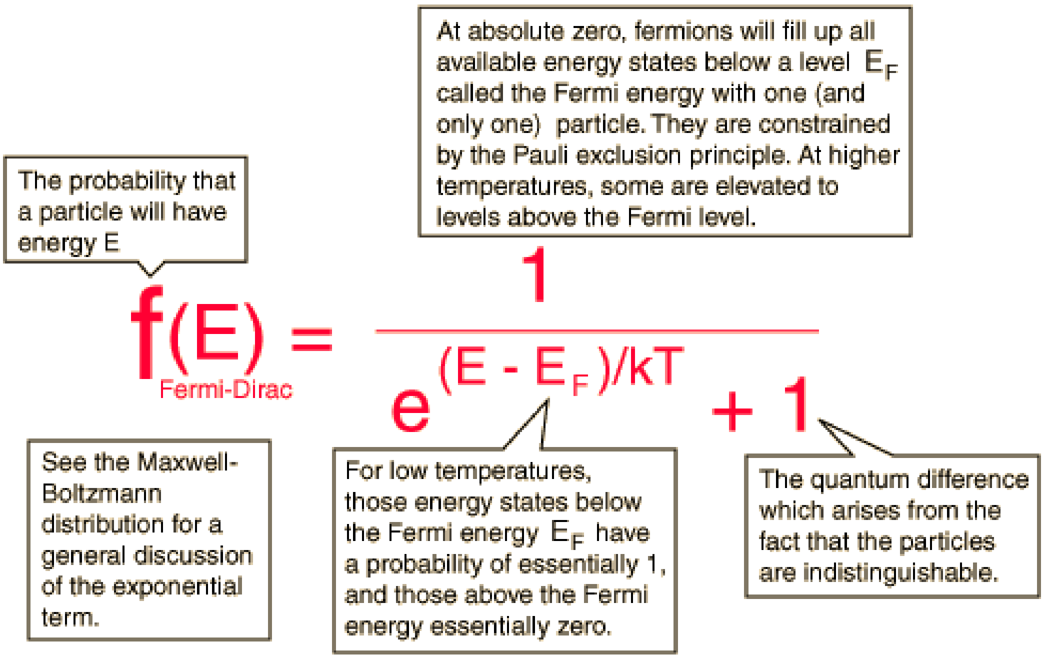
Fermi-Dirac Details



The Fermi-Dirac distribution.

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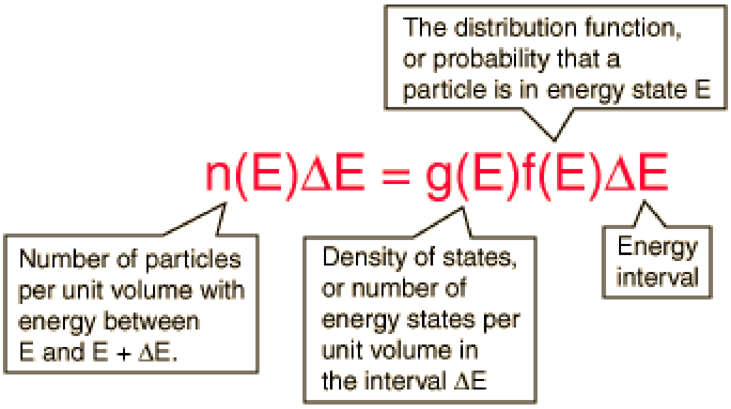
Applied statistics concepts

Electron Energy Density

The behavior of electrons in solids depends upon the distribution of energy among the electrons:



Since electrons are fermions, the distribution function is the Fermi-Dirac distribution

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

Explain the symbols

This distribution determines the probability that a given energy state will be occupied, but must be multiplied by the density of states function to weight the probability by the number of states available at a given energy.

The determination of how many ways there are to obtain an energy in an incremental energy range dE can be approached with the quantum mechanical particle in a box. The energy for an infinite walled box is

$$E = \frac{(n_1^2 + n_2^2 + n_3^2)h^2}{8mL^2}$$

Treating the "quantum numbers" n as a space such that a given set of n

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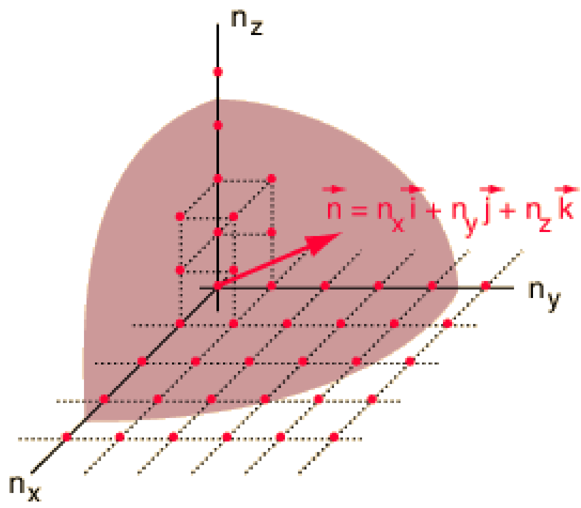
Quantum statistics

values determines a point in that space, you can argue that the number of possible states is proportional to the "volume" in "n-space".



The Rayleigh scheme for counting modes.
After Richtmyer, et al.

It is convenient to define a radius R in n-space:

$$R = \sqrt{n_1^2 + n_2^2 + n_3^2}$$

The energy can be expressed in terms of R and vice versa.

$$R = \frac{2\sqrt{2mE}\,L}{h} \quad or \quad E = \frac{h^2R^2}{8mL^2}$$

The n-space associated with the particle-in-a-box involves only positive values of n, so the volume must be divided by 8. It then must be multiplied by 2 to account for the two possible spin values of the electron. The number of values is then

$$N = (2)\left(\frac{1}{8}\right)\frac{4}{3}\pi R^3 = \left(\frac{8\pi}{3}\right)(2mE)^{3/2}\frac{L^3}{h^3}$$

The number of states per unit volume is

$$n_s = \frac{N}{L^3} = \left(\frac{8\pi}{3}\right)\frac{(2mE)^{3/2}}{h^3}$$

The final density of states as a function of energy is then the deriviative of this population with respect to energy

$$\rho(E) = \frac{dn_s}{dE} = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$$

This represents the number of electron states per unit volume per unit energy at energy E. This energy density is a factor in many of the electical properties of solids. Note that the result is independent of the dimension L which was chosen above,

concepts

References
Rohlf
Sec 12.6

Richtmyer, et al.
Ch. 5

showing that the expression can be applied to the bulk material.

Application in solids

HyperPhysics***** Quantum Physics	Go Back

Spin Classification

One essential parameter for classification of particles is their "spin" or ~~intrinsic angular momentum~~. Half-integer spin ~~fermions~~ are constrained by the ~~Pauli exclusion principle~~ whereas integer spin ~~bosons~~ are not. The ~~electron~~ is a fermion with ~~electron spin~~ 1/2.

The spin classification of particles determines the nature of the ~~energy distribution~~ in a collection of the particles. Particles of integer spin obey ~~Bose-Einstein statistics~~, whereas those of half-integer spin behave according to ~~Fermi-Dirac statistics~~.

Fermions

Fermions are particles which have half-integer ~~spin~~ and therefore are constrained by the ~~Pauli exclusion principle~~. Particles with integer spin are called ~~bosons~~. Fermions include ~~electrons~~, ~~protons~~, ~~neutrons~~. The wavefunction which describes a collection of fermions must be antisymmetric with respect to the exchange of identical particles, while the wavefunction for a collection of bosons is symmetric.

The fact that electrons are fermions is foundational to the buildup of the periodic table of the elements since there can be only one electron for each state in an atom (only one electron for each possible set of quantum numbers). The fermion nature of electrons also governs the behavior of ~~electrons in a metal~~ where at low temperatures all the low energy states are filled up to a level called the ~~Fermi energy~~. This filling of states is described by ~~Fermi-Dirac statistics~~.

Bosons

Bosons are particles which have integer spin and which therefore are not constrained by the Pauli exclusion principle like the half-integer spin fermions. The energy distribution of bosons is described by Bose-Einstein statistics. The wavefunction which describes a collection of bosons must be symmetric with respect to the exchange of identical particles, while the wavefunction for a collection of fermions is antisymmetric.

At low temperatures, bosons can behave very differently than fermions because an unlimited number of them can collect into the same energy state. The collection into a single state is called condensation, or Bose-Einstein condensation. It is responsible for the phenomenon of superfluidity in liquid helium. Coupled particles can also act effectively as bosons. In the BCS Theory of superconductivity, coupled pairs of electrons act like bosons and condense into a state which demonstrates zero electrical resistance.

Bosons include photons and the characterization of photons as particles with frequency-dependent energy given by the Planck relationship allowed Planck to apply Bose-Einstein statistics to explain the thermal radiation from a hot cavity.

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Bose-Einstein Condensation

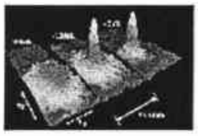
In 1924 Einstein pointed out that bosons could "condense" in unlimited numbers into a single ground state since they are governed by Bose-Einstein statistics and not constrained by the Pauli exclusion principle. Little notice was taken of this curious possibility until the anomalous behavior of liquid helium at low temperatures was studied carefully.

When helium is cooled to a critical temperature of 2.17 K, a remarkable discontinuity in heat capacity occurs, the liquid density drops, and a fraction of the liquid becomes a zero viscosity "superfluid". Superfluidity arises from the fraction of helium atoms which has condensed to the lowest possible energy.

A condensation effect is also credited with producing superconductivity. In the BCS Theory, pairs of electrons are coupled by lattice interactions, and the pairs (called Cooper pairs) act like bosons and can condense into a state of zero electrical resistance.

The conditions for achieving a Bose-Einstein condensate are quite extreme. The participating particles must be considered to be identical, and this is a condition that is difficult to achieve for whole atoms. The condition of indistinguishability requires that the deBroglie wavelengths of the particles overlap significantly. This requires extremely low temperatures so that the deBroglie wavelengths will be long, but also requires a fairly high particle density to narrow the gap between the particles.

Since the 1990s there has been a surge of research into Bose-Einstein condensation since it was discovered that Bose-Einstein condensates could be formed with ultra-cold atoms. The use of laser cooling and the trapping of ultra-cold atoms with magnetic traps has produced temperatures in the nanokelvin range. Cornell and Wieman along with Ketterle of MIT received the 2001 Nobel Prize in Physics "for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates". Cornell and Wieman led an active group at the University of Colorado, Boulder which has produced Bose-Einstein condensates with rubidium atoms. Other groups at MIT, Harvard and Rice have been very active in this rapidly advancing field.



Liquid Helium

Kamerlingh Onnes worked for many years to liquify the element which persisted as a gas to the lowest temperature. Using liquid air to produce liquid hydrogen and then the hydrogen to jacket the liquification apparatus, he produced about 60 cubic centimeters of liquid helium on July 10, 1908. Its boiling point was found to be 4.2 K. Onnes received the Nobel Prize in 1913 for his low temperature work leading to this achievement.

When helium is cooled to a critical temperature of 2.17 K (called its lambda point), a remarkable discontinuity in heat capacity occurs, the liquid density drops, and a fraction of the liquid becomes a zero viscosity "superfluid". Superfluidity arises from the fraction of helium atoms which has condensed to the lowest possible energy.

An important application of liquid helium has been in the study of superconductivity and for the applications of superconducting magnets.

Liquid helium working range

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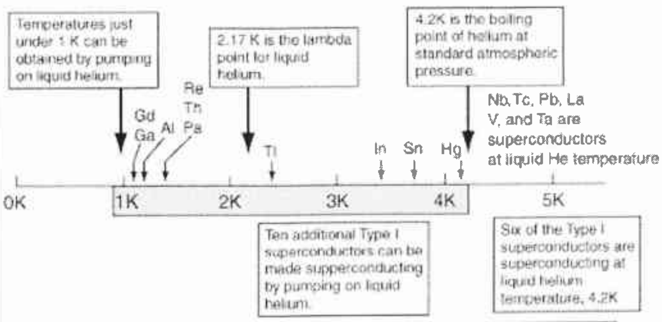
Liquid Helium Working Range



Liquid helium	**Lambda point**	**Superconductors**

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Reference: Blatt

Superfluidity

A remarkable transition occurs in the properties of liquid helium at the temperature 2.17K, called the "lambda point" for helium. Part of the liquid becomes a "superfluid", a zero viscosity fluid which will move rapidly through any pore in the apparatus.

A vacuum container which seemed to be leak tight could suddenly leak helium rapidly as the superfluid moved out through a microscopic hole. A vertical tube could produce a fountain effect as the superfluid moved up the walls and out the top.

In 1938, F. London proposed a "two-fluid" model to explain the behavior of the liquid: normal liquid and the superfluid fraction consisting of those atoms which have "condensed" to the ground state and make no contribution to the entropy or heat capacity of the liquid. This condensed fraction is the standard example of Bose-Einstein condensation.

Another remarkable characteristic of the the superfluid is its very high heat conductivity, 30 times that of copper!

Application in IRAS Satellite

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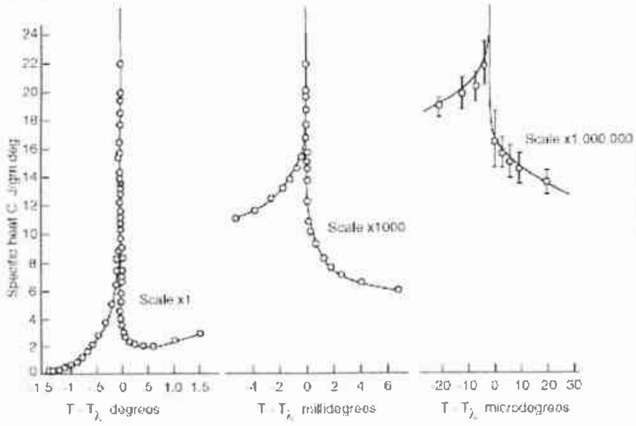
Reference: Blatt

Lambda Point for Liquid Helium

When helium is cooled to a critical temperature of 2.17 K , a remarkable discontinuity in heat capacity occurs, the liquid density drops, and a fraction of the liquid becomes a zero viscosity "superfluid". It is called the lambda point because the shape of the specific heat curve is like that Greek letter. Superfluidity arises from the fraction of helium atoms which has condensed to the lowest possible energy by a process called Bose-Einstein condensation.

When helium is cooled to a critical temperature of 2.17 K , a remarkable discontinuity in heat capacity occurs, the liquid density drops, and a fraction of the liquid becomes a zero viscosity "superfluid". It is called the lambda point because the shape of the specific heat curve is like that Greek letter. Superfluidity arises from the fraction of helium atoms which has condensed to the lowest possible energy by a process called Bose-Einstein condensation.



Specific heat data from Buckingham and Fairbank, 1961, p. 138.

Liquid helium working range

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Reference: Blatt